

APPLIED QUANTUM PHYSICS

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chapter - 7



Introduction - Harmonic Motion - Harmonic Oscillator - Comparison between Particle in a well and Harmonic Oscillator (One Dimensional) - Tunnelling - Barrier Penetration and quantum tunnelling - Scanning tunnelling microscope (STM) - Resonant tunnelling diode - Finite square potential well - Electron in periodic potential - Energy spectra in atoms, molecules and solids - Bloch's theorem for particles in a periodic potential - Behaviour of an electron in a periodical potential the kroning penney model (Qualitative treatment) - Brillouin Zones - Energy band theory of solids - Origin of energy band formation in solids - Classification of materials into metals, semiconductors and insulators on the basis of band theory - Important Solved Problems - Assignment Problems -Important Part - A Questions & Answers - University Part B Questions - Additional Part B Questions.



CHAPTER

7

UNIT
5

7.1 INTRODUCTION

In the previous chapter we have discussed about the basics of quantum mechanics like dual nature of a particle, how the particle behaves when it is enclosed in a one dimensional infinite potential well. In this chapter we are going to extend and apply the concept of the same in a three dimensional box.

Further we are going to discuss about the barrier penetration and quantum tunnelling, for a particle having lesser energy than the barrier potential.

Movement of electron in a periodic potential, rather than in a constant potential leads to zone theory of solids, which in turn helps in distinguishing the behaviour of solids as conductors, semiconductors and insulators.

7.2 HARMONIC MOTION

We know harmonic motion occurs when atoms vibrates about their equilibrium positions.

Examples

1. An atom in a crystal lattice.
2. An object supported by a spring.
3. An object floating in a liquid.
4. A diatomic molecule etc.

7.3 HARMONIC OSCILLATOR

In classical mechanics, a harmonic oscillator is a system in which when a particle is displaced from its equilibrium position, it experiences a restoring force to get back to its original position. The quantum harmonic oscillator is similar to a classical harmonic oscillator.

In quantum harmonic oscillator the molecular atomic vibrations are quantized and the allowed energies of a quantum harmonic oscillator are discrete and evenly spaced.

Derivation / Proof

Let us consider a particle of mass ' m ' executing simple harmonic motion along the ' x ' direction as shown in *Fig. 7.1*

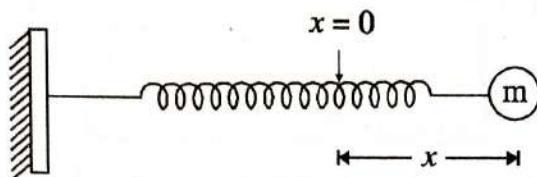


Fig. 7.1

As the particle is displaced through a distance ' x ' from its equilibrium position ($x = 0$), a restoring force (F) acts so as to return the particle again to its equilibrium position.

$$\therefore \text{The restoring force } F = -kx$$

Where ' k ' is called the force constant.

We know the Schroedinger time independent wave equation for a particle moving in one dimension is

$$\frac{d^2 \Psi}{dx^2} + \frac{2m}{\hbar^2} (E - V) \Psi = 0 \quad \dots(1)$$

$$\text{Here, the potential energy of the particle is } V = \frac{1}{2} kx^2 \quad \dots(2)$$

Thus, the Schroedinger's equation for the harmonic oscillator with potential energy ' V ' shall be written by substituting equation (2) in equation (1).

$$\therefore \frac{d^2 \Psi}{dx^2} + \frac{2m}{\hbar^2} \left[E - \frac{1}{2} kx^2 \right] \Psi = 0 \quad \dots(3)$$

$$\text{We know } \hbar = \frac{h}{2\pi} \text{ (or) } \hbar^2 = \frac{h^2}{4\pi^2}$$

\therefore Equation (3) becomes

$$\frac{d^2 \Psi}{dx^2} + \frac{2m}{\left(\frac{h^2}{4\pi^2}\right)} \left[E - \frac{1}{2} kx^2 \right] \Psi = 0$$

$$\text{(or)} \quad \frac{d^2 \Psi}{dx^2} + \frac{8\pi^2 m}{h^2} \left[E - \frac{1}{2} kx^2 \right] \Psi = 0$$

$$\text{(or)} \quad \frac{d^2 \Psi}{dx^2} + \left[\frac{8\pi^2 m E}{h^2} - \frac{4\pi^2 m k}{h^2} x^2 \right] \Psi = 0 \quad \dots(4)$$

$$\text{Let } \frac{8\pi^2 m E}{h^2} = \alpha \text{ and } \left[\frac{4\pi^2 m k}{h^2} \right]^{1/2} = \beta$$

\therefore Equation (4) becomes

$$\frac{d^2 \Psi}{dx^2} + [\alpha - \beta^2 x^2] \Psi = 0 \quad \dots(5)$$

Now, let us introduce a dimensionless independent variable $y = x \sqrt{\beta}$ $\dots(6)$

$$\text{(or)} \quad x = \frac{y}{\sqrt{\beta}} \quad \text{(or)} \quad x^2 = \frac{y^2}{\beta} \quad \dots(7)$$

Differentiating equation (6) with respect to 'x' we get

$$\frac{dy}{dx} = \sqrt{\beta} \quad \dots(8)$$

$$\text{We can write } \frac{d\Psi}{dx} \text{ as } \frac{d\Psi}{dx} = \frac{d\Psi}{dy} \cdot \frac{dy}{dx} \quad \dots(9)$$

Substituting eqn (8) in eqn (9) we get

$$\frac{d\Psi}{dx} = \frac{d\Psi}{dy} \cdot \sqrt{\beta}$$

Differentiating, we get

$$\begin{aligned} \frac{d^2 \Psi}{dx^2} &= \frac{d^2 \Psi}{dy^2} \sqrt{\beta} \cdot \sqrt{\beta} \\ \text{(or)} \quad \frac{d^2 \Psi}{dx^2} &= \beta \cdot \frac{d^2 \Psi}{dy^2} \end{aligned} \quad \dots(10)$$

Substituting eqn (7) and (10) in eqn (5) we get

$$\begin{aligned} \beta \cdot \frac{d^2 \Psi}{dy^2} + \left[\alpha - \beta^2 \cdot \frac{y^2}{\beta} \right] \Psi &= 0 \\ \text{(or)} \quad \frac{d^2 \Psi}{dy^2} + \left[\frac{\alpha}{\beta} - \frac{\beta^2 y^2}{\beta^2} \right] \Psi &= 0 \\ \text{(or)} \quad \frac{d^2 \Psi}{dy^2} + \left[\frac{\alpha}{\beta} - y^2 \right] \Psi &= 0 \end{aligned} \quad \dots(11)$$

The solution for this equation (11) is

$$\Psi = f(y) e^{-y^2/2} \quad \dots(12)$$

By introducing ψ of equation (12) in equation (11) we can write

$$\frac{d^2 f}{dy^2} - 2y \frac{df}{dy} + \left[\frac{\alpha}{\beta} - 1 \right] f = 0 \quad \dots(13)$$

Let $\frac{\alpha}{\beta} - 1 = 2n$

\therefore Equation (13) becomes

$$\frac{d^2 f}{dy^2} - 2y \frac{df}{dy} + 2nf = 0 \quad \dots(14)$$

This is the standard mathematical equation, which is similar to the **Hermite's equation**

$$\text{i.e., } \frac{d^2 H}{dy^2} - 2y \frac{dH}{dy} + 2nH = 0 \quad \dots(15)$$

\therefore The solution for equation (15) are called **Hermite polynomials**, which is similar to equation (12), i.e.,

$$\Psi_n(y) = NH_n(y) e^{-y^2/2} \quad \dots(16)$$

Where 'N' is the Normalization constant.

\therefore Equation (16) represents the **Eigen function of the harmonic oscillator**

Energy Eigen values

The energy eigen values (permitted energy values) shall be obtained as follows:

We know $\frac{\alpha}{\beta} - 1 = 2n$

(or) $\frac{\alpha}{\beta} = 2n + 1$

(or) $\alpha = (2n + 1)\beta$ $\dots(17)$

Since $\alpha = \frac{8\pi^2 mE}{h^2}$ and $\beta = \left[\frac{4\pi^2 mk}{h^2} \right]^{1/2}$, we can write equation (17) as

$$\frac{8\pi^2 mE}{h^2} = (2n + 1) \left[\frac{4\pi^2 mk}{h^2} \right]^{1/2}$$

$$(or) E = \frac{h^2}{8\pi^2 m} (2n + 1) \frac{2\pi [mk]^{1/2}}{h}$$

$$(or) E = \frac{h}{4\pi} (2n + 1) \frac{[mk]^{1/2}}{m}$$

$$(or) E = 2 \left[n + \frac{1}{2} \right] \cdot \frac{h}{4\pi} \left[\frac{k}{m} \right]^{1/2} \quad \left[\therefore \frac{m^{1/2}}{m} = \frac{\sqrt{m}}{\sqrt{m} \sqrt{m}} = \frac{1}{\sqrt{m}} \right]$$

$$\therefore E = \left[n + \frac{1}{2} \right] \frac{h}{2\pi} \sqrt{\frac{k}{m}} \quad ... (18)$$

We know that, for harmonic oscillator, the frequency of oscillation

$$\nu = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

\therefore Equation (18) becomes

$$E = \left[n + \frac{1}{2} \right] h \nu$$

Thus, the energy of the harmonic oscillator is quantised in steps of $h\nu$

$$\therefore E_n = \left[n + \frac{1}{2} \right] h \nu \quad ... (19)$$

Where $n = 0, 1, 2, 3 \dots$

Equation (19) represents the Energy Eigen value for a harmonic oscillator.

Eigen Values plot

Energy Eigen values for various values of ' n ' shall be obtained as follows.

(i) When $n=0$ equation (19) becomes

$$E_0 = \frac{1}{2} h \nu$$

... (20)

Equation (20) represents the lowest value of energy, that the oscillator can have and is called **zero point energy**.

(ii) When $n=1$, Equation (19) becomes

$$E_1 = \left[1 + \frac{1}{2} \right] \hbar\nu$$

(or) $E_1 = \frac{3}{2} \hbar\nu$

(iii) When $n=2$, Equation (19) becomes

$$E_2 = \left[2 + \frac{1}{2} \right] \hbar\nu$$

(or) $E_2 = \frac{5}{2} \hbar\nu$

Similarity for various values of 'n'. The energy eigen values of an harmonic oscillator are plotted as shown in *Fig. 7.2*

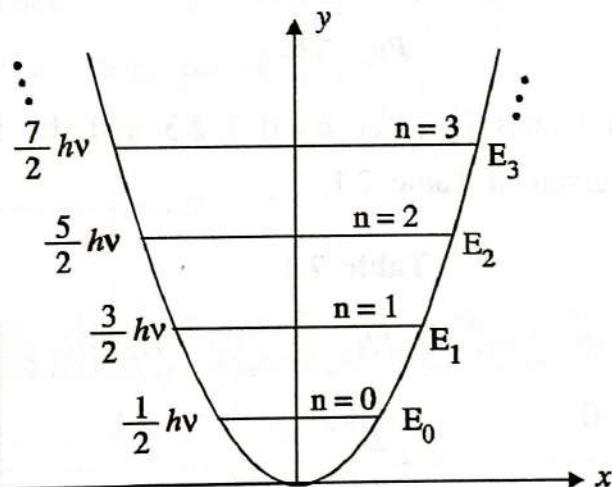


Fig. 7.2

Normalized wave function

The normalized wave function of the Harmonic oscillator is given by

$$\int_{-\infty}^{+\infty} |\psi_n|^2 dy = 1 \quad \text{where } n = 0, 1, 2, 3 \dots$$

The general formula for the n^{th} normalized wave function in terms of Hermite polynomial is given by

$$\psi_n = \frac{1}{\sqrt{2^n n! \sqrt{\pi}}} e^{-y^2/2} H_n(y) \quad \dots(20)$$

The corresponding normalized wave function for various values of 'n' is as shown in *Fig. 7.3*

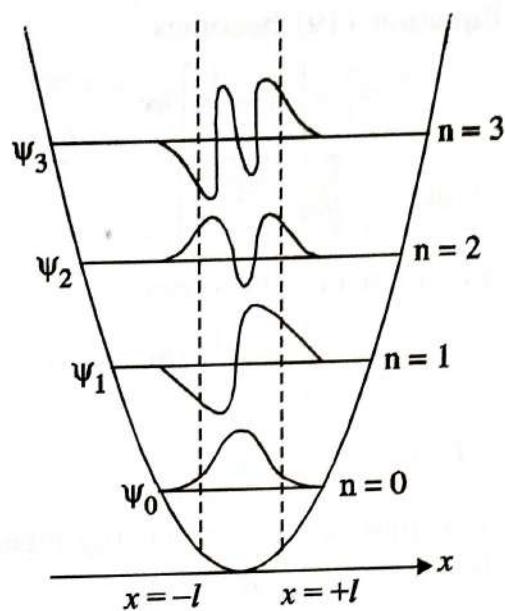


Fig. 7.3

The energy eigen values [E_n] for $n = 0, 1, 2, 3$ and the first four hermite polynomials [$H_n(y)$] is given in Table 7.1.

Table 7.1

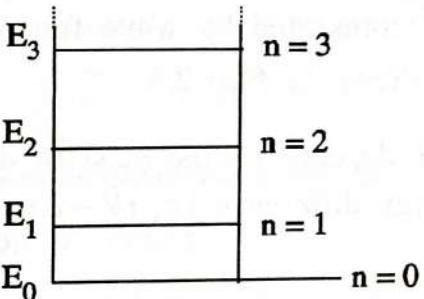
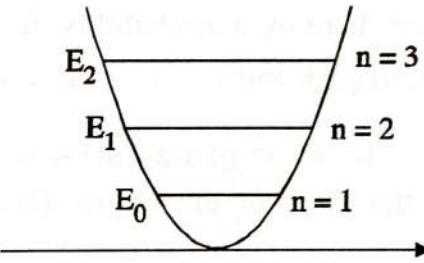
n	E_n	$H_n(y)$
0	$\frac{1}{2} h\nu$	1
1	$\frac{3}{2} h\nu$	$2y$
2	$\frac{5}{2} h\nu$	$4y^2 - 2$
3	$\frac{7}{2} h\nu$	$8y^3 - 12y$

Conclusions

1. The particles executing SHM will have discrete energy values.
2. The energy values are equidistant and are separated by $h\nu$.
3. The minimum energy (for $n = 0$) is not zero.
4. Energy levels are Non-degenerate.

7.4 COMPARISON BETWEEN PARTICLE IN A WELL AND HARMONIC OSCILLATOR (ONE DIMENSIONAL)

The comparison between the energy eigen values for a particle in a one dimensional potential well and in a harmonic oscillator are appended below.

S.No.	Particle in a 1D Potential Well		Harmonic oscillator
1.	Energy eigen value is $E_n = \frac{n^2 h^2}{8ml^2}$	1.	Energy eigen value is $E_n = \left(n + \frac{1}{2} \right) h \nu$
2.	The minimum energy (for $n = 0$) is zero.	2.	The minimum energy (for $n = 0$) is not zero.
3.	The energy values are not at equal distance.	3.	The energy values are at equal distances.
4.	Energy levels are as shown in Fig. 7.4	4.	Energy levels are as shown in Fig. 7.5
	 <p>Fig. 7.4</p>		 <p>Fig. 7.5</p>

7.5 TUNNELLING

Tunnelling

In quantum mechanics a particle having lesser energy (E) than the barrier potential (V) can easily cross over the potential barrier having a finite width ' l ' even without climbing over the barrier by tunnelling through the barrier. This process is called Tunnelling.

Explanation

Let us consider a beam of particles having kinetic energy E , less than the barrier potential (V), incident from the region (1) as shown in Fig. 7.6.

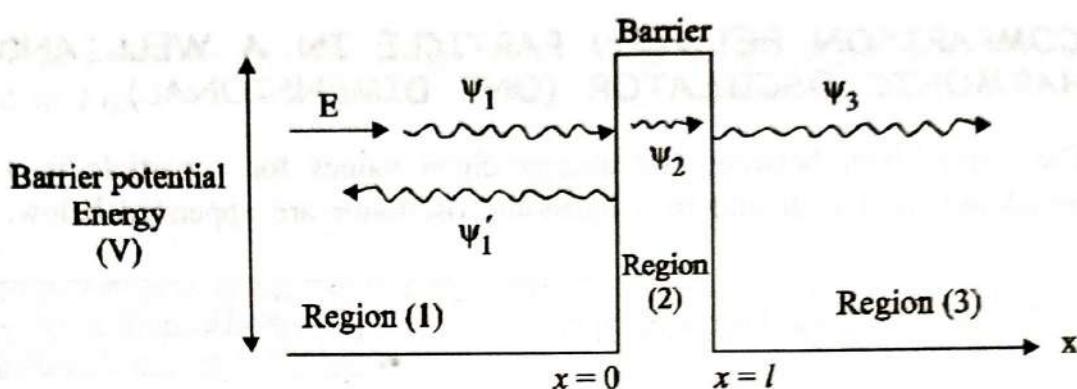


Fig. 7.6

According to classical mechanics the probability for the incident particles to cross the barrier is zero, because it has very less energy (E) when compared to the barrier potential (V).

But according to quantum mechanics, the incident wave from region (1) having the wave function ψ_1 can cross the barrier (Region (2)) by the process of tunnelling, represented by wave function ψ_2 and comes out to the region (3), represented by the wave function ψ_3 . However based on the width of the potential barrier there is a probability for the wave, represented by wave function ψ'_1 to get reflected within the same region (1) as shown in *Fig. 7.6*.

The value of transmission coefficient T depends on the mass of the particle (m), thickness of the barrier (l) and the energy difference i.e., $(V - E)$.

Examples

1. The tunnelling effect is observed in Josephson junction, in which electron pairs in the super conductors tunnel through the barrier layer, giving rise to Josephson current.)
2. This effect is also observed in the case of emission of alpha particles by radioactive nuclei.

Here, though the ‘ α ’ particle has very less kinetic energy they are able to escape from the nucleus whose potential wall is around 25 MeV high.

3. Tunnelling also occurs in certain semiconductor diodes called resonant tunnelling diodes.
4. Electron tunnels through insulating layer and act as a switch by tunnelling effect.

7.6 BARRIER PENETRATION AND QUANTUM TUNNELLING

Barrier Penetration

(If a particle with energy ' E ' is incident on a thin energy barrier of height ' V ', greater than ' E ', then there is a finite probability of the particle to penetrate the barrier. This phenomenon is called barrier penetration and this effect is called **Tunnelling Effect (or) Quantum Tunnelling**.) According to classical mechanics, the probability of a particle to penetrate / tunnel the barrier is zero, but according to quantum mechanics it is finite.

Concept of Barrier Penetration

Let us consider a particle of kinetic energy ' E ' which moves from the left (Region-1) and strikes the potential barrier of height ' V ' as shown in *Fig. 7.7*. If the kinetic energy (E) is lesser than the potential energy (V) (i.e., if $E < V$), then according to classical mechanics there is no chance for the particle to cross the potential barrier (V).

But, according to quantum mechanics, the particle has certain probability (few chances) to penetrate (or) cross the potential barrier (V) and comes out to Region-3, by tunnelling the Region-2.

Note: For wider and higher potential barrier, the chance of crossing the barrier through tunnelling process is very less.

Derivation / Proof

Let us consider such a particle with energy $E < V$ incident from left side (Region-1) and tunnel the Region-2 of width ' l ' [$x = 0$ to l] and comes out as a transmitted wave in Region-3, as shown in *Fig. 7.7*.

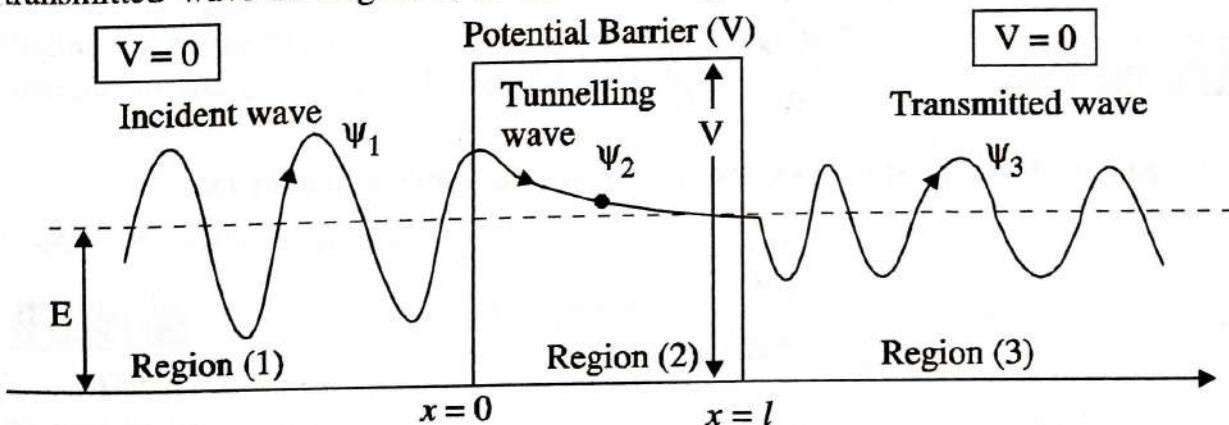


Fig. 7.7

Here, the potential energy $V=0$, on both sides of the barrier, which means, no forces will act upon the particle in Region-1 and in Region-3.

∴ The boundary conditions shall be written for various regions as

For Region-1: When $x < 0$; $V = 0$

For Region-2: When $0 < x < l$; $V = V$

For Region-3: When $x > l$; $V = 0$

Let ψ_1 , ψ_2 and ψ_3 be the wavefunctions in Regions 1, 2 and 3 respectively.

Then, the Schroedinger's wave equations for all the three regions shall be written as

For Region-1

$$\frac{d^2 \psi_1}{dx^2} + \frac{2m}{\hbar^2} [E - V] \psi_1 = 0$$

Since $V=0$ in Region-1, we can write the above equation as

$$\frac{d^2 \psi_1}{dx^2} + \frac{2m}{\hbar^2} E \psi_1 = 0 \quad \dots(1)$$

For Region-2

$$\frac{d^2 \psi_2}{dx^2} + \frac{2m}{\hbar^2} [E - V] \psi_2 = 0$$

Since $V > E$ in Region-2, we can write the above equation as

$$\frac{d^2 \psi_2}{dx^2} - \frac{2m}{\hbar^2} [V - E] \psi_2 = 0 \quad \dots(2)$$

For Region-3

$$\frac{d^2 \psi_3}{dx^2} + \frac{2m}{\hbar^2} [E - V] \psi_3 = 0$$

Since $V=0$ in Region-3, we can write the above equation as

$$\frac{d^2 \psi_3}{dx^2} + \frac{2m}{\hbar^2} E \psi_3 = 0 \quad \dots(3)$$

Let $\frac{2mE}{\hbar^2} = \alpha^2$ and $\frac{2m}{\hbar^2} (V - E) = \beta^2$

\therefore Equations (1), (2) and (3) shall be written as

$$\boxed{\text{For Region-1}} \quad \frac{d^2 \psi_1}{dx^2} + \alpha^2 \psi_1 = 0 \quad \dots(4)$$

$$\boxed{\text{For Region-2}} \quad \frac{d^2 \psi_2}{dx^2} - \beta^2 \psi_2 = 0 \quad \dots(5)$$

$$\boxed{\text{For Region-3}} \quad \frac{d^2 \psi_3}{dx^2} + \alpha^2 \psi_3 = 0 \quad \dots(6)$$

The solutions for eqns (4), (5) and (6) shall be written as

$$\boxed{\text{For Region-1}} \quad \psi_1 = A e^{i \alpha x} + B e^{-i \alpha x} \quad \dots(7)$$

$$\boxed{\text{For Region-2}} \quad \psi_2 = F e^{\beta x} + G e^{-\beta x} \quad \dots(8)$$

$$\boxed{\text{For Region-3}} \quad \psi_3 = C e^{i \alpha x} + D e^{-i \alpha x} \quad \dots(9)$$

Here A, B, C, D, F, G are the amplitudes of corresponding waves in various regions as shown in Fig. 7.7.

Let us discuss the behaviour of wave function and the amplitudes in each Region, viz.,

Region-1

The wave function of the incident wave in Region-1 shall be written from equation (7) as

$$\boxed{\psi_1 (\text{Incident}) = A e^{i \alpha x}} \quad \dots(10)$$

Where 'A' is the amplitude of the *incident wave* in Region-1

Since there are ample chances for the wave to get reflected within the Region-1, due to higher potential barrier (or) larger width of the barrier, the wave function of the reflected wave in Region-1 shall be written from equation (7) as

$$\boxed{\psi_1 (\text{Reflected}) = B e^{-i \alpha x}} \quad \dots(11)$$

Where 'B' is the amplitude of the *reflected wave* in Region-1.

Region-2

The wave function of the transmitted (or) tunnelling wave at Region-2 shall be written from equation (8) as

$$\boxed{\psi_2 = F e^{\beta x} + G e^{-\beta x}} \quad \dots(12)$$

Where $\beta = \frac{\sqrt{2m(V-E)}}{\hbar}$.

F – is the amplitude of the **barrier penetrating wave** (or) tunnelling wave in Region-2.

G – is the amplitude of the **reflected wave at the boundary** between Region-1 and Region-2.

From eqn (12) we can see that the exponents are real quantities, so the wave function ψ_2 will not oscillate and therefore does not represent a moving particle at Region-2.

Thus, the particle can either penetrate (or) tunnel through Region-2 and transmitted to Region-3 (or) it shall be reflected back to Region-1 itself. [Given by equation (11)].

Region-3

The wave function of the transmitted wave in Region-3 shall be written from equation (9) as

$$\Psi_3(\text{Transmitted}) = Ce^{i\alpha x} \quad \dots(13)$$

Where 'C' is the amplitude of the **transmitted wave** in Region-3.

In Region-3, i.e., $x > l$, there can be only transmitted wave and there will not be any reflected wave and therefore the amplitude of the reflected wave in Region-3 i.e., $D = 0$

Thus, the wave function of the reflected wave in Region-3 is also zero.

$$\therefore \Psi_3(\text{Reflected}) = 0 \quad \dots(14)$$

The transmission and reflection co-efficient shall be obtained as follows:

Transmission Co-efficient

We know that the probability density is the square of the amplitude of that function. Therefore the barrier transmission co-efficient (T) is the ratio between the square of the amplitude of the transmitted wave $|C|^2$ and the square of the amplitude of the incident wave $|A|^2$.

$$\therefore \text{The transmission co-efficient } T = \frac{|C|^2}{|A|^2} = \frac{4\sqrt{E}\sqrt{E-V}}{[\sqrt{E} + \sqrt{E-V}]^2} \quad \dots(15)$$

Equation (15) is also called as the "**Penetrability**" of the barrier.

Reflection Co-efficient

Similarly, the reflection co-efficient (R) for the barrier surface at $x=0$ is the ratio between the square of the amplitude of the reflected wave $|B|^2$ and the square of the amplitude of the incident wave $|A|^2$

$$\therefore \text{The reflection co-efficient } R = \frac{|B|^2}{|A|^2} = \left[\frac{\sqrt{E} - \sqrt{E-V}}{\sqrt{E} + \sqrt{E-V}} \right]^2 \quad \dots(16)$$

7.7 SCANNING TUNNELLING MICROSCOPE (STM)

Introduction

A microscope is a device which is used to view the magnified image of a smaller object, which cannot be clearly seen through our naked eye.

In 1980, Gerd Binning and Heinrich Rohrer invented a new type of microscope called scanning tunnelling microscope (STM). In this microscope, we can obtain the images of electrically conducting surfaces in atomic scale, i.e., STM is used to observe the surfaces in terms of atom by atom.

Principle

The basic principle used in scanning tunnelling microscope (STM) is the tunnelling of electron between the sharp metallic tip of the probe and the surface of a sample.

Here, constant tunnelling current is maintained by adjusting the distance between the tip and the sample, with an air gap for electron to tunnel. In a similar way the tip is used to scan atom by atom and line by line of the sample and the topography of the sample is recorded in the computer.

Construction

1. The experimental setup consists of a probe as shown in *Fig. 7.8* in which a small thin metal wire is etched in such a way that the tip of the probe will have only one atom as shown in *Fig. 7.9*.
2. The tip is tapered down to a single atom, so that it can follow even a small change in the contours of the sample.
3. The tip is connected to the scanner and it can be positioned to X , Y , Z co-ordinates using a personal computer, as shown in *Fig. 7.8*.
4. The sample for which the image has to be recorded is kept below the tip of the probe at a particular distance (Atleast to a width of 2 atoms spacing) in a such a way that the tip should not touch the sample. i.e., A small air gap should always be maintained between the tip of the probe and the sample, as shown in *Fig. 7.9*.

5. The computer is also used to record the path of the probe and the topography of the sample in a grey-scale (or) colour.
6. Necessary circuit connections along with an amplifier are provided to measure the tunnelling current in the circuit.

Working

1. Circuit is switched ON and necessary biasing voltage is given to the probe.
2. Due to biasing the electrons will tunnel (or) jump between the tip of the probe and the sample and therefore produces a small electric current called tunnelling current, as shown in *Fig. 7.9*.
3. The tunnelling current flows through the circuit only if the tip is in contact with the sample through the small air gap at a distance ' d ' between them.

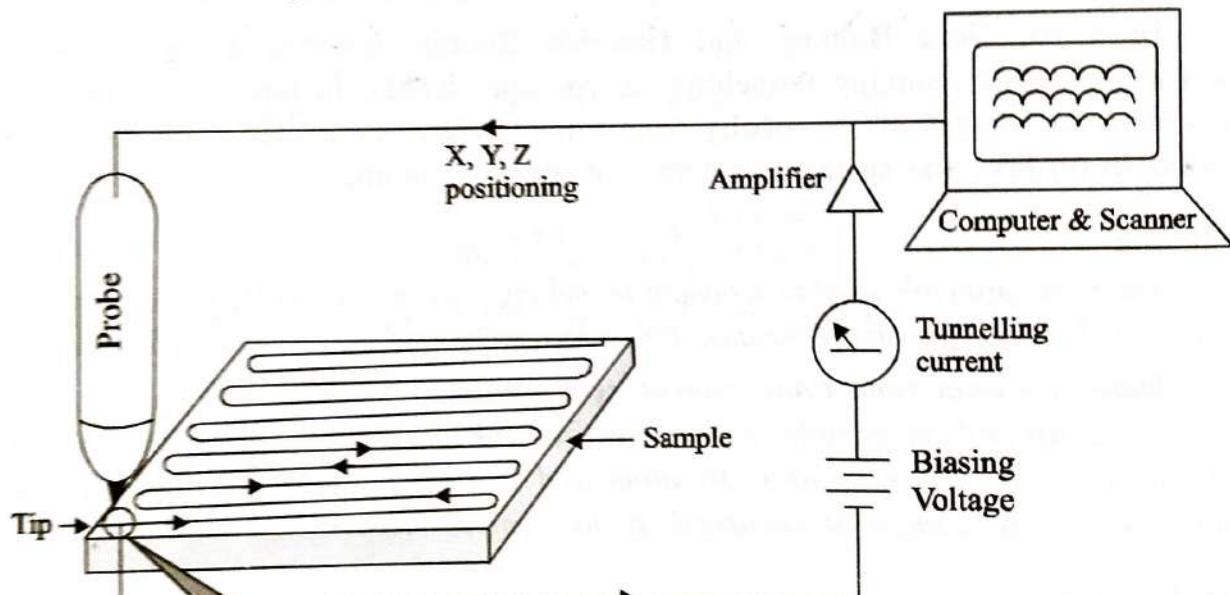
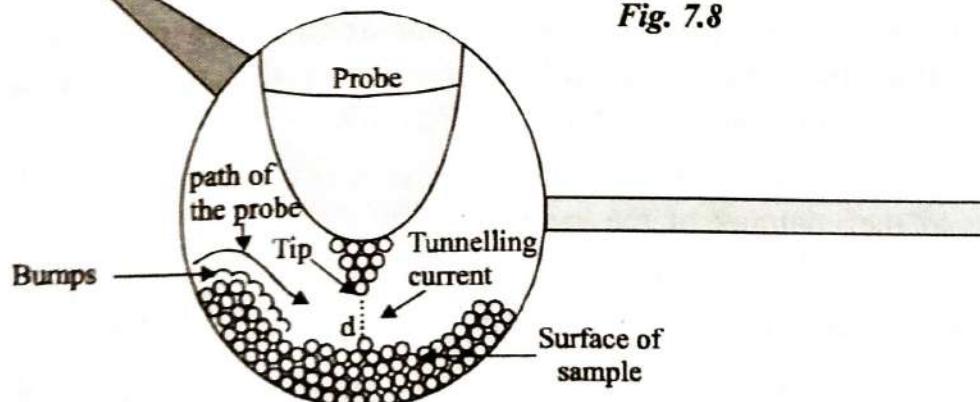


Fig. 7.8



Scanning Tunneling
Microscope (STM)

Fig. 7.9

4. The current produced is amplified and measured in the computer.
5. It is found that the current increases (or) decreases based on the distance between the tip of the probe and the sample.
6. The current in the circuit should be monitored in such a way that it should be maintained constant.
7. Therefore, for maintaining the constant current, the distance (d) between the tip and the sample should be continuously adjusted, whenever the tip moves over the surface of the sample.
8. The height fluctuations (d) between the tip and the sample is accurately recorded and as a resultant, a map of 'bumps' is obtained in the computer as shown in *Fig. 7.8*.
9. In a similar way the tip is scanned atom by atom and line by line of the sample and the topography of the sample is recorded in the computer.
10. The STM does not show the picture of the atom, rather it records only the exact position of the atoms, more precisely the position of electrons.

Advantages

1. It can scan, the positions & topography atom by atom (or) even electrons.
2. It is the Latest technique used in Research laboratories for scanning the materials.
3. Very accurate measurement shall be obtained.
4. Magnification is upto nano-scale.

Disadvantages

1. Even a very small sound (or) vibrations will disturb the measurement setup.
2. It should be kept in vacuum, as even a single dust particle may damage the tip of the probe
3. Cost is high.
4. More complexity.

Applications

1. It is used to produce Integrated circuit.
2. It is used in Biomedical devices.
3. Chemical and material sciences research labs are the major areas in which it is used.
4. They are used in material science studies for both bump and flat surfaces.

- Note:**
1. For bump surfaces, the Feed back loop should be constantly monitored and such a setup is called "constant current" image setup.
 2. For flat surfaces, the feed back loop shall be turned off and such a setup is called "constant height" image setup.

7.8 RESONANT TUNNELLING DIODE

Introduction

So far we have discussed about the quantum tunnelling through a single potential barrier. Now we are going to discuss about the quantum tunnelling through two potential barriers (or) a potential profile.

Resonant tunnelling occurs through a potential profile which consists of two potential barriers, so called double barrier structure which are located very close to each other.

Principle

Resonant tunnelling diode works on the principle of tunnelling effect, in which the charge carriers cross the energy barrier(s) even with lesser energy than the barrier potential, quantum mechanically. The probability of tunnelling increases with the decreasing barrier energy.

Symbol and Circuit diagram

The symbol and circuit diagram of a resonant tunnelling diode is as shown in Fig. 7.10 and Fig. 7.11, respectively.

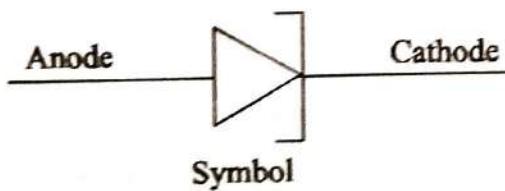


Fig. 7.10

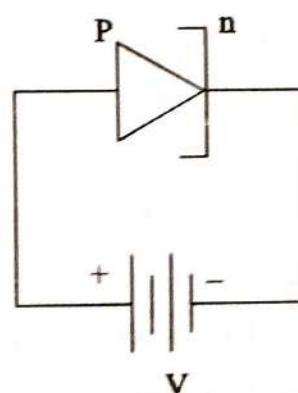


Fig. 7.11

Theory

A resonant tunnelling diode also called as Essaki diode is formed using *p* and *n*-materials, with heavy doping say 1000 times larger than the conventional *p*–*n*-junction diode. Due to heavy doping, the barrier potential decreases drastically, in turn will help the charge carriers to easily tunnel the junctions, quantum mechanically.

Quantum Well Structure

A resonant tunnelling diode (RTD) consists of a quantum well structure with discrete energy values E_1, E_2 etc, surrounded by two thin layer of potential barriers (V_1 and V_2) with emitter (in *n*-region) and collector (in *p*-region) on either side as shown in *Fig. 7.12*.

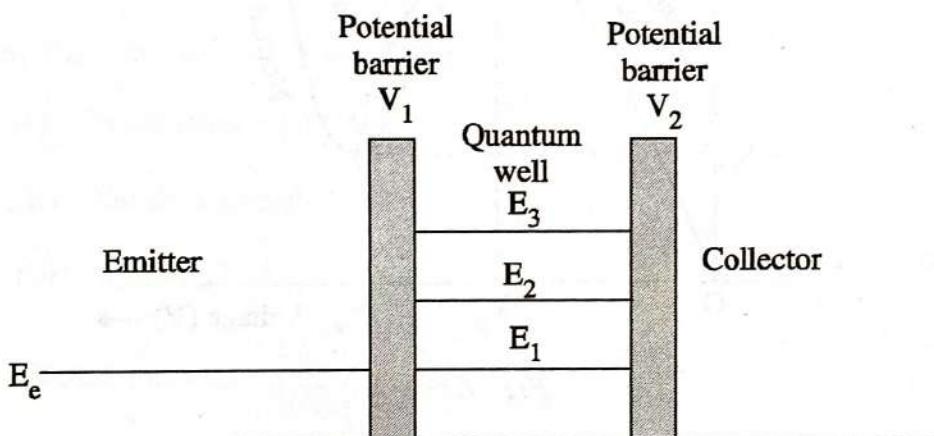


Fig. 7.12

V – *I* Characteristics

The diode is forward biased as shown in *Fig. 7.11*. During forward bias, when voltage is increased, then the current in the diode varies at different resistance regions as follows.

Positive Resistance Region

When a voltage is applied across the resonant tunnelling diode, a terahertz wave is emitted and therefore at resonance, the energy value (E_1) in the quantum well becomes equal to the energy value (E_e) in the emitter side.

i.e., At low voltage and at resonance $\Rightarrow E_1 \approx E_e$

Thus at $E_1 = E_e$, i.e., at resonance, the charge carriers tunnel the potential barriers (V_1 and V_2) and reaches collector region by the process called **resonant tunnelling**.

Therefore the current increases rapidly due to tunnelling effect and reaches the peak-point 'P' as shown in Fig. 7.13 and this current, is called **peak current** (I_p). The voltage at which the diode reaches peak current is called **peak voltage** (V_p).

This region, where the current increases due to the increase in applied voltage is called **positive resistance region**.

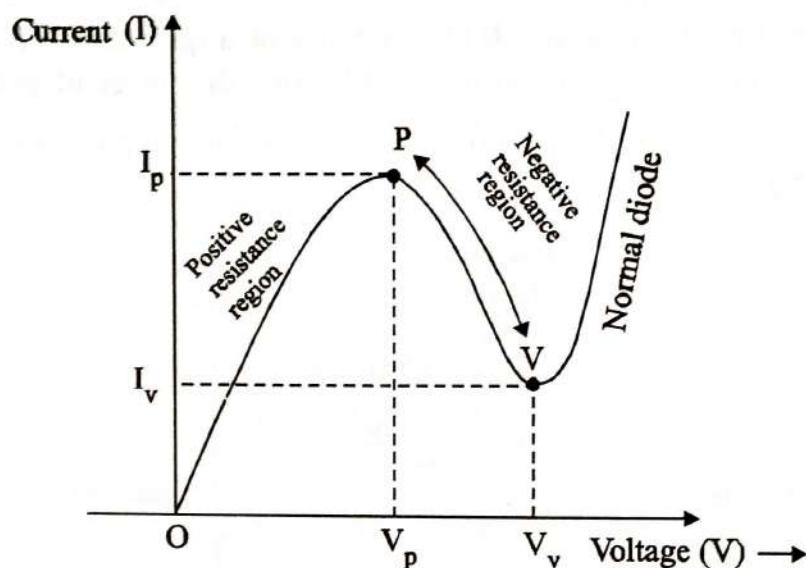


Fig. 7.13

Negative Resistance Region

When the voltage is further increased, then the terahertz wave dies out and now the energy value (E_1) in the quantum well becomes lesser than the energy value (E_e) in the emitter side.

i.e., At higher voltage $\Rightarrow E_1 < E_e$

However, since the quantum well has discrete energy values, the energy value (E_2) in the quantum well is still larger than the energy value (E_e) in the emitter side as shown in Fig. 7.12.

i.e., $E_2 > E_e$

Therefore, the charge carriers cannot tunnel the potential barriers and thus the current in the diode decreases, and reaches the valley point V as shown in Fig. 7.13.

This region where the current decreases due to increase in applied voltage is called **negative resistance region**.

This minimum current is called **valley current (I_v)** and the corresponding voltage is called **valley voltage (V_v)**.

Normal Diode

Now, when the applied voltage is further increased beyond the valley point voltage in such a way that energy value (E_2) in the quantum well becomes equal to the energy value (E_e) in the emitter side, then the current again increases and therefore the resonant tunnelling diode behaves as a normal diode as shown in **Fig. 7.13.**

Thus, the current in resonant tunnelling diode is due to 3 components *viz.*,

- (i) Tunnelling current (I_T)
- (ii) Diode current (I_D)

and (iii) Excess current (I_E)

\therefore Total current

$$I_{\text{Total}} = I_T + I_D + I_E$$

Advantages

1. Cost and noise is low.
2. Fabrication is very simple.
3. Operation speed is very high.
4. Power dissipation is low and hence it is environmental friendly device.

Disadvantages

1. Since it is a two terminal device, it is difficult to isolate the input and output.
2. It is a low output swing device.

Applications

1. As resonant tunnelling diode has both positive resistance [From point O to P] and negative resistance [From point P to V], it has many applications in the switching devices.

2. It can be used as normal diodes also.
3. They are used as high frequency microwave oscillators.
4. When resonant tunnelling diode is operated under negative resistance region, then it can be used as an oscillator (or) a switch.
5. Resonant tunnelling diode (RTD) are used in memory calls, multivalued logic circuit devices etc.

Note: RTD is will replace transistors as the workhorse of IC's is near future.

7.9 FINITE SQUARE POTENTIAL WELL

Let us consider a one dimensional finite square potential well of length ' l ' in which a particle of energy ' E ' is moving along the 'x' axis as shown in Fig. 7.14.

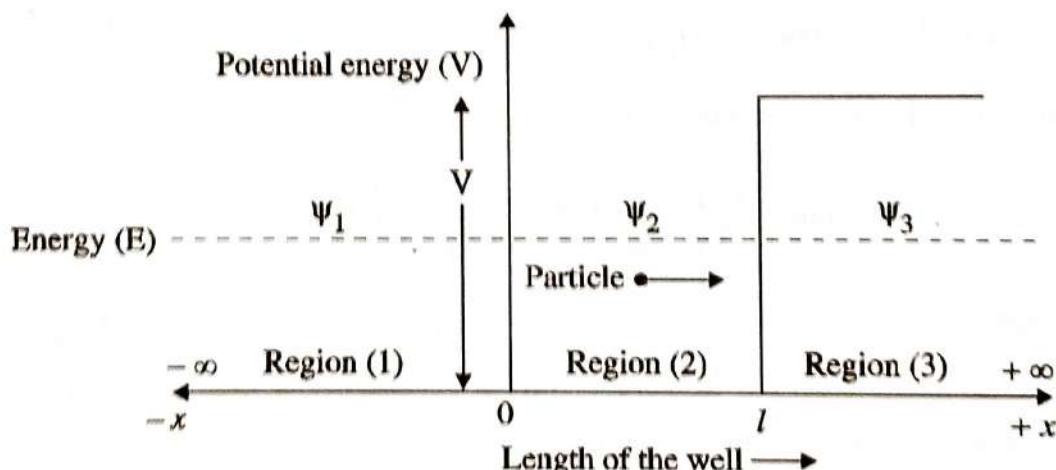


Fig. 7.14

If the energy of the trapped particle (E) is less than the potential energy (V) of the potential barrier, then the Schrödinger time independent wave equation for Region-1 and Region-3 can be written as

$$\frac{d^2 \psi}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

Since $V > E$, we can write the above equation as

$$\frac{d^2 \psi}{dx^2} - \frac{2m}{\hbar^2} (V - E) \psi = 0 \quad \dots(1)$$

$$\text{Let } \frac{2m}{\hbar^2} (V - E) = k^2$$

\therefore We can write equation (1) as

$$\frac{d^2 \Psi}{dx^2} - k^2 \Psi = 0 \quad \dots(2)$$

The above equation holds good for the following boundary conditions, viz.,

- (i) $x < 0$ [For Region (1)]
 - and (ii) $x > l$ [For Region (3)]
- \therefore The solution for equation (2) can be written as

For Region (1) $\Psi_1 = A e^{kx} + B e^{-kx} \quad \dots(3)$

For Region (3) $\Psi_3 = C e^{kx} + D e^{-kx} \quad \dots(4)$

Where A, B, C, D are called arbitrary constants, which can be found by using the boundary conditions.

Wave Function at Region - 1

At $x = -\infty$, the chance for finding the particle is zero, $\therefore \Psi_1 = 0$.

\therefore Equation (3) becomes

$$0 = A e^{-\infty} + B e^{+\infty}$$

As $\lim_{x \rightarrow -\infty} e^{-kx} = 0$ and $\lim_{x \rightarrow +\infty} e^{kx} = \infty$

$$0 = 0 + B$$

(or) **$B = 0$** $\dots(5)$

Substituting the value of B from equation (5) in equation (3), we get

$$\Psi_1 = A e^{kx} \quad \dots(6)$$

Wave Function at Region-3

At $x = \infty$, here also the chance for finding the particle is zero, $\therefore \Psi_3 = 0$

\therefore Equation (4) becomes

$$0 = C e^{+\infty} + D e^{-\infty}$$

$$0 = C + 0$$

(or) **$C = 0$** $\dots(7)$

Substituting the value of C from equation (7) in equation (4), we get

$$\Psi_3 = D e^{-kx} \quad \dots(8)$$

The equation (6) and equation (8) shows that the wave function at Region-1 and Region-3 decrease exponentially from the sides of the walls (i.e.) at $x < 0$ in Region-1 and $x > L$ in Region-3, respectively.

Thus, there are certain (or) few probability for finding the particle outside the region between 0 to ' l ' as shown in *Fig. 7.15.* and *Fig. 7.16.* However, the particle is finite i.e., definitely present inside the Region-2, between 0 to ' l '.

Wave Function at Region-2

\therefore The Schroedinger's wave equation for Region-2 shall be written as

$$\frac{d^2 \Psi_2}{dx^2} + \frac{2m}{\hbar^2} (E - V) \Psi = 0 \quad \dots(9)$$

Since the particle is moving between 0 to ' l ' the potential energy (V) is zero, i.e., $V=0$

\therefore Equation (9) becomes

$$\frac{d^2 \Psi_2}{dx^2} + \frac{2m}{\hbar^2} E \Psi = 0 \quad \dots(10)$$

Solution for equation (10) shall be written as

$$\Psi_2 = F \sin \frac{\sqrt{2mE}}{\hbar} x + G \cos \frac{\sqrt{2mE}}{\hbar} x \quad \dots(11)$$

Where F and G are the arbitrary constants, which shall be found by applying the boundary conditions.

Boundary conditions at Region-2

In Region-2, at the boundary i.e., when $x=0$ and $x=l$, the wave functions will have the same values as that of outside (near the wall of the potential well) Region-1 and Region-3.

At $x=0$

When $x=0$, the wave function at Region-2 is equal to the wave function at Region-1. i.e., $\Psi_2 = \Psi_1$ and \therefore we can write $\frac{d\Psi_2}{dx} = \frac{d\Psi_1}{dx}$

(i) To find $\psi_2 = \psi_1$

When $x = 0$; $\psi_2 = \psi_1$

\therefore Equation (11) = Equation (6)

Applying the boundary condition, i.e., $x = 0$ in eqn. (11) and eqn. (6), we get

$$\therefore F \sin \frac{\sqrt{2mE}}{\hbar} (0) + G \cos \frac{\sqrt{2mE}}{\hbar} (0) = A e^{k0}$$

$$\therefore O + G = A$$

(or)

$$G = A$$

...(12)

(ii) To find $\frac{d\psi_2}{dx} = \frac{d\psi_1}{dx}$

When $x = 0$; $\frac{d\psi_2}{dx} = \frac{d\psi_1}{dx}$

\therefore Differentiating equation (6) with respect to 'x' we get

$$\frac{d\psi_1}{dx} = A e^{kx} \cdot k \quad \dots(13)$$

Differentiating equation (11) with respect to 'x' we get

$$\frac{d\psi_2}{dx} = F \cos \frac{\sqrt{2mE}}{\hbar} (x) \left[\frac{\sqrt{2mE}}{\hbar} \right] - G \sin \frac{\sqrt{2mE}}{\hbar} (x) \left[\frac{\sqrt{2mE}}{\hbar} \right] \quad \dots(14)$$

Applying the boundary condition, i.e., $x = 0$ in eqn (14) and eqn (13) and equating we get

$$F \cos \frac{\sqrt{2mE}}{\hbar} (0) \left[\frac{\sqrt{2mE}}{\hbar} \right] - G \sin \frac{\sqrt{2mE}}{\hbar} (0) \left[\frac{\sqrt{2mE}}{\hbar} \right] = A e^{k0} \cdot k$$

$$\therefore F \frac{\sqrt{2mE}}{\hbar} = A k \quad \dots(15)$$

Substituting the value of A from equation (12) in equation (15) we get

$$F \frac{\sqrt{2mE}}{\hbar} = G k$$

$$\therefore F = \frac{G k \hbar}{\sqrt{2mE}}$$

...(16)

At $x = l$

When $x = l$ the wave function at Region-2 is equal to the wave function at Region-3. i.e., $\psi_2 = \psi_3$ and \therefore we can write $\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx}$

(i) To find $\psi_2 = \psi_3$

When $x = l$; $\psi_2 = \psi_3$

\therefore Equation (11) = Equation (8)

Applying the boundary condition, i.e., $x = l$ in eqn (11) and eqn (8), we get

$$F \sin \frac{\sqrt{2mE}}{\hbar} (l) + G \cos \frac{\sqrt{2mE}}{\hbar} (l) = D e^{-kl} \quad \dots(17)$$

Substituting the value of 'F' from equation (16) in equation (17) we get

$$\frac{G k \hbar}{\sqrt{2mE}} \sin \frac{\sqrt{2mE}}{\hbar} (l) + G \cos \frac{\sqrt{2mE}}{\hbar} (l) = D e^{-kl}$$

$$(or) G \left[\frac{k \hbar}{\sqrt{2mE}} \sin \frac{\sqrt{2mE}}{\hbar} (l) + \cos \frac{\sqrt{2mE}}{\hbar} (l) \right] = D e^{-kl}$$

$$\therefore D = e^{kl} G \left[\frac{k \hbar}{\sqrt{2mE}} \sin \frac{\sqrt{2mE}}{\hbar} (l) + \cos \frac{\sqrt{2mE}}{\hbar} (l) \right] \quad \dots(18)$$

(ii) To find $\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx}$

When $x = l$; $\frac{d\psi_2}{dx} = \frac{d\psi_3}{dx}$

$$\psi_3 = D e^{-kx} \quad \dots(18)$$

Differentiating eqn. (8), with respect to 'x', we get

$$\frac{d\psi_3}{dx} = D e^{-kx} (-k) \quad \dots(19)$$

Applying the boundary condition i.e., $x = l$ in eqn (14) and eqn (19) and equating we get

$$F \cos \frac{\sqrt{2mE}}{\hbar} (l) \left[\frac{\sqrt{2mE}}{\hbar} \right] - G \sin \frac{\sqrt{2mE}}{\hbar} (l) \left[\frac{\sqrt{2mE}}{\hbar} \right] = D e^{-kl} (-k)$$

$$(or) F \left[\frac{\sqrt{2mE}}{\hbar} \cos \frac{\sqrt{2mE}}{\hbar} l \right] - G \left[\frac{\sqrt{2mE}}{\hbar} \sin \frac{\sqrt{2mE}}{\hbar} l \right] = -k D e^{-kl} \quad \dots(20)$$

Substituting the value of 'F' from equation (16) in equation (20) we get

$$\frac{Gk\hbar}{\sqrt{2mE}} \left[\frac{\sqrt{2mE}}{\hbar} \cos \frac{\sqrt{2mE}}{\hbar} l \right] - G \left[\frac{\sqrt{2mE}}{\hbar} \sin \frac{\sqrt{2mE}}{\hbar} l \right] = -kDe^{-kl}$$

(or)
$$G \left[k \cos \frac{\sqrt{2mE}}{\hbar} l - \frac{\sqrt{2mE}}{\hbar} \sin \frac{\sqrt{2mE}}{\hbar} l \right] = -kDe^{-kl} \quad \dots(21)$$

Substituting equation (18) in equation (21) and solving, we get

$$\tan \frac{\sqrt{2mE}}{\hbar} l = \frac{2\sqrt{E(V-E)}}{2E-V}$$

Thus the energy values are quantised and the allowed energy levels shall be found by numerical (or) graphical methods.

Energy levels

The wave functions for the first three allowed energy levels are shown in Fig. 7.15.

The probability densities (After normalization) for finding the particle at different locations are shown in Fig. 7.16.

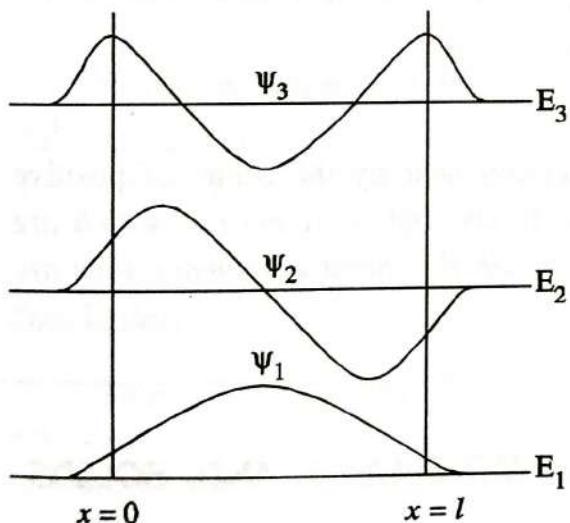


Fig. 7.15

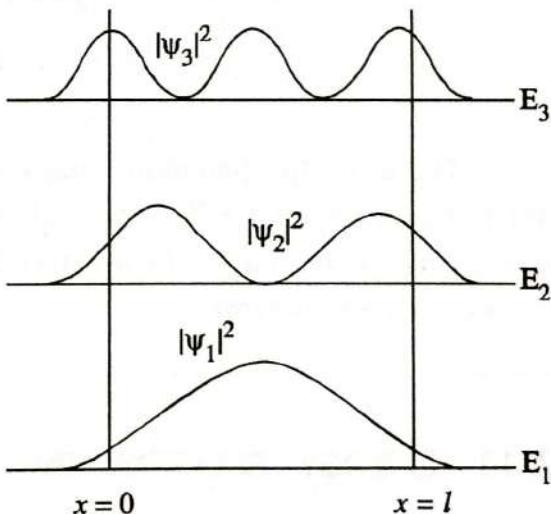


Fig. 7.16

Conclusion

From the above figures we can conclude that when a particle is enclosed in a finite potential well, then there *are some chances for finding the particle outside the potential well.*

7.10 ELECTRON IN PERIODIC POTENTIAL

In quantum free electron theory of metals the electrons were assumed to be moving in a region of constant potential and hence it moves freely about the crystal. But it fails to explain why some solids behaves as conductors, some as insulators and some as semiconductors etc.

Therefore instead of considering an electron to move in a constant potential, *In Zone theory of solids the electrons are assumed to move in a field of periodic potential.*

In a metal piece, the positive ions are arranged in a regular and proper order, therefore a periodic potential (i.e) the potential field which varies periodically with the same period as the lattice, exists in the metal. Also the potential is minimum near the centre of positive ions and is maximum between the centres of ions as shown in *Fig. 7.17*.

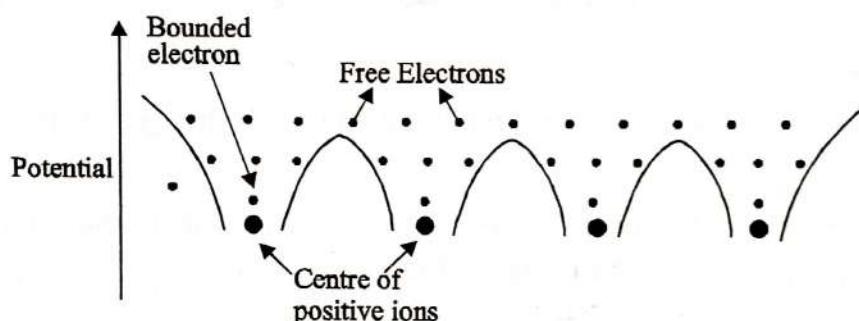


Fig. 7.17

Therefore the potential energy of the electron near by the centre of positive ion is maximum and will not be able to move freely, but *the electrons which are above these potential peaks are free to move inside the metal and hence they are termed as free electrons.*

7.11 ENERGY SPECTRA IN ATOMS, MOLECULES AND SOLIDS

To picture the energy spectra in atoms, molecules and solids let us consider a metal say sodium, which consists of 11 electrons with electronic configuration of $1s^2 2s^2 2p^6 3s^1$. The energy spectrum of a single atom is as shown in *Fig. 7.18*. When two sodium atoms are assembled to form a sodium molecule, the energy spectrum of the molecule is as shown in *Fig. 7.19*. It is found that for a sodium molecule each atomic levels are splitted into two closely spaced levels.

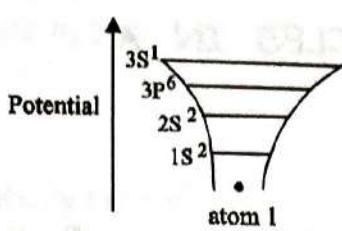


Fig. 7.18

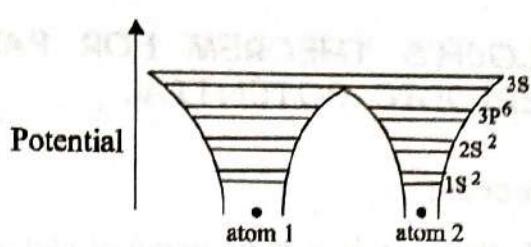


Fig. 7.19

Similarly if ' N ' number of atoms are assembled to form a solid, then we have N number of very closely spaced sub levels so called as ***energy band*** as shown in **Fig. 7.20**.

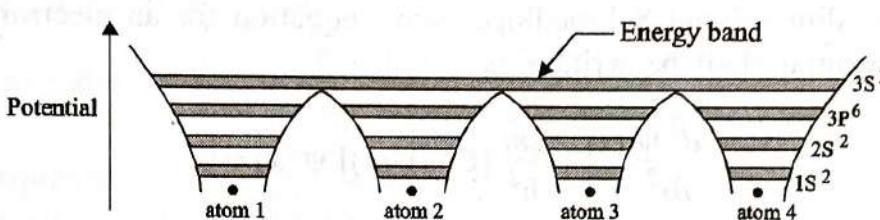


Fig. 7.20

It can be found that each energy band is separated by the gaps and are known as ***Energy gap (or) forbidden band gap energy***.

It can be seen that the electrons present in the outermost energy band ($3s^1$) are mixed together and they are free to move over the metals. These free electrons are responsible for the conduction to occur.

Conclusion

Thus we can conclude that the electron moving in a periodic potential lattice will have discontinuous energy values (i.e) they are separated as allowed and forbidden zones (or) bands.

Note: In the case of electron moving in constant potential lattice (quantum theory) it has quasi continuous energy values.

7.12 BLOCH'S THEOREM FOR PARTICLES IN A PERIODIC POTENTIAL

Bloch Theorem

Bloch theorem is a mathematical statement of an electron (Particle) wave function moving in a perfectly periodic potential. These functions are called bloch functions.

Explanation

Let us consider a particle, say an electron moving in a periodic potential.

The one dimensional Schroedinger wave equation for an electron moving in a periodic potential shall be written as

$$\frac{d^2 \psi(x)}{dx^2} + \frac{2m}{\hbar^2} [E - V(x)] \psi(x) = 0 \quad \dots(1)$$

Suppose the electron moves along X-direction in a one dimensional crystal, then the potential energy of the electron should satisfy the condition

$$V(x) = V(x+a) \quad \dots(2)$$

Where a is the periodicity of the potential.

The solution for equation (1) is

$$\psi(x) = e^{ikx} u_k(x) \quad \dots(3)$$

where

$$u_k(x) = u_k(x+a) \quad \dots(4)$$

Here e^{ikx} represents the plane wave and $u_k(x)$ represents the periodic function.

∴ Equation (3) is called Bloch theorem and Equation (4) is called Bloch function.

Proof

If equation (1) has the solutions with the property of equation (2), we can write the property of the Bloch functions i.e., equation (3) as

$$\psi(x+a) = e^{ik(x+a)} u_k(x+a)$$

(or)

$$\psi(x+a) = e^{ikx} \cdot e^{ika} u_k(x+a)$$

Since $u_k(x+a) = u_k(x)$, we can write the above equation as

$$\Psi(x+a) = e^{ikx} e^{ika} \cdot u_k(x)$$

Since $\Psi(x) = e^{ikx} u_k(x)$, we can write the above equation as

$$\Psi(x+a) = e^{ika} \cdot \Psi(x) \quad \dots(5)$$

(or)

$$\Psi(x+a) = Q \Psi(x) \quad \dots(6)$$

Where $Q = e^{ika}$

If $\Psi(x)$ is a single-valued function, then

we can write

$$\Psi(x) = \Psi(x+a) \quad \text{Thus Bloch theorem is proved.}$$

This equation is similar to that of eqn (2) and eqn (4) i.e., If the potential is a function of 'x' and 'a', then the wave function is also a function of 'x' and 'a'.

7.13 BEHAVIOUR OF AN ELECTRON IN A PERIODIC POTENTIAL -THE KRONIG - PENNEY MODEL (QUALITATIVE TREATMENT)

Kronig and Penney treated a simplest example for one dimensional periodic potential. In this model it is assumed that the potential energy of an electron has the form of a periodic array of square wells as shown in Fig. 7.21.

Here we have two regions. viz.

Region (i) In this region, between the limits $0 < x < a$, the potential energy is zero and hence the electron is assumed to be a free particle.

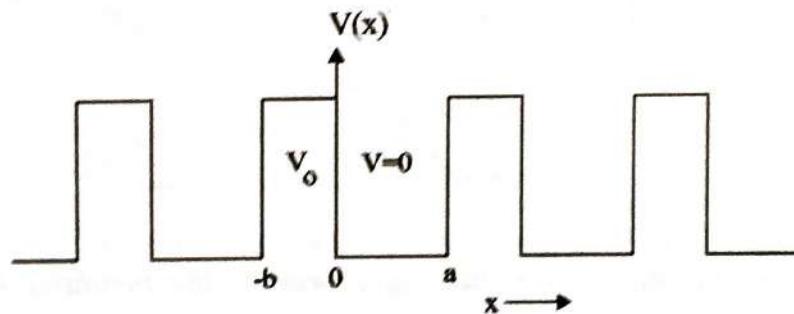


Fig. 7.21

∴ The one dimensional Schroedinger wave equation for a free particle is

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} [E - 0] \psi = 0$$

$$\text{(or)} \quad \frac{d^2\psi}{dx^2} + \alpha^2 \psi = 0$$

...(1)

$$\text{Where } \alpha^2 = \frac{2m}{\hbar^2} E$$

Region (ii) In this region between the limits $-b < x < 0$, the potential energy of the electron is V_o

∴ The one dimensional Schroedinger wave equation is

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} [E - V_o] \psi = 0$$

$$\text{(or)} \quad \frac{d^2\psi}{dx^2} - \beta^2 \psi = 0$$

...(2)

$$\text{Where } \beta^2 = \frac{2m}{\hbar^2} (V_o - E) \quad (\text{Since } E < V_o)$$

For both the region, the appropriate solution suggested by Bloch is of the form,

$$\psi(x) = e^{ikx} u_k(x)$$

...(3)

Differentiating equation (3) and substituting it in equation (1) and (2) and then further solving it under the boundary conditions, we get

$$P \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka$$

...(4)

where $P = \frac{mV_0 ba}{\hbar^2}$, is called as **Scattering power of the potential barrier**, which is the measure of the strength with which the electrons are attracted by the positive ions.

In equation (4), there are only two variables (i.e) α and k . We know $\cos ka$ can take values only from -1 to 1 , Therefore the left hand side of equation (4) must also fall in this range. A plot is made between the LHS of equation (4) and αa for a value of $P = \frac{3\pi}{2}$ (arbitrary) as shown in *Fig. 7.22*.

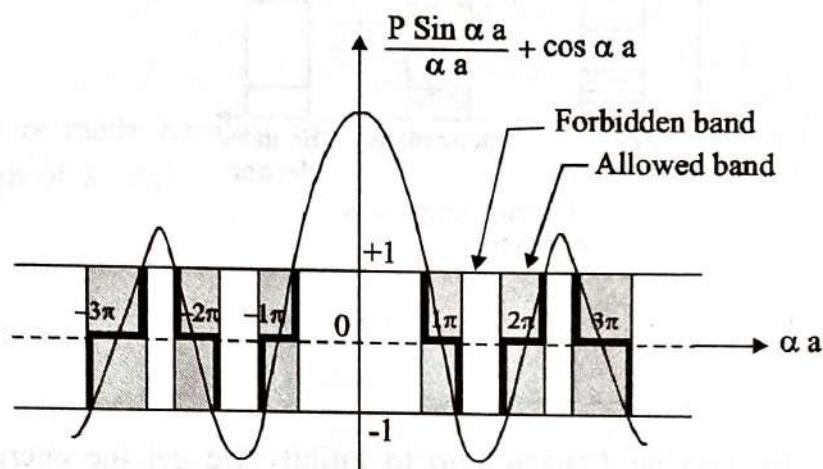


Fig. 7.22

Conclusions

From the *Fig. 7.22* the following conclusions can be made.

- The energy spectrum has a number of allowed energy bands denoted by solid horizontal line separated by forbidden band gaps denoted by dotted lines.
- The width of allowed energy band (shaded portion) increases with the increase in αa .
- When P is increased, the binding energy of the electrons with the lattice points is also increased. Therefore the electron will not be able to move freely and hence the width of the allowed energy band is decreased. Especially for $P \rightarrow \infty$, then allowed energy band becomes infinitely narrow and the energy spectrum becomes a line spectrum as shown in *Fig. 7.23*.
- When P is decreased, the binding energy of the electron decreases and thus it moves freely over the lattice points and hence we get a wide range of allowed energy levels as shown in *Fig. 7.23*.

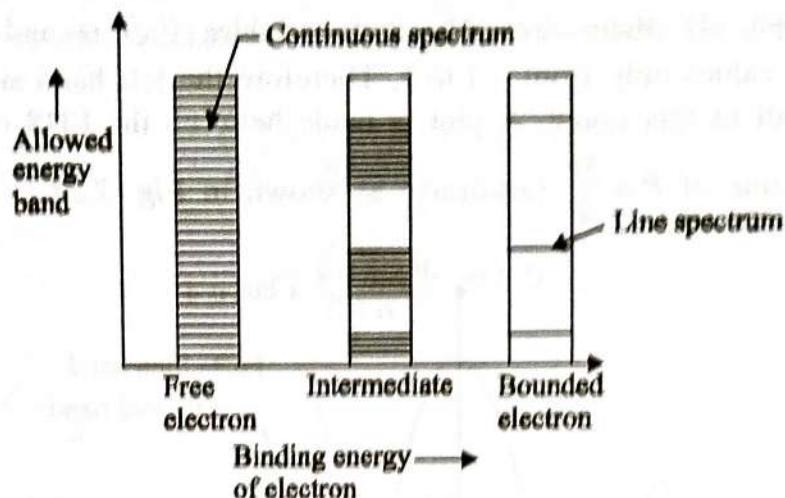


Fig. 7.23

- (v) Thus by varying P from zero to infinity we get the energy spectra of all ranges.

7.14 BRILLOUIN ZONES

Definition: *Brillouin Zones are the boundaries that are marked by the values of propagation wave vector \vec{k} in which the electrons can have allowed energy values without diffraction. Since k is a vector, it has different values along different directions.*

Explanation: The relation between the wave vector and the energy of the electron in a constant potential field can be got as follows.

We know, in a length of potential box ' l ' the energy of the electron in a constant potential field

$$E = \frac{n^2 h^2}{8ml^2} \quad \dots(1)$$

$$\text{Also, we know wave vector } k = \frac{n\pi}{l} \quad \dots(2)$$

$$(\text{or}) \quad l^2 = \frac{n^2 \pi^2}{k^2} \quad \dots(3)$$

Substituting equation (3) in (1), we get

$$E = \frac{n^2 h^2}{8m \left(\frac{n^2 \pi^2}{k^2} \right)}$$

$$E = \frac{k^2 h^2}{8m \pi^2} \quad \dots(4)$$

A plot is made between the total energy 'E' and the wave vector k , for various values of k with $n = \pm 1, \pm 2, \dots$ (since $k = n\pi/l$)

$$\text{(i.e) for } n = \pm 1 \ k = \pm \frac{\pi}{l}$$

$$\text{for } n = \pm 2 \ k = \pm \frac{2\pi}{l}$$

$$\text{for } n = \pm 3 \ k = \pm \frac{3\pi}{l}$$

For the above values of k , the curve is obtained as shown in Fig. 7.24, which is in the form of a parabola with discontinuities.

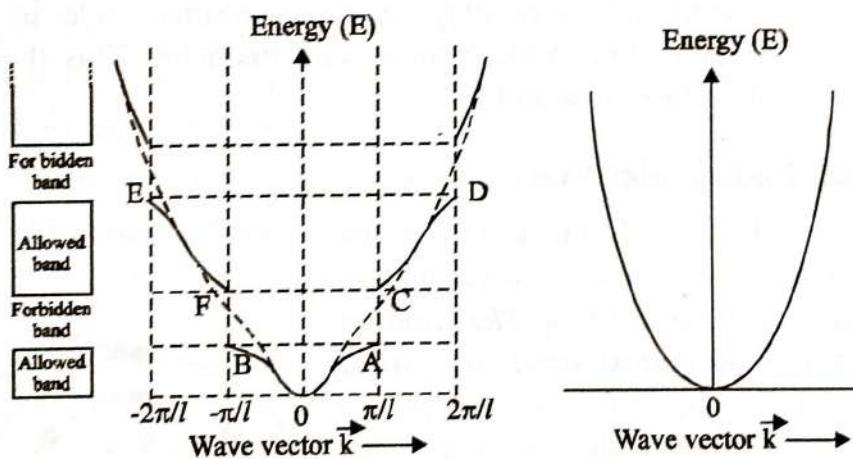


Fig. 7.24



Fig. 7.25

From the figure it can be seen that the energy of the electron increases continuously from zero to $\pm \frac{\pi}{l}$ then the electron meets the wall and is reflected.

This range of allowed energy values in the region between $-\frac{\pi}{l}$ to $\frac{\pi}{l}$ is called First Brillouin Zone.

The second allowed energy values consists of two parts; one from $\frac{\pi}{l}$ to $\frac{2\pi}{l}$ and another form $-\frac{\pi}{l}$ to $-\frac{2\pi}{l}$ is called **Second Brillouin Zone**.

Similarly there will be 3rd, 4th etc, Brillouin Zones for various ranges of k -values.

Also from Fig. 7.24 we can see that each Brillouin Zone is separated by breakup of energy values known as **Forbidden Zone (or) Energy Gap**.

Conclusion

Therefore, we can conclude that the electron can go from one brillouin zone to the other only if it is supplied with an energy equal to forbidden gap energy.

Thus the forbidden gap is the one which decides whether the solid is a conductor (or) insulator (or) semiconductor.

Note: For comparison purpose the same E-K curve for a free electron in a constant potential is given in Fig. 7.25 in which the curve obtained is continuous.

7.15 ENERGY BAND THEORY OF SOLIDS

Energy band theory of solids plays a very important role in determining whether a solid is a conductor, insulator or a semiconductor. This theory explains how an energy band occurs in a solid.

(i) Free and bound electrons

In an isolated atom all the electrons are tightly bounded with the central positive nucleus and revolves around various orbits. **The number of electrons at the outermost orbit are called valence electrons**. In the outermost orbits, the attractive force between the nucleus and electrons will be very less, so that the electrons can be easily detached from the nucleus. **These detached electrons from the outer most orbits are called free electrons**. But as far as the innermost orbits are concerned, the electrons are tightly bounded with positive nucleus, and hence they are termed as **bound electrons**. The free and bounded electrons are shown in Fig. 7.26.

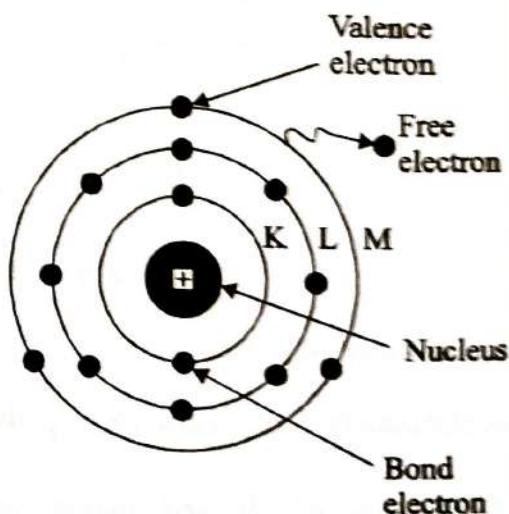


Fig. 7.26

(ii) Energy levels

We know that each orbit of an atom has fixed amount of energy associated with it. The electrons moving in a particular orbit possess the energy of that orbit. The larger the orbit, the greater is its energy. So, the outermost orbit electrons possess more energy than the inner orbit electrons.

A convenient way of representing the energy of different orbits are called **energy levels**, as shown in Fig. 7.27. Let E_1 be the energy level of the K -shell, E_2 be the energy level of the M -shell, E_3 be the energy level of the L -shell and so on. The larger the orbit of an electron, the greater is its energy and higher is the energy level.

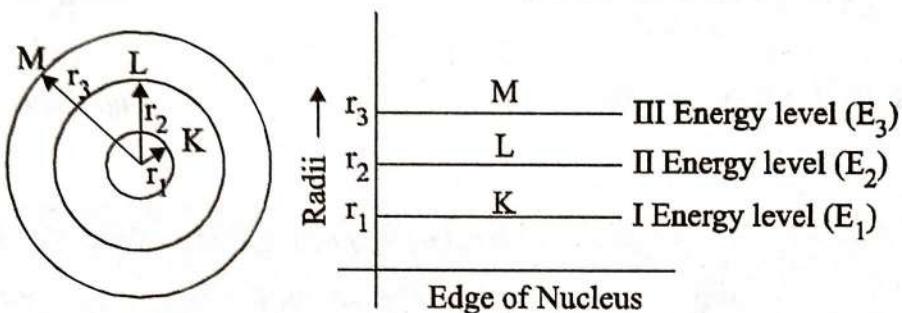


Fig. 7.27

From the Fig. 7.27 it is clear that the electrons can revolve only in certain permitted orbits of radii $r_1, r_2, r_3 \dots$ and not in any arbitrary orbit. Since the electrons are not allowed inbetween the radii r_1 and r_2 or between the radii r_2 and r_3 , etc., there won't be any electronic energy levels inbetween those radii, so called *forbidden radii*. These unallowed energy levels are called *forbidden energy levels*.

(iii) Energy bands

It has to be noted that as long as the atoms are widely separated, they have identical energy levels. But, once the atoms are brought together the interatomic force of attraction between the atoms in the solid may modify the energy levels of a solid as energy bands.

Now let us discuss how energy levels of single free atom becomes energy bands in solids.

Let us consider two identical atoms of diameter(d) separated at a distance (r), so that the electronic energy levels of one atom [E_1^1 (K -shell) and E_2^1 (L -shell)]

do not affect the electronic energy levels of the other atom [E_1^2 (K -shell) and E_2^2 (L -shell)] as shown in Fig. 7.28.

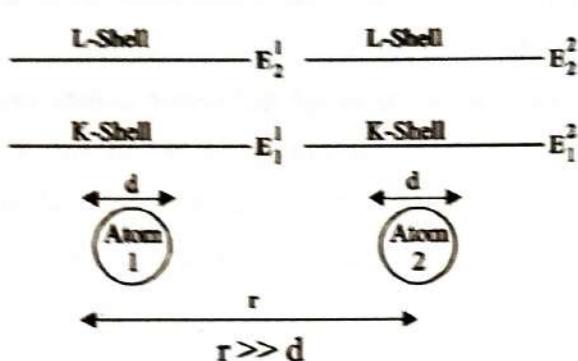


Fig. 7.28

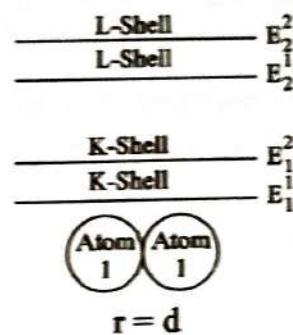


Fig. 7.29

Note: In E_1^1 , E_2^1 , E_1^2 , E_2^2 ..., The subscript represents the Energy levels E_1 , E_2 etc and superscript represents the atom- 1, atom- 2 etc.

Now, when we bring the atoms closer together, some force of attraction occurs between them and according to Quantum mechanics, their wave functions will start overlapping. Therefore *when two atoms are brought closer, it does not remain as two independent atoms, rather it forms a single two-atom system with two different energy levels to form energy band* as shown in Fig. 7.29.

7.16 ORIGIN OF ENERGY BAND FORMATION IN SOLIDS

We know that when two atoms of equal energy levels are brought closer together, the original energy levels viz E_1 and E_2 splits each into two energy levels i.e., the K - shell energy E_1 splits into E_1^1 and E_1^2 , similarly the L - shell energy E_2 splits into E_2^1 and E_2^2 (Fig. 7.28 and Fig. 7.29). Now, when three atoms are brought closer together, the original energy levels viz E_1 and E_2 splits each into three energy levels viz E_1^1 , E_1^2 , E_1^3 and E_2^1 , E_2^2 , E_2^3 respectively as shown in Fig. 7.30. These type of transformations from the original energy levels into two (or) more energy levels is known as **Energy Level Splitting**.

Energy Bands

Therefore, if 'N' number of atoms of equal energy levels are brought closer to form a solid, then it forms a closely spaced continuous energy levels, so called **energy bands** as shown in Fig. 7.31.

Hence an energy band can be defined as, the range of energies possessed by an electron in a solid. The magnified view of the energy band which consists of a large number of very closely spaced energy levels is shown in Fig. 7.32.

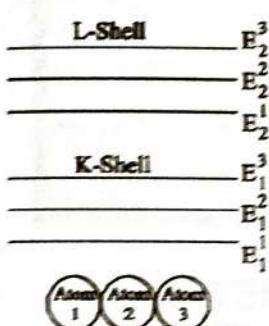


Fig. 7.30

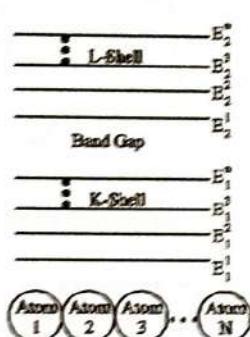


Fig. 7.31

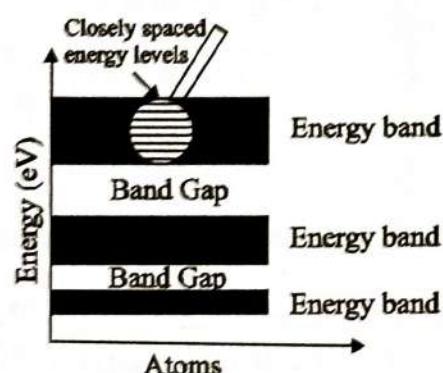


Fig. 7.32

Note: The overall range of energies from the lowest energy level to the highest energy level of a band is called the width of band. From Fig. 7.32, we can also see that the width of band increases gradually due to the increase of electrons from the inner most orbit towards the outer most orbit.

Inner Filled Bands, Valence Band and Conduction Band

During the formation of energy bands, the inner filled energy levels forms a energy band, called **Inner filled bands**.

Similarly the electrons in the outermost shells of atoms forms a energy band, called **valence band**. The valence band will be of completely filled (or) partially filled with electrons, based on the type of the materials.

If an electron comes out from valence band for conduction, then they form an energy level corresponding to the energy band called **conduction band** as shown in Fig. 7.33.

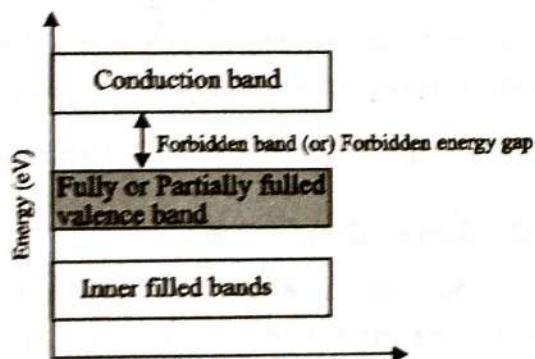


Fig. 7.33

Forbidden Band

While referring to energy bands, they are separated by small regions which does, not allow any energy levels. Such regions between the energy bands are called **forbidden bands** (or) **forbidden energy gap** E_g as shown in *Fig. 7.33.*

Note: Though there are number of energy bands in solids, valence band (The range of energies possessed by valence electrons) and conduction band (The range of energies possessed by conduction electrons) are particularly important, because these valence electrons (or) so called free electrons are capable of electrical conduction, when it moves from valence band to conduction band.

7.17 CLASSIFICATION OF MATERIALS INTO METALS, SEMICONDUCTORS AND INSULATORS ON THE BASIS OF BAND THEORY

Based on band theory, and on the basis of the presence of forbidden band gap the materials are classified into three categories, viz.

- (i) Metals (or) Conductors,
- (ii) Semi-conductors and
- (iii) Insulators.

(i) Metals (or) Conductors

In conductors, there is **no forbidden band gap**. Here the valence and conduction band overlap each other as shown in *Fig. 7.34*. In metals the availability of free electrons will be very high due to the overlapping of conduction band and valence band. Hence, even when a small external field is applied to it, the electrons from valence band freely enters into the conduction band and produces current.

Examples: Copper, Aluminium, Iron etc.,

(ii) Semi Conductors

In semiconductors, **the forbidden band gap is very small**, in the order 0.5 eV to 1.5 eV and hence there will be a very small gap between the valence and conduction band as shown in *Fig. 7.35.*

Examples: Germanium, Silicon etc.,

Generally, in semiconductors the availability of free electrons in conduction band will be less compared to metals, due to the presence of forbidden band gap between the valence band and conduction band. Therefore, when external field of energy, equal to (or) greater than forbidden band gap energy (E_g) is applied to a semiconductor, immediately the conduction will take place.

(iii) Insulators

In Insulators, the *forbidden gap is very wide*, in the range of 3 eV to 5.47 eV and hence there will be a very large gap between the valence band and conduction band as shown in Fig. 7.36. Since the forbidden band gap energy is very very high in the case of insulators, very large amount of energy (external field) is required for conduction to occur.

Examples: Diamond, Dielectrics etc.,

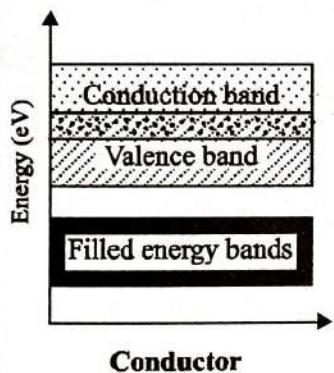


Fig. 7.34

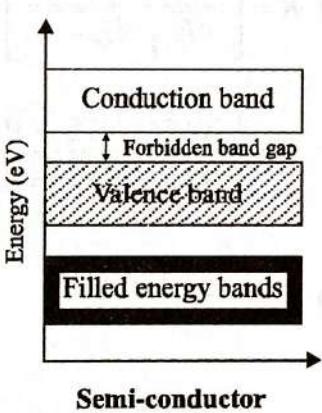


Fig. 7.35

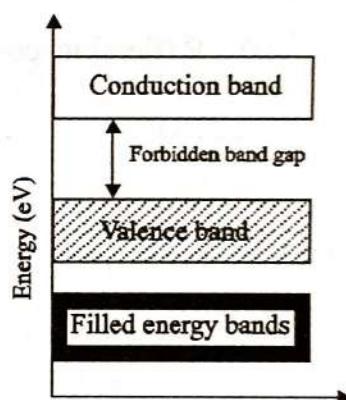


Fig. 7.36

IMPORTANT SOLVED PROBLEMS

1. Calculate the reflection and transmission co-efficient of an electron of energy 15 eV incident on a potential barrier of height 10 eV.

Solution

Given data

$$E = 15 \text{ eV}$$

$$V = 10 \text{ eV}$$

Formula

$$(i) \text{ Reflection co-efficient } R = \left[\frac{\sqrt{E} - \sqrt{E - V}}{\sqrt{E} + \sqrt{E - V}} \right]^2$$

$$\therefore R = \left[\frac{\sqrt{15} - \sqrt{15 - 10}}{\sqrt{15} + \sqrt{15 - 10}} \right]^2$$

$$(\text{or}) \quad R = \left[\frac{\sqrt{15} - \sqrt{5}}{\sqrt{15} + \sqrt{5}} \right]^2$$

$$(\text{or}) \quad R = \left[\frac{3.873 - 2.236}{3.873 + 2.236} \right]^2$$

$$\therefore R = 0.072$$

$$(ii) \text{ Transmission co-efficient } T = 1 - R$$

$$\therefore T = 1 - 0.072$$

$$\therefore T = 0.928$$

**∴ The Reflection co-efficient $R = 0.072$
& The transmission co-efficient $T = 0.928$**

2. Calculate the reflection co-efficient for a particle of kinetic energy 9 eV incident on a potential step of height 5 eV.

Solution

Given data

$$E = 9 \text{ eV}$$

$$V = 5 \text{ eV}$$

Formula

$$\begin{aligned} \text{Reflection co-efficient } R &= \left[\frac{\sqrt{E} - \sqrt{E-V}}{\sqrt{E} + \sqrt{E-V}} \right]^2 \\ \therefore R &= \left[\frac{\sqrt{9} - \sqrt{9-5}}{\sqrt{9} + \sqrt{9+5}} \right]^2 \\ \therefore R &= \left[\frac{1}{5} \right]^2 \\ \therefore R &= \frac{1}{25} \end{aligned}$$

$$(\text{or}) \quad R = 0.04$$

∴ Reflection co-efficient $R = 0.04$

3. Show that the sum of reflection and transmission co-efficient is one for a particle scattered by a potential step.

Solution

$$\text{We know Reflection co-efficient } R = \left[\frac{\sqrt{E} - \sqrt{E-V}}{\sqrt{E} + \sqrt{E-V}} \right]^2$$

$$\text{and Transmission co-efficient } T = \frac{4\sqrt{E}\sqrt{E-V}}{[\sqrt{E} + \sqrt{E-V}]^2}$$

If $k = \sqrt{E}$ and $k' = \sqrt{E - V}$, we can write

$$\text{Reflection co-efficient } R = \left[\frac{k - k'}{k + k'} \right]^2$$

$$\text{and Transmission co-efficient } T = \frac{4kk'}{[k + k']^2}$$

(i) For $E > V$

$$R + T = \frac{[k - k']^2}{[k + k']^2} + \frac{4kk'}{[k + k']^2}$$

$$\text{(or)} \quad R + T = \frac{k^2 + k'^2 - 2kk' + 4kk'}{[k + k']^2}$$

$$\text{(or)} \quad R + T = \frac{k^2 + k'^2 + 2kk'}{[k + k']^2}$$

$$\text{(or)} \quad R + T = \frac{[k + k']^2}{[k + k']^2} = 1$$

$$\therefore R + T = 1$$

(ii) For $E < V$

Here for $E < V$, transmission will not be there

$$\therefore T = 0$$

$$\therefore R + T = 1 + 0$$

$$\therefore R + T = 1$$

ASSIGNMENT PROBLEMS

- Calculate the probability of transmission that a 1 eV electron will penetrate a potential barrier of 4 eV, when the width of the barrier is 2 Å.
(Ans: $T = 0.084$ (or) $T = 8.4\%$)
- A particle of energy E is incident on a potential step of barrier height V . What should be the ratio E/V so that the probability of transmission is 50%?

$$\text{(Ans: } \frac{E}{V} = 1.03)$$

Quantum Tunneling

Quantum tunneling is a phenomenon in which the particles penetrate through a potential energy barrier with a height greater than the total energy of the particles. In quantum tunneling, the particles will have a finite probability to cross an energy barrier, so that as the energy needed to break a bond with another particle is attained, even though the particle's energy is less than the energy barrier.

Example: The emission of alpha rays in radioactive decay is an example for quantum tunneling.

2. Give a note on the origin of energy bands.

Origin of Energy Bands

Different substances will have different band structures which determines the characteristics of that substance in terms of electrical conduction.

During the formation of energy bands, the inner filled energy levels forms a energy band, called **Inner filled bands**.

Similarly the electrons in the outermost shells of atoms forms a energy band, called **valence band**. The valence band will be of completely filled (or) partially filled with electrons, based on the type of the materials.

If an electron comes out from valence band for conduction, then they form an energy level corresponding to the energy band called **conduction band**.

For a metal (conductor) the conduction and valence band overlap, and so the additional energy is more enough for the free electrons to cause current.

For insulators there is often a wide band gap between the conduction band and valence band, and so a huge amount of energy is required to cause the electron to be freed and accelerated as current.

For semiconductors there is still some band gap, however it is lot smaller and hence it act as insulator at 0 K and as conductor above 0 K.

ADDITIONAL PART-A QUESTIONS & ANSWERS

1. List out the outcome (or) Characteristics of the particle executing simple harmonic motion in a harmonic oscillator.
 1. The particles executing SHM will have discrete energy values.
 2. The energy values are equidistant and are separated by $\hbar\nu$.
 3. The minimum energy (for $n=0$) is not zero.
 4. Energy levels are Non-degenerate.

2. Define zero point energy of a harmonic oscillator.

The energy of the harmonic oscillator is quantised in steps of $\hbar\nu$

$$\therefore E_n = \left[n + \frac{1}{2} \right] \hbar\nu$$

Where $n = 0, 1, 2, 3 \dots$

When $n=0$, the above equation becomes

$$E_0 = \frac{1}{2} \hbar\nu$$

Here E_0 represent the lowest energy value (or) zero point energy of the harmonic oscillator.

3. Give the energy eigen values and Hermite polynomials for a harmonic oscillator.

The energy eigen values (E_n) for $n = 0, 1, 2, 3$ and the first four hermite polynomials [$H_n(y)$] is given in table below.

n	E_n	$H_n(y)$
0	$\frac{1}{2} h\nu$	1
1	$\frac{3}{2} h\nu$	$2y$
2	$\frac{5}{2} h\nu$	$4y^2 - 2$
3	$\frac{7}{2} h\nu$	$8y^3 - 12y$

4. Compare the energy values for a particle in a infinite potential well and the particle in a harmonic oscillator.

S.No.	Particle in a 1D Potential Well	Harmonic oscillator
1.	Energy eigen value is $E_n = \frac{n^2 h^2}{8ml^2}$	1. Energy eigen value is $E_n = \left(n + \frac{1}{2} \right) h\nu$
2.	The minimum energy (for $n = 0$) is zero.	2. The minimum energy (for $n = 0$) is not zero.
3.	The energy values are not at equal distance	3. The energy values are at equal distances
4.	Energy levels are as shown in Fig. 7(a)	4. Energy levels are as shown in Fig. 7(b)

Fig. 7(a)

Fig. 7(b)

5. What do you understand by the term Transmission and reflection co-efficient?

Transmission Co-efficient

We know that the probability density is the square of the amplitude of that function. Therefore the barrier transmission co-efficient (T) is the ratio between the square of the amplitudes of the transmitted wave $|C|^2$ and the incident wave $|A|^2$.

$$\therefore \text{The transmission co-efficient } T = \frac{|C|^2}{|A|^2} \quad \dots(15)$$

The above equation is also called as the "Penetrability" of the barrier.

Reflection Coefficient

The reflection co-efficient (R) for the barrier surface at $x=0$ is the ratio between the square of the amplitude of the reflected wave $|B|^2$ and the square of the amplitude of the incident wave $|A|^2$

$$\therefore \text{The reflection co-efficient } R = \frac{|B|^2}{|A|^2} \quad \dots(16)$$

6. What is meant by quantum tunnelling (or) tunnelling effect?

In quantum mechanics a particle having lesser energy (E) than the barrier potential (V) can easily cross over the potential barrier having a finite width ' T ' even without climbing over the barrier by tunnelling through the barrier. This process is called Tunnelling.

7. Mention any four occurrences of tunnelling effect.

1. The tunnelling effect is observed in Josephson junction, in which electron pairs in the super conductors tunnel through the barrier layer, giving rise to Josephson current.
2. This effect is also observed in the case of emission of alpha particles by radioactive nuclei.
Here, though the ' α ' particle has very less kinetic energy they are able to escape from the nucleus whose potential wall is around 25 MeV high.
3. Tunnelling also occurs in certain semiconductor diodes called resonant tunnelling diodes.
4. Electron tunnels through insulating layer and act as a switch by tunnelling effect.

8. What is the principle used in scanning tunnelling microscope?

The basic principle used in scanning tunnelling microscope (STM) is the tunnelling of electron between the sharp metallic tip of the probe and the surface of a sample.

Here, constant tunnelling current is maintained by adjusting the distance between the tip and the sample, with an air gap for electron to tunnel. In a similar way the tip is used to scan atom by atom and line by line of the sample and the topography of the sample is recorded in the computer.

9. What are the advantages and disadvantages of STM?

Advantages

1. It can scan, the positions & topography atom by atom (or) even electrons.
2. It is the Latest techniques used in Research laboratories for scanning the materials.
3. Very accurate measurement shall be obtained.
4. Magnification is upto nano-scale.

Disadvantages

1. Even a very small sound (or) vibrations will disturb the measurement setup.
2. It should be kept in vacuum, as even a single dust particle may damage the tip of the probe
3. Cost is high.
4. More complexity.

10. Given any two applications of STM.

1. It is used to produce Integrated circuit.
2. It is used in Biomedical devices.
3. Research labs are the major areas in which it is used.
4. They are used in material science studies for both bump and flat surfaces.

11. What is the principle used in Resonant tunnelling diode.

Resonant resonant tunnelling diode works on the principle of tunnelling effect, in which the charge carriers cross the energy barrier even with lesser energy than the barrier potential, Quantum mechanically. The probability of tunnelling increases with the decreasing barrier energy.

12. Draw the symbol and write the theory behind resonant tunnelling diode.

The symbol of a resonant tunnelling diode is as shown in Fig. 7(c)

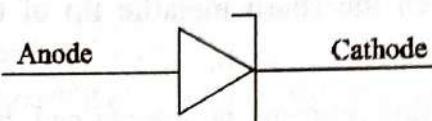


Fig. 7(c)

A resonant tunnelling diode also called as Essaki diode is formed using *p* and *n*-materials, with heavy doping say 1000 times larger than the conventional *p*–*n*-junction diode. Due to heavy doping, the barrier potential decreases drastically, in turn will help the charge carriers to easily tunnel the junctions, quantum mechanically.

13. When a resonant tunnelling diode behaves as a normal diode?

When a resonant tunnelling diode (or) resonant diode is forward biased, the current increases and then decreases at a voltage called valley voltage point.

Now, when the applied voltage is further increased *beyond the valley-point voltage, the resonant tunnelling diode behaves as a normal diode.*

14. List out the current components in a resonant diode.

The current in resonant diode is due to 3 components *viz.,*

- (i) Tunnelling current (I_T)
- (ii) Diode current (I_D)
- and (iii) Excess current (I_E)

∴ Total current

$$I_{\text{Total}} = I_T + I_D + I_E$$

15. What is meant by Peak Current and Valley Current?

When the forward biasing voltage is increased the electrons from the *n*-region tunnel into the potential barrier and reaches the *p*-region.

Therefore the current increases rapidly due to tunnelling effect and reaches the peak-point ‘p’ as shown in Fig. 7(d) and this current, is called peak current (I_p). The voltage at which the diode reaches peak current is called peak voltage (V_p).

Now when the applied voltage is further increased, the tunnelling effect decreases as most of the electrons would have exhausted in tunnelling process. Therefore the current decreases and reaches the valley point ‘V’ as shown in Fig. 7(d).

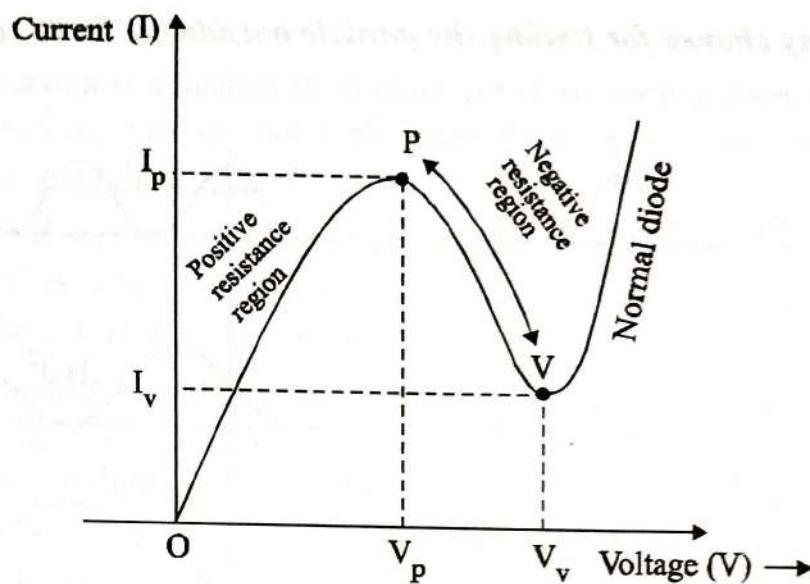


Fig. 7(d)

This minimum current is called valley current (I_v) and the corresponding voltage is called valley voltage (V_v).

16. What are the advantages and disadvantages of a resonant diode.

Advantages

1. Cost and noise is low.
2. Fabrication is very simple.
3. Operation speed is very high.
4. Power dissipation is low and hence it is environmental friendly device.

Disadvantages

1. Since it is a two terminal device, it is difficult to isolate the input and output.
2. It is a low output swing device.

17. What are the applications of resonant tunnelling diode.

1. As resonant tunnelling diode has both positive resistance [From point O to P] and negative resistance [From point P to V], it has many applications in the switching devices.
2. It can be used as normal diodes also.
3. They are used as high frequency microwave oscillators.
4. When resonant tunnelling diode is operated under negative resistance region, then it can be used as an oscillator (or) a switch.

18. Is there any chance for finding the particle outside the finite square potential well?

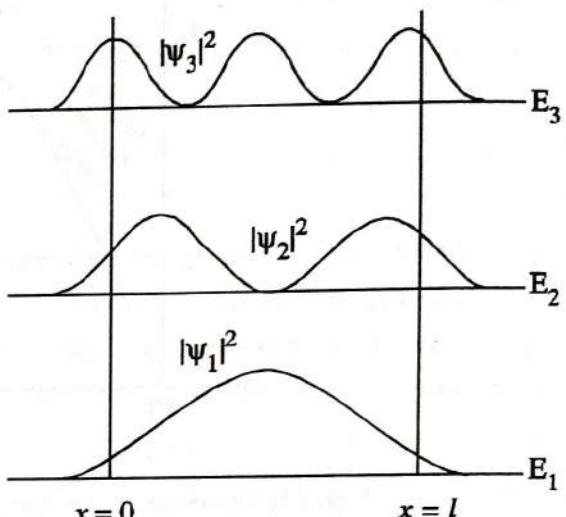
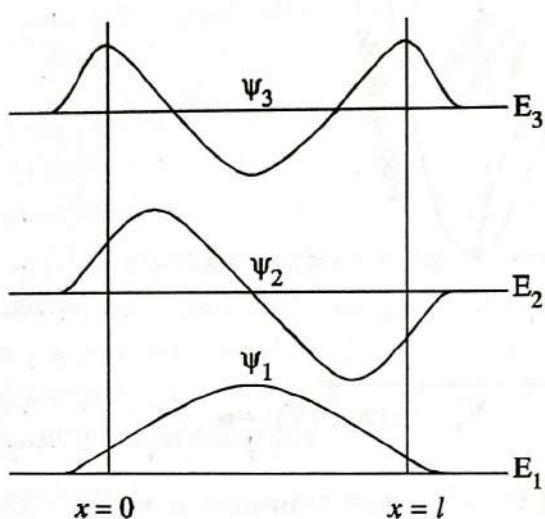


Fig. 7(e)

Yes, from the above figures fig. 7(e) we can see that when a particle is enclosed in a finite potential well, then the probability densities of the wave functions shows that there *are some chances for finding the particle outside the well.*

19. What are the differences between quantum theory and zone theory?

S.No.	Quantum theory	Zone theory
1.	Here the electron is assumed to move in a region of constant potential.	In zone theory the electron is assumed to move in a region of periodic potential
2.	According to this theory the mass of the electron remains constant, when it moves through constant potential	Accordingly to zone theory the mass of the electron varies when it moves through periodic potential and is called effective mass of an electron.
3.	If fails to explain why some solids behaves as conductors, some behave as insulators and semi conductors.	It explains the behaviour of solids as conductors, semiconductors and insulators

20. State Bloch's Theorem.

Bloch theorem is a mathematical statement of an electron (Particle) wave function moving in a perfectly periodic potential. These functions are called bloch functions.

21. What are Brillouin Zones?

Brillouin Zones are the boundaries that are marked by the values of propagation wave vector \vec{k} in which the electrons can have allowed energy values without diffraction. Since k is a vector, it has different values along different directions.

22. What is meant by valence band, conduction band and forbidden band gap?

During the formation of energy bands, the inner filled energy levels forms a energy band, called **Inner filled bands**.

Similarly the electrons in the outermost shells of atoms forms a energy band, called **valence band**. The valence band will be of completely filled (or) partially filled with electrons, based on the type of the materials.

If an electron comes out from valence band for conduction, then they form an energy level corresponding to the energy band called **conduction band** as shown in Fig. 7(f).

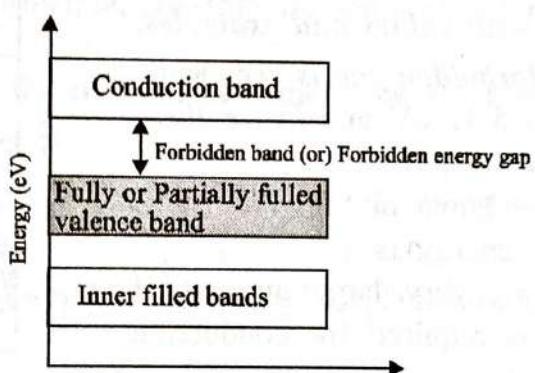


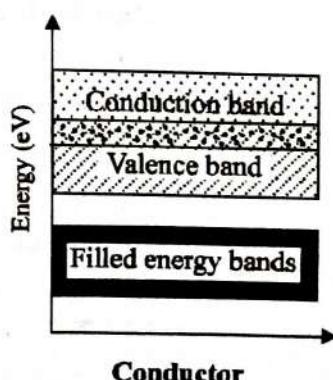
Fig. 7(f)

23. With examples, discuss the energy band diagram of a conductor.

In conductors, there is **no forbidden band gap**.

Here the valence and conduction band overlap each other as shown in Fig. 7(g). In metals the availability of free electrons will be very high due to the overlapping of conduction band and valence band. Hence, even when a small external field is applied to it, the electrons from valence band freely enters into the conduction band and produces current.

Examples: Copper, Aluminium, Iron etc.,



Conductor
Fig. 7(g)

- 24. Draw and discuss about the energy band diagram of semiconductor? Give examples.**

In semiconductors, the **forbidden band gap is very small, in the order 0.5 eV to 1.5 eV** and hence there will be a very small gap between the valence and conduction band as shown in Fig. 7(h).

Examples: Germanium, Silicon etc.,

Generally, in semiconductors the availability of free electrons in conduction band will be less compared to metals, due to the presence of forbidden band gap between the valence band and conduction band. Therefore, when external field of energy, equal to (or) greater than forbidden band gap energy (E_g) is applied to a semiconductor, immediately the conduction will take place.

- 25. Whether insulator will have small (or) large band gap? Justify with values and examples.**

In Insulators, the **forbidden gap is very wide**, in the range of 3 eV to 5.47 eV and hence there will be a very large gap between the valence band and conduction band as shown in Fig. 7(i). Since the forbidden band gap energy is very very high in the case of insulators, very large amount of energy (external field) is required for conduction to occur.

Examples: Diamond, Dielectrics etc.,

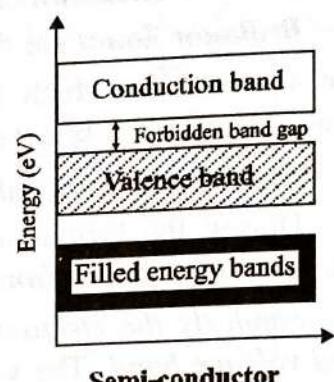


Fig. 7(h)

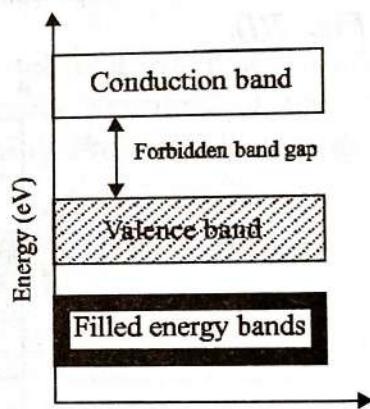


Fig. 7(i)

ANNA UNIVERSITY PART-B QUESTIONS

- (i) Write short notes on tunnelling effect.
 (ii) Describe the principle, construction and working of scanning tunnelling microscope. (April 2019)
- Describe the principle and working of a resonant tunnelling diode. (April 2020)
- Explain the origin of band gap when the electron is moving in a periodic potential. Also explain the effective mass of electron in a periodic potential. (EI April 2002)

4. Discuss qualitatively how band theory of solids leads to the classification of solids into conductors, semi conductors and insulators. (ECE April 2002)
5. Explain the concepts of Barrier penetration and quantum tunneling in detail with necessary sketch. (Jan. 2022)
6. Write a brief note on Bloch's theorem for particles in a periodic potential and Kronig Penney model. (Jan. 2022)

ADDITIONAL PART B QUESTIONS

1. Obtain the energy values and eigen functions of a harmonic oscillator.
2. Compare the energy values of a particle in an infinite potential well and a harmonic oscillator.
3. Describe the barrier penetration process and quantum tunnelling of an electron.
4. Derive the transmission and reflection coefficients for a particle which undergoes quantum tunnelling.
5. (i) Describe the principle, construction and working of a scanning tunnelling microscope.
 (ii) List some of the merits and demerits of STM with applications.
6. (i) With necessary theory, describe the quantum structure and V-I characteristics of a resonant tunnelling diode along with its applications.
 (ii) Give the advantages and disadvantages of a resonant tunnelling diode.
7. What is meant by quantum tunnelling? Explain how this principle is used in resonant diodes?
8. Derive the Eigen values for a particle in a finite square potential well and show that they are quantised, using graphical methods.
9. Discuss Bloch Theorem and prove the same for a particle in a periodic potential.
10. Explain the behaviour of an electron moving in a field of periodic potential using Kronig and Penny model.
11. Explain zone theory and get the relation between wave vector and the energy of the electron in a constant potential field.
12. Describe the energy band theory of solids and the origin of band formation in solids.
13. Write notes on
 (i) Energy levels (ii) Energy bands (iii) Bound and free electrons
14. Describe how zone theory is used to classify the solids as conductors, semi conductors and insulators.

