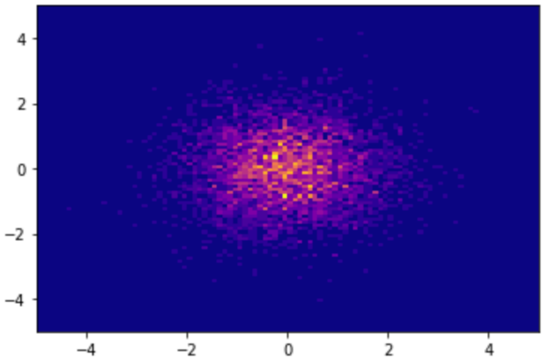
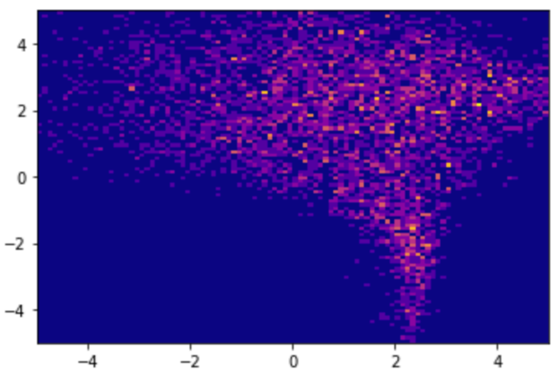
Run all the blocks of "Task 1" notebook for the results of Task 1
Run all the blocks of "Task 2" notebook for the results of Task 2 and Extra Credits Part

# Task 1

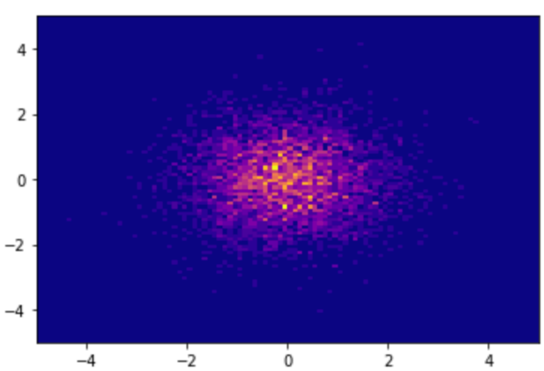
For the random samples, given the distribution:




Generated the following distribution on Forward flow:



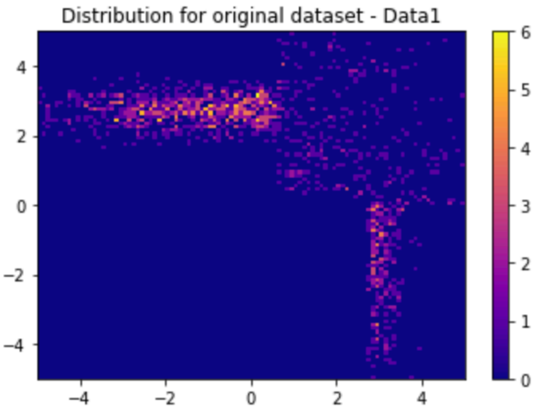

Inverse Flow gave back the original distribution:



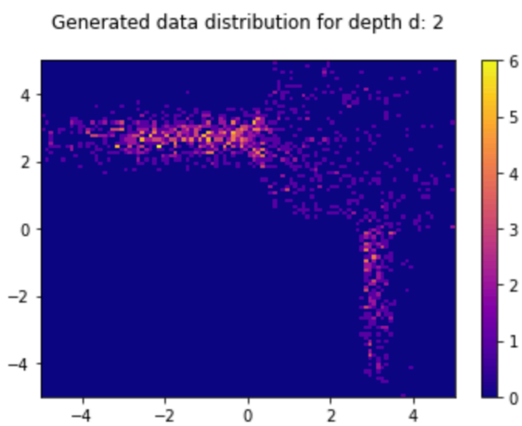
# Task 2

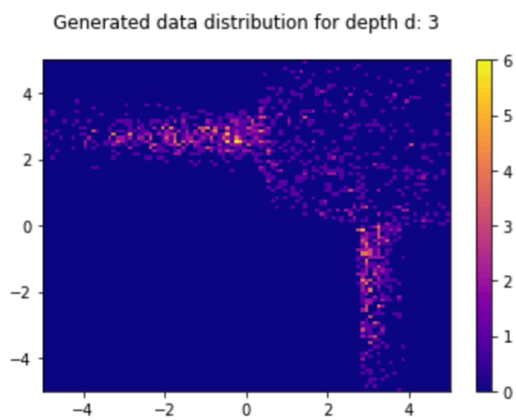
## Results on Data1 dataset:

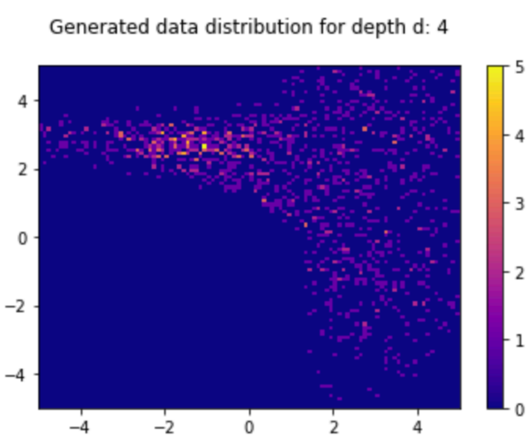
Distribution for original dataset:

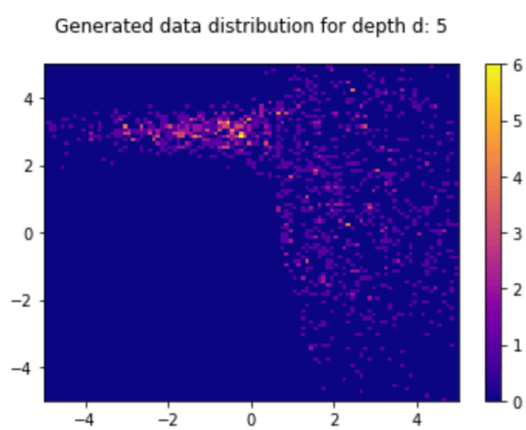



Following are the distributions generated for different depth d:



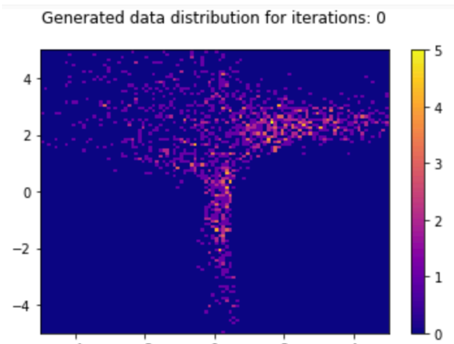



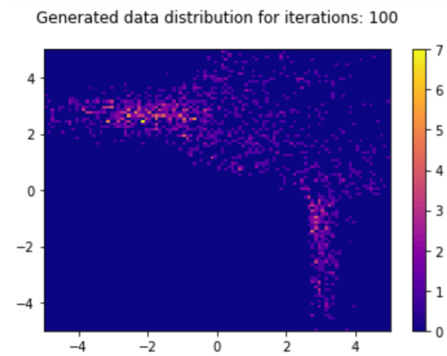



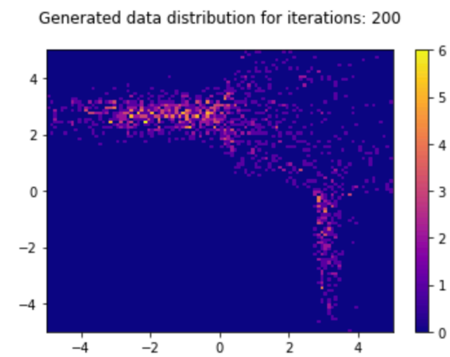



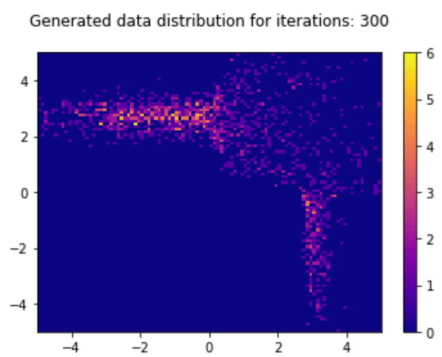
From the above results we can see that, for depth d = 2, 3, the distributions are more similar to the original distribution. And further increase in depth, leads to overfitting the dataset. And also from the experiments with dataset data2 shown in the report, we can see that similarity of distribution obtained depends on random seed used for initialization.

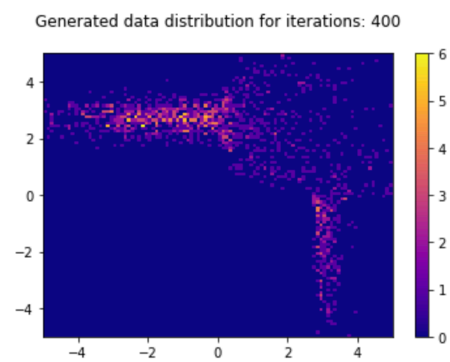
By picking depth d = 2, we perform the training on different iterations and obtain the following results:

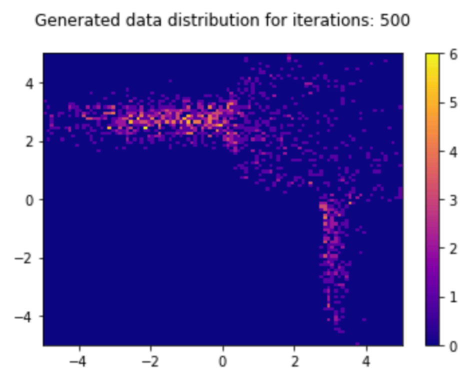


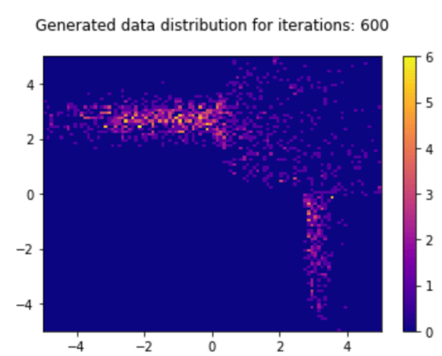



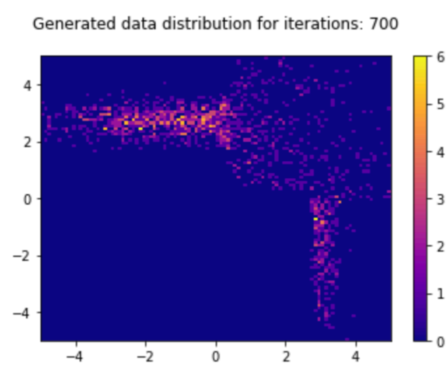



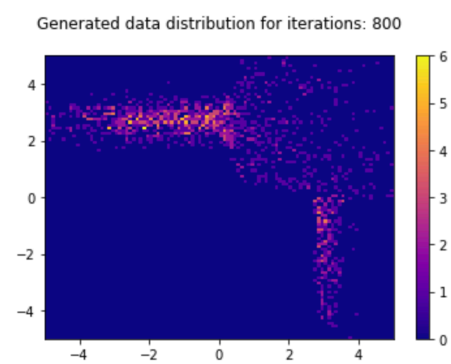



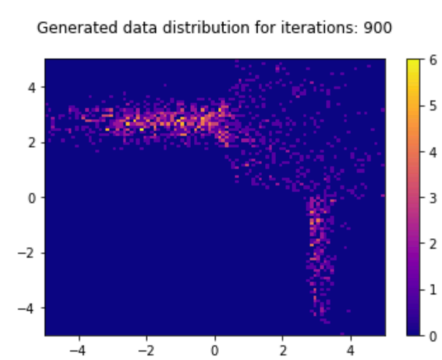



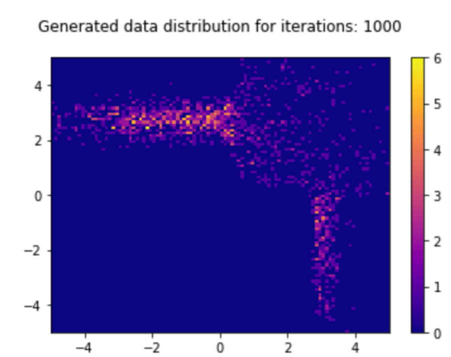



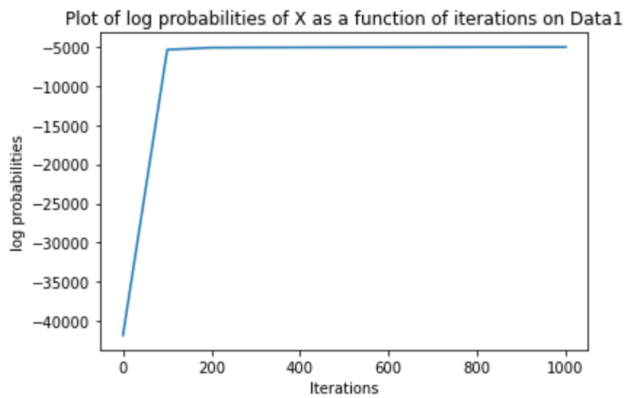



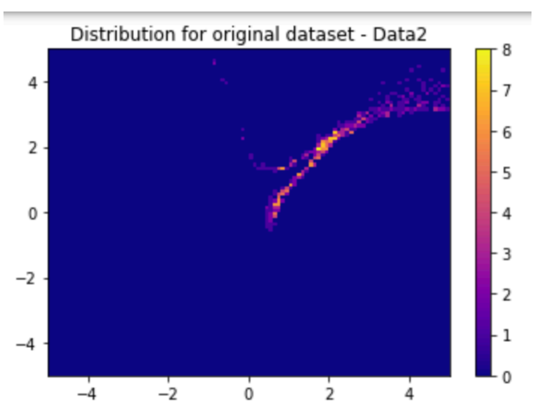
From the above figures we can see that the approximate distribution improves with increase in number of iterations. Below is the plot of log probability of data X as a function of iterations. We can see that the log probability of data increases with the training.

```
log probabilities on Data1:
[tensor(-41779.7227), tensor(-5407.3677), tensor(-5153.8540), tensor(-5136.9263), tensor(-5123.6685), tensor(-5113.
0664), tensor(-5104.3857), tensor(-5097.1304), tensor(-5090.9668), tensor(-5085.6641), tensor(-5081.0547)]
```

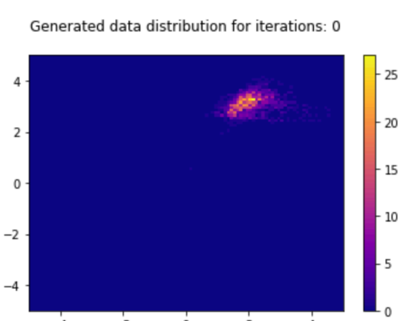


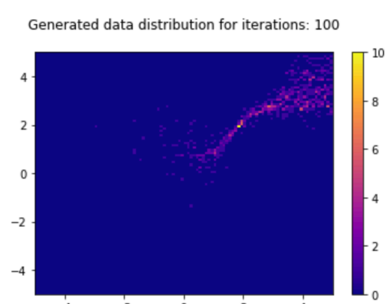

# Results on Data2 dataset:

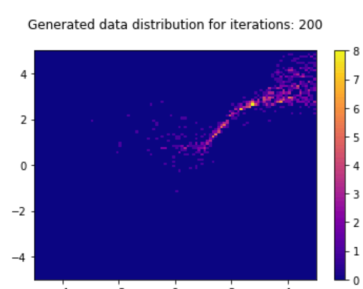
Distribution for original dataset:

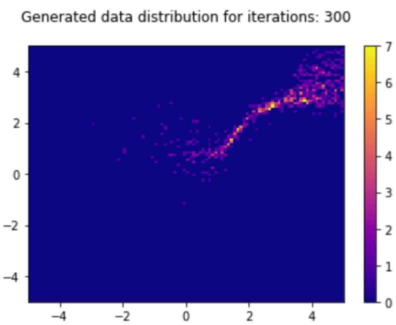


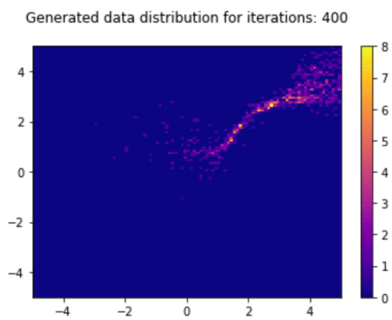

By picking depth d = 5 and seed = 0 for initialization, we perform the training on different iterations and obtain the following results:

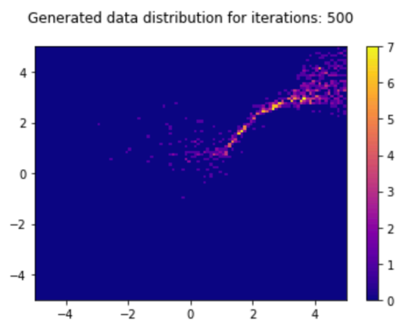


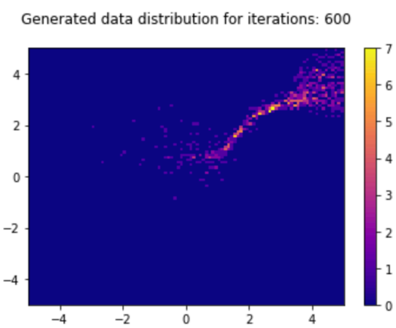



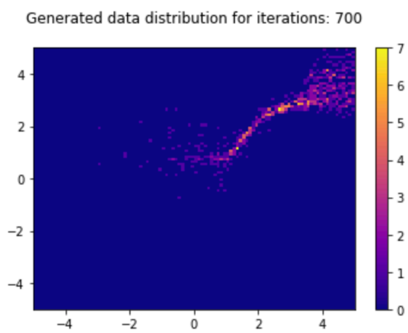



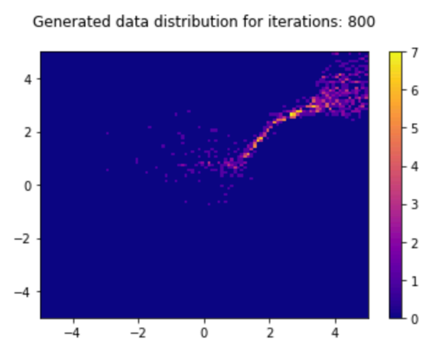


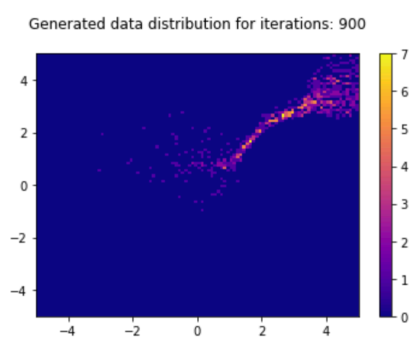



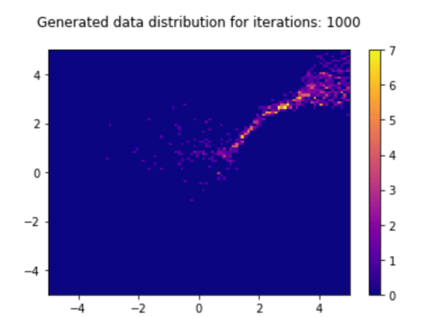



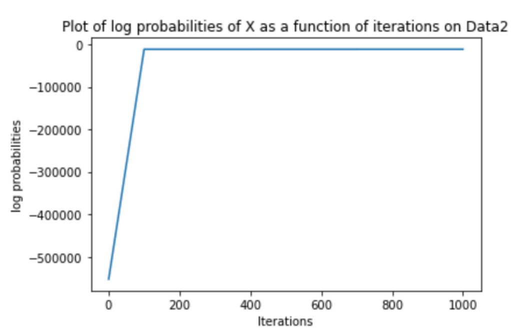



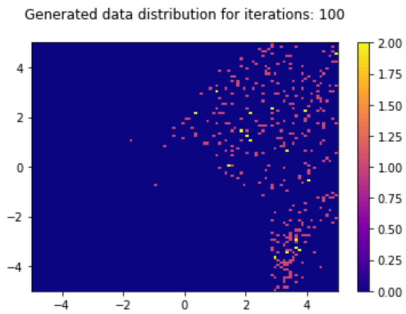



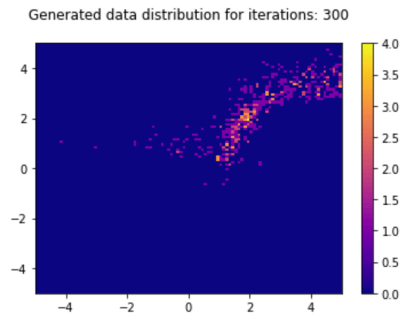

From the above figures we can see that the approximate distribution improves with increase in number of iterations. Below is the plot of log probability of data X as a function of iterations. We can see that the log probability of data increases with the training.

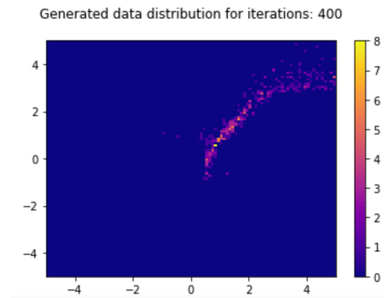
```
log probabilities on Data2:
[tensor(-552505.4375), tensor(-11608.2617), tensor(-11590.4629), tensor(-11584.8848), tensor(-11580.2969), tensor(-
11575.6934), tensor(-11570.8301), tensor(-11569.2666), tensor(-11558.5947), tensor(-11553.1562), tensor(-11543.3604
)]
```

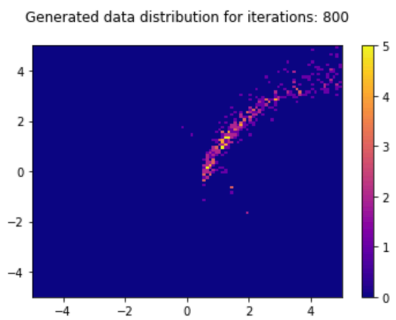


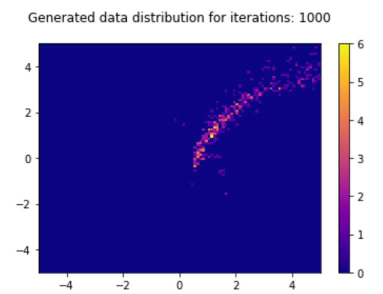
For seed = 325, following results were obtained:



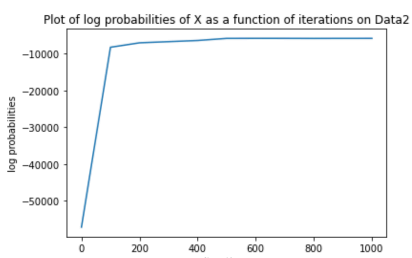



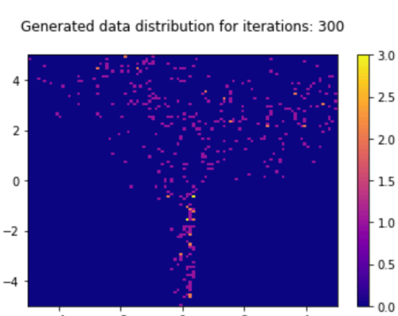



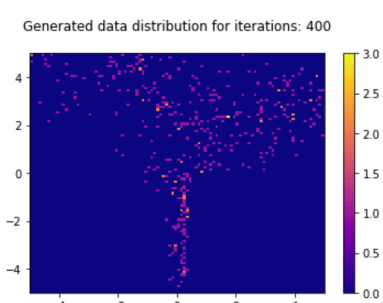



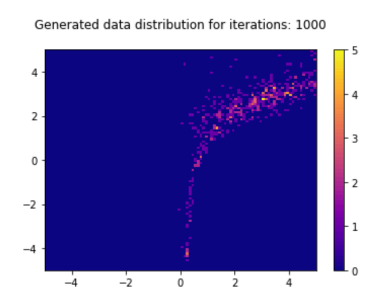

```
log probabilities on Data2:
[tensor(-57248.1094), tensor(-8198.1709), tensor(-6991.9932), tensor(-6677.4756), tensor(-6360.1562), tensor(-5782.
6221), tensor(-5770.4980), tensor(-5773.7212), tensor(-5794.9971), tensor(-5779.9375), tensor(-5776.4067)]
```



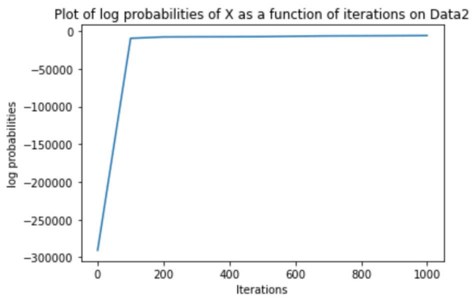

For seed = 4567, following results were obtained:



```
log probabilities on Data2:
[tensor(-290547.5938), tensor(-9666.3174), tensor(-7845.8848), tensor(-7697.9399), tensor(-7616.9526), tensor(-7471
.1357), tensor(-6921.5986), tensor(-6498.4639), tensor(-6381.4121), tensor(-6259.9199), tensor(-6032.8013)]
```



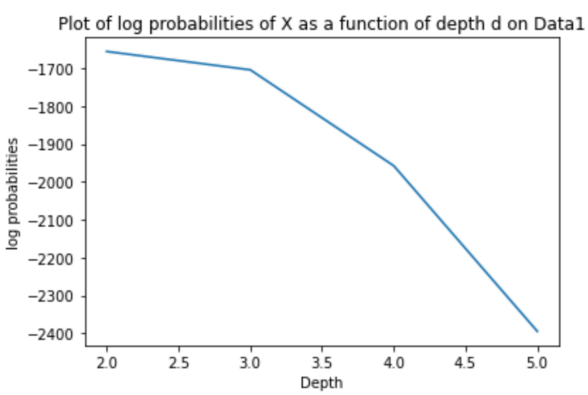

From the above experimentation, we can see that the similarities to distribution to the original distribution depends on seeds during initialization of parameters. Produces different similarities for different seeds.

# Extra Fun and Credits:

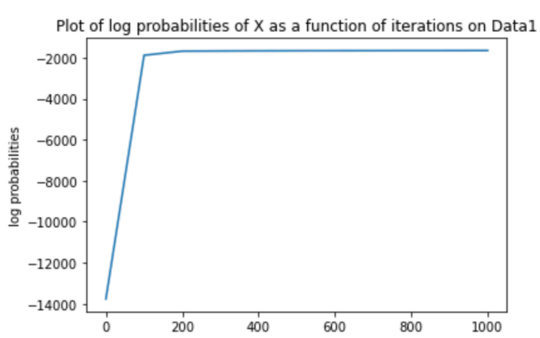
Dataset was split into train and test data, 2/3 of the dataset is used as training data and 1/3rd as test data. Log Probabilities were calculated on test data for various depth and iterations.

Results on Data1 Dataset:

```
log probabilities on Data1:
[tensor(-1655.6016), tensor(-1704.0582), tensor(-1957.4718), tensor(-2394.4875)]
```



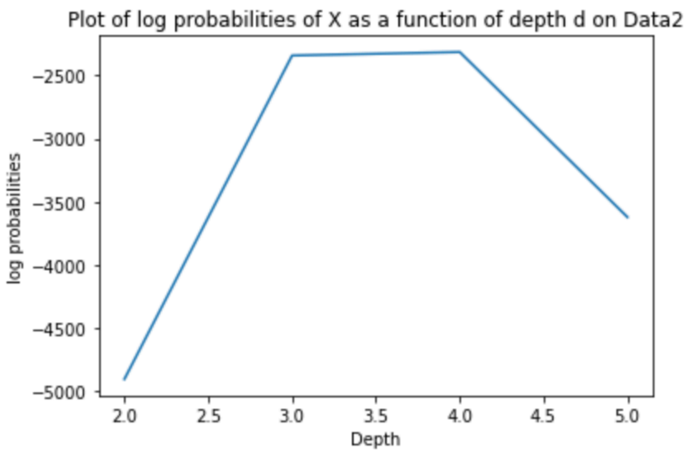

```
log probabilities on Data1:
[tensor(-13768.1455), tensor(-1885.5620), tensor(-1683.3137), tensor(-1675.3910), tensor(-1670.0334), tensor(-1666.
1263), tensor(-1663.1188), tensor(-1660.7136), tensor(-1658.7301), tensor(-1657.0519), tensor(-1655.6016)]
```



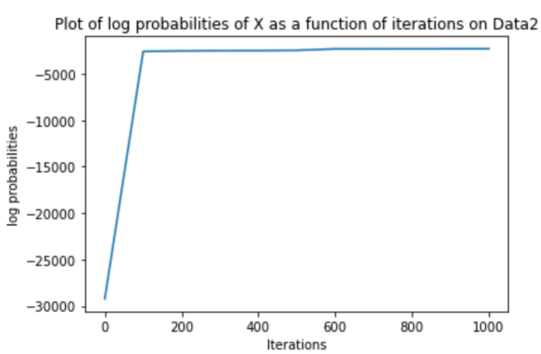
Experiment on dataset data1 shows that log probability is high at d=2 and decreases further due to overfitting of data and also log probability increases with increase in number of iterations. Therefore for we can see that quantitative results agree with qualitative(Visual) results.

Results on Dataset Data2:

```
log probabilities on Data2:
[tensor(-4904.8975), tensor(-2342.2649), tensor(-2314.8252), tensor(-3622.0884)]
```




```
log probabilities on Data2:
[tensor(-29236.0605), tensor(-2604.7759), tensor(-2546.4441), tensor(-2514.4326), tensor(-2511.1411), tensor(-2486.
1794), tensor(-2330.3091), tensor(-2327.0747), tensor(-2321.5166), tensor(-2317.4138), tensor(-2314.8252)]
```




Experiment on dataset data2 shows that log probability increases with depth and is highest at d=4 and decreases further due to overfitting of data and also log probability increases with increase in number of iterations. Therefore for we can see that quantitative results agree with qualitative(Visual) results.
(Didn't get time to explore second part of the Extra credit part, will definitely experiment on it)

In [204…
```
import numpy as np
import torch
import torch.nn as nn
import matplotlib.pyplot as plt
import math
```

In [205…
```
def coupling_layer(w, b, m, z):
    X = torch.zeros((z.shape[0], z.shape[1]))
    X[:, m] = z[:, m]
    z_m = torch.reshape(z[:, m], (z[:, m].shape[0], 1))
    w = torch.reshape(w, (1, w.shape[0]))
    theta = torch.exp(torch.tanh(torch.mm(z_m, w) + b))
    X[:, 1-m] = z[:, 1-m] * theta[:, 0] + theta[:, 1]
    return X

def forward_flow(W, B, M, z0):
    Z = z0
    for i in range(len(M)):
        Z = coupling_layer(W[i], B[i], M[i], Z)
    return Z

def inverse_coupling_layer(w, b, m, x):
    Z = torch.zeros((x.shape[0], x.shape[1]))
    Z[:, m] = x[:, m]
    x_m = torch.reshape(x[:, m], (x[:, m].shape[0], 1))
    w = torch.reshape(w, (1, w.shape[0]))
    theta = torch.exp(torch.tanh(torch.mm(x_m, w) + b))
    Z[:, 1-m] = (x[:, 1-m] - theta[:, 1])/ theta[:, 0]
    return Z

def inverse_flow(W, B, M, X):
    for i in range(len(M)):
        X = inverse_coupling_layer(W[i], B[i], M[i], X)
    return X
```

In [206…
```
M = torch.tensor([0,1])
W = torch.tensor(([[-1, 1], [1, -1]]), dtype = torch.float32)
B = torch.tensor(([[1, 1], [1,1]]), dtype = torch.float32)
z0 = torch.tensor(([[1, 2],[3, 4]]), dtype = torch.float32)
X = forward_flow(W, B, M, z0)
print(X)
```

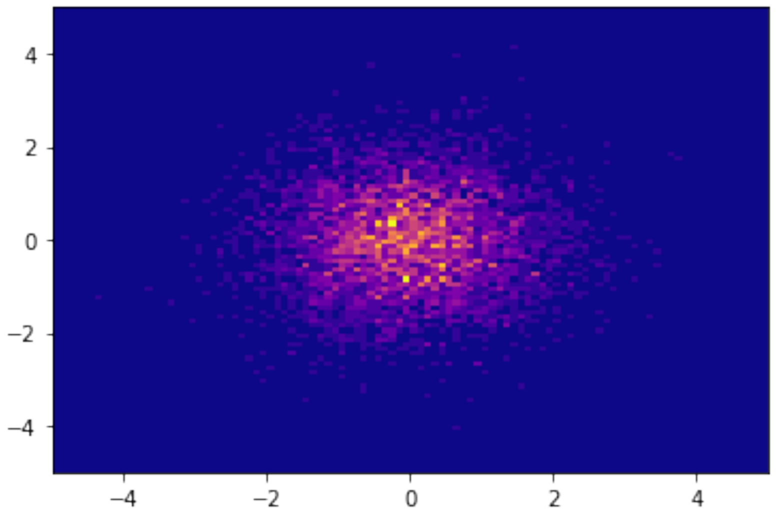
```
tensor([[3.0866, 4.6222],
        [8.5234, 4.2419]])
```

In [207…
```
Z0 = inverse_flow(torch.flipud(W), torch.flipud(B), torch.flipud(M), X)
print(Z0)
```

```
tensor([[1.0000, 2.0000],
        [3.0000, 4.0000]])
```

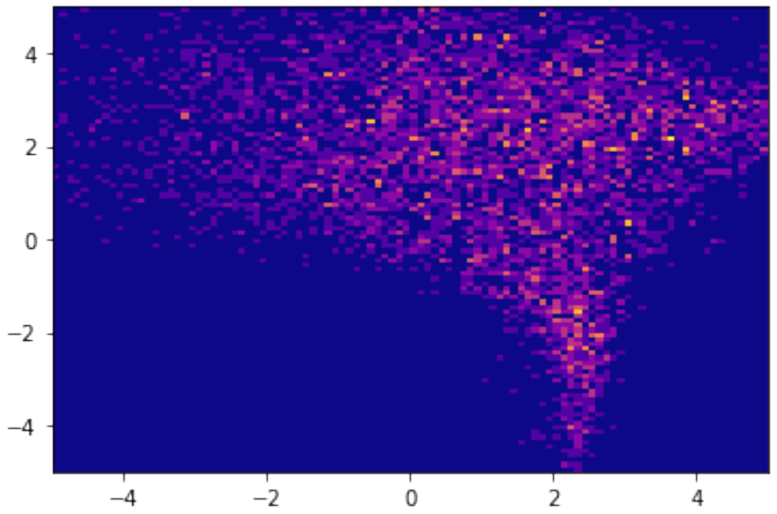
In [220…
```
torch.manual_seed(0)
d = 2
N = 5000
z0_iid = torch.randn(N, 2)
```

In [221…
```
plt.hist2d(list(z0_iid[:,0]), list(z0_iid[:,1]), bins=100, range=[[-5,5],[-5,5]],  cmap = 'plasma')
plt.show()
```



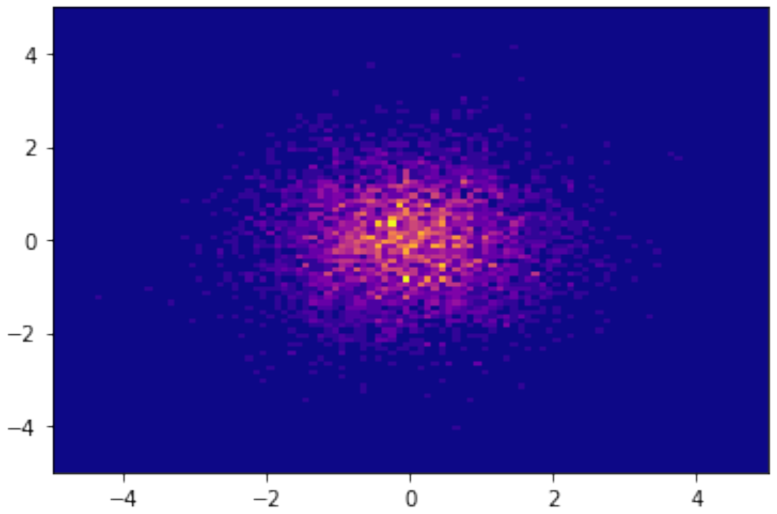
In [222…
```
X_d = forward_flow(W, B, M, z0_iid)
```

In [223…
```
plt.hist2d(list(X_d[:,0]), list(X_d[:,1]), bins=100, range=[[-5,5],[-5,5]],  cmap = 'plasma')
plt.show()
```



In [224…
```
Z0_inv = inverse_flow(torch.flipud(W), torch.flipud(B), torch.flipud(M), X_d)
```

In [225…
```
plt.hist2d(list(Z0_inv[:,0]), list(Z0_inv[:,1]), bins=100, range=[[-5,5],[-5,5]],  cmap = 'plasma')
plt.show()
```




In [ ]:

In [ ]:

In [527…

```
import numpy as np
import torch
import torch.nn as nn
import matplotlib.pyplot as plt
import math

steps = []
for i in range(11):
    steps.append(i * 100)
```

In [528…

```
def load_dataset(filename):
    data = open(filename, 'rt')
    x = np.loadtxt(data, delimiter = ",")
    X = torch.from_numpy(x)
    X = X.to(torch.float32)
    return X
```

In [529…

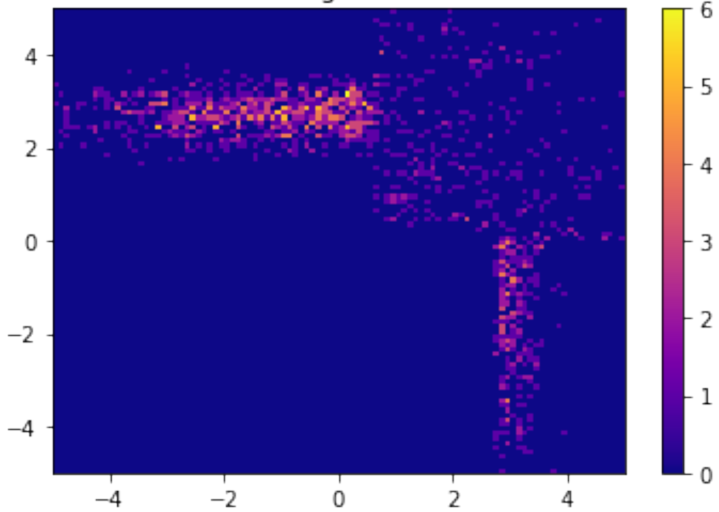
```
def coupling_layer(w, b, m, z):
    X = torch.zeros((z.shape[0], z.shape[1]))
    X[:, m] = z[:, m]
    z_m = torch.reshape(z[:, m], (z[:, m].shape[0], 1))
    w = torch.reshape(w, (1, w.shape[0]))
    theta = torch.exp(torch.tanh(torch.mm(z_m, w) + b))
    X[:, 1-m] = z[:, 1-m] * theta[:, 0] + theta[:, 1]
    return X

def forward_flow(W, B, M, z0):
    Z = z0
    for i in range(len(M)):
        Z = coupling_layer(W[i], B[i], M[i], Z)
    return Z

def inverse_coupling_layer(w, b, m, x):
    Z = torch.zeros((x.shape[0], x.shape[1]))
    Z[:, m] = x[:, m]
    x_m = torch.reshape(x[:, m], (x[:, m].shape[0], 1))
    w = torch.reshape(w, (1, w.shape[0]))
    theta = torch.exp(torch.tanh(torch.mm(x_m, w) + b))
    Z[:, 1-m] = (x[:, 1-m] - theta[:, 1])/ theta[:, 0]
    log_det = torch.log(torch.abs(theta[:, 0]))
    return Z, log_det

def inverse_flow(W, B, M, X):
    log_jacob_det = 0
    for i in range(len(M)):
        X, log_det = inverse_coupling_layer(W[i], B[i], M[i], X)
        log_jacob_det += log_det
    return X, log_jacob_det

def log_likelihood(W, B, M, X):
    Z, log_jacob_det = inverse_flow(W, B, M, X)
    log_Pz = -((1/2)* Z.shape[1] * math.log(2 * math.pi)) - ((1/2) * torch.sum(Z * Z, dim = 1))
    log_Qx = log_Pz - log_jacob_det
    return log_Qx
```

In [530…
```
def density_estimation(d, X, opt_steps = 1000):
    torch.manual_seed(0) #Results for different seed is included in the report
    W = torch.randn((d, 2), requires_grad=True, dtype = torch.float32)
    B = torch.randn((d, 2), requires_grad=True, dtype = torch.float32)
    M = torch.zeros(d, dtype = torch.int16)
    for i in range(len(M)):
        if i % 2 == 0:
            M[i] = 1

    optimizer = torch.optim.Adam([W, B], lr=0.3)

    for i in range(opt_steps):
        optimizer.zero_grad()
        loss = -torch.sum(log_likelihood(W, B, M, X))
        loss.backward()
        optimizer.step()

    return W, B, M
```

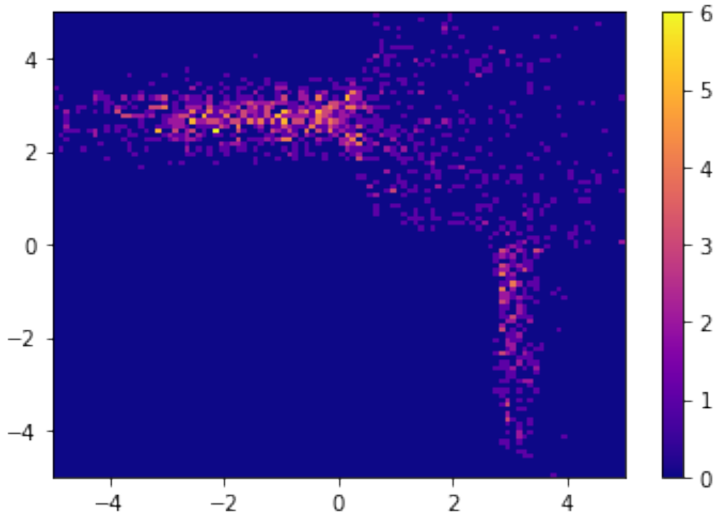
In [531…
```
def model_depth(X):
    d = [2, 3, 4, 5]
    for i in range(len(d)):
        W, B, M = density_estimation(d[i], X)
        Z0 = torch.randn(1500, 2)
        X_estimated = forward_flow(torch.flipud(W.detach()), torch.flipud(B.detach()), torch.flipud(M), Z0)
        plt.title(f"Generated data distribution for depth d: {d[i]}\n")
        plt.hist2d(list(X_estimated[:,0]), list(X_estimated[:,1]), bins=100, range=[[-5,5],[-5,5]],  cmap = 'plasma')
        cb = plt.colorbar()
        plt.show()

def model_iterations(X, steps, d):

    log_prob = []

    for i in range(len(steps)):
        W, B, M = density_estimation(d, X, steps[i])
        Z0 = torch.randn(1500, 2)
        X_estimated = forward_flow(torch.flipud(W.detach()), torch.flipud(B.detach()), torch.flipud(M), Z0)
        plt.title(f"Generated data distribution for iterations: {steps[i]}\n")
        plt.hist2d(list(X_estimated[:,0]), list(X_estimated[:,1]), bins=100, range=[[-5,5],[-5,5]],  cmap = 'plasma')
        cb = plt.colorbar()
        plt.show()

        log_Qx = torch.sum(log_likelihood(W.detach(), B.detach(), M, X))
        log_prob.append(log_Qx)
    return log_prob
```

In [532…
```
X_data1 = load_dataset('hw3data/data1.csv')
plt.title('Distribution for original dataset - Data1')
plt.hist2d(list(X_data1[:,0]), list(X_data1[:,1]), bins=100, range=[[-5,5],[-5,5]],  cmap = 'plasma')
cb = plt.colorbar()
plt.show()
model_depth(X_data1)
```

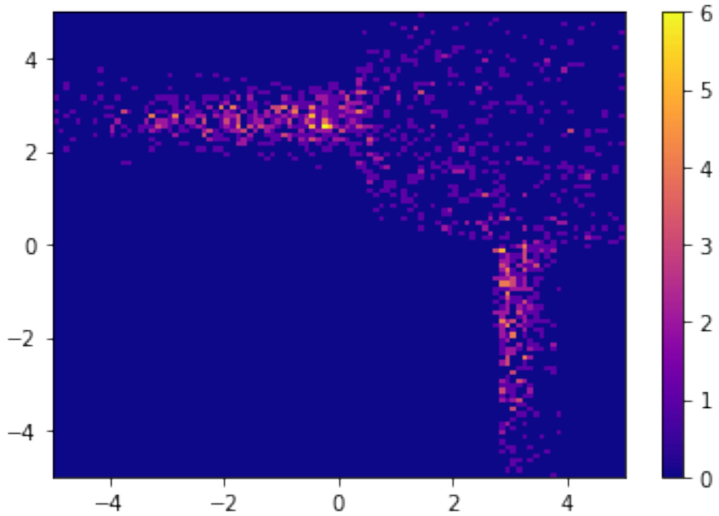
Distribution for original dataset - Data1



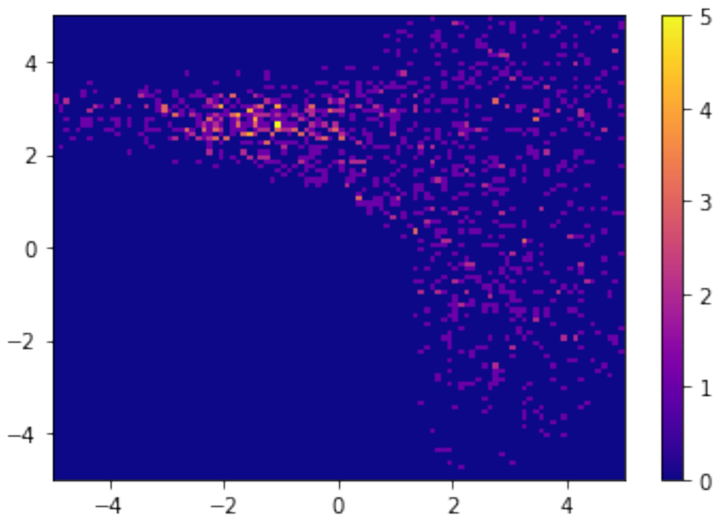
Generated data distribution for depth d: 2



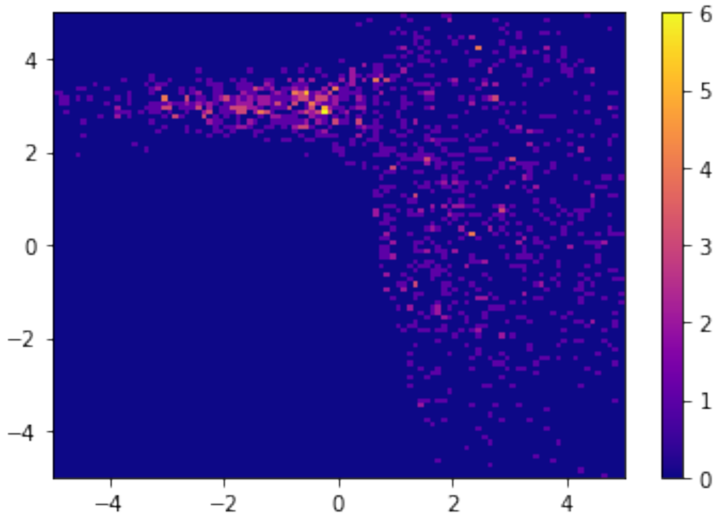
Generated data distribution for depth d: 3



Generated data distribution for depth d: 4



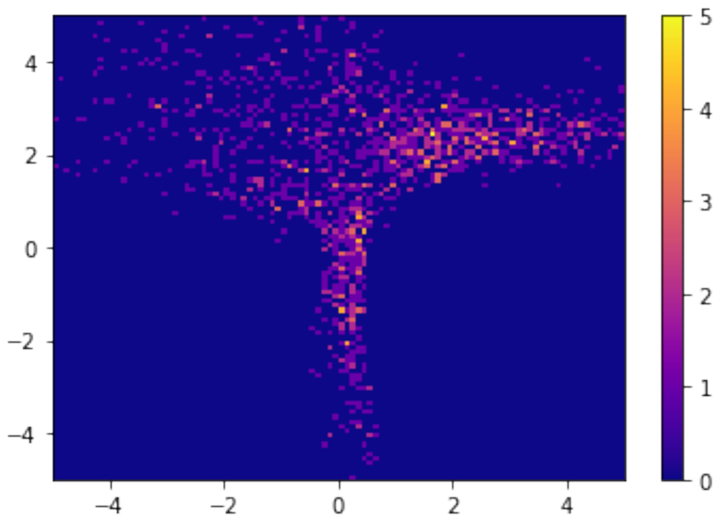
Generated data distribution for depth d: 5



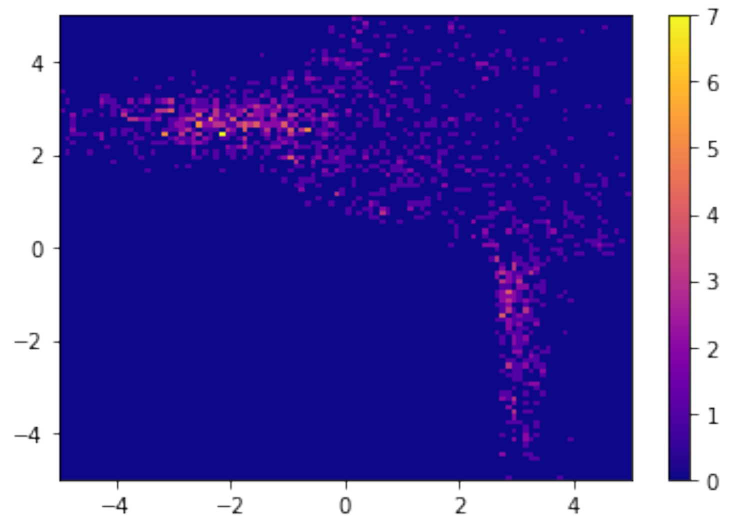
In [533…

```
d = 2
log_prob = model_iterations(X_data1, steps, d)
```

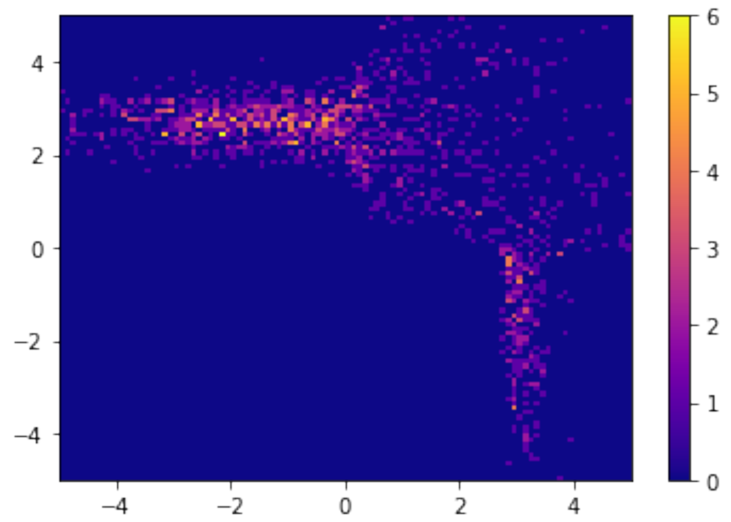
Generated data distribution for iterations: 0



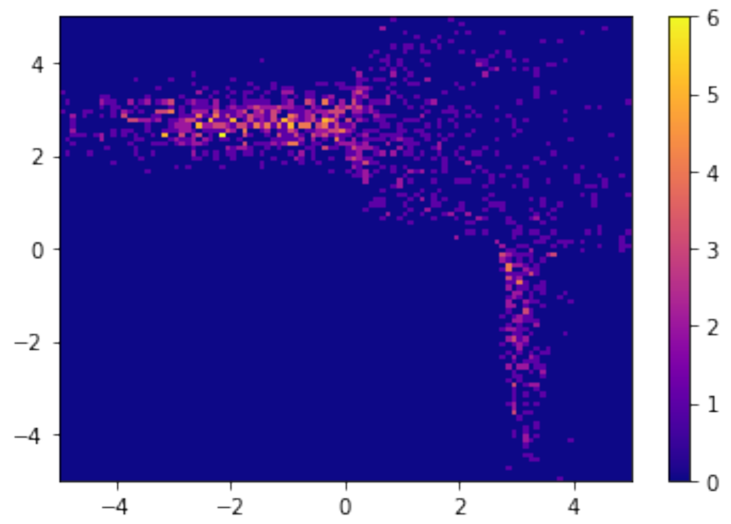
Generated data distribution for iterations: 100



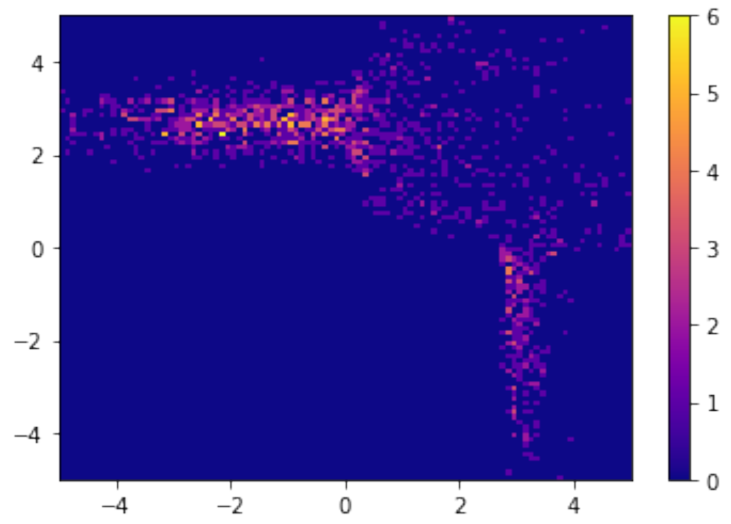

Generated data distribution for iterations: 200



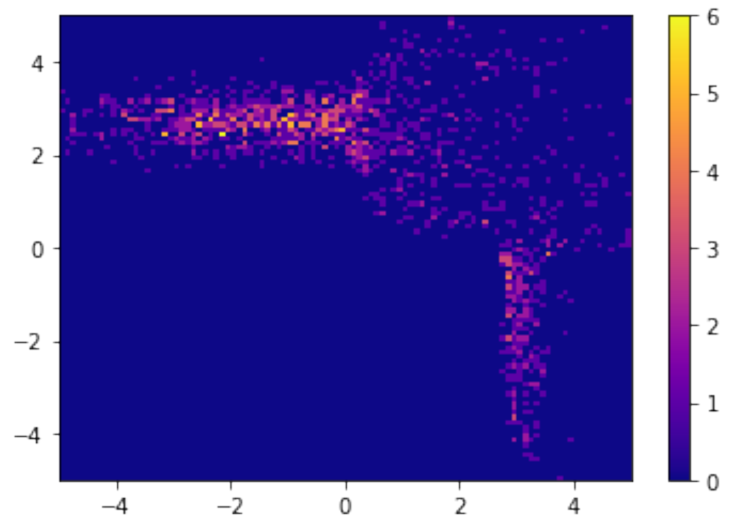

Generated data distribution for iterations: 300



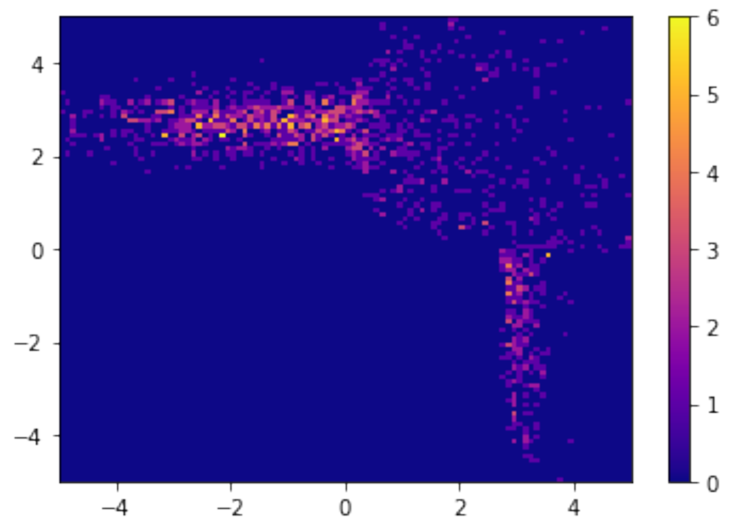
Generated data distribution for iterations: 400



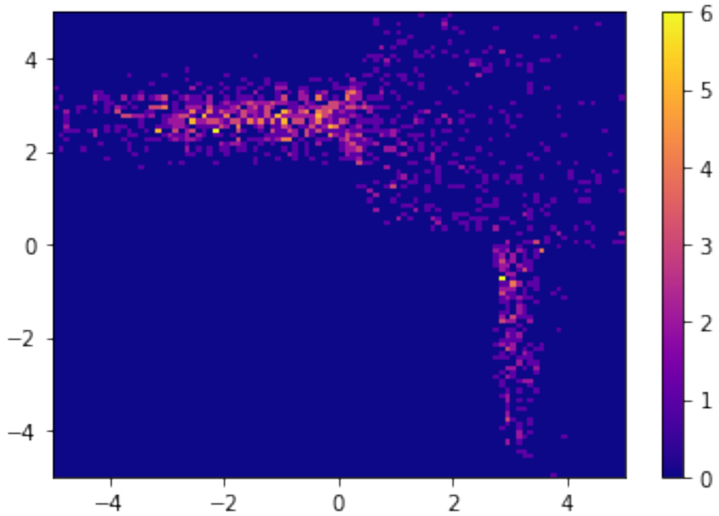
Generated data distribution for iterations: 500



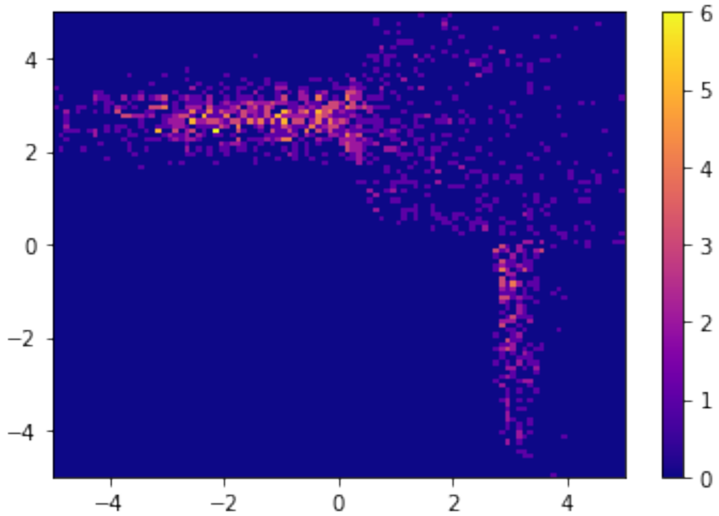
Generated data distribution for iterations: 600



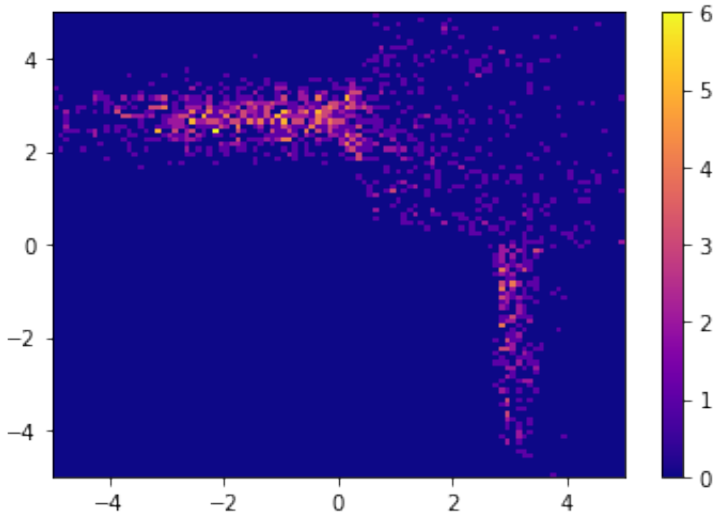
Generated data distribution for iterations: 700

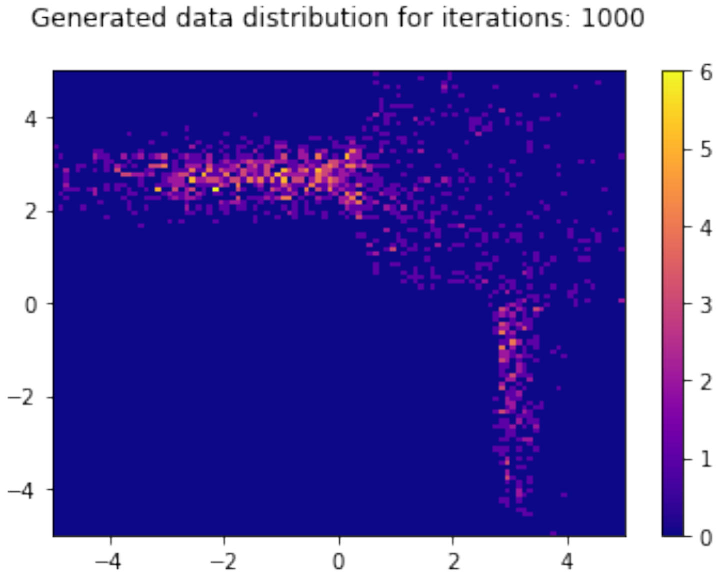



Generated data distribution for iterations: 800




Generated data distribution for iterations: 900

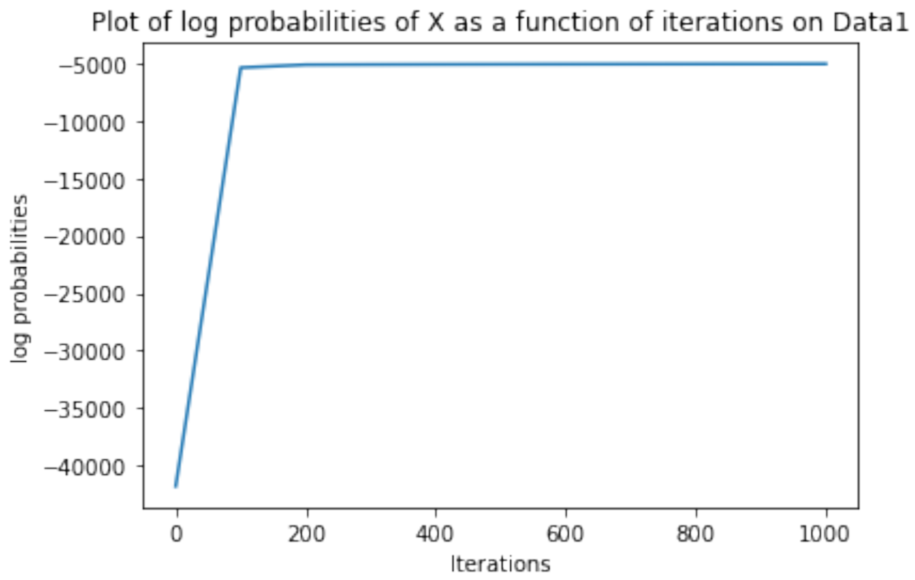





In [534…

```
print(f'log probabilities on Data1:\n{log_prob}\n')
plt.title('Plot of log probabilities of X as a function of iterations on Data1')
plt.plot(steps, log_prob)
plt.ylabel('log probabilities')
plt.xlabel('Iterations')
plt.show()
```

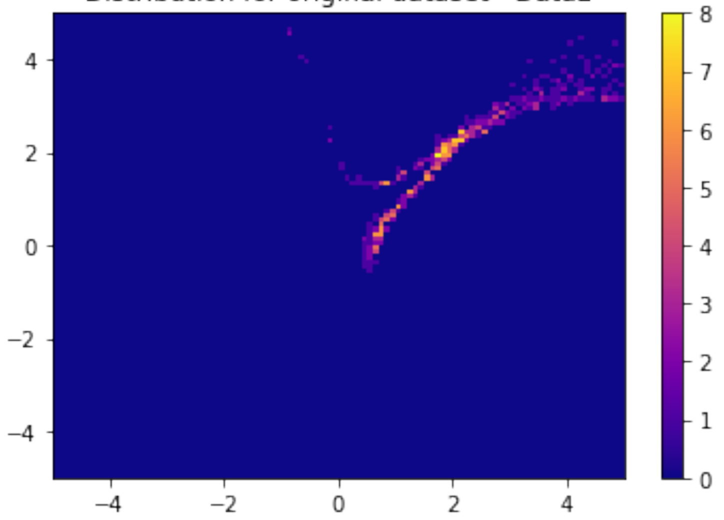
```
log probabilities on Data1:
[tensor(-41779.7227), tensor(-5407.3677), tensor(-5153.8540), tensor(-5136.9263), tensor(-5123.6685), tensor(-5113.0664), tensor(-5104.3857), tensor(-5097.1304), tensor
(-5090.9668), tensor(-5085.6641), tensor(-5081.0547)]
```



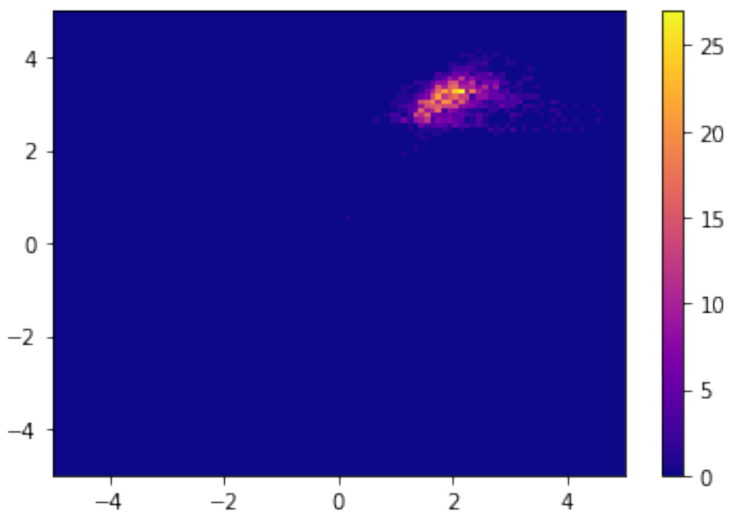

In [535…

```
#Note: Please check plots for different iterations either on jupyter notebook or report
X_data2 = load_dataset('hw3data/data2.csv')
plt.title('Distribution for original dataset - Data2')
plt.hist2d(list(X_data2[:,0]), list(X_data2[:,1]), bins=100, range=[[-5,5],[-5,5]],  cmap = 'plasma')
cb = plt.colorbar()
plt.show()

d = 5
log_prob_data2 = model_iterations(X_data2, steps, d)
```

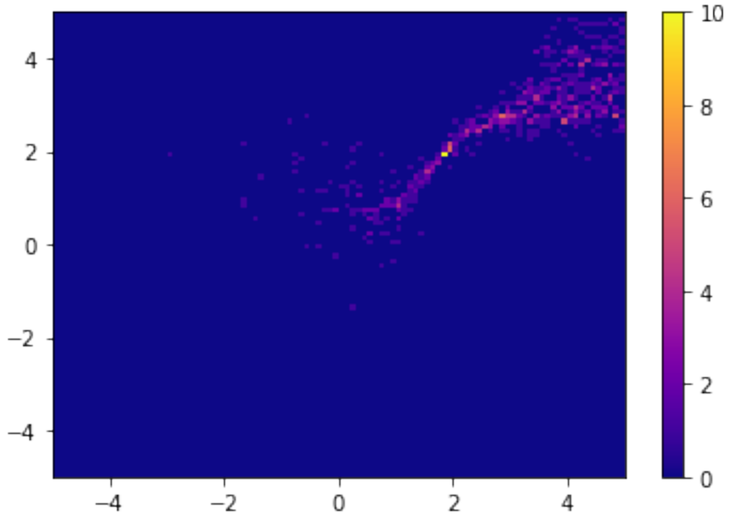
Distribution for original dataset - Data2



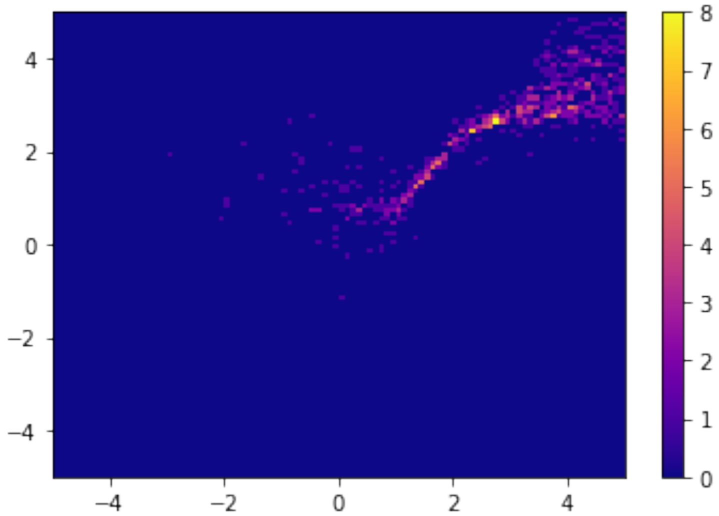
Generated data distribution for iterations: 0



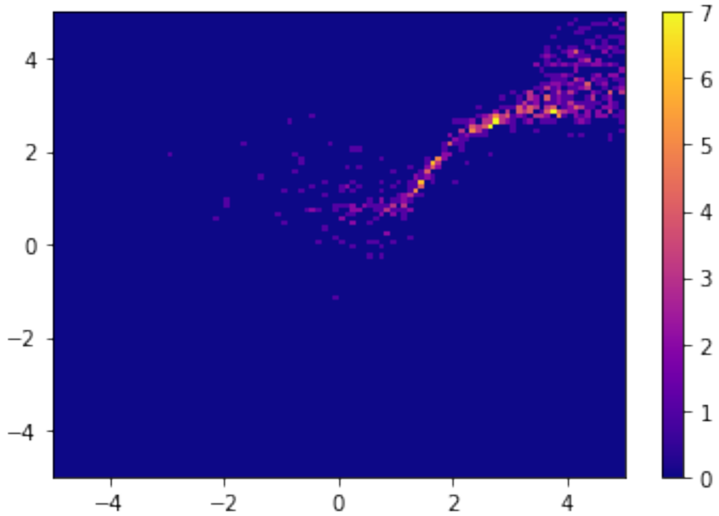
Generated data distribution for iterations: 100



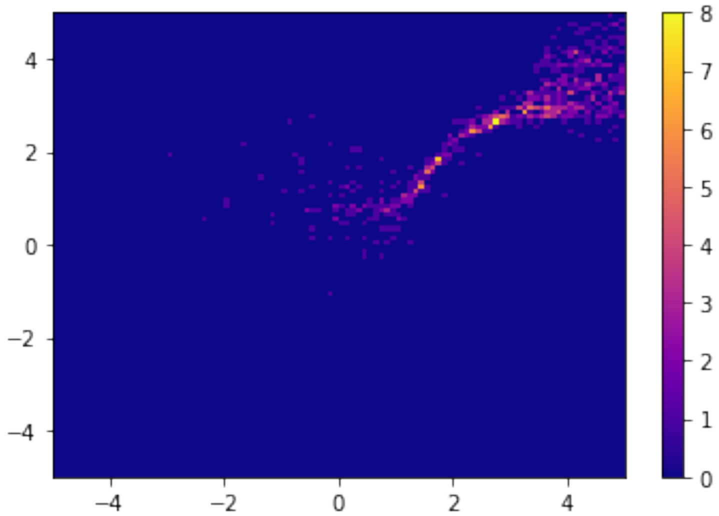
Generated data distribution for iterations: 200



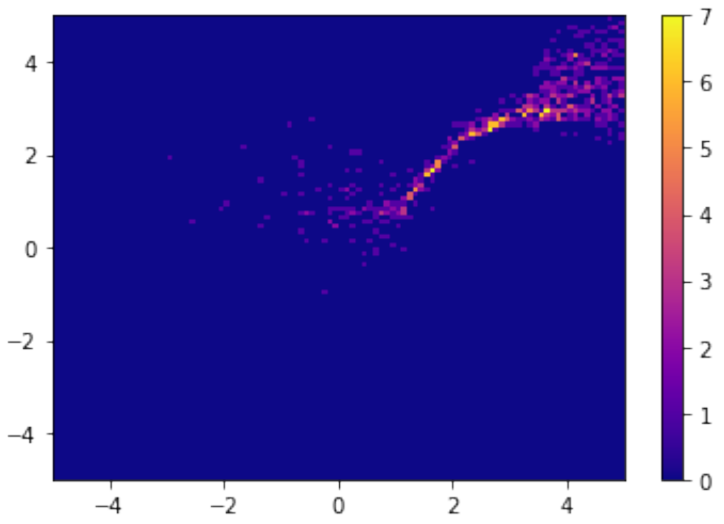
Generated data distribution for iterations: 300



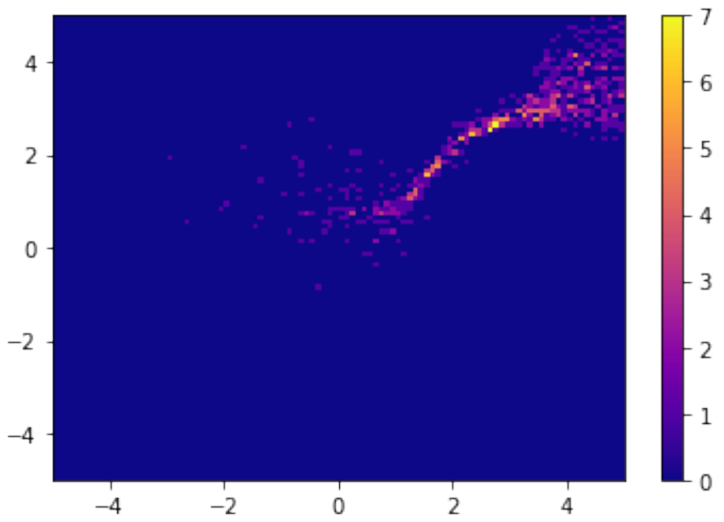
Generated data distribution for iterations: 400



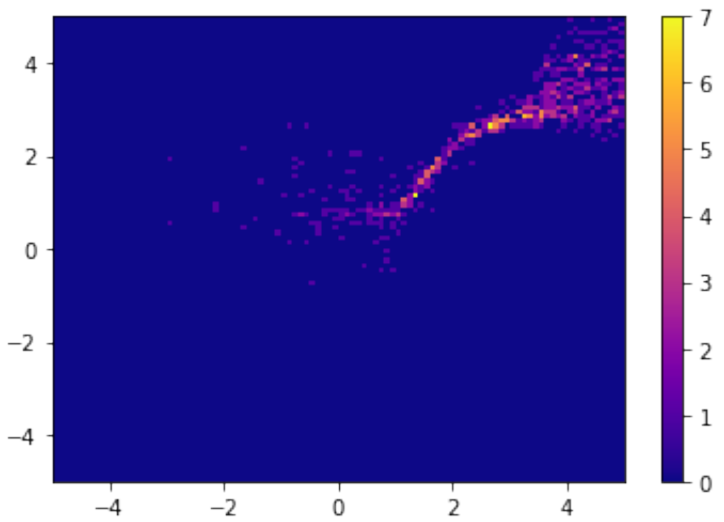
Generated data distribution for iterations: 500



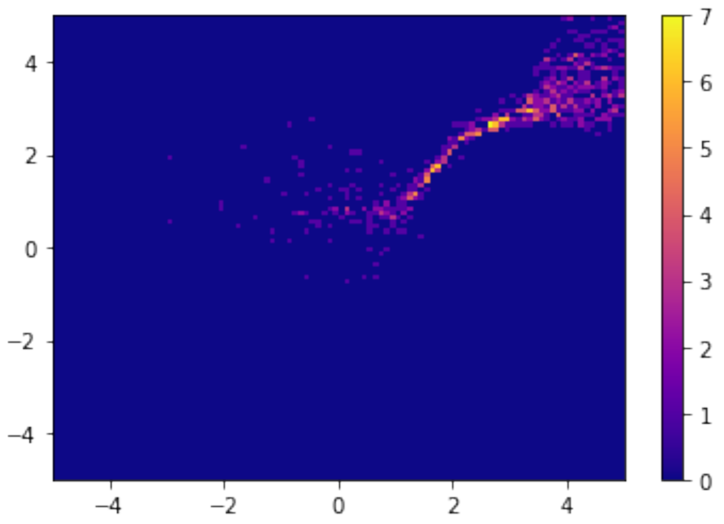

Generated data distribution for iterations: 600



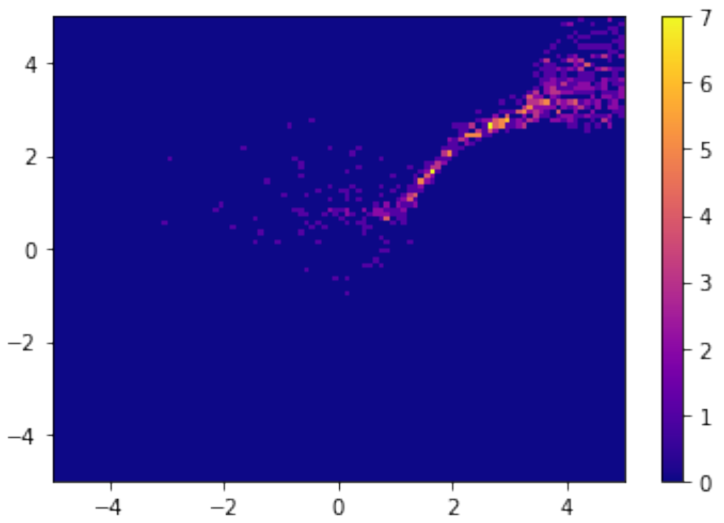

Generated data distribution for iterations: 700



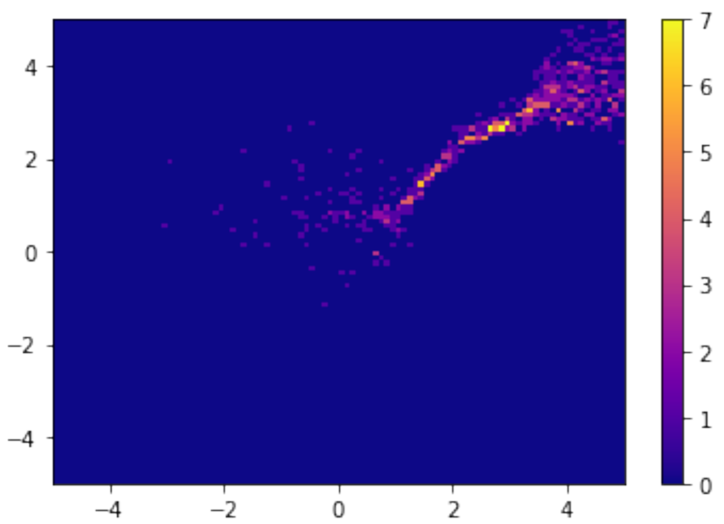
Generated data distribution for iterations: 800




Generated data distribution for iterations: 900



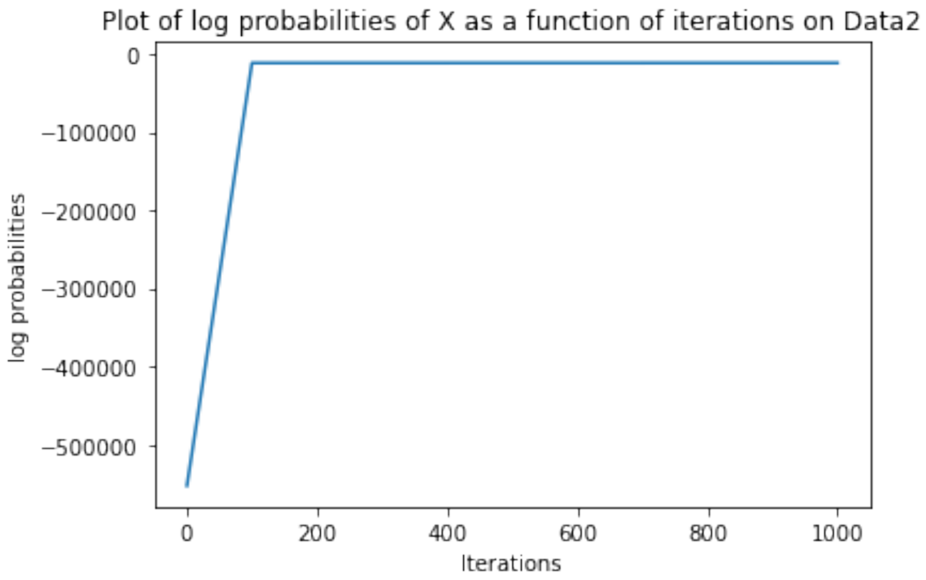

Generated data distribution for iterations: 1000



In [536…

```
print(f'log probabilities on Data2:\n{log_prob_data2}\n')
plt.title('Plot of log probabilities of X as a function of iterations on Data2')
plt.plot(steps, log_prob_data2)
plt.ylabel('log probabilities')
plt.xlabel('Iterations')
plt.show()
```

```
log probabilities on Data2:
[tensor(-552505.4375), tensor(-11608.2617), tensor(-11590.4629), tensor(-11584.8848), tensor(-11580.2969), tensor(-11575.6934), tensor(-11570.8301), tensor(-11569.2666)
, tensor(-11558.5947), tensor(-11553.1562), tensor(-11543.3604)]
```



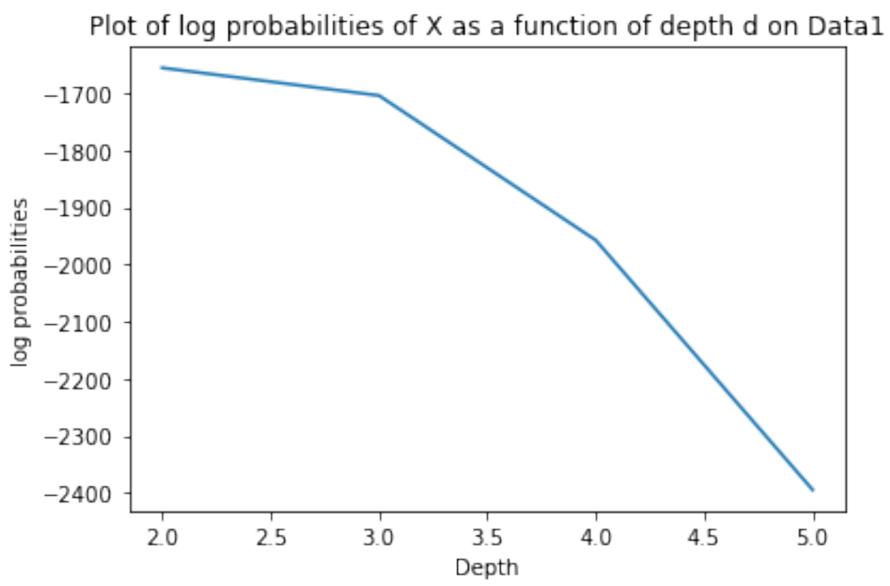

In [537…

```
#Extra Fun

def model_depth_extra(train_data, test_data, d):
    log_prob = []
    for i in range(len(d)):
        W, B, M = density_estimation(d[i], train_data)
        log_Qx = torch.sum(log_likelihood(W.detach(), B.detach(), M, test_data))
        log_prob.append(log_Qx)
    return log_prob

def model_iterations_extra(train_data, test_data, steps, d):
    log_prob = []
    for i in range(len(steps)):
        W, B, M = density_estimation(d, train_data, steps[i])
        log_Qx = torch.sum(log_likelihood(W.detach(), B.detach(), M, test_data))
        log_prob.append(log_Qx)
    return log_prob
```

In [538…

```
d = [2, 3, 4, 5]
split = int((2/3) * X_data1.shape[0])
datasets1 = torch.split(X_data1, split, dim=0)
train_data1 = datasets1[0]
test_data1 = datasets1[1]

log_p_data1_depth = model_depth_extra(train_data1, test_data1, d)
print(f'log probabilities on Data1:\n{log_p_data1_depth}\n')
plt.title('Plot of log probabilities of X as a function of depth d on Data1')
plt.plot(d, log_p_data1_depth)
plt.ylabel('log probabilities')
plt.xlabel('Depth')
plt.show()
```

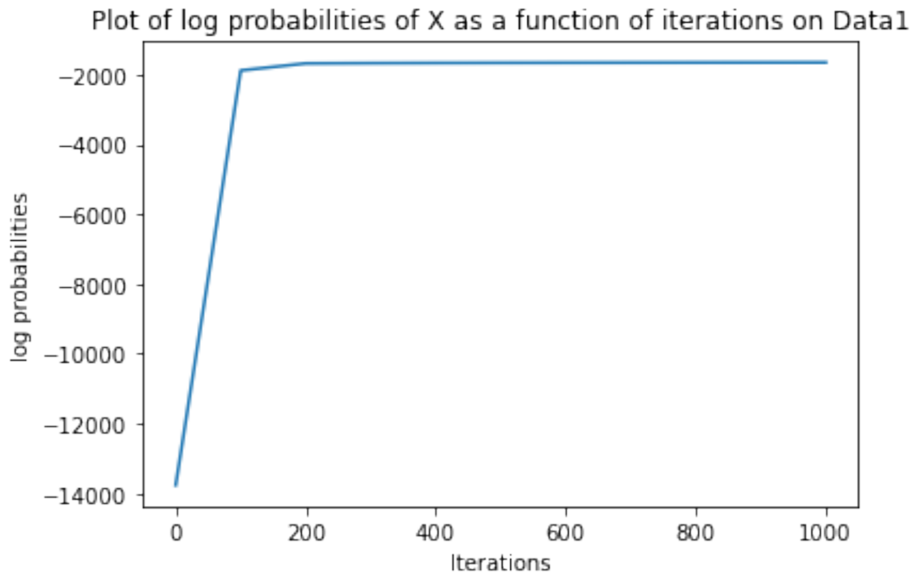
```
log probabilities on Data1:
[tensor(-1655.6016), tensor(-1704.0582), tensor(-1957.4718), tensor(-2394.4875)]
```




In [539…

```
depth = 2
log_p_data1_iterations = model_iterations_extra(train_data1, test_data1, steps, depth)
print(f'log probabilities on Data1:\n{log_p_data1_iterations}\n')
plt.title('Plot of log probabilities of X as a function of iterations on Data1')
plt.plot(steps, log_p_data1_iterations)
plt.ylabel('log probabilities')
plt.xlabel('Iterations')
plt.show()
```

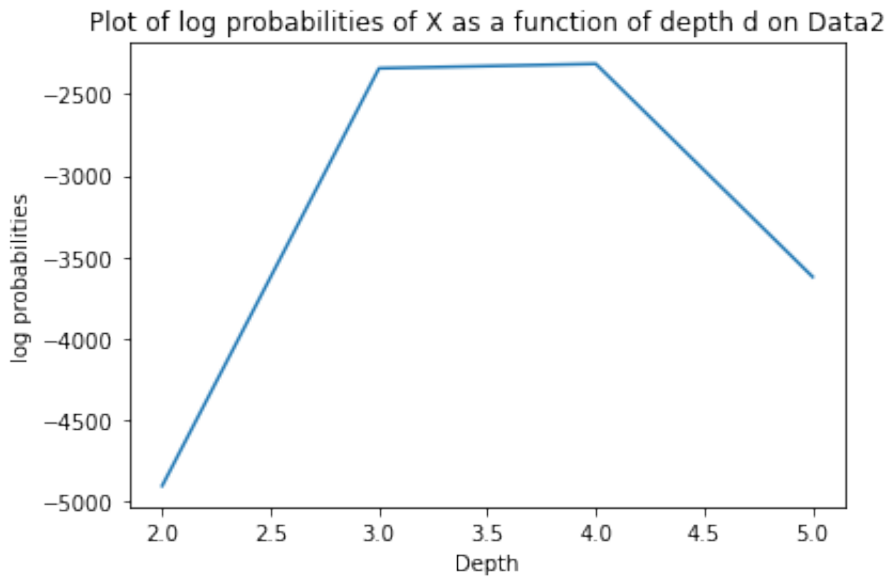
```
log probabilities on Data1:
[tensor(-13768.1455), tensor(-1885.5620), tensor(-1683.3137), tensor(-1675.3910), tensor(-1670.0334), tensor(-1666.1263), tensor(-1663.1188), tensor(-1660.7136), tensor
(-1658.7301), tensor(-1657.0519), tensor(-1655.6016)]
```




In [540…

```
d = [2, 3, 4, 5]
split = int((2/3) * X_data2.shape[0])
datasets2 = torch.split(X_data2, split, dim=0)
train_data2 = datasets2[0]
test_data2 = datasets2[1]

log_p_data2_depth = model_depth_extra(train_data2, test_data2, d)
print(f'log probabilities on Data2:\n{log_p_data2_depth}\n')
plt.title('Plot of log probabilities of X as a function of depth d on Data2')
plt.plot(d, log_p_data2_depth)
plt.ylabel('log probabilities')
plt.xlabel('Depth')
plt.show()
```

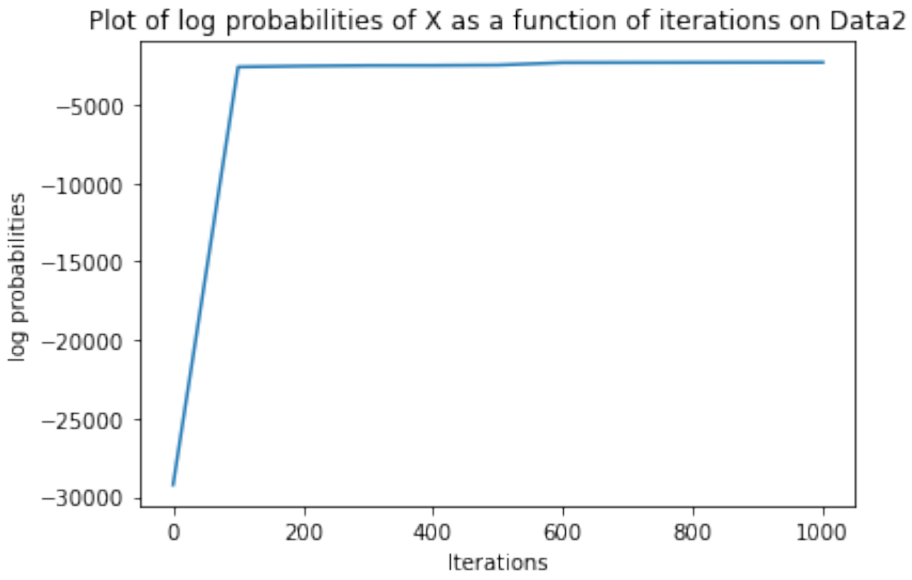
```
log probabilities on Data2:
[tensor(-4904.8975), tensor(-2342.2649), tensor(-2314.8252), tensor(-3622.0884)]
```



In [541…

```
depth = 4
log_p_data2_iterations = model_iterations_extra(train_data2, test_data2, steps, depth)
print(f'log probabilities on Data2:\n{log_p_data2_iterations}\n')
plt.title('Plot of log probabilities of X as a function of iterations on Data2')
plt.plot(steps, log_p_data2_iterations)
plt.ylabel('log probabilities')
plt.xlabel('Iterations')
plt.show()
```

```
log probabilities on Data2:
[tensor(-29236.0605), tensor(-2604.7759), tensor(-2546.4441), tensor(-2514.4326), tensor(-2511.1411), tensor(-2486.1794), tensor(-2330.3091), tensor(-2327.0747), tensor
(-2321.5166), tensor(-2317.4138), tensor(-2314.8252)]
```




In [ ]:

In [ ]: