“Quantitative Macroeconomics & Social Insurance - TA 4”

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1 Income Dynamics and Life-Cycle

Consider the labor income process

$$y_{ijt} = \beta' f(X_{ijt}) + \tilde{y}_{ijt},$$
$$\tilde{y}_{ijt} = z_{ijt} + \epsilon_{ijt},$$
$$z_{ijt} = \rho z_{ij-1t-1} + \nu_{ijt}.$$

- y_{ijt} is log-income for household i with household head of age j at time t.
- $\epsilon_{ijt} \sim f_\epsilon, \nu_{ijt} \sim f_\nu$ are iid shocks with standard deviations $\sigma_\nu, \sigma_\epsilon$.
- Note that f_ϵ, f_ν might be non-gaussian densities.
- Two-stage estimation:
  - estimate β using fixed effects
  - estimate $\theta = (\rho, \sigma_\nu, \sigma_\epsilon)$ with GMM.

Table 1: First-stage estimation

y_{ijt}	(1)	(2)
Age	.128	.075
	(.078)	(.062)
Age2	-.002	-.001
	(.002)	(.001)
College	-1.672	-1.426
	(1.161)	(0.956)
High School	.749	.465
	(1.102)	(0.888)
Famsize	-.038***	-.004
	(.004)	(.004)
Married	.491***	.509***
	(.018)	(.017)

Notes: Data source PSID. Fixed effects estimates. (1) before tax log-income, (2) after tax log-income. Standard errors are clustered at the household level. Sample size is 94,996 households.

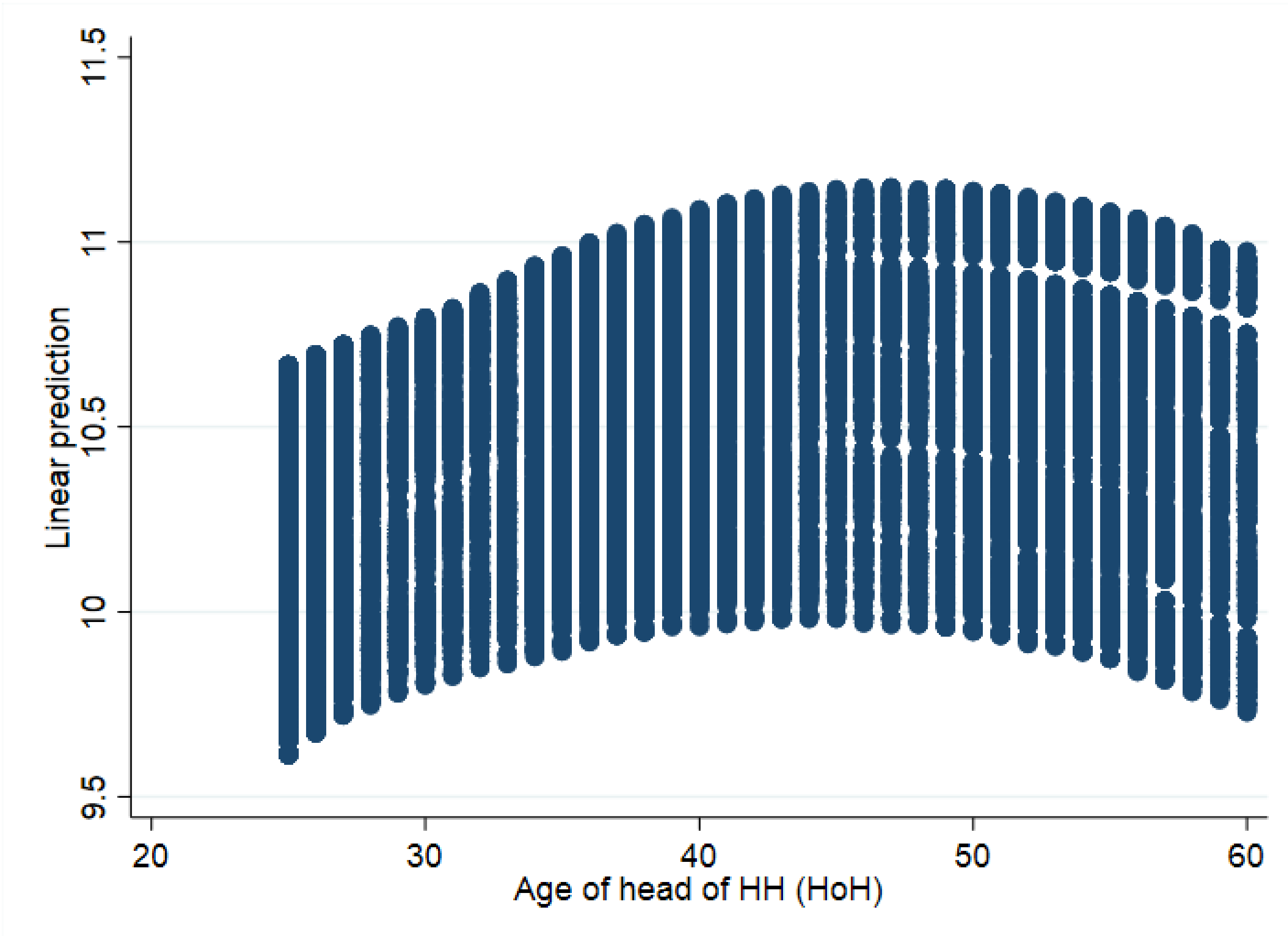

Figure 1: Earnings profiles over life-cycle

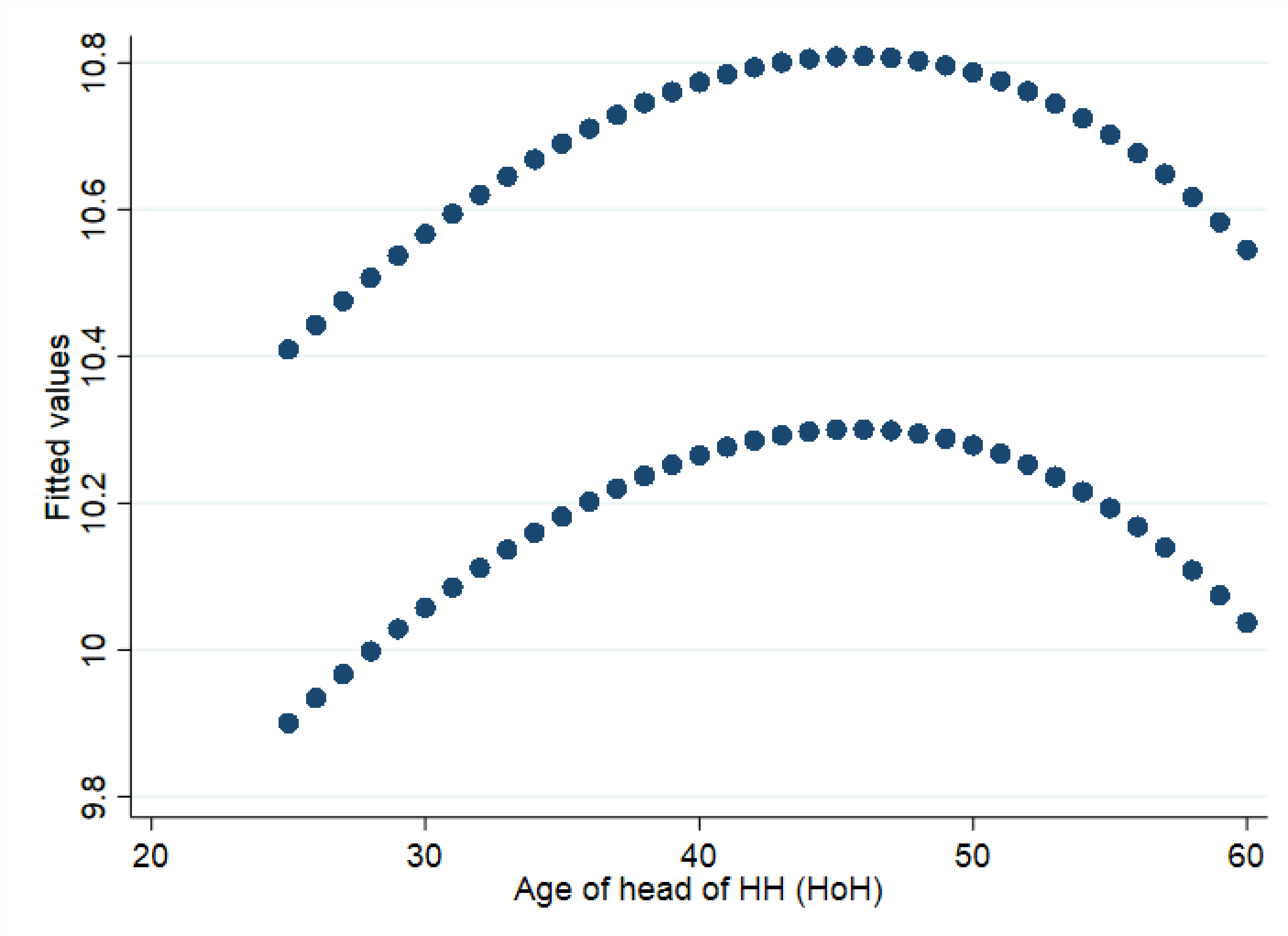

Figure 2: Smoothed earnings profiles by age and marital status

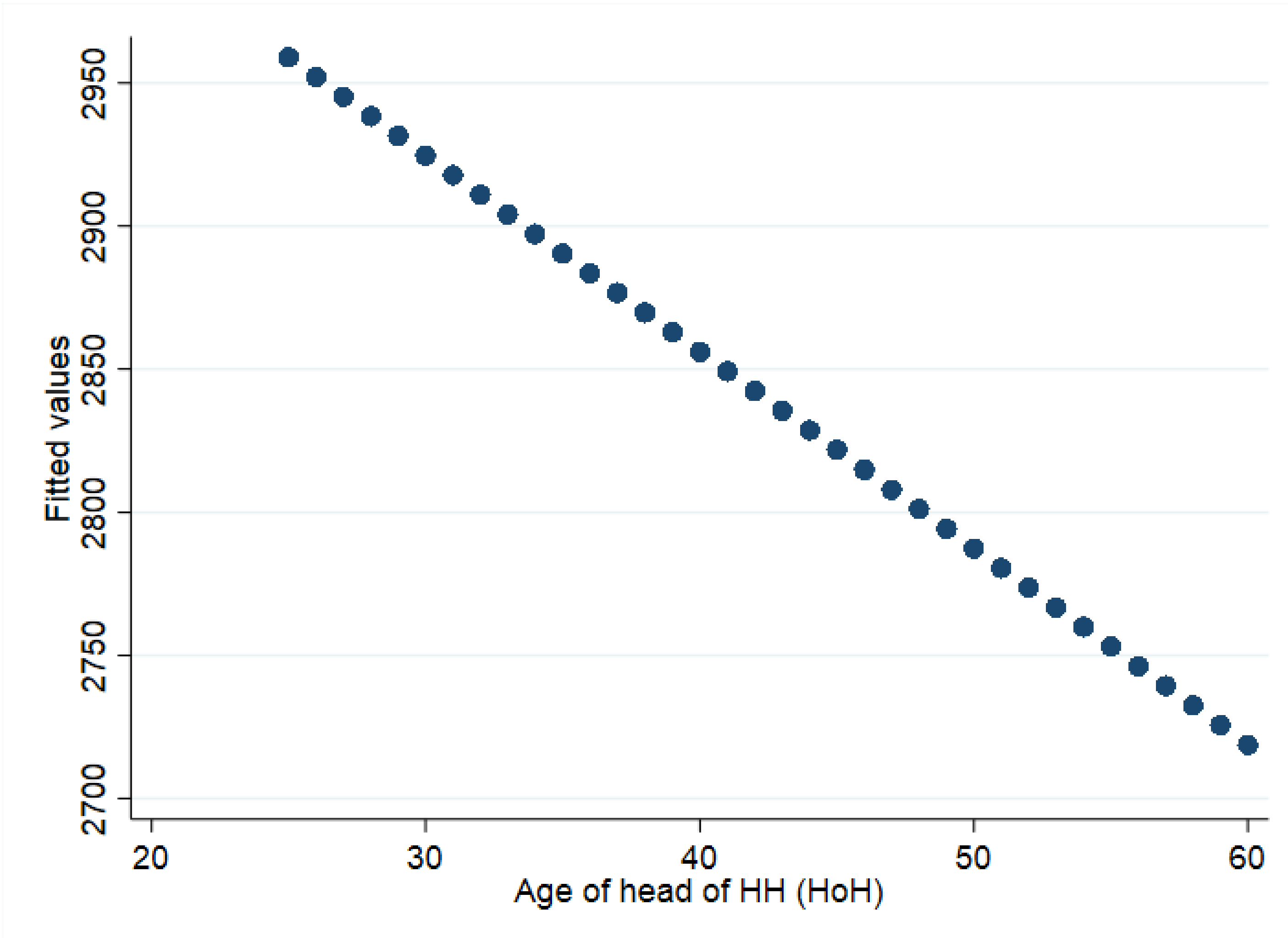

Figure 3: Age-Hours Correlation

At the second stage we rely on the following moment conditions

$$n^{-1}\sum_i(\tilde{y}^2_{ijt}) - \mu^1(\theta),$$
$$n^{-1}\sum_i(\tilde{y}^2_{ij+1t+1}) - \mu^2(\theta),$$
$$n^{-1}\sum_i(\tilde{y}_{ijt}\tilde{y}_{ij+1t+1}) - \mu^3(\theta),$$
$$n^{-1}\sum_i(\tilde{y}_{ijt}\tilde{y}_{ij+2t+2}) - \mu^4(\theta).$$

- Theoretical moments $\mu^k(\theta)$ can be derived solving backward z_{ijt}.

$$\tilde{y}_{ijt} = \rho^{j-j_0+1} z_{j-(j-j_0)-1,t-(t-t_0)-1} + \sum_{s=j_0}^{j} \rho^{j-s}\nu_{j-(j-s),t-(t-s)} + \epsilon_{ijt}.$$

- We instrument all the equations with age of the household head and each equation with the sample counterparts of theoretical moments.

- Note that
$$\mathrm{Var}(\tilde{y}_{ijt}) = \sigma^2_z + \sigma^2_\epsilon = \frac{\sigma^2_\nu}{1-\rho^2} + \sigma^2_\epsilon.$$

Rewrite the moment conditions in matrix notation, e.g.

$$\mathrm{E}(m_i(\theta)) = \mathrm{E}(g(z_i)(\varepsilon_i(\theta))) = \mathrm{E}(g(z_i)(\tilde{y}_i^2 - \mu^2(\theta)).$$

Remarks:

- Are the instruments z_i exogenous? We want $\mathrm{E}(z_i \epsilon_i) = \mathrm{E}(z_i \nu_i) = 0$
- Are the instruments z_i relevant? We want $\mathrm{E}(m_i(\theta)) = 0$ when $\theta = \theta_0$.

Note that $\tilde{y}_{ijt}$ follows an ARIMA(1,1)

$$\tilde{y}_{ijt} = \rho \tilde{y}_{ij-1t-1} + \mu_t + \epsilon_t - \rho \epsilon_t.$$

We need $|\rho| < 1$ for both stationarity and invertibility.

Table 2: Second-stage estimation

$\tilde{y}_{ijt}$	(1)	(2)
ρ	.930	.908
	(.011)	(.013)
σ_z^2	21.389	41.877
	(24.17)	(68.175)
σ_ν^2	.062	.064
	(.017)	(.015)
σ_ϵ^2	.194	.148
	(.014)	(.012)

Notes: Data source PSID up to 1997. GMM estimates. (1) before tax log-income, (2) after tax log-income. Sample size is 952 households.

- 1st Stage: Before government $\sigma_{\hat{y}}^2 = .860$, after government $\sigma_{\hat{y}}^2 = .775$.
- 2nd Stage: Before $\sigma_{\hat{y}}^2 = .66$, after $\sigma_{\hat{y}}^2 = .516$.
- Persistent shocks are very persistent, almost random walk.