Homework 4
Quantitative Macroeconomics

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1. A simple wealth model

Firstly we have to solve the following maximization of the Stochastic endowment process:

$$\max_{\{ct\}_t^T} E_o \sum_{t=0}^{T} \beta^t u(c_t) s.t. c_t + a_{t+1} = w_t y_t + (1+r_t) a_t \tag{1}$$

The endowment efficiency over units of labor is a Markov Process with $\pi(y'/y)$ denoting the probability of tomorrow endowment taking into consideration today's endowment.

Taking into consideration two possible borrowing constrains defined as:

1) Natural borrowing constraint:

$$a_{t+1} \geq -y_{min} \sum_{s=0}^{T-(t+1)} (1+r)^{-s} \tag{2}$$

2) Prevention for borrowing altogether:

$$a_{t+1} \geq 0 \tag{3}$$

And two possible utility functions specified below:

1) Quadratic Utility

$$u(c_t) = -\frac{1}{2}(c_t - \overline{c})^2 \tag{4}$$

2) CRRA Utility

$$u(c_t) = \frac{c^{1-\sigma} - 1}{1-\sigma} \tag{5}$$

Additionally we have to take into consideration the two-state process by $Y = \{1-\sigma_y, 1+\sigma_y\}$ which allow us to obtain a persistent variance of σ_y^2.

2. Solving the ABHI Model

Firstly we want to formulate the problem of the agent recursively, to obtain the stochastic Euler equation. Then, following the lecture notes we want to obtain:

$$v(a,y;\Phi) = \max_{c\geq 0, a'\geq 0} u(c) + \beta \sum_{y'} \pi(y'|y) v(a',y';\Phi) \tag{6}$$

That in our case can be specified as:

$$V(t,T) = \max_{a_{t+1}} u(y_t + (1+r)a_t - a_{t+1}) + \beta \sum_{l'=1}^{q} \pi_{l,l'} V(t+1,T) \tag{7}$$

Then applying our borrowing constraint we can obtain:

$$\sum_{t=0}^{T} \sum_{s^t} [\beta^t \pi_t(s^t) U(y_t(s^t) + (1+r)a_t(s^{t-1}) - a_{t+1}(s^t), s^t) + \mu(s^t)(a_{t+1}(s^t) - y_{min}\frac{r+1}{r})] \tag{8}$$

From which we generalize the two following Euler equations taking into consideration the duration of the model:

1) Infinite:

$$u_c'(c^t) = \max \left\{ u(y_t + a_t + y_{min}\frac{r+1}{r}), \frac{1+r}{1+\rho} E_t u(c_{t+1}) \right\} \tag{9}$$

2) Finite:

$$u_c'(c^t) = \max \left\{ u(y_t + a_t + y_{min}\frac{r+1-(r+1)^{t-T+1}}{r}), \frac{1+r}{1+\rho} E_t u(c_{t+1}) \right\} \tag{10}$$

3. Partial Equilibrium

INFINITE HORIZON

Firstly, we will present the results for the Partial Equilibrium model taking into consideration the statement II.2 that specifies a Infinitely lived household economy. Additionally we will differentiate by certainty/uncertainty case and utility function, then we can obtain the following results:

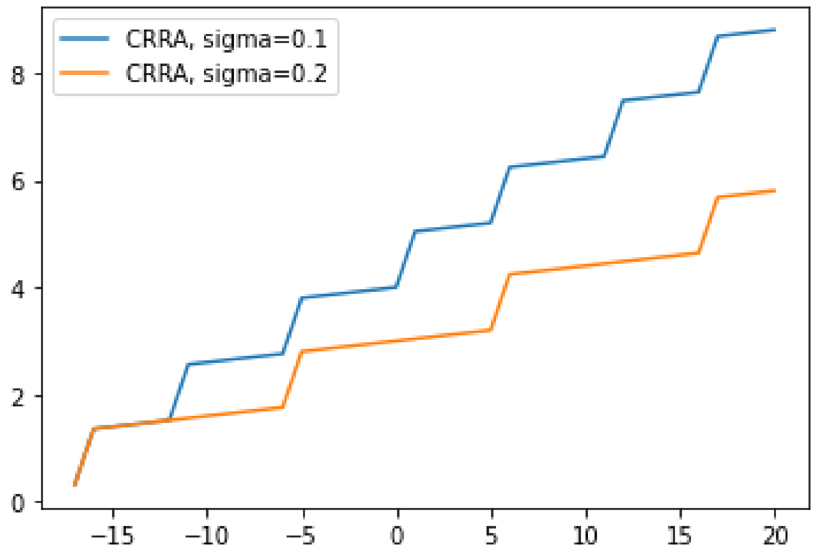

Figure 1: Consumption Function - CRRA sensibility over σ

1) Certainty - CRRA utility

In the Figure 1, we can observe how the Consumption function varies for different values of Capital. Then taking into consideration that $\sigma_y = 0$ for all the following values, we can conclude that the consumption is higher the higher the capital is. For being more exhaustive in the analysis we elaborate two different estimates taking into consideration different values of σ, from which we can conclude that for bigger values of σ the slope of is less explosive and maintaining lower values.

2) Certainty - Quadratic utility

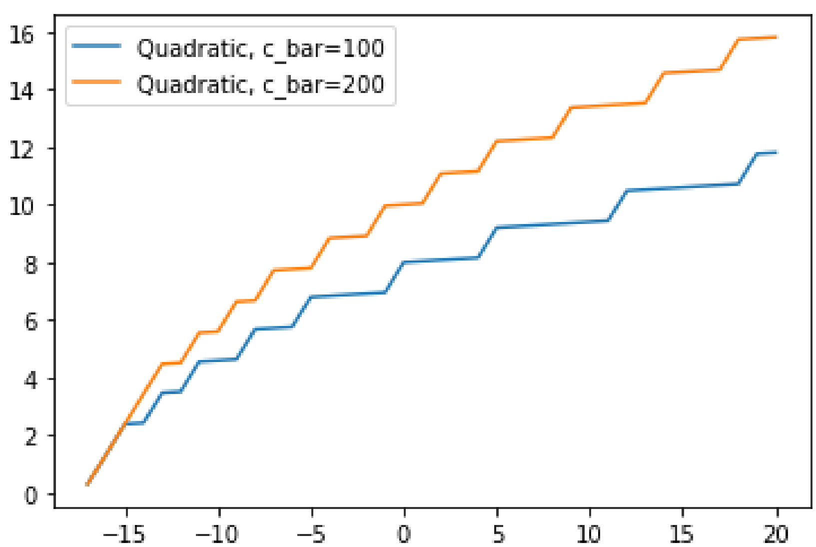

Figure 2: Consumption Function - Quadratic utility sensibility over $\bar{c}$

Similar to the previous graph in Figure 2 we can observe the different values of the consumption estimate but this time taking into consideration a Quadratic utility function for the certainty case. On the contrary to the previous case we can observe how the variation of the slope change with respect of the $\bar{c}$ value, obtaining lower values the bigger the $\bar{c}$.

3) Uncertainty - Quadratic utility

Following, we present the results for the uncertainty case of quadratic utility with it's respective shocks with the Quadratic utility in the certainty case:

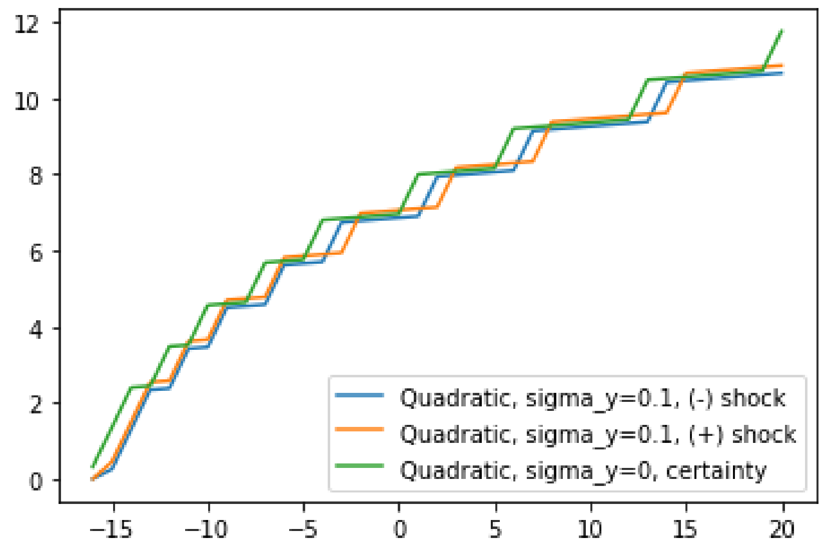

Figure 3: Consumption Function - Quadratic utility sensibility over shocks

In the Figure 3 we estimate the consumption policy for the uncertainty cases, regarding the possible different shocks. Additionally we compare this two functions with respect of the previous certainty case. Then, we can conclude from that the introduction of shocks in savings that the consumption of the individual does not change significantly but there is a small positive difference in the case of certainty.

4) Uncertainty/Certainty - Assets policy

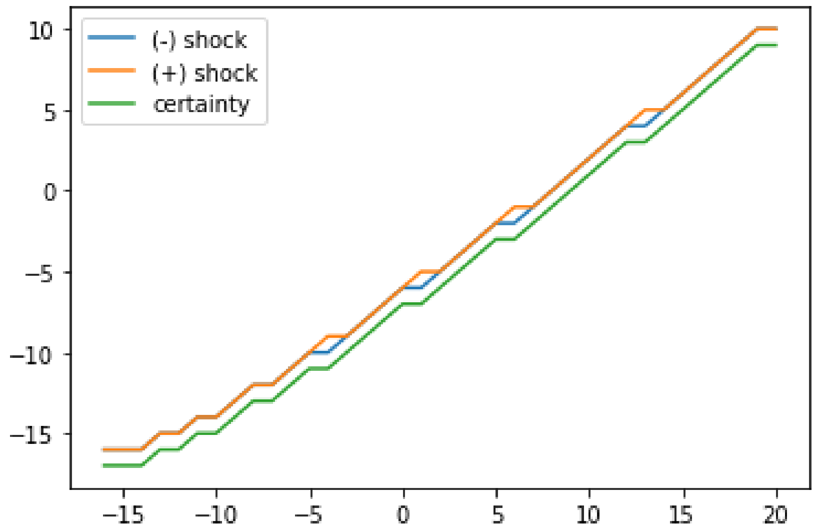

Figure 4: Consumption Function - Quadratic utility sensibility over shocks

For last, we present the policy function for the assets taking in consideration the previously explained cases for uncertainty/certainty. In the Figure 4 we can observe how the policy function of assets is coherent with the consumption policy previously seen. On the contrary that in the case of consumption we can observe that the policy for assets is higher when we have the shock, this change may be due to the necessity of been more precarious.

FINITE HORIZON

Secondly, we present the results for the Partial Equilibrium model taking into consideration a finitely lived household economy, specifically using a T=45. Once again we will differentiate by utility function, then we can obtain the following results:

1) Consumption and Assets - CRRA utility for certainty case

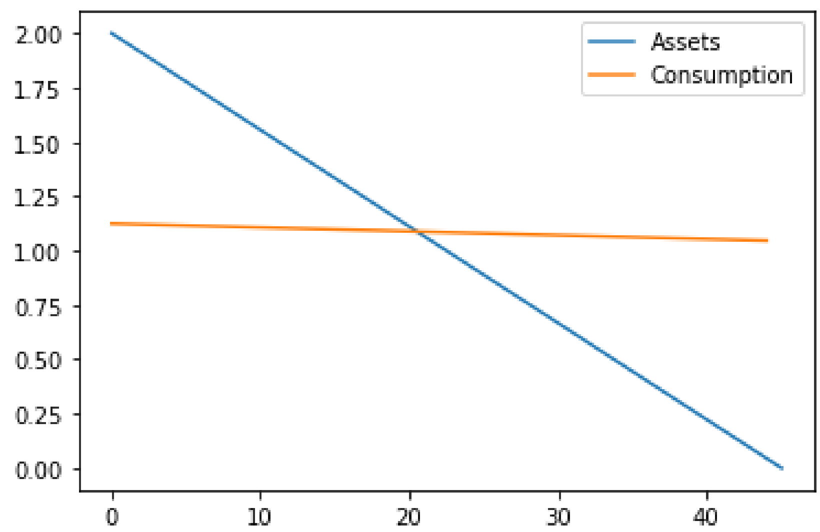

Figure 5: Consumption and Assets Policy Function - CRRA utility

In the Figure 5, we obtain the Consumption and Assets path for the finite case taking into consideration the T=45. The values observed are coherent to the literature because while the consumption reduce in a small proportion during the time of estimation the value of assets drastically change passing from 2 to 0 at the end of the finite period, the individual end the studied period with 0 assets.

2) Consumption sensitivity over initial Capital - CRRA utility for certainty case

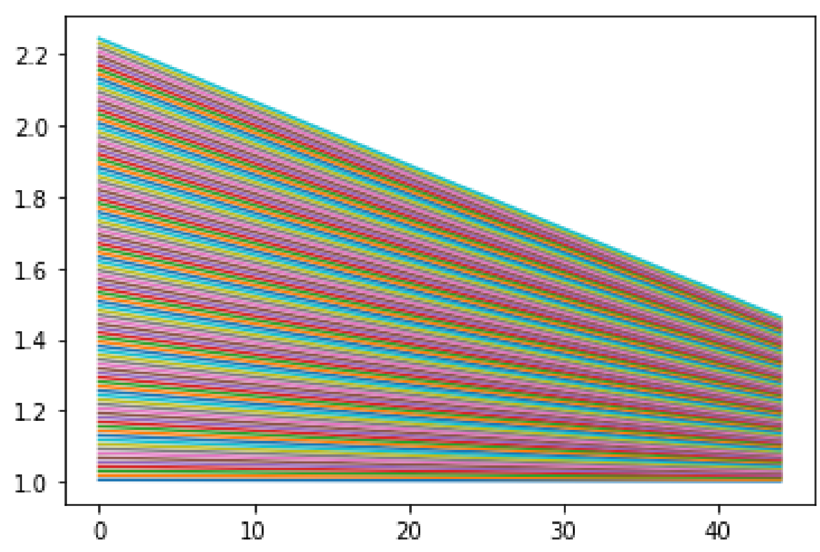

Figure 6: Consumption - CRRA utility

For better understanding the sensibility of consumption this time we take into consideration different measures of initial capital from the individuals. In Figure 6 we can observe the reduction of consumption over time, the lower the initial capital the lower the negative slope.

3) Consumption sensitivity taking different ages - CRRA utility for certainty case

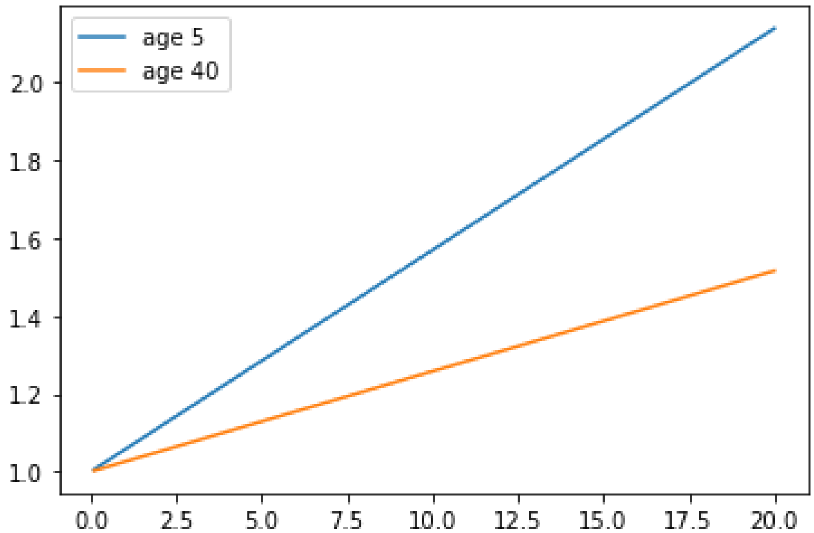

Figure 7: Consumption - CRRA utility

Additionally we represent the difference in policy consumption policy function over capital taking into consideration different ages specified in the statements. Then, in Figure 7, we can observe that even both values starts in close initial capital near 1 both ages the value diverge substantially when the capital is bigger, maintaining higher values for people with lower age.

4) Consumption and Assets - Quadratic utility for certainty case

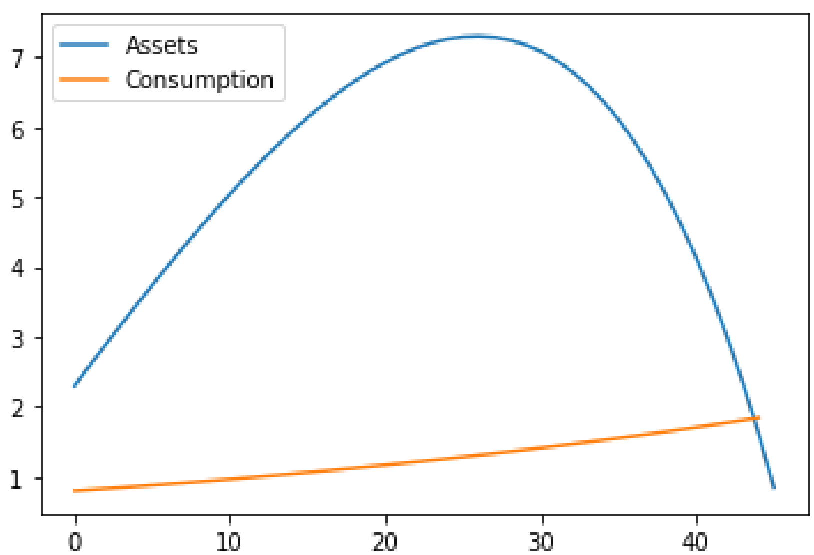

Figure 8: Consumption and Assets Policy Function - Quadratic utility

In the Figure 8, we observe the same functions that in the case of CRRA but taking into consideration a Quadratic utility. In contrast from the previous results we can observe that the consumption function has a slightly increase over time and the assets function has a Inverted “U” shape having the highest point around t=25.

5) Consumption sensitivity over initial Capital - Quadratic utility for certainty case

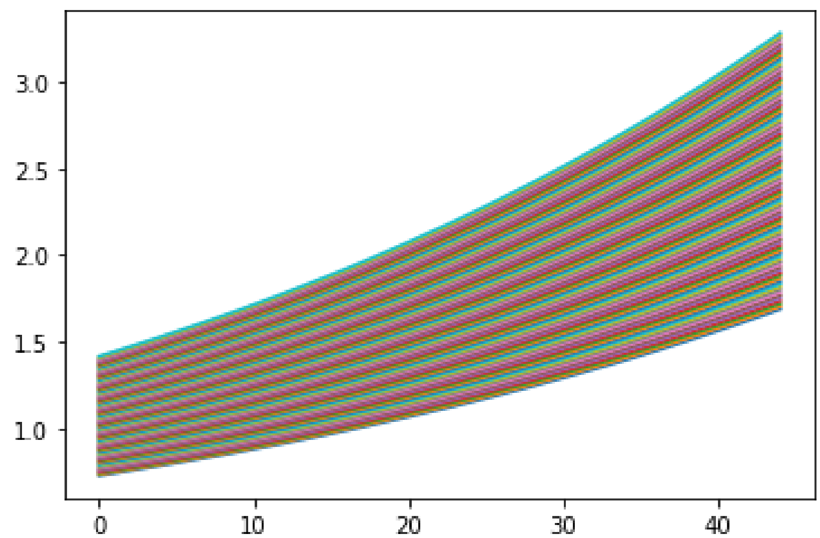

Figure 9: Consumption - Quadratic utility

Now proving the result taking into consideration different levels of initial capital we obtain Figure 9 where we can observe that in contrast to the previous case the consumption function have a positive slope with a convex shape, having grater increment for values with high initial capital.

6) Consumption sensitivity taking different ages - Quadratic utility for certainty case

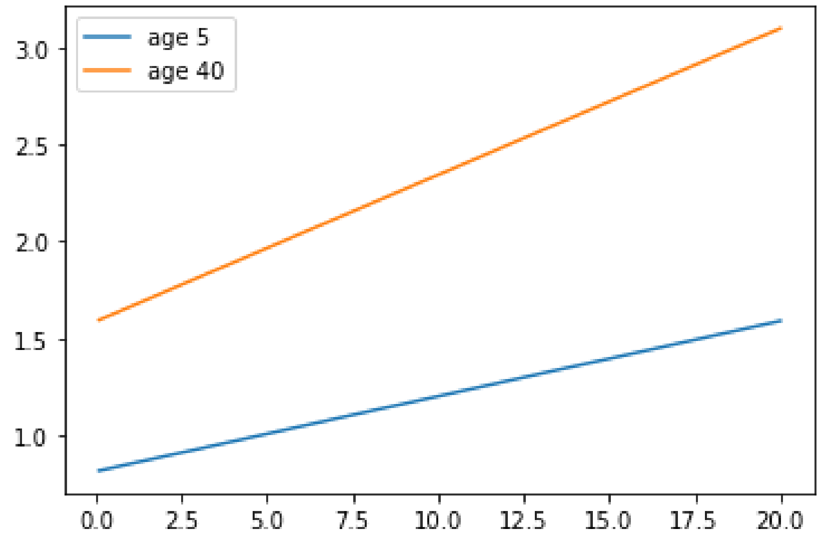

Figure 10: Consumption - Quadratic utility

Finally we can observe that the case of consumption over capital taking into consideration different ages, so in Figure 10 we can see that in this case the initial value is different for both cases. The individual for a age of 40 have a permanent higher consumption in comparison with the individuals at age 5.

GENERAL EQUILIBRIUM

Finally, we will present the results for the General Equilibrium case considering the statement of II.5. For this exercise we will assume a Cobb Douglas production function maintaining the values of $\alpha = 0.33$. Once again we will be using a Value function iteration for solving the problem specified. For better understanding of the following results firstly we will be studying the Supply and Demand individually and then we will pass to the Equilibrium point.

1) Supply

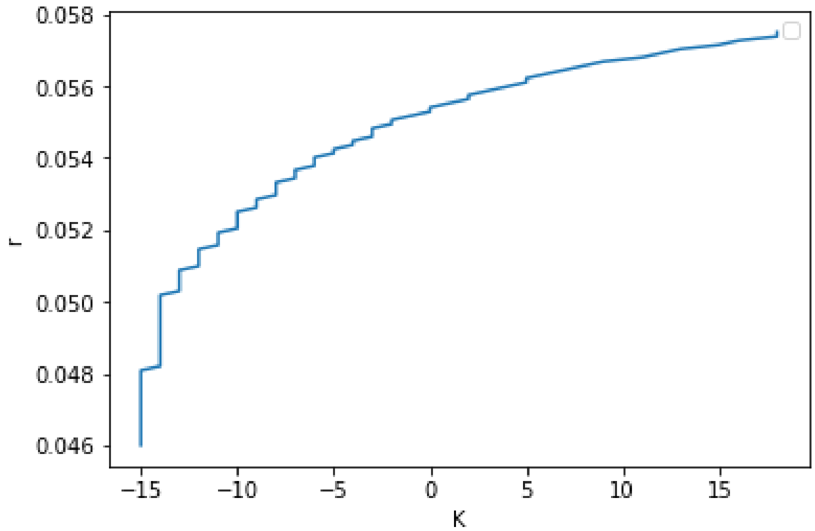

Figure 11: Supply of Capital

In the Figure 11 we can observe the supply of capital for a representative agent from the model. Then we obtain the supply of capital from the level of assets obtained in the steady state. Like we can observe the Supply trend is coherent with the literature maintaining a positive slope and concave shape. The higher the interest rate, the higher the supply of capital.

2) Demand

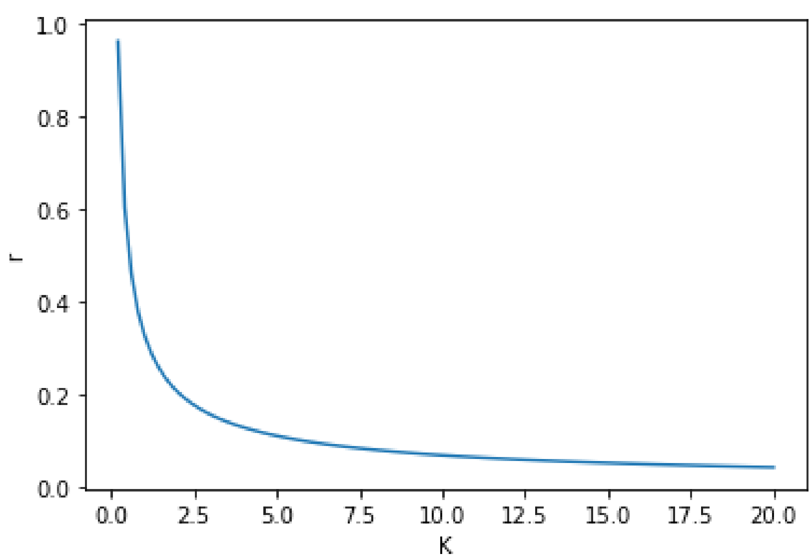

Figure 12: Demand of Capital

On the other hand now we can observe Figure 12 in which we have the capital demand for a representative agent. The obtained demand has a convex shape with a negative slope asymptotic to the figure axis. Like we can observe the lower the Interest rate, the lower the demand of capital.

3) Equilibrium of the ABHI model

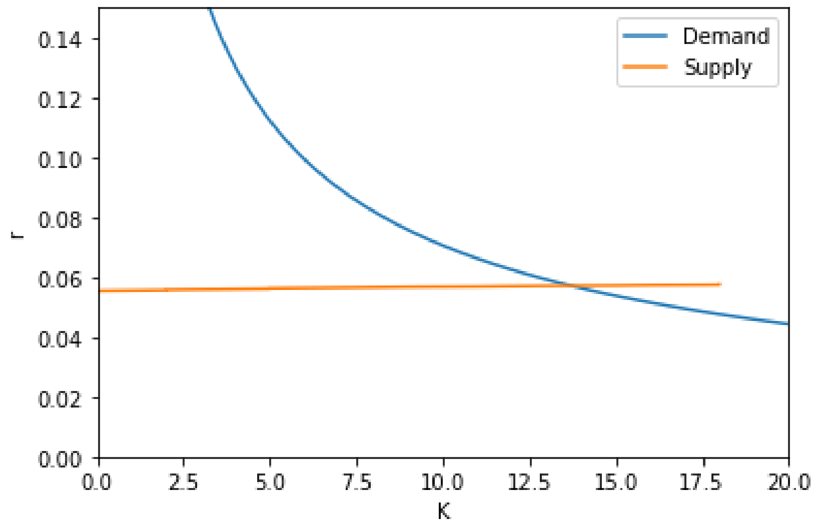

Figure 13: Equilirbium ABHI model

Then merging both of the previous graphs we can obtain the Figure 13 in which we emphasises the Equilibrium point between Supply and Demand of capital. Like we can observe the Equilibrium is achieved around the 6% of interest rate of return, which is higher than the studied literature. Additionally, one main concern is that the supply of assets is limited which makes our supply of capital maintain similar values at different levels of interest rate.

4) Example of Aiyagari model

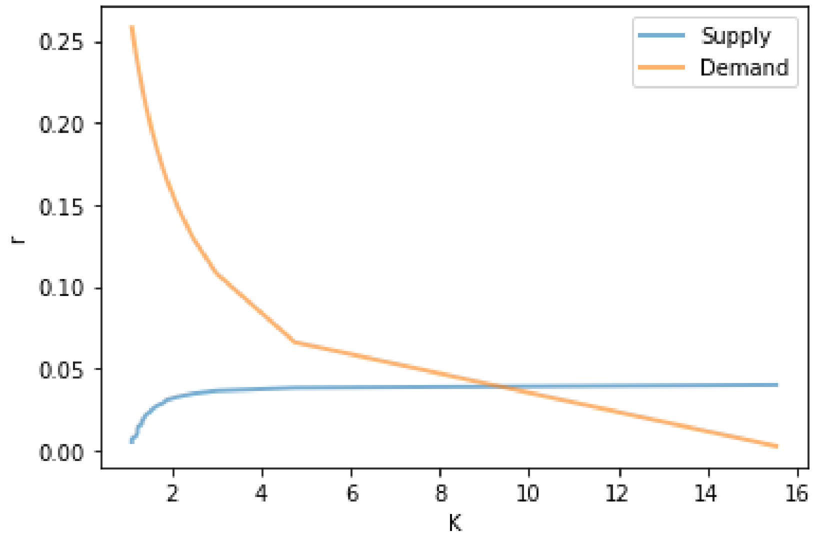

Figure 14: Aiyagari model

For obtaining the Aiyagari Model (1994), we will be using the code presented in: "Quantitative Economics with Python. Sargent and Stachurski. 2020", because when we try to implement the parameters specified in the model we don't obtain the wanted results. Then in the Figure 14 we can observe how implementing more realistic feature we can also simulate an Equilibrium point. Due to the change of methodology this Figure is not comparable to the previous ones but is comparable with the Krueger, Mitman and Perri to which the equilibrium is significantly similar.