Logistic Regression - Complete Breakdown

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1. In-Depth and Specific Intuitive Understanding

Logistic Regression is a **classification algorithm**, despite its name containing "regression." Unlike linear regression, which predicts continuous values, logistic regression predicts **probabilities** for binary or multi-class classification problems.

Key Idea

- Instead of fitting a straight line, logistic regression fits an **S-shaped curve (sigmoid function)** that maps any real-valued number into the **range [0,1]**.
- This allows us to interpret the output as a **probability** of belonging to a certain class.
- We then apply a **threshold** (e.g., 0.5) to make a classification decision.

Mathematical Model

Instead of using a linear function:

$$y = w_0 + w_1x_1 + w_2x_2 + ... + w_nx_n$$

we apply the **sigmoid function** $\sigma(z)$ to convert it into a probability:

$$p = \sigma(z) = \frac{1}{1 + e^{-z}}$$

where:

$$z = w_0 + w_1x_1 + w_2x_2 + ... + w_nx_n$$

- If p is **greater than 0.5**, predict **class 1**.
- If p is **less than 0.5**, predict **class 0**.

This prevents the model from predicting invalid probabilities like in linear regression.

2. When Logistic Regression is Used and When It Should Be Avoided

✅ When to Use It

- When the target variable is **binary (0/1, yes/no, spam/not spam, etc.)**.
- When the relationship between the features and output is **approximately linear in log-odds space**.
- When **interpretability** is important (coefficients indicate feature importance).
- When data is **not too large**, making optimization via gradient descent feasible.

❌ When to Avoid It

- If the data is **non-linearly separable**, logistic regression will not work well.
- If there are **many correlated features**, the model may become unstable.
- If the dataset is **highly imbalanced**, logistic regression can be biased toward the majority class.
- If we need a **highly complex decision boundary**, models like decision trees or neural networks may perform better.

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3. When It Fails to Converge and How to Avoid That

When Logistic Regression Fails

- **Learning rate (α) is too high**: Causes divergence due to large updates in gradient descent.
- **Features are highly correlated (multicollinearity)**: Makes weight updates unstable.
- **Severely imbalanced classes**: If one class dominates, the model may not learn meaningful decision boundaries.
- **Outliers**: Logistic regression is sensitive to extreme values, leading to slow or incorrect convergence.

How to Ensure Convergence

✅ Use **feature scaling** (Standardization: mean = 0, variance = 1) to stabilize training.
✅ Use **regularization (L1/L2)** to handle multicollinearity.
✅ Adjust **learning rate** dynamically (e.g., using Adam optimizer).
✅ Use **oversampling (SMOTE)** or class weighting for imbalanced datasets.

When Logistic Regression Always Converges

- If the dataset is **well-conditioned (scaled, no multicollinearity)**.
- If an **appropriate learning rate** is used.
- If the **decision boundary is approximately linear**.

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4. Advantages and Disadvantages

Advantages

✅ **Simple and interpretable** (coefficients represent feature importance).

✅ **Computationally efficient** for small to medium-sized datasets.

✅ **Outputs probabilities**, which can be useful for decision-making.

✅ **Can be regularized** (L1/L2) to prevent overfitting.

Disadvantages

❌ **Assumes a linear decision boundary in log-odds space** (fails for non-linearly separable data).

❌ **Sensitive to outliers.**

❌ **Not ideal for large feature spaces** (SVMs and Neural Networks perform better).

❌ **Struggles with imbalanced data.**

5. Intuitive Algorithm / Pseudo Code

1. **Initialize weights** w and bias w_0 randomly.
2. **For each training epoch**:
   - Compute z:

     $$z = w_0 + \sum_{j=1}^{n} w_j x_j$$

   - Apply **sigmoid function**:

     $$p = \frac{1}{1 + e^{-z}}$$

   - Compute **binary cross-entropy loss**:

     $$J(w) = -\frac{1}{m} \sum_{i=1}^{m} [y_i \log p_i + (1 - y_i) \log(1 - p_i)]$$

   - Compute **gradients**:

     $$w_j := w_j - \alpha \frac{\partial J}{\partial w_j}$$

   - Update weights and bias.
3. **Repeat until convergence**.

6. Mathematical and Logical Breakdown

Loss Function (Binary Cross-Entropy)

$$J(w) = -\frac{1}{m}\sum_{i=1}^{m}\left[y_i \log p_i + (1 - y_i)\log(1 - p_i)\right]$$

where:

- y_i is the actual label (0 or 1).
- p_i is the predicted probability.

Gradient Descent Update Rules

$$\frac{\partial J}{\partial w_j} = \frac{1}{m}\sum_{i=1}^{m}(p_i - y_i)x_{ij}$$

$$w_j := w_j - \alpha\frac{\partial J}{\partial w_j}$$

$$w_0 := w_0 - \alpha\frac{\partial J}{\partial w_0}$$

where α is the **learning rate**.

7. Manual Implementation in Python

```python
import numpy as np

class LogisticRegressionGD:
    def __init__(self, learning_rate=0.01, epochs=1000):
        self.learning_rate = learning_rate
        self.epochs = epochs
        self.weights = None
        self.bias = None

    def sigmoid(self, z):
        """Compute sigmoid function"""
        return 1 / (1 + np.exp(-z))

    def fit(self, X, y):
        """Train logistic regression using gradient descent"""
        m, n = X.shape
        self.weights = np.zeros(n)
        self.bias = 0

        for _ in range(self.epochs):
            # Compute linear combination (z)
            z = np.dot(X, self.weights) + self.bias

            # Apply sigmoid activation
            p = self.sigmoid(z)

            # Compute gradient
            dw = (1/m) * np.dot(X.T, (p - y))
            db = (1/m) * np.sum(p - y)

            # Update weights
            self.weights -= self.learning_rate * dw
            self.bias -= self.learning_rate * db

    def predict(self, X):
        """Predict class labels (0 or 1)"""
        probabilities = self.sigmoid(np.dot(X, self.weights) + self.bias)
        return (probabilities >= 0.5).astype(int)
```

⌘K for Command, ⌘L for Cascade

8. Scikit-Learn Implementation (Fully Commented)

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```python
from sklearn.linear_model import LogisticRegression
from sklearn.model_selection import train_test_split
from sklearn.datasets import make_classification

# Generate synthetic dataset
X, y = make_classification(n_samples=100, n_features=2, n_classes=2, random_state=4

# Split into training (80%) and testing (20%)
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, random_sta

# Initialize Logistic Regression model
model = LogisticRegression()

# Train the model
model.fit(X_train, y_train)

# Predict class labels
y_pred = model.predict(X_test)

# Print accuracy
print("Model Accuracy:", model.score(X_test, y_test))
```

Final Summary

- Logistic Regression is a classification algorithm that predicts probabilities.
- It uses the sigmoid function to map outputs to [0,1].
- It is best for binary classification with a linear decision boundary.
- Gradient Descent optimizes the cross-entropy loss function.
- Regularization helps with multicollinearity and overfitting.