Memory updated

Independent Component Analysis (ICA) – A Deep Dive

1. What is Independent Component Analysis (ICA)?

Independent Component Analysis (ICA) is a **blind source separation** method used to **extract independent signals from mixed data**.

Intuitive Example

Imagine you have **multiple microphones in a room** recording **a mix of people talking**. Each microphone captures a combination of voices.
ICA can **separate** the voices by assuming they are **statistically independent**.

Key Idea: Unlike PCA (which maximizes variance), ICA looks for signals that are **statistically independent**.

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2. When to Use & Avoid ICA

✅ Use ICA When:

- You need to **separate mixed signals** (e.g., audio processing, EEG data, financial signals).
- You assume that **sources are independent**.

❌ Avoid ICA When:

- Data is **not independent** → ICA fails if the signals are correlated.
- There are **too many noisy components** → ICA is sensitive to noise.
- You need **ordered components** → ICA does not rank components by importance like PCA.

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3. How ICA Works – Step-by-Step Algorithm

ICA tries to find a **linear transformation** that makes the transformed components **statistically independent**.

1. **Center & Whiten the Data** – Ensure zero mean and decorrelation.
2. **Initialize Random Unmixing Matrix** W.
3. **Iteratively Maximize Non-Gaussianity** (because independent sources are non-Gaussian).
4. **Compute Independent Components**.

5. Step-by-Step Pseudo Code for ICA

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```plaintext
# Step 1: Preprocessing
1. Subtract the mean to center the data.
2. Whiten the data using PCA.

# Step 2: Initialize Unmixing Matrix
1. Randomly initialize W.

# Step 3: Optimize W using Non-Gaussianity
FOR each iteration:
    a. Compute output S = W X.
    b. Apply non-linearity (e.g., tanh).
    c. Update W using gradient ascent.
    d. Normalize W.

# Step 4: Extract Independent Components
1. Compute final S = W X.
```

4. Mathematical Breakdown

ICA assumes that **observed signals** X are mixtures of **independent source signals** S:

$$X = AS$$

where:

- X is the **observed data matrix** $(n \times d)$.
- A is the **mixing matrix**.
- S is the **unknown independent source matrix**.

We aim to **recover** S **from** X by finding an **unmixing matrix** W:

$$S = WX$$

Step 1: Center & Whiten the Data

Before applying ICA, we **center** the data:

$$X' = X - \mu$$

where μ is the mean.

Next, we **whiten** the data using Principal Component Analysis (PCA):

$$X_{\text{whitened}} = D^{-1/2} E^T X'$$

where:

- E is the eigenvector matrix of XX^T.
- D is the diagonal matrix of eigenvalues.

This ensures the features are **uncorrelated** and have unit variance.

Step 2: Estimate the Unmixing Matrix W

ICA finds W by **maximizing statistical independence** using **non-Gaussianity** (since independent signals are non-Gaussian).

A common function to maximize is the **Kurtosis**:

$$K(y) = E[y^4] - 3(E[y^2])^2$$

where $y = WX$.

Alternatively, we can use **Negentropy**:

$$J(y) = H(y_{\text{Gaussian}}) - H(y)$$

where $H(y)$ is the entropy of y.

Step 3: Iterative Optimization

We update W using **gradient ascent**:

$$W^{(t+1)} = W^{(t)} + \alpha \left[I - g(W^{(t)}X)W^{(t)} \right] W^{(t)}$$

where:

- $g(y)$ is a **non-linearity function** (e.g., $\tanh(y)$).
- α is the learning rate.

Step 4: Compute Independent Components

Once W converges, we extract the independent sources:

$$S = WX$$

6. Fully Commented Manual Implementation (From Scratch)

```python
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_moons

class ICA:
    """
    Implementation of Independent Component Analysis (ICA) using FastICA algorithm.
    """
    def __init__(self, n_components, max_iters=200, tol=1e-4):
        """
        Initialize ICA.
        :param n_components: Number of independent components.
        :param max_iters: Maximum number of iterations.
        :param tol: Convergence threshold.
        """
        self.n_components = n_components
        self.max_iters = max_iters
        self.tol = tol
        self.W = None  # Unmixing matrix

    def fit(self, X):
        """
        Fit ICA to the dataset.
        :param X: Data matrix (n_samples, n_features).
        """
        # Step 1: Center the data
        X -= X.mean(axis=0)

        # Step 2: Whiten the data
        cov = np.cov(X, rowvar=False)
        eigvals, eigvecs = np.linalg.eigh(cov)
        X_whitened = np.dot(X, eigvecs / np.sqrt(eigvals + 1e-5))

        # Step 3: Initialize random unmixing matrix
        self.W = np.random.randn(self.n_components, self.n_components)

        # Step 4: Fast ICA Iteration
        for _ in range(self.max_iters):
            W_new = np.dot(np.tanh(np.dot(X_whitened, self.W.T)).T, X_whitened) / X.shape[0] - np.eye(self.n_components)
            if np.linalg.norm(W_new - self.W) < self.tol:
                break
            self.W = W_new

```

```python

    def transform(self, X):
        """
        Transform dataset using learned independent components.
        :param X: Data matrix (n_samples, n_features).
        :return: Independent components.
        """
        X -= X.mean(axis=0)
        return np.dot(X, self.W.T)

# Generate synthetic dataset for ICA
X, _ = make_moons(n_samples=500, noise=0.1, random_state=42)

# Apply ICA manually
ica = ICA(n_components=2)
ica.fit(X)
X_ica = ica.transform(X)

# Visualize ICA results
plt.figure(figsize=(8, 6))
plt.scatter(X_ica[:, 0], X_ica[:, 1], alpha=0.7, c='blue', edgecolor='k')
plt.xlabel('Independent Component 1')
plt.ylabel('Independent Component 2')
plt.title('ICA Transformation on Synthetic Data')
plt.show()
 ⌘K for Command, ⌘L for Cascade
```

7. Scikit-Learn Implementation

python

```python
from sklearn.decomposition import FastICA
from sklearn.datasets import make_moons
import matplotlib.pyplot as plt

# Generate synthetic dataset
X, _ = make_moons(n_samples=500, noise=0.1, random_state=42)

# Apply ICA
ica = FastICA(n_components=2, random_state=42)
X_ica = ica.fit_transform(X)

# Plot results
plt.scatter(X_ica[:, 0], X_ica[:, 1], alpha=0.7)
plt.xlabel('Independent Component 1')
plt.ylabel('Independent Component 2')
plt.title('ICA Transformation')
plt.show()
```

8. Advantages & Disadvantages

✅ Advantages

- Useful for **blind source separation** (e.g., EEG, audio signals).
- Works well when **sources are truly independent**.

❌ Disadvantages

- Sensitive to **noise and outliers**.
- Requires **independent sources**, which is not always realistic.

Final Takeaways

✅ ICA is best for separating mixed signals into independent sources.
✅ Unlike PCA, it focuses on statistical independence, not variance.
✅ FastICA is the most common implementation using non-Gaussianity.
