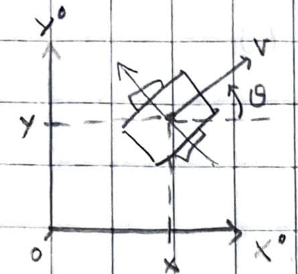
Pregunta 2

OJO:

a)



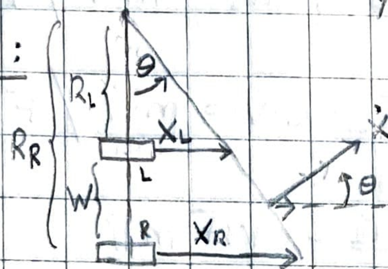
$$\vec{X} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix} \qquad \dot{\vec{X}} = \begin{pmatrix} \dot x \\ \dot y \\ \dot\theta \end{pmatrix}$$

$$\left.\begin{aligned} \dot x &= V\cdot\cos\theta \\ \dot y &= V\cdot\sin\theta \end{aligned}\right\} V = \frac{\dot\phi_R + \dot\phi_L}{2}\cdot r \quad / \; r = \text{radio de ruedas}$$

$$X = \frac{r}{2}\int_0^T (\dot\phi_R + \dot\phi_L)\cos\theta\, dt + X_0$$

$$Y = \frac{r}{2}\int_0^T (\dot\phi_R + \dot\phi_L)\sin\theta\, dt + Y_0$$

$\underline{\dot\theta}$:



$$\dot x = \frac{\dot x_L + \dot x_R}{2}$$

$$W = R_R - R_L$$

$$X_L = R_L\cdot\theta \quad \Big| \quad X_R = R_R\cdot\theta$$

$$\dot X_L = R_L\cdot\dot\theta \quad \Big| \quad \dot X_R = R_R\cdot\dot\theta$$

$$R_L = \dot X_L/\dot\theta$$

$$R_R = \dot X_R/\dot\theta$$

$$W = R_R - R_L$$

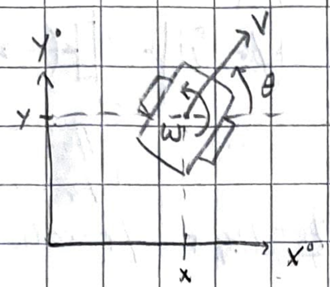
$$W = \frac{\dot X_R - \dot X_L}{\dot\theta} \quad \Big| \quad \begin{aligned} \dot X_R &= \dot\phi_R\cdot r \\ \dot X_L &= \dot\phi_L\cdot r \end{aligned}$$

$$\dot\theta = \frac{\dot\phi_R\cdot r - \dot\phi_L\cdot r}{W}$$

$$\theta = \frac{r}{W}\int_0^T (\dot\phi_R - \dot\phi_L)\, dt + \theta_0$$

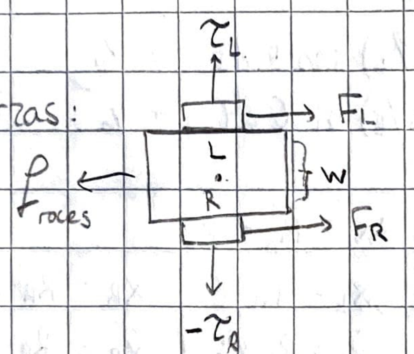
$$\therefore \begin{pmatrix} X \\ Y \\ \theta \end{pmatrix} = \begin{pmatrix} \frac{r}{2}\cdot\int_0^T (\dot\phi_R + \dot\phi_L)\cos\theta\, dt \\ \frac{r}{2}\cdot\int_0^T (\dot\phi_R + \dot\phi_L)\sin\theta\, dt \\ \frac{r}{W}\int_0^T (\dot\phi_R - \dot\phi_L)\, dt \end{pmatrix} + \begin{pmatrix} X_0 \\ Y_0 \\ \theta_0 \end{pmatrix}$$

b)



$$\vec{X} = \begin{pmatrix} x \\ y \\ \theta \\ v \\ w \end{pmatrix} \qquad \dot{\vec{X}} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{v} \\ \dot{w} \end{pmatrix} \qquad \begin{aligned} \dot{x} &= V \cdot \cos\theta \\ \dot{y} &= V \cdot \sin\theta \\ \dot{\theta} &= \omega \end{aligned}$$

Fuerzas:



$$\sum F = ma$$

$$F_R + F_L - f_{roces} = m \cdot \dot{v} \;/\; f_{roces} = c \cdot v$$

$$F_R + F_L - c \cdot v = m \cdot \dot{v} \;/\; F = \frac{\tau}{r}$$

$$\frac{\tau_R + \tau_L}{r} - cv = m \dot{v}$$

$$\dot{v} = \frac{1}{m} \cdot \left(\frac{\tau_R + \tau_L}{r} - c \cdot v \right)$$

$$v = \frac{1}{m} \int_0^T \left(\frac{\tau_R + \tau_L}{r} - c \cdot v \right) dt + V_0$$

$$\sum \tau = J \cdot \alpha$$

$$\frac{W}{2} F_R + \frac{W}{2} F_L - b \cdot w = J \cdot \dot{w}$$

$$\frac{W}{2} \cdot \left(\frac{\tau_R + \tau_L}{r} \right) - b \cdot w = J \dot{w}$$

$$\dot{w} = \frac{\frac{W}{2r} \cdot (\tau_R + \tau_L) - b \cdot w}{J}$$

$$w = \frac{1}{J} \int_0^T \left[\frac{W}{2r} (\tau_R + \tau_L) - b \cdot w \right] dt + w_0$$

$$\therefore \begin{pmatrix} x \\ y \\ \theta \\ v \\ w \end{pmatrix} = \begin{pmatrix} \int_0^T V \cdot \cos\theta \, dt \\ \int_0^T V \cdot \sin\theta \, dt \\ \int_0^T w \, dt \\ \frac{1}{m} \cdot \int_0^T \left(\frac{\tau_R + \tau_L}{r} - c \cdot v \right) dt \\ \frac{1}{J} \cdot \int_0^T \left(\frac{W}{2r} (\tau_R + \tau_L) - b \cdot w \right) dt \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \\ \theta_0 \\ V_0 \\ w_0 \end{pmatrix}$$