5
a)

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma_0 = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\ell = \begin{vmatrix} 0 & 0 & 1 & 1 \\ 1 & -i & 0 & 0 \\ 1 & i & 0 & 0 \\ 0 & 0 & -1 & 1 \end{vmatrix} \qquad \text{Det}(\ell) = 4i \neq 0$$

Luego es L.I

b) $\langle \sigma_0 | \sigma_1 \rangle, \langle \sigma_0 | \sigma_2 \rangle, \langle \sigma_0 | \sigma_3 \rangle$
$\langle \sigma_1 | \sigma_2 \rangle, \langle \sigma_1 | \sigma_3 \rangle, \langle \sigma_2 | \sigma_3 \rangle$

sea $\langle a | b \rangle = Tr(A^{\dagger} B)$

- $\langle \sigma_0 | \sigma_1 \rangle = Tr\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right) = Tr \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = 0$

- $\langle \sigma_0 | \sigma_2 \rangle = Tr\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \right) = Tr \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = 0$

- $\langle \sigma_0 | \sigma_3 \rangle = Tr\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right) = Tr\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = 0$

- $\langle \sigma_1 | \sigma_2 \rangle = Tr\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\right) = Tr\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = 0$

- $\langle \sigma_1 | \sigma_3 \rangle = Tr\left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right) = Tr\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = 0$

- $\langle \sigma_2 | \sigma_3 \rangle = Tr\left(\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right) = Tr\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = 0$

Luego $\sigma_0, \sigma_1, \sigma_3, \sigma_4$ son ortogonales entre sí.