1.6.6

(2) Demostrar

- $\cos(3\alpha) = \cos^3(\alpha) - 3\cos(\alpha)\,\text{sen}^2\alpha$
- $\text{sen}(3\alpha) = 3\cos^2(\alpha)\,\text{sen}(\alpha) - \text{sen}^3(\alpha)$

Tenemos

$$\cos(3\alpha) + i\,\text{sen}(3\alpha) = [\cos\alpha + i\,\text{sen}\,\alpha]^3$$

$$= \cos^3\alpha + 3i\cos^2\alpha\,\text{sen}\,\alpha + 3\cos\alpha\,[i\,\text{sen}\,\alpha]^2 + [i\,\text{sen}\,\alpha]^3$$

$$= \cos^3\alpha + (3\cos^2\alpha\,\text{sen}\,\alpha)i - 3\cos\alpha\,\text{sen}^2\alpha - i\,\text{sen}^3\alpha$$

$$= \underbrace{[\cos^3\alpha - 3\cos\alpha\,\text{sen}^2\alpha]}_{\text{Parte real}} + \underbrace{[3\cos^2\alpha\,\text{sen}\,\alpha - \text{sen}^3\alpha]}_{\text{Parte real}}i$$

luego Parte real $= \cos(3\alpha)$

Parte imaginaria $= \text{sen}(3\alpha)$

Asi:

$$\cos(3\alpha) = \cos^3\alpha - 3\cos\alpha\,\text{sen}^2\alpha$$

$$\text{sen}(3\alpha) = 3\cos^2\alpha\,\text{sen}\,\alpha - \text{sen}^3\alpha$$

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(5) Encontrar las raices de:

a) $(2i)^{1/2}$

$$z = 2e^{i\frac{\pi}{2}}$$

$$z^{1/2} = 2^{1/2} e^{i\left(\frac{\frac{\pi}{2}+2\pi k}{2}\right)} =$$

$$z_1^{1/2} = \sqrt{2}\, e^{i\left(\frac{\frac{\pi}{2}+2\pi\cdot 0}{2}\right)} = \underline{\sqrt{2}\, e^{\frac{i\pi}{4}}}$$

$$z_2^{1/2} = \sqrt{2}\, e^{i\left(\frac{\frac{\pi}{2}+2\pi\cdot 1}{2}\right)} = \underline{\sqrt{2}\, e^{\frac{5}{4}\pi i}}$$

b) $(1-\sqrt{3}i)^{1/2}$

$$|z| = \sqrt{1^2+\sqrt{3}^2} = 2$$

$$\alpha = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\frac{1}{3}\pi$$

$$z = 2e^{i\cdot -\frac{1}{3}\pi}$$

$$z^{1/2} = \sqrt{2}\, e^{i\left(\frac{-\frac{1}{3}\pi+2\pi k}{2}\right)}$$

$$z_1^{1/2} = \sqrt{2}\, e^{i\left(\frac{-\frac{1}{3}\pi+2\pi\cdot 0}{2}\right)} = \sqrt{2}\, e^{i\left(\frac{1}{6}\pi\right)} = \underline{\sqrt{2}\, e^{-\frac{\pi}{6}i}}$$

$$z_2^{1/2} = \sqrt{2}\, e^{i\left(\frac{-\frac{1}{3}\pi+2\pi\cdot 1}{2}\right)} = \sqrt{2}\, e^{i\left(\frac{\frac{5}{3}\pi}{2}\right)} = \underline{\sqrt{2}\, e^{\frac{5\pi}{6}i}}$$

c) $(-1)^{1/3}$

$$z = 1e^{i\pi}$$

$$z^{1/3} = 1^{1/3}\cdot e^{\left(\frac{\pi+2\pi k}{3}\right)}$$

$$z_1^{1/3} = 1\cdot e^{\left(\frac{\pi+2\pi\cdot 0}{3}\right)i} = 1e^{\frac{\pi}{3}i} = \underline{e^{\frac{\pi i}{3}}}$$

$$z_2^{1/3} = 1\cdot e^{\left(\frac{\pi+2\pi\cdot 1}{3}\right)i} = 1e^{\pi i} \quad \underline{e^{\pi i}}$$

$$z_3^{1/3} = 1e^{\left(\frac{\pi+2\pi\cdot 2}{3}\right)i} = \underline{e^{\frac{5\pi}{3}i}}$$

d) $8^{1/6}$

$$z = 8e^{0i}$$

$$z_1^{1/6} = 8^{1/6} e^{\left(\frac{0+2\pi\cdot 0}{6}\right)i} = 8^{1/6} e^{0} = \underline{8^{1/6}}$$

$$z_2^{1/6} = 8^{1/6} e^{\left(\frac{0+2\pi\cdot 1}{6}\right)i} = \underline{8^{1/6} e^{\left(\frac{\pi}{3} i\right)}}$$

$$z_3^{1/6} = 8^{1/6} e^{\left(\frac{0+2\pi\cdot 2}{6}\right)i} = \underline{8^{1/6} e^{\frac{2}{3}\pi i}}$$

$$z_4^{1/6} = 8^{1/6} e^{\left(\frac{0+2\pi\cdot 3}{6}\right)i} = \underline{8^{1/6} e^{\pi i}}$$

$$z_5^{1/6} = 8^{1/6} e^{\left(\frac{0+2\pi\cdot 4}{6}\right)i} = \underline{8^{1/6} e^{\frac{4}{3}\pi i}}$$

$$z_6^{1/6} = 8^{1/6} e^{\left(\frac{0+2\pi\cdot 5}{6}\right)i} = \underline{8^{1/6} e^{\frac{5}{3}\pi i}}$$

e) $(-8-8\sqrt{3}\,i)^{1/4}$

$$|z| = \sqrt{(-8)^2+(-8\sqrt{3})^2} = 16$$

$$\theta = \tan^{-1}\left(\frac{-8\sqrt{3}}{-8}\right) = \frac{1}{3}\pi$$

$$\alpha = \pi + \frac{1}{3}\pi = \frac{4}{3}\pi$$

$$z = 16e^{\left(\frac{4}{3}\pi i\right)}$$

$$z_1^{1/4} = 16^{1/4} e^{\left(\frac{\frac{4}{3}\pi+2\pi\cdot 0}{4}\right)i} = \underline{2e^{\frac{1}{3}\pi i}}$$

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$$z_2^{1/4} = 2e^{\left(\frac{\frac{4}{3}\pi + 2\pi\cdot 1}{4}\right)i} = 2e^{\frac{5}{6}\pi i}$$

$$z_3^{1/4} = 2e^{\left(\frac{\frac{4}{3}\pi + 2\pi\cdot 2}{4}\right)i} = 2e^{\frac{4}{3}\pi i}$$

$$z_4^{1/4} = 2e^{\left(\frac{\frac{4}{3}\pi + 2\pi\cdot 3}{4}\right)} = 2e^{\frac{11}{6}\pi i}$$

(6) Demuestre que:

a) $\mathrm{Log}(-ie) = 1 - \frac{\pi}{2} i$

$$\mathrm{Log}(-ie) = \mathrm{Log}\left(e \cdot e^{-\frac{1}{2}\pi i + 2n\pi}\right) = \ln e + i\left(-\frac{1}{2}\pi + 2n\pi\right)$$

$$= 1 + i\left(-\frac{1}{2}\pi + 2n\pi\right)$$

Si $n = 0 \longrightarrow \underline{1 + \frac{1}{2}\pi i}$

b) $\mathrm{Log}(1-i) = \frac{1}{2}\ln(2) - \frac{\pi}{4} i$

$$\mathrm{Log}\left(\sqrt{2}\, e^{-\frac{\pi}{2} i + 2\pi k}\right) = \ln\sqrt{2} + i\left(\frac{\pi}{2} + 2\pi k\right)$$

$$= \ln 2^{1/2} + i\left(-\frac{\pi}{2} + 2\pi k\right)$$

$$\longrightarrow \underline{\frac{1}{2}\ln 2 - \frac{\pi}{2} i}$$

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c) $\operatorname{Log}(e) = 1 + 2n\pi i$

$\operatorname{Log}(e^{0i}) = \operatorname{Log}(e \cdot e^{0i + 2\pi n}) = \operatorname{Ln}(e) + i(2\pi n)$

$= \underline{1 + 2\pi n i}$

d) $\operatorname{Log}(i) = (2n + \frac{1}{2})\pi i$

$\operatorname{Log}(e^{\frac{\pi}{2} i + 2\pi n}) = \operatorname{Ln}(1) + i\left(\frac{\pi}{2} + 2\pi n\right)$

$= \underline{\left(\frac{1}{2} + 2n\right)\pi i}$