$\ldots |q_\alpha\rangle = a^0 + a^i|q_i\rangle = a^0 + a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

(a)

- $|a\rangle + |b\rangle = a^\alpha|q_\alpha\rangle + b^\alpha|q_\alpha\rangle$
  $= (a^\alpha + b^\alpha)|q_\alpha\rangle = c^\alpha|q_\alpha\rangle = |c\rangle \in V$

- $|a\rangle + |b\rangle = a^\alpha|q_\alpha\rangle + b^\alpha|q_\alpha\rangle = (a^\alpha + b^\alpha)|q_\alpha\rangle$
  $= (b^\alpha + a^\alpha)|q_\alpha\rangle = b^\alpha|q_\alpha\rangle + a^\alpha|q_\alpha\rangle$

- $|a\rangle + (|b\rangle + |c\rangle) = a^\alpha|q_\alpha\rangle + (b^\alpha|q_\alpha\rangle + c^\alpha|q_\alpha\rangle)$
  $= a^\alpha|q_\alpha\rangle + ((b^\alpha + c^\alpha)|q_\alpha\rangle)$
  $= a^\alpha|q_\alpha\rangle + (b^\alpha + c^\alpha)|q_\alpha\rangle = (a^\alpha + b^\alpha + c^\alpha)|q_\alpha\rangle$
  $= (a^\alpha + b^\alpha)|q_\alpha\rangle + c^\alpha|q_\alpha\rangle$
  $= (|a\rangle + |b\rangle) + |c\rangle$

- $|a\rangle + |0\rangle = a^\alpha|q_\alpha\rangle + 0^\alpha|q_\alpha\rangle = (a^\alpha + 0^\alpha)|q_\alpha\rangle$
  $= a^\alpha|q_\alpha\rangle = |a\rangle$

- $|a\rangle + |-a\rangle = a^\alpha|q_\alpha\rangle + (-a)^\alpha|q_\alpha\rangle = (a^\alpha - a^\alpha)|q_\alpha\rangle$
  $= 0|q_\alpha\rangle = |0\rangle$

* $\alpha |a\rangle = \alpha\, a^{x} |q_x\rangle = (\alpha a^{x}) |q_x\rangle = |\alpha a\rangle$

* $\alpha(|a\rangle + |b\rangle) = \alpha(a^{x} |q_x\rangle + b^{x} |q_x\rangle)$
$= \alpha\,(a^{x} + b^{x}) |q_x\rangle = (\alpha a^{x} + \alpha b^{x}) |q_x\rangle$
$= (\alpha a^{x} |q_x\rangle + \alpha b^{x} |q_x\rangle) = |\alpha a\rangle + |\alpha b\rangle$

* $(\alpha + \beta) |a\rangle = (\alpha+\beta)(a^{x}) |q_x\rangle$
$= (\alpha a^{x} + \beta a^{x}) |q_x\rangle = (\alpha a^{x}) |q_x\rangle + (\beta a^{x}) |q_x\rangle$
$= |\alpha a\rangle + |\beta a\rangle$

* $\alpha(\beta |a\rangle) = \alpha(\beta (a^{x}) |q_x\rangle) = \alpha(\beta a^{x}) |q_x\rangle$
$= (\alpha\beta a^{x}) |q_x\rangle = |\alpha\beta a\rangle$

* $1 |a\rangle = 1 (a^{x}) |q_x\rangle = (1 \cdot a^{x}) |q_x\rangle = (a^{x}) |q_x\rangle$
$= |a\rangle$

(b) $|b\rangle = b^0 + b^1|q_1\rangle + b^2|q_2\rangle + b^3|q_3\rangle$

$|r\rangle = r^0 + r^1|q_1\rangle + r^2|q_2\rangle + r^3|q_3\rangle$

$|b\rangle \odot |r\rangle = b^0 r^0 + b^0 r^1|q_1\rangle + b^0 r^2|q_2\rangle + b^0 r^3|q_3\rangle$

$+ b^1|q_1\rangle r^0 + b^1|q_1\rangle r^1|q_1\rangle + b^1|q_1\rangle r^2|q_2\rangle + b^1|q_1\rangle r^3|q_3\rangle$

$+ b^2|q_2\rangle r^0 + b^2|q_2\rangle r^1|q_1\rangle + b^2|q_2\rangle r^2|q_2\rangle + b^2|q_2\rangle r^3|q_3\rangle$

$+ b^3|q_3\rangle r^0 + b^3|q_3\rangle r^1|q_1\rangle + b^3|q_3\rangle r^2|q_2\rangle + b^3|q_3\rangle r^3|q_3\rangle$

$= b^0 r^0 + b^0 r^1|q_1\rangle + b^0 r^2|q_2\rangle + b^0 r^3|q_3\rangle$

$+ b^1|q_1\rangle r^0 - b^1 r^1 + b^1 r^2|q_3\rangle - b^1 r^3|q_2\rangle$

$+ b^2|q_2\rangle r^0 - b^2|q_2\rangle r^1|q_1\rangle - b^2 r^2 + b^2 r^3|q_1\rangle$

$+ b^3|q_3\rangle r^0 - b^3 r^1|q_2\rangle - b^3 r^2|q_1\rangle - b^3 r^3$

$= (b^0 r^0 - b^1 r^1 - b^2 r^2 - b^3 r^3) + r^0(b^1|q_1\rangle + b^2|q_2\rangle + b^3|q_3\rangle)$

$+ b^0(r^1|q_1\rangle + r^2|q_2\rangle + r^3|q_3\rangle)$

$+$

$[\,|q_1\rangle(b^2 r^3 - b^3 r^2) + |q_2\rangle(b^1 r^3 - b^3 r^1)$

$+ |q_3\rangle(b^1 r^2 - b^2 r^1)\,]$

$$(b^0 - \vec{r} \cdot \vec{b}) + r^0 \vec{b} + b^0 \vec{r} + \vec{b} \times \vec{r}$$

$$\boxed{(b^0 r^0 - \vec{r} \cdot \vec{b},\ r^0 \vec{b} + r^0 \vec{r} + \vec{b} \times \vec{r})}$$

(c)

$$\langle d|\Theta|r\rangle = a\langle q_0\rangle + S^{(\alpha j)} f^0_\alpha \langle q_j\rangle + A^{(jk)i} b_j r_k \langle q_i\rangle$$

$$= a\langle q\rangle + (S^{\alpha j} f^0_\alpha + S^{j\alpha} f^0_\alpha)\langle q_j\rangle + (A^{jki} b_j r_k - A^{kji} b_j r_k)$$

luego

$$a = b^0 r^0 - \vec{b}\cdot\vec{r} = a\langle q_0\rangle$$

$$S^{\alpha j} f^0 \langle q_j\rangle = S^{0j}\langle q_j\rangle = S^{0j}\langle q_1\rangle + S^{03}\langle q_3\rangle$$

expresándose como

$$S = [\vec{b}\ \vec{r}] = \begin{bmatrix} b^0 & r^0 \\ b^1 & r^1 \\ b^2 & r^2 \\ b^3 & r^3 \end{bmatrix}$$

- $S^{0j}\langle q_j\rangle = b^0 r^1\langle q_1\rangle + b^0 r^2\langle q_2\rangle + b^0 r^3\langle q_3\rangle = b^0\vec{r}$
- $S^{j0}\langle q_j\rangle = r^0\vec{b}$

Entendiendo a A^{jki} como levi-civita

$A^{jki} b_j r_k |a_i\rangle$

$$|b\rangle \odot |r\rangle = b^0 r^0 - \vec{b}\cdot\vec{r} + b^0\vec{r} + r^0\vec{b} + \vec{b}\times\vec{r}$$

d)

$$a = b^0 r^0 - \vec{b}\cdot\vec{r}$$

$$S^{ij} = \begin{pmatrix} b^0 & r^0 \\ b^1 & r^1 \\ b^2 & r^2 \\ b^3 & r^3 \end{pmatrix}$$ Con $A^{ijk} = \varepsilon^{ijk}$

$$|a\rangle \odot |r\rangle = (a^0 r^0 - \vec{a}\cdot\vec{r}, r^0\vec{a} + a^0\vec{r} + \vec{a}\times\vec{r})$$

Haciendo reflexión

$a^\alpha = -a^\alpha$

$r^\alpha = -r^\alpha$

$|a_\alpha\rangle = -|a_\alpha\rangle$

$$|d\rangle = (a^0 r^0 - \vec{a}\cdot\vec{r} - r^0\vec{a} - a^0\vec{r} - \vec{a}\times\vec{r})$$

No puede decirse si es vector o pseudovector.

F)

$|q_1\rangle \odot |q_1\rangle = -1$

$|q_1\rangle \odot |q_2\rangle = |q_3\rangle$

$|q_1\rangle \odot |q_3\rangle = -|q_2\rangle$

$|q_2\rangle \odot |q_1\rangle = -|q_3\rangle$

$|q_2\rangle \odot |q_2\rangle = -1$

$|q_2\rangle \odot |q_3\rangle = |q_1\rangle$

$|q_3\rangle \odot |q_1\rangle = |q_2\rangle$

$|q_3\rangle \odot |q_2\rangle = -|q_1\rangle$

$|q_3\rangle \odot |q_3\rangle = -1$

Forma un grupo isomorfo a la base de cuaternios

i) $|\vec{a}\rangle = \dfrac{|a\rangle^*}{\| \, |a\rangle \|^2} = \dfrac{a_0 - a_j |q_j\rangle}{|a\rangle^* \odot |a\rangle} = \dfrac{a_0 - a_j |q_j\rangle}{a_0^2 + a_j^2}$

$= |a\rangle \odot \overline{|a\rangle} = (a_0 + a_j |q_j\rangle) \odot \dfrac{(a_0 - a_j |q_j\rangle)}{a_0^2 + a_j^2}$

$= \dfrac{(a_0^2 \cancel{+ a_0 a_j |q_j\rangle} \cancel{- a_0 a_j |q_j\rangle} - a_j |q_j\rangle a_j |q_j\rangle}{a_0^2 + a_j^2}$

$= \dfrac{a_0^2 + a_j^2}{a_0^2 + a_j^2} = 1$