⑥ a. Demostrar que: $e^i = \dfrac{e_j \times e_k}{e_i \cdot (e_j \times e_k)}$

$i, j, k = 1, 2, 3$ y permutaciones

Por construcción $a_i \cdot a^j = \delta_i^j$

$e^i = \alpha (e_j \times e_k)$

$e^i \cdot e_i = \alpha (e_j \times e_k) \cdot e_i$

$1 = \alpha\, e_i \cdot (e_j \times e_k)$

$1 = \dfrac{e^i}{(e_j \times e_k)}\, e_i \cdot (e_j \times e_k)$

$e^i = \dfrac{(e_j \times e_k)}{e_i (e_j \times e_k)}$

b. $V = e_1 \cdot (e_2 \times e_3)$, $\tilde{V} = e^1 \cdot (e^2 \times e^3)$

$$\tilde{V} = \frac{e_2 \times e_3}{e_1 \cdot (e_2 \times e_3)} \cdot (e^2 \times e^3)$$ $\langle e_i | e^j \rangle = \delta_i^j$

$$\tilde{V} = \frac{1}{e_1 \cdot (e_2 \times e_3)}$$

$$V \tilde{V} = e_1 \cdot (e_2 \times e_3) \frac{1}{e_1 \cdot (e_2 \times e_3)} = 1$$

c. $a \cdot e^i = 1$

$a \cdot e^i = 1$

$$a \cdot \frac{e_j \times e_k}{e_i \cdot (e_j \times e_k)} = 1$$

$$a \cdot e_j \times e_k = e_i (e_j \times e_k)$$

$\rightarrow a \cdot e_j \times e_k - e_i (e_j \times e_k) = 0$ $\underbrace{e_j \times e_k}_{\neq 0} (a - e_i) = 0$ $a = e_i$

1) d $w_1 = 4\hat{\imath} + 2\hat{j} + \hat{k}$

$w_2 = 3\hat{\imath} + 3\hat{j}$

$w_3 = 2\hat{k}$

$$w^1 = \frac{(w_2 \times w_3)}{w_1 \cdot (w_2 \times w_3)}$$

$$w_2 \times w_3 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 0 \\ 0 & 0 & 2 \end{vmatrix} = (6)\hat{i} - (6)\hat{j}$$

$(1, 2\ 3)$

$$w^1 = \frac{(6\hat{i} - 6\hat{j})}{(4\hat{i} + 2\hat{j} + \hat{k}) \cdot (6\hat{i} - 6\hat{j})} = \frac{1}{2}\hat{i} - \frac{1}{2}\hat{j}$$

$$w^2 = \frac{(w_3 \times w_1)}{w_2 \cdot (w_3 \times w_1)}$$

$$w_3 \times w_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2 \\ 4 & 2 & 1 \end{vmatrix} = (-4)\hat{i} - (-8)\hat{j} + $$

$$= -4\hat{i} + 8\hat{j}$$

$$w^2 = \frac{(-4\hat{i} + 8\hat{j})}{(3\hat{i} + 3\hat{j}) \cdot (-4\hat{i} + 8\hat{j})} = -\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j}$$

$$w^3 = \frac{(w_1 \times w_2)}{w_3 \cdot (w_1 \times w_3)}$$

$$w_1 \times w_2 : \begin{array}{ccc} \hat{\imath} & \hat{\jmath} & \hat{k} \end{array} \begin{vmatrix} 4 & 2 & 1 \\ 3 & 3 & 0 \end{vmatrix} = -3\hat{\imath} - (-3)\hat{\jmath} + (12-6)\hat{k}$$
$$= -3\hat{\imath} + 3\hat{\jmath} + 6\hat{k}$$

$$w^3 = \frac{-3\hat{\imath} + 3\hat{\jmath} + 6\hat{k}}{2\hat{k}(-3 + 3\hat{\jmath} + 6\hat{k})} = -\frac{1}{4}\hat{\imath} + \frac{1}{4}\hat{\jmath} + \frac{1}{2}\hat{k}$$

11) $a = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

B.F $e_i = \left[\left(\frac{4}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{1}{\sqrt{21}}\right), \left(\frac{3}{\sqrt{18}}, \frac{3}{\sqrt{18}}, 0\right), (0, 0, 1)\right]$

Dual $e^i = \left[\left(\frac{1}{2}, -\frac{1}{2}, 0\right), \left(-\frac{1}{3}, \frac{2}{3}, 0\right), \left(-\frac{1}{4}, \frac{1}{4}, \frac{1}{2}\right)\right]$

Componentes Contravariantes

$$\begin{pmatrix} v^1 \\ v^2 \\ v^3 \end{pmatrix} = \begin{pmatrix} 4 & 3 & 0 \\ 2 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}^{-1} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 1 \\ 7/4 \end{pmatrix} \rightarrow \langle w^i | a \rangle$$

$$\begin{pmatrix} v^1 \\ v^2 \\ v^3 \end{pmatrix} = \begin{pmatrix} -1/2 \\ 1 \\ 7/4 \end{pmatrix} \leftarrow \langle w^i | a \rangle$$

$\langle \tilde{a} | w_i \rangle = \quad \tilde{a}_1 = 11 \quad \tilde{a}_2 = 9 \quad \tilde{a}_3 = 6$

$2a + 2b + c = 1$ $\rightarrow$ $2a + b = -1$

$3a + 3b = 2$ $\quad 3a + 3b = 2$

$c^1 = 3$

$3a + 3(-1-2a) = 2$

$3a - 3 - 6a = 2$

$-3a = 5$

$a^2 = -\frac{5}{3}$, $b^3 = \frac{7}{3}$

componentes covariantes

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 & -2 & -1 \\ -1 & 4 & 3 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} a_1 \\ b_2 \\ c_3 \end{pmatrix}$$

$a - 2b - c = 1$

$-a + 4b + 3c = 2$

$c = 2$

$a - 2b = 3$ $\quad a = 3 + 2b$

$-a + 4b = -4$

$-3 - 2b + 4b = -4$

$2b = -1$

$b_2 = -1/2$

$a_1 = 2$

componentes v

$v^i = [(-\frac{5}{3}, \frac{7}{3}, 3)]$, $v_i = [(2, -\frac{1}{2}, 2)]$

7) Matriz hermítica: Matriz = Transpuesta conjugada.

$\langle a|b\rangle = Tr(A^{\dagger}B)$

$$\sigma_0 = I = \overset{e_1}{\begin{pmatrix}1&0\\0&1\end{pmatrix}},\quad \sigma_1 = \overset{e_2}{\begin{pmatrix}0&1\\1&0\end{pmatrix}},\quad \sigma_2 = \overset{e_3}{\begin{pmatrix}0&-i\\i&0\end{pmatrix}},\quad \sigma_3 = \overset{e_4}{\begin{pmatrix}1&0\\0&-1\end{pmatrix}}$$

· Hallar base dual $\langle e^i|e_j\rangle = \delta^i_j$

I)
$\langle e^1|e_1\rangle = 1$
$\langle e^1|e_2\rangle = 0$
$\langle e^1|e_3\rangle = 0$
$\langle e^1|e_4\rangle = 0$

II)
$\langle e^2|e_1\rangle = 0$
$\langle e^2|e_2\rangle = 1$
$\langle e^2|e_3\rangle = 0$
$\langle e^2|e_4\rangle = 0$

III)
$\langle e^3|e_1\rangle = 0$
$\langle e^3|e_2\rangle = 0$
$\langle e^3|e_3\rangle = 1$
$\langle e^3|e_4\rangle = 0$

IV)
$\langle e^4|e_1\rangle = 0$
$\langle e^4|e_2\rangle = 0$
$\langle e^4|e_3\rangle = 0$
$\langle e^4|e_4\rangle = 1$

sea $A = \begin{pmatrix}a^i & b^i\\ c^i & d^i\end{pmatrix}$

1)

$$Tr((e^1)^{\dagger}e_1) = 1 = Tr\left[\begin{pmatrix}a^1 & c^1\\ b^1 & d^1\end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix}1&0\\0&1\end{pmatrix}\right] = Tr\,\frac{1}{\sqrt{2}}\begin{pmatrix}a & c\\ b & d\end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(a^1 + d^1) = 1$$

$$Tr\left[\begin{pmatrix}a^1 & c^1\\ b^1 & d^1\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}\right] = Tr\begin{pmatrix}c^1 & a^1\\ d^1 & b^1\end{pmatrix} = \frac{1}{\sqrt{2}}(c^1 + b^1) = 0$$

$$Tr\left[\begin{pmatrix}a^1 & c^1\\ b^1 & d^1\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix}\right] = Tr\begin{pmatrix}c^1 i & -a^1 i\\ d^1 i & -b^1 i\end{pmatrix} = \frac{1}{\sqrt{2}i}(c^1 i - b^1 i) = 0$$

$$\|e\| = \sqrt{\langle}$$

$$Tr\left[\begin{pmatrix} a^1 & c^1 \\ b^1 & d^1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right] = Tr\begin{pmatrix} a^1 & -c^1 \\ b^1 & -d^1 \end{pmatrix} = \frac{1}{\sqrt{2}}(a^1 - d^1) = 0$$

$\frac{1}{\sqrt{2}}(a^1 + d^1) = 1$ $\quad c^1 + b^1 = 0$

$a^1 - d^1 = 0$ $\quad c^1 i - b^1 i = 0$

$a^1 = d^1$

$a^1 + a^1 = \sqrt{2}$ $\quad a^1 = \frac{\sqrt{2}}{2}$ $\quad c^1 = -b^1$

$2a^1 = \sqrt{2}$

$d^1 = \frac{\sqrt{2}}{2}$ $\quad (-b^1) i - b^1 i = 0$

$(-b^1 - b^1) = 0$

$-b^1 = b^1$

$b^1 = 0 \quad c^1 = 0$

Luego $\varsigma e'1 \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} \end{pmatrix}$

2)

$$Tr\left[\begin{pmatrix} a^2 & c^2 \\ b^2 & d^2 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right] = Tr\begin{pmatrix} a^2 & c^2 \\ b^2 & d^2 \end{pmatrix} = \frac{1}{\sqrt{2}}(a^2 + d^2) = 0$$

$$Tr\left[\begin{pmatrix} a^2 & c^2 \\ b^2 & d^2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right] = Tr\begin{pmatrix} c^2 & a^2 \\ d^2 & b^2 \end{pmatrix} = \frac{1}{\sqrt{2}}(c^2 + b^2) = 1$$

$c^2 i - b^2 i = 0$

$a^2 - d^2 = 0$ $\quad c^2 + b^2 = \sqrt{2}$ $\quad c = \sqrt{2} - b^2$

$c^2 i - b^2 i = 0$

$a^2 + d^2 = 0$ $\quad (\sqrt{2} - b^2) i - b^2 i = 0$

$a^2 - d^2 = 0$ $\quad a^2 + a^2 = 0$ $\quad (\sqrt{2} - 2b^2) i = 0$

$a^2 = d^2$ $\quad 2a^2 = 0$ $\quad \sqrt{2} = 2b^2$

$c = \sqrt{2} - \frac{\sqrt{2}}{2}$ $\quad b^2 = \frac{\sqrt{2}}{2}$

$a^2 = 0 \quad d^2 = 0$

$c = \frac{\sqrt{2}}{2}$

$|e^3\rangle \begin{pmatrix} 0 & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$

3)

$a^3 + d^3 = 0$

$c^3 + b^3 = 0$

$c^3 i - b^3 = \sqrt{2} i$

$a^3 - d^3 = 0$

$a^3 = -d^3 \quad (-d^3) - d^3 = 0$

$-2d^3 = 0$

$a^3 = 0 \quad d^3 = 0$

$c^3 = -b^3 \quad (-b^3) i - b^3 i = \sqrt{2} i$

$-2b^3 i = \sqrt{2} i$

$c^3 = \frac{1}{2} i \quad b^3 = \frac{-\sqrt{2}}{2} i$

$\langle e^3| \begin{pmatrix} 0 & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & 0 \end{pmatrix}$

4)

$a^4 + d^4 = 0$

$c^4 + b^4 = 0$

$c^4 i - b^4 i = 0$

$a^4 - d^4 = \sqrt{2}$

$a^4 = -d^4 \quad (-d^4) - d^4 = \sqrt{2}$

$-2d^4 = \sqrt{2}$

$a^4 = \frac{\sqrt{2}}{2} \quad d^4 = -\frac{\sqrt{2}}{2}$

$c^4 = 0$

$b^4 = 0$

$\langle e^4| = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 \\ 0 & -\frac{\sqrt{2}}{2} \end{pmatrix}$

$$\|\sigma_0\| = \sqrt{\langle\sigma_0|\sigma_0\rangle} = \sqrt{Tr(\sigma_0^{\dagger}\sigma_0)} = \sqrt{Tr\begin{pmatrix}1&0\\0&1\end{pmatrix}} = \sqrt{2}$$

$$\|\sigma_1\| = \sqrt{Tr(\sigma_1^{\dagger}\sigma_1)} = \sqrt{Tr\begin{pmatrix}1&0\\0&1\end{pmatrix}} = \sqrt{2}$$

$$\|\sigma_2\| = \sqrt{Tr(\sigma_2^{\dagger}\sigma_2)} = \sqrt{Tr\begin{pmatrix}-1&0\\1&-1\end{pmatrix}} = \sqrt{-2} = \sqrt{2}i$$

$$\|\sigma_3\| = \sqrt{Tr(\sigma_3^{\dagger}\sigma_3)} = \sqrt{Tr\begin{pmatrix}1&0\\0&1\end{pmatrix}} = \sqrt{2}$$

Sea el vector $v = u^1 e_1 + u^2 e_2 + u^3 e_3$

con componentes contravariantes u^i

Sea el vector $a = x$:

$$|a\rangle = a^0\begin{pmatrix}1&0\\0&1\end{pmatrix} + a^1\begin{pmatrix}0&1\\1&0\end{pmatrix} + a^2\begin{pmatrix}0&-i\\i&0\end{pmatrix} + a^3\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$|a\rangle = \begin{pmatrix}a^0+a^3 & a^1-a^2 i\\ a^1+a^2 i & a^0-a^3\end{pmatrix}$$

$$\langle a| = a_0\begin{pmatrix}\frac{\sqrt{2}}{2} & 0\\ 0 & \frac{\sqrt{2}}{2}\end{pmatrix} + a_1\begin{pmatrix}0 & \frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & 0\end{pmatrix} + a_2\begin{pmatrix}0 & \frac{-\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & 0\end{pmatrix}$$

$$+ a_3\begin{pmatrix}\frac{\sqrt{2}}{2} & 0\\ 0 & -\frac{\sqrt{2}}{2}\end{pmatrix}$$