Preface & Chapter 1

Course Textbook



Bayesian Multilevel Models for Repeated Measures Data

A Conceptual and Practical Introduction in R

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Course Outline

Week	Date	Topics
1	Sep. 27	Chapter 1: Introduction: Experiments and Variables
2	Oct. 4	Chapter 2: Probabilities, likelihood, and inference
3	Oct. 11	Chapter 3: Fitting Bayesian regression models with brms
4	Oct. 18	Chapter 4: Inspecting a 'single group' of observations using a Bayesian multilevel model
5	Oct. 25	Chapter 5: Comparing two groups of observations: Factors and contrasts
6	Nov. 1	Chapter 6: Variation in parameters ('random effects') and model comparison
7	Nov. 8	Chapter 7: Comparing many groups, interactions, and posterior predictive checks
8	Nov. 15	Chapter 9: Quantitative predictors and their interactions with factors
9	Nov. 22	Chapter 10: Logistic regression and signal detection theory models
10	Nov. 29	Thanksgiving
11	Dec. 6	Chapter 13: Writing up Experiments

Motivation

• Leave out common information: No boring 'origin story'.

All repeated measures, all the time.

Only useful examples, and present complex examples.

No frequentism, only Bayesian.

Other Books

5	Linear Regression 2	97	•		242			
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Mixed-Effects Models 2: Logistic Regression

9.1 Preliminaries 9.2 Introduction

5.1 Preliminaries

313 313

315

Motivation

• Learn by doing.

• Practice

• Statistics is procedural not declarative knowledge.

Why Bayesian Modeling?

 It works whenever frequentist approaches work, and many times when they don't.

You get a lot more information from Bayesian models.

 Frequentist approaches offer off-the shelf, one-size-fits all hypothesis testing machines.

Bayesian modelling let's you build the model you want.

Class Structure

• One chapter per week.

Quiz on each chapter.

• Short assignment due before next class.

• Final paper.

Chapter 1

Introduction: Experiments and Variables

Experiments and Science

• Experiments: Procedures to help answer some research question.

• Experiments are scientific when they adhere to the 'scientific method'*.

'The' Scientific Method

- 1. Ask questions based on gaps in their knowledge about the world.
- 2. Collect data using codified procedures developed to avoid certain pitfalls and maximize the chance that the collected data can answer their questions.
- 3. Evaluate their questions in light of their data.
- 4. Reach conclusions, where possible, and synthesize their conclusions with their previous knowledge about the world.

Modern Science = Math

- Mostly empirical, and results in the collection of a lot of numbers.
- We then need to describe and quantify patterns in those numbers.

"[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word." - Galileo

"When you can measure what you are speaking about, and express it in numbers, you know something about it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely, in your thoughts advanced to the stage of science."- Lord Kelvin

Modern Science = Math Statistics

"As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality." - Einstein

 We can very rarely be exact and deterministic in our knowledge gathering.

• Statistics allows us to think about the amount of uncertainty or variability in the conclusions we reach.

Controlled Experiments and Effects

- A controlled experiment is one where the experimenter interferes to ensure the fairness of an experiment.
 - Think of 'control' as continuous rather than discrete.

- Effects: Associations between changes in experimental conditions and outcomes.
- Effects may be causal but may not be.

Controlled Experiment: Example

- Two groups of listeners are asked to read a passage.
- One is given caffeine and the other is not. We check for a difference in average reading times.
- If there is, we conclude caffeine has 'an effect' on reading times.
- Does the experiment 'prove' this is true? Can anything?

Experiments and Inference

• Inference: Going from premises to conclusions.

• Deduction: Arguments whose conclusions <u>must</u> be true if the premises are true.

• Induction: Arguments whose conclusions <u>may</u> be true if the premises are true.

Induction

• Induction = Probabilistic reasoning

Most reasoning is inductive

• Almost all experiments are indcutvie.

Problems with Induction I

• The problem of induction: Induction works if and only if the things we did not observe are just like those we did, i.e. if the future is like the past.

"Domestic animals expect food when they see the person who usually feeds them. We know that all these rather crude expectations of uniformity are liable to be misleading. The man who has fed the chicken every day throughout its life at last wrings its neck instead, showing that more refined views as to the uniformity of nature would have been useful to the chicken." – Bertrand Russel

Problems with Induction II

Affirming the consequent: Taking a true if/then statement and flipping it.

Example 1: If you live in Davis, then you Live in California. [wrong] Joe lives in California, therefore, Joe lives in Davis.

Example 2: If caffeine speeds up reading times, the caffeine group will read faster. [wrong] The caffeine group read faster, therefore, caffeine speeds up reading times.

Example 3: If some linguistic theory is true, I will observe some result. [wrong] I observed some result, therefore, some linguistic theory is true.

Problems with Induction III

• Much of modern science may consist of affirming the consequent (especially in linguistics!).

• It is difficult/impossible to 'prove' a general truth based on limited observations.

Let's be humble!

The problem of 'inverse probability'

Probability of data given some hypothesis

$$\rightarrow P(D|H)$$

$$P(H|D)$$
 given some data

Probability of hypothesis given some data

$$P(H|D) \neq P(D|H)$$

Basic Probability

$$P(A)$$
 = Probability that A is true

$$P(B)$$
 = Probability that B is true

$$P(A \& B) = P(B|A) \cdot P(A)$$

$$P(B \& A) = P(A|B) \cdot P(B)$$

How likely is a very tall man to play in the NBA?

- Here is some relevant information:
 - 100,994,367 males over 18 in the USA
 - 3,199 men over 6' 10" in USA
 - 486 Active NBA players
 - 88 players in the NBA are over 6' 10"

How likely is a very tall man to play in the NBA?

Marginal ('overall') probabilities

P(Tall) = Probability that a man is over 6'10"

P(NBA) = Probability of playing in the NBA

Joint probabilities

P(Tall & NBA) = Probability of being tall AND playing in the NBA P(NBA & Tall) = Probability of playing in the NBA AND being tall.

Conditional ('if') probabilities

P(Tall|NBA) = Probability of being tall given/if that you play in the NBA P(NBA|Tall) = Probability of playing in the NBA given/if that you are tall.

How likely is a very tall man to play in the NBA?

of Tall adult males / # of adult males in the USA

$$P(Tall) = 3199 / 100,994,367 = 0.000032$$

of NBA Players / # of adult males in the USA

$$P(NBA) = 486 / 100,994,367 = 0.0000048$$

of Tall, adult male NBA players / # of Tall adult males

$$P(NBA|Tall) = 88 / 3199 = 0.028$$

of Tall, adult male NBA players / # of NBA players

$$P(Tall|NBA) = 88 / 486 = 0.18$$

Conditional Probabilities = Not Reversible

NOT reversible!
$$P(NBA|Tall) = 88 / 3199 = 0.028 \neq P(Tall|NBA) = 88 / 486 = 0.18$$

<u>UNLESS</u> you consider the base rate!

$$P(NBA|T) \cdot P(Tall) = 0.028 \cdot 0.000032 = 0.00000087$$

$$P(Tall|NBA) \cdot P(NBA) = 0.18 \cdot 0.0000048 = 0.00000087$$

Why??

• This inequality is basic logic: $P(NBA|Tall) \neq P(Tall|NBA)$.

Why does the 'base rate' fix this?

$$P(NBA|Tall) \cdot P(Tall) = P(Tall|NBA) \cdot P(NBA)$$

$$P(NBA \& Tall) = P(Tall \& NBA)$$

Is this a Lion?



Data: The visual image.

Hypothesis: This is not a dog, it's a Lion.

Is this a Lion?



P(Image|Lion)

P(Lion|Image)

 $P(Image|Lion) \neq P(Lion|Image)$

Is this a Lion?



 $P(\text{Lion}|\text{Image}) \neq P(\text{Image}|\text{Lion})$

 $P(\text{Lion}|\text{Image}) \cdot P(\text{Image}) = P(\text{Image}|\text{Lion}) \cdot P(\text{Lion})$

 $P(\text{Lion}|\text{Image}) \cdot P(\text{Image}) \propto P(\text{Image}|\text{Lion}) \cdot P(\text{Lion})$



The Bayesian Approach

- Let's say that P(Image|Lion) = 0.8.
- Does this mean you will think this is a lion?
- Does this mean that P(Lion|Image)
 is high?
- What should P(Lion) be?



The Bayesian Approach

- On campus, P(Lion) = 0.0001.
- On an African safari, P(Lion) = 0.1.
- So:

 $P(\text{Lion}|\text{Image}) \cdot P(\text{Image}) \propto P(\text{Image}|\text{Lion}) \cdot P(\text{Lion})$

- ls:
- $P(\text{Lion}|\text{Image}) \propto 0.8 \cdot 0.0001 \propto 0.0008$ on campus.
- $P(\text{Lion}|\text{Image}) \propto 0.8 \cdot 0.1 \propto 0.08 \text{ on safari.}$



The Bayesian Approach

 The same evidence leads to different conclusions in different situations.

 This is how we reason "in real life"!

 Bayesian reasoning helps us work "backwards" from data to hypothesis!



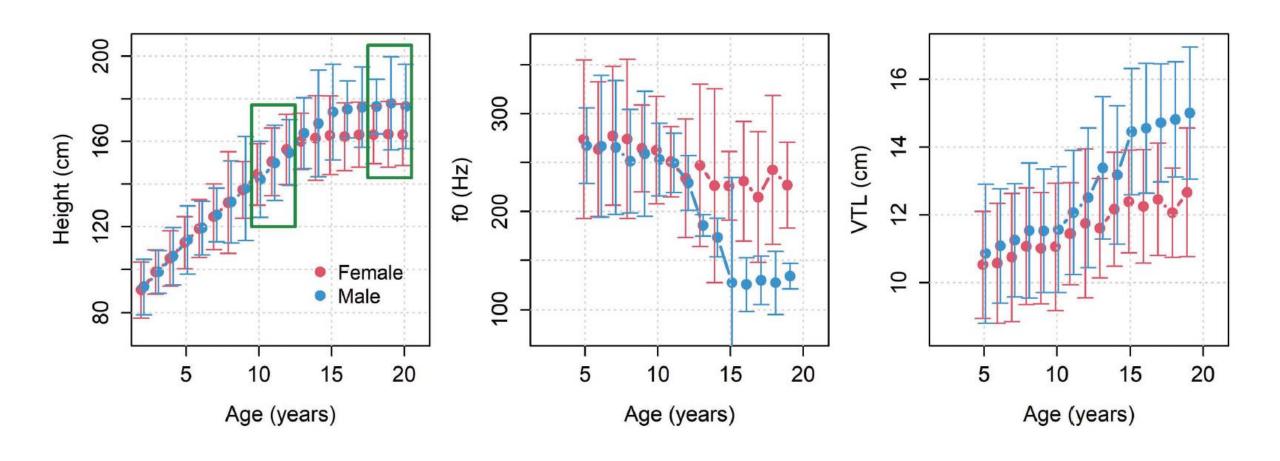
Our Experiment

• An investigation of <u>apparent</u> height, age, and Gender.

• Examining the relationship between apparent speaker characteristics and speech acoustics.

 We played people speech stimuli and collected behavioral measures.

Our Experiment



Our Experimental Methods

- Listeners were played recordings of 139 boys, girls, men, and women saying the word 'heed'.
- Were asked to judge:
 - The height of the speaker in feet and inches.
 - The gender of the speaker (male or female).
 - The age of the speaker (child 10-12, or adult >18).

```
# load package
library ("bmmb")
# load data
data (exp data all)
# see first 6 rows
head (exp data all)
## L C height R S C v vtl f0 dur G A G v A v
## 1 1 g 165.6 a 1 b 12.2 277 237 f c m c
## 2 1 w 173.2 b 1 b 12.2 277 237 f a m c
## 3 1 w 165.6 a 2 b 12.4 287 317 f a m c
## 4 1 q 147.8 b 2 b 12.4 287 317 f c
## 5 1 q 165.6 a 3 b 11.6 219 277 f c
## 6 1 g 158.8 b 3 b 11.6 219 277 f c
```

```
str (exp data all)
## 'data.frame': 4170 obs. of 13 variables:
## $ L : int 1 1 1 1 1 1 1 1 1 ...
## $ C : chr "g" "w" "w" "g" ...
## $ height: num 166 173 166 148 166 ...
       : Factor w/ 2 levels "a", "b": 1 2 1 2 1 2 1 2 1 2 ...
## $ R
## $ S : int 1122334455...
           : Factor w/ 4 levels "b", "g", "m", "w": 1 1 1 1 1 1 1 1 1 1 ...
## $ C v
## $ vtL
           : num 12.2 12.2 12.4 12.4 11.6 11.6 11.9 11.9 12.1 12.1 ...
## $ f0
          : int 277 277 287 287 219 219 260 260 244 244 ...
## $ dur
           : int 237 237 317 317 277 277 318 318 242 242 ...
## $ G
         : chr "f" "f" "f" "f" ...
         : chr "c" "a" "a" "c" ...
   $ A
   $ G V
         : Factor w/ 2 levels "f", "m": 2 2 2 2 2 2 2 2 2 2 ...
## $ A v : Factor w/ 2 levels "a", "c": 2 2 2 2 2 2 2 2 2 2 ...
```

```
# show the first six
head(exp_data_all$height)
## [1] 165.6 173.2 165.6 147.8 165.6 158.8

# show the first element
exp_data_all$height[1]
## [1] 165.6

# show elements 2 to 6
exp_data_all$height[2:6]
## [1] 173.2 165.6 147.8 165.6 158.8
```

```
head(exp_data_all[[3]])
## [1] 165.6 173.2 165.6 147.8 165.6 158.8
head(exp_data_all[["height"]])
## [1] 165.6 173.2 165.6 147.8 165.6 158.8
```

```
head(exp_data_all[,3])
## [1] 165.6 173.2 165.6 147.8 165.6 158.8

head(exp_data_all[,"height"])
## [1] 165.6 173.2 165.6 147.8 165.6 158.8
```

```
exp_data_all[1,]
## L C height R S C_v vtl f0 dur G A G_v A_v
## 1 1 g 165.6 a 1 b 12.2 277 237 f c m c

exp_data_all[1,2]
## [1] "g"
```

Variables

• Placeholders for some value (known or unknown).

• Random variables: Variables whose values aren't known a priori (before observation).

Populations and Samples

• The population: The entire set of values or possible outcomes.

The sample: The set of outcomes/values you observe.

Dependent and Independent Variables

• Dependent/outcome variables: Variables you want to understand.

• Independent/predictor variables: Variables you use to explain.

Types of Variables

 Quantitative: Numeric values on at least an internal scale. Usually a large number of possible outcomes.

• Categorical/factors: Non-numeric values, usually a 'smaller' number of possible outcomes. Possible values are called levels.

 Ordinal: Categorical variables the preserve an order but not a distance between elements.

Categorical Variables

```
# see the first 6 observations
head (exp data all$C v)
## [1] b b b b b b
## Levels: b q m w
# it has levels
levels (exp data all$C v)
## [1] "b" "g" "m" "w"
# each level has numerical values
table (exp data all$C v, as.numeric (exp data all$C v))
##
##
   b 810 0 0 0
##
   g 0 570 0 0
##
    m 0 0 1350 0
    w 0 0 0 1440
```

Continuous Variables?

- Is the variable on a ratio or interval scale? This is a prerequisite for a quantitative value to be used as a dependent variable. An interval scale means that differences between values are meaningful, and a ratio scale additionally means that 0 is meaningful.
- Is the underlying value continuous? Many variables are discrete in practice due to limitations in measurement. However, if the underlying value is continuous (e.g. height, time), then this can motivate treating the measurement as a quantitative dependent variable since fractional values 'make sense'. For example, even if you measure time only down to the nearest millisecond, a value of 0.5 milliseconds is possible and interpretable. In contrast, a value of 0.5 people is not.
- Are there a large number (>50) of possible values the measured variable can take? For example, a die can only take on 6 quantitative values, which is not enough.
- Are most/all of the observed values far from their bounds? Human height does not really get much smaller than about 50 cm and longer than about 220 cm, so it is technically bounded. However, in most cases, our observations are expected to not be away from these boundaries.

Logical Variables

```
2 == 1
## [1] FALSE

"hello" == "hello"
## [1] TRUE

"hello" != "hello"
## [1] FALSE
```

```
# are the values less than or equal to 3?
c(1,2,3,4,5,6,7,8,9,10) <= 3
## [1] TRUE TRUE TRUE FALSE FALSE FALSE FALSE FALSE</pre>
```

```
logical_vector = c(1,2,3,4,5,6,7,8,9,10) <= 3

as.numeric (logical_vector)
## [1] 1 1 1 0 0 0 0 0 0 0

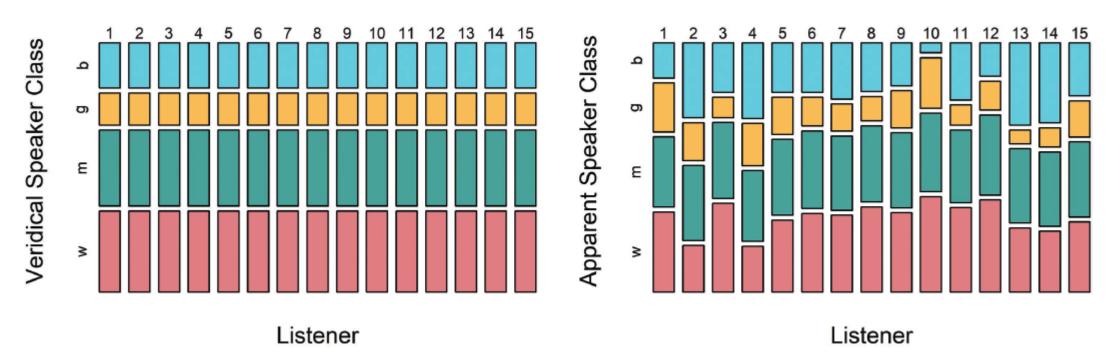
sum (logical_vector)
## [1] 3

sum (c(1,2,3,4,5,6,7,8,9,10) <= 3)
## [1] 3</pre>
```

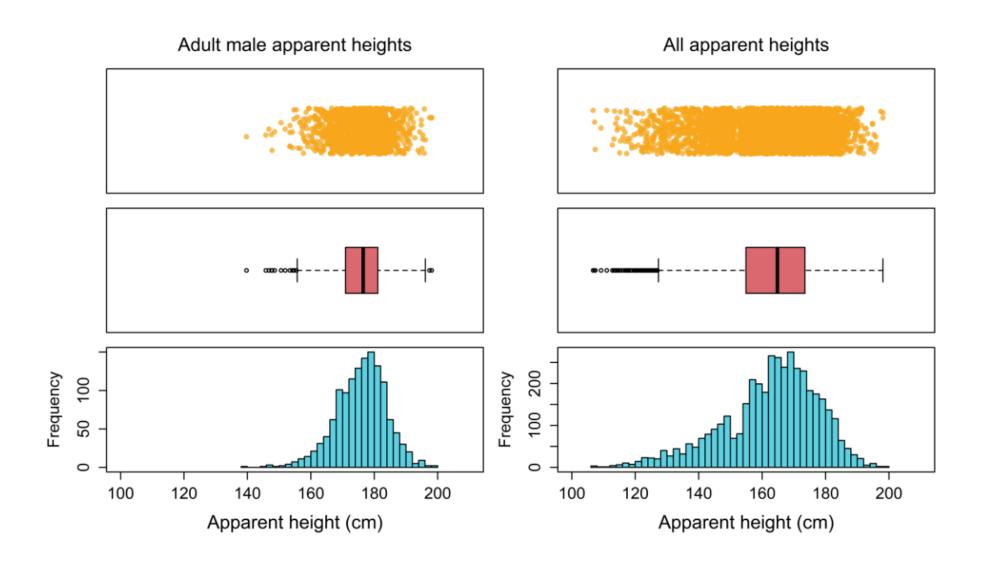
Logical Variables

```
# TRUE if f0 < 175
f0 idx = exp data all$f0 < 175
str (f0 idx)
## logi [1:4170] FALSE FALSE FALSE FALSE FALSE ...
sum (f0 idx)
## [1] 1290
                                # get only rows where f0 < 175, i.e. where f0 idx is TRUE
                                 low f0 = \exp data \ all[f0 \ idx,]
                                nrow(low f0)
                                 ## [1] 1290
                                \max(low f0\$f0)
                                 ## [1] 172
                                 # get only rows where f0 >= 175, i.e. where f0 idx is FALSE
                                 high f0 = \exp data \ all[!f0 \ idx,]
                                 nrow(high f0)
                                 ## [1] 2880
                                 min(high f0$f0)
                                 ## [1] 175
```

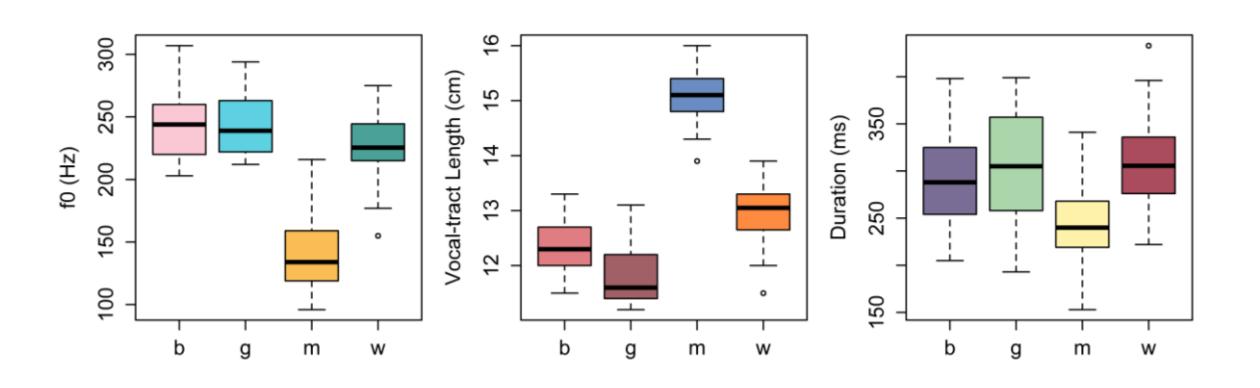
Plotting Distributions



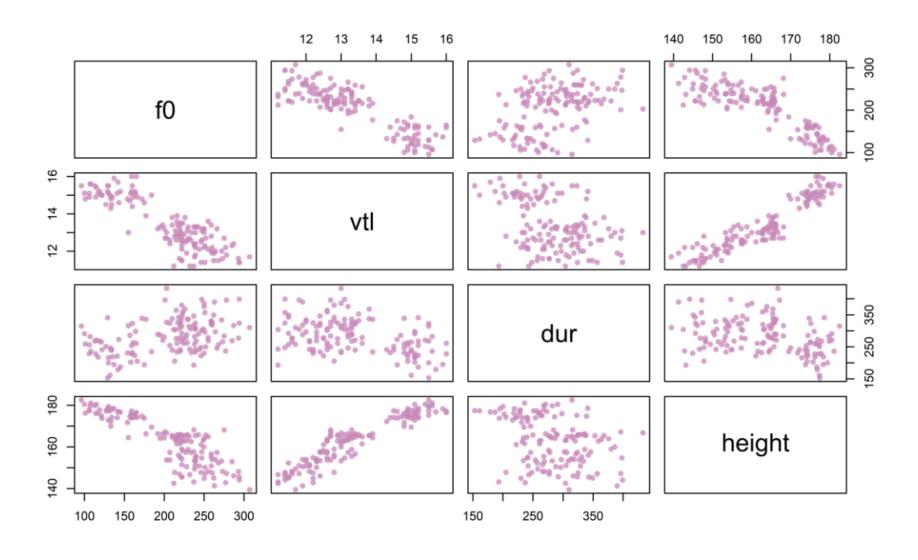
Plotting Distributions



Box Plots

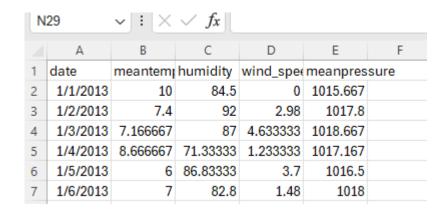


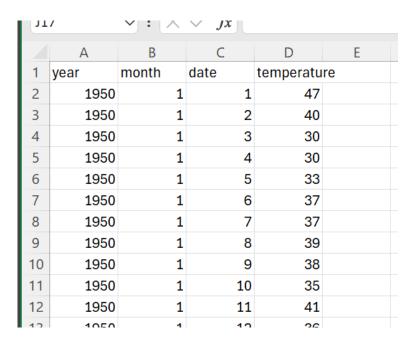
Scatter Plots



Data

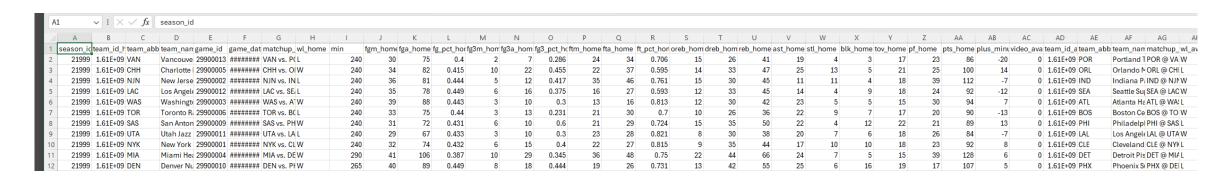
- The book experiment.
- Davis and Delhi temperatures.
- GDP per capita
- Male and female height





Data

NFL, NBA, Soccer data



	A B	С	D	E	F	G	Н	1	J	K	L	M	N	0	Р	Q	R
Rk	Player	Tm	Age	Pos	G	GS	Att	Yds	TD	X1D	Succ.	Lng	Y.A	Y.G	Fmb	code	
	1 DeMarco	I DAL	26	RB	16	16	392	1845	13	89	53.1	51	4.7	115.3	5	MurrDe00	
	2 LeSean M	PHI	26	RB	16	16	312	1319	5	68	44.6	53	4.2	82.4	4	McCoLe01	
	3 Le'Veon B	PIT	22	RB	16	16	290	1361	8	75	49.3	81	4.7	85.1	. 0	BellLe00	
	4 Marshaw	r SEA	28	RB	16	14	280	1306	13	67	52.1	79	4.7	81.6	3	LyncMa00	
	5 Matt Forte	CHI	29	RB	16	16	266	1038	6	65	48.9	32	3.9	64.9	2	FortMa00	
	6 Alfred Mo	r WAS	26	RB	16	16	265	1074	8	53	46	30	4.1	67.1	. 2	MorrAl00	
	7 Arian Fos	t HOU	28	RB	13	13	260	1246	8	55	45	51	4.8	95.8	2	FostAr00	
	8 Frank Gor	SFO	31	RB	16	16	255	1106	4	55	48.2	52	4.3	69.1	. 2	GoreFr00	
	9 Eddie Lac	GNB	24	RB	16	16	246	1139	9	60	45.9	44	4.6	71.2	3	LacyEd00	
	10 Justin For	BAL	29	RB	16	14	235	1266	8	57	42.6	52	5.4	79.1	. 1	ForsJu00	
	11 Mark Ingr	NOR	25	RB	13	9	226	964	9	52	49.1	31	4.3	74.2	3	IngrMa01	
1	40	D.C.T			4.5	_	000		_		10.5				-		

Assignment 1

- Write a report using a qmd file that investigates:
- 1. Apparent height, f0, or VTL (vocal-tract length) in the book data.
- 2. Or investigate any data you want that is 'appropriate' for the next few classes (see next slide).
- Submit the report and the qmd file.

Assignment 1

Use the paper to establish that the variable you are investigating is:

- 1. Quantitative
- 2. Is not highly skewed
- 3. Values are not close to its upper or lower bounds
- Include at least two plots that corroborate this and describe the information they represent in the figure legend.

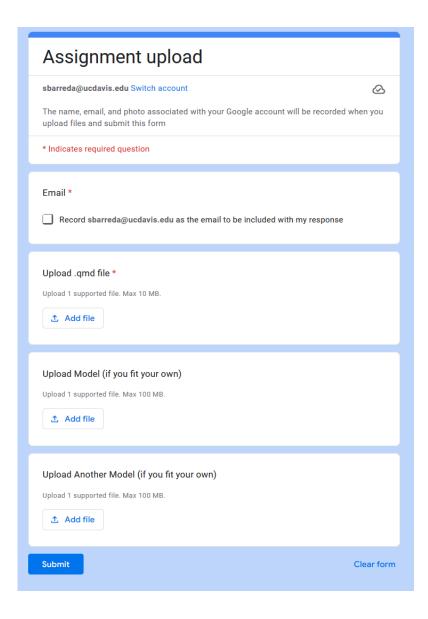
Assignment Submission

	⊘ ▼ + :
	⊘ :
	⊘ :
∷ ⊘ references.bib	⊘ :
	⊘ :

.Rmd Files

- Two options:
 - Use the 'minimal_bookdown_example.zip' example.
 - Use the default new Rmd file and 'knit' it.

Assignment Submission



 Please submit the qmd files and model (if you fit one) using this form.

 Submit the word or pdf document version on Canvas.