Chapter 9

Lin 205C
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Chapter Precap

- Introduce the use of quantitative predictors to model variation along lines.
- First, a model with only a single slope and intercept is introduced.
- Then, models with a single quantitative predictor and a single categorical predictor, but no interaction, are introduced.
- We describe models with an interaction between a quantitative predictor and a categorical predictor, which can represent group-specific intercepts and slopes.
- We present a random effects model including a single quantitative predictor but featuring ‘random slopes’ for each participant.
- Finally, a larger model where variation in both the intercepts and slopes of lines is predicted using two categorical predictors and their interaction, in addition to including listener-dependent ‘random’ effects for all predictors and their interaction.

Data

```
library (brms)
library (bmmb)

data (exp_data)

options (contrasts = c('contr.sum','contr.sum'))

# make a copy of the VTL predictor and center it
exp_data$vtl_original = exp_data$vtl
exp_data$vtl = exp_data$vtl - mean(exp_data$vtl)
```

- `height`: A number representing the *height* (in centimeters) reported for the speaker on each trial.
- `vtl`: An estimate of the speaker's *vocal-tract length* (VTL) in centimeters.

- `C`: A letter representing the speaker *category* (`b`=boy, `g`=girl, `m`=man, `w`=woman) reported by the listener for each trial.
- `G`: The *apparent gender* of the speaker indicated by the listener, `f` (female) or `m` (male).
- `A`: The *apparent age* of the speaker indicated by the listener, `a` (adult) or `c` (child).

Research Questions

(Q1) Is there a linear relationship between speaker VTL and apparent height?
(Q2) Can we predict expected apparent height responses given only knowledge of speaker VTL?

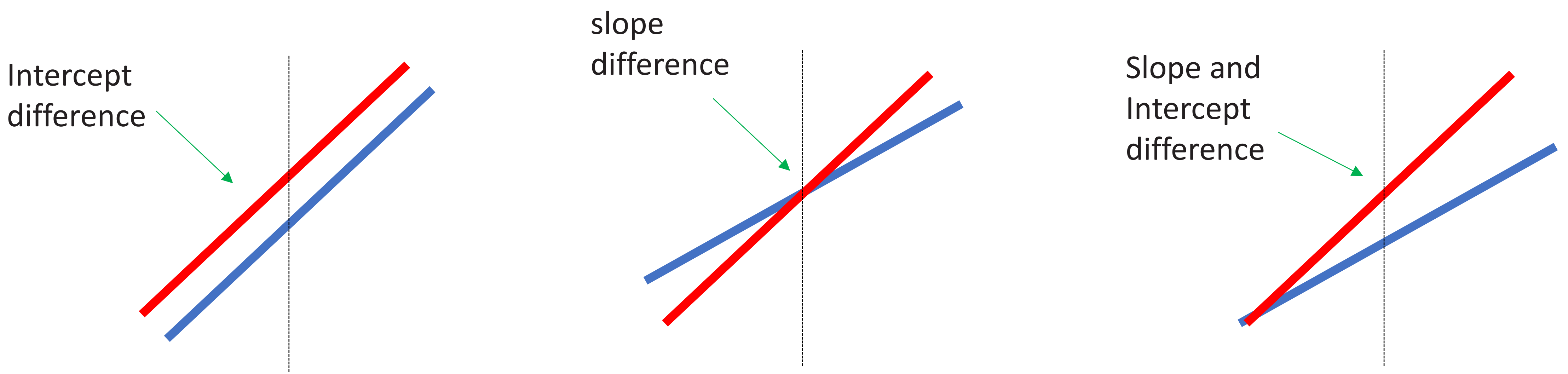
- For the first time we:
  - Include a quantitative predictor.
  - Model variation along lines.

Lines

- Lines have two parameters:
  - An intercept determines the value of the line where x = 0.
  - A slope determines how much the line rises for every unit change in x.

$$y = a + b \cdot x$$

$$\mu = \text{Intercept} + \text{slope} \cdot x$$

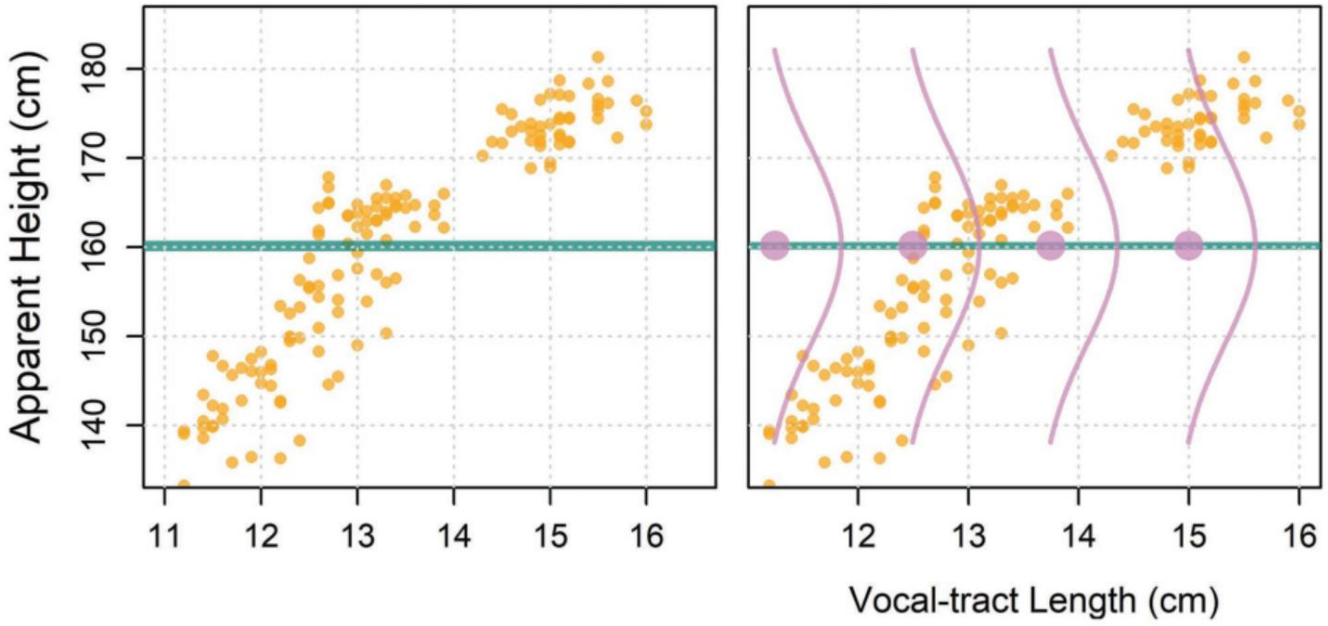


Secret Lines

- We've 'secretly' been modeling variation along lines this whole time.
- We have been fixing all slopes at 0.
- Is there a 'better' slope for our data?

Intercept = 160, slope = 0

$$\mu = \text{Intercept} + \text{slope} \cdot \text{vtl}$$
$$\mu = \text{Intercept} + 0 \cdot \text{vtl}$$
$$\mu = \text{Intercept}$$



The ‘Best’ Line

- What does ‘better’ mean?

- In general*, the ‘best’ slope is the one that minimizes the residuals: The distance of points to the line.

$$r_{[i]} = \left(\text{Intercept} + \text{slope} \cdot \text{vtl}_{[i]}\right) - \text{height}_{[i]}$$

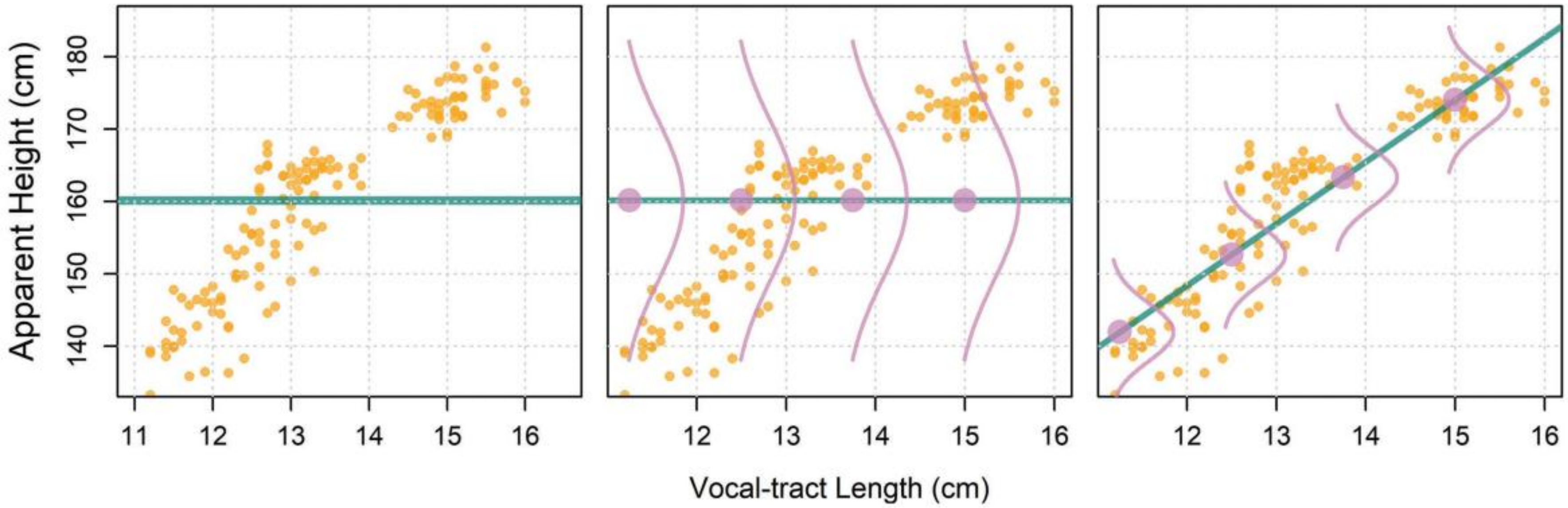
- Minimizing the residuals means maximizing the likelihood, i.e., making the data more probable given our model.

*For multilevel models the ‘best’ line won’t necessarily be the one that minimizes the residuals.

Minimizing Residuals

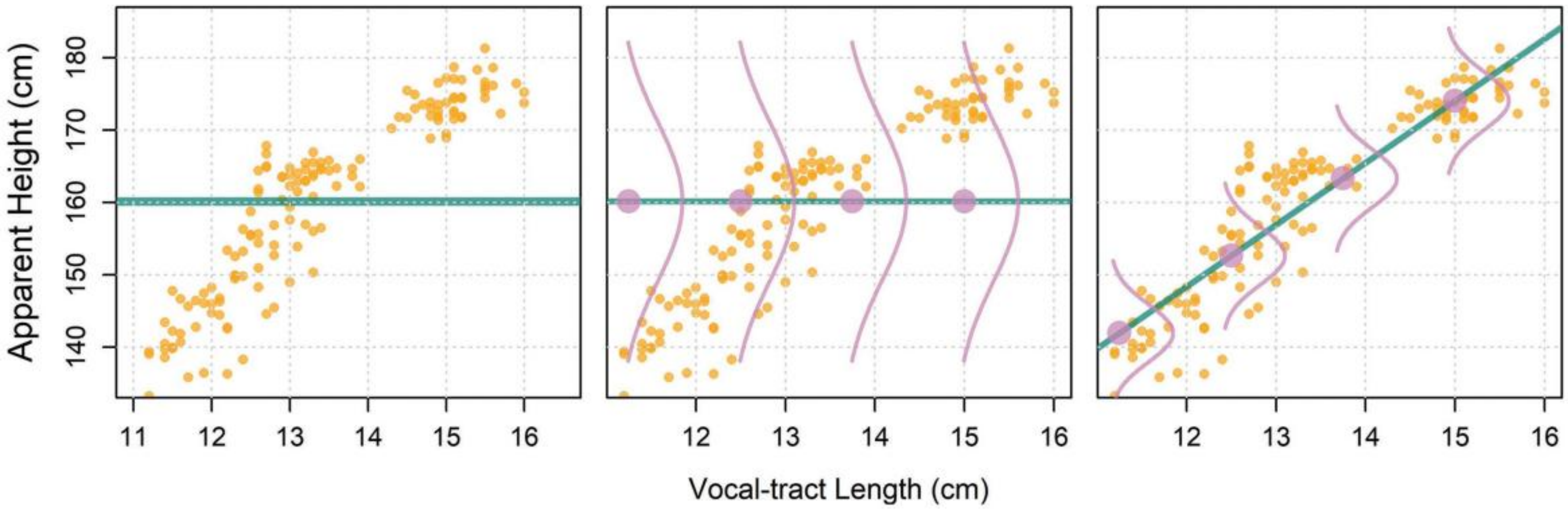
- A slope of zero leads to huge error variance (it’s bad).

- The non-zero slope makes our residuals and error variance much smaller (that’s good).



Variation along Lines

- Before our normal distribution sat in one place, generating data.
- Now our distribution slides around generating data based on the value of x.



Description of the Model

- We'll begin with 'fixed effects unilevel models' just to understand the geometry of these.

```
height ~ vtl
```

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \text{Intercept} + VTL \cdot \text{vtl}_{[i]}$$

slope parameter

predictor

Priors:

$$\text{Intercept} \sim \mathrm{t}(3, 160, 12)$$

$$VTL \sim \mathrm{t}(3, 0, 12)$$

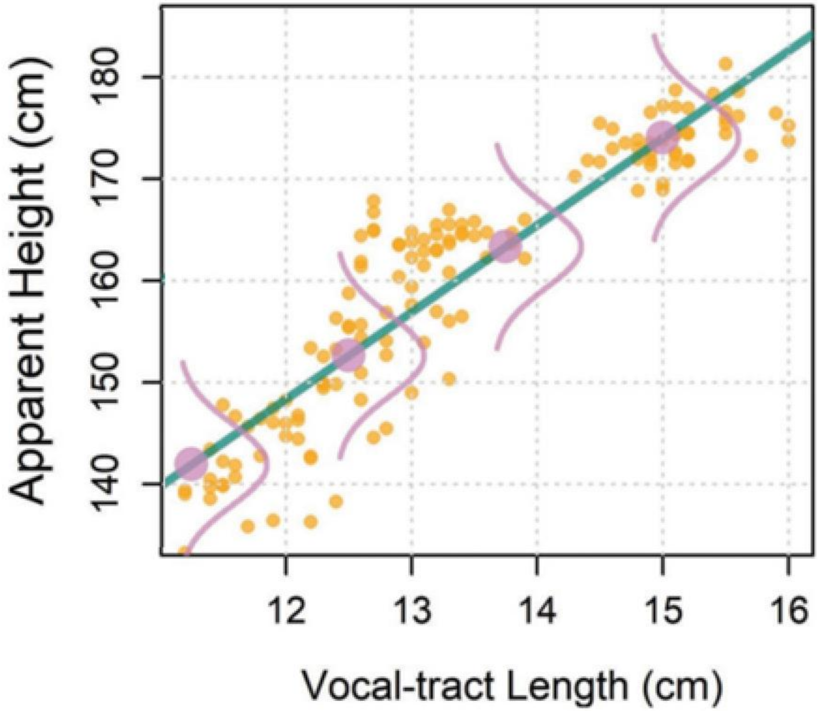
$$\sigma \sim \mathrm{t}(3, 0, 12)$$

Description of the Model

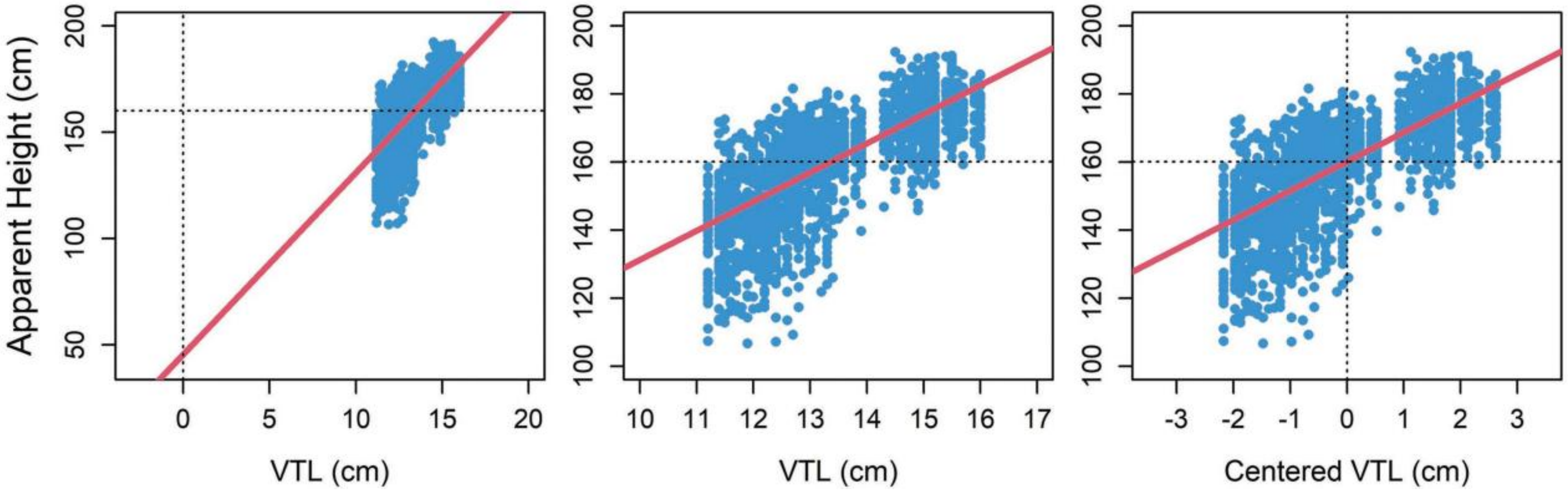
- Two ways to think of this model.

$$\text{height}_{[i]} = \text{Intercept} + VTL \cdot \text{vtl}_{[\text{i}]} + \text{N}(0, \sigma)$$

$$\text{height}_{[i]} = \text{N}\left(\text{Intercept} + VTL \cdot \text{vtl}_{[\text{i}]}, \sigma\right)$$



Centering Predictors



Centering

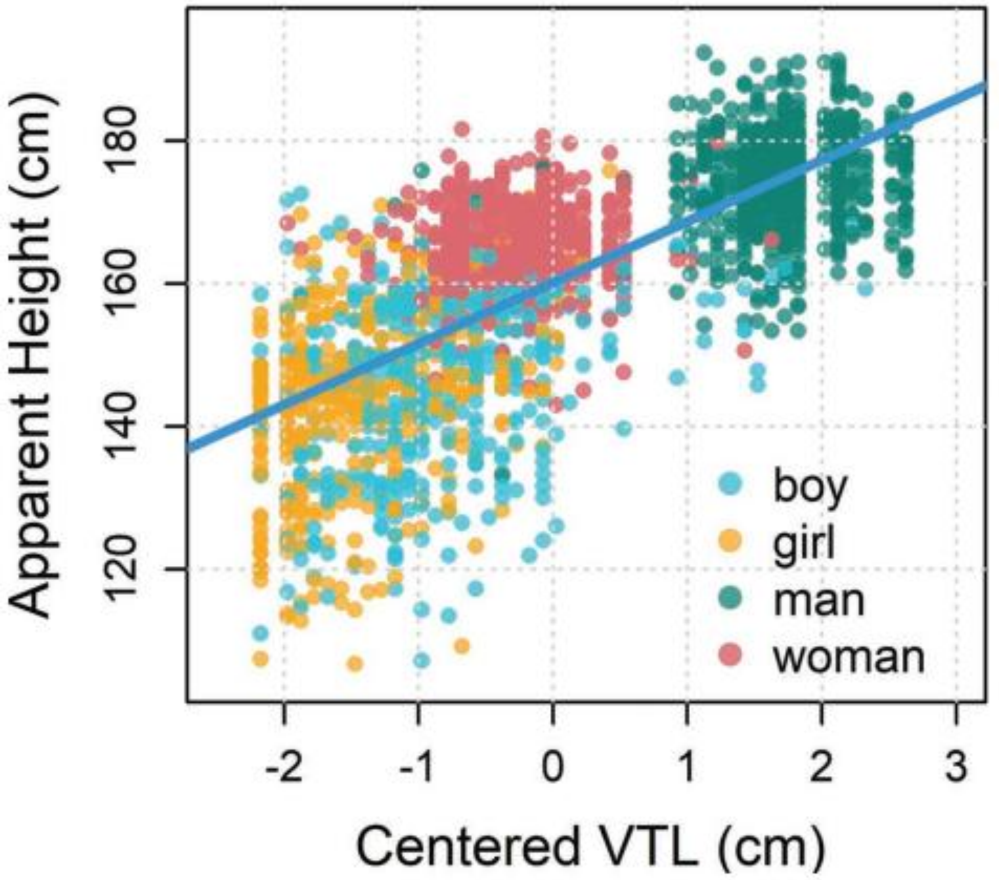
- If 0 is not meaningful for the predictor, centering can:
  - Make the intercept represent a more meaningful value.
  - Facilitate specifying priors.
  - Facilitate the comparison of main effects in the presence of interactions in some cases.
- Recommendation: If 0 is not a meaningful value for your predictor, consider centering it.

Fitting the Model

```
# Fit the model yourself
model_single_line =
  brm (height ~ vtl, data = exp_data, chains=1, cores=1,
       warmup=1000, iter = 6000,
       prior = c(set_prior("student_t(3, 160, 12)", class = "Intercept"),
                 set_prior("student_t(3, 0, 12)", class = "b"),
                 set_prior("student_t(3, 0, 12)", class = "sigma")))

# Or download it from the GitHub page:
model_single_line = bmmb::get_model ('9_model_single_line.RDS')
```

Interpreting the Model

```
bmmb::short_summary (model_single_line)
## Formula:  height ~ vtl
## Population-Level Effects:
##           Estimate Est.Error l-95% CI u-95% CI
## Intercept   160.13      0.23   159.68   160.60
## vtl           8.56      0.18     8.21     8.89
##
## Family Specific Parameters:
##       Estimate Est.Error l-95% CI u-95% CI
## sigma    10.79      0.17    10.47    11.12
```



Models with Many Intercepts but One Slope

- These models can have a different intercept for the line representing each group. However, all lines have the same slope.

```
height ~ C + vtl
```

$$\text{height}_{[i]} \sim \text{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \text{Intercept} + C_{\left[\mathsf{C}_{[i]}\right]} + VTL \cdot \text{vtl}_{[i]}$$

Priors:

$$\text{Intercept} \sim \text{t}(3, 160, 12)$$

$$VTL \sim \text{t}(3, 0, 12)$$

$$C_{[\bullet]} \sim \text{t}(3, 0, 12)$$

$$\sigma \sim \text{t}(3, 0, 12)$$

What do we need for many possible slope values?

Models with Many Intercepts but One Slope

- Useful perspective: All predictors can be divided into those affecting slopes, and those affecting intercepts.

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$
$$\mu_{[i]} = \text{Intercept} + C_{\left[\mathrm{C}_{[i]}\right]} + VTL \cdot \text{vtl}_{[i]}$$

Priors:

$$\text{Intercept} \sim \mathrm{t}(3,160,12)$$
$$VTL \sim \mathrm{t}(3,0,12)$$
$$C_{[\bullet]} \sim \mathrm{t}(3,0,12)$$
$$\sigma \sim \mathrm{t}(3,0,12)$$

$$\text{(A)} \xleftrightarrow{\text{the same model}} \text{(B)}$$

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$
$$\mu_{[i]} = a_{[i]} + b_{[i]} \cdot \text{vtl}_{[i]}$$
$$a_{[i]} = \text{Intercept} + C_{\left[\mathrm{C}_{[i]}\right]}$$
$$b_{[i]} = VTL$$

Priors:

$$\text{Intercept} \sim \mathrm{t}(3,160,12)$$
$$VTL \sim \mathrm{t}(3,0,12)$$
$$C_{[\bullet]} \sim \mathrm{t}(3,0,12)$$
$$\sigma \sim \mathrm{t}(3,0,12)$$

Models with Many Intercepts but One Slope

- This parametrization makes it clear why there is only one slope, but many possible intercept values.

$$\text{height}_{[i]} \sim \text{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = a_{[i]} + b_{[i]} \cdot \text{vtl}_{[i]}$$

$$a_{[i]} = \text{Intercept} + C_{\left[\mathsf{C}_{[i]}\right]}$$

$$b_{[i]} = VTL$$

Priors:

$$\text{Intercept} \sim \text{t}(3, 160, 12)$$

$$VTL \sim \text{t}(3, 0, 12)$$

$$C_{[\bullet]} \sim \text{t}(3, 0, 12)$$

$$\sigma \sim \text{t}(3, 0, 12)$$

Fitting the Model

```
# Fit the model yourself
model_many_intercept_one_slope =
  brm (height ~ C + vtl,
       data=exp_data, chains=1,cores=1,warmup=1000,iter=6000,
       prior = c(set_prior("student_t(3, 160, 12)", class = "Intercept"),
                 set_prior("student_t(3, 0, 12)", class = "b"),
                 set_prior("student_t(3, 0, 12)", class = "sigma")))

# Or download it from the GitHub page:
model_many_intercept_one_slope =
  bmmb::get_model ('9_model_many_intercept_one_slope.RDS')
```

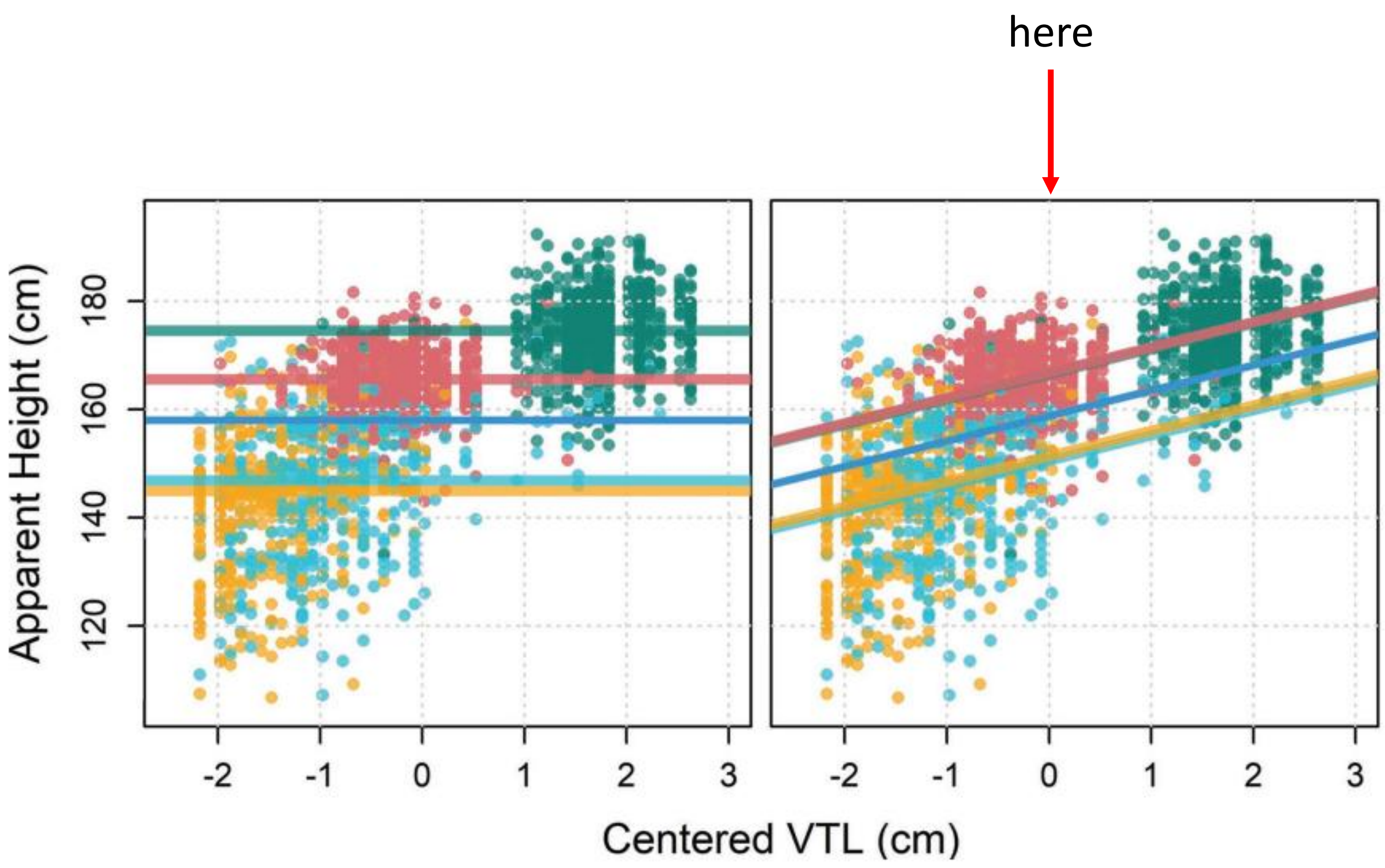
Interpreting the Model

```r
many_intercept_one_slope_hypothesis = bmmb::short_hypothesis (
  model_many_intercept_one_slope,
  hypothesis =
    c("Intercept = 0",                # overall intercept
      "Intercept + C1 = 0",           # group 1 intercept
      "Intercept + C2 = 0",           # group 2 intercept
      "Intercept + C3 = 0",           # group 3 intercept
      "Intercept + -(C1+C2+C3)=0",    # group 4 intercept
      "vtl = 0"))                     # overall slope

many_intercept_one_slope_hypothesis
##     Estimate Est.Error    Q2.5   Q97.5                  hypothesis
## H1   158.796    0.2138 158.374 159.214              (Intercept) = 0
## H2   150.245    0.5184 149.246 151.258           (Intercept+C1) = 0
## H3   151.503    0.6145 150.317 152.712           (Intercept+C2) = 0
## H4   166.752    0.6352 165.515 167.977           (Intercept+C3) = 0
## H5   166.685    0.3883 165.916 167.450 (Intercept+-(C1+C2+C3)) = 0
## H6   4.604      0.3169   3.974   5.219                    (vtl) = 0
```

Interpreting the Model

- When lines have a slope of zero, group effects reflect the spacing of the lines.

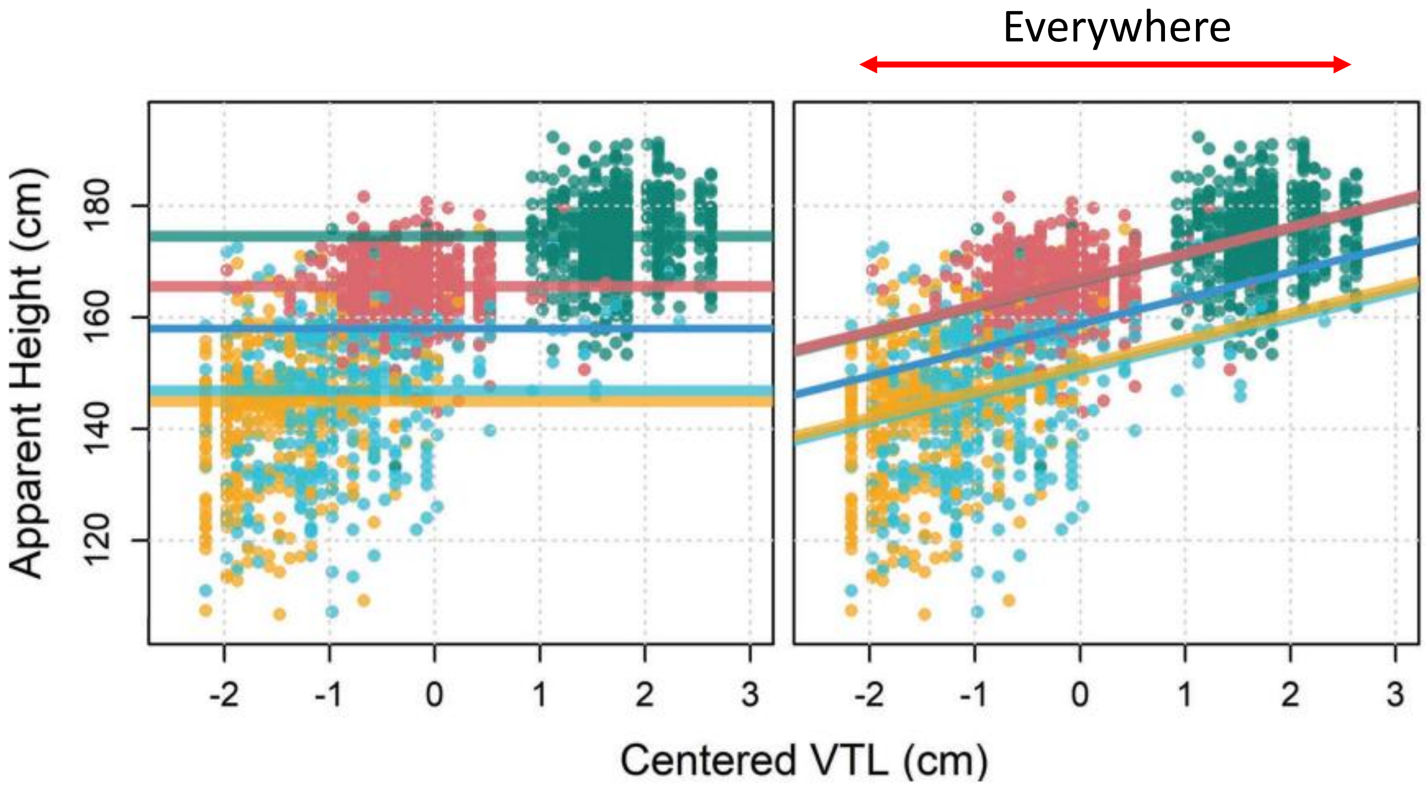
- When models have a non-zero slope, group effects reflect the spacing of the lines <u>only when x is exactly zero</u>.



Interpreting the Model

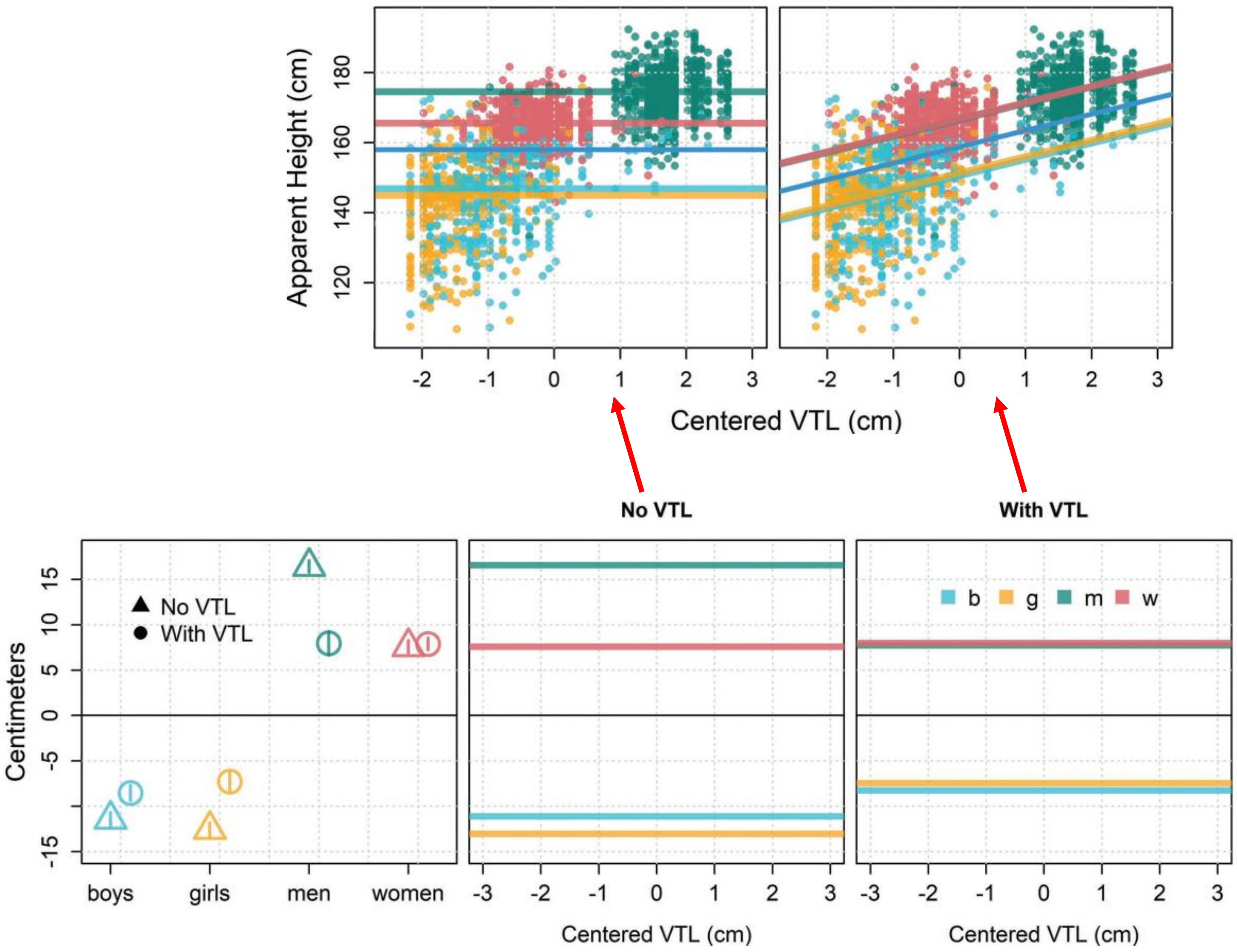
- However, when lines share a slope, they are parallel.

- So, group effects reflect line spacing *everywhere*.



Interpreting Group Effects with Shared Slopes

- Group effects will likely be different in the presence of non-zero slopes.



Models with Varying Slopes and Intercepts

- These models can have a different intercept and slope for the line representing each group.

```
height ~ C * vtl
height ~ C + vtl + vtl:C
```

category-dependent intercept

overall ‘marginal’ slope

category-dependent slope

Models with Varying Slopes and Intercepts

$$\mu_{[i]} = \text{Intercept} + C_{[\mathsf{c}_{[i]}]} + VTL \cdot \text{vtl}_{[i]} + VTL : C_{[\mathsf{c}_{[i]}]} \cdot \text{vtl}_{[i]}$$

$$\mu_{[i]} = \left(\text{Intercept} + C_{[\mathsf{c}_{[i]}]} \right) + \left(VTL \cdot \text{vtl}_{[i]} + VTL : C_{[\mathsf{c}_{[i]}]} \cdot \text{vtl}_{[i]} \right)$$

$$\mu_{[i]} = \left(\text{Intercept} + C_{[\mathsf{c}_{[i]}]} \right) + \left(VTL + VTL : C_{[\mathsf{c}_{[i]}]} \right) \cdot \text{vtl}_{[i]}$$

$$\mu_{[i]} = a_{[i]} + b_{[i]} \cdot \text{vtl}_{[i]}$$

$$a_{[i]} = \text{Intercept} + C_{[\mathsf{c}_{[i]}]}$$

$$b_{[i]} = VTL + VTL : C_{[\mathsf{c}_{[i]}]})$$

Models with Varying Slopes and Intercepts

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \left(\text{Intercept} + C_{\left[\mathsf{C}_{[i]}\right]}\right) + \left(VTL \cdot \text{vtl}_{[i]} + VTL : C_{\left[\mathsf{C}_{[i]}\right]} \cdot \text{vtl}_{[i]}\right)$$

Priors:

$$\text{Intercept} \sim \mathrm{t}(3,160,12)$$

$$VTL \sim \mathrm{t}(3,0,12)$$

$$C_{[\bullet]} \sim \mathrm{t}(3,0,12)$$

$$VTL : C_{[\bullet]} \sim \mathrm{t}(3,0,12)$$

$$\sigma \sim \mathrm{t}(3,0,12)$$

the same model

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = a_{[i]} + b_{[i]} \cdot \text{vtl}_{[i]}$$

$$a_{[i]} = \text{Intercept} + C_{\left[\mathsf{C}_{[i]}\right]}$$

$$b_{[i]} = VTL + VTL : C_{\left[\mathsf{C}_{[i]}\right]}$$

Priors:

$$\text{Intercept} \sim \mathrm{t}(3,160,12)$$

$$VTL \sim \mathrm{t}(3,0,12)$$

$$C_{[\bullet]} \sim \mathrm{t}(3,0,12)$$

$$VTL : C_{[\bullet]} \sim \mathrm{t}(3,0,12)$$

$$\sigma \sim \mathrm{t}(3,0,12)$$

Fitting the Model

```
# Fit the model yourself
options (contrasts = c('contr.sum','contr.sum'))
model_multi_slope =
  brm (height ~ C + vtl + vtl:C,
       data=exp_data, chains=1,cores=1,warmup=1000,iter=6000,
       prior = c(set_prior("student_t(3, 160, 12)", class = "Intercept"),
                 set_prior("student_t(3, 0, 12)", class = "b"),
                 set_prior("student_t(3, 0, 12)", class = "sigma")))
```

Inspecting the Fixed Effects

```
# inspect fixed effects
brms::fixef (model_multi_slope)
##              Estimate  Est.Error    Q2.5    Q97.5
## Intercept     160.800    0.4525  159.935  161.690
## C1             -9.420    0.6208  -10.632   -8.221
## C2             -4.834    0.8904   -6.580   -3.090
## C3              9.279    1.0062    7.316   11.283
## vtl             4.271    0.3463    3.582    4.937
## C1:vtl          1.438    0.5067    0.448    2.433
## C2:vtl          3.747    0.6181    2.544    4.937
## C3:vtl         -1.666    0.6069   -2.913   -0.507
```

Intercept main effect

Category-dependent intercepts

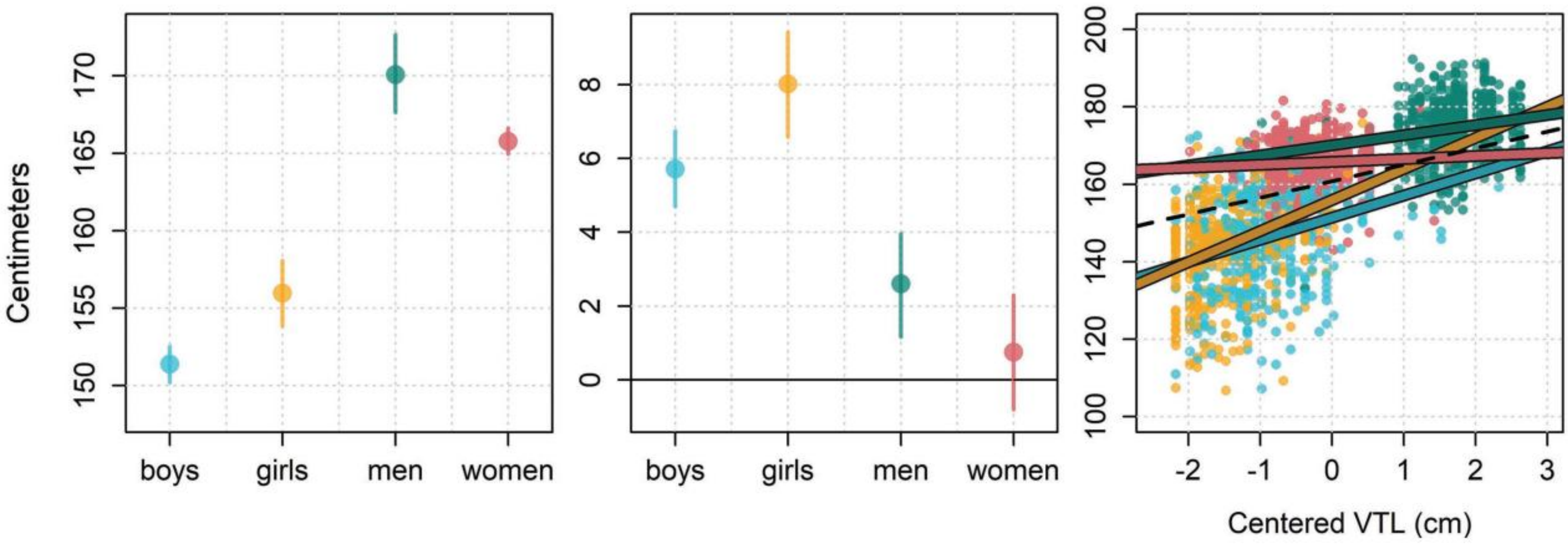
Slope main effect

Category-dependent slopes

Group-Specific Line Parameters

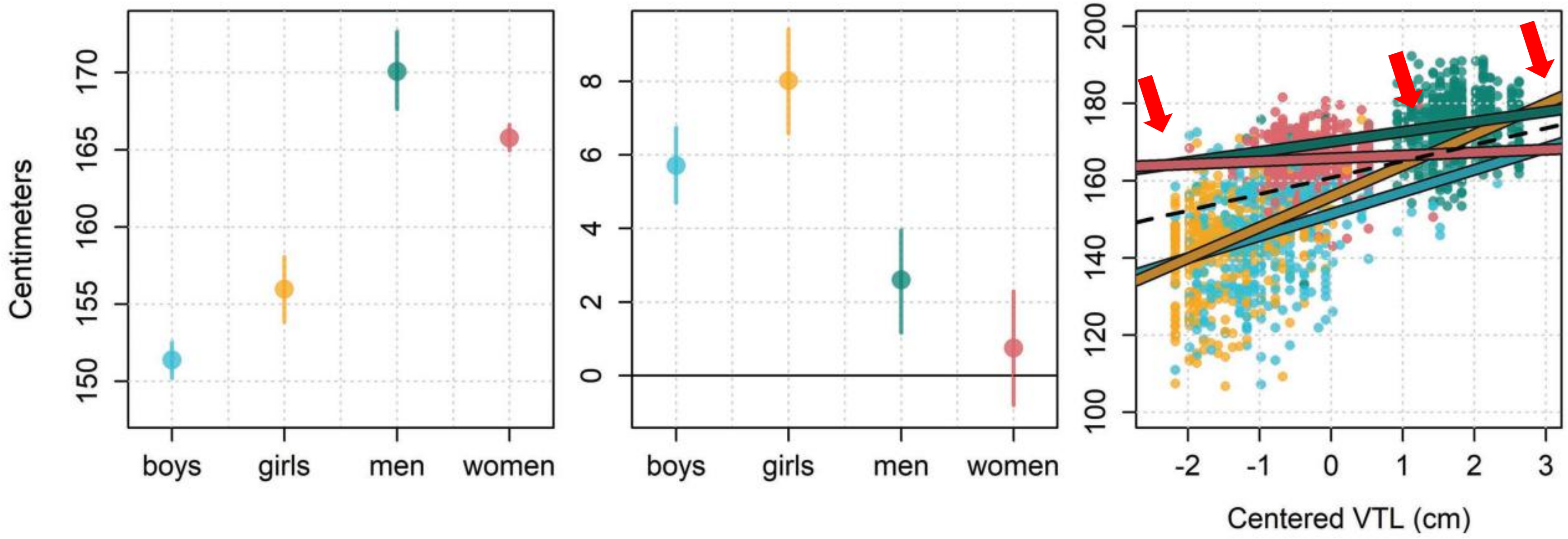
```
multi_slope_hypothesis = bmmb::short_hypothesis (
  model_multi_slope,
  hypothesis =
    c("Intercept = 0",                          # overall intercept
      "Intercept + C1 = 0",                       # group 1 mean
      "Intercept + C2 = 0",                       # group 2 mean
      "Intercept + C3 = 0",                       # group 3 mean
      "Intercept + -(C1+C2+C3) = 0",              # group 4 mean
      "vtl = 0",                                  # overall slope
      "vtl + C1:vtl = 0",                         # group 1 slope
      "vtl + C2:vtl = 0",                         # group 2 slope
      "vtl + C3:vtl = 0",                         # group 3 slope
      "vtl + -(C1:vtl+C2:vtl+C3:vtl) = 0"))       # group 4 slope
```

Group-Specific Line Parameters



Interpreting Group Effects with Varying Slopes

- Group effects reflect separation of lines only at exactly zero.
- Separation (i.e., group effects) may be substantially different elsewhere.



Multilevel Models

- Multilevel models with random slopes or intercepts effectively have different slopes or intercepts for different levels of the grouping factor (e.g. listener).

```
height ~ C + vtl + vtl:C

height ~ L + vtl + vtl:L

height ~ vtl + (vtl|L)
```

Data and Research Questions

- `L`: A number from 1 to 15 indicating which *listener* responded to the trial.
- `height`: A number representing the *height* (in centimeters) reported for the speaker on each trial.
- `S`: A number from 1 to 139 indicating which *speaker* produced the trial stimulus.
- `vtl`: An estimate of the speaker's *vocal-tract length* in centimeters.
- `G`: The *apparent gender* of the speaker indicated by the listener, `f` (female) or `m` (male).
- `A`: The *apparent age* of the speaker indicated by the listener, `a` (adult) or `c` (child).

(Q1) What is the linear relationship between speaker VTL and apparent height?
(Q2) Is the effect of VTL on apparent height affected by the apparent age and gender of the speaker?

Description of the Model

```
height ~ vtl + (vtl|L) + (1|S)
```

$$\text{height}_{[i]} \sim \mathrm{t}\left(\nu, \mu_{[i]}, \sigma\right)$$
$$\mu_{[i]} = a + b \cdot \text{vtl}_{[i]}$$
$$a_{[i]} = \text{Intercept} + L_{\left[\mathsf{L}_{[i]}\right]} + S_{\left[\mathsf{S}_{[i]}\right]}$$
$$b_{[i]} = VTL + VTL:L_{\left[\mathsf{L}_{[i]}\right]}$$

Priors :

$$S_{[\bullet]} \sim \mathrm{t}(3, 0, \sigma_S)$$
$$\begin{bmatrix} L_{[\bullet]} \\ VTL:L_{[\bullet]} \end{bmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \Sigma\right)$$

$$\text{Intercept} \sim \mathrm{t}(3, 156, 12)$$
$$VTL \sim \mathrm{t}(3, 0, 12)$$
$$\sigma_L, \sigma_{VTL:L}, \sigma_S \sim \mathrm{t}(3, 0, 12)$$
$$\sigma \sim \mathrm{t}(3, 0, 12)$$
$$\nu \sim \text{gamma}(2, 0.1)$$
$$R \sim \text{LKJCorr}(2)$$

Fitting the Model

```r
# Fit the model yourself
priors = c(brms::set_prior("student_t(3,160, 12)", class = "Intercept"),
           brms::set_prior("student_t(3,0, 12)", class = "b"),
           brms::set_prior("student_t(3,0, 12)", class = "sd"),
           brms::set_prior("lkj_corr_cholesky (2)", class = "cor"),
           brms::set_prior("gamma(2, 0.1)", class = "nu"),
           brms::set_prior("student_t(3,0, 12)", class = "sigma"))

model_random_slopes_simple =
  brms::brm (height ~ vtl + (vtl|L) + (1|S), data = exp_data, chains = 4,
                cores = 4, warmup = 1000, iter = 5000, thin = 4,
                prior = priors, family = "student")
```

Interpreting the Model

```
bmmb::short_summary(model_random_slopes_simple)
## Formula:  height ~ vtl + (vtl | L) + (1 | S)
##
## Group-Level Effects:
## ~L (Number of levels: 15)
##                     Estimate Est.Error l-95% CI u-95% CI
## sd(Intercept)           3.67      0.80     2.44     5.56
## sd(vtl)                 2.84      0.62     1.93     4.31
## cor(Intercept,vtl)     -0.41      0.22    -0.76     0.07
##
## ~S (Number of levels: 139)
##               Estimate Est.Error l-95% CI u-95% CI
## sd(Intercept)     4.53      0.34     3.89     5.25
##
## Population-Level Effects:
##           Estimate Est.Error l-95% CI u-95% CI
## Intercept   160.60      1.07   158.56   162.76
## vtl           8.39      0.82     6.73     9.94
##
## Family Specific Parameters:
##       Estimate Est.Error l-95% CI u-95% CI
## sigma     6.83      0.22     6.41     7.27
## nu        5.24      0.73     4.05     6.82
```

Interpreting the Model

```
bmmb::short_summary(model_random_slopes_simple)
## Formula:  height ~ vtl + (vtl | L) + (1 | S)
##
## Group-Level Effects:
## ~L (Number of levels: 15)
##                      Estimate Est.Error l-95% CI u-95% CI
## sd(Intercept)            3.67      0.80     2.44     5.56
## sd(vtl)                  2.84      0.62     1.93     4.31
## cor(Intercept,vtl)      -0.41      0.22    -0.76     0.07
##
## ~S (Number of levels: 139)
##                 Estimate Est.Error l-95% CI u-95% CI
## sd(Intercept)       4.53      0.34     3.89     5.25
##
## Population-Level Effects:
##            Estimate Est.Error l-95% CI u-95% CI
## Intercept    160.60      1.07   158.56   162.76
## vtl            8.39      0.82     6.73     9.94
##
## Family Specific Parameters:
##       Estimate Est.Error l-95% CI u-95% CI
## sigma     6.83      0.22     6.41     7.27
## nu        5.24      0.73     4.05     6.82
```

```
bmmb::short_summary(model_single_line)
## Formula:  height ~ vtl
## Population-Level Effects:
##            Estimate Est.Error l-95% CI u-95% CI
## Intercept    160.13      0.23   159.68   160.60
## vtl            8.56      0.18     8.21     8.89
##
## Family Specific Parameters:
##       Estimate Est.Error l-95% CI u-95% CI
## sigma    10.79      0.17    10.47    11.12
```

Inspecting the Random Effects

```
# get listener effects
listener_effects = short_hypothesis(
  model_random_slopes_simple, c("Intercept = 0","vtl = 0"),
  scope = "ranef",group="L")
```

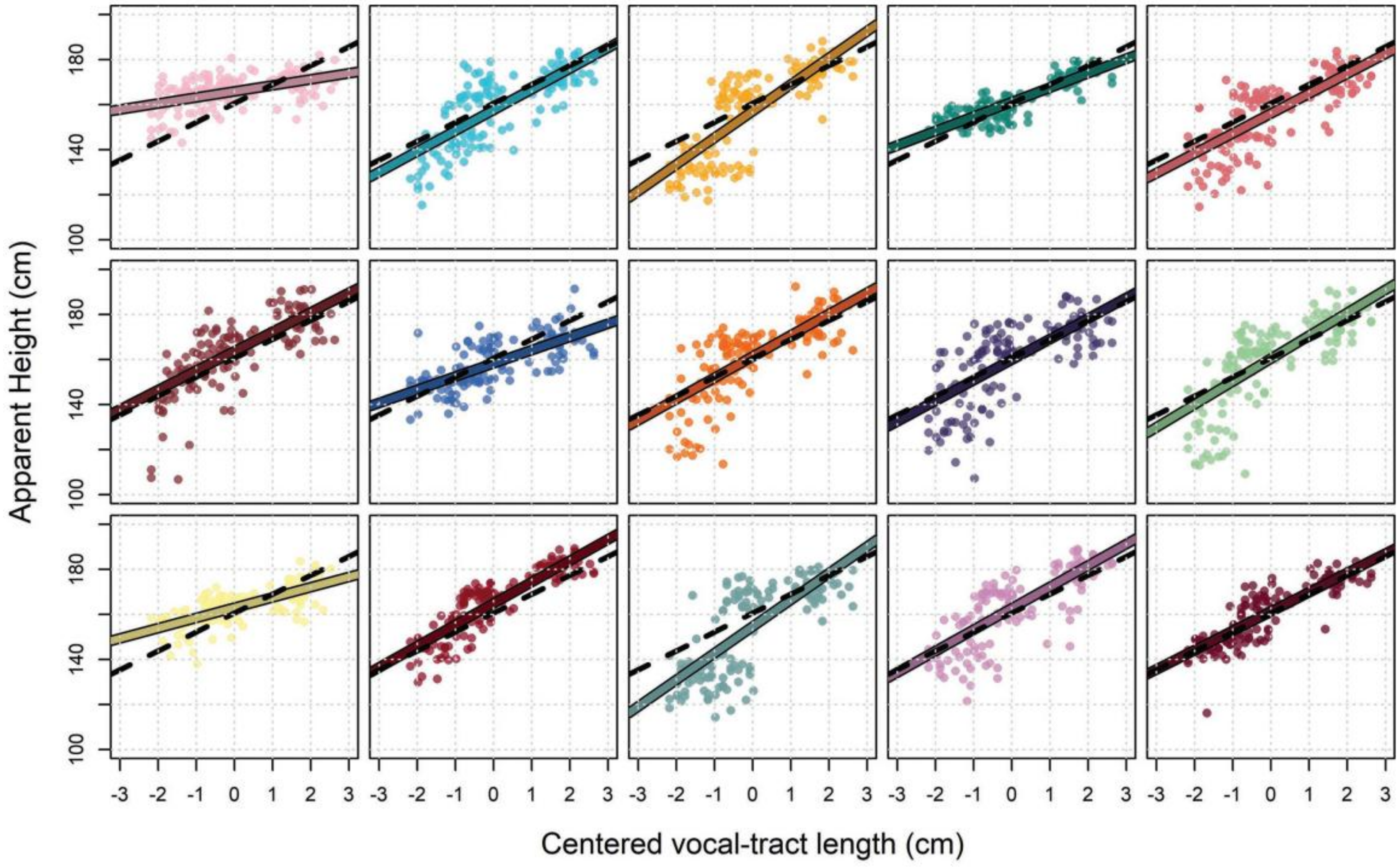
```
# get 3rd and 18th row of our listener effects
# these are the intercept and slope effects for listener 3
listener_effects[c(3,18),]
##       Estimate Est.Error   Q2.5  Q97.5       hypothesis group
## H3      -3.482    1.1836 -5.887 -1.186 (Intercept) = 0     3
## H18      3.448    0.8921  1.699  5.226         (vtl) = 0     3
```

Inspecting the Listener Coefficients

```
# get listener coefficients
listener_coefficients = short_hypothesis(
  model_random_slopes_simple, c("Intercept = 0","vtl = 0"),
  scope = "coef",group="L")
```

```
# get 3rd and 18th row of our listener coefficients
# these are the intercept and slope coefficients for listener 3
listener_coefficients[c(3,18),]
##      Estimate Est.Error   Q2.5  Q97.5       hypothesis group
## H3     157.12    0.7981 155.56 158.73 (Intercept) = 0     3
## H18     11.84    0.5667  10.71  12.95         (vtl) = 0     3
```

Inspecting the Listener Coefficients



A Model with 'Random Slopes'

```
height ~ vtl*A*G + (vtl*A*G|L) + (1|S)
```

```
height ~ vtl + A + G + A:G + vtl:A + vtl:G + vtl:A:G +
        (vtl + A + G + A:G + vtl:A + vtl:G + vtl:A:G|L) + (1|S)
```

Description of the Model

$$\text{height}_{[i]} \sim \text{t}\left(\nu, \mu_{[i]}, \sigma\right)$$
$$\mu_{[i]} = a + b \cdot \text{vtl}_{[i]}$$

$$a_{[i]} = \text{Intercept} + A + G + A:G +$$
$$A:L_{\left[\mathsf{L}[i]\right]} + G:L_{\left[\mathsf{L}[i]\right]} + A:G:L_{\left[\mathsf{L}[i]\right]} + L_{\left[\mathsf{L}[i]\right]} + S_{\left[\mathsf{S}[i]\right]}$$

$$b_{[i]} = VTL + VTL:A + VTL:G + VTL:A:G +$$
$$VTL:A:L_{\left[\mathsf{L}[i]\right]} + VTL:G:L_{\left[\mathsf{L}[i]\right]} + VTL:A:G:L_{\left[\mathsf{L}[i]\right]} + VTL:L_{\left[\mathsf{L}[i]\right]}$$

$$\text{Priors}:$$
$$S_{[\bullet]} \sim \text{t}\left(3, 0, \sigma_S\right)$$

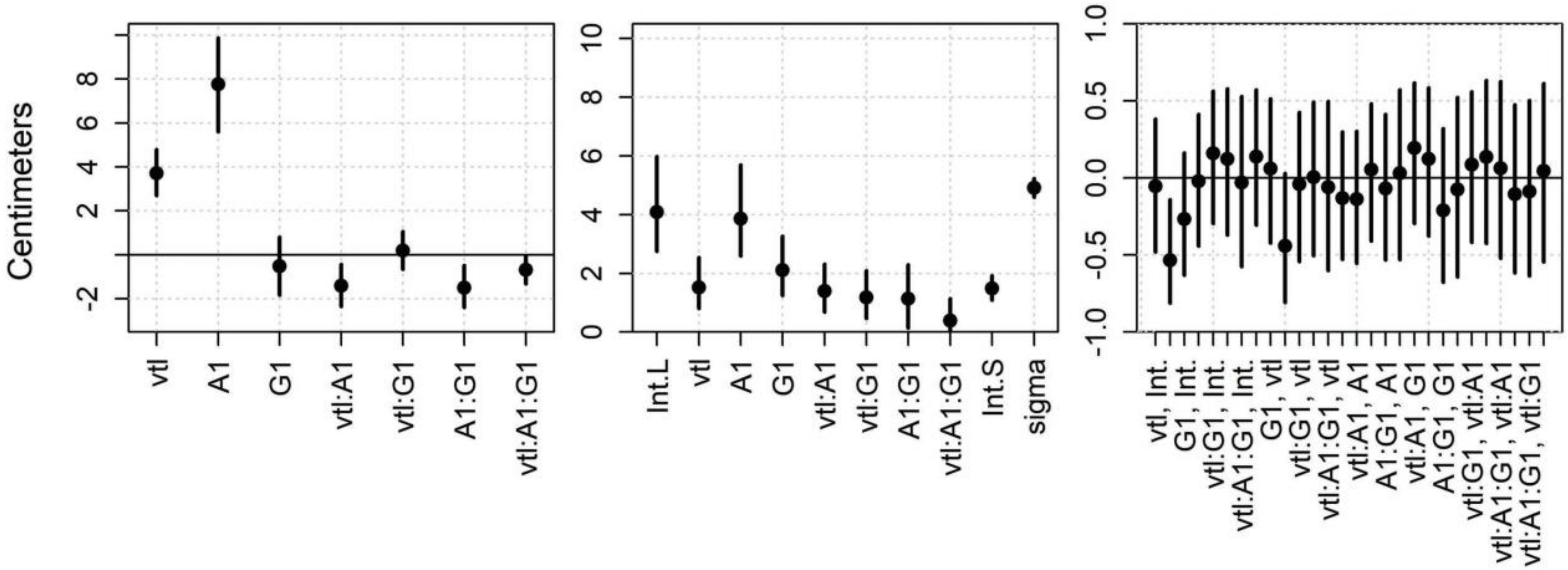
$$\begin{bmatrix} A:L_{[\bullet]} \\ G:L_{[\bullet]} \\ A:G:L_{[\bullet]} \\ L_{[\bullet]} \\ VTL:A:L_{[\bullet]} \\ VTL:G:L_{[\bullet]} \\ VTL:A:G:L_{[\bullet]} \\ VTL:L_{[\bullet]} \end{bmatrix} \sim \text{MVNormal}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \Sigma \right)$$

Fitting the Model

```
# Fit the model yourself
priors = c(brms::set_prior("student_t(3,160, 12)", class = "Intercept"),
           brms::set_prior("student_t(3,0, 12)", class = "b"),
           brms::set_prior("student_t(3,0, 12)", class = "sd"),
           brms::set_prior("lkj_corr_cholesky (2)", class = "cor"),
           brms::set_prior("gamma(2, 0.1)", class = "nu"),
           brms::set_prior("student_t(3,0, 12)", class = "sigma"))

model_random_slopes_complex =
  brms::brm (height ~ vtl*A*G + (vtl*A*G|L) + (1|S), data = exp_data,
             chains = 4, cores = 4, warmup = 1000, iter = 5000, thin = 4,
             prior = priors, family = "student")
```

Interpreting the Model

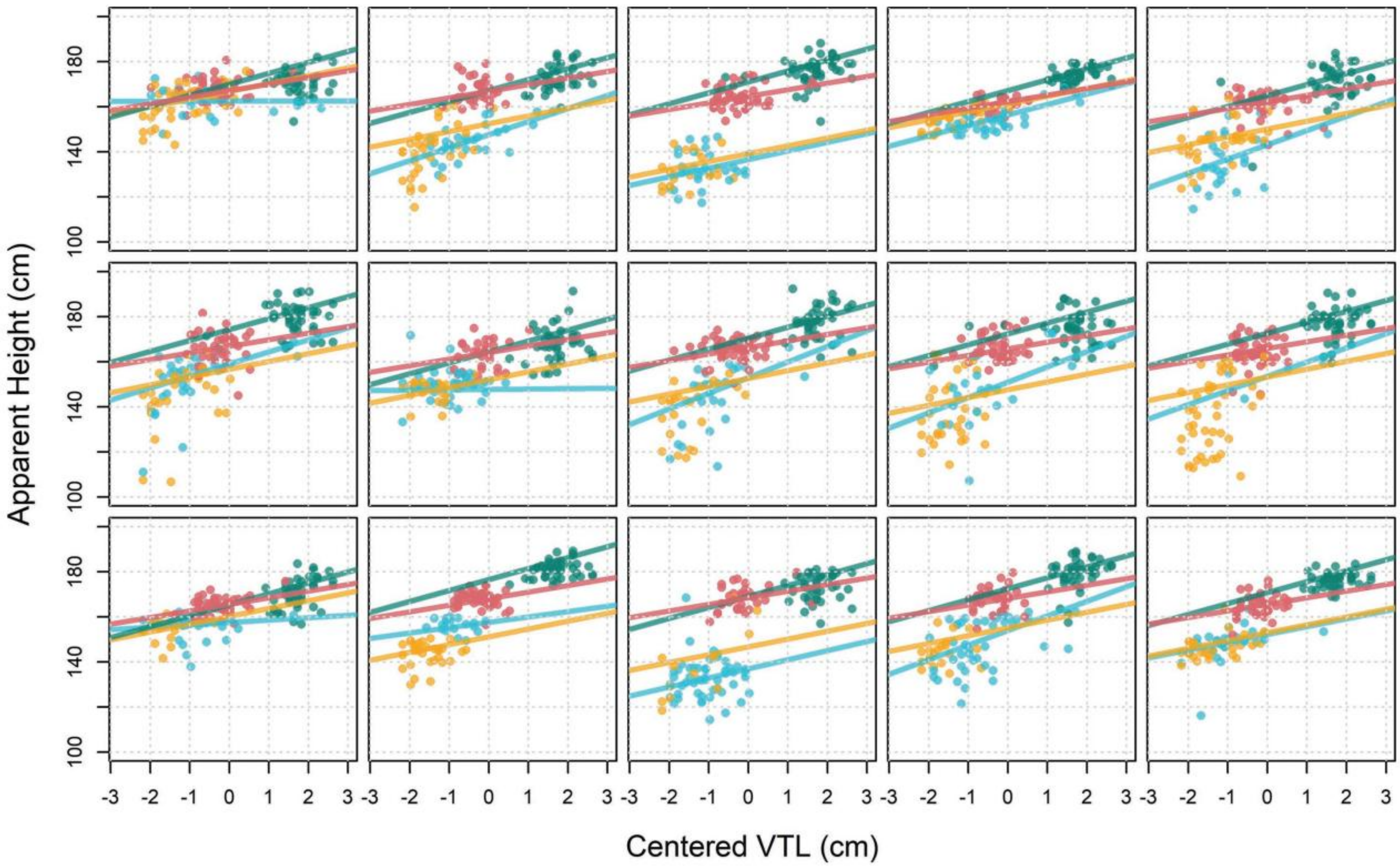
```
# model fixed effects
fixef_effects = brms::fixef (model_random_slopes_complex)

# model random effect standard deviations
sds = bmmb::get_sds (model_random_slopes_complex)

# model random effect correlations
correlations = bmmb::get_corrs (model_random_slopes_complex, factor="L")
```



Listener and Group-Specific Parameters

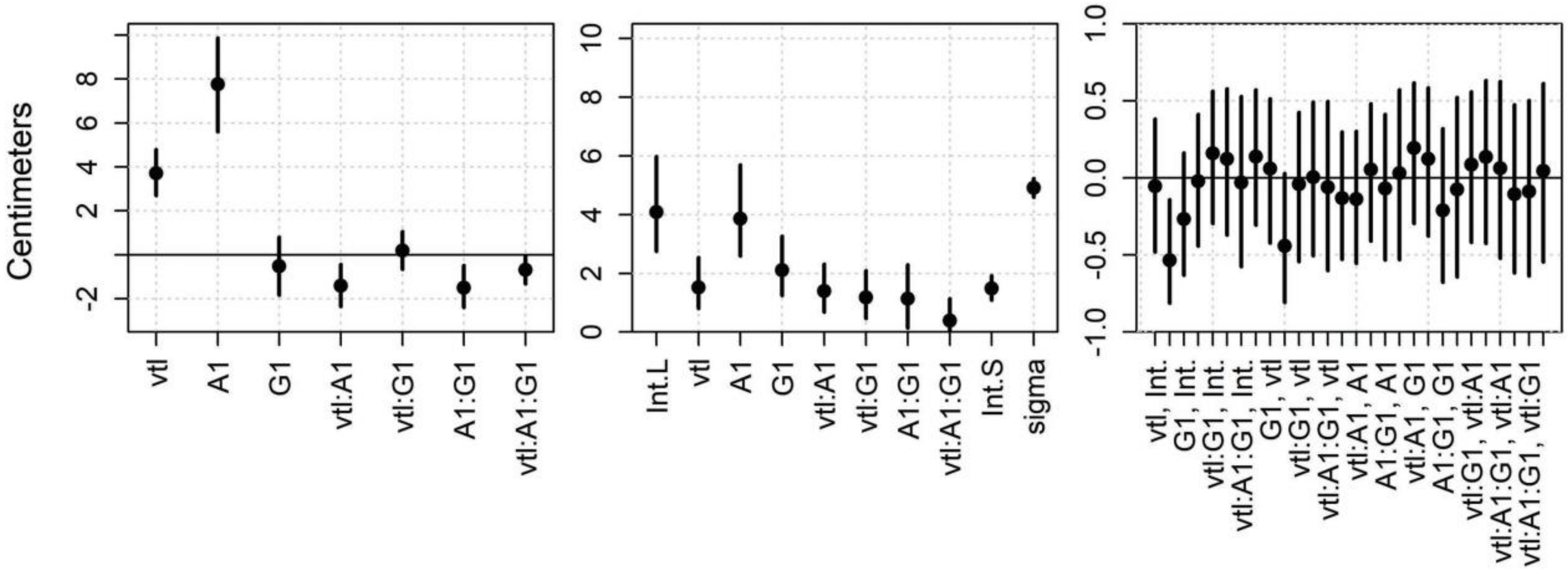
```r
random_slopes_complex_hypothesis_listener = bmmb::short_hypothesis (
  model_random_slopes_complex,
  hypothesis =
    c("Intercept - A1 - G1 + A1:G1 = 0",          # group 1 mean
      "Intercept - A1 + G1 - A1:G1 = 0",          # group 2 mean
      "Intercept + A1 - G1 - A1:G1 = 0",          # group 3 mean
      "Intercept + A1 + G1 + A1:G1 = 0",          # group 4 mean
      "vtl - vtl:A1 - vtl:G1 + vtl:A1:G1 = 0",    # group 1 slope
      "vtl - vtl:A1 + vtl:G1 - vtl:A1:G1 = 0",    # group 2 slope
      "vtl + vtl:A1 - vtl:G1 - vtl:A1:G1 = 0",    # group 3 slope
      "vtl + vtl:A1 + vtl:G1 + vtl:A1:G1 = 0"),   # group 4 slope
  scope = "coef",group="L")
```

Listener and Group-Specific Parameters



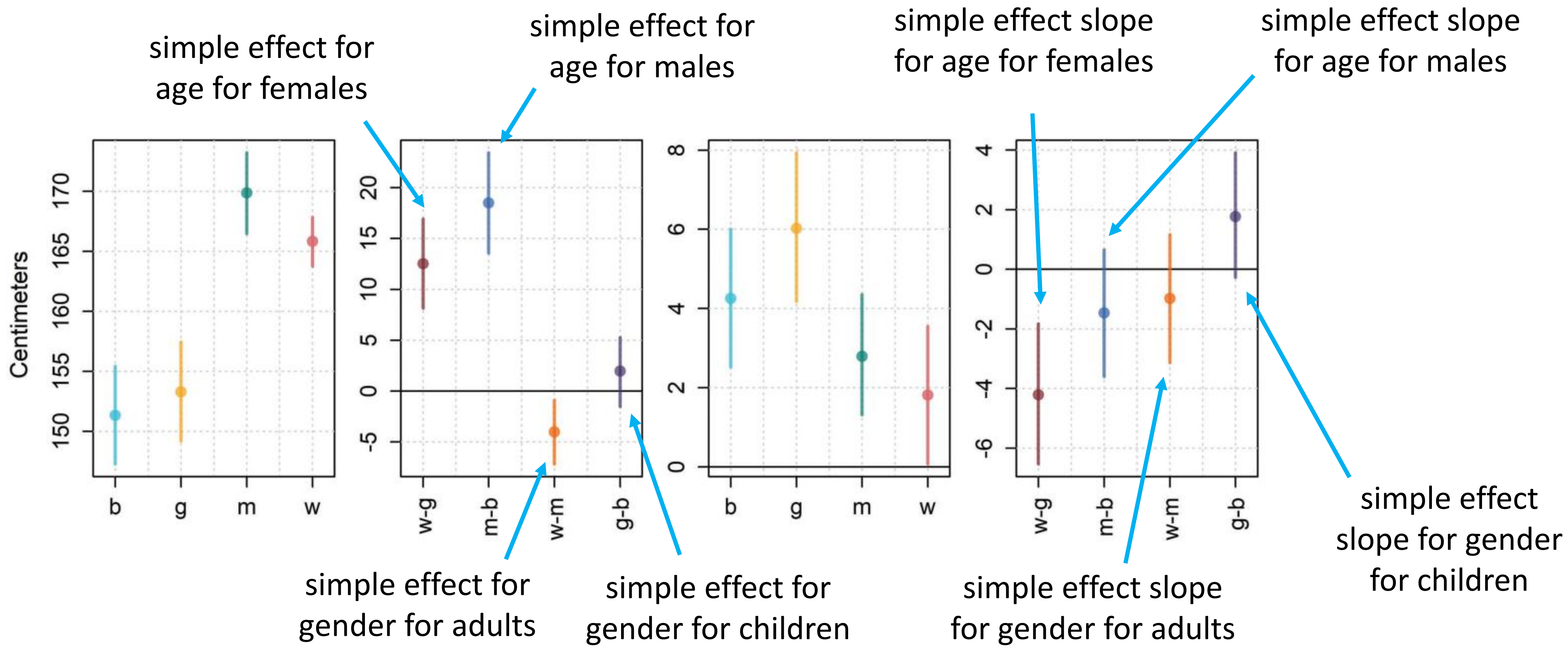
Answering our Research Questions

- Don’t: List of a bunch of numbers.
- Do: Tell the reader the story of the numbers, what do they all mean.



Answering our Research Questions

- Find group-specific slopes and intercepts and inspect the simple effects.



Answering our Research Questions

```
simple_effects = bmmb::short_hypothesis (
  model_random_slopes_complex,
  hypothesis =
   c("2*(A1 + A1:G1) = 0",          # age difference for women
     "2*(A1 - A1:G1) = 0",          # age difference for men
     "2*(G1 + A1:G1) = 0",          # gender difference for adults
     "2*(G1 - A1:G1) = 0",          # gender difference for children
     "2*(vtl:A1 + vtl:A1:G1) = 0",  # VTL difference by age for women
     "2*(vtl:A1 - vtl:A1:G1) = 0",  # VTL difference by age for men
     "2*(vtl:G1 + vtl:A1:G1) = 0",  # VTL difference by gender for adults
     "2*(vtl:G1 - vtl:A1:G1) = 0")) # VTL difference by gender for children
```

Causality

- Difficult if not impossible to ‘prove’ something causes another thing.

- Many examples of mistaken inferences regarding causality. Impossible to say what we’ll be wrong about going forward.

- Our models can help us understand relationships but not causes or exact mechanisms.

Exercises: Week 8

Use the data in ‘exp_ex’ or use your own data to answer a related question. In any case, describe the model, present and explain the results. More requirements:

- You must include a model at least this complex: y ~ A*B + (A*B|L), where either A or B is a quantitative predictor.
- Fit a new model.
- Include at least two non-book figures.
- Include a table of your fixed effects.
- Report and interpret at least one simple effect.