Week 4 – Chapter 4

Lin 205C
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Chapter Precap

- Discutir el análisis de datos compuestos por múltiples observaciones de los miembros de “un grupo".

- Introducir y explicar los siguientes conceptos:
  - Modelos 'multinivel’.
  - Datos de "medidas repetidas".
  - Sin agrupación, agrupación completa y agrupación parcial adaptable.
  - Efectos 'aleatorios' y 'fijos’.

- Ajustamos un modelo multinivel con una estructura más apropiada para nuestros datos de medidas repetidas que los modelos del capítulo anterior.
- Simulamos datos de medidas repetidas en función de los parámetros estimados por nuestro modelo y vemos cómo la exclusión de diferentes componentes afecta a nuestros datos simulados.

Repeated Measures Data

- Observaciones múltiples de una "fuente" o "unidad experimental" determinada.

- Las observaciones no son independientes, lo que causa un problema si actuamos como si lo fueran.

- Por ejemplo, nuestro experimento contó con 139 observaciones de 15 oyentes diferentes (y 15 observaciones para cada hablante).

Last Chapter: One Big Pile of Data

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

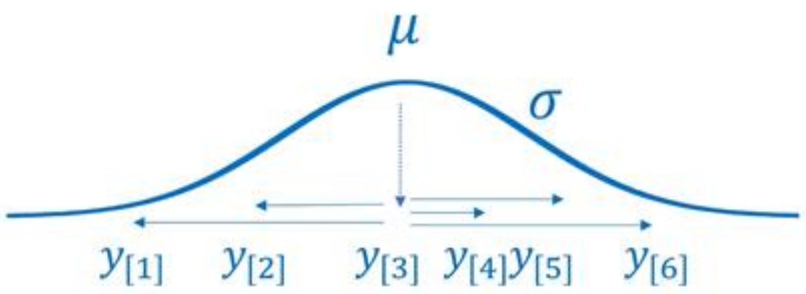
$$\mu_{[i]} = \text{Intercept}$$

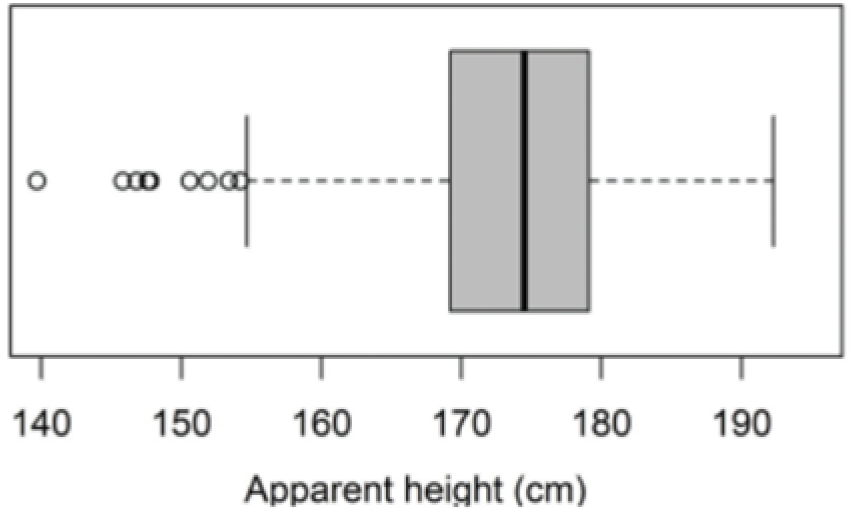
Priors:

$$\text{Intercept} \sim \mathrm{N}(176, 15)$$

$$\sigma \sim \mathrm{N}(0, 15)$$

"Unilevel" Model





Repeated Measures Data

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \text{Intercept}$$

?

Priors:

$$\text{Intercept} \sim \mathrm{N}(176, 15)$$

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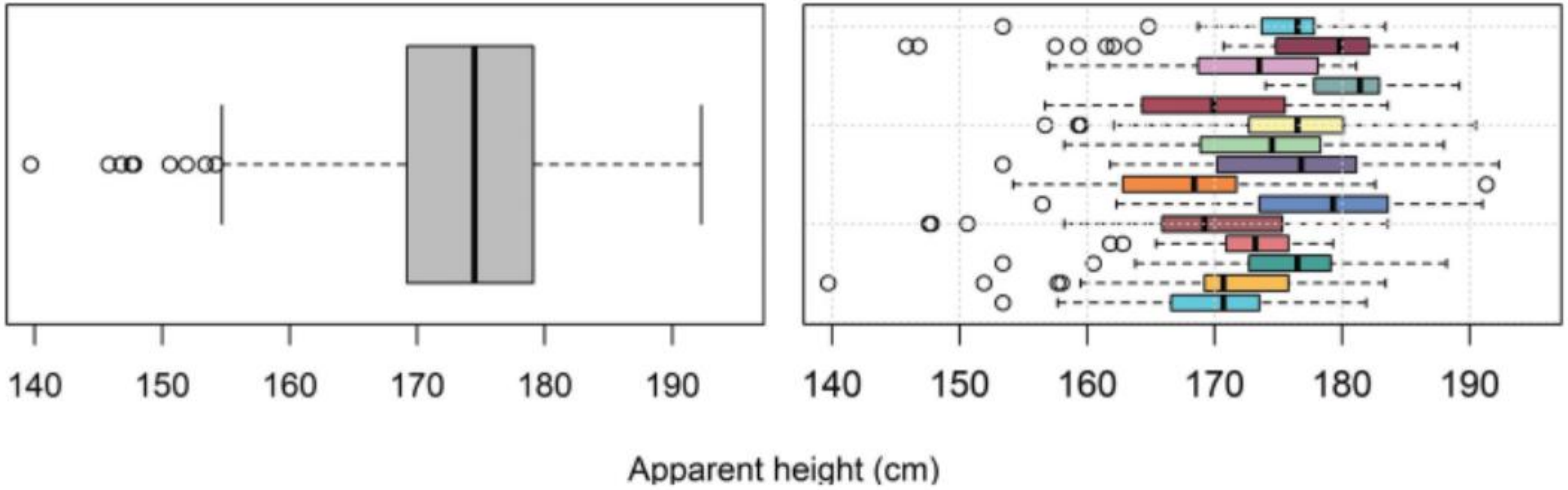
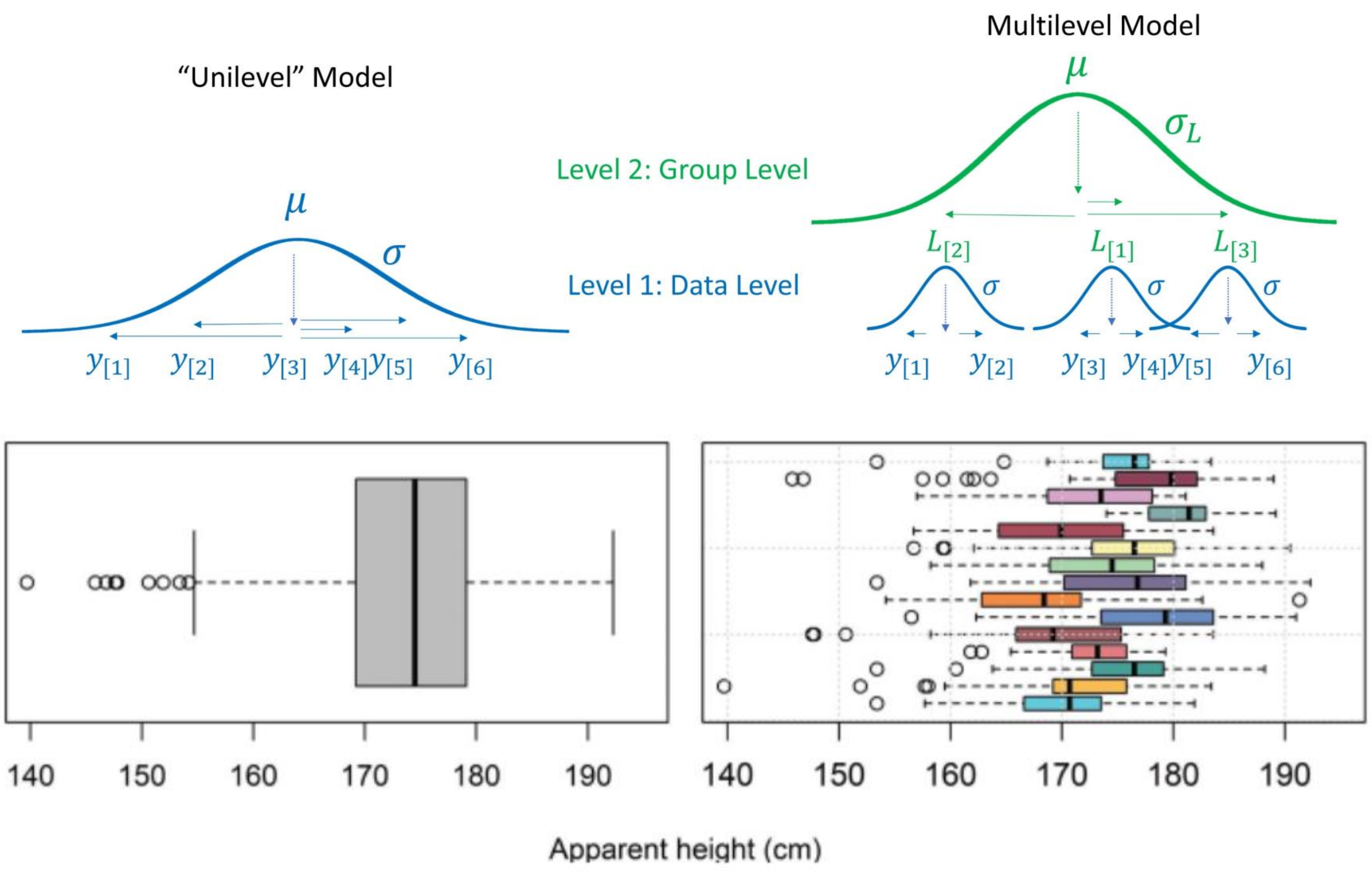


Repeated Measures Data: Levels of Variation

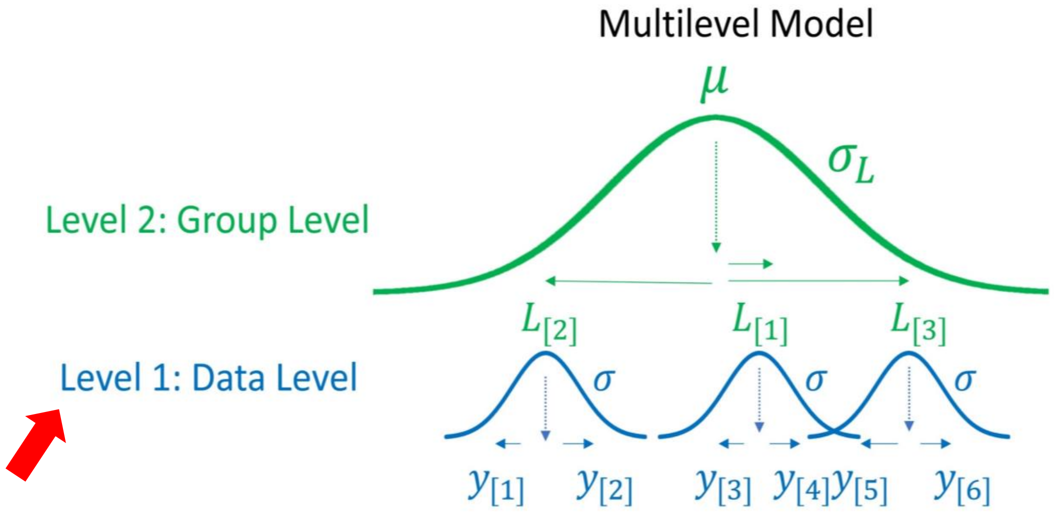


‘Levels’ of Variation: The ‘Lower’ level

- La distribución “data level” *dentro* de un clúster/unidad/sujeto determinado.

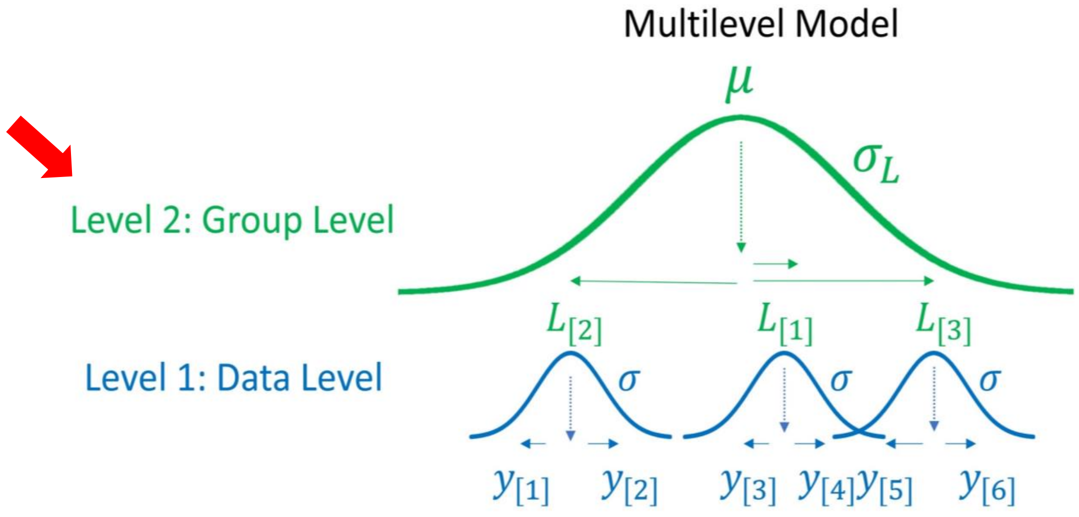
- La distribución condicional de datos dada una fuente específica de datos.

- Ejemplo: Una sola persona tendrá una distribución de tiempos de reacción para una tarea experimental.



‘Levels’ of Variation: The ‘Upper’ level

- La distribución de parámetros “group level" en diferentes clústeres/unidades/sujetos.
- Distribución marginal de los parámetros entre todas las fuentes de datos.
- Ejemplo: Habrá una distribución de los tiempos de reacción promedio para una tarea experimental, entre los participantes.



Multilevel Models

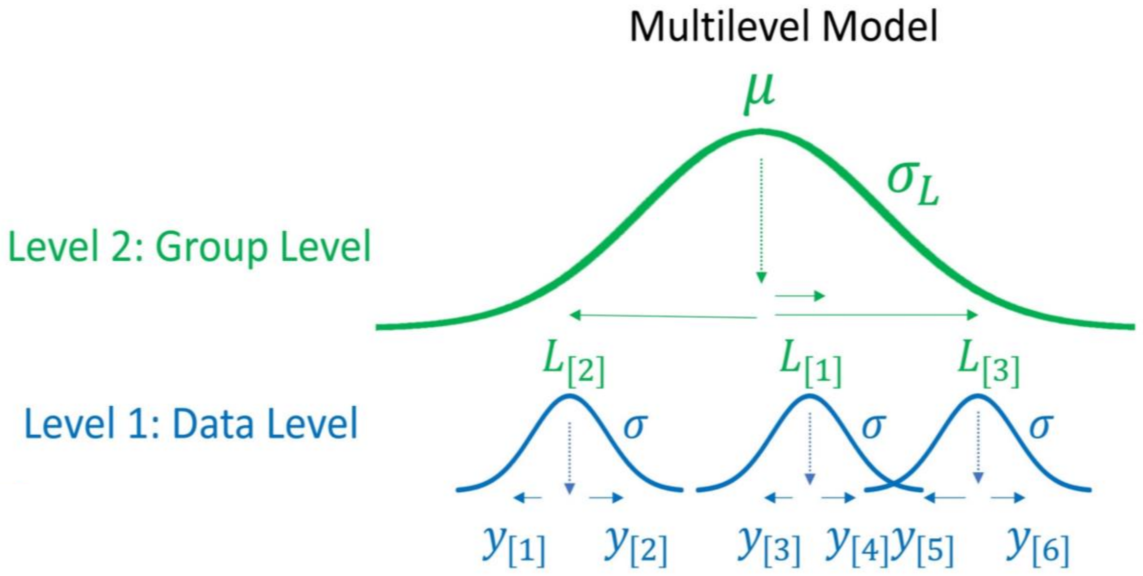
- Los modelos multinivel pueden estimar parámetros de "múltiples niveles" simultáneamente.

- Por ejemplo, podemos modelar:

  - Variación dentro del sujeto (σ).

  - Promedios para cada sujeto ($L_{[i]}$).

  - Variación entre sujetos (σ_{L}).



Representing Factors with Many Levels

- Cada nivel de un factor necesita un predictor.
- Matemáticamente, esto es una matriz de 1s y 0s.
- Tenemos (aproximadamente) 15 predictores para nuestros 15 niveles de oyente.

$$\mu_{[i]} = \text{Intercept} + L_{[1]} \cdot 0 + L_{[2]} \cdot 1 + L_{[3]} \cdot 0 + \ldots + L_{[15]} \cdot 0$$

$$\mu_{[i]} = \text{Intercept} + L_{[2]}$$

Representing Factors with Many Levels

- Representaremos los parámetros que representan los niveles de un factor como vectores.
- Estos serán seleccionados por nuestros predictores del mismo nombre.
- Esto significa: "Nuestro valor previsto para la prueba i, $\mu_{[i]}$, es la suma del intercepto del modelo y el coeficiente L indexado por el valor del predictor L para el ensayo i ($\mathsf{L}_{[i]}$)."

$$\mu_{[i]} = \text{Intercept} + L_{[\mathsf{L}_{[i]}]}$$

$$\mathsf{L}_{[1]} = 2, \mathsf{L}_{[2]} = 4, \mathsf{L}_{[3]} = 1, \ldots$$

$$\mu_{[i=1]} = \text{Intercept} + L_{[\mathsf{L}_{[i=1]}]} = \text{Intercept} + L_{[2]}$$

$$\mu_{[i=2]} = \text{Intercept} + L_{[\mathsf{L}_{[i=2]}]} = \text{Intercept} + L_{[4]}$$

$$\mu_{[i=3]} = \text{Intercept} + L_{[\mathsf{L}_{[i=3]}]} = \text{Intercept} + L_{[1]}$$

$$\ldots$$

Strategies for Estimating Factors with Many Levels

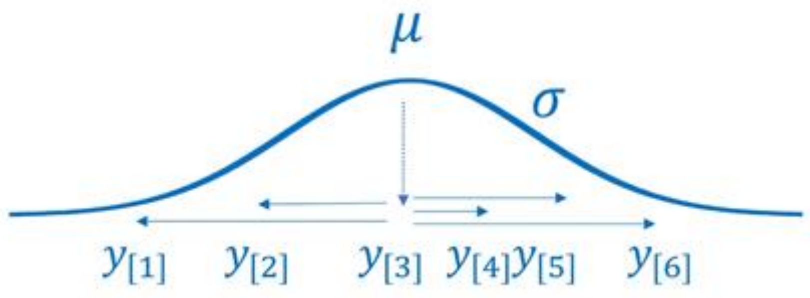
- Sus factores pueden tener mas que 10 niveles (¡o más de 100!).

- Esto puede resultar en un gran número de parámetros.

- Hay tres maneras generales para tratar los factores con muchos niveles:
  - Agrupación completa.
  - Sin agrupación.
  - Agrupación parcial (adaptativa).

Complete Pooling

- Tiras todo en una gran pila y actúas como si no tuvieras clústeres de datos individuales.
- Pierdes información útil (e.g., σ_{L}).
- Gran problema: es probable que los datos no sean independientes y los IC no serán fiables.



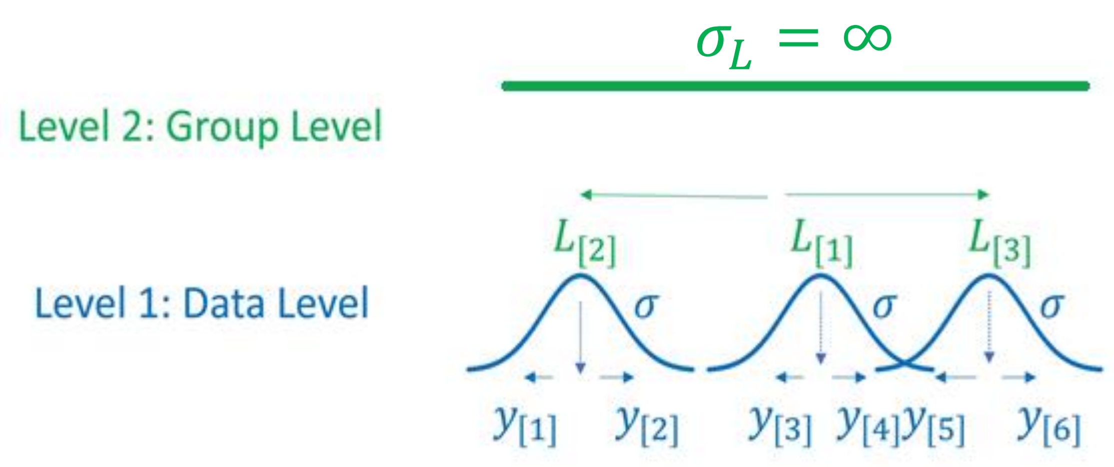
$$\mathrm{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$
$$\mu_{[i]} = \mathrm{Intercept}$$

Priors:

$$\mathrm{Intercept} \sim \mathrm{N}(176, 15)$$
$$\sigma \sim \mathrm{N}(0, 15)$$

No Pooling

- Reconoce los clústeres, pero no impone restricciones en sus valores.
- Pierdes información útil (e.g., σ_L).
- Problema mediano: Valores de parámetros entre clústeres (i.e. L) por lo general, no son totalmente independientes.



$$\text{height}_{[i]} \sim \text{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \text{Intercept} + L_{\left[\text{L}[i]\right]}$$

Priors:

$$L_{[\bullet]} \sim \text{uniform}\left(-\infty, \infty\right)$$

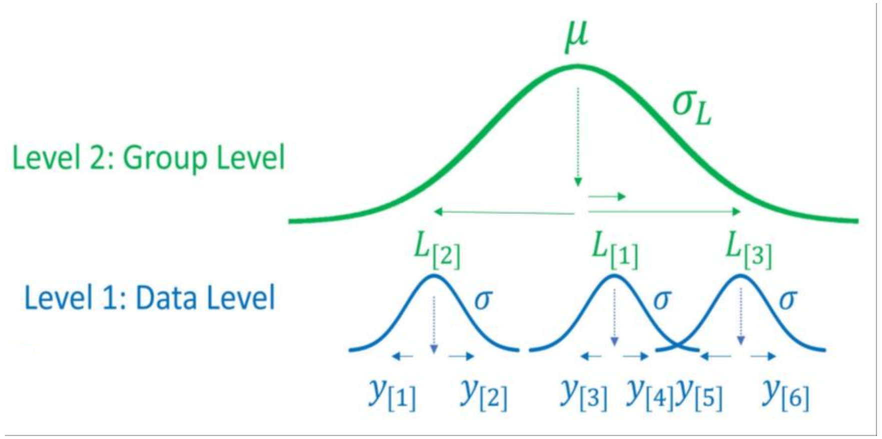
$$\text{Intercept} \sim \text{N}\left(176, 15\right)$$

Partial Pooling

- Reconoce los clústeres e impone restricciones probabilísticas a sus valores.

- Utiliza información en el "segundo" nivel (e.g, σ_L) para modelar.

- Gran beneficio: Más información y modelos que se comportan mejor.



$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \text{Intercept} + L_{\left[\mathrm{L}[i]\right]}$$

Priors :

$$L_{[\cdot]} \sim \mathrm{N}(0, \sigma_L)$$

$$\text{Intercept} \sim \mathrm{N}(176, 15)$$

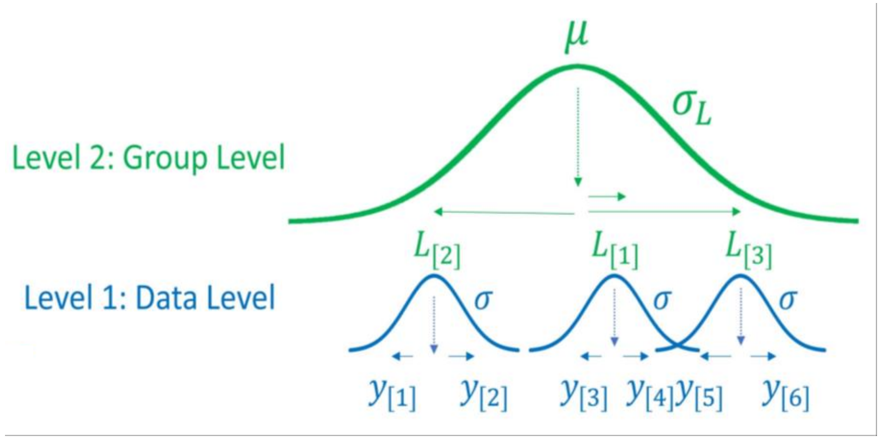
$$\sigma \sim \mathrm{N}(0, 15)$$

$$\sigma_L \sim \mathrm{N}(0, 15)$$

Adaptive Partial Pooling

- La agrupación parcial es cualquier modelo que utilice la misma información (e.g. σ_{L}) para diferentes parámetros.

- La agrupación parcial *adaptativa* es cuando un modelo estima estas restricciones(e,g, σ_{L}) de los datos.



$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \text{Intercept} + L_{\left[\mathrm{L}_{[i]}\right]}$$

Priors :

$$L_{[\bullet]} \sim \mathrm{N}(0, \sigma_L)$$

$$\text{Intercept} \sim \mathrm{N}(176, 15)$$

$$\sigma \sim \mathrm{N}(0, 15)$$

$$\sigma_L \sim \mathrm{N}(0, 15)$$

Comparison of Approaches

Agrupación completa

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$
$$\mu_{[i]} = \text{Intercept}$$

Priors:

$$\text{Intercept} \sim \mathrm{N}(176,15)$$
$$\sigma \sim \mathrm{N}(0,15)$$

Sin agrupación

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$
$$\mu_{[i]} = \text{Intercept} + L_{\left[\mathrm{L}[i]\right]}$$

Priors:

$$L_{[\cdot]} \sim \text{uniform}(-\infty, \infty)$$

$$\text{Intercept} \sim \mathrm{N}(176,15)$$

Agrupación parcial

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$
$$\mu_{[i]} = \text{Intercept} + L_{\left[\mathrm{L}[i]\right]}$$

Priors :

$$L_{[\cdot]} \sim \mathrm{N}\left(0, \sigma_L\right)$$

$$\text{Intercept} \sim \mathrm{N}(176,15)$$
$$\sigma \sim \mathrm{N}(0,15)$$
$$\sigma_L \sim \mathrm{N}(0,15)$$

Hyperpriors

- Los priores para tus priores.

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \text{Intercept} + L_{\left[\mathsf{L}_{[i]}\right]}$$

$$\text{Priors}:$$

$$L_{[\bullet]} \sim \mathrm{N}(0, \sigma_L)$$

$$\text{Intercept} \sim \mathrm{N}(176, 15)$$

$$\sigma \sim \mathrm{N}(0, 15)$$

$$\sigma_L \sim \mathrm{N}(0, 15)$$

$$P\left(L_{[\bullet]} \mid y\right) = \frac{P\left(y \mid L_{[\bullet]}\right) \cdot P\left(L_{[\bullet]}\right)}{P(y)}$$

$$P\left(L_{[\bullet]}, \sigma_L \mid y\right) = \frac{P\left(y \mid L_{[\bullet]}, \sigma_L\right) \cdot P\left(L_{[\bullet]}, \sigma_L\right)}{P(y)}$$

$$P\left(L_{[\bullet]}, \sigma_L \mid y\right) = \frac{P\left(y \mid L_{[\bullet]}\right) \cdot P\left(L_{[\bullet]} \mid \sigma_L\right) \cdot P(\sigma_L)}{P(y)}$$

What Gets a Prior?

- Todos los parámetros <u>estimados</u> necesitan priores.
- Por ejemplo:
  - L y σ_L necesitan priores porque están estimados.
  - La desviación estándar de σ_L es 15. <u>No</u> se estima.

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \text{Intercept} + L_{\left[\mathsf{L}[i]\right]}$$

$$\text{Priors:}$$

$$L_{[\bullet]} \sim \mathrm{N}(0, \sigma_L)$$

$$\text{Intercept} \sim \mathrm{N}(176, 15)$$

$$\sigma \sim \mathrm{N}(0, 15)$$

$$\sigma_L \sim \mathrm{N}(0, 15)$$

Data and Research Questions

```
# load book package and brms
library (bmmb)
library (brms)

# load and subset experimental data
data (exp_data)
men = exp_data[exp_data$C_v=='m',]
```

- L: An integer from 1 to 15 indicating which *listener* responded to the trial.
- height: A floating-point number representing the *height* (in centimeters) reported for the speaker on each trial.

```
head (men)
##      L C height R  S C_v  vtl  f0 dur G A G_v A_v
## 93   1 m  169.9 a 47   m 14.8 172 339 m a   m   a
## 95   1 m  173.5 a 48   m 15.6 108 236 m a   m   a
## 97   1 m  172.0 a 49   m 15.5  96 315 m a   m   a
```

Data and Research Questions

Vamos a responder de nuevo a las siguientes preguntas, esta vez con un modelo multinivel legítimo:

(1) ¿Qué tan alto suena el hombre adulto promedio?

(2) ¿Podemos establecer límites a las alturas aparentes medias creíbles en función de los datos que hemos recopilado?

The Model Formula

- Nuestro modelo anterior decía: "predecimos la altura usando un solo intercepto general".

```
height ~ 1
```

- No queremos modelar un intercepto, sino un intercepto para cada oyente.

- Queremos entender la distribución del intercepto dado el sujeto.

```
P(Intercept|Listener)
```

The Model Formula

- Para modelar interceptos dependientes del oyente con agrupación parcial adaptable, usamos esta fórmula:

```
height ~ 1 + ( 1 | L)
```

- Todo lo que pongas entre paréntesis se ajusta con agrupación parcial.

```
(Predictor|Grouping factor)
```

Description of the Model

We expect height judgments to be normally distributed around the expected value for any given trial, $\mu_{[i]}$, with some unknown standard deviation σ. The expected value for a trial is equal to a fixed overall average (Intercept) and some value associated with the individual listener who made a perceptual judgment on the trial $\left(L_{[\mathsf{L}_{[i]}]}\right)$. The listener coefficients $\left(L_{[\bullet]}\right)$ were modeled as coming from a normal distribution with a mean of zero and a standard deviation (σ_L) that was estimated from the data.

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \text{Intercept} + L_{[\mathsf{L}_{[i]}]}$$

Priors :

$$L_{[\bullet]} \sim \mathrm{N}(0, \sigma_L)$$

$$\text{Intercept} \sim \mathrm{N}(176, 15)$$

$$\sigma \sim \mathrm{N}(0, 15)$$

$$\sigma_L \sim \mathrm{N}(0, 15)$$

Decomposing Variation

Modelo 'Uninivel' $\longrightarrow$ $\sigma_{\text{total}}^2 = \sigma^2$

Modelo multinivel $\longrightarrow$ $\sigma_{\text{total}}^2 = \sigma_L^2 + \sigma^2$

Specifying Priors

- `Intercept`: This is a unique class, only for intercepts.
- `sd`: This is for standard deviation parameters related to 'batches' of parameters, e.g. `sd(Intercept)` for `L` (σ_L).
- `sigma`: The data-level error term.

```
bmmb::get_prior (height ~ 1 + (1|L), data = men)[,-c(7:9)]
##                   prior     class      coef group resp dpar  source
##  student_t(3, 174.5, 7.1) Intercept                         default
##       student_t(3, 0, 7.1)     sd                           default
##       student_t(3, 0, 7.1)     sd               L           default
##       student_t(3, 0, 7.1)     sd  Intercept    L           default
##       student_t(3, 0, 7.1)     sigma                        default
```

Fitting the Model

```
# Fit the model yourself
model_multilevel =  brms::brm (
  height ~ 1 + (1|L), data = men, chains = 4, cores = 4,
  warmup = 1000, iter = 3500, thin = 2,
  prior = c(brms::set_prior("normal(176, 15)", class = "Intercept"),
          brms::set_prior("normal(0, 15)", class = "sd"),
          brms::set_prior("normal(0, 15)", class = "sigma")))

# Or download it from the GitHub page:
model_multilevel = bmmb::get_model ('4_model_multilevel.RDS')
```

Interpreting the Model

```
# inspect model
bmmb::short_summary (model_multilevel)
## Formula:  height ~ 1 + (1 | L)
##
## Group-Level Effects:
## ~L (Number of levels: 15)
##               Estimate Est.Error l-95% CI u-95% CI
## sd(Intercept)     3.78      0.87     2.47     5.84
##
## Population-Level Effects:
##           Estimate Est.Error l-95% CI u-95% CI
## Intercept    173.8      1.02    171.8    175.8
##
## Family Specific Parameters:
##       Estimate Est.Error l-95% CI u-95% CI
## sigma     7.03      0.19     6.67     7.41
```

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \text{Intercept} + L_{\left[\mathsf{L}_{[i]}\right]}$$

Priors :

$$L_{[\bullet]} \sim \mathrm{N}(0, \sigma_L)$$

$$\text{Intercept} \sim \mathrm{N}(176, 15)$$

$$\sigma \sim \mathrm{N}(0, 15)$$

$$\sigma_L \sim \mathrm{N}(0, 15)$$

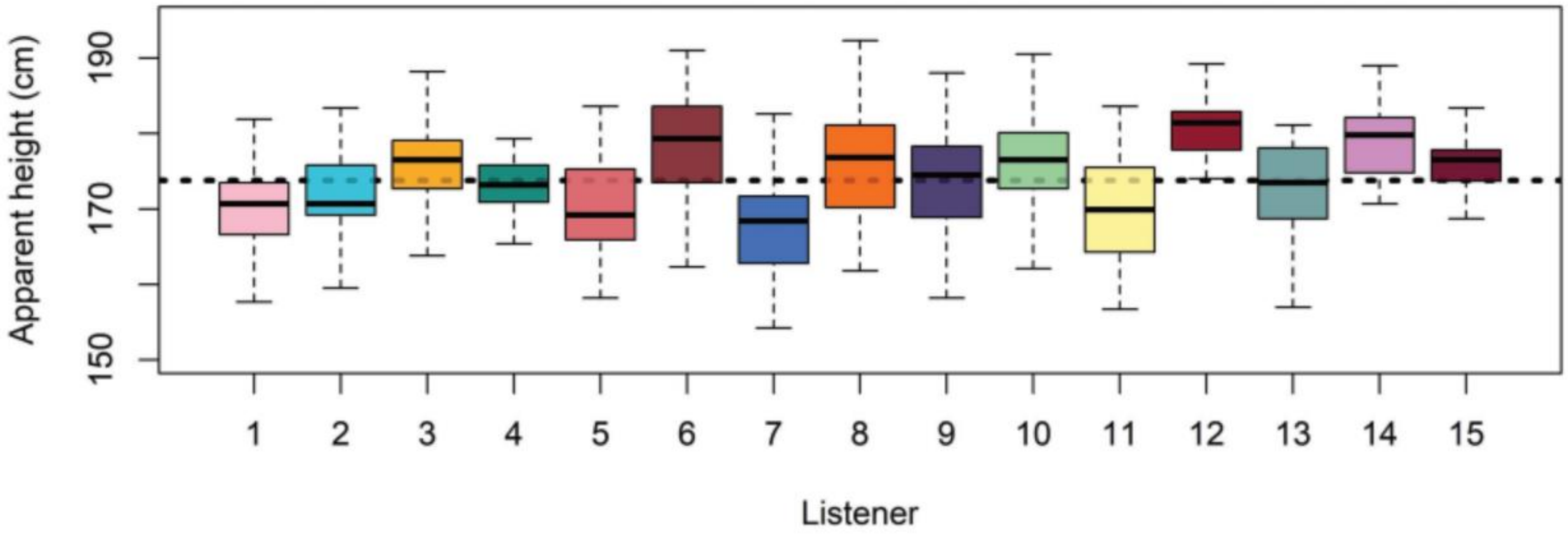
Interpreting the Model

```
# inspect model
bmmb::short_summary (model_multil
## Formula:  height ~ 1 + (1 | L)
##
## Group-Level Effects:
## ~L (Number of levels: 15)
##               Estimate Est.Err
## sd(Intercept)     3.78      0.
##
## Population-Level Effects:
##           Estimate Est.Error l
## Intercept    173.8      1.02
##
## Family Specific Parameters:
##       Estimate Est.Error l-95%
## sigma     7.03      0.19      6
```

```
# find mean height for each listener
listener_means = aggregate (height ~ L, data = men, FUN = mean)

# find the within listener standard deviation
# This is the within-talker 'error'.
listener_sigmas = aggregate (height ~ L, data = men, FUN = sd)

# the mean of the listener means corresponds to our Intercept
mean (listener_means$height)
## [1] 173.8

# the standard deviation of the listener means corresponds
# to 'sd(Intercept)', the estimate of the standard deviation
# of listener intercepts
sd (listener_means$height)
## [1] 3.594

# the average within-listener standard deviation corresponds
# to sigma, the estimated error
mean (listener_sigmas$height)
## [1] 6.822
```

Interpreting the Model

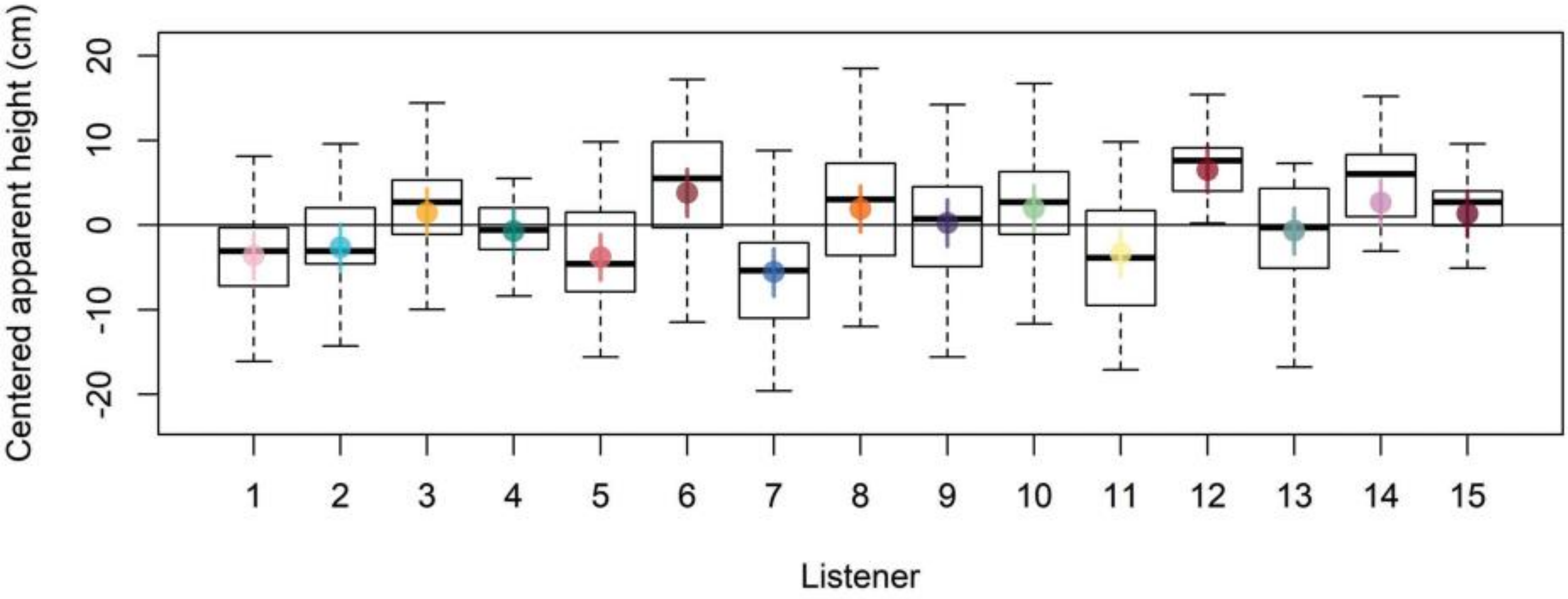
```
## Group-Level Effects:
## ~L (Number of levels: 15)
##               Estimate Est.Error l-95% CI u-95% CI
## sd(Intercept)     3.78      0.87     2.47     5.84
##
## Population-Level Effects:
##           Estimate Est.Error l-95% CI u-95% CI
## Intercept    173.8      1.02    171.8    175.8
##
## Family Specific Parameters:
##       Estimate Est.Error l-95% CI u-95% CI
## sigma     7.03      0.19     6.67     7.41
```



‘Random’ and ‘Fixed’ Effects

- Muchas definiciones inconsistentes de estos términos.
- En la práctica:
  - Los efectos "fijos" se estiman utilizando ninguna agrupación, o la agrupación es mínima.
  - Los efectos aleatorios suelen ajustarse con la agrupación parcial adaptativa.

Inspecting the 'Random' Effects



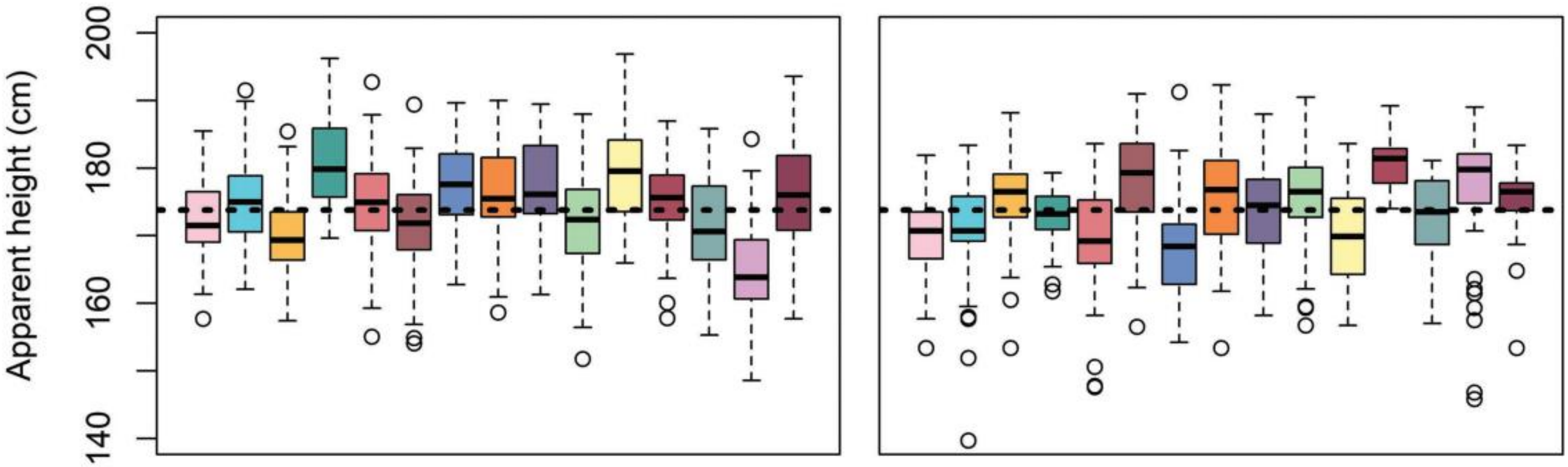
Simulating Data

```
# skip this line if you want a new simulated data set.
set.seed(1)
# this is the value of our intercept
Intercept = 174
# this is a vector of 15 listener effects
L_ = rnorm (15, 0, 3.8 )
# vector indicating which listener produced which utterance
L = rep (1:15, each = 45)
# this vector contains the error
error = rnorm (45 * 15, 0, 7)
```

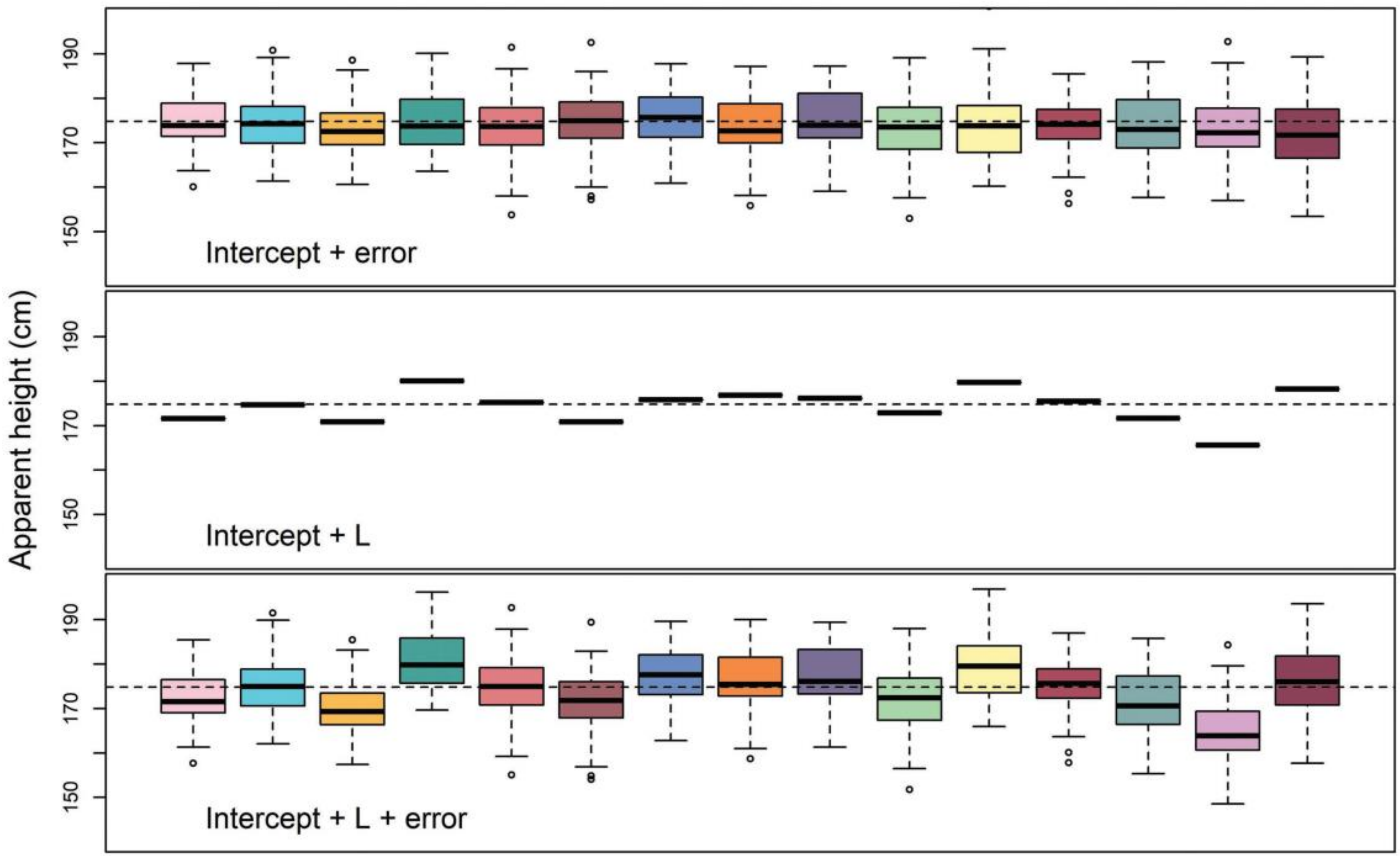
```
# the sum of an intercept, listener effects and random error
height_rep = Intercept + L_[L] + error
```

```
# this fake data is missing between listener variation
height_rep_1 = Intercept + error
# this fake data is missing within listener variation
height_rep_2 = Intercept +  L_[L]
```

Simulating Data



Simulating Data

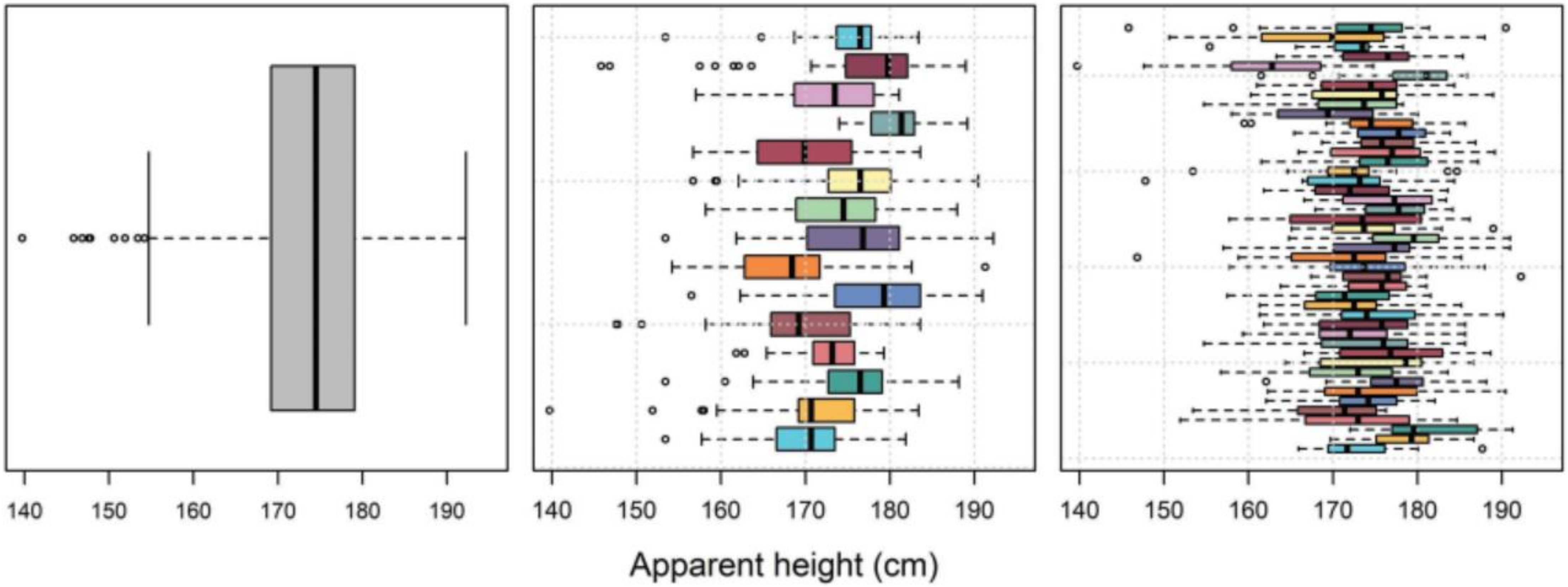


Simulating Data

```
set.seed(1)
# do 10,000 replications
reps = 10000

# hold the replicated values of sigma_L
sigma_L_rep = rep(0,reps)
for ( i in 1:reps){
  Intercept = 173.8 # set the intercept
  L_L = rnorm (15, 0, 0)  # zero between-listener variance
  L = rep (1:15, each = 45) # 45 responses from each of 15 listeners
  epsilon = rnorm (45 * 15, 0, 7.78) # generate random noise
  height_rep = Intercept + L_L[L] + epsilon # add up to components

# get replicated listener means
  L_rep_means = tapply(height_rep, L, mean)
  sigma_L_rep[i] = sd (L_rep_means) # find sigma of listener effects
}
```

```
quantile(sigma_L_rep)
##      0%    25%    50%    75%   100%
## 0.4429 0.9862 1.1337 1.2810 2.0813
```

Adding a Second Random Effect



Updating our Model Description

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \text{Intercept} + L_{\left[\mathsf{L}_{[i]}\right]} + S_{\left[\mathsf{S}_{[i]}\right]}$$

$$\text{Priors:}$$

$$L_{[\bullet]} \sim \mathrm{N}(0, \sigma_L)$$

$$S_{[\bullet]} \sim \mathrm{N}(0, \sigma_S)$$

$$\text{Intercept} \sim \mathrm{N}(176, 15)$$

$$\sigma \sim \mathrm{N}(0, 15)$$

$$\sigma_L \sim \mathrm{N}(0, 15)$$

$$\sigma_S \sim \mathrm{N}(0, 15)$$

Fitting our Model

```
model_multilevel_L_S = brms::brm (
  height ~ 1 + (1|L) + (1|S), data = men, chains = 4, cores = 4,
  warmup = 1000, iter = 3500, thin = 2,
  prior = c(brms::set_prior("normal(176, 15)", class = "Intercept"),
            brms::set_prior("normal(0, 15)", class = "sd"),
            brms::set_prior("normal(0, 15)", class = "sigma")))
```

Interpreting the New Information

```
bmmb::short_summary (model_multilevel_L_S)
## Formula:  height ~ 1 + (1 | L) + (1 | S)
##
## Group-Level Effects:
## ~L (Number of levels: 15)
##                Estimate Est.Error l-95% CI u-95% CI
## sd(Intercept)      3.81      0.86     2.51     5.87
##
## ~S (Number of levels: 45)
##                Estimate Est.Error l-95% CI u-95% CI
## sd(Intercept)      2.83      0.42      2.1     3.72
##
## Population-Level Effects:
##            Estimate Est.Error l-95% CI u-95% CI
## Intercept     173.8      1.12    171.6    176.1
##
## Family Specific Parameters:
##        Estimate Est.Error l-95% CI u-95% CI
## sigma      6.47      0.19     6.11     6.85
```

$$\text{height}_{[i]} \sim \mathrm{N}\left(\mu_{[i]}, \sigma\right)$$

$$\mu_{[i]} = \text{Intercept} + L_{\left[\mathsf{L}_{[i]}\right]} + S_{\left[\mathsf{S}_{[i]}\right]}$$

Priors :

$$L_{[\cdot]} \sim \mathrm{N}(0, \sigma_L)$$

$$S_{[\cdot]} \sim \mathrm{N}(0, \sigma_S)$$

$$\text{Intercept} \sim \mathrm{N}(176, 15)$$

$$\sigma \sim \mathrm{N}(0, 15)$$

$$\sigma_L \sim \mathrm{N}(0, 15)$$

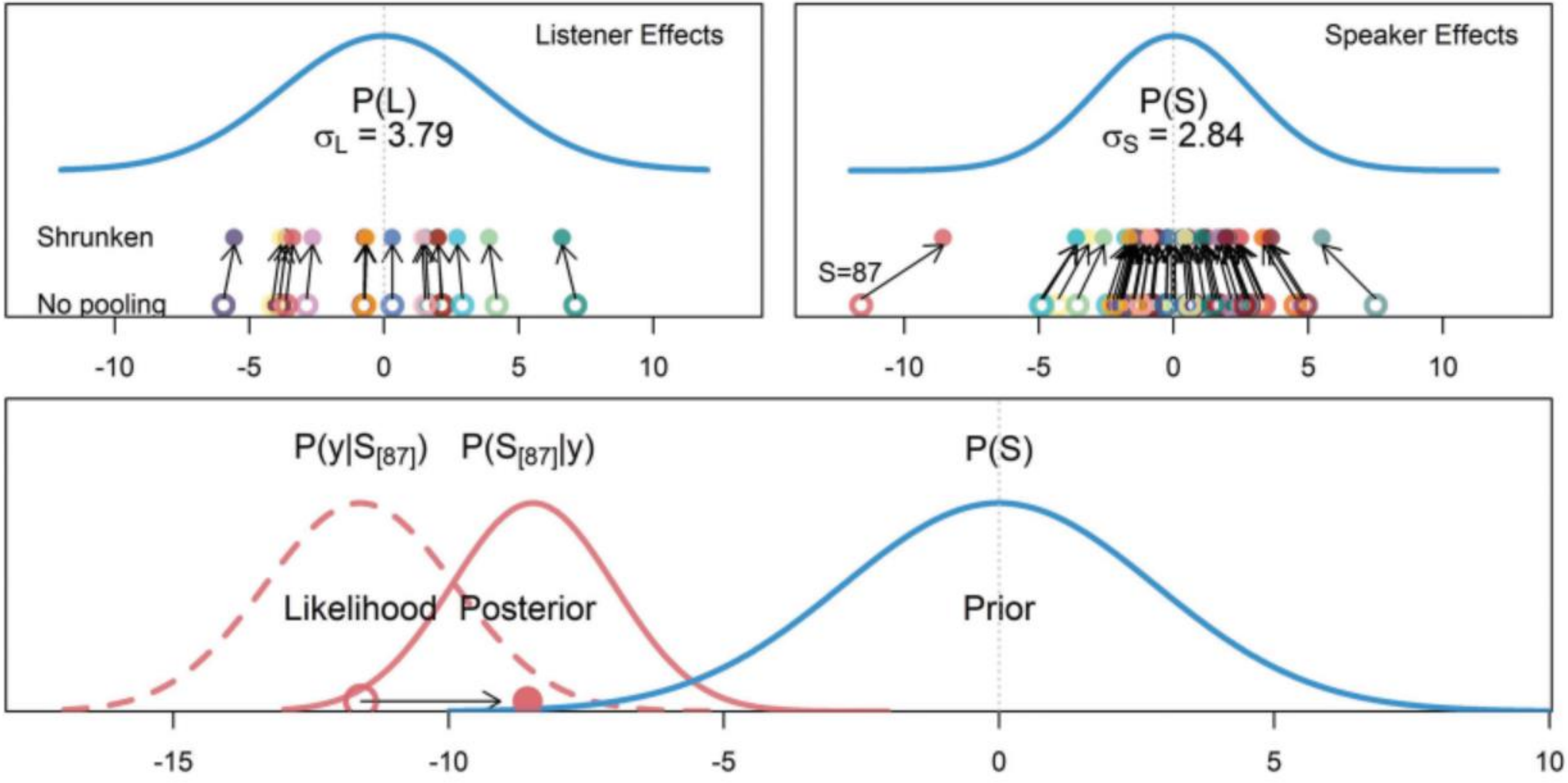
$$\sigma_S \sim \mathrm{N}(0, 15)$$

Comparing Models

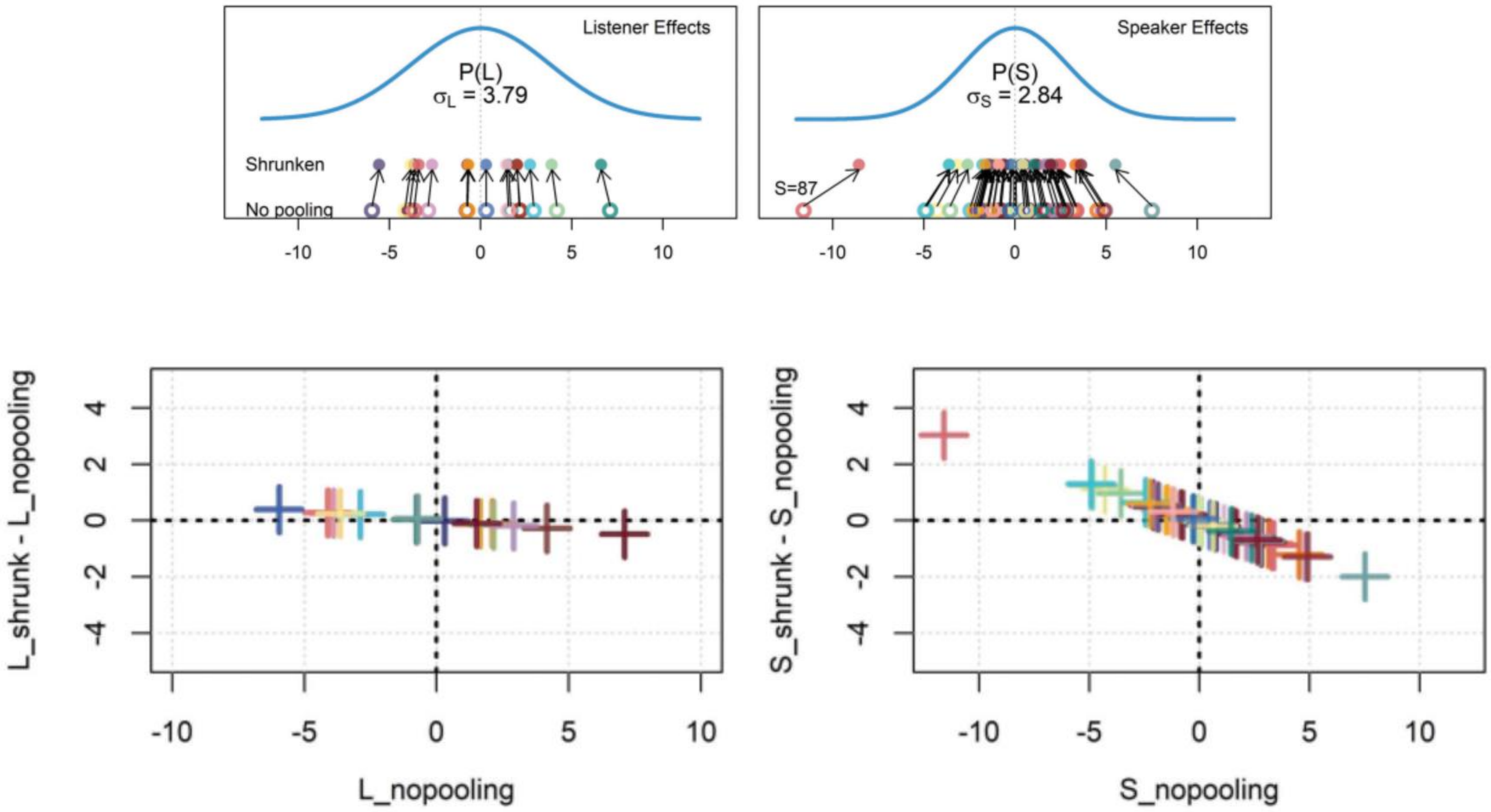
```
bmmb::short_summary (model_priors)
## Formula:  height ~ 1
## Population-Level Effects:
##            Estimate Est.Error l-95% CI u-95% CI
## Intercept     173.8      0.31    173.2    174.4
##
## Family Specific Parameters:
##        Estimate Est.Error l-95% CI u-95% CI
## sigma      7.77      0.21     7.37     8.19
```

```
bmmb::short_summary (model_multilevel_L_S)
## Formula:  height ~ 1 + (1 | L) + (1 | S)
##
## Group-Level Effects:
## ~L (Number of levels: 15)
##               Estimate Est.Error l-95% CI u-95% CI
## sd(Intercept)     3.81      0.86     2.51     5.87
##
## ~S (Number of levels: 45)
##               Estimate Est.Error l-95% CI u-95% CI
## sd(Intercept)     2.83      0.42      2.1     3.72
##
## Population-Level Effects:
##           Estimate Est.Error l-95% CI u-95% CI
## Intercept    173.8      1.12    171.6    176.1
##
## Family Specific Parameters:
##       Estimate Est.Error l-95% CI u-95% CI
## sigma     6.47      0.19     6.11     6.85
```

Investigating ‘Shrinkage’



Investigating ‘Shrinkage’



Answering our Research Questions

```
bmmb::short_summary (model_multilevel_L_S)
## Formula:  height ~ 1 + (1 | L) + (1 | S)
##
## Group-Level Effects:
## ~L (Number of levels: 15)
##               Estimate Est.Error l-95% CI u-95% CI
## sd(Intercept)     3.81      0.86     2.51     5.87
##
## ~S (Number of levels: 45)
##               Estimate Est.Error l-95% CI u-95% CI
## sd(Intercept)     2.83      0.42      2.1     3.72
##
## Population-Level Effects:
##           Estimate Est.Error l-95% CI u-95% CI
## Intercept    173.8      1.12    171.6    176.1
##
## Family Specific Parameters:
##       Estimate Est.Error l-95% CI u-95% CI
## sigma     6.47      0.19     6.11     6.85
```

Based on our model the average apparent height of adult males is likely to be 173.8 cm (s.d. = 1.1, 95% CI = [171.6, 176.1]). The estimated magnitude of the random error was 6.5 cm (s.d. = 0.2, 95% CI = [6.1, 6.9]). Systematic between-listener variation averages about 3.8 cm (s.d. = 0.9, 95% CI = [2.5, 5.8]), while systematic between-speaker variation averages about 2.8 cm (s.d. = 0.4, 95% CI = [2.1, 3.8]).

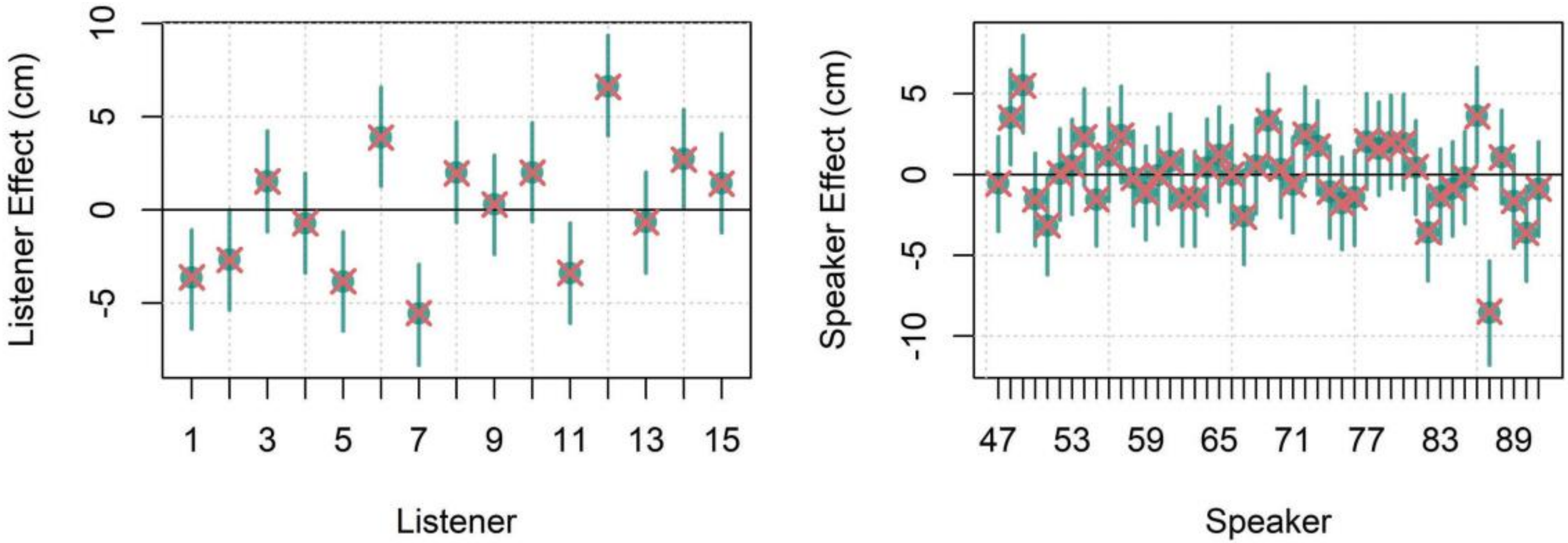
Traditionalists Corner: lmer

```r
lmer_model = lme4::lmer (height ~ 1 + (1|L) + (1|S), data = men)
```

```r
bmmb::short_summary(model_multilevel_L_S)
## (1) Formula:  height ~ 1 + (1 | L) + (1 | S)
##
## (2) Group-Level Effects:
##     ~L (Number of levels: 15)
##                    Estimate Est.Error l-95% CI u-95% CI
## (3) sd(Intercept)      3.81      0.86     2.51     5.87
##
##     ~S (Number of levels: 45)
##                    Estimate Est.Error l-95% CI u-95% CI
## (4) sd(Intercept)      2.83      0.42      2.1     3.72
##
##     Population-Level Effects:
##                Estimate Est.Error l-95% CI u-95% CI
## (5) Intercept     173.8      1.12    171.6   176.05
##
##     Family Specific Parameters:
##            Estimate Est.Error l-95% CI u-95% CI
## (6) sigma     6.47      0.19     6.11     6.85
```

```r
summary (lmer_model)
##     Linear mixed model fit by REML ['lmerMod']
## (1) Formula: height ~ 1 + (1 | L) + (1 | S)
##        Data: men
##
##     REML criterion at convergence: 4527.4
##
##     Scaled residuals:
##         Min      1Q  Median      3Q     Max
##     -4.6205 -0.4868  0.0722  0.5700  2.7179
##
## (2) Random effects:
##      Groups   Name        Variance Std.Dev.
## (3)  S        (Intercept)  7.593   2.756
## (4)  L        (Intercept) 11.990   3.463
## (6)  Residual             41.630   6.452
##     Number of obs: 675, groups:  S, 45; L, 15
##
##     Fixed effects:
##                 Estimate Std. Error t value
## (5) (Intercept)  173.788      1.015   171.3
```

Traditionalists Corner: lmer



Exercises

Use the data in 'exp_ex' to do one of the following. You may also use your own data to answer a related question. In any case, describe the model, present and explain the results, and include at least two figures.

1. Easy: Analyze the (pre-fit) model that's exactly like model_multilevel_L_S, except using the data in exp_ex (bmmb::get_model("model_multilevel_L_S_ex.RDS")).

2. Medium: Fit a model just like model_multilevel_L_S, but for the data from some other group, for either the original or big resonance levels.

3. Hard: Fit two models like model_multilevel_L_S for two groups, or for one group across resonance levels, and compare results across models.