Quantum Eraser

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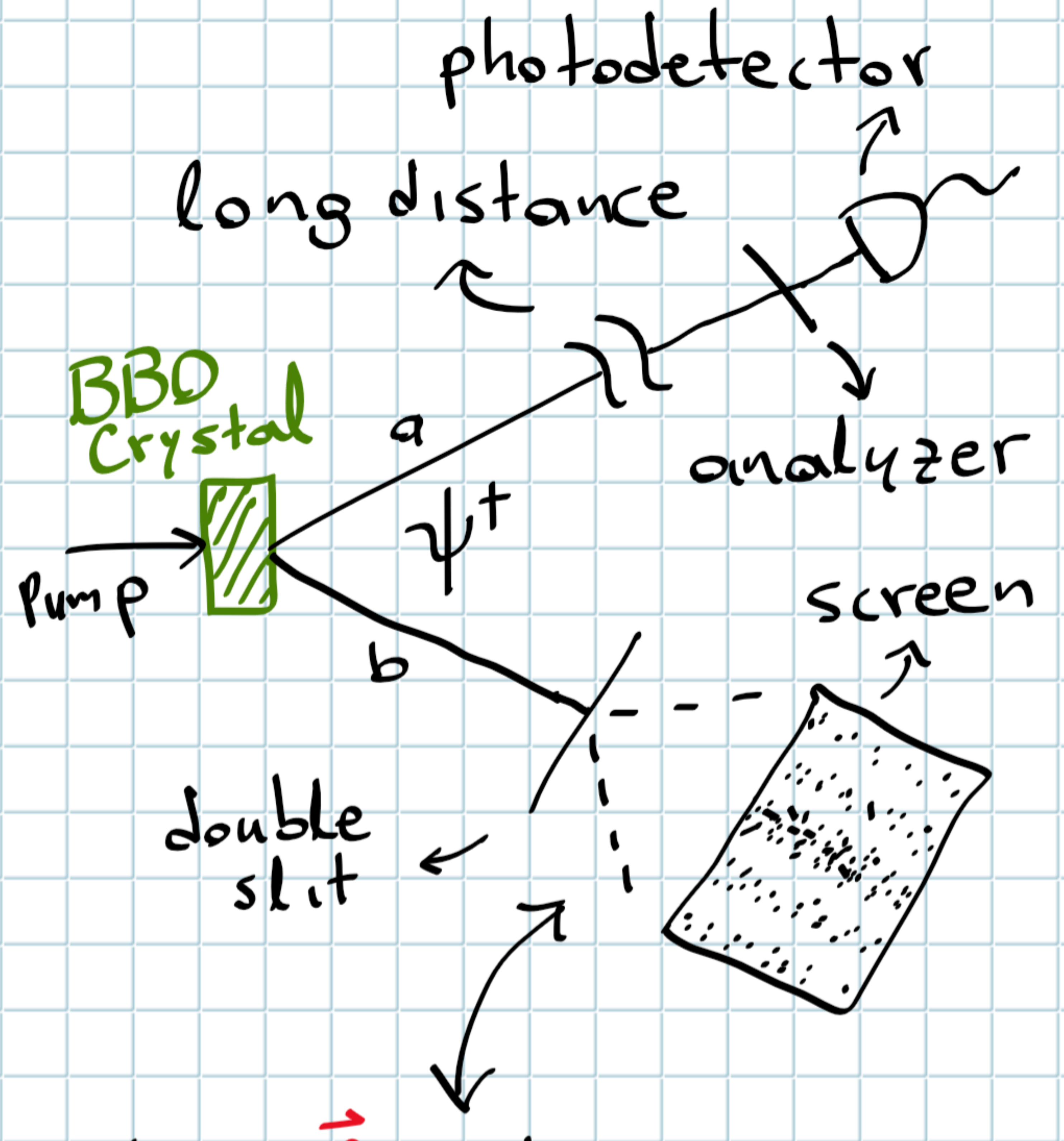


two-photon state generated by the BBO crystal

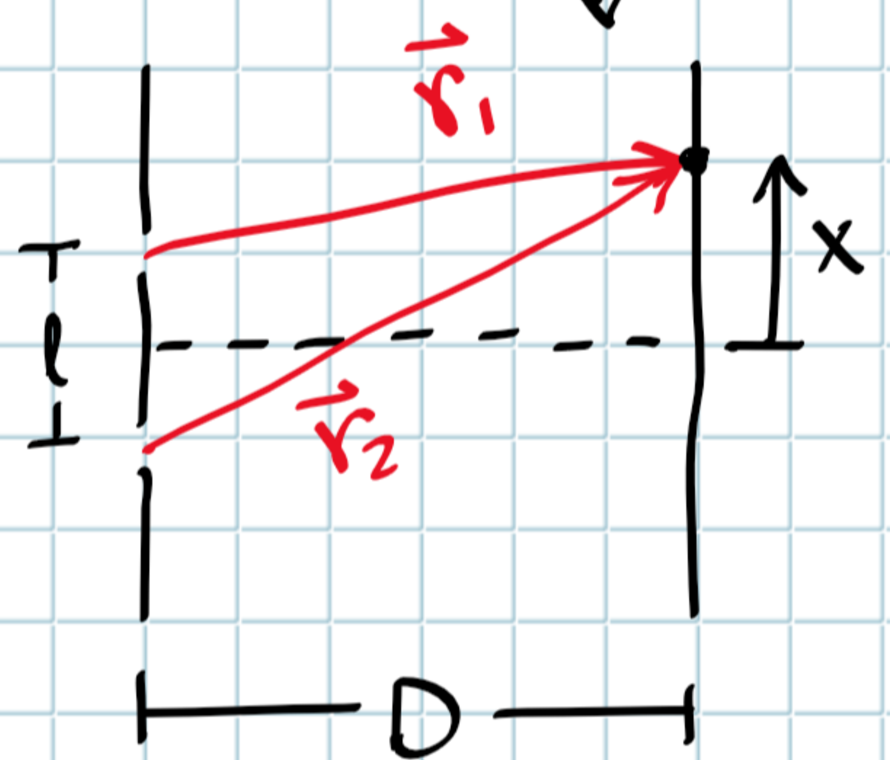
$$|\psi^+\rangle_{ab} = \frac{1}{\sqrt{2}}\left(|H\rangle_a |V\rangle_b + |V\rangle_a |H\rangle_b\right)$$

state immediately after the double slit (no QWPs)

$$|\psi^+\rangle_{ab} \otimes \frac{1}{\sqrt{2}}\left(|s_1\rangle_b + |s_2\rangle_b\right) \quad \text{which-path states}$$

the effect of free-space propagation is

$$|s_j\rangle_b \rightarrow \int_{-\infty}^{\infty} dx \, \frac{e^{ikr_j}}{r_j} |x\rangle_b , \quad \text{screen states}$$



normalizing...

$$U_{fsp}|s_j\rangle = \sqrt{\frac{D}{\pi}} \int_{-\infty}^{\infty} dx \, \frac{e^{ikr_j}}{r_j} |x\rangle_b$$

thus the final state is

$$|\psi_f\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle_a |V\rangle_b + |V\rangle_a |H\rangle_b\right) \otimes \sqrt{\frac{D}{2\pi}} \int dx \left(\frac{e^{ikr_1}}{r_1} + \frac{e^{ikr_2}}{r_2} \right) |x\rangle .$$

In this state, polarization and position degrees of freedom are separable (not entangled).

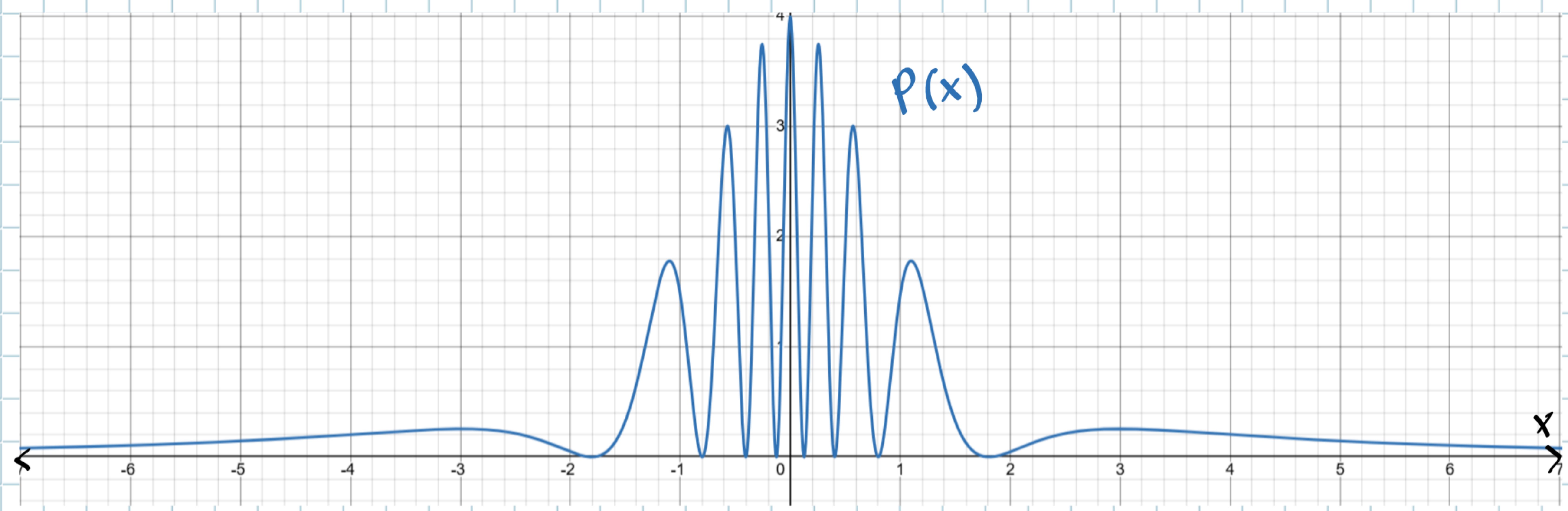
Since we do not measure polarization of the b photon, we can trace out this degree of freedom, which gives us

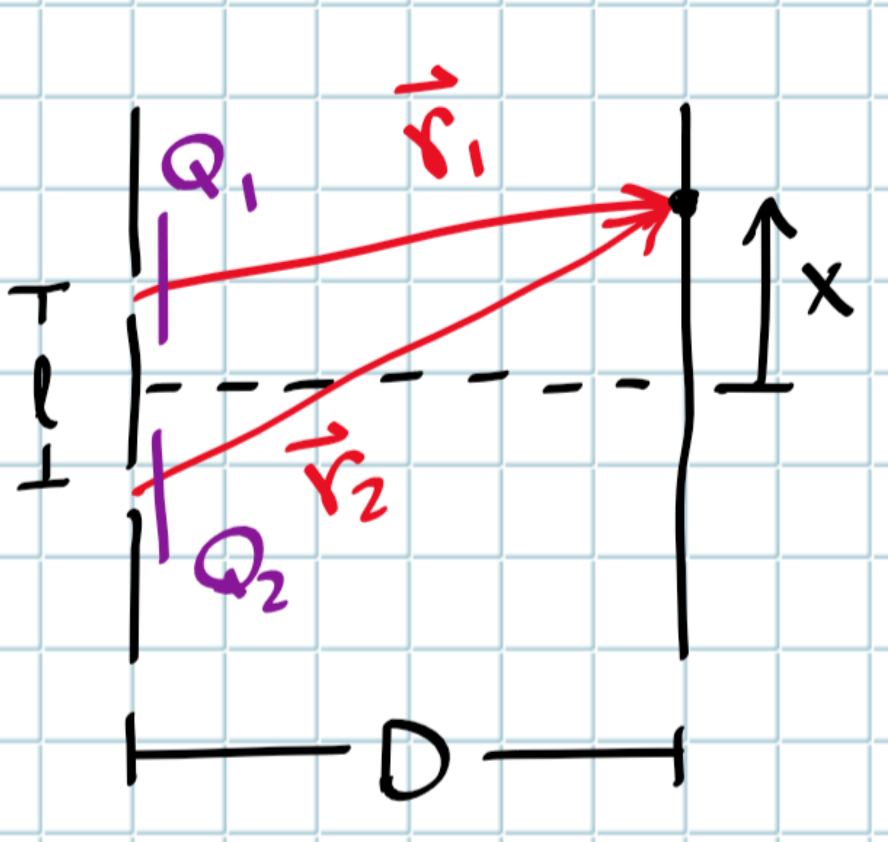
$$\boxed{\begin{aligned} \rho_f &= \left(\tfrac{1}{2}|H\rangle\langle H|_a + \tfrac{1}{2}|V\rangle\langle V|_a\right) \\ &\otimes \frac{D}{2\pi} \int dx \int dx' \left[\frac{e^{ik(r_1 - r_1')}}{r_1 r_1'} + \frac{e^{ik(r_1 - r_2')}}{r_1 r_2'} + \text{same } 1 \leftrightarrow 2 \right] |x\rangle\langle x'|_b \end{aligned}}$$

thus, the photon detection probability density at the screen is

$$P(x) = \langle x | \mathrm{Tr}_a(\rho_f) | x \rangle = \frac{D}{2\pi}\left(\frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{2}{r_1 r_2} \cos[k(r_1 - r_2)] \right)$$

where $r_j = \sqrt{D^2 + (x \pm l/2)}$, regardless of what happens to photon a.





What if we introduced two QWPs Q_1 and Q_2 with their fast axes at $\pm 45°$ with respect to the horizontal? That is, such that

$$Q_1|H\rangle = |R\rangle \qquad Q_1|V\rangle = i|L\rangle$$

$$Q_2|H\rangle = |L\rangle \qquad Q_2|V\rangle = -i|R\rangle$$

$$Q_1 = \frac{1}{2}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$Q_2 = \frac{1}{2}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

Then, the state immediately after the double slit would be

$$\frac{1}{2}\Big(i|H\rangle_a \otimes \big(|L,S_1\rangle_b - |R,S_2\rangle_b\big) + |V\rangle_a \otimes \big(|R,S_1\rangle_b + |L,S_2\rangle_b\big)\Big)$$

After free-space propagation we get

$$|\psi_f\rangle = \frac{i}{2}|L\rangle_b \otimes \sqrt{\frac{D}{\pi}}\int dx \left(\frac{e^{ikr_1}}{r_1}|H\rangle_a - i\frac{e^{ikr_2}}{r_2}|V\rangle_a\right)|x\rangle_b$$

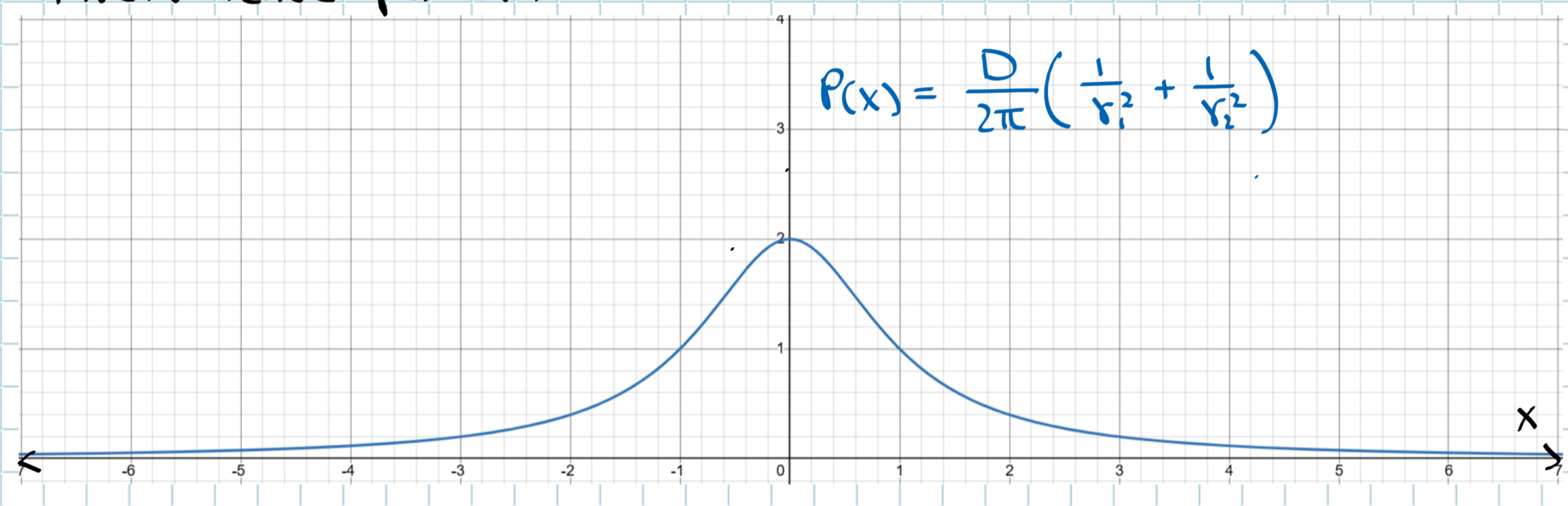
$$+\frac{1}{2}|R\rangle_b \otimes \sqrt{\frac{D}{\pi}}\int dx \left(\frac{e^{ikr_1}}{r_1}|V\rangle_a - i\frac{e^{ikr_2}}{r_2}|H\rangle_a\right)|x\rangle_b$$

Tracing out unmeasured degrees of freedom...

$$\rho_f = \frac{D}{4\pi}\int dx \int dx' \Bigg[\left(\frac{e^{ik(r_1-r_1')}}{r_1 r_1'} + \frac{e^{ik(r_2-r_2')}}{r_2 r_2'}\right)\overbrace{\big(|H\rangle\langle H|_a + |V\rangle\langle V|_a\big)}^{\mathbb{1}_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}$$

$$+ i\left(\frac{e^{ik(r_1-r_2')}}{r_1 r_2'} - \frac{e^{ik(r_2-r_1')}}{r_1' r_2}\right)\overbrace{\big(|H\rangle\langle V|_a + |V\rangle\langle H|_a\big)}^{X_a = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}\Bigg]|x\rangle\langle x'|_b$$

Here, regardless of what happens to photon a, we see no interference pattern



If photon a is measured in the HV-basis then we can post-select the outcomes of the measurements of photon b based on the results. This leads to

$$P_H(x) = {}_b\langle x|\,{}_a\langle H|\,\rho_f\,|H\rangle_a|x\rangle_b = \frac{D}{4\pi}\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}\right)$$

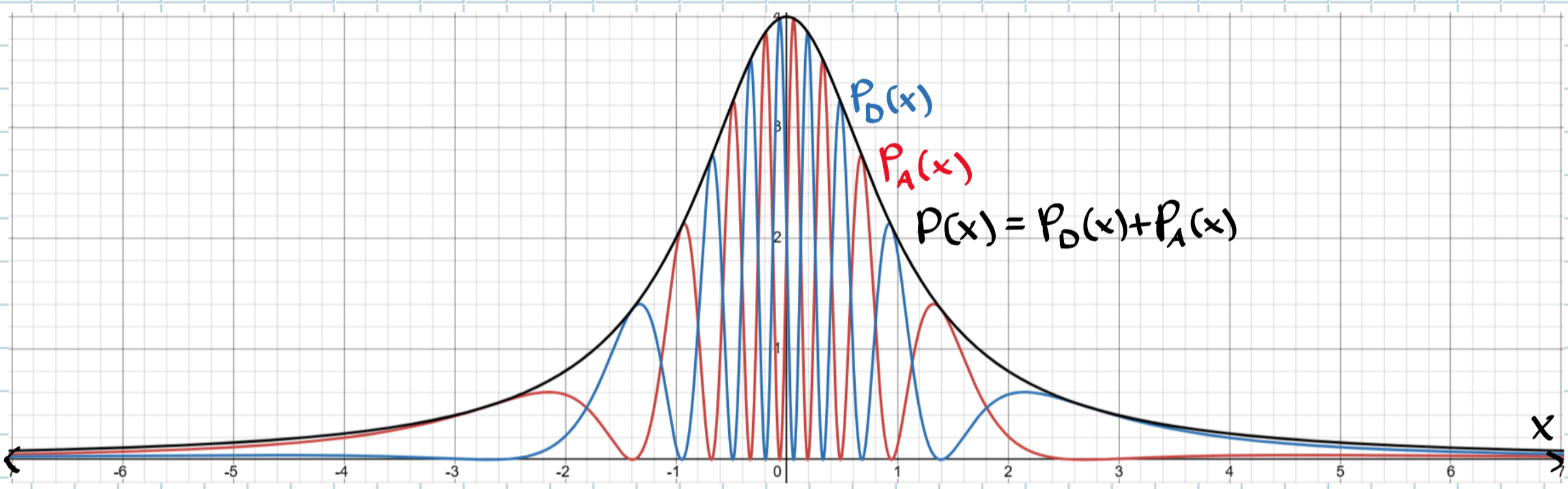
$$P_V(x) = {}_b\langle x|\,{}_a\langle V|\,\rho_f\,|V\rangle_a|x\rangle_b = \frac{D}{4\pi}\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}\right)$$

these are $P(x,H)$ and $P(x,V)$

Again, no interference is observed. What if Bob measured in the DA-basis instead? Then, post-selection allow us to "recover" the interference pattern

$$P_D(x) = \frac{D}{4\pi}\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}-\frac{2}{r_1r_2}\sin[k(r_1-r_2)]\right)$$

$$P_A(x) = \frac{D}{4\pi}\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{2}{r_1r_2}\sin[k(r_1-r_2)]\right)$$



Is "Quantum erasing" simply post-selection?

What would happen if Bob measured in the LR-basis?