Λ
Λ
$_{s}table_{2}003, takahashi_{r}evising_{2}012ormachinelearningestimatorslikethecosmologicalemulator(CosmicEmu)heitmann_{c}$
$bodysimulationsandtypicallypermitanexplorationoftheparameterspacejustaroundafiducialcosmologyorcosmologywit$
$linearregimelikebaryoniceffectssuchasin[\mathbf{?}]ortoparametrizethecosmologicaldependenceofnon-$
$linearclustering, beyondtheZeldovichapproximation(see[\mathbf{?}]).Thesefittingfunctionshave, however, anintrinsicerrortha$
$linear; thereforeonehastobeawareoftherangeinscalesandredshiftstheyweredesignedtoworkon, inordertokeeptheerrors$
$_{r}enormalized_{2}006andallotherresummationmethodsderivedfromit(forexample[\mathbf{?}]), thetimerenormalisationgroup(TRG$
$linearscales, whenthesinglestreamapproximationdoesnotholdanylonger.$
Λ
$_{1}995, Amendola_{2}000, Amendola_{2}004, pettorino_{b}accigalupi_{2}008, involveanextradegreeoffreedom, associatedtoascalarfi$
$analyticalnon-$
$linearanalysis[\mathbf{?},\mathbf{?}]andcosmologicalN-$
$bodysimulationswithincoupledDarkEnergyhavebeenperformedbymanydifferentgroupsbaldi_{e}tal_{2}010, Li_{B}arrow_{2}011, m$
$_{m}ass_{2}006, maccio_{c}oncentration_{2}008, baldi_{c}larifying_{2}011, cui_{h}alo_{2}012, baldi_{e}ffect_{2}011, sutter_{o}bservability_{2}014andthere$
$scalestructurecreminelli_{s}ingle-field_{2}014, CMBmorris_{c}osmicmicrowave_{2}014orlaboratoryexperimentshamilton_{a}tom$
$0.3h/$
$_{c}odecs_{2}012hasshownthatCDEmodelshavecharacteristicandmeasurablefeaturesinthemorphologyandhistoryofnon-$
$linearstructures, suchashalos, subhalosandvoids, blackandthereforeinthenon-linearpowerspectrum.$
?
?
$_{2}000, Amendola_{2}004, pettorino_{b}accigalupi_{2}008, baldi_{e}tal_{2}010.$
r
b
c
ϕ

$$\ddot{\phi}+3H\dot{\phi}+\frac{dV}{d\phi}=\sqrt{\frac{2}{3}}\beta(\phi)\frac{\rho_c}{M_{Pl}}\ , \qquad (1)$$

$$\dot{\rho}_c+3H\rho_c=-\sqrt{\frac{2}{3}}\beta(\phi)\frac{\rho_c\dot{\phi}}{M_{Pl}}\ , \qquad (2)$$

$$\dot{\rho}_b+3H\rho_b=0\ , \qquad (3)$$

$$\dot{\rho}_r+4H\rho_r=0\ , \qquad (4)$$

$$3H^2=\frac{1}{M_{Pl}^2}(\rho_b+\rho_c+\rho_r+\rho_\phi) \qquad (5)$$

ϕ
$M_{pl}\equiv$
$1/\sqrt{8\pi G}$
$V(\phi)$
$V(\phi)=$
$Ae^{-\alpha\phi}$
$_{M}atarrese_{1}984, Wetterich_{1}988.Thecouplingfunction\beta(\phi)$
$\beta(\phi)=$
β
?
?
$\bar{\gamma}$
?
$_{2}005, Bertotti_{I}ess_{T}ortora_{2}003)roughlyto\bar{\gamma}|\leq$
$4\cdot$
10^{-5}
$G_{eff}=$
$G_N(1-$
$\bar{\gamma}/2)$
$G_{eff}=$
$G_N(1+$
$4\beta^2/3)$
$\beta^2=$
$-3\bar{\gamma}/8$
$\beta^2_{baryons}$
10^{-5}
??
$z=$
0
z

$$m_c(z)=m_{c,0}e^{-\beta(\phi(z)-\phi(0))}\ . \qquad (6)$$

Parameter	Explanation	Value	Reference ΛCDM
A		0.0218	
α		0.08	
$\phi(z=0)$		0	
β		{0.05, 0.10, 0.15}	
$w_\phi(z=0)$		{−0.997, −0.995, −0.992}	−1
$\sigma_8(z=0)$		{0.825, 0.875, 0.967}	0.809

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Parameter	Value
H_0	70.3^{-1-1}
Ω_{CDM}	0.226
Ω_{DE}	0.729
A_s	2.42×10^{-9}

k_{Ny}
k_{Ny}
$P(k)$
P_{top}
P_{fold}
P_{top}
P_{fold}
$P_{shot} =$
$1/N$
??
??
$P(k)$
k_{Ny}
σ_8^2
β
$z =$
0
$z =$
0.55
$z =$
1.0
$z =$
1.61
$\beta =$
0.15
$\beta =$
0.10
$\beta =$
0.05
β
σ_8^2
Λ
Λ
??
$R(k;\beta,z) \equiv$
$P_{Exp}(k;\beta,z)/P_{LCDM}(k;\beta,z)$
$z =$
$\{0,\ 0.55,\ 1.0,\ 1.61\}$
$\beta =$
$\{0.05,\ 0.10,\ 0.15\}$
σ_8
$z =$
0
$_{c}odecs_{2}012.Therefore,inordertoseethenete ffectofthecouplingatnon-$
$linearscales,eachpowerspectrumratiohasbeenre-$
$normalizedbydividingeachmodelbyitsrespectiveallth eratiosareunity.Thenete ffectofthefifthforceisa``bump''atnon-$
$linearscales,whoseamplitudeincreaseswithhighercouplingsandwhosemaximumisshiftedintohigherwavenumbersforh$
$linearpowerspectrumiswhatwewanttousetoimprovetheestimationofparametersusingfuturesurveys.Toachievethis,we$
z
k
β
?
k
$R(k;\beta,z) \equiv$
$P_{EXP}(k;\beta,z)/P_{LCDM}(k;\beta,z)$
k
$R(k;\beta,z)$
??
β
z
$a_i =$
$a_0,\ a_1,\ c,\ b,\ k_0$
$i =$
$0,...,5$
k_0
b
a_i
β
z
β
z
2
?
$R^2 =$
$1-$
S_{res}/S_{tot}
S_{res}
S_{tot}
2
??
2
??

$$f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot \arctan((k-k_0)\cdot b)^{2}$$
$$f(k) = 1 + a_0 + a_1 \cdot k + c \cdot k \cdot \frac{b\cdot(k-k_0)}{\sqrt{1+b^2\cdot(k-k_0)^2}}$$

2
a_0
a_1
c
b
k_0
β
z
??
2
β
σ_8
k
k

Λ
$k \approx$
$0.1-$
$1 h/$
β
$_c odecs_2 012 we build different fitting function models, such that when multiplied by the \Lambda$
$_c oyote_2 014) they reproduce the N-$
$body results to an accuracy of 1\%, for scales between 0.1 and 5 and a range in, between 0 and 1.8. To achieve this accuracy in the fitt$
β
β
β^2
0.0025
σ
$\Delta\beta^2 \approx$
$8\cdot$
10^{-5}
?
$\beta^2 \leq$
$2\cdot$
10^{-5}
σ
β
z
$_i(\beta,\, z) =$
$q_{i1}+$
$q_{i2}\beta+$
$q_{i3}\, z+$
$q_{i4}\beta z$
$+$
$q_{i5}\beta^2+$
$q_{i6}\, z^2+$
$q_{i7}\, z^3+$
$q_{i8}\, \beta^3$
q_{ij}
$A_i =$
$q_{ij} B_j$
$A_i =$
$a_i(\beta,\, z)$
$B_j =$
$(1,\, \beta,\, z,\, \beta z,\, \beta^2,\, z^2,\, \beta^3,\, z^3)$
$i =$
$1, ..., 5,\ j =$
$1, ..., 8$
q_{ij}