Universidad Champagnat

Statiscal learning and portfolio optimizacion under skewness probability distribution

Santiago Emiliano Eguren

Rodri

Cristian

July 2022

1 Introduction

The aim of this paper is comprobate if is posible to optimize effectively portfolios under the assuption skewness probability distribution function.

2 Basic Concepts

2.1 Multivariate Normal Variance Mixtures

A random vector $\mathbf{Z} = [Z_1, Z_2, ..., Z_n]$ follows a normal variance mixture, if:

$$\mathbf{Z} = \mu + \sqrt{W} AU \tag{1}$$

or

$$\begin{bmatrix} Z_1 \\ Z_2 \\ . \\ . \\ . \\ Z_n \end{bmatrix} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ . \\ . \\ . \\ \mu_n \end{bmatrix} \sqrt{W} \begin{bmatrix} a_{11} & a_{12} & . & . & . & a_{1n} \\ a_{21} & a_{22} & . & . & . & a_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ a_{n1} & a_{n2} & . & . & . & a_{nn} \end{bmatrix} U \tag{2}$$

Where:

$\mu \in \mathbb{R}^n$, is the location vector.

W is a non-negative random variable independent of U.

$A \in \mathbb{R}^{nxn}$ denotes the scale matrix. Where $\Sigma = AA^T$ is the covarince matrix and $\Sigma \in \mathbb{R}^{nxn}$:

$$\Sigma = \begin{bmatrix} Var[Z_1] & Cov[Z_1, Z_2] & . & . & . & Cov[Z_1, Z_n] \\ Cov[Z_2, Z_1] & Var[Z_2] & . & . & . & Cov[Z_2, Z_n] \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ Cov[Z_n, Z_1] & Cov[Z_n, Z_2] & . & . & . & Var[Z_n] \end{bmatrix} \tag{3}$$

$\mathbf{U} \in \mathbb{R}^n$ denotes a standard normal random vector:

$$U \sim N(\begin{bmatrix} 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & . & . & . & 0 \\ 0 & 1 & . & . & . & 0 \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ 0 & 0 & . & . & . & 1 \end{bmatrix}) \tag{4}$$

2.2 Expected Shortfall

Expected Shortfall is defined as the average loss beyond VaR:

$$ES_\varepsilon[Z_t] = E[-Z_t | -Z_t > VaR_\varepsilon[Z_t]] \tag{5}$$

ES is also know as Expected Tail Loss (ETL) or Conditional Value-at-Risk (CVAR). Usually ε is equal to $0,01$ or $0,05$. ES is a convex function weights and hence is usefull to optimizate portfolios (see Rockafellar and Uryasev [1]).

3 Portfolio Optimizacion

To optimize, we consider daily return data from the S&P 500 index between $2008-01-01$ and $2018-01-01$. We fit marginal ARMA(1, 1)-GARCH(1, 1) and then fit normal variance mixture models to the resulting standardized residuals.

We consider the inverse-gamma distribution for W. The portfolio will contain 3 random stock.

4 Conclusion

References

[1] Rockafellar, R.T. and Uryasev, S., *Optimization of conditional value-at-risk*, Journal of Risk 3, 21-41,2000.

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