Published in Towards Data Science

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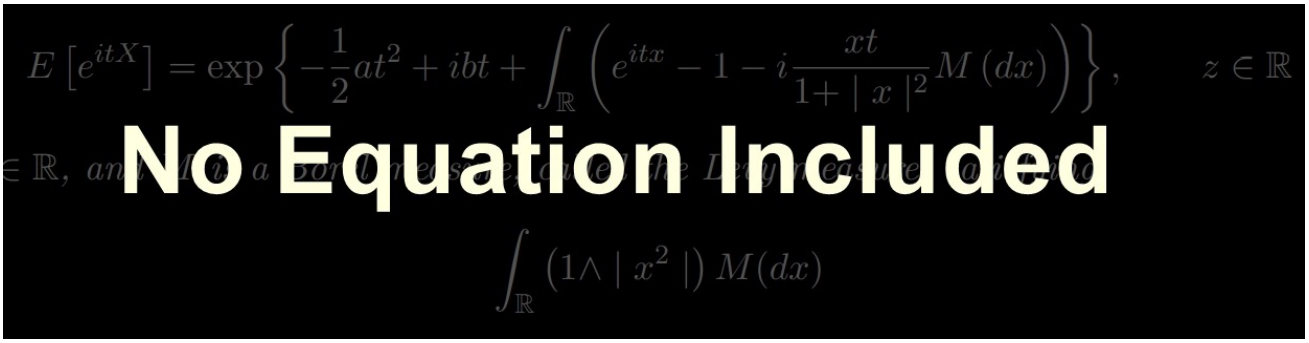
Jun 25, 2020 · 5 min read · Listen



Source: Infinitely Divisible Distribution? [Click]

JOURNEY TO TEMPERED STABLE DISTRIBUTION #0

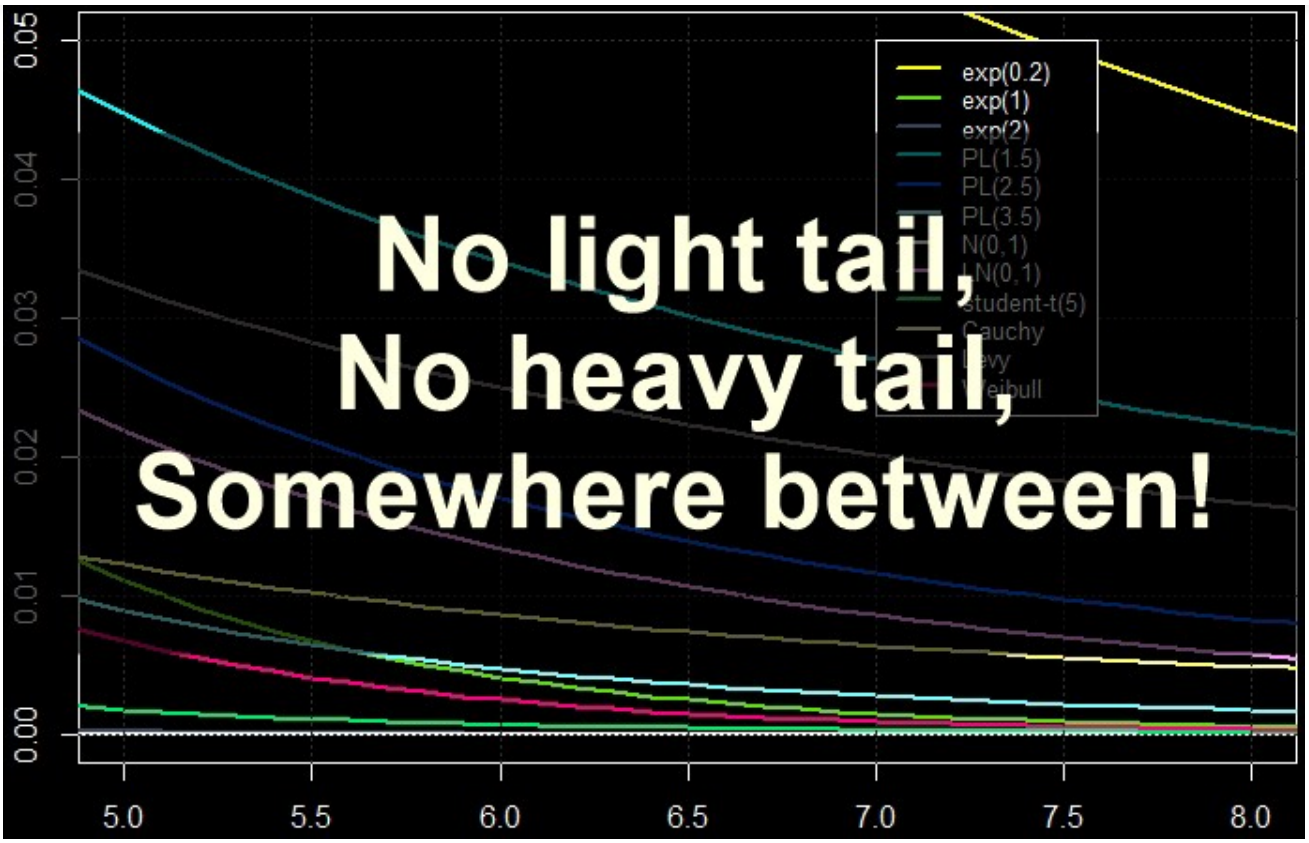
Why Tempered Stable Distribution?

Application on Asset Returns: Why not normal distribution and stable distribution?

Hi, this is Roh (ρ)! Welcome to the series "Journey to Tempered Stable Distribution"

- **Part 0 : Why Tempered Stable (TS) Distribution?**
- Part 1 : What is fat-tailed distribution? [Click]
- Part 2 : Infinitely Divisible Distribution? [Click]

In the middle of writing this series, **people around me asked me to write an article explaining why it is needed without math involved** and I decided to summarize it and put it in Part 0. Therefore, here, we are going to discuss why do we need " Tempered Stable Distribution ". Moreover, I will try to explain without math



Source : What is fat-tailed distribution? [Click]

I would rather start why (not) normal and stable distribution, because tempered stable distribution is developed to overcome the shortcomings from those two distributions specifically in modeling asset returns.

> Tails of the distribution for asset returns are heavier than the normal distribution but thinner than the stable distribution (Grabchak and Samorodnitsky, 2010)

Why (not) Normal distribution?

- Normal distribution is convenient to use. The sum of two normal variables is again normal, which is essential in finance application 1) **sum** of two stocks

- Easy to calculate various risk measures in a closed form (e.g. VaR and ES)
- ***Asset returns are not normally distributed.*** *Not appropriate to model asset returns whose distribution has a non-symmetric, skewed and leptokurtic properties*
- ***Tail is light, i.e., it decays too fast*** *(If you are not familiar with tail, please check the first part of this article)*

Why (not) non-normal Stable distribution?

- Superior to the normal distribution. It allows to control the level of skewed, fat tail, and symmetric of the distribution.
- ***Not consistent to the sampling frequency!*** *The empirically estimated tail index appears to be lower for more frequent returns and higher for less frequent returns (Gencay et al., 2001).*
- This means that you may find that daily (less frequent) returns have a finite variance, while return for micro-secondly sampled series have infinite variance. [Tail Paradox]
- ***For non-normal stable distribution, variance and higher moments are infinite! Then what happened?***

Issues with infinite variance in finance application

What happens if we do not have a finite variance in finance application? The two most heavily used finance application becomes meaningless.

Mean-variance portfolio construction

- If variance (and higher moments) is not finite, a mean-variance portfolio cannot be defined.

Modelling of option prices

- In order to obtain a well-defined model for pricing options, the mean, variance, and higher moments of the return distribution have to exist.


tail part of stable distribution so that we have “stable-like” distribution with lighter tail.

Edited by Roh, Source: Pixabay

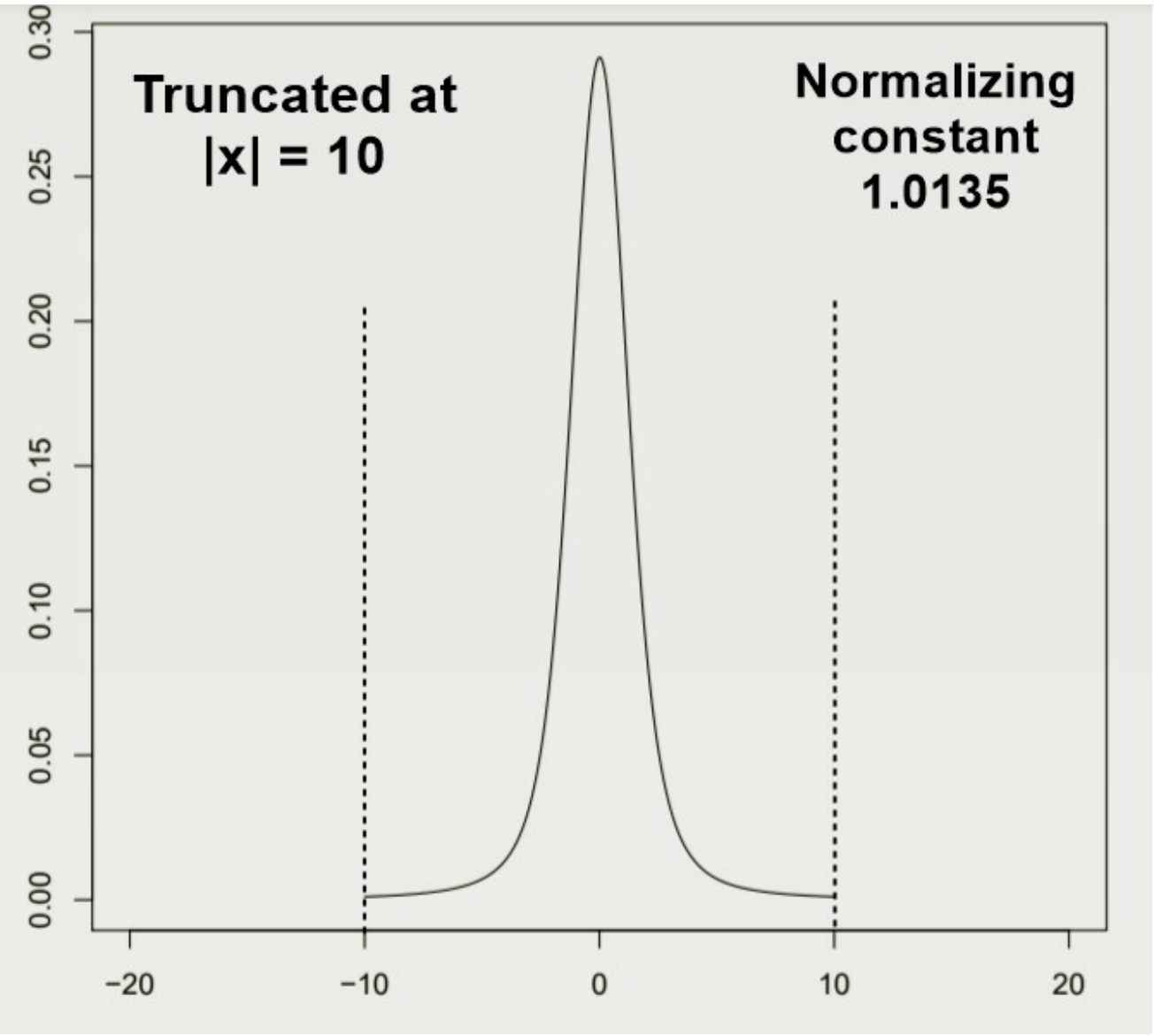
How to do it? There are many ways!

> ***Modifying Tails!! How?***
>
> *1. Truncating the Tails*
>
> *2. Multiply faster decaying functions on tails*
>
> *3. Tilting the Tails*

For more details, see [Click]

Some illustrations

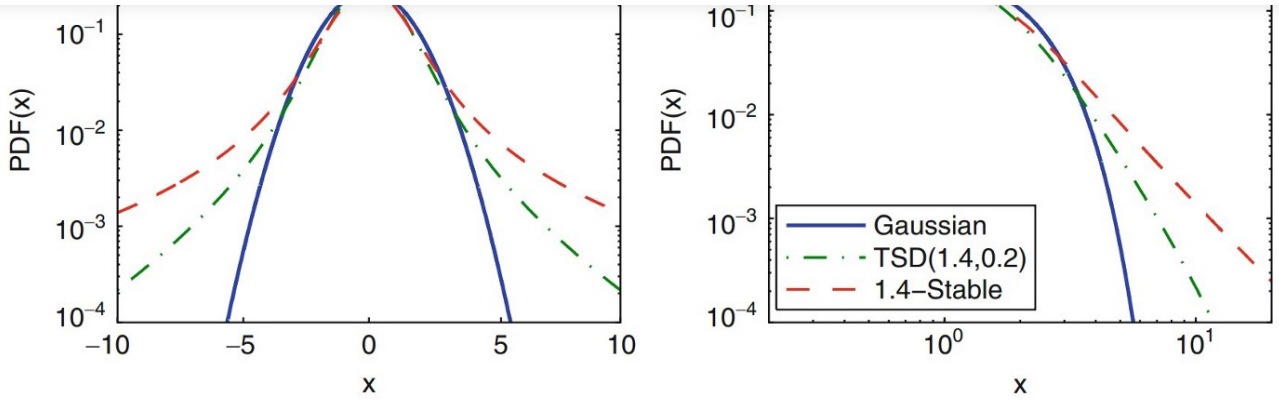
I provide two illustrations. The first one is truncated tails and the second one is that TS distribution has a lighter tail than stable distribution and heavier tail than the normal distribution.



Source: M. Grabchak, Edited by Roh

Normal < TS < Stable

We can see from the below graphs (log scale) that the **Tempered Stable distribution is located in the middle of normal distribution and stable distribution in the tail part.**



Source: Handbooks of Computational Statistics (p. 1031)

Why (not) Tempered Stable Distribution?

- **Able to control the level of a skewed, fat tail, and symmetric of the distribution**
- **It has finite moments!**
- *Like for stable distributions, only the characteristic function is known.*
- *No closed-form formulas exist for the density or the distribution functions.*
- *The difficulty of statistical inference when it is done by maximum likelihood utilizing the fast Fourier (FFT) technique for approximating*

Lastly, here is a list of some types of tempered stable distribution.

1 + 6 Types of TS

- Smoothly Truncated Stable Distribution (STS)
- Classical TS (CTS)
- Generalized Classical TS(GTS)
- Modified TS (MTS)
- Normal TS (NTS)
- Rapidly Decreasing TS (RDTS)

was helpful to you. Please comment below if you have any question. I hope you are curious about what is to come next. Next time, I will be back with " Journey to Tempered Stable Distribution[Part. 1: What is fat-tailed distribution?" [Click]

Reference

1. Heavy-Tailed Distributions in VaR Calculations, 2010, A. Misiorek and R. Weron
2. M. Grabchak and G. Samorodnitsky (2010). "Do Financial Returns Have Finite or Infinite Variance? A Paradox and an Explanation." Quantitative Finance 10(8): 883–893.
3. Handbook Of Heavy-tailed Distributions In Asset Management And Risk Management, 2019, S. Stoyanov, et al.
4. Encyclopedia of Financial Models (Volume 3), pp 241-258,2012, Frank. J. Fabozzi

I also added hyperlinks for all of the references above. Please check the references for the detailed information. I will update the reference later if there is anything that I missed.

Thank you for reading this document. Do not forget to share this document if you find it useful.

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