Homework 3 Sol. (OCDR)

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By:

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To:

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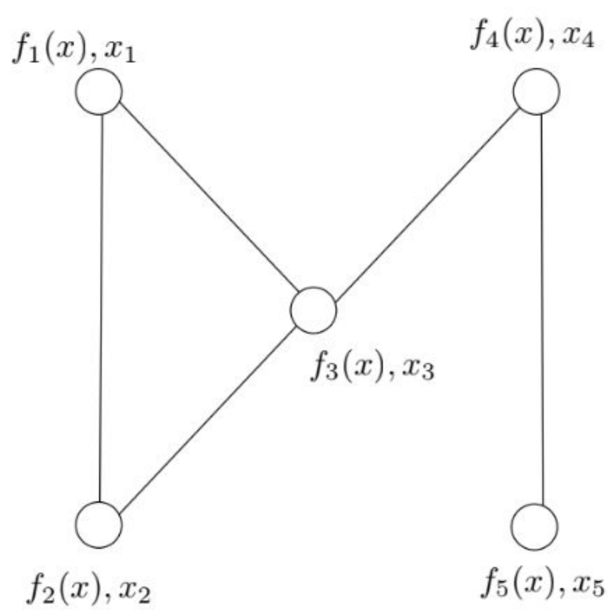
1) Consensus-Based Optimization and Distributed ADMM

1. Consider the graph in Figure 1. Assume that each x_i is a copy of the variable for that node, $x_i \epsilon \Re$ and each node has a local cost function as follows: $f_i(x) = 2\text{ix} + \text{ix}^2$ with $i = 1, 2, \ldots, 5$. (For instance if $i = 1, f_1(x) = 2x + x^2$).

Consider the optimization problem

$$\min F(x) = \sum_{i=1}^{m} f_i(x)$$
$$s.t. \ x \, \epsilon \, X$$



$$x_1(t+1) = \sum_{j \in \mathcal{N}_1} A_{1j} x_j(t) - \alpha(t) \nabla f_1\left(\sum_{j \in \mathcal{N}_1} A_{1j} x_j(t)\right)$$

$$\nabla f_i(x) = 2i + 2\text{ix} = 2i(1+x)$$

(a) Consider that $X = \mathfrak{R}$. Obtain a suitable matrix A to guarantee convergence of the distributed **consensus-based** algorithm and solve the problem using Matlab/Python

Because of $\{G_t\}$ is a sequence of undirected graphs, {A(t)} can be a sequence of m × m matrices such that the following conditions hold:

- Each A(t) is a **doubly stochastic matrix** that is compliant with the graph Gt , i.e., Aij(t) > 0 whenever {i, j} ∈ Et for t ≥ 0.
- **(Aperiodicity)** The diagonal entries of each A(t) are positive, Aii(t) > 0 for all t and i.
- **(Uniform Posivity)** There is a scalar β > 0 such that Aij(t) ≥ β whenever Aij(t) > 0.
- **(Irreducibility)** Each graph Gt is connected.

The Laplacian is:

$$\boxed{L(\mathcal{G}) = \Delta(\mathcal{G}) - Y(\mathcal{G})}$$

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ -1 & -1 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 01 & 0 & -1 & 1 \end{bmatrix}$$

$$A = (I - \alpha L)$$

$$A = \begin{bmatrix} 1-2\alpha & \alpha & \alpha & 0 & 0 \\ \alpha & 1-2\alpha & \alpha & 0 & 0 \\ \alpha & \alpha & 1-3\alpha & \alpha & 0 \\ 0 & 0 & \alpha & 1-2\alpha & \alpha \\ 0 & 0 & 0 & \alpha & 1-\alpha \end{bmatrix}$$

Choosing any α, the matrix remains double stochastic. However to acomplish the requeriment of aperiodicity and uniform posivity, the following has to be true:

$$\alpha > 0$$
$$1 - 2\alpha > 0$$
$$0 < \alpha < \frac{1}{2}$$

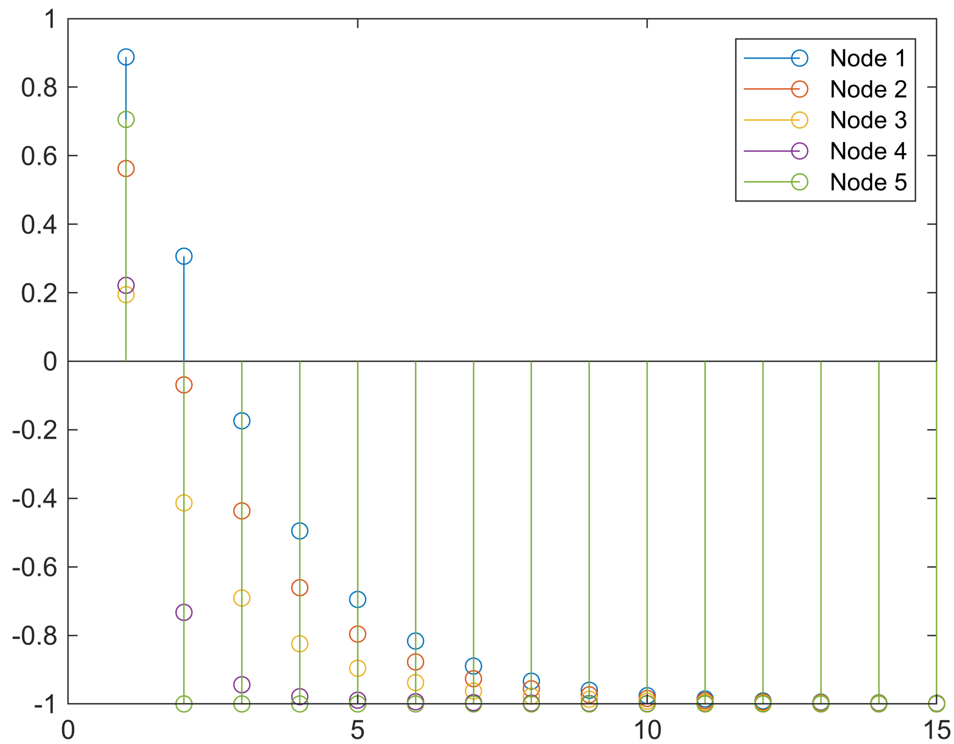
I choose $\alpha = \frac{1}{4}$ arbitrary.

The algorithm for finding x is the following:

$$x_i^{k+1} = A_{\mathrm{ij}} \sum_{j \epsilon \mathrm{N}_i} x_j^k - \alpha \nabla f_i \left(A_{\mathrm{ij}} \sum_{j \epsilon \mathrm{N}_i} x_j^k \right) x_j^k$$

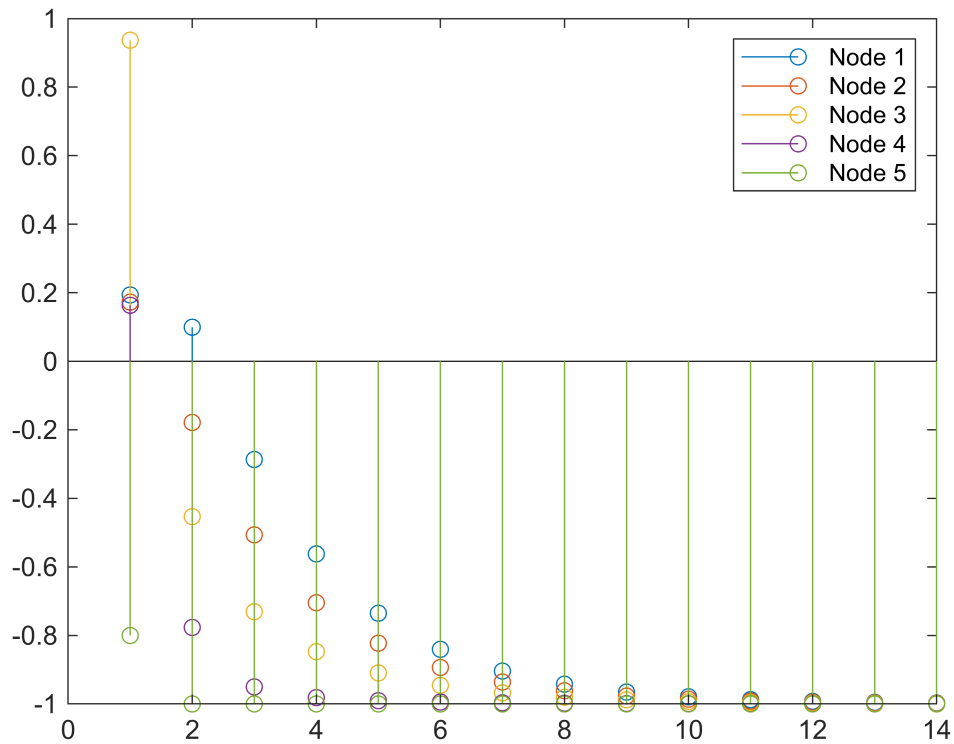
It's seen that it only takes account the vecinity nodes.

```
clc
clear

alpha=0.25;
A = [(1-2*alpha), alpha, alpha, 0, 0;
     alpha, 1-2*alpha, alpha, 0, 0;
     alpha, alpha, 1-3*alpha, alpha, 0;
     0, 0, alpha, 1-2*alpha    , alpha;
     0, 0, 0, alpha, 1-alpha];
A_ij=A.*[0,1,1,1,1;1,0,1,1,1;1,1,0,1,1;1,1,1,0,1;1,1,1,1,0];

% Number of nodes (5)
m = size(A, 1);
% Generate distributed initial conditions randomly in the interval [-5, 5]
x0 = rand(m, 1);
% The solution to the concensus equation in discrete time is iterative
max_it=500;
x=zeros(m,max_it);
x(:,1)=x0;
epsilon=1E-3;
k=1;
a=0.1;
%define the gradient for each node
gradf=@(i,arg)2*i*(1+arg);
%Till reaching an error < epsilon any(abs(diff(x(:,k)))>epsilon)
while(any(abs(diff(x(:,k)))>epsilon))

        w(:,k)=A*x(:,k);
        %gradient evaluated in w(i)
        grad=[gradf(1,w(1,k)) gradf(2,w(2,k)) gradf(3,w(3,k)) gradf(4,w(4,k)) gradf(5,w(5,k)) ]
        %grad=[df1 df df3 df4 df5]';
        x(:,k+1)=A*x(:,k)-a*grad;
        k=k+1;
end


% Plot the discrete time response for each node
figure
for i = 1:m
    stem(1:k,x(i,1:k))
    hold on;
end

legend('Node 1','Node 2','Node 3','Node 4','Node 5');
```



(b) Consider now that $X = [-1, 2]$ and repeat

```
% Generate distributed initial conditions randomly in the interval [-1, 2]
x0 = -1+2*rand(m, 1);
% The solution to the concensus equation in discrete time is iterative
x=zeros(m,max_it);
x(:,1)=x0;
k=1;
%Till reaching an error < epsilon
while(any(abs(diff(x(:,k)))>epsilon))
        w(:,k)=A*x(:,k);
        %gradient evaluated in w(i)
        grad=[gradf(1,w(1,k)) gradf(2,w(2,k)) gradf(3,w(3,k)) gradf(4,w(4,k)) gradf(5,w(5,k)) ]
        x(:,k+1)=A*x(:,k)-a*grad;
        k=k+1;
end

% Plot the discrete time response for each node

figure
for i = 1:m
    stem(1:k,x(i,1:k))
    hold on;
end
%}
legend('Node 1','Node 2','Node 3','Node 4','Node 5');
```



(c) Consider again that $X = \Re$, but $A(t)$ is time-varying. Solve the problem using Matlab/Python. Show the evolution of the iteration $x_i(k)$ vs k

Consider the case of time-varying undirected graphs {Gt}. The algorithm of the updates is

$$\mathbf{w}_i(t+1) = \sum_{j=1}^{m} A_{ij}\mathbf{x}_j(t) \qquad (a_{ij} = 0 \text{ when } j \notin N_i).$$

$$\mathbf{x}_i(t+1) = \mathcal{P}_X[\mathbf{w}_i(t+1) - \alpha(t)\tilde{\nabla} f_i(\mathbf{w}_i(t+1))]$$

where $\mathcal{P}_X[y]$ is the Euclidean projection of y on X and $\alpha(t) > 0$ is a stepsize.

This time, α and $A$are changing with time. So in each iteration for x_i i produce a new double stochastic matrix A.

```
% Generate distributed initial conditions randomly
x0 = rand(m, 1);
% The solution to the concensus equation in discrete time is iterative
x=zeros(m,max_it);
x(:,1)=x0;
k=1;
%Till reaching an error < epsilon
while(any(abs(diff(x(:,k)))>epsilon))

    alpha=0.5*rand(1);
```

```
    A = [(1-2*alpha), alpha, alpha, 0, 0;
     alpha, 1-2*alpha, alpha, 0, 0;
     alpha, alpha, 1-3*alpha, alpha, 0;
     0, 0, alpha, 1-2*alpha    , alpha;
     0, 0, 0, alpha, 1-alpha];
    A_ij=A.*[0,1,1,1,1;1,0,1,1,1;1,1,0,1,1;1,1,1,0,1;1,1,1,1,0];
    %find x(k+1) for all the nodes
    w(:,k)=A*x(:,k);
    %gradient evaluated in w(i)
    grad=[gradf(1,w(1,k)) gradf(2,w(2,k)) gradf(3,w(3,k)) gradf(4,w(4,k)) gradf(5,w(5,k)) ]';
    x(:,k+1)=A*x(:,k)-a*grad;
    k=k+1;

end

% Plot the discrete time response for each node

figure
for i = 1:m
    stem(1:k,x(i,1:k))
    hold on;
end
%}
legend('Node 1','Node 2','Node 3','Node 4','Node 5');
```

With the convergence assured, the matrix A is finally:

```
A
```

```
A = 5×5
    0.3730    0.3135    0.3135         0         0
    0.3135    0.3730    0.3135         0         0
    0.3135    0.3135    0.0596    0.3135         0
         0         0    0.3135    0.3730    0.3135
         0         0         0    0.3135    0.6865
```

2. Randomly generate an undirected graph of 25 nodes. Set the dimension of the state $x \in \Re^3$. For each agent create a 3 × 3 matrix M_i and vector b_i with entries from an interval [0.1, 1]. Solve the optimization problem:

$$\min \sum_{i=1}^{m} f_i(x)$$

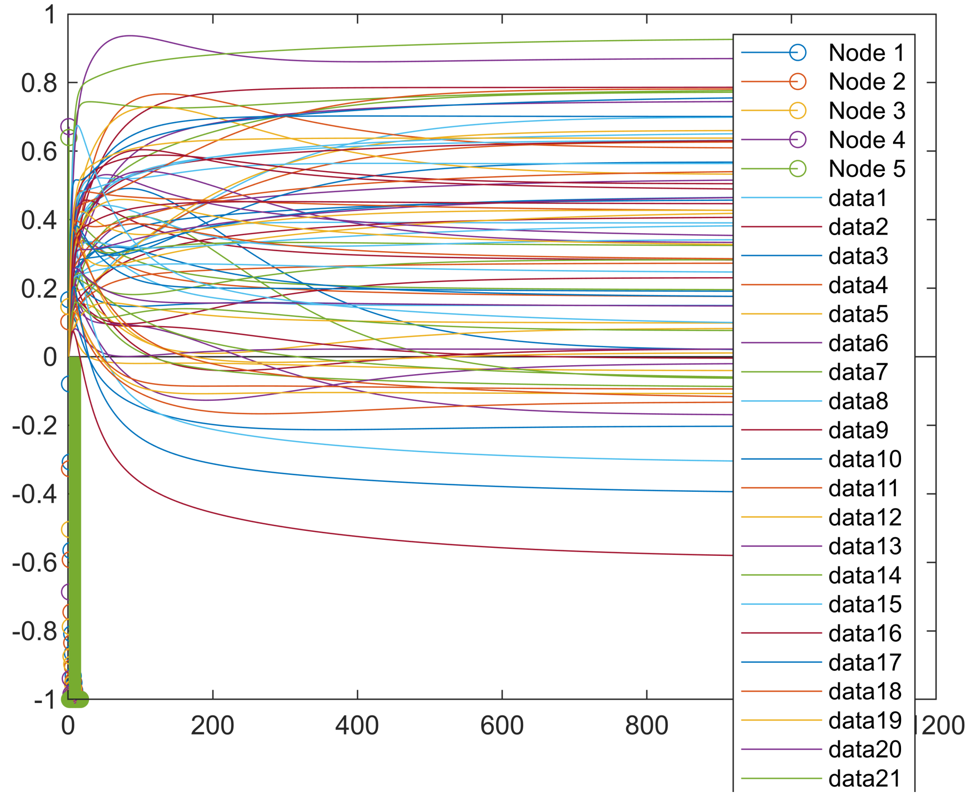
$$f_i(x) = ||xM_i - b_i||^2$$

which is a distributed optimization problem. Solve using Matlab/Python and analyze the convergence from a **computational viewpoint** (no theoretical analysis). Show a step by step figure of each x_i (only two coordinates are needed).

```
clc
clear

a=0.05;

M1=[.5 .2 .4;.7 .9 .5;1 .3 .8];
M2=[.5 .4 .4;.1 .7 .4;.7 .4 1];
M3=[.7 .4 .6;.7 .9 .4;.4 .5 1];
M4=[1 1 .8;1 .1 .9;.7 1 .1];
M5=[.6 .8 .2;.5 1 .5;.2 .4 .3];
M6=[1 1 .4;1 .2 .4;.6 .5 .8];
M7=[.8 .9 .2;.1 .9 .7;.9 .2 .3];
M8=[.9 .2 .1;.7 .5 .3;.2 .7 1];
M9=[1 .1 .7;.1 .5 1;.2 1 .5];
M10=[.9 .3 .8;.6 .6 .2;.1 .2 .3];
M11=[.6 .5 .9;.7 1 .5;.1 .2 .6];
M12=[.2 .3 .1;.2 .9 .4;.5 .7 .1];
M13=[.7 .3 .6;.9 .7 .4;.5 .4 .6];
M14=[.8 .6 .9;.7 .9 .1;.8 .8 .8];
M15=[.5 .4 .7;.1 .1 .9;1 .3 .5];
M16=[.7 .6 1;.9 .5 .9;.4 .8 .2];
M17=[.4 .1 1;.7 .1 .7;1 .2 .2];
M18=[.7 .8 .7;.6 .9 .6;.6 .9 .1];
M19=[.7 .3 .9;1 .1 .7;.9 .8 .5];
M20=[.7 .3 .1;1 .1 .7;.9 .8 .5];
M21=[.8 .3 .2;.7 .4 .7;.2 .5 .2];
M22=[.4 .6 .7;.5 .3 .4;.7 .1 .7];
M23=[.9 .6 1;.4 .6 .6;.6 .6 .5];
M24=[.8 .1 1;.6 .5 .8;.5 .2 .3];
M25=[.9 .6 1;.9 .9 .2;.1 .4 1];

b1=[.5 .5 .6]';
b2=[.1 .1 .3]';
b3=[.9 .4 .4]';
b4=[.4 .3 .8]';
b5=[.2 .3 .4]';
b6=[.2 .3 .7]';
b7=[.3 .3 .2]';
b8=[.8 1 .3]';
b9=[.2 .1 .1]';
b10=[.9 .6 .9]';
b11=[.1 .4 1]';
b12=[.6 .6 .2]';
b13=[.2 .5 .1]';
```

```
b14=[.9 .8 .7]';
b15=[.9 .2 .2]';
b16=[.8 .9 .6]';
b17=[.9 .4 .2]';
b18=[.2 .5 .7]';
b19=[.4 .3 .6]';
b20=[.3 .9 .4]';
b21=[.8 .1 1]';
b22=[1 .7 .5]';
b23=[.3 .6 1]';
b24=[.8 1 1]';
b25=[.9 1 .9]';

ceros=zeros(75);

x=ceros(:,1);
df=ceros(:,1);

L=[1 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
   0 2 -1 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
   0 -1 2 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
   0 0 0 2 -1 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
   0 0 0 -1 2 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
   0 0 0 0 0 1 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
   -1 -1 0 0 0 0 3 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
   0 0 -1 -1 0 0 0 5 -1 0 0 -1 -1 0 0 0 0 0 0 0 0 0 0 0 0
   0 0 0 0 0 0 0 -1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
   0 0 0 0 -1 0 0 0 0 2 0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0
   0 0 0 0 0 -1 -1 0 0 0 4 0 0 0 0 -1 -1 0 0 0 0 0 0 0 0
   0 0 0 0 0 0 0 -1 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
   0 0 0 0 0 0 0 -1 0 0 0 0 2 0 0 0 0 -1 0 0 0 0 0 0 0
   0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 -1 0 0 0 0 0 0
   0 0 0 0 0 0 0 0 0 -1 0 0 0 0 2 0 0 0 -1 0 0 0 0 0 0
   0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 1 0 0 0 0 0 0 0 0 0
   0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 0 2 0 0 0 -1 0 0 0 0
   0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0 2 0 0 0 -1 0 0 0
   0 0 0 0 0 0 0 0 0 0 0 0 0 -1 -1 0 0 0 4 0 0 0 -1 0 -1
   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 -1
   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 1 0 0 0 0
   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 2 -1 0 0
   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 -1 2 0 0
   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 -1
   0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 -1 0 0 0 -1 3];

A=eye(25)-1/25*L;

kr=kron(A,eye(3));

for i=1:1000
```

```
    df1=2*M1*(M1*x(1:3,i)-b1);
    df2=2*M2*(M2*x(4:6,i)-b2);
    df3=2*M3*(M3*x(7:9,i)-b3);
    df4=2*M4*(M4*x(10:12,i)-b4);
    df5=2*M5*(M5*x(13:15,i)-b5);
    df6=2*M6*(M6*x(16:18,i)-b6);
    df7=2*M7*(M7*x(19:21,i)-b7);
    df8=2*M8*(M8*x(22:24,i)-b8);
    df9=2*M9*(M9*x(25:27,i)-b9);
    df10=2*M10*(M10*x(28:30,i)-b10);
    df11=2*M11*(M11*x(31:33,i)-b11);
    df12=2*M12*(M12*x(34:36,i)-b12);
    df13=2*M13*(M13*x(37:39,i)-b13);
    df14=2*M14*(M14*x(40:42,i)-b14);
    df15=2*M15*(M15*x(43:45,i)-b15);
    df16=2*M16*(M16*x(46:48,i)-b16);
    df17=2*M17*(M17*x(49:51,i)-b17);
    df18=2*M18*(M18*x(52:54,i)-b18);
    df19=2*M19*(M19*x(55:57,i)-b19);
    df20=2*M20*(M20*x(58:60,i)-b20);
    df21=2*M21*(M21*x(61:63,i)-b21);
    df22=2*M22*(M22*x(64:66,i)-b22);
    df23=2*M23*(M23*x(67:69,i)-b23);
    df24=2*M24*(M24*x(70:72,i)-b24);
    df25=2*M25*(M25*x(73:75,i)-b25);

    df=[df1;df2;df3;df4;df5;df6;df7;df8;df9;df10;df11;df12;df13;df14;df15;df16;df17;df18;df19;d

    x(:,i+1)=kr*x(:,i)-a*df;

end

k=1:1001;

plot(k,x)
```



3. Consider again Problem in 1. (Figure 1) with $X = \Re$. Assume all you need to solve the problem using a **Distributed ADMM**. Solve the problem using Matlab/Python. Show the evolution of the iteration $x_i(k)$ vs k. Make a comparative analysis of performance (in words) between consensus-based and DADMM.

(a) Consider that $X = \Re$. Obtain a suitable matrix A to guarantee convergence of the distributed **consensus-based** algorithm and solve the problem using Matlab/Python

It's possible to reformule the problem of optimization using the ADMM form :

$$\min \sum_{i=1}^{m} f_i(x_i)$$
$$s.t. \;\; x_i - z = 0$$

Where x is the local vairable, and z is the global variable. $x - z = 0$ is the concensus constraint.

$$L_p = \sum_{i=1}^{5} (f_i(x_i) + \mu_i(x_i - z)) + \rho||x_i - z||_2^2 + g(z)$$

Where g(z) is a term that uses regulatization for improving the optimization, or overfitting. In this case I choose $g(z) = 0$.

$$x_i^{k+1} = \arg\min_x L_p\left(x_i, z^k, \mu^k\right) = \arg_x \left(\nabla L_{p_x} \right) = 0$$

$$0 = 2i + 2i\mathrm{x}_i + \mu_i + 2\rho(\mathrm{x}_i - z)$$

$$x_i^{k+1} = \frac{2\rho z^k - 2i - \mu_i^k}{2i + 2\rho}$$

$$z^{k+1} = \arg\min L_p\left(x_i^{k+1}, z, \mu_i^k\right) = \frac{1}{5}\sum_{i=1}^{5}\left(x_i^{k+1} + \left(\frac{1}{\rho}\right)\mu_i^k\right)$$

$$\mu_i^{k+1} = \mu_i^k + \rho\left(x_i^{k+1} - z^{k+1}\right)$$

The matrix A can be obtained this way:

$$x_i^{k+1} = A_{\mathrm{ij}}\left(\sum_{j\,\epsilon\,\mathrm{N}_i} x_j^k - \alpha \nabla f_i x_j^k\right)$$

The matrix is caracterized by the null diagonal values beacuse only take account the neighborhood N_i, and the conections among the nodes, hence if there's no connection the element in the matrix is null. In this case:

$$x_i^{k+1} = A_{\mathrm{ij}} \sum_{j\,\epsilon\,\mathrm{N}_i} x_j^k\left(1 - \alpha\nabla f_i\right) = \begin{bmatrix} 0 & x_2^k(1-\alpha\nabla f_1) & x_3^k(1-\alpha\nabla f_1) & 0 & 0 \\ x_1^k(1-\alpha\nabla f_2) & 0 & x_3^k(1-\alpha\nabla f_2) & 0 & 0 \\ x_1^k(1-\alpha\nabla f_3) & x_2^k(1-\alpha\nabla f_3) & 0 & x_4^k(1-\alpha\nabla f_3) & 0 \\ 0 & 0 & x_3^k(1-\alpha\nabla f_4) & 0 & x_5^k(1-\alpha\nabla f_4) \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix}$$

$$A_i = \left(\sum_{j\,\epsilon\,\mathrm{N}_i} x_j^k - \alpha\nabla f_i x_j^k\right)^{-1} x_i^{k+1}$$

After making the inverse and make the multiplication, due to the equivalence we can obtain the matrix A definitely:

$$A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{bmatrix} \begin{bmatrix} 0 & (1-\alpha\nabla f_1) & (1-\alpha\nabla f_1) & 0 & 0 \\ (1-\alpha\nabla f_2) & 0 & (1-\alpha\nabla f_2) & 0 & 0 \\ (1-\alpha\nabla f_3) & (1-\alpha\nabla f_3) & 0 & (1-\alpha\nabla f_3) & 0 \\ 0 & 0 & (1-\alpha\nabla f_4) & 0 & (1-\alpha\nabla f_4) \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

```
clc
clear
%parameters
rho=0.6; %arbitrary choice
N=5; %number of nodes
alpha=0.5; %stepsize of gradient ascend
```

```
%Initial conditions
z=rand(1,100);
mu=zeros(5,100);
mu(:,1)=max(0,rand(5,1)); %Initial values for each mu(<0). There's a mu for each restriction x:
x=zeros(5,100);
x(:,1)=rand(5,1);
%Error
epsilon=1E-3; %Minimum difference between x and mu
k=1;

while(any(abs(x(:,k)-z)>epsilon))
    %First, find x(k+1) for all the nodes
    for i=1:N
        x(i,k+1)=((2*rho)*z(k)-2*i-mu(i,k))/(2*i+rho*2);
    end
    Matrix=zeros(5);
    gradf=@(i)2*i*(1-alpha*x(i,k));
    M=2*[0 gradf(1) gradf(1) 0 0;
     gradf(2) 0 gradf(2) 0 0;
     gradf(3) gradf(3) 0 gradf(3) 0;
     0 0 gradf(4) 0 gradf(4);
     0 0 0 gradf(5) 0];

    z(k+1)=(1/N)*(sum(x(:,k+1))+(1/rho)*mu(i,k));
    for i=1:N
        mu(i,k+1)=mu(i,k)+alpha*(x(i,k+1)-z(k+1));
        mu(i,k+1)=max(0,mu(i,k+1));
    end
    k=k+1;

end
```

With the convergence assured, the A is finally:

```
Ai=inv(M.*x(:,k-1))*x(:,k);
A=M.*Ai
```

```
A = 5×5
         0   -0.1835   -0.1835         0         0
    0.6331         0    0.6331         0         0
    2.0518    2.0518         0    2.0518         0
         0         0    0.8002         0    0.8002
         0         0         0   -2.1680         0
```

(b) Consider now that $X = [-1, 2]$ and repeat

```
%parameters
rho=0.6; %arbitrary choice
N=5; %number of nodes
alpha=0.5; %stepsize of gradient ascend
%Initial conditions
z=rand(1);
```

```
mu=max(0,zeros(5,100));
mu(:,1)=rand(5,1); %Initial values for each mu. There's a mu for each restriction xi-z=0
x=zeros(5,100);
lowerl=-1;
upperl=2;
x(:,1)=lowerl+(upperl-lowerl)*rand(1,1);
%Error
epsilon=1E-3; %Minimum difference between x and mu
k=1;

while(any(abs(x(:,k)-z)>epsilon))
    %First, find x(k+1) for all the nodes
    for i=1:N
        x(i,k+1)=((rho/2)*z(k)-2*i-mu(i,k))/(2*i+rho/2);
    end
    Matrix=zeros(5);
    gradf=@(i)2*i*(1-alpha*x(i,k));
    M=2*[0 gradf(1) gradf(1) 0 0;
     gradf(2) 0 gradf(2) 0 0;
     gradf(3) gradf(3) 0 gradf(3) 0;
     0 0 gradf(4) 0 gradf(4);
     0 0 0 gradf(5) 0];

    z(k+1)=(1/N)*(sum(x(:,k+1))+(1/rho)*mu(i,k));
    for i=1:N
        mu(i,k+1)=mu(i,k)+alpha*(x(i,k+1)-z(k+1));
        mu(i,k+1)=max(0,mu(i,k+1));
    end
    k=k+1;
end
```

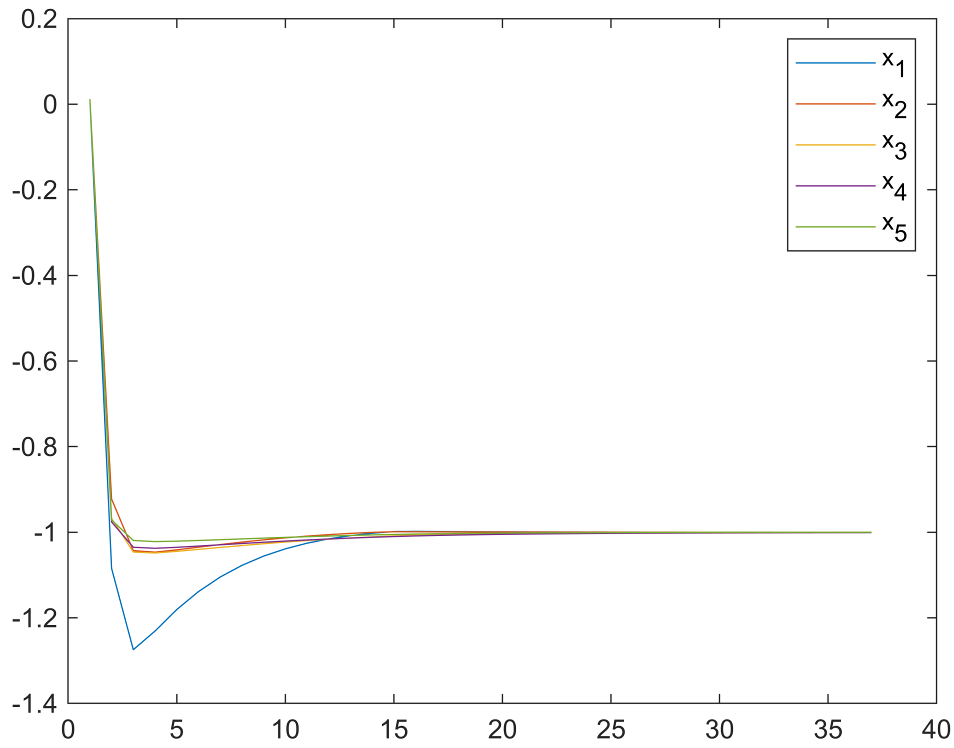
With the convergence assured, the A is finally:

```
Ai=inv(M.*x(:,k-1))*x(:,k);
A=M.*Ai
```

```
A = 5×5
         0   -0.1833   -0.1833         0         0
    0.6334         0    0.6334         0         0
    2.0500    2.0500         0    2.0500         0
         0         0    0.8002         0    0.8002
         0         0         0   -2.1672         0
```

The states in the nodes throw time are:

```
x=x(:,1:k-1);
t=1:k-1;
figure()
plot(t,x)
legend('x_1','x_2','x_3','x_4','x_5');
```



(c) Consider again that $X = \Re$, but $A(t)$ is time-varying. Solve the problem using Matlab/Python. Show the evolution of the iteration $x_i(k)$ vs k

```
%parameters
rho=0.6; %arbitrary choice
N=5; %number of nodes
alpha=0.5; %stepsize of gradient ascend
%Initial conditions
z=rand(1,100);
mu=zeros(5,100);
mu(:,1)=max(0,rand(5,1)); %Initial values for each mu. There's a mu for each restriction xi-z=0
x=zeros(5,100);
x(:,1)=rand(5,1);
%Error
epsilon=1E-3; %Minimum difference between x and mu
k=1;
A=ones(5);
while(any(abs(x(:,k)-z)>epsilon))
    %First, find x(k+1) for all the nodes
    for i=1:N
        x(i,k+1)=((rho/2)*z(k)-2*i-mu(i,k))/(2*i+rho/2);
    end
    Matrix=zeros(5);
    alpha=rand(1);
```

```
    gradf=@(i)2*i*(1-alpha*x(i,k));
    M=2*[0 gradf(1) gradf(1) 0 0;
     gradf(2) 0 gradf(2) 0 0;
     gradf(3) gradf(3) 0 gradf(3) 0;
     0 0 gradf(4) 0 gradf(4);
     0 0 0 gradf(5) 0];
    Ai=inv(M.*x(:,k))*x(:,k+1);
    A=(M.*Ai).*A;

    z(k+1)=(1/N)*(sum(x(:,k+1))+(1/rho)*mu(i,k));
    for i=1:N
        mu(i,k+1)=mu(i,k)+alpha*(x(i,k+1)-z(k+1));
        mu(i,k+1)=max(0,mu(i,k+1));
    end
    k=k+1;

end
```

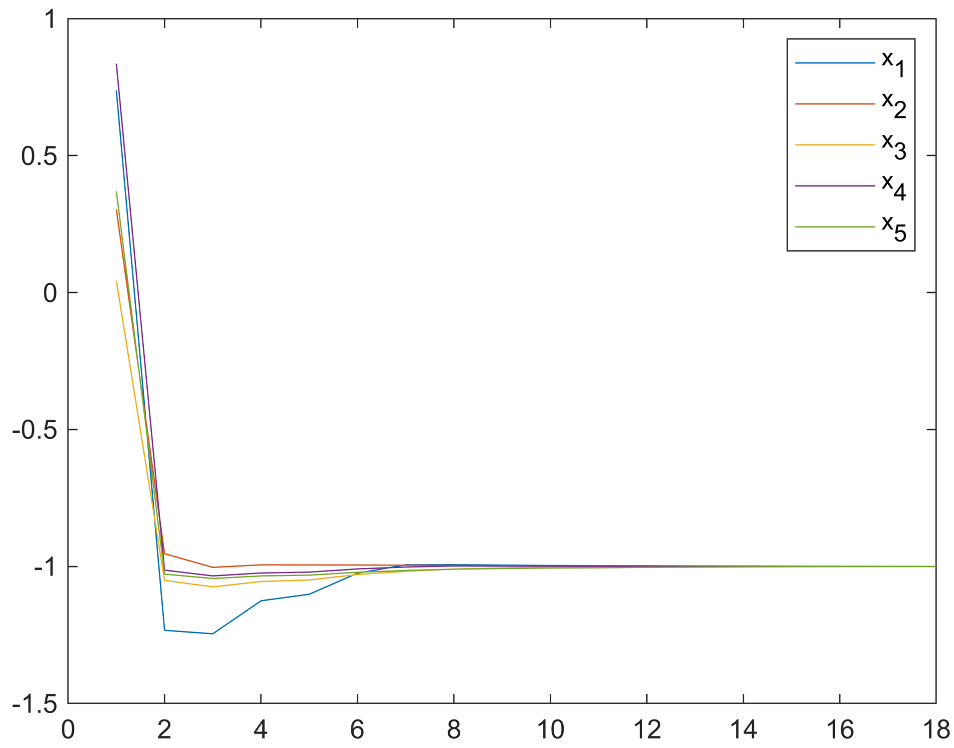
With the convergence assured, the A is finally:

```
A
```

```
A = 5×5
10^6 ×
        0    0.0000    0.0000         0         0
  -0.0000         0   -0.0000         0         0
   0.5964    0.5964         0    0.5964         0
        0         0   -0.0000         0   -0.0000
        0         0         0    2.6020         0
```

The states in the nodes throw time are:

```
x=x(:,1:k-1);
t=1:k-1;
figure()
plot(t,x)
legend('x_1','x_2','x_3','x_4','x_5');
```

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```

```
                            evalin('debug', cmd);

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            viewPeerNode = this.getViewPeerNode(viewId);

Error in internal.matlab.variableeditor.peer.MF0ViewModelVEProvider/setPropertyOnClient (line 195)
                this.setViewProperty(propertyName, propertyValue, viewKey);

Error in internal.matlab.variableeditor.peer.RemoteArrayViewModel/setProperty (line 437)
            this.Provider.setPropertyOnClient(propertyName, propertyValue, this, this.viewID);

Error in internal.matlab.variableeditor.peer.RemoteNumericArrayViewModel/handleDataChangedOnDataModel (line
183)
            this.setProperty('Slice', this.DataModel.Slice);

Error in internal.matlab.variableeditor.ViewModel>@(e,d)this.handleDataChangedOnDataModel(e,d) (line 42)
```

```
                this.DataChangeListeners = event.listener(dataModel,'DataChange',@(e,d)
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