Between-group competition also enhances co-operation in public goods appropriation games
Supplementary Material

Juan Camilo Cárdenas, Santiago Gómez, and César Mantilla

August 1, 2017

1 Additional table

Table S.1: OLS regression to test treatment differences in cooperation decay within each block. Statistically insignificant coefficients for the interaction term reflect lack of differences.

Dependent variable: Appropriation level	Block 1	Block 2 Rounds 1-5	Block 2 Rounds 6-10	Block 2 Rounds 1-10	Block 3
	(1)	(2)	(3)	(4)	(5)
GPM	0.439	-0.799**	-1.140***	-0.898***	-0.370**
	(0.361)	(0.232)	(0.293)	(0.126)	(0.131)
Round	0.328***	0.127**	0.102***	0.0762***	0.213***
	(0.0770)	(0.0495)	(0.0255)	(0.0143)	(0.0280)
GPM x Round	-0.108	-0.0304	0.0335	0.00310	0.0103
	(0.109)	(0.0700)	(0.0361)	(0.0202)	(0.0396)
Constant	2.332***	3.360***	3.247***	3.481***	3.340***
	(0.255)	(0.164)	(0.207)	(0.0888)	(0.0930)
Observations	10	10	10	20	10
R-squared	0.818	0.938	0.982	0.948	0.963

Standard errors in parentheses. *** p<0.01, ** p<0.05

2 A theoretical perspective on the appropriation dilemma and between-group competition

2.1 The game model

We have $g \in \{1, ..., G\}$ groups of $n \in \{1, ..., N\}$ players. In the appropriation dilemma each group member chooses the desired resource to take for himself $x_i \in A = \{\underline{a}, \overline{a}\}$ according to the profit function

$$\pi_i(x_i, x_{-i}) = ax_i + \beta \left(e - \left(x_i + \sum_{j \neq i} x_j \right) \right). \tag{1}$$

The private benefits from appropriation, and the collective benefits from the non-appropriated units, correspond to the first and second terms, respectively. Since $a > \beta$ the individual returns from appropriating a resource unit are larger than the returns from not appropriating this unit. Nevertheless, from the society's perspective, as long as $a < n\beta$ this is inefficient since the social returns from the non-appropriated unit exceed the private returns from an appropriated one. Due to the linearity in the profit function each player has incentives to appropriate $x_i = \overline{a}$. By contrast, the socially desired per capita appropriation level will be $x_i = \underline{a}$.

We introduce between-group competition throughout a ranking function $U : R^G \to R^G$. The input of the ranking function is the vector $\boldsymbol{E}$ which defines the aggregate appropriation of every group $E_g = \sum_{i \in g} x_i$. The greater the appropriation the lower the position in the ranking. The reason is that social benefits decrease with each unit appropriated within the group. The output is a vector $\boldsymbol{\mu}$ defining, for each group, a multiplier for every individual profit, such that $(\sum_g \mu_g)/G = 1$. In this case the final profit for each individual i in group g will be $\overline{\pi}_{ig} = \mu_g \cdot \pi_i$. The tie-breaking rule is to average the group's multipliers.

2.2 Equilibrium with group competition

As is shown above, the equilibrium conditions with a unique group (i.e., no competition) are straightforward due to the linearity in the dilemma. By contrast, with multiple groups the condition for having a Nash Equilibrium is:

$$\begin{aligned} &\mu_g(E^e) \left\{ ax_i^e + \beta \left[ne - \left(x_i^e + \textstyle\sum_{j \neq i} x_j^e \right) \right] \right\} \geq \\ &\qquad \mu_g \left(x_i + \textstyle\sum_{j \neq i, j \in g} x_j, E_{-i}^e \right) \left\{ ax_i + \beta \left[ne - \left(x_i + \textstyle\sum_{j \neq i} x_j^e \right) \right] \right\} \\ &\qquad\qquad \forall i,\ \forall g,\ \forall x_i \in A \end{aligned} \tag{2}$$

The left side is the final profit of the agent, which is composed of the winnings of the within-group appropriation game (the expression between braces), and the multiplier of the respective group (μ_g). To have an equilibrium it is needed that for each agent there are no incentives to choose another appropriation level, producing a deviation of the potential equilibrium. The right side describe the possible winnings for choosing a different appropriation level, maintaining constant the appropriation level of all other agents. A deviation will change the total appropriation of the group and may or may not change the multiplier (μ_g). At the same time a deviation will change the winnings of the within-group appropriation game (the expression between braces). This is what eq. 2 imposes. If the group multiplier is affected, a deviation will produce opposite effects on the group multiplier and the withing-group appropriation game: one increasing while the other decreases. Since the ranking function U is not continuous, equilibrium solutions are more difficult to define. Let us start by discussing the conditions for having a symmetric equilibrium.

2.2.1 Symmetric Equilibrium

In a symmetric equilibrium every player chooses the same appropriation level x^e. If this is the case every group has the same appropriation level E^e and, consequently, every group receives the same multiplier $\mu^e = 1$. Equation 2 simplifies to:

$$\{ax^e + \beta n(e - x^e)\} \geq \mu_g(x_i + (n-1)x^e, E^e_{-g})\{ax_i + \beta[ne - (x_i + (n-1)x^e)]\}$$
$$\forall i,\ \forall g,\ \forall x_i \in A$$

Notice that a decrease in the appropriation will have opposite effects in the ranking function U and in the pre-multiplier payoff π_i. A unilateral deviation consisting in a lower appropriation level will increase the multiplier μ_g but at the cost of decreasing the pre-multiplier profits. Notice that due to the discontinuous nature of U and the linear nature of the within-group appropriation game the optimal decrease in appropriation will be the smallest decrease available. On the other hand, and again due to the discontinuity in U, if the ranking function provides incentives to increase appropriation then the optimal response is to maximize its value at $x_i = \overline{a}$.

For simplicity, and without loss of generality, we will redefine the multiplier for the top ranked group G as $\mu_G = 1 + \phi$ and the symmetric multiplier for the bottom ranked group as $\mu_1 = 1 - \phi$. We will normalize the individual returns from appropriation as well. That is $a = 1$. Below we describe the conditions that provide incentives to decrease, and increase, the appropriation level in presence of group competition. Afterwards, we show that for our game parameters, the maximum appropriation level is the unique symmetric equilibrium.

Incentives to decrease the appropriation level Assume a symmetric appropriation level $\tilde{x}$. A downward deviation imposes the following trade-off to the subject: the group-ranking multiplier will increase from $\mu^e = 1$ to $\mu_G = 1 + \phi$, but it will reduce the private gains from appropriation in $1 - \beta$ per unit not appropriated. The most profitable downward deviation occurs by reducing the appropriation level one unit, the minimun possible. Therefore, the condition describing the incentives to decrease the appropriation level is

$$\{\tilde{x} + \beta n(e - \tilde{x})\} \leq (1 + \phi)\{\tilde{x} - 1 + \beta[ne - ((\tilde{x} + 1) + (n - 1)\tilde{x})]\}$$

This simplifies to:

$$\frac{1 - \beta}{\tilde{x} + \beta n(e - \tilde{x})} \leq \frac{\phi}{1 + \phi} \tag{3}$$

Intuitively, condition 3 implies that the earned multiplier, expressed as a percentage increase ($\phi/(1 + \phi)$), must be greater than the benefit of the last appropriated unit ($1 - \beta$), normalized to the realized earnings ($\tilde{x} + \beta n(e - \tilde{x})$).

Incentives to increase the appropriation level Assume again a symmetric appropriation level $\tilde{x}$. An upward deviation imposes a different trade-off to the subject: the private gains will increase by $1 - \beta$ per each additional unit appropriated, but the group ranking multiplier will drop from $\mu^e = 1$ to $\mu_G = 1 - \phi$. Given the discontinuous nature of the multiplier function, the most profitable upward deviation would be to maximize the appropriation or $\tilde{x} = e$. Therefore, incentives to deviate upwards are given by

$$\{\tilde{x} + \beta n(e - \tilde{x})\} \leq (1 - \phi)\{e + \beta[ne - (n - 1)\tilde{x} - e]\}$$

Which simplifies to:

$$\frac{(1 - \beta)(e - \tilde{x})}{e + \beta(n - 1)(e - \tilde{x})} \geq \phi \tag{4}$$

Condition 4 is satisfied as long as the additional benefits from increasing appropriation by $e - \tilde{x}$ units, normalized with respect to the payoff from maximizing appropriation ($e + \beta(n - 1)(e - \tilde{x})$), are larger than the foregone multiplier ϕ.

Maximum appropriation as the unique symmetric Nash Equilibrium The aim of our experimental design is to introduce between-group competition without altering the symmetric equilibrium from the single-group social dilemma. In terms of the conditions derived above, we need a set of parameters such that condition 3 does not hold for any appropriation level, and condition 4 holds for every appropriation level below the maximum.

Table S.2: Parameters employed in the experiment

β	e	n	ϕ
0.35	5	4	0.10

The parameters listed in Table S.2 imply that the experimental setting satisfies condition 4, and never satisfies condition 3. This guarantees that the only symmetric equilibrium is maximum appropriation. Notice that this is independent of the number of groups g.

2.2.2 Asymmetric Equilibria

We will now argue that there are no asymmetric equilibria given the parameters in our experimental setting. One can initially consider two types of asymmetric equilibria. The first type consists on within-group symmetric strategies that differ between groups. We will label this case "between-group asymmetry." The second type considers different appropriation levels within at least one group, and is labeled as "within-group asymmetry."

Let us start by analyzing between-group asymmetry. It implies that there will be at least two groups with a multiplier different to $\mu_g = 1$, one of them with $\mu_g < 1$ and the other with a multiplier $\mu_g > 1$. That is nonetheless not possible. Condition 4 is strong enough that subjects with the multiplier $\mu_g > 1$ awarded by having a lower appropriation level than other groups, will have incentives to increase their appropriation since the earnings per additional unit $1 - \beta$ are more profitable than the current multiplier above unity. This is true even in a setting in which the profit from the multiplier is maximized: only two groups in competition. Therefore, any appropriation level symmetric within the group, but asymmetric between groups, cannot be an equilibrium due to the incentives described in condition 4.

Consider now the within-group asymmetry. First, assume a subject belonging to a group receiving μ_G with an aggregate appropriation level low enough that, even if he maximizes its appropriation level, the group would still be ranked on top. In this case there will be no trade-off between increasing the pre-multiplier earnings and obtaining a greater multiplier. Hence, the fact that $\beta < 1$ provides the incentives for every group member to maximize appropriation and shift towards the symmetric equilibrium already discussed.

Second, consider the opposite group performance. Assume a subject within a group the multiplier $\mu_1 = 1 - \phi$ with an aggregate appropriation level high enough that the subject himself cannot alter the ranking function even if he minimizes his appropriation level. If the penalty for the worst performing group is at its maximum, or $\phi = 1$, then every action is a symmetric Nash equilibrium of the game because the multiplier will be zero and, regardless of the appropriation level, the payoff will be zero as well. However, notice that a direct consequence of $\phi = 1$ is that the Nash equilibrium is altered toward minimum appropriation. Nonetheless, as long as $\phi < 1$ the multiplier is positive, and the group with the lowest multiplier has no other incentive than to maintain maximum appropriation.

On the other hand, groups receiving a multiplier greater than $1 - \phi$ must have an appropriation level below the maximum. This is only sustainable in equilibrium if increases in the appropriation do not change the multiplier. However, even if a single player has an appropriation level $x = e - 1$ and the others appropriate $x = e$, it would require that condition 3 holds and we know that for our experimental parameters this is never true.

Summing up, with the particular values reported in Table S.2, the only equilibrium in our social dilemma with group competition, regardless of the number of groups, is to maximize appropriation.

3 Additional material in online repository

- The oTree code to execute this experiment can be found here
- The screenshots for a session (in Spanish) can be found here
- The raw data, the final dataset, as well as the Stata do-files to obtain the final dataset and to produce the manuscript's figures and tables can be found here