6 Error Correction Model

6.1 COINTEGRATION AND ERROR CORRECTION MECHANISM

Engle and Granger (1987) show that cointegrated variables can always be transformed into an error correction mechanism (ECM) and vice versa. This bi-directional transformation is often called the 'Granger Representation Theorem' and implies that there is some adjustment process that prevents economic variables from drifting too far away from their long-run equilibrium time path. The cointegration and error correction models are very useful in situations where both long-run equilibrium and short-run disequilibrium behaviour are of interest. In tourism demand analysis, the long-run equilibrium behaviour of tourists is expected to be a major concern of policy makers and planners whilst the short-run dynamics are likely to provide useful information for short-term business forecasting and managerial decisions.

This chapter explores how cointegration and error correction are linked, and the ways in which error correction models are estimated within a single equation framework.

6.1.1 From ADLM to Error Correction Model

We started in Chapter 4 with the general ADLM of the form

$$y_t = \alpha + \sum_{j=1}^{k}\sum_{i=0}^{p}\beta_{ji}x_{jt-i} + \sum_{i=1}^{p}\phi_i y_{t-i} + \varepsilon_t \tag{6.1}$$

Equation (6.1) can be reparameterised into an ECM of the form

$$\begin{aligned}\Delta y_t &= (\textit{current and lagged } \Delta x_{jt}\text{s}, \textit{lagged } \Delta y_t\text{s}) \\ &\quad - (1-\phi_1)[y_{t-1} - \sum_{j=1}^{k}\xi_j x_{jt-1}] + \varepsilon_t\end{aligned} \tag{6.2}$$

We shall demonstrate this in the case of an ADLM(1,1) model, but the derivation can be extended to a general ADLM(p,q) process. The ADLM(1,1) model takes the form

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \phi_1 y_{t-1} + \varepsilon_t \tag{6.3}$$

Subtracting y_{t-1} from both sides of Equation (6.3) yields

$$\Delta y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} - (1-\phi_1) y_{t-1} + \varepsilon_t$$

$$\text{or} \quad \Delta y_t = \alpha + \beta_0 \Delta x_t + (\beta_0 + \beta_1) x_{t-1} - (1-\phi_1) y_{t-1} + \varepsilon_t. \tag{6.4}$$

Equation (6.4) can be further reparameterised to give

$$\Delta y_t = \beta_0 \Delta x_t - (1-\phi_1)[y_{t-1} - k_0 - k_1 x_{t-1}] + \varepsilon_t \tag{6.5}$$

where $k_0 = \alpha/(1 - \phi_1)$, $k_1 = (\beta_0 + \beta_1)/(1 - \phi_1)$.

The parameter β_0 is called the impact parameter, $(1 - \phi_1)$ is the feedback effect, k_0 and k_1 are the long-run response coefficients, and the combination of the terms in the square brackets is called the error correction mechanism. Since the coefficient ϕ_1 is less than 1 and greater than 0, the coefficient of the error correction term, $-(1 - \phi_1)$, is greater than –1 and less than 0. This implies that the system will adjust itself toward equilibrium by removing $(1 - \phi_1)$ of a unit from the error made in the previous period. Although Equations (6.3) and (6.5) are different in their functional forms, they actually represent the same data generating process.

Equation (6.5) has the following advantages over Equation (6.3):

First, Equation (6.5) reflects both the long-run and short-run effects in a single model. The specification indicates that changes in y_t depend on changes in x_t and the disequilibrium error in the previous period.

Second, Equation (6.5) overcomes the problem of spurious correlation by employing differenced variables. It can be easily shown that the term $[y_{t-1} - k_0 - k_1 x_{t-1}]$ is a stationary process if y_t and x_t are cointegrated. Therefore, it is unlikely that the residuals in (6.5) will be correlated. Moreover, the use of (6.5) avoids the problems associated with the growth rate model in which only differenced data are used.

Third, the ECM fits in well with the general-to-specific methodology. Since the ECM is another way of writing the general ADLM, acceptance of the ADLM is equivalent to acceptance of the error correction model.

Fourth, the estimation of Equation (6.5) reduces the problem of data mining, since in the model reduction process one is permitted to eliminate the differenced variables according to statistical significance. However, the elimination of lagged levels variables is not permitted since they represent the cointegration relationship. For example, suppose that a cointegration relationship is found between the variables y_t, x_t and z_t. In the estimation of the ECM in which both differenced and lagged levels forms of y, x and z are involved, the researcher is free to eliminate Δy, Δx and/or Δz, but the lagged levels variables y, x and z should always appear in the final ECM.

Finally, estimation of the general ADLM (6.3), which involves a large number of explanatory variables, tends to suffer from the problem of multicollinearity, that is, several of the explanatory variables are likely to be highly correlated, which will result in abnormally large standard errors and hence the calculated t statistics cannot be used as a reliable criterion for hypothesis testing. However, the corresponding variables in the ECM are less likely to be highly correlated. In fact, Engle and Granger (1987) show that the explanatory variables in the ECM are almost orthogonal (that is, the correlation is almost zero). This is a desirable property, as the t statistics provide a reliable guide for the elimination of differenced variables. Consequently, it is easier for a researcher to arrive at a sufficiently parsimonious final preferred model using the testing down procedure of the general-to-specific methodology.

6.1.2 From ECM to Cointegration Regression

Engle and Granger (1987) demonstrate that if a pair of economic variables is cointegrated, they can always be represented by an ECM and vice versa. This can be shown by the following transformation. The long-run steady state suggests that $y_t = y_{t-1}$ and $x_t = x_{t-1}$, that is, $\Delta y_t = \Delta x_t = 0$. Therefore, the ECM (6.5) becomes

$$0 = -(1-\phi_1)[y_t - k_0 - k_1 x_t]$$

$$\text{i.e.,} \quad y_t = k_0 + k_1 x_t \tag{6.6}$$

Equation (6.6) is the long-run cointegration regression with k_0 and k_1 being the long-run cointegration coefficients. Now $k_0 = \alpha/(1 - \phi_1)$ and $k_1 = (\beta_0 + \beta_1)/(1 - \phi_1)$, and therefore the long-run cointegration coefficients (vector) can be obtained from the estimates of the general ADLM model (6.1).

6.2 ESTIMATING THE ECM

Various procedures for estimating the single equation ECM have been proposed. In this section three estimation methods are explained: the Engle-Granger two-stage, the Wickens-Breusch one-stage and the ADLM procedures.

6.2.1 Engle-Granger Two-Stage Approach

In their seminal paper Engle and Granger (1987) suggest testing for the long-run equilibrium relationship between a set of economic variables and modelling their short-run dynamics via an ECM in a two-step procedure. The first step is to test for a cointegration relationship between the two $I(1)$ variables, y_t and x_t, based on the static long-run equilibrium regression:

$$y_t = k_0 + k_1 x_t + \varepsilon_t \tag{6.7}$$

If the estimated residual term is a stationary process, y_t and x_t are said to be cointegrated. According to Stock (1987) if y_t and x_t are cointegrated, OLS estimates of the cointegration vector (k_0, k_1) will be 'super-consistent'.

After confirming the acceptance of a cointegration relationship, the second step is to estimate the ECM

$$\Delta y_t = \sum_{i=0}^{p} \beta_i \Delta x_{t-i} + \sum_{j=1}^{p} \phi_j \Delta y_{t-j} - \lambda \hat{\varepsilon}_{t-1} + u_t \tag{6.8}$$

where $\hat{\varepsilon}_{t-1} = y_{t-1} - \hat{k}_0 - \hat{k}_1 x_{t-1}$ is the OLS residuals from (6.7).

As we can see, the error correction equation (6.8) consists of differenced variables with appropriate lags, and the lag structure is determined by experimentation. In this stage the short-run parameters are obtained. According to Engle and Granger (1987), the estimates of the short-run parameters are consistent and efficient. The estimated standard errors of the parameters in the second stage are the true standard errors, and therefore the model can be used for forecasting and policy evaluation.

One of the concerns about the Engle and Granger two-stage approach is its first step. As we already know that the variables in the cointegration regression are $I(1)$ variables, the estimated standard errors of the cointegration coefficients are not standard normal. That is why many researchers do not even bother to report them in their empirical studies. Moreover, the Engle-Granger two-stage approach does not prove that the cointegration regression is really a long-run one. This is an assumption and cannot be tested statistically. Researchers therefore have to have a good justification that the variables in the cointegration regression represent the long-run equilibrium relationship, and normally the justification is the relevant economic theory. Another problem associated with the Engle-Granger method is that it tends to produce biased estimates of the long-run coefficients in small samples.

6.2.2 Wickens-Breusch One-Stage Approach

It has been shown by Wickens and Breusch (1988) that although the estimates of the short-run ECM of the Engle-Granger two-stage approach are consistent and efficient in large samples, they are biased in small samples. An alternative estimation method which overcomes this problem has been suggested by Wickens and Breusch, and involves estimating both the long-run and short-run parameters in a single step using OLS. The estimation is based on

$$\Delta y_t = \alpha + \sum_{i=0}^{p-1} \beta_i \Delta x_{t-i} + \sum_{i=1}^{p-1} \phi_i \Delta y_{t-i} + \lambda_1 y_{t-1} + \lambda_2 x_{t-1} + u_t \tag{6.9}$$

Again the lag lengths of the differenced variables are determined by the statistical significance of the estimated coefficients in (6.9). After estimating (6.9), the long-run cointegration parameters may be derived from

$$y_t = -\frac{\hat{\alpha}}{\hat{\lambda}_1} - \frac{\hat{\lambda}_2}{\hat{\lambda}_1} x_t$$

$$\text{or} \quad y_t = k_0^* + k_1^* x_t \tag{6.10}$$

where $k_0^* = -\hat{\alpha}/\hat{\lambda}_1$ and $k_1^* = -\hat{\lambda}_2/\hat{\lambda}_1$.

Wickens and Breusch (1988) show that the OLS estimates of both the long-run and short-run parameters in Equation (6.9) are consistent, efficient and unbiased.

6.2.3 ADLM Approach

This method of estimating the ECM was developed by Pesaran and Shin (1995), and involves estimating both the long-run cointegration relationship and the short-run ECM based on a general ADLM. Our discussion of this estimation approach is based on Equation (6.3), but the principle is the same for the more general form of the ADLM, as given in Equation (6.1).

The ADLM approach first estimates Equation (6.3) with extended lag structure using OLS.

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \ldots + \beta_p x_{t-P} + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \ldots + \phi_P y_{t-P} + \varepsilon_t \tag{6.11}$$

The optimal lag structure of the ADLM from which the ECM is derived is determined by the following criteria: $\overline{R}^2$, *AIC* and *SBC* as described in Chapter 4. The long-run cointegration regression is $y_t = \hat{k}_0 + \hat{k}_1 x_t$, where

$$\hat{k}_1 = \frac{\hat{\beta}_0 + \hat{\beta}_1 + \ldots + \hat{\beta}_p}{1 - \hat{\phi}_1 - \hat{\phi}_2 - \ldots - \hat{\phi}_p} \tag{6.12}$$

$$\text{and} \quad \hat{k}_0 = \frac{\hat{\alpha}}{1 - \hat{\phi}_1 - \hat{\phi}_2 - \ldots - \hat{\phi}_p} \tag{6.13}$$

The corresponding short-run ECM may be derived from Equation (6.11). Since

$$\Delta y_t = y_t - y_{t-1}$$

$$y_t = \Delta y_t + y_{t-1}$$

$$\text{Also} \quad x_t = \Delta x_t + x_{t-1}$$

Substituting these relationships into Equation (6.11) and rearranging the terms gives the following ECM representation:

$$\Delta y_t = \sum_{i=0}^{p-1} \beta_i^* \Delta x_{t-i} + \sum_{i=1}^{p-1} \phi_i^* \Delta y_{t-i} - \lambda EC_{t-1} + \varepsilon_t \tag{6.14}$$

where $\lambda = 1 - \hat{\phi}_1 - \hat{\phi}_2 - \ldots - \hat{\phi}_{p-1}$, $EC_{t-1} = y_{t-1} - \alpha - \beta_1 x_{t-1}$, which is the error correction term, and the coefficients β_i^* and ϕ_i^* are calculated from:

$$\begin{aligned}
\beta_0^* &= \hat{\beta}_p + \hat{\beta}_{p-1} + \ldots + \hat{\beta}_2 + \hat{\beta}_1 \\
\beta_1^* &= \hat{\beta}_p + \hat{\beta}_{p-1} + \ldots + \hat{\beta}_2 \\
&\vdots \\
\beta_{p-1}^* &= \hat{\beta}_p
\end{aligned}$$

and

$$\begin{aligned}
\phi_1^* &= \hat{\phi}_p + \hat{\phi}_{p-1} + \ldots + \hat{\phi}_3 + \hat{\phi}_2 \\
\phi_2^* &= \hat{\phi}_p + \hat{\phi}_{p-1} + \ldots + \hat{\phi}_3 \\
&\vdots \\
\phi_{p-1}^* &= \hat{\phi}_p
\end{aligned}$$

The $\hat{\beta}_i$s and $\hat{\phi}_i$s are the estimated parameters from the ADLM (6.11). The standard errors of these estimated coefficients can be calculated using the formula provided by Bewley (1979).

As noted by Pesaran and Shin (1995), however, the ADLM approach also tends to suffer from small-sample bias, although the consistency improves with increasing sample size.

6.3 WORKED EXAMPLES

In this section we use inbound tourism demand data for South Korea to illustrate the error correction modelling procedure. The demand for tourism to Korea by two major tourism-generating countries, the UK and USA, is examined. The total number of tourist arrivals by country ranging from 1962 to 1994 (33 observations) is used as the dependent variable and the data are obtained from the Korea National Tourism Corporation (KNTC) Annual Statistical Report jointly published by KNTC and the Ministry of Culture and Sports. Since the tourist arrivals variable includes both business and leisure travellers, the gross domestic product (GDP) of the tourism-generating country is used as the income variable, rather than personal disposable income. To reflect the influence of business activities on tourism demand, a trade volume variable measured by the sum of total imports and exports between Korea and the tourist-generating country is also included in the model. The tourism price variable is measured by the relative consumer price index (CPI) of Korea to that of the tourism-generating country, adjusted by the corresponding exchange rate (see Equation (6.16)). The data are provided in Table A6.1 in Appendix 6.1.

6.3.1 Testing for Cointegration

The proposed long-run demand model is of the form

$$\ln TA_{it} = \alpha_0 + \alpha_1 \ln GDP_{it} + \alpha_2 \ln TV_{it} + \alpha_3 \ln RCPI_{it} + u_t \tag{6.15}$$

where TA_{it} is total tourist arrivals in South Korea from country i (i = 1 represents UK and i = 2 represents USA) in year t; GDP_{it} is the index of the real gross domestic product of origin i in year t; TV_{it} is real trade volume measured by total imports and exports between South Korea and country i in year t; $RCPI_{it}$ is the tourism price variable calculated from

$$RCPI_{it} = \frac{CPI_{Korea,t} \,/\, EX_{Korea/i,t}}{CPI_{it}} \tag{6.16}$$

This takes into account the effects of both relative inflation and the exchange rate on the demand for tourism to Korea.

The null hypothesis is that the variables in Equation (6.15) are cointegrated. The integration orders (or the numbers of unit roots) of the variables are examined prior to the cointegration test. In testing for unit roots, the Dickey-Fuller and Phillips-Perron tests discussed in Chapter 5 are used. The results of these tests show that all the series (in logarithms) are likely to be $I(1)$ with the possibility that the USA GDP is a trended stationary process. Since a trended stationary series still needs to be differenced to get rid of the trend, the inclusion of this variable in the cointegration regression is not unjustified.

The long-run cointegration regressions are estimated using OLS as

$$\begin{aligned}\text{UK:}\quad \ln TA_{1t} &= -4.257 + 2.097 \ln GDP_{1t} + 0.473 \ln TV_{1t} \\ &\quad - 0.213 \ln RCPI_{1t}\end{aligned} \tag{6.17}$$

$$R^2 = 0.995 \quad \text{CIDW} = 1.516$$

$$\begin{aligned}\text{USA:}\quad \ln TA_{2t} &= -6.158 + 1.655 \ln GDP_{2t} + 0.576 \ln TV_{2t} \\ &\quad - 0.861 \ln RCPI_{2t}\end{aligned} \tag{6.18}$$

$$R^2 = 0.971 \quad \text{CIDW} = 0.490$$

As a rough guide, if there is a cointegration relationship between a set of variables, the CIDW statistic in the cointegration regression should be

Table 6.1 Dickey-Fuller (DF) and Phillips-Perron (PP) Test Results

Tests	*UK*	*USA*
DF	–4.733(0)	–3.490(3)
PP	–4.707(3)	–3.188(3)

Note: Figures in parentheses are the lag lengths used in the tests.

greater than its R-squared value. According to this criterion the variables in the UK model are likely to be cointegrated whilst those in the USA model are not. This can be further confirmed by the Dickey-Fuller and Phillips-Perron tests on the OLS residuals from Equations (6.17) and (6.18). The results are presented in Table 6.1.

The critical value for non-cointegration with four $I(1)$ variables in the long-run model with 25 degrees of freedom is –4.56 at the 5% level and –4.15 at the 10% level. The results in Table 6.1 show that the tourism demand variables are cointegrated in the UK model, but not in the USA equation.

The long-run equations (6.17 and 6.18) show that all the demand elasticities are correctly signed and significant at the 5% level. The income elasticities are positive and greater than 1 suggesting that travelling to South Korea is a luxury for tourists from the UK and USA. However, since the variables in the USA long-run model are not cointegrated, the estimated long-run elasticities are biased.

The significance of the trade volume variable is not surprising since a relatively large proportion (about 20%) of tourist arrivals in South Korea is related to business travel. According to Lee *et al.* (1996), business travellers dominated Korean inbound tourism in the 1960s, and they paved the way for the surge of pleasure tourists to South Korea since the early 1970s.

The relative price elasticity in the UK and USA long-run models is less than 1. This indicates that tourism demand is price inelastic.

6.3.2 Error Correction Models

This section presents the Engle and Granger two-stage, the Wickens and Breusch one-stage and the ADLM procedures for ECM estimation. Although a cointegration relationship does not exist in the USA model, we still try to model the short-run dynamics using the ECM. This will allow us to compare the consequences of ECMs with and without cointegrated variables.

6.3.2.1 Engle-Granger Two-Stage Approach

As mentioned earlier, if a set of variables is found to be cointegrated, the cointegration regression can always be transformed into an error correction model of the form

$$\Delta \ln TA_{it} = \beta_0 + \beta_1 \Delta \ln GDP_{it} + \beta_2 \Delta \ln TV_{it} + \beta_3 \Delta \ln RCPI_{it} + \delta \hat{u}_{t-1} + \varepsilon_t \quad (6.19)$$

where $\hat{u}_{t-1}$ is the estimated error term obtained from the cointegration regression Equation (6.15) and Δ is the first difference operator. The coefficient of $\hat{u}_{t-1}$ is expected to be negative and significant. In estimating Equation (6.19) lagged dependent and independent variables are also included and a 'test-down' procedure is employed repeatedly until the most parsimonious specification is achieved.

The initial estimates of the general ECMs for the UK and USA are given in Equations (6.20) and (6.21):

$$\begin{aligned}
\text{UK:} \quad \Delta \ln TA_{1t} = {} & \underset{(1.736)}{0.048} - \underset{(-0.349)}{0.072} \Delta \ln TA_{1t-1} + \underset{(1.645)}{0.718} \Delta \ln GDP_{1t} \\
& + \underset{(0.220)}{0.126} \Delta \ln GDP_{1t-1} + \underset{(4.837)}{0.340} \Delta \ln TV_{1t} \\
& + \underset{(0.993)}{0.074} \Delta \ln TV_{1t-1} + \underset{(0.417)}{0.055} \Delta \ln RCPI_{1t} \\
& - \underset{(2.201)}{0.280} \Delta \ln RCPI_{1t-1} - \underset{(-1.891)}{0.384} \hat{u}_{1t-1}
\end{aligned} \quad (6.20)$$

$R^2 = 0.608 \quad \sigma = 0.082$

$$\begin{aligned}
\text{USA:} \quad \Delta \ln TA_{2t} = {} & -\underset{(-0.411)}{0.017} + \underset{(1.450)}{0.224} \Delta \ln TA_{2t-1} - \underset{(-0.419)}{0.431} \Delta \ln GDP_{2t} \\
& + \underset{(0.488)}{0.561} \Delta \ln GDP_{2t-1} + \underset{(3.168)}{0.858} \Delta \ln TV_{2t} \\
& + \underset{(0.368)}{0.092} \Delta \ln TV_{2t-1} - \underset{(-1.827)}{0.665} \Delta \ln RCPI_{2t} \\
& - \underset{(-0.067)}{0.026} \Delta \ln RCPI_{2t-1} - \underset{(-3.056)}{0.561} \hat{u}_{2t-1}
\end{aligned} \quad (6.21)$$

$R^2 = 0.610 \quad \sigma = 0.105$

The values in parentheses are t ratios. The insignificant variables judged by the t values are eliminated one by one starting with the most insignificant ones until all remaining coefficients are statistically significant. The final ECMs for the UK and USA are given as below:

$$\begin{aligned}
\text{UK:} \quad \Delta \ln TA_{1t} = {} & \underset{(2.595)}{0.052} + \underset{(1.887)}{0.752} \Delta \ln GDP_{1t} + \underset{(5.553)}{0.341} \Delta \ln TV_{1t} \\
& - \underset{(-2.176)}{0.239} \Delta \ln RCPI_{1t-1} - \underset{(-2.976)}{0.468} \hat{u}_{1t-1}
\end{aligned} \quad (6.22)$$

$R^2 = 0.584 \quad \sigma = 0.078 \quad \chi^2_{Auto}(2) = 1.467$

$\chi^2_{Norm}(2) = 0.952 \quad \chi^2_{ARCH}(1) = 0.049 \quad \chi^2_{White}(14) = 15.720$

$\chi^2_{RESET}(2) = 1.388 \quad F_{Forecast}(5,21) = 0.260$

$$\text{USA:} \quad \Delta \ln TA_{2t} = \underset{(-0.357)}{-0.013} + \underset{(1.530)}{1.356}\,\Delta \ln GDP_{2t-1} + \underset{(4.195)}{0.823}\,\Delta \ln TV_{2t} - \underset{(-2.142)}{0.666}\,\Delta \ln RCPI_{2t} - \underset{(-3.155)}{0.440}\,\hat{u}_{2t-1} \tag{6.23}$$

$R^2 = 0.554 \quad \sigma = 0.104 \quad \chi^2_{Auto}(2) = 3.340$

$\chi^2_{Norm}(2) = 8.843^{**} \quad \chi^2_{ARCH}(1) = 0.585 \quad \chi^2_{White}(14) = 24.440^{**}$

$\chi^2_{RESET}(2) = 23.292^{***} \quad F_{Forecast}(5,21) = 0.886$

** and *** denote that the statistics are significant at the 5% and 1% levels, respectively. The diagnostic statistics are explained in Chapter 4.

The estimation results given in Equations (6.22) and (6.23) show that the UK short-run ECM passes all the diagnostic statistics whilst the USA equivalent fails the normality, homoscedasticity and the functional form tests. This is not surprising since no cointegration relationship was found in the USA long-run model based on the Engle-Granger procedure. In practice, if a cointegration relationship does not exist between a set of economic variables, the ECM specification should not be used. The solution to this problem is either to try an alternative modelling strategy or to re-specify the long-run model in terms of a different functional form and/or inclusion of new explanatory variables based on economic theory.

The estimated coefficients in the short-run error correction model are short-run demand elasticities. Since the USA ECM is not statistically acceptable, there is no point in trying to interpret the economic meanings of the coefficients. However, the estimated demand elasticities in the UK ECM are unbiased, efficient and consistent, and therefore economic interpretations of the estimated coefficients are possible. The short-run income and trade elasticities in Equation (6.22) are lower than those in the long-run model, as expected on the base of economic theory, but the short-run price elasticity of the demand for Korean tourism by UK residents remains more or less the same as that in the long-run model. This suggests that the relative cost of living for UK tourists in Korea has a similar influence on both long-run and short-run decision making for UK residents. The coefficient of the error correction term in the UK ECM is correctly signed and significant at the 1% level as expected.

6.3.2.2 Wickens-Breusch One-Stage Approach

The general tourism demand models are first estimated using the Wickens-Breusch method, based on the same data set, and the results are given below:

$$
\begin{aligned}
\text{UK:}\quad \Delta\ln TA_{1t} = & \underset{(-0.939)}{-1.339} - \underset{(-0.689)}{0.146}\,\Delta\ln TA_{1t-1} + \underset{(1.230)}{0.638}\,\Delta\ln GDP_{1t} \\
& - \underset{(-0.220)}{0.021}\,\Delta\ln GDP_{1t-1} + \underset{(3.935)}{0.297}\,\Delta\ln TV_{1t} \\
& + \underset{(1.012)}{0.080}\,\Delta\ln TV_{1t-1} + \underset{(0.702)}{0.115}\,\Delta\ln RCPI_{1t} \\
& - \underset{(-1.965)}{0.290}\,\Delta\ln RCPI_{1t-1} - \underset{(-1.630)}{0.342}\ln TA_{1t-1} \\
& + \underset{(1.414)}{0.752}\ln GDP_{1t-1} + \underset{(1.369)}{0.143}\ln TV_{1t-1} \\
& - \underset{(-0.506)}{0.058}\ln RCPI_{1t-1}
\end{aligned}
$$

$R^2 = 0.656 \quad \sigma = 0.083$

$$
\begin{aligned}
\text{USA:}\quad \Delta\ln TA_{2t} = & \underset{(-2.150)}{-4.908} - \underset{(-0.164)}{0.028}\,\Delta\ln TA_{2t-1} - \underset{(-0.932)}{0.957}\,\Delta\ln GDP_{2t} \\
& + \underset{(0.100)}{0.116}\,\Delta\ln GDP_{2t-1} + \underset{(2.274)}{0.634}\,\Delta\ln TV_{2t} \\
& - \underset{(-0.165)}{0.045}\,\Delta\ln TV_{2t-1} - \underset{(-0.951)}{0.353}\,\Delta\ln RCPI_{2t} \\
& + \underset{(0.652)}{0.262}\,\Delta\ln RCPI_{2t-1} - \underset{(-3.151)}{0.561}\ln TA_{2t-1} \\
& + \underset{(1.313)}{0.084}\ln GDP_{2t-1} + \underset{(1.701)}{0.263}\ln TV_{2t-1} \\
& - \underset{(-2.810)}{0.821}\ln RCPI_{2t-1}
\end{aligned}
$$

$R^2 = 0.727 \quad \sigma = 0.095$

Since some of the coefficients in the models are insignificant, a model reduction process is needed. Following the general-to-specific methodology, the insignificant variables are eliminated based on the t statistic values and the final ECMs become:

$$
\begin{aligned}
\text{UK:}\quad \Delta\ln TA_{1t} = & \underset{(-0.837)}{-0.858} + \underset{(4.156)}{0.281}\,\Delta\ln TV_{1t} - \underset{(-2.669)}{0.443}\ln TA_{1t-1} \\
& + \underset{(1.488)}{0.604}\ln GDP_{1t-1} + \underset{(2.376)}{0.208}\ln TV_{1t-1} \\
& - \underset{(-2.305)}{0.178}\ln RCPI_{1t-1}
\end{aligned}
\tag{6.24}
$$

$R^2 = 0.541 \qquad \sigma = 0.082 \qquad \chi^2_{Auto}(2) = 0.717$

$\chi^2_{Norm}(2) = 0.974 \qquad \chi^2_{ARCH}(1) = 0.156 \qquad \chi^2_{White}(20) = 27.363$

$\chi^2_{RESET}(2) = 4.358 \qquad F_{Forecast}(5,21) = 0.137$

$$
\begin{aligned}
\text{USA:}\quad \Delta\ln TA_{2t} = & \underset{(-2.510)}{-4.171} + \underset{(2.209)}{0.450}\,\Delta\ln TV_{2t} + \underset{(1.600)}{0.135}\,\Delta\ln RCPI_{2t-1} \\
& - \underset{(-3.791)}{0.531}\ln TA_{2t-1} + \underset{(1.875)}{0.836}\ln GDP_{2t-1} \\
& + \underset{(1.875)}{0.235}\ln TV_{2t-1} - \underset{(-3.210)}{0.696}\ln RCPI_{2t-1}
\end{aligned}
\tag{6.25}
$$

$R^2 = 0.703$ $\sigma = 0.088$ $\chi^2_{Auto}(2) = 1.865$

$\chi^2_{Norm}(2) = 0.004$ $\chi^2_{ARCH}(1) = 2.659$ $\chi^2_{White}(26) = 29.438$

$\chi^2_{RESET}(2) = 22.235^{***}$ $F_{Forecast}(5,19) = 0.467$

Again, the UK model is highly satisfactory in terms of the diagnostic statistics, whereas the USA model suffers from the same problems as those in the Engle-Granger two-stage model. Although the coefficient of the variable $\Delta \ln RCPI_{2t-1}$ is not significant at the 5% level, we keep it in the final specification since the elimination of this variable will drastically reduce the explanatory power of the model. To obtain the long-run parameters, Equations (6.24) and (6.25) can be transformed to give:

$$\text{UK:} \quad \Delta \ln TA_{1t} = 0.304\Delta \ln TV_{it} - 0.443(\ln TA_{1t-1} + 1.937 \\ - 1.363 \ln GDP_{1t-1} - 0.469 \ln TV_{1t-1} \\ + 0.402 \ln RCPI_{1t-1}) \tag{6.26}$$

$$\text{USA:} \quad \Delta \ln TA_{2t} = 0.450\Delta \ln TV_{2t} + 0.135\Delta \ln RCPI_{2t-1} \\ - 0.530(\ln TA_{2t-1} + 7.870 - 1.577 \ln GDP_{2t-1} \\ - 0.443 \ln TV_{2t-1} + 1.313 \ln RCPI_{2t-1}) \tag{6.27}$$

Theoretically, the Engle-Granger method and the Wickens-Breusch approach should produce similar results for large samples if the cointegration relationship exists in the levels variables. However, since the lagged levels variables in the parentheses in the UK model (6.24) represent the cointegration relationship and the combination of these levels variables tends to be stationary, the OLS estimates of the long-run parameters in this method are more reliable. In contrast, OLS estimation in the first stage of the Engle-Granger method tends to be biased and these biases are likely to be carried over to the second stage of the estimation. For the USA model, since no cointegration relationship has been found, the Wickens-Breusch ECM still suffers from similar problems to those of the corresponding Engle-Granger ECM.

The long-run relationships for the UK and USA can therefore be derived as:

$$\text{UK:} \quad \ln TA_{1t-1} = -1.937 + 1.363 \ln GDP_{1t-1} + 0.469 \ln TV_{1t-1} \\ - 0.402 \ln RCPI_{1t-1} \tag{6.28}$$

$$\text{USA:} \quad \ln TA_{2t-1} = -7.870 + 1.577 \ln GDP_{2t-1} + 0.443 \ln TV_{2t-1} \\ - 1.313 \ln RCPI_{2t-1} \tag{6.29}$$

Comparing Equations (6.28) and (6.29) with (6.17) and (6.18), respectively, we see that the long-run parameters estimated from the Engle-Granger method are not the same as those from the Wickens-Breusch approach for both the UK and USA models.

To conclude, the Engle-Granger and the Wickens-Breusch cointegration-ECM approaches should produce similar results for large samples. In the case of small samples, the Wickens-Breusch method is preferred due to the consistent and unbiased nature of the estimation.

6.3.2.3 ADLM Method

Since the data are annual, we introduce one lag for each of the variables in the general ADLM[1], that is:

$$\begin{aligned}\ln TA_{it} &= \alpha + \beta_{11}\ln GDP_{it} + \beta_{12}\ln GPD_{it-1} + \beta_{21}\ln TV_{it} + \beta_{22}\ln TV_{it-1} \\ &\quad + \beta_{31}\ln RCPI_{it} + \beta_{32}\ln RCPI_{it-1} + u_t\end{aligned} \tag{6.30}$$

The estimates of the ADLM (6.30) using the UK and USA data are given below[2]:

$$\begin{aligned}\text{UK:}\quad \ln TA_{1t} &= \underset{(-1.089)}{-1.321} + \underset{(3.116)}{0.544}\ln TA_{1t-1} + \underset{(1.217)}{0.582}\ln GDP_{1t} \\ &\quad + \underset{(0.342)}{0.194}\ln GPD_{1t-1} + \underset{(4.017)}{0.281}\ln TV_{1t} \\ &\quad - \underset{(-0.956)}{0.073}\ln TV_{1t-1} - \underset{(-0.027)}{0.004}\ln RCPI_{1t} \\ &\quad - \underset{(-1.089)}{0.144}\ln RCPI_{1t-1}\end{aligned} \tag{6.31}$$

$R^2 = 0.997$ $\sigma = 0.082$ $\chi^2_{Auto}(1) = 0.022$

$\chi^2_{Norm}(2) = 0.920$ $\chi^2_{Hetro}(1) = 1.912^3$ $\chi^2_{RESET}(1) = 0.886$

$$\begin{aligned}\text{USA:}\quad \ln TA_{2t} &= \underset{(-3.290)}{-5.140} + \underset{(6.238)}{0.617}\ln TA_{2t-1} - \underset{(-0.772)}{0.644}\ln GDP_{2t} \\ &\quad + \underset{(1.641)}{1.555}\ln GDP_{2t-1} + \underset{(1.957)}{0.440}\ln TV_{2t} \\ &\quad - \underset{(-1.614)}{0.334}\ln TV_{2t-1} - \underset{(-1.434)}{0.428}\ln RCPI_{2t} \\ &\quad - \underset{(-0.836)}{0.289}\ln RCPI_{2t-1}\end{aligned} \tag{6.32}$$

$R^2 = 0.994$ $\sigma = 0.091$ $\chi^2_{Auto}(1) = 0.889$

$\chi^2_{Norm}(2) = 7.950^{**}$ $\chi^2_{Hetro}(1) = 4.279^{**}$ $\chi^2_{RESET}(1) = 8.914^{***}$

The ADLM method uses the full lag structure to derive the long-run and short-run parameters, and the insignificant parameters are therefore not eliminated. However, Pesaran and Shin (1995) show that the optimal lag structure can be determined by a number of criteria such as the Akaike information criterion (AIC), Schwarz Bayesian criterion (SBC) and the Hannan-Quinn criterion (HQC). For illustrative purposes we impose the restriction that lag length equals one.

The results show that the UK model is well specified according to the diagnostics, but the USA model still appears to have problems with regard to functional form, heteroscedasticity and normality. Based on the formulae (6.12) and (6.13), we can derive the long-run static parameters. Assuming there are no dynamics in the long run, Equation (6.31) becomes

$$(1-0.544)\ln TA_{1t} = -1.321 + (0.582 + 0.194)\ln GDP_{1t} + (0.281 - 0.072)\ln TV_{1t} - (0.004 + 0.144)\ln RCPI_{1t}$$

If both sides of the above equation are divided by (1 – 0.544), we get

$$\ln TA_{1t} = -2.897 + 1.702\ln GDP_{1t} + 0.456\ln TV_{1t} - 0.322\ln RCPI_{1t} \quad (6.33)$$

This is the long-run model for the UK. Using the same procedure, the USA long-run model can be derived as follows:

$$\ln TA_{2t} = -13.425 + 2.380\ln GDP_{2t} + 0.277\ln TV_{2t} - 1.871\ln RCPI_{2t} \quad (6.34)$$

Although the standard errors (or t ratios) of the coefficients in (6.33) and (6.34) are omitted, they can be calculated according to the method proposed by Bewley (1979).

The corresponding ECMs are estimated based on the estimated long-run regressions (6.33) and (6.34).

$$\text{UK:}\quad \Delta\ln TA_{1t} = \underset{(-1.089)}{-1.321} + \underset{(1.216)}{0.582}\,\Delta\ln GDP_{1t} + \underset{(4.017)}{0.281}\,\Delta\ln TV_{1t} - \underset{(-0.026)}{0.004}\,\Delta\ln RCPI_{1t-1} - \underset{(-2.612)}{0.456}\,\hat{EC}_{1t-1} \quad (6.35)$$

where $\hat{EC}_{1t-1} = \ln TA_{1t-1} + 2.897 - 1.702\ln GDP_{1t-1} - 0.456\ln TV_{1t-1} + 0.322\ln RCPI_{1t-1}$

$$\text{USA:}\quad \Delta\ln TA_{2t} = \underset{(-3.290)}{-5.140} - \underset{(-0.772)}{0.644}\,\Delta\ln GDP_{2t} + \underset{(1.957)}{0.440}\,\Delta\ln TV_{2t} - \underset{(-1.433)}{0.428}\,\Delta\ln RCPI_{2t-1} - \underset{(-3.871)}{0.383}\,\hat{EC}_{2t-1} \quad (6.36)$$

where $\hat{EC}_{2t-1} = \ln TA_{2t-1} + 13.425 - 2.379\ln GDP_{2t-1} - 0.277\ln TV_{2t-1} + 1.871\ln RCPI_{2t-1}$

Apart from the R-squared values, the other diagnostic statistics for (6.35) and (6.36) are the same as those in (6.31) and (6.32).

As can be seen, the ADLM estimation method is also a two-stage process. The advantage of this method over the Engle-Granger two-stage approach is that in the first stage the error term in the ADLM tends to be independent and identically distributed, and therefore the diagnostic statistics and the standard errors of the coefficients are standard and the normal critical values can be used for inference.

As to which method performs best, there is no clear cut answer to this question. But if the sample is sufficiently large and there is only one cointegration relationship between a number of variables, then all three methods should produce similar results. However, in the case of multi-cointegration relationships, a more advanced approach, such as the Johansen (1988) method, should be used. This approach will be discussed in the next chapter.

Another way of assessing the validity of these estimation methods is to look at the *ex post* forecasting performance. In the next sub-section we will re-estimate the ECMs using the data from 1962 to 1990 and reserve the final four observations for forecasting comparison purposes. Since the USA ECMs are badly specified for all three methods, we will not carry out the forecasting exercise for the USA.

6.3.2.4 Forecasting Performance of ECMs

The forecasts from 1991 to 1994 are generated from all three ECMs. The forecasting errors are calculated based on the differences between the actual and forecast values. In order to see which model performs best, two measures of forecasting performance are used: the mean absolute forecasting error (MAE) and the root mean square forecasting error (RMSE). The results are presented in Table 6.2.

The results in Table 6.2 show that the ADLM approach produces the smallest forecasting errors.

Table 6.2 Forecast Errors Generated from the Three UK ECMs

Model	*MAE*	*RMSE*
Engle-Granger	0.04378	0.04690
Wickens-Breusch	0.05928	0.06167
ADLM	0.03221	0.03633

APPENDIX 6.1

Table A6.1 Data Used in Chapter 6

Year	*UKTA*	*USTA*	*UKGDP*	*USGDP*	*UKTV*	*USTV*	*UKCPI*	*USCPI*	*KCPI*	*UKEXR*[1]	*KEXR*[2]
1962	602	7328	48.89	42.83	28.623	841.67	10.7	23.1	3.8	2.8	130
1963	705	10178	51.02	44.47	24.727	1113.80	10.9	23.4	4.5	2.8	130
1964	737	11530	53.69	46.54	35.145	861.232	11.3	23.7	5.8	2.8	213.85
1965	828	14152	56.05	50.71	17.266	877.689	11.8	24.1	6.6	2.8	266.4
1966	1052	30226	57.11	53.76	25.436	1218.47	12.2	24.9	7.5	2.8	271.34
1967	1522	39274	58.42	55.16	45.993	1542.16	12.5	25.5	8.3	2.77	270.52
1968	1924	41823	60.80	57.43	76.610	2331.86	13.1	26.6	9.3	2.4	276.65
1969	2564	49606	62.05	58.99	139.70	2824.90	13.8	28.1	10.4	2.4	288.16
1970	2680	55352	63.47	58.96	144.54	3076.25	14.7	29.7	12.1	2.4	310.56
1971	3029	58003	64.73	60.79	214.36	3689.30	16.1	31.0	13.7	2.43	347.15
1972	3671	63578	67.00	63.70	299.91	4121.35	17.2	32.0	15.3	2.5	392.89
1973	4980	77537	71.93	67.01	371.83	5744.44	18.8	34.0	15.8	2.45	398.32
1974	5345	80621	70.71	66.59	428.48	6941.30	21.8	37.8	19.6	2.34	404.47
1975	6446	97422	70.20	66.05	566.20	6794.04	27.1	41.2	24.6	2.22	484
1976	8899	102199	72.15	69.31	808.37	8470.53	31.6	43.6	28.4	1.81	484
1977	9970	113710	73.85	72.44	810.83	9991.04	36.6	46.4	31.3	1.75	484
1978	12566	118039	76.40	75.93	1004.98	11827.24	39.6	49.9	35.8	1.92	484

(*Continued*)

Table A6.1 Data Used in Chapter 6 (*Continued*)

Year	*UKTA*	*USTA*	*UKGDP*	*USGDP*	*UKTV*	*USTV*	*UKCPI*	*USCPI*	*KCPI*	*UKEXR*[1]	*KEXR*[2]
1979	13395	127355	78.54	77.84	1537.67	13282.13	44.9	55.6	42.3	2.12	484
1980	12414	121404	76.84	77.43	1134.54	12307.89	53.0	63.1	54.5	2.33	607.43
1981	14874	130402	75.85	78.79	1308.42	13924.08	59.3	69.6	66.1	2.03	681.03
1982	16140	151249	77.16	77.09	1659.30	14236.05	64.4	73.9	70.8	1.75	731.08
1983	18598	176488	80.00	80.09	1688.86	16693.46	67.4	76.2	73.3	1.52	775.75
1984	19213	212986	81.86	85.05	1699.55	19512.33	70.7	79.5	75.0	1.34	805.98
1985	21414	239423	94.93	87.75	1604.28	19552.42	75.0	82.4	76.8	1.30	870.02
1986	23481	284571	88.57	90.30	1726.22	23744.78	77.6	83.9	78.9	1.47	881.45
1987	24606	326330	92.83	93.08	2541.86	30704.75	80.8	87.0	81.3	1.64	822.57
1988	33276	347281	97.48	96.74	3129.35	37156.52	84.7	90.5	87.1	1.78	731.47
1989	34423	317133	99.60	99.19	2774.33	36902.69	91.3	94.9	92.1	1.64	671.46
1990	36054	325388	100.00	100.00	2976.00	36392.00	100.0	100.0	100.0	1.78	707.76
1991	35848	315828	98.04	98.84	3320.36	37437.13	105.9	104.2	109.3	1.77	733.35
1992	36284	333805	97.52	102.09	3159.72	36088.30	109.8	107.4	116.1	1.77	780.65
1993	35923	325366	99.72	105.27	2993.16	35255.13	111.5	110.6	121.7	1.50	802.67
1994	40999	332428	103.56	109.57	3325.29	40667.95	114.3	113.4	129.3	1.53	803.45

Notes: [1]Exchange rate in terms of pound sterling per US dollar.
[2]Exchange rate in terms of Korean won per US dollar.