← MA1029, section León Gpo. 401, Spring 2023

Tarea 6 Criterio de la Segunda Derivada (Homework)

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Print Assignment

Current Score

QUESTION	1	2	3	4	5
POINTS	12.5/12.5 ✓	12.5/12.5 ✓	12.5/12.5 ✓	12.5/12.5 ✓	12.5/12.5 ✓

TOTAL SCORE
100/100 100.0%

Due Date

WED, APR 12, 2023

Request Extension

Assignment Submission & Scoring

Assignment Submission

For this assignment, you submit answers by question parts. The number of submissions remaining for each question part only changes if you submit or change the answer.

Assignment Scoring

Your last submission is used for your score.

1. [12.5/12.5 Points] DETAILS | PREVIOUS ANSWERS | SCALCET9M 14.7.003.

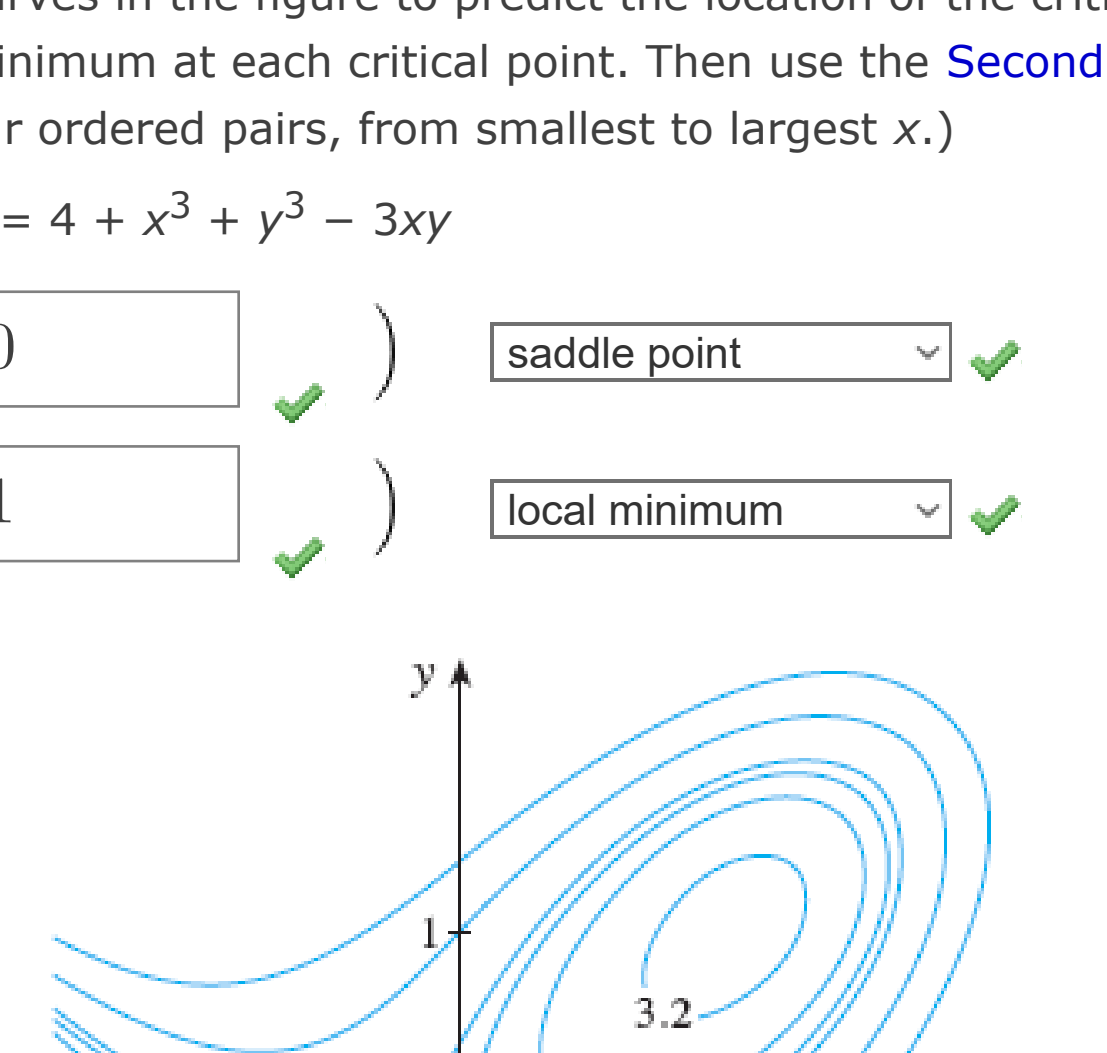
MY NOTES | ASK YOUR TEACHER

Use the level curves in the figure to predict the location of the critical points of f and whether f has a saddle point or a local maximum or minimum at each critical point. Then use the Second Derivatives Test to confirm your predictions. (Order your answers by their ordered pairs, from smallest to largest x.)

$f(x, y) = 4 + x^3 + y^3 - 3xy$

$(x, y) = ($ 0,0 $)$ ✓ saddle point ✓

$(x, y) = ($ 1,1 $)$ ✓ local minimum ✓



2. [12.5/12.5 Points] DETAILS | PREVIOUS ANSWERS | SCALCET9M 14.7.005.

MY NOTES | ASK YOUR TEACHER | PRACTICE ANOTHER

Find the local maximum and minimum values and saddle point(s) of the function. If you have three-dimensional graphing software, graph the function with a domain and viewpoint that reveal all the important aspects of the function. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

$f(x, y) = x^2 + xy + y^2 + 7y$

local maximum value(s) DNE ✓

local minimum value(s) $-\frac{49}{3}$ ✓

saddle point(s) $(x, y, f) =$ DNE ✓

Need Help? Watch It

3. [12.5/12.5 Points] DETAILS | PREVIOUS ANSWERS | SCALCET9M 14.7.043.MI.SA.

MY NOTES | ASK YOUR TEACHER | PRACTICE ANOTHER

This question has several parts that must be completed sequentially. If you skip a part of the question, you will not receive any points for the skipped part, and you will not be able to come back to the skipped part.

Tutorial Exercise

Find the shortest distance, d, from the point $(2, 0, -3)$ to the plane $x + y + z = 3$.

Step 1

The distance from $(2, 0, -3)$ to a point (x, y, z) is given by

$d = \sqrt{(x - 2)^2 + (y - 0)^2 + (z + 3)^2}$. ✓ (2) ✓ (0) ✓ (3)

Step 2

We will minimize $d^2 = (x - 2)^2 + y^2 + (z + 3)^2$. Since $z = 3 - x - y$, we can simplify this to

$d^2 = (x - 2)^2 + y^2 + (6 - x - y)^2$. ✓ ($6 - x - y$)

Step 3

For $f(x, y) = (x - 2)^2 + y^2 + (6 - x - y)^2$, we get

$f_x(x, y) = 4x + 2y - 16$ ✓ ($4x + 2y - 16$) and

$f_y(x, y) = 4y + 2x - 12$ ✓ ($2x + 4y - 12$).

Step 4

Solving $4x + 2y - 16 = 0$ and $2x + 4y - 12 = 0$ simultaneously gives

$x = 10/3$ ✓ (10/3)

$y = 4/3$ ✓ (4/3).

Recall that $z = 3 - x - y$, so for this x and y we have $z = -\frac{5}{3}$.

Step 5

Since the shortest possible distance must occur at this critical point $(x, y, z) = \left(\frac{10}{3}, \frac{4}{3}, -\frac{5}{3}\right)$ we can conclude that the minimal distance is

$$d = \sqrt{\left(\frac{10}{3} - 2\right)^2 + \left(\frac{4}{3}\right)^2 + \left(6 - \frac{10}{3} - \frac{4}{3}\right)^2} = \frac{4\sqrt{3}}{3}$$ ✓ ($\frac{4}{\sqrt{3}}$).

You have now completed the Master It.

Submit Answer

4. [12.5/12.5 Points] DETAILS | PREVIOUS ANSWERS | SCALCET9M 14.7.045.

MY NOTES | ASK YOUR TEACHER | PRACTICE ANOTHER

Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(8, 2, 0)$.

$(x, y, z) = ($ $4,1,-\sqrt{17}$ $)$ ✓ (smaller z-value)

$(x, y, z) = ($ $4,1,\sqrt{17}$ $)$ ✓ (larger z-value)

Need Help? Watch It

5. [12.5/12.5 Points] DETAILS | PREVIOUS ANSWERS | SCALCET9M 14.7.047.MI.SA.

MY NOTES | ASK YOUR TEACHER | PRACTICE ANOTHER

This question has several parts that must be completed sequentially. If you skip a part of the question, you will not receive any points for the skipped part, and you will not be able to come back to the skipped part.

Tutorial Exercise

Find three positive numbers whose sum is 340 and whose product is a maximum.

Step 1

Let the three numbers be x, y, and z. Then $x + y + z = 340$ ✓ (340).

This can be rewritten as $z = 340$ ✓ (340) $- x - y$.

Step 2

We wish to maximize the product xyz. Substituting $z = 340 - x - y$ gives us

$f(x, y) = 340xy - x^2y - xy^2$ ✓ ($340xy - x^2y - xy^2$).

Step 3

Taking the partial derivatives, we have $f_x(x, y) = 340y - 2xy - y^2$ ✓ ($340y - 2xy - y^2$) and

$f_y(x, y) = 340x - x^2 - 2xy$ ✓ ($340x - x^2 - 2xy$).

Step 4

$f_x = 0$ implies that $y = 0$ ✓ (0) or $y = -2$ ✓ (-2) $x +$ 340 ✓ (340).

Step 5

Substituting $y = 0$ into $f_y = 0$ gives us $x = 0$ and $x = 340$ ✓ (340).

Step 6

Substituting $y = 340 - 2x$ into $f_y = 0$ gives $3x^2 - 340$ ✓ (340) $x = 0$, and so $x = 0$ and

$x = 340/3$ ✓ (340/3).

Step 7

We therefore have the four critical points: $A(0, 0)$, $B(340, 0)$, $C(0, 340)$, and $D\left(\frac{340}{3}, \frac{340}{3}\right)$.

We now check the value of $f(x, y) = 340xy - x^2y - xy^2$ for each critical point to see where it is maximal. Recalling that $z = 340 - x - y$, we conclude that the three numbers are as follows. (Enter your answers as a comma-separated list.)

$\frac{340}{3}, \frac{340}{3}, \frac{340}{3}$ ✓ ($\frac{340}{3}, \frac{340}{3}, \frac{340}{3}$)

You have now completed the Master It.

6. [12.5/12.5 Points] DETAILS | PREVIOUS ANSWERS | SCALCET9M 14.7.050.

MY NOTES | ASK YOUR TEACHER | PRACTICE ANOTHER

Find the dimensions of the box with volume 2744 cm^3 that has minimal surface area. (Let x, y, and z be the dimensions of the box.)

$(x, y, z) = ($ 14,14,14 $)$ ✓

7. [12.5/12.5 Points] DETAILS | PREVIOUS ANSWERS | SCALCET9M 14.XP.7.003.

MY NOTES | ASK YOUR TEACHER | PRACTICE ANOTHER

Find the local maximum and minimum values and saddle point(s) of the function. If you have three-dimensional graphing software, graph the function with a domain and viewpoint that reveal all the important aspects of the function. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

$f(x, y) = x^4 + y^4 - 4xy + 3$

local maximum value(s) DNE ✓

local minimum value(s) 1,1 ✓

saddle point(s) $(x, y, f) =$ (0,0,3) ✓

Need Help? Watch It

8. [12.5/12.5 Points] DETAILS | PREVIOUS ANSWERS | SCALCET9M 14.7.051.

MY NOTES | ASK YOUR TEACHER | PRACTICE ANOTHER

Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 15$.

125/6 ✓

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