1. Prove (by giving a formal proof) or disprove (by giving a counter example) the following statement: the Manhattan distance $df(x,y) =: \Sigma^k_{i=1} |x_i - y_i|$ is metric.

   Answer: It is a metric distance function. A proof is given below.

   (a) (reflexivity) $df(x,x) = \Sigma^k_{i=1} |x_i - x_i| = 0$.

   (b) (symmetry) $df(x,y) = \Sigma^k_{i=1} |x_i - y_i| = \Sigma^k_{i=1} |y_i - x_i| = df(y,x)$.

   (c) (triangle inequality) for any 3 objects x, y, z, we need to show that $df(x,z) \le df(x,y) + df(y,z)$.
   $df(x,z) = \Sigma^k_{i=1} |x_i - z_i| = \Sigma^k_{i=1} |x_i - y_i + y_i - z_i| = \Sigma^k_{i=1} |(x_i - y_i) + (y_i - z_i)|$
   Using the inequality that for any real numbers a, b, we have $|a + b| \le |a| + |b|$,
   $df(x,z) = \Sigma^k_{i=1} |(x_i - y_i) + (y_i - z_i)| \le \Sigma^k_{i=1} |(x_i - y_i)| + \Sigma^k_{i=1} |(y_i - z_i)| = df(x,y) + df(y,z)$.
   QED.

2. Prove (by giving a formal proof) or disprove (by giving a counter example) the following statement: the relative entropy distance (Kullback-Liebier divergence) $df(x,y) =: \Sigma^k_{i=1} x_i \ln (x_i / y_i)$ is metric.

   Answer: It is not metric because it is not symmetric. Consider the following counter example. Suppose k = 1, x = e and y = 1.
   Then $df(x,y) = e \ln (e/1) = e$. But $df(y,x) = 1 \ln (1/e) = -1$.
   (We use the straight definition with ln being the natural log. The same example works for logarithms of any base.)

3. Since relative entropy is not symmetric, is the following extension symmetric: $df(x,y) =: \Sigma^k_{i=1} x_i \ln (x_i / y_i) + y_i \ln (y_i / x_i)$?

   Answer: Yes.