General Information

Prof. Antonella Ferrara https://scholar.google.com/citations?user=r5JuMskAAAAJ&hl=en

Course Teaching Material: KIRO UNIPV https://elearning.unipv.it/

- 504462 - PROCESS CONTROL 2025-26 - PROF.SSA FERRARA ANTONELLA

Lecture Time-table: http://www-3.unipv.it/ingserv/orario2526/1sem/insegnamenti/ProCont.html

Exams: https://studentionline.unipv.it/esse3/Home.do https://kirotesting.unipv.it/

Introduction

- Program of the course:

Advanced SISO control schemes:
Pre-filters and parallel compensators, two degrees of freedom control schemes, compensation of measurable disturbances, systems with delays and Smith Predictor, Padé approximation, decoupling in the frequency domain, control of open loop unstable systems.

Advanced MIMO control schemes:
Decoupling based control schemes, decentralized control, relative gain array.

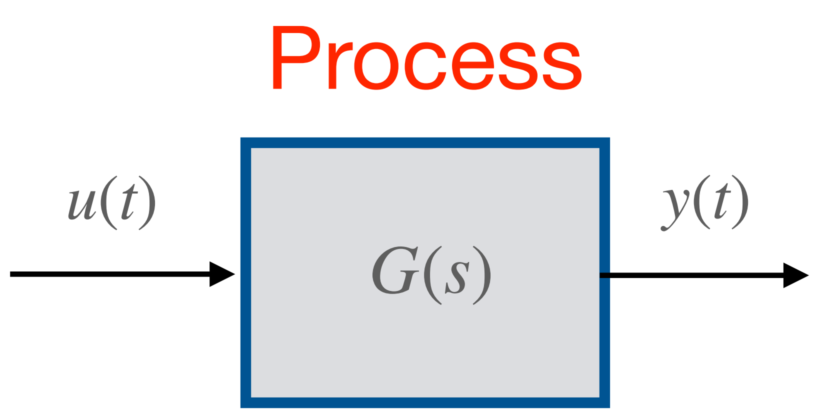
PID controllers:
Features and properties. Rules for the empirical calibration. Wind-up and anti wind-up schemes.

Digital control:
Discrete-time systems. The concept of equilibrium for discrete-time systems. Stability of linear time-invariant discrete-time systems. Jury test. Digital control schemes. Zeta transform and its properties. Transfer functions in the z domain. Sampling and aliasing. Choice of the sampling time. Zero-order-Hold. Discretization of continuous-time controllers. Bilinear transformation, Euler and Tustin methods.

Introduction

- Some of the figures in these slides, kindly provided by McGraw-Hill, are those of the Textbook:

Control of MIMO Systems

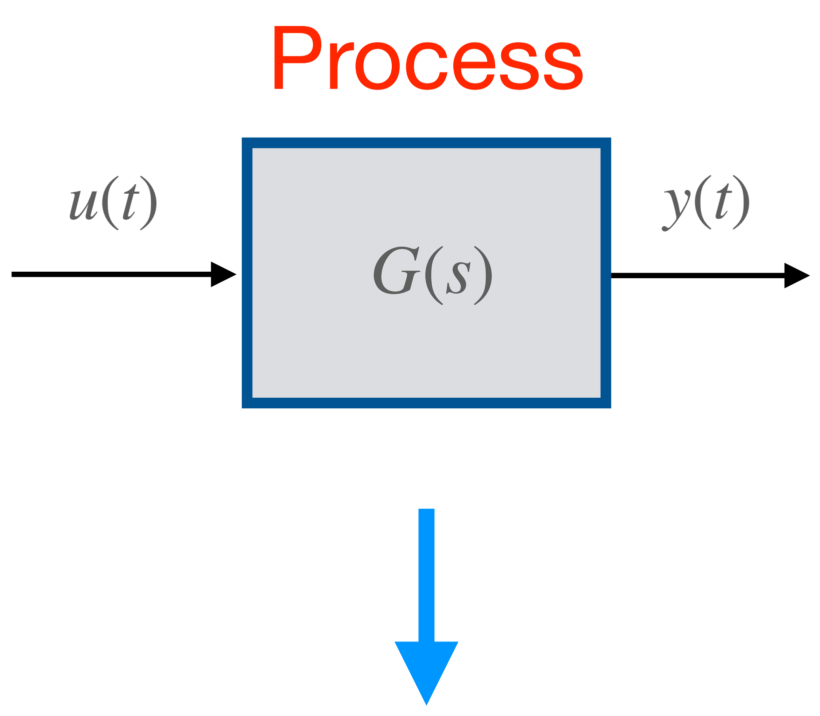


$$u(t) \in \mathbb{R}^m$$

$$y(t) \in \mathbb{R}^m$$

$$G(s) \in \mathbb{R}^{m \times m}$$

Control of MIMO Systems



$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1m}(s) \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}(s) & G_{m2}(s) & \cdots & G_{mm}(s) \end{bmatrix}$$

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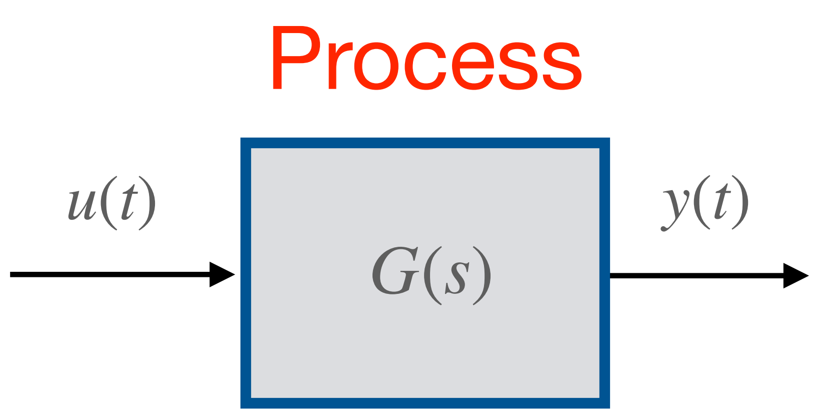

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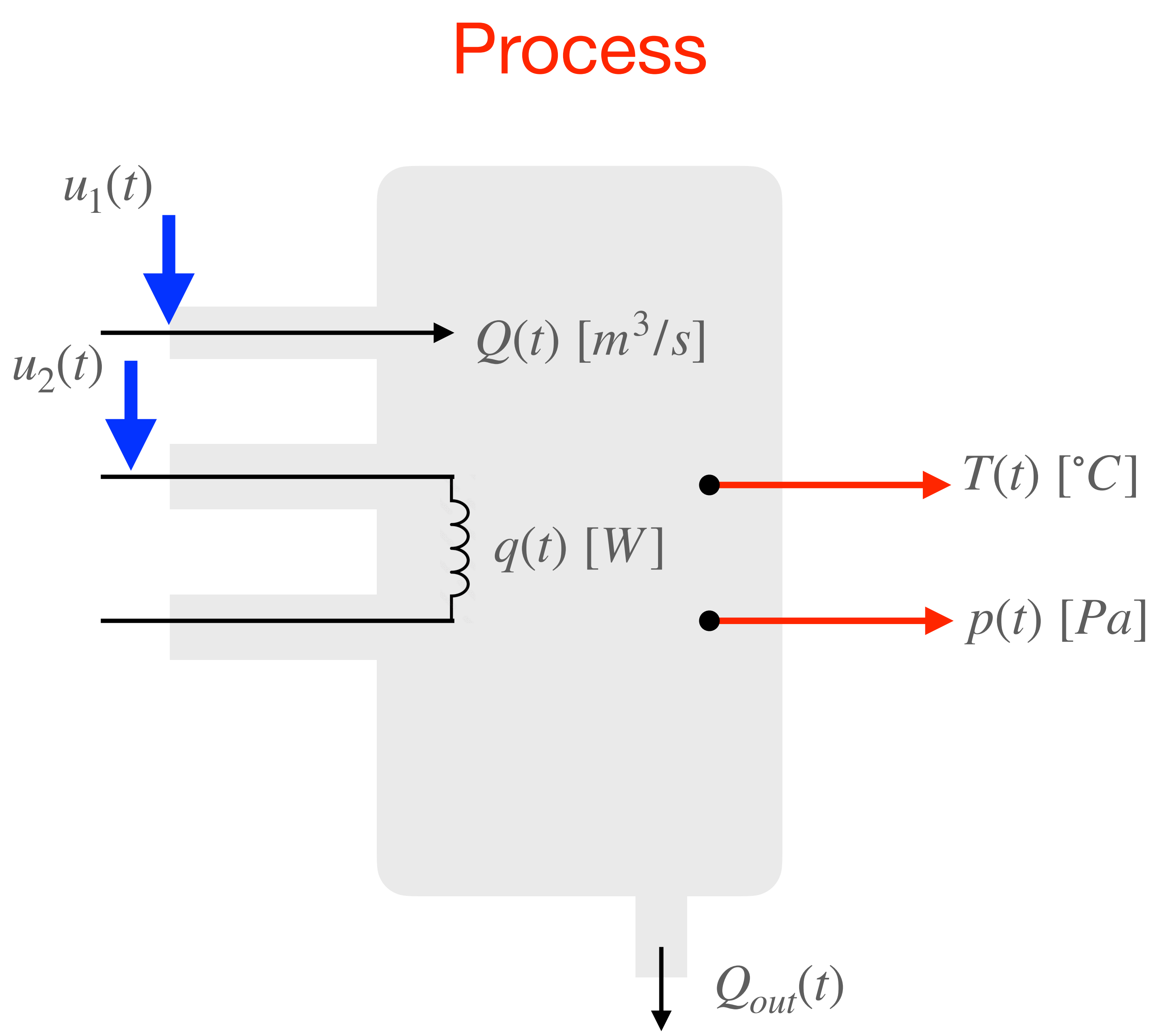
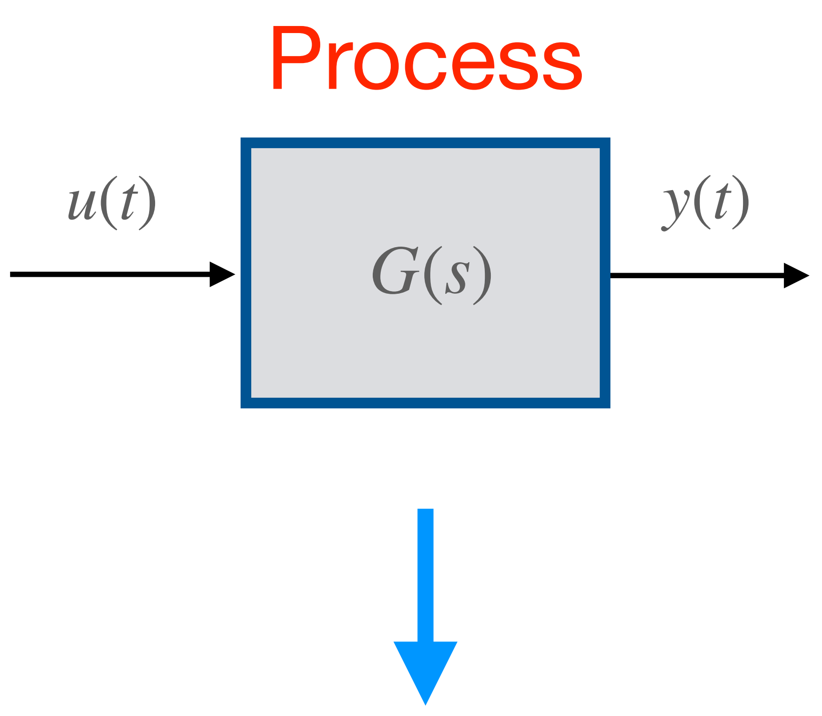


Thermal fluid heating system

Control of MIMO Systems



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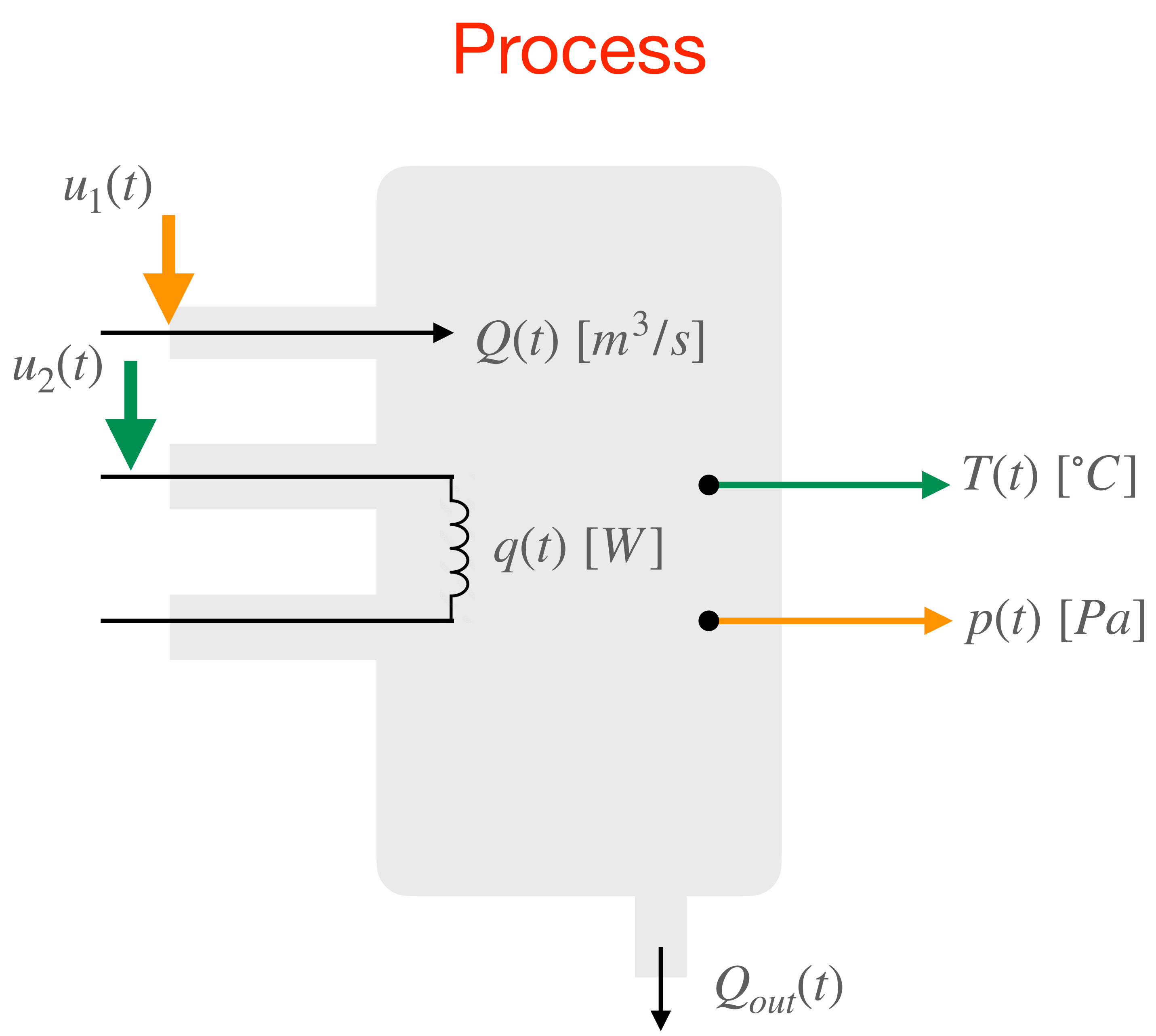
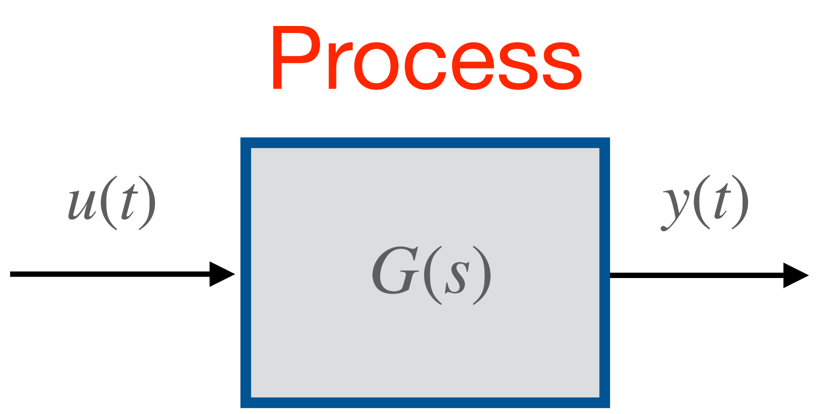


Thermal fluid heating system

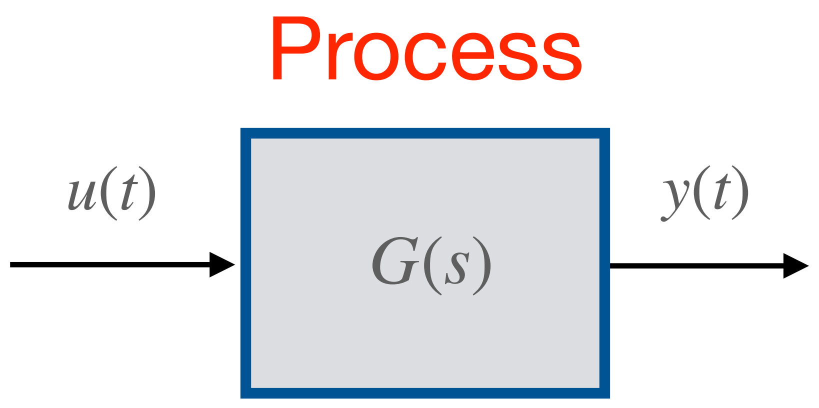
Decoupling Based Control Schemes



Assumptions:

- $G(s) \in \mathbb{R}^{2\times2}$

Decoupling Based Control Schemes



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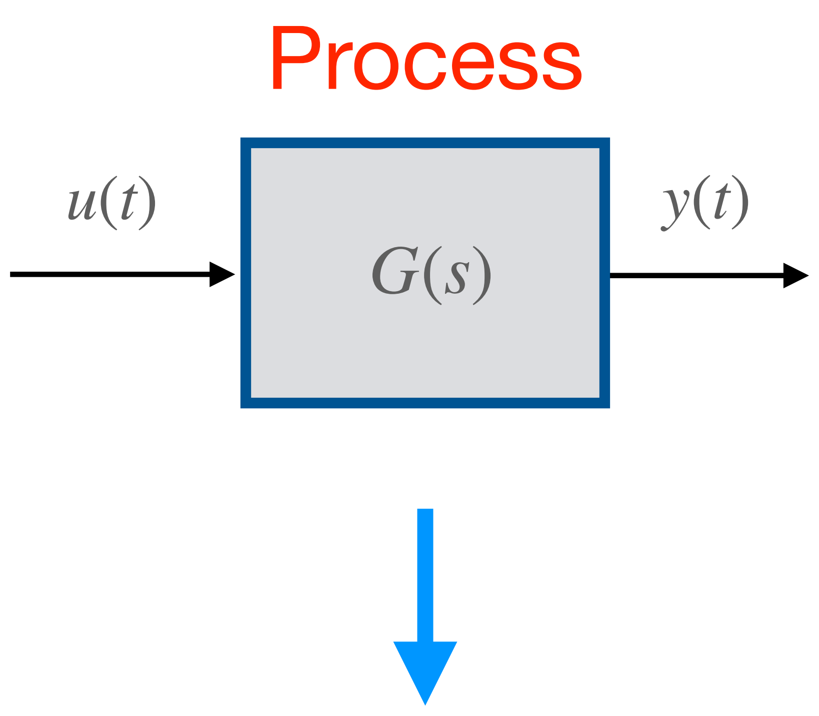

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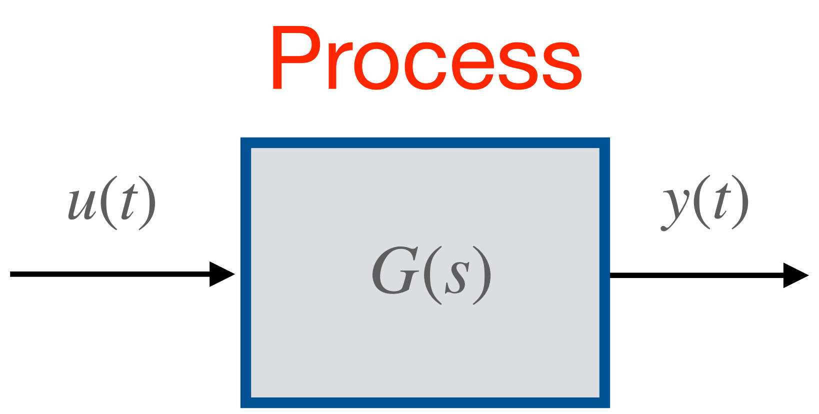
- $G(s)$ triangular matrix

Decoupling Based Control Schemes



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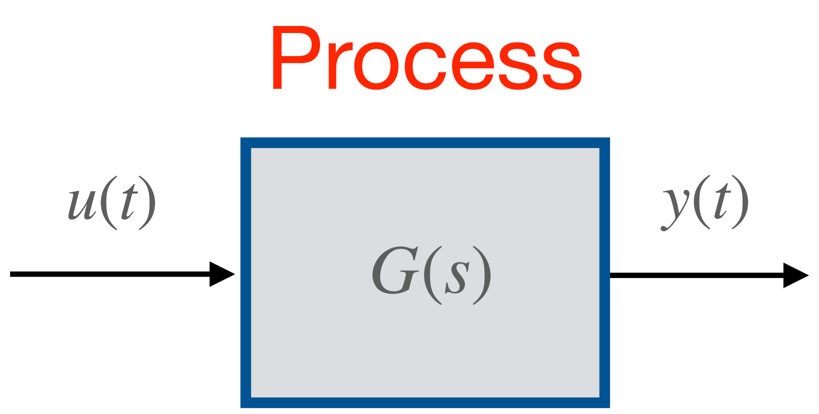

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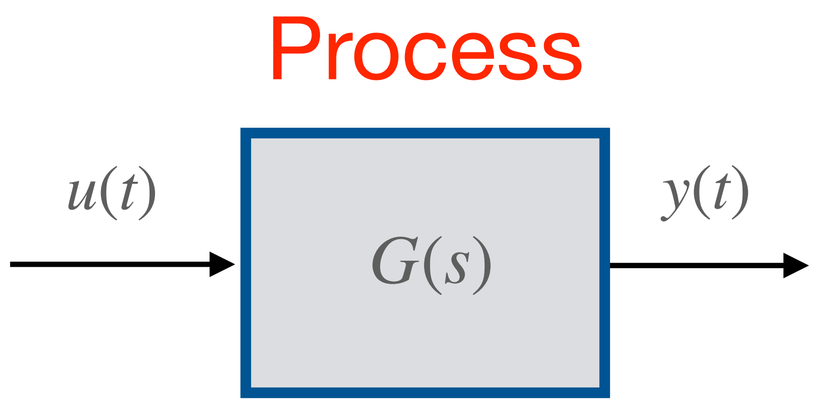

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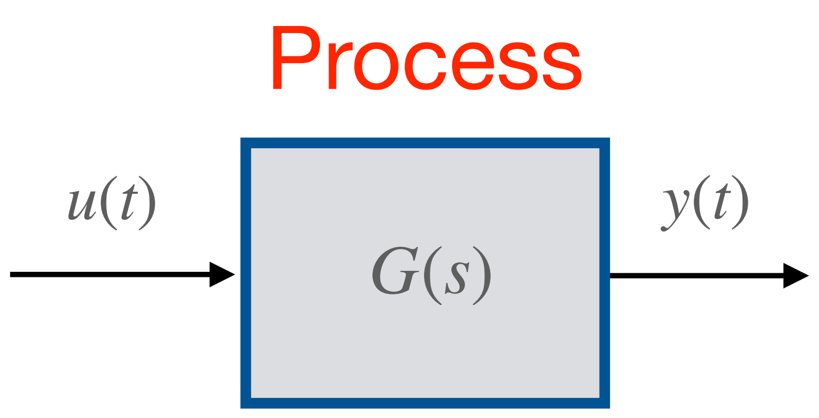

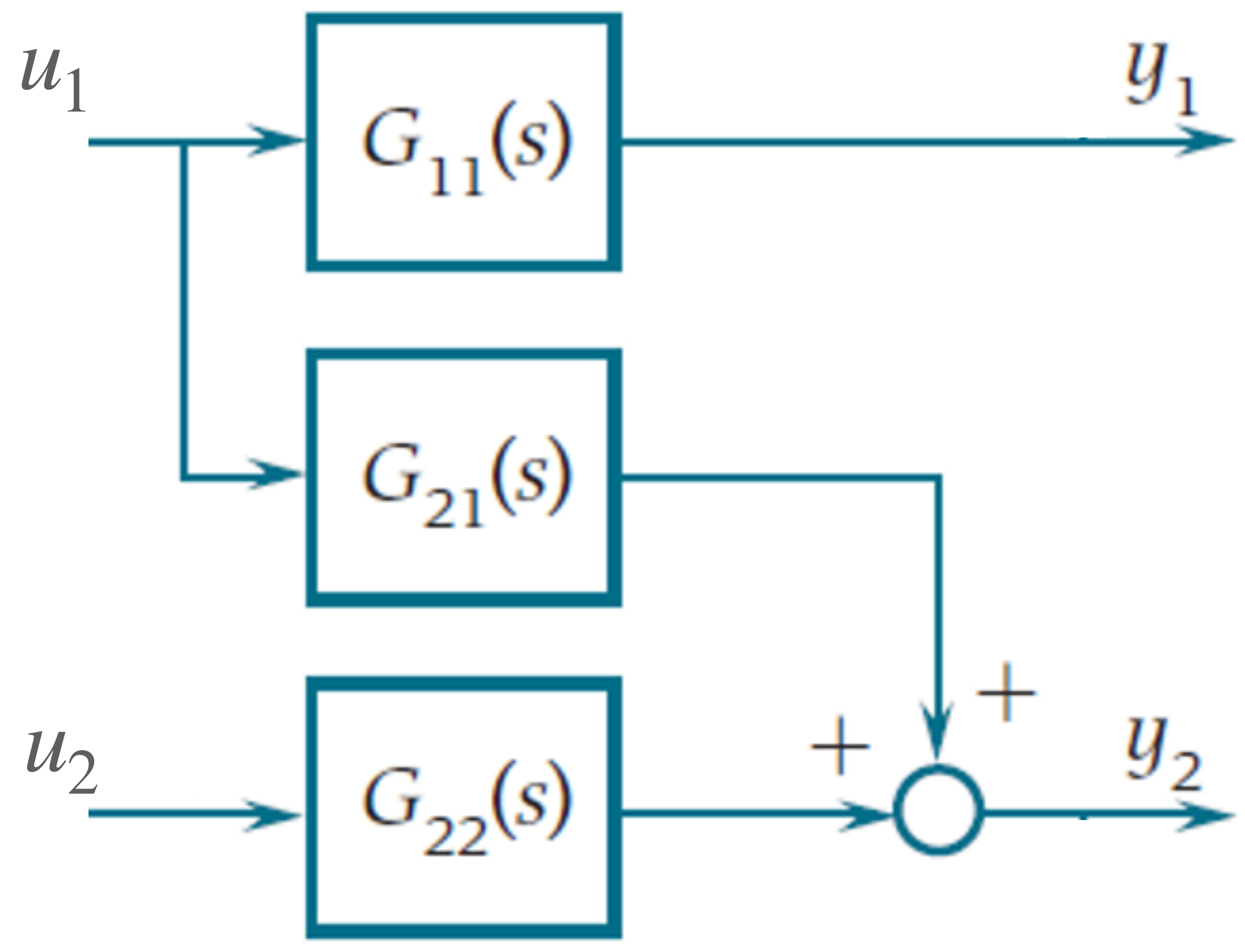

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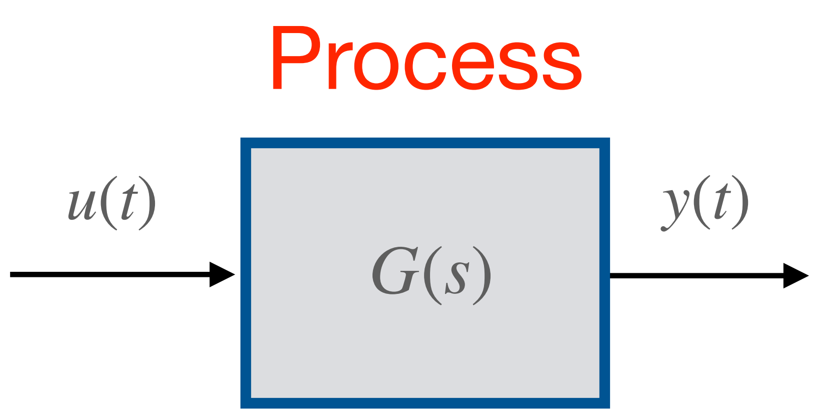

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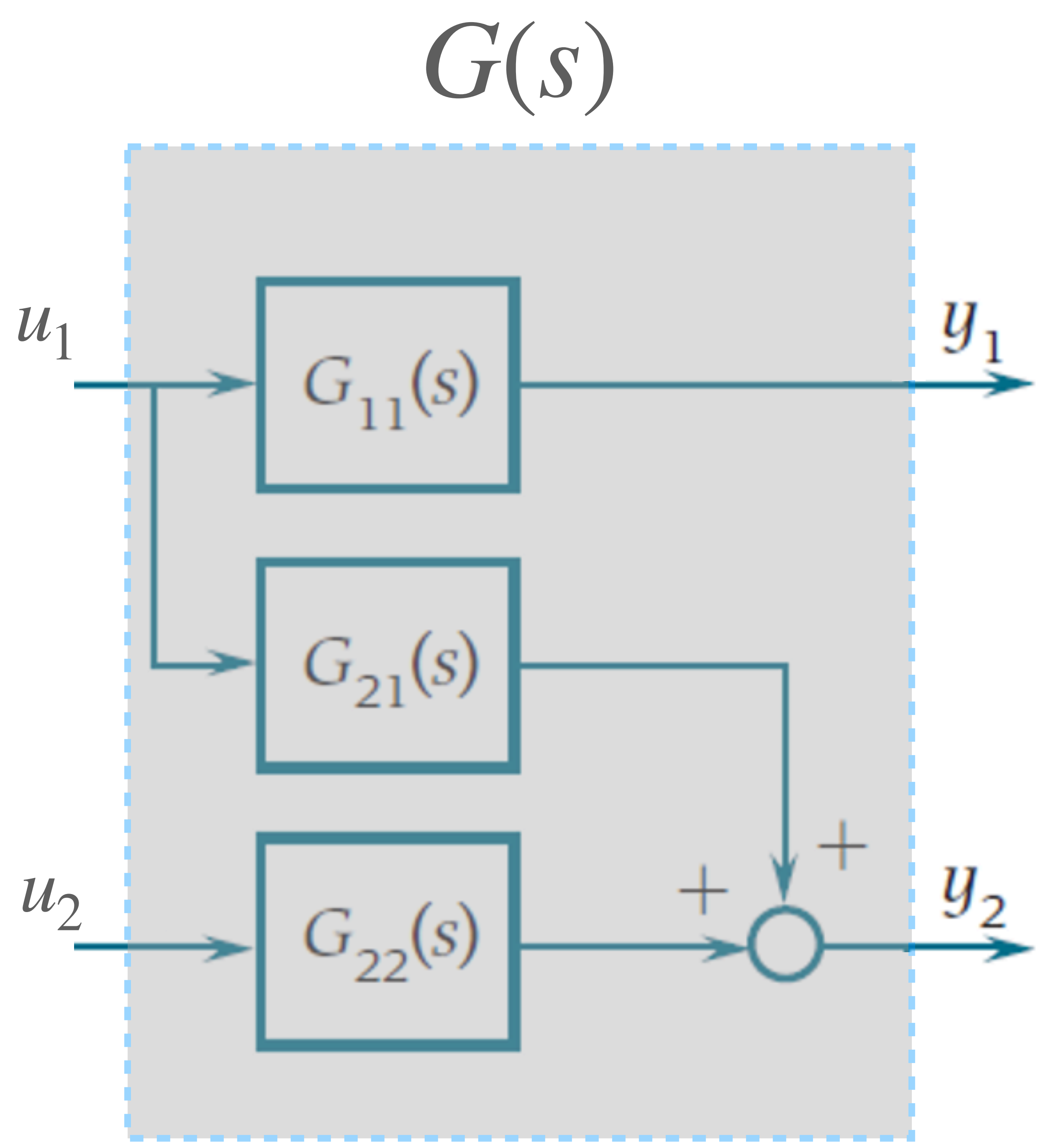

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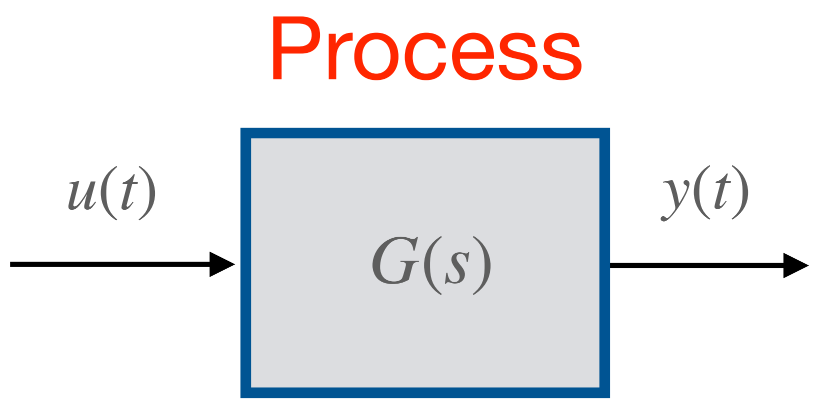

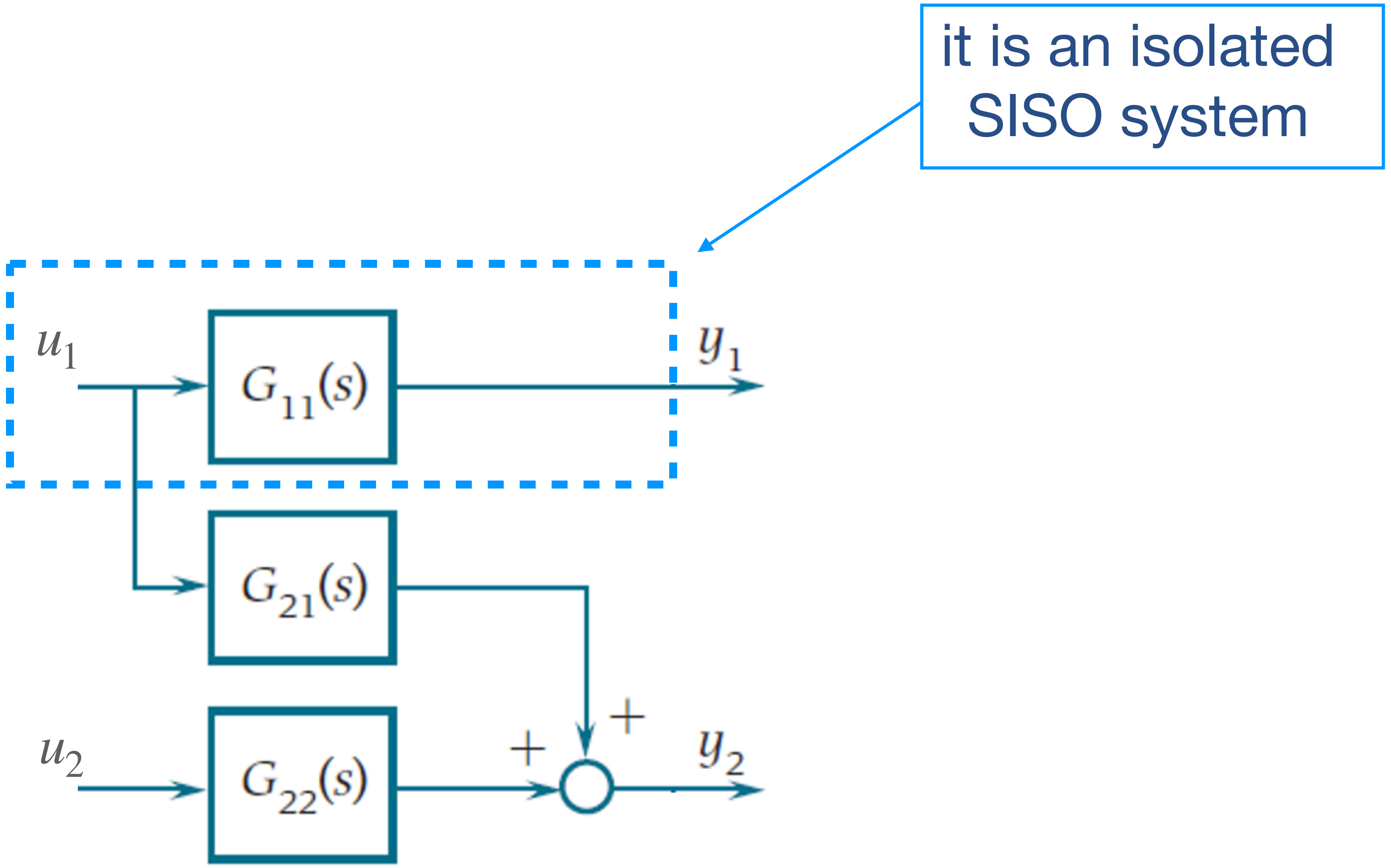

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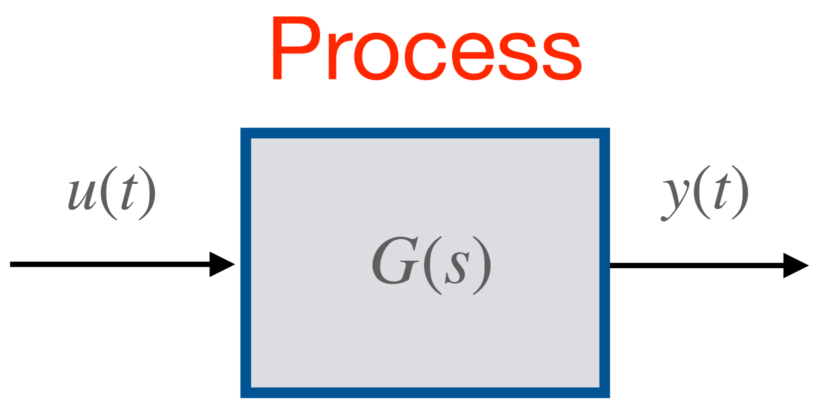

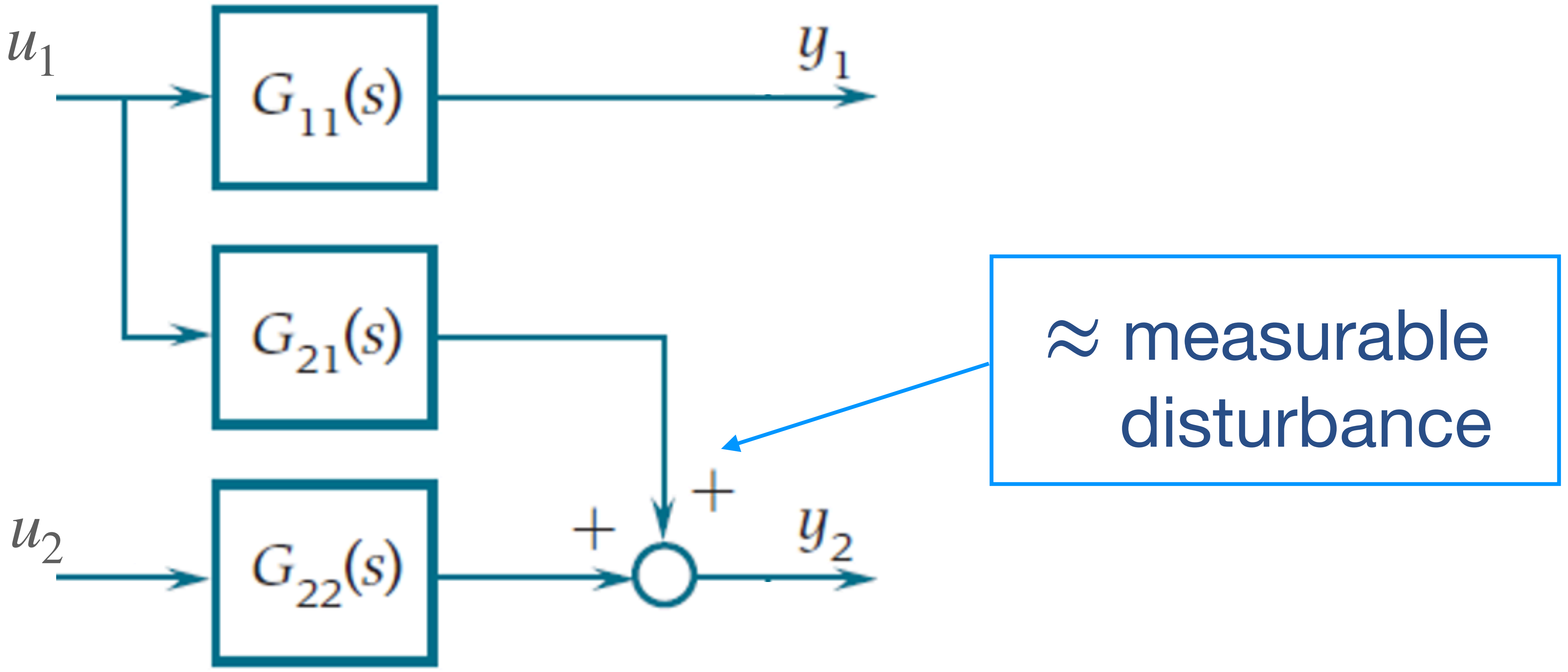

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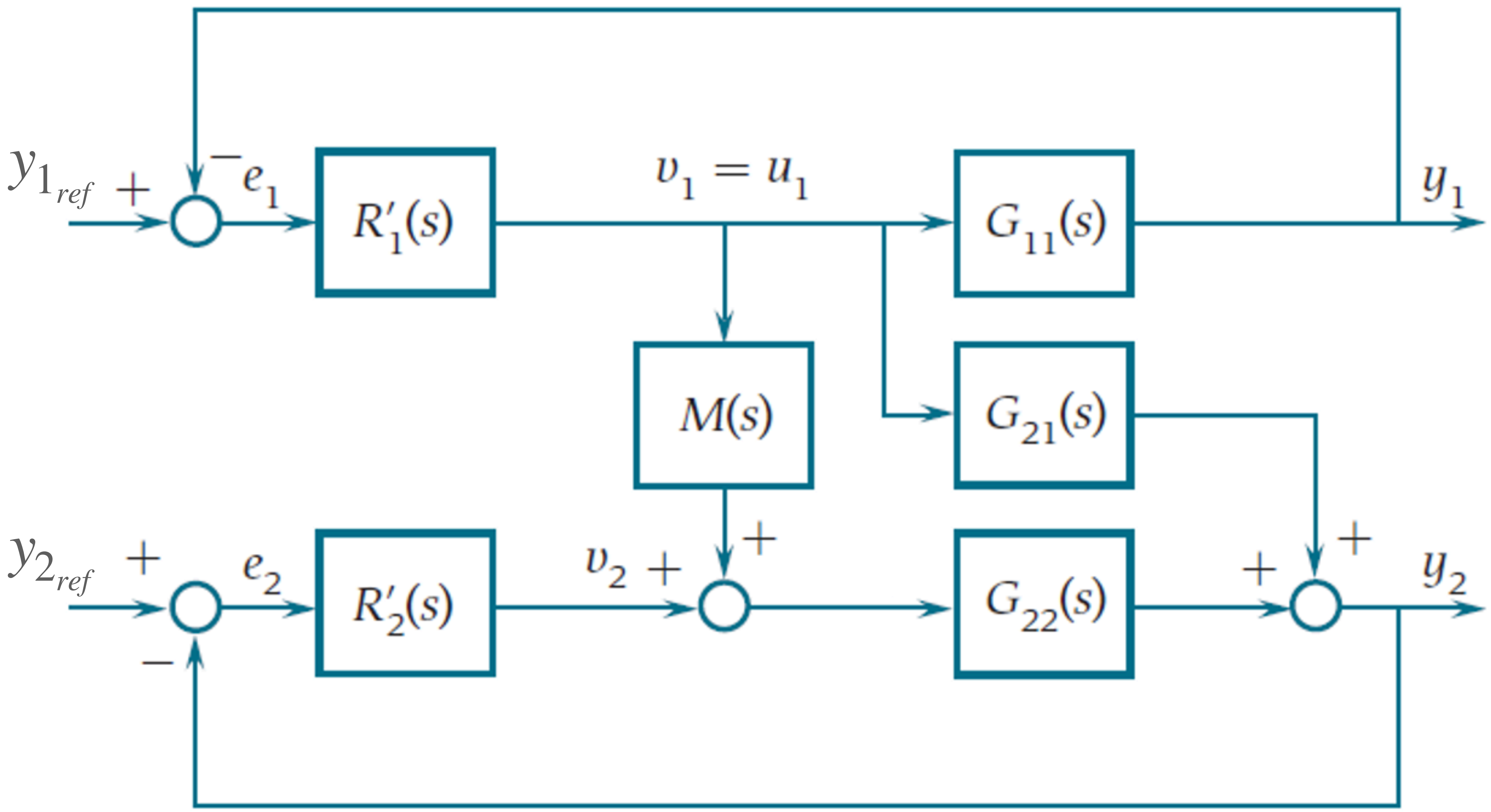
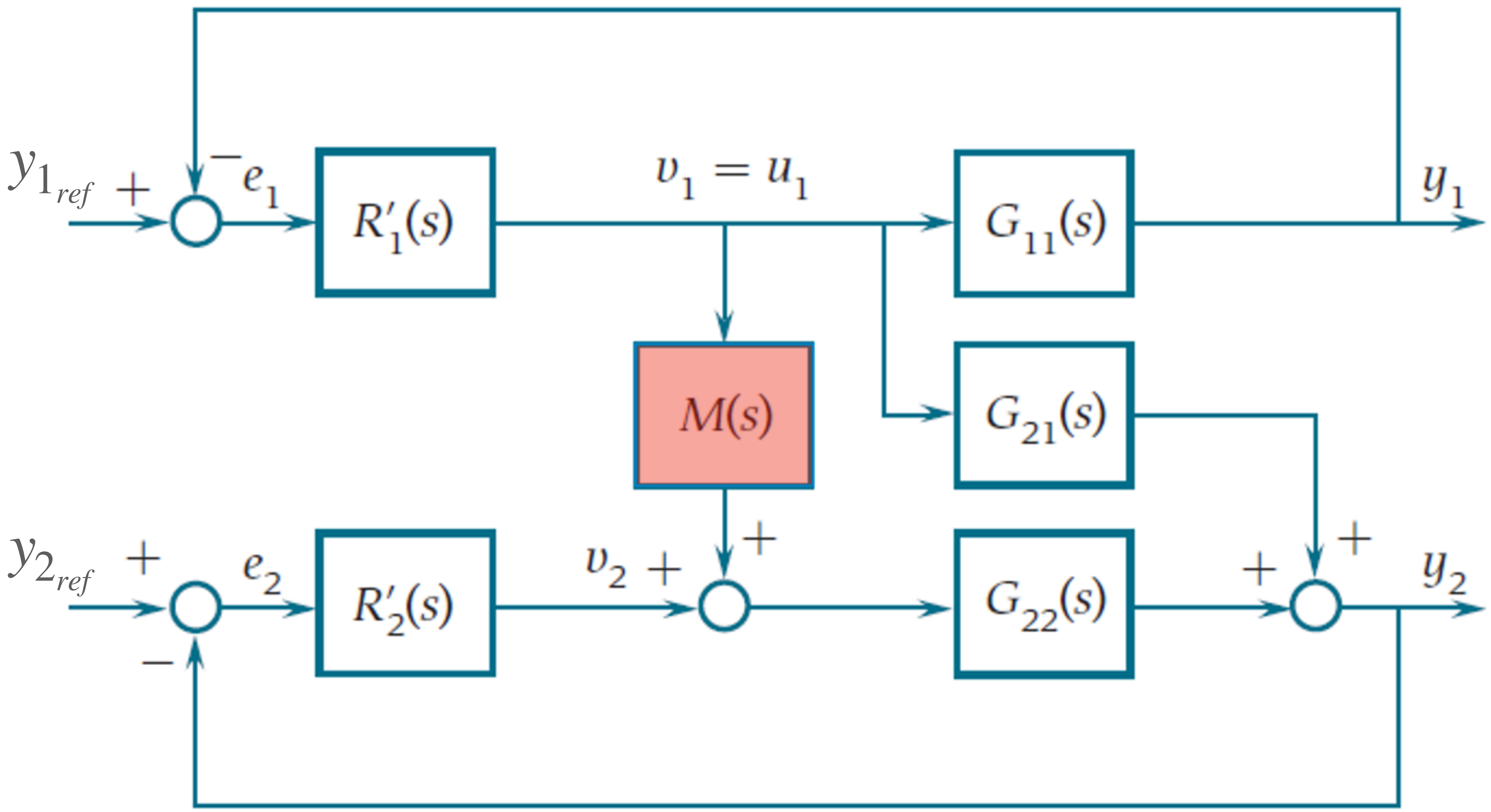


Decoupling Based Control Schemes



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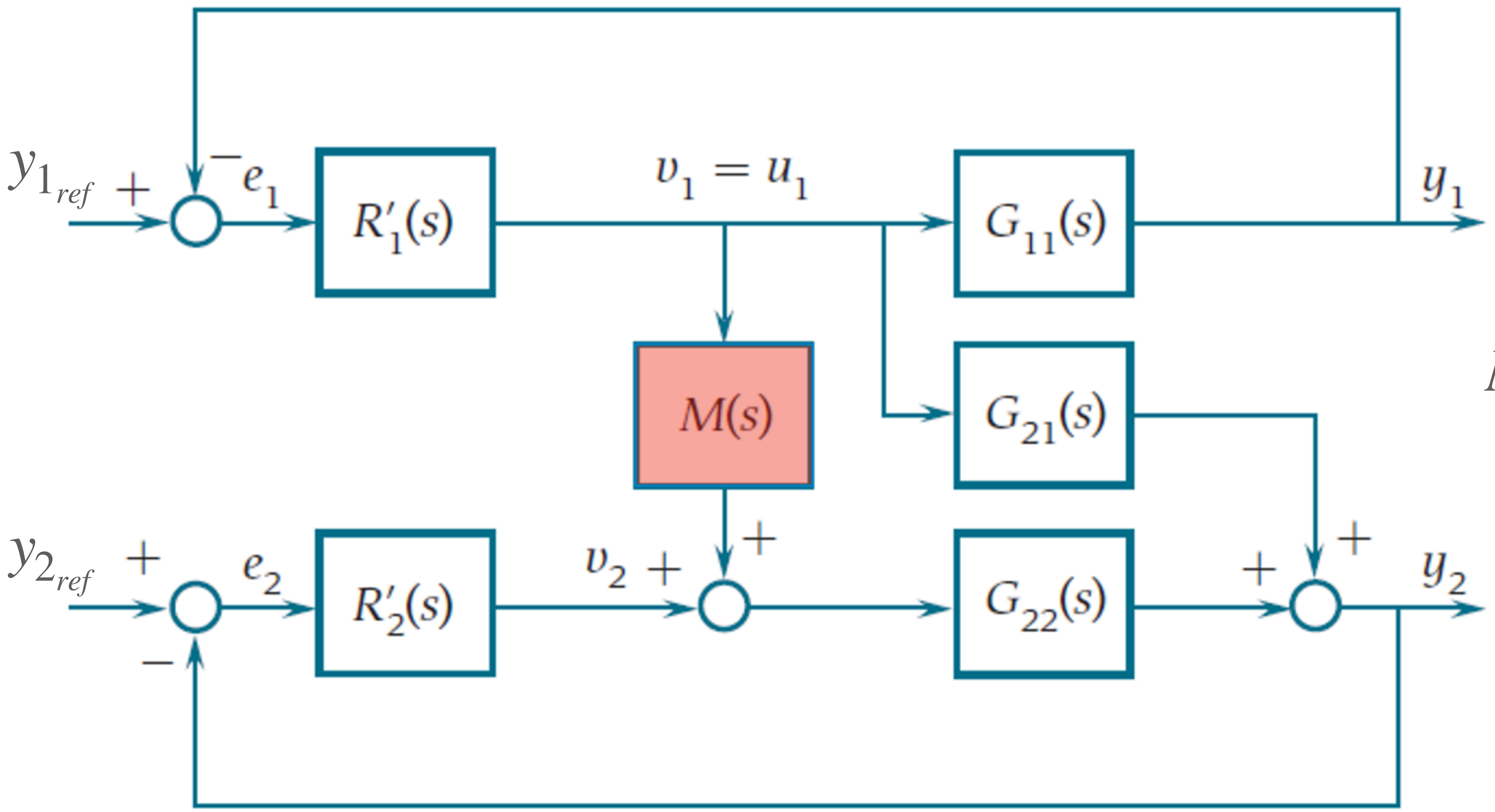
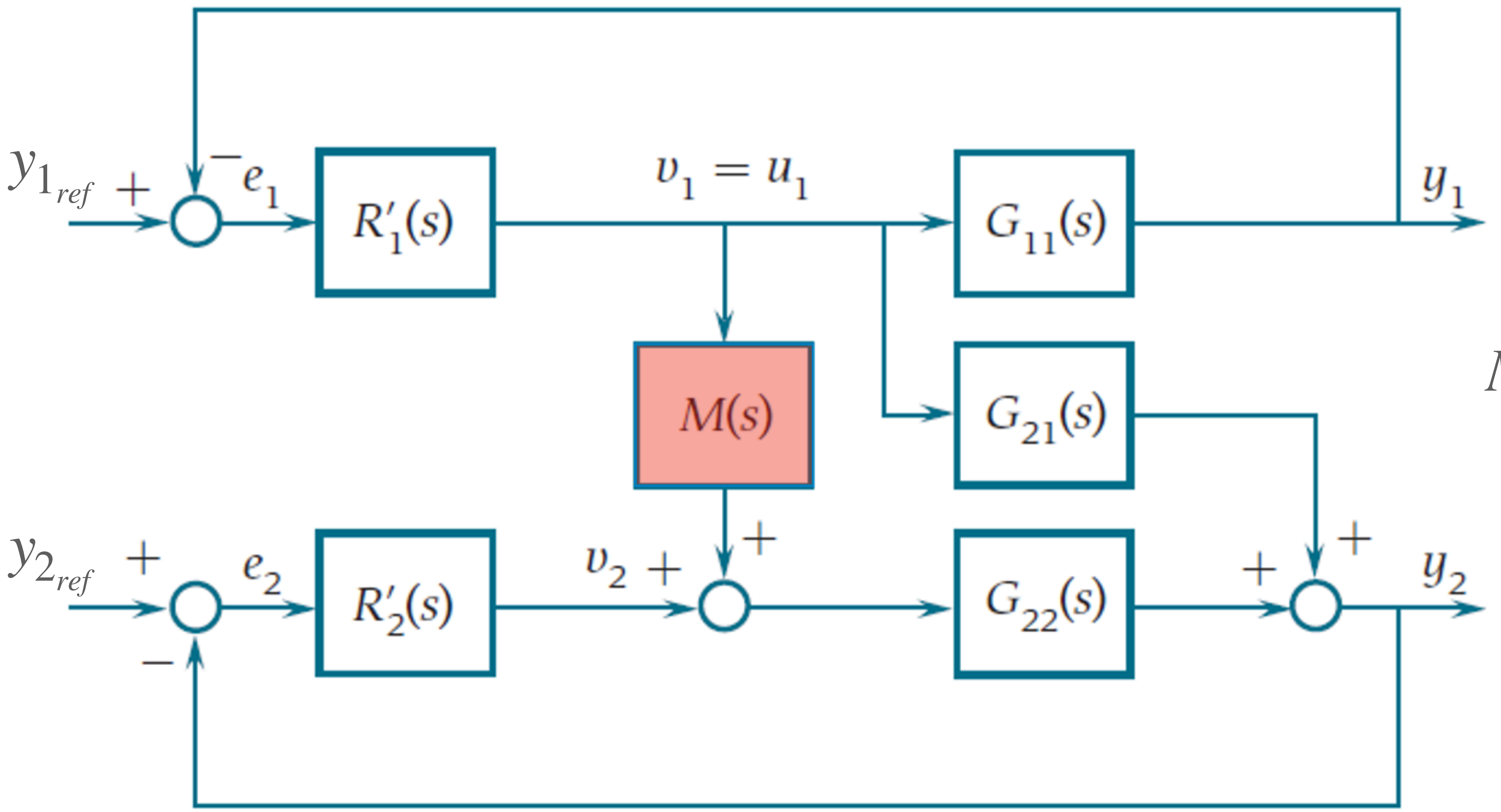
Decoupling Based Control Schemes



$$M(s)G_{22}(s) + G_{21}(s) = 0$$

Decoupling Based Control Schemes



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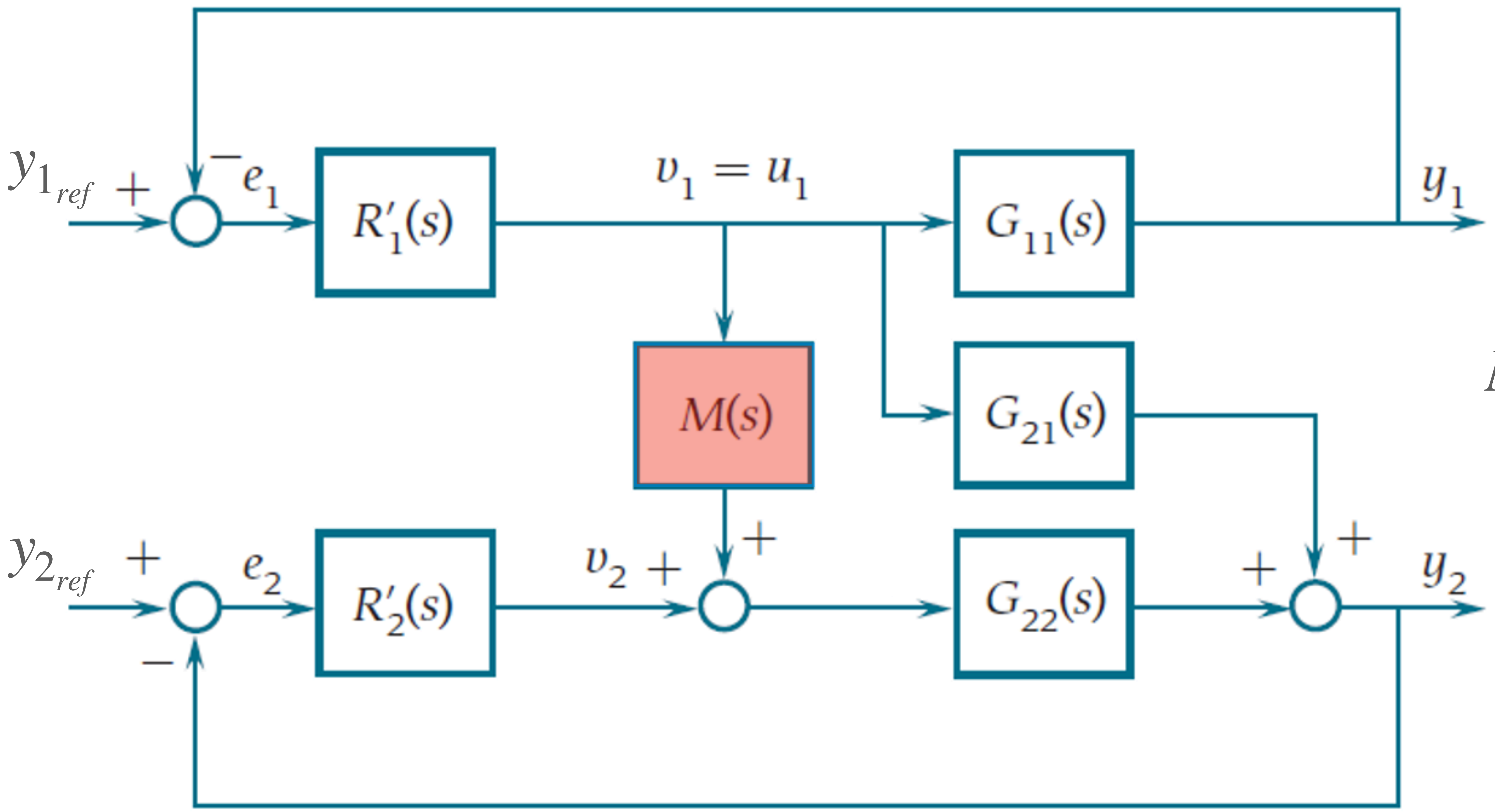
$$\boxed{M(s) = -\frac{G_{21}(s)}{G_{22}(s)}}$$

Decoupling Based Control Schemes



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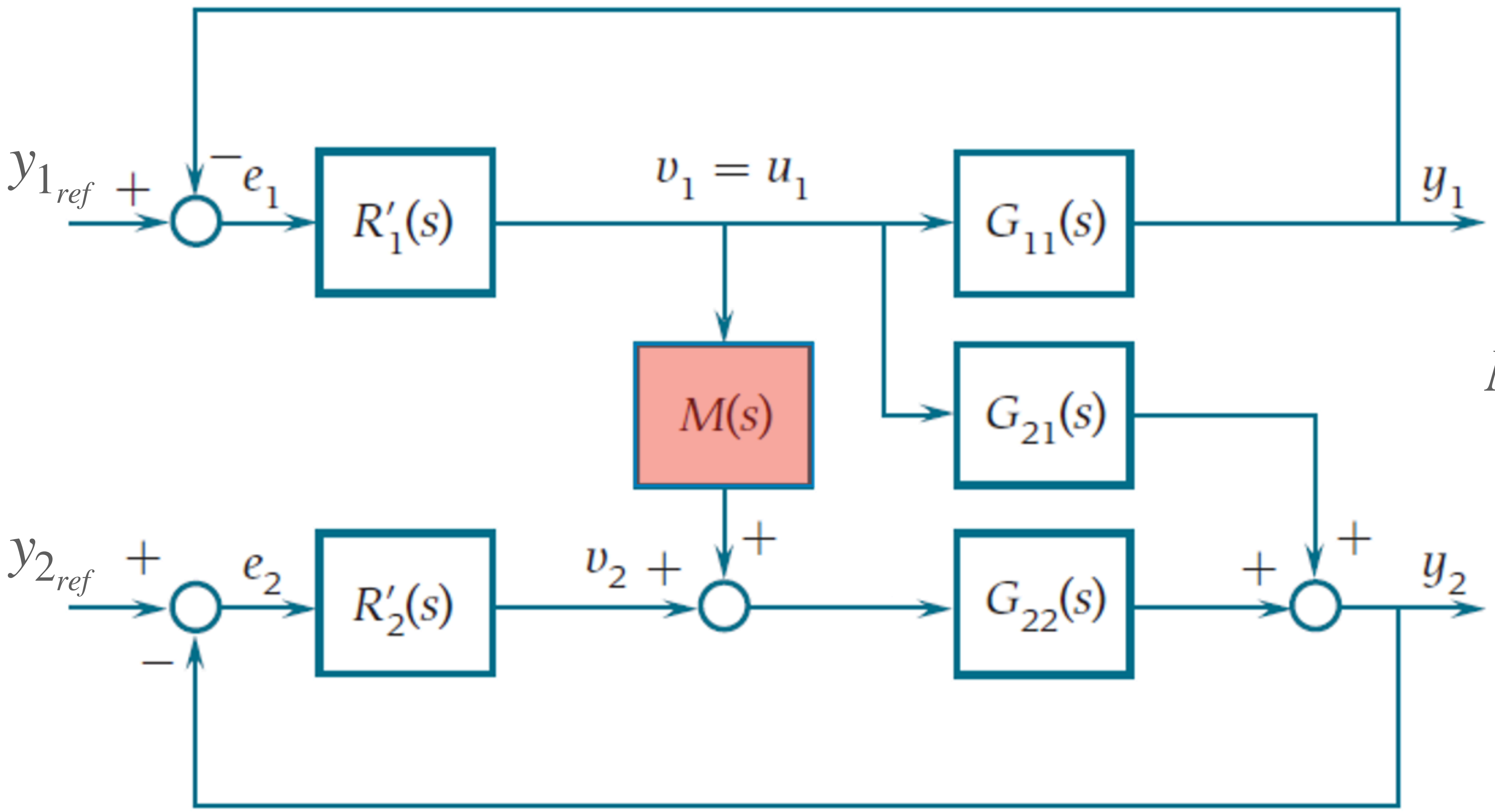
or an approximation of it
in the frequency domain
if the system is not causal

Decoupling Based Control Schemes



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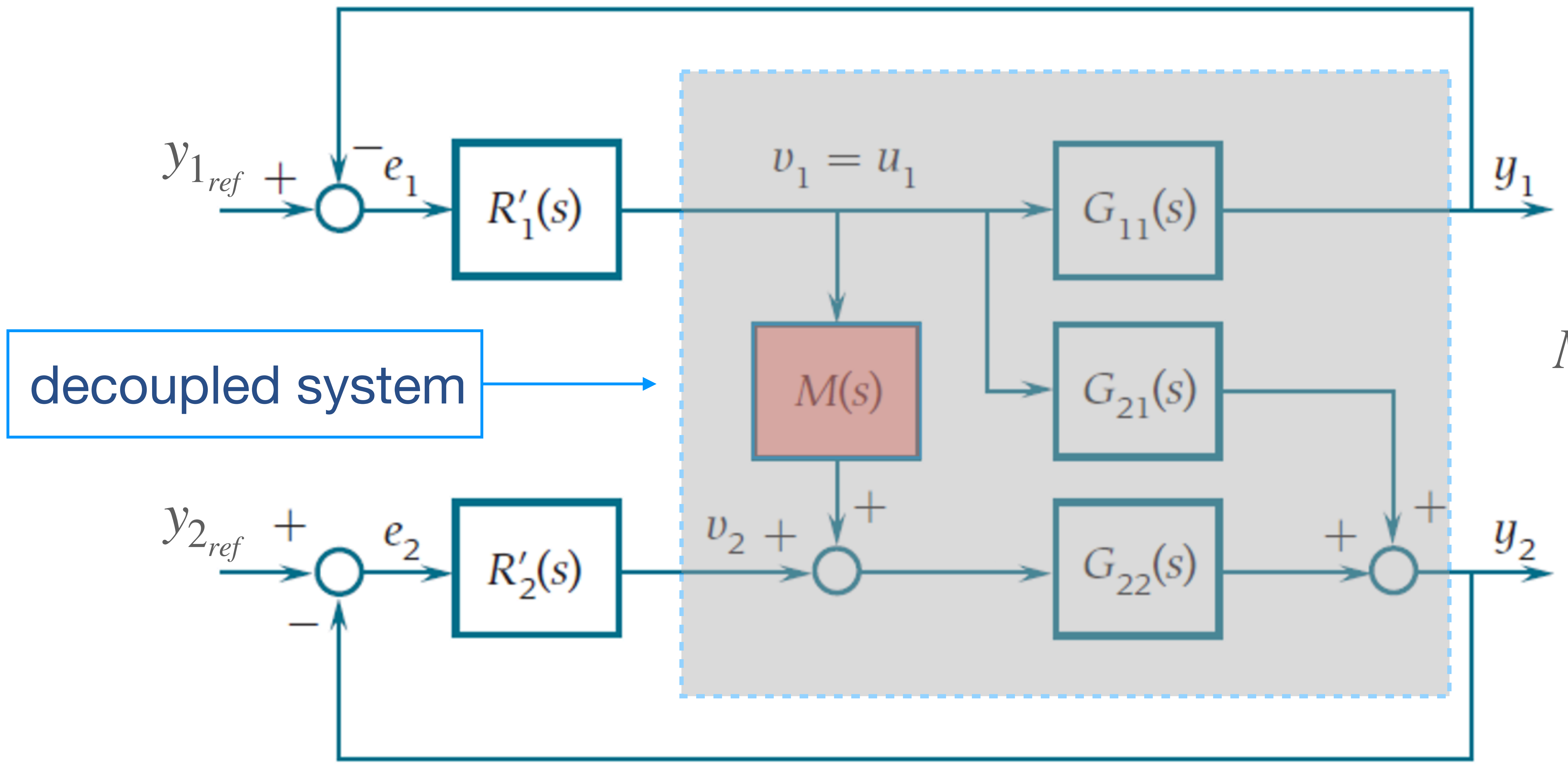
$M(s)$ must be as. stable: some assumptions on $G_{ij}(s)$ are needed

Decoupling Based Control Schemes



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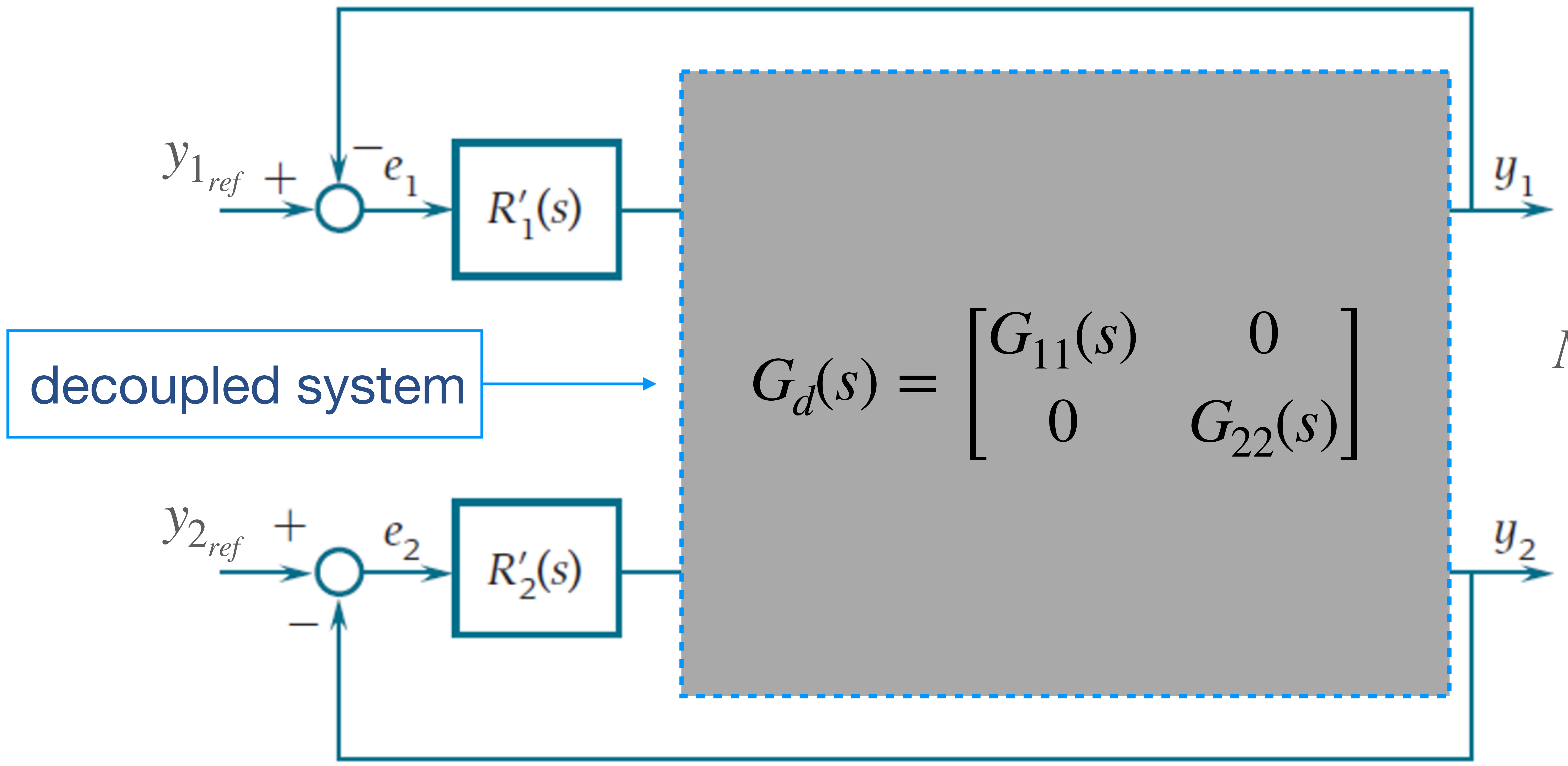

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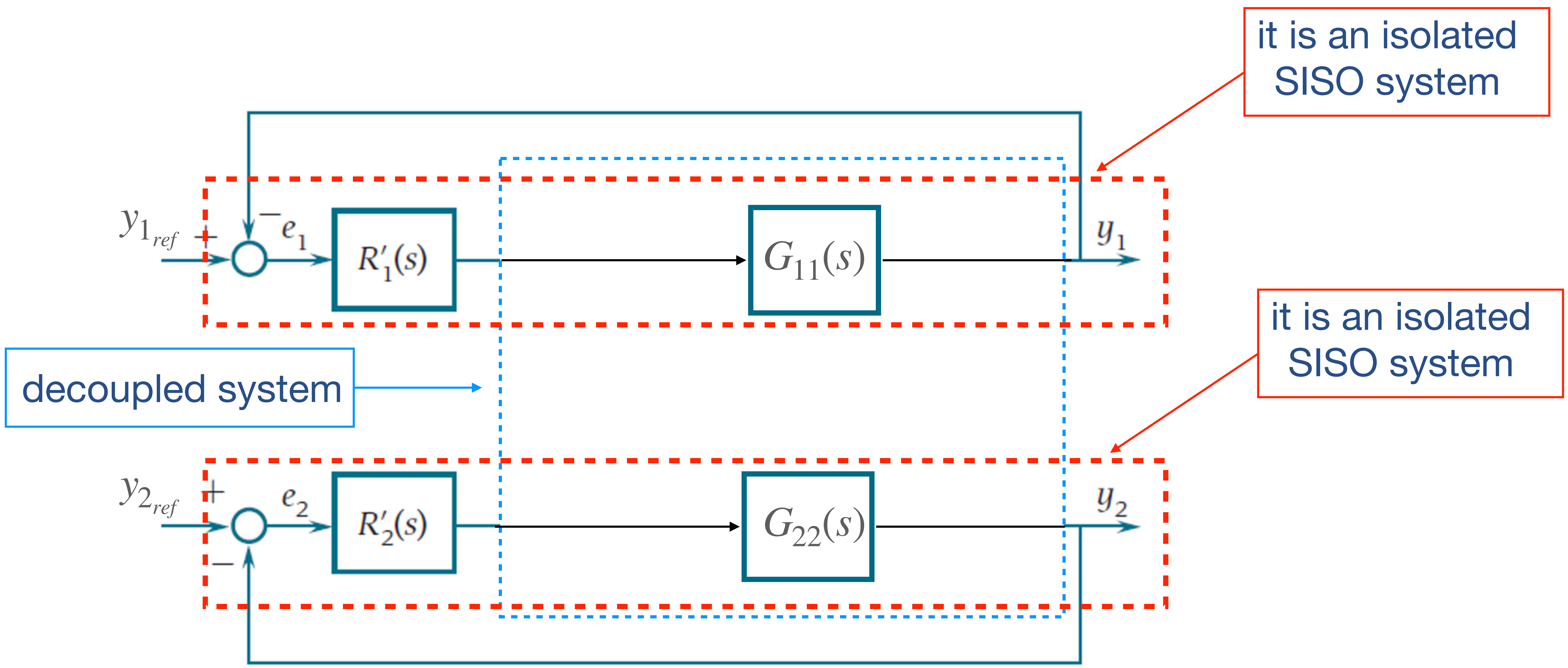
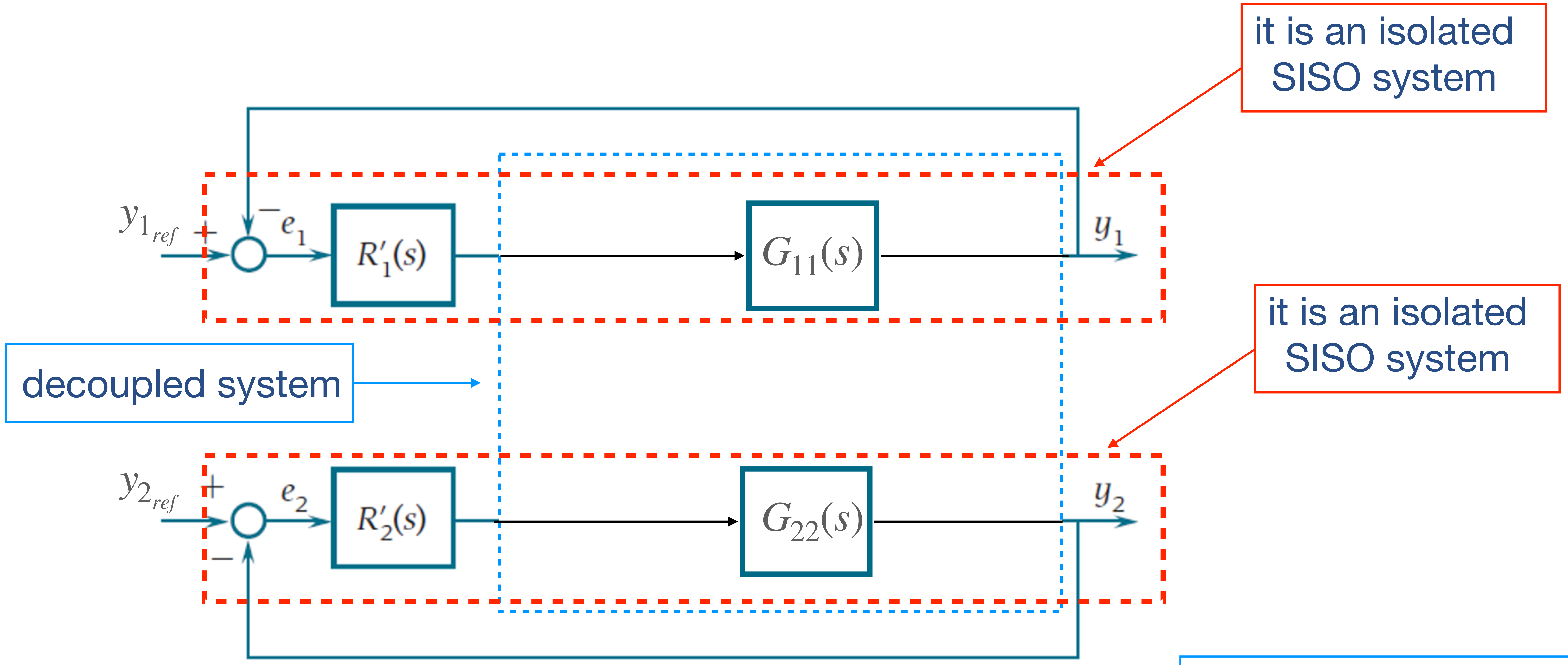


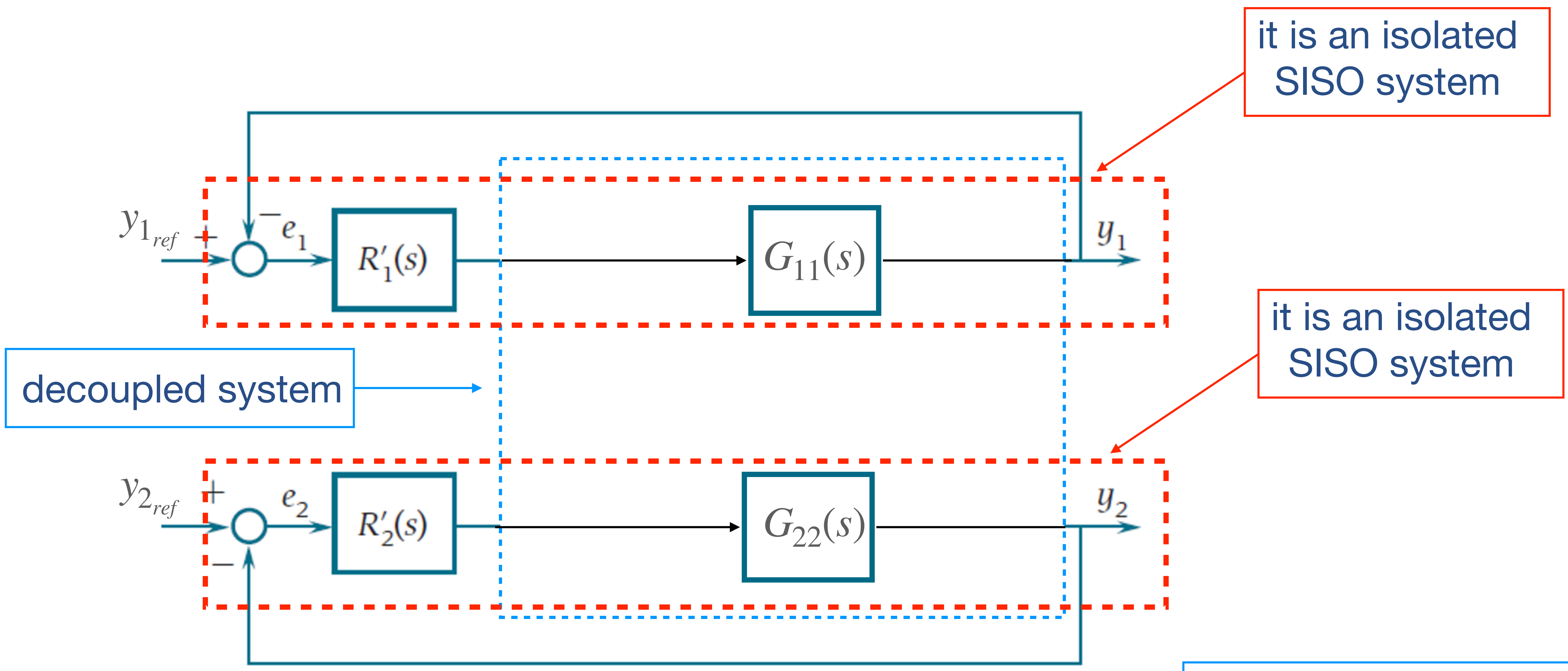
Decoupling Based Control Schemes



Homework:

Find $M(s)$ when $G(s)$ is upper-triangular

Decoupling Based Control Schemes



Draw the MIMO control scheme

Homework:

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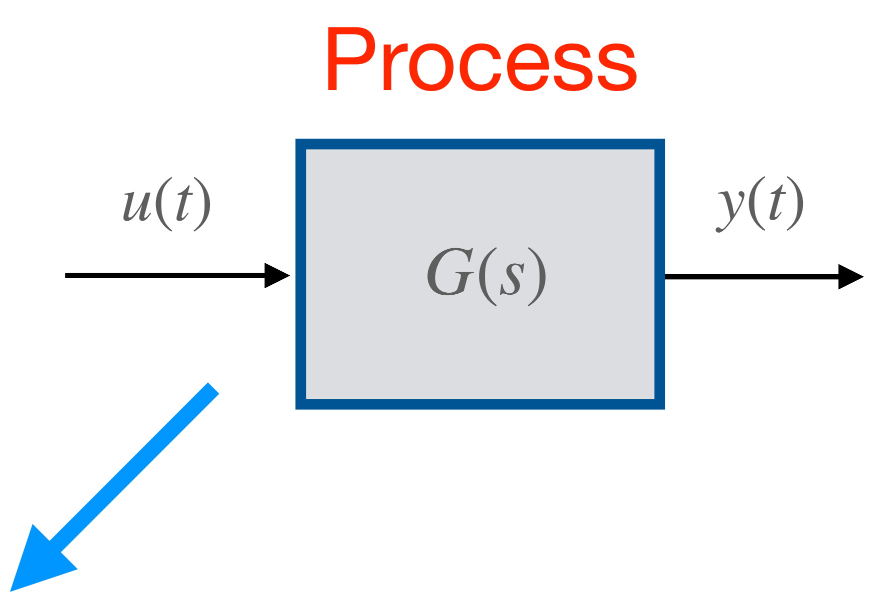

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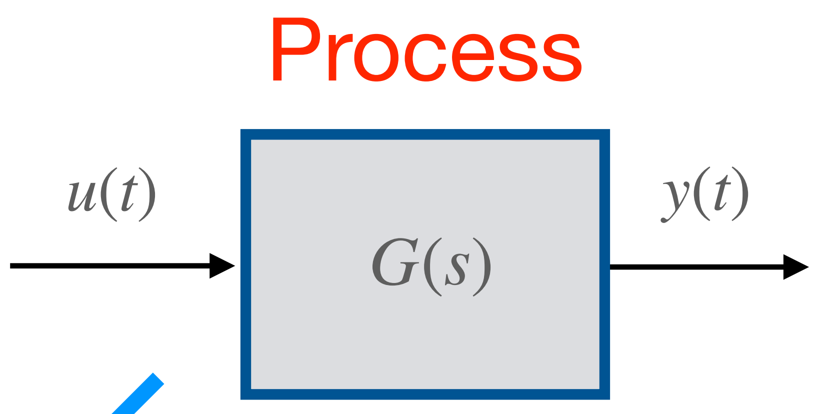
Decoupling Based Control Schemes



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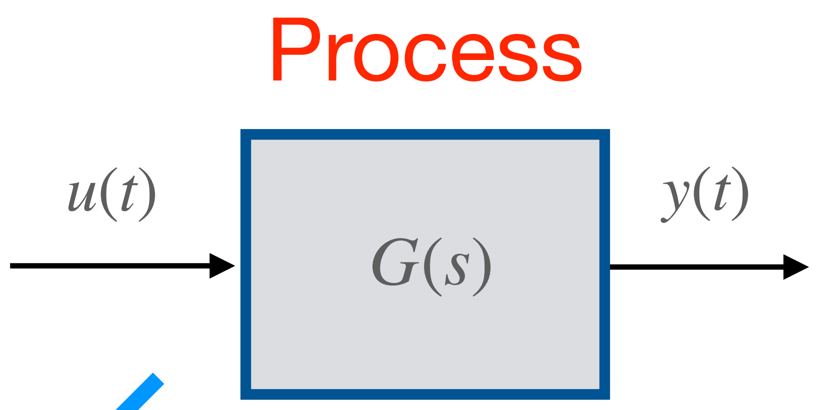
$\downarrow$

Sufficient conditions:

- $G(s)$ rational function
- $G(s)$ As. Stable
- $\det G(s) \neq 0 \ \forall\, s$ s.t. $\mathrm{Re}(s) \geq 0$

Decoupling Based Control Schemes



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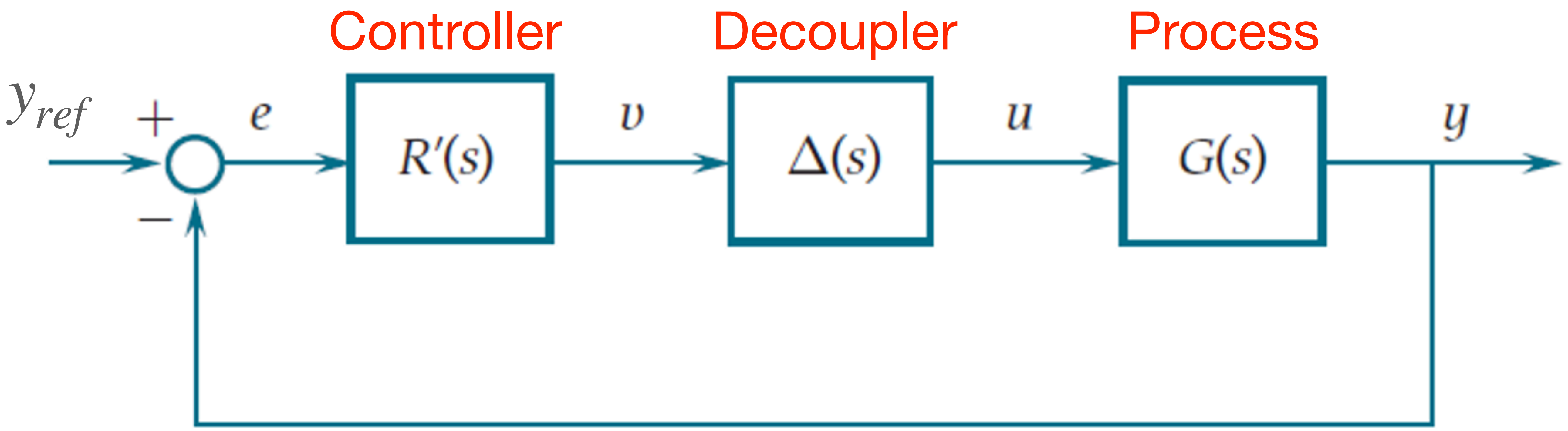

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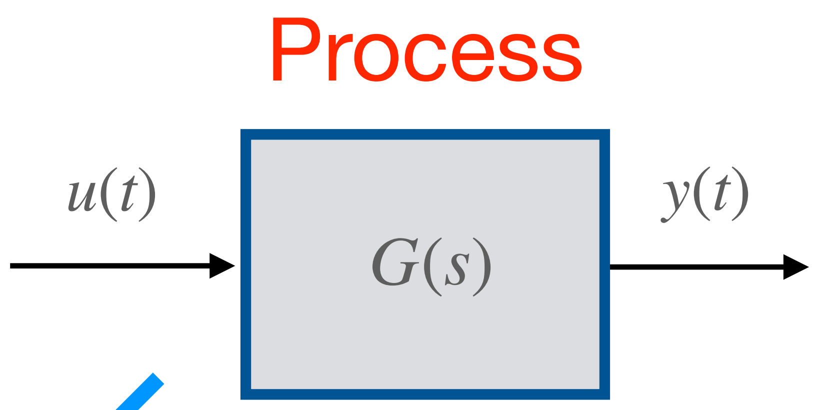

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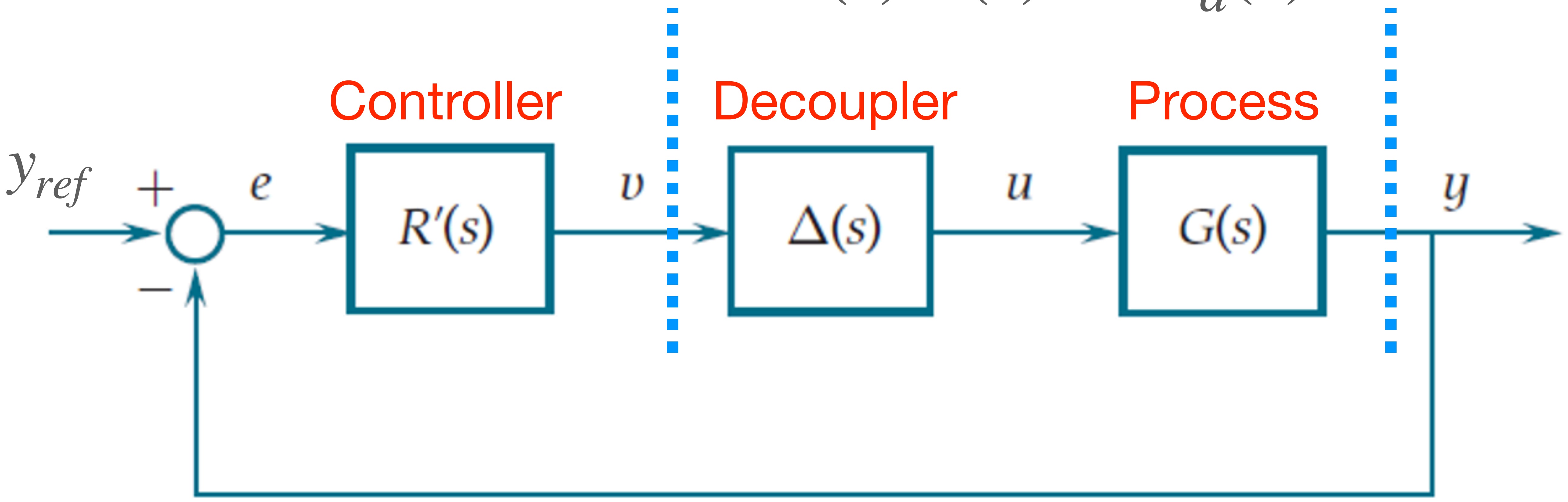

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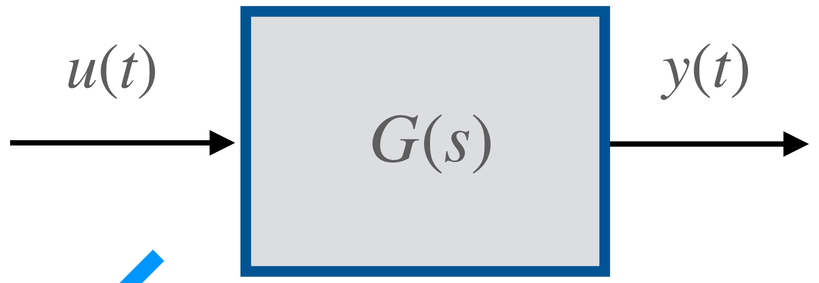
Decoupling Based Control Schemes

Process



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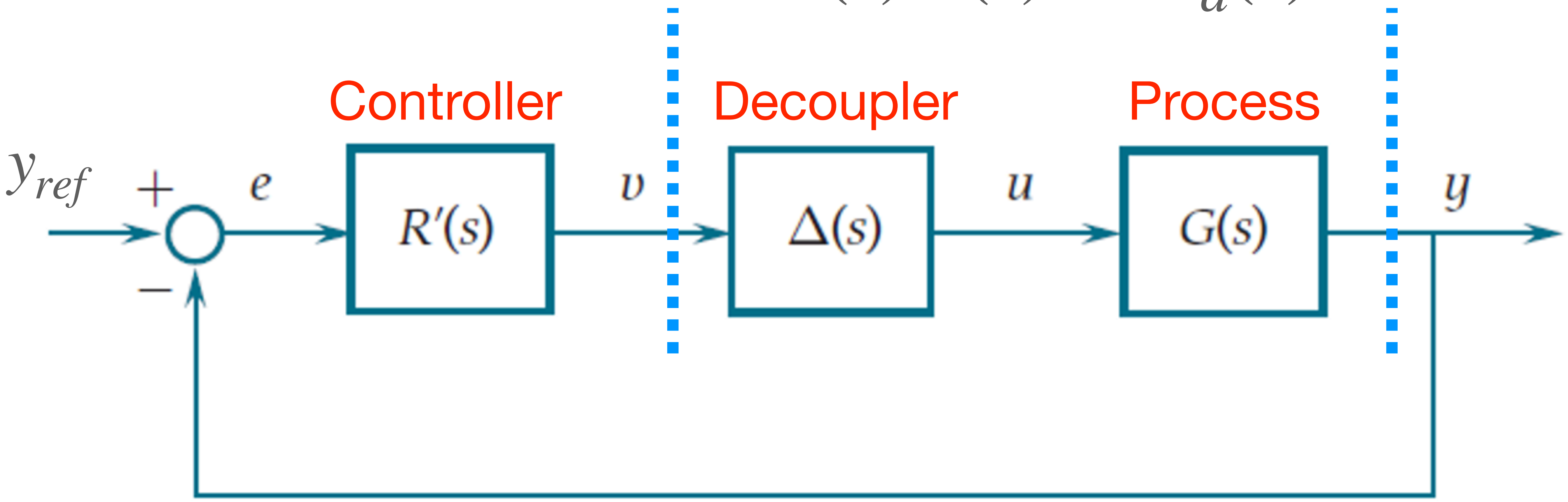
diagonal system

$$G(s)\Delta(s) = G_d(s)$$

Controller Decoupler Process



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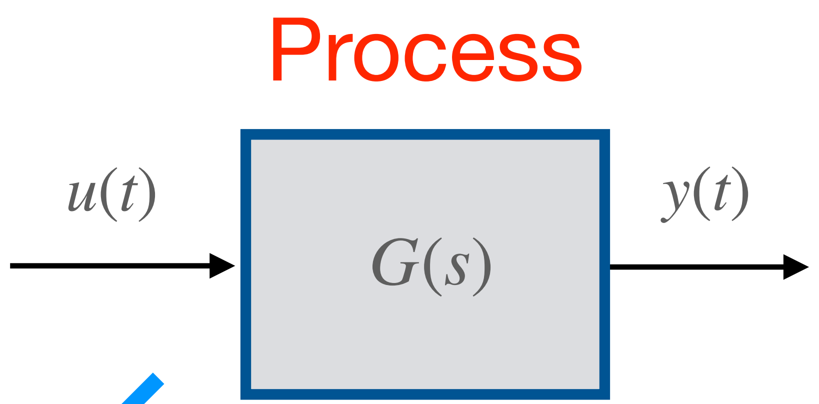
Decoupling Based Control Schemes



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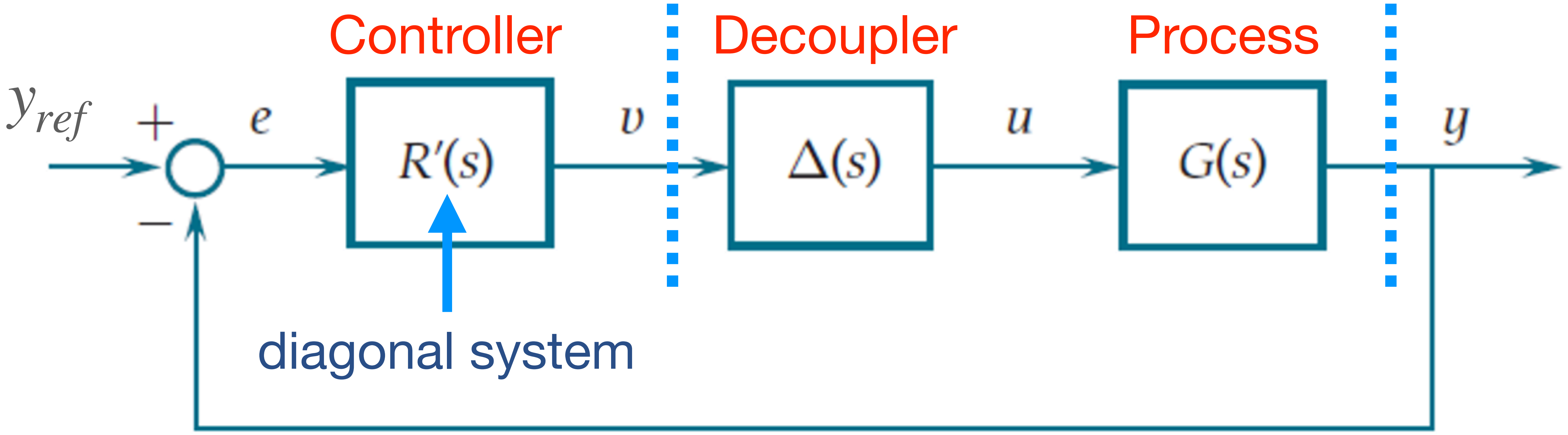


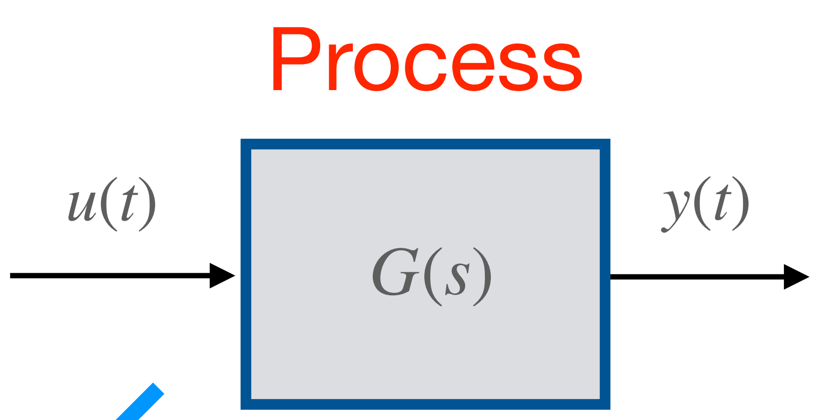
Assumptions:

- $G(s) \in \mathbb{R}^{2\times2}$
- $G(s)$ full matrix

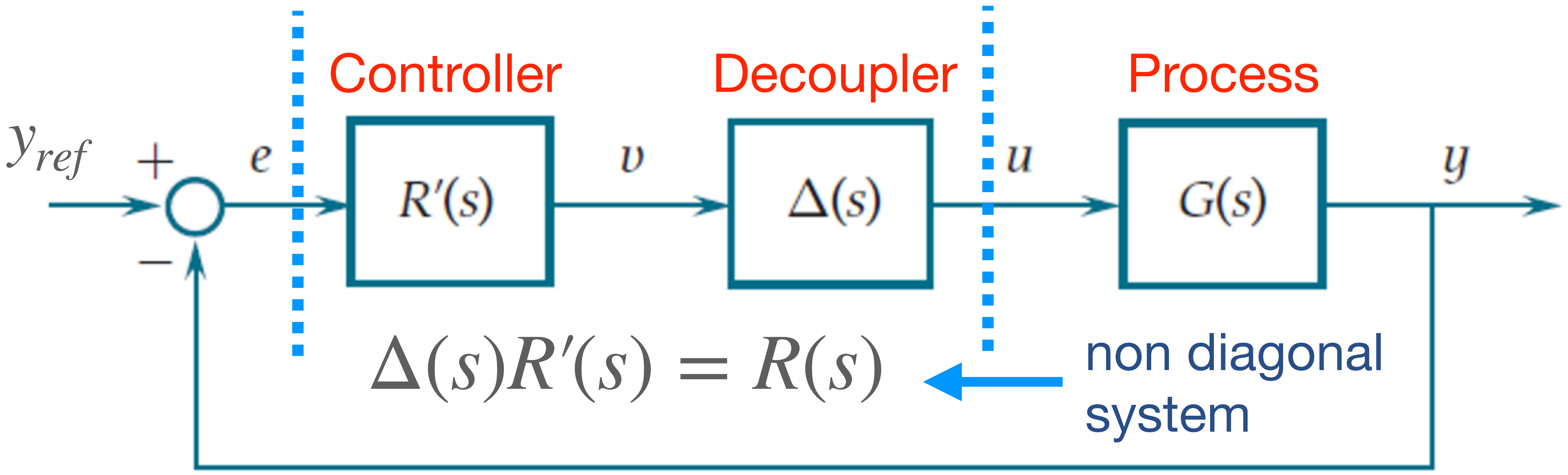
Sufficient conditions:

- $G(s)$ rational function
- $G(s)$ As. Stable
- $\det G(s) \neq 0 \ \forall\, s$ s.t. $\mathrm{Re}(s) \geq 0$

Decoupling Based Control Schemes



$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$



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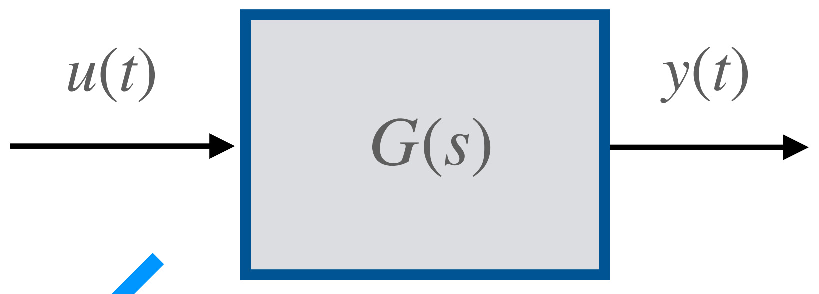
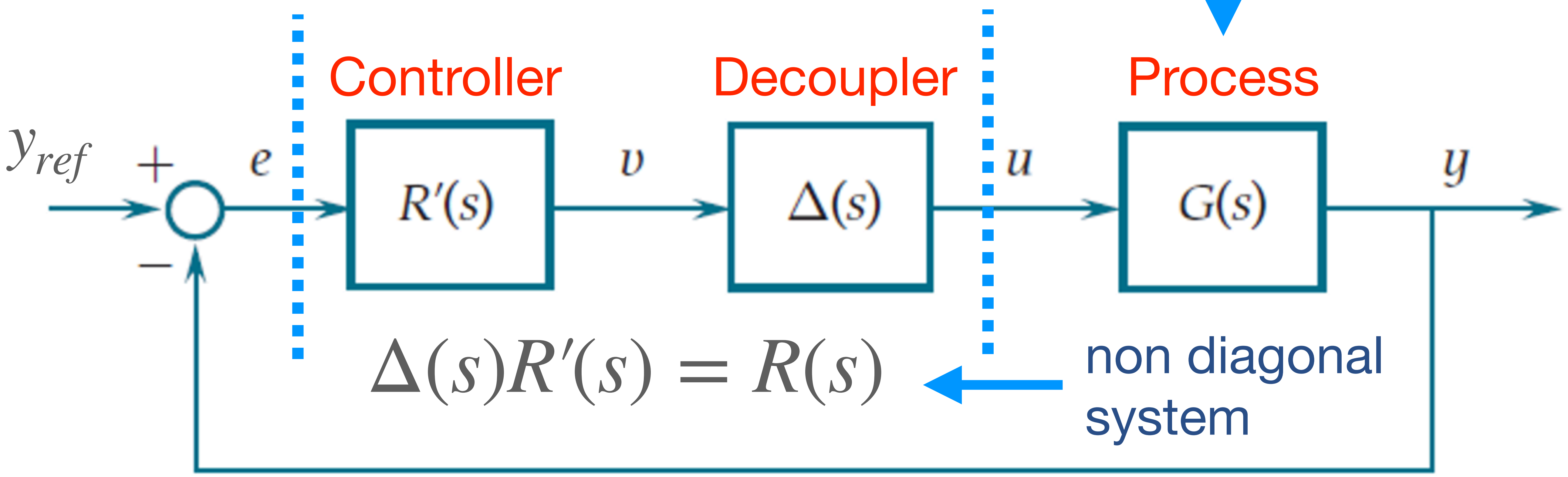
Decoupling Based Control Schemes

Process



$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

non diagonal system



$$\Delta(s)R'(s) = R(s)$$

non diagonal system

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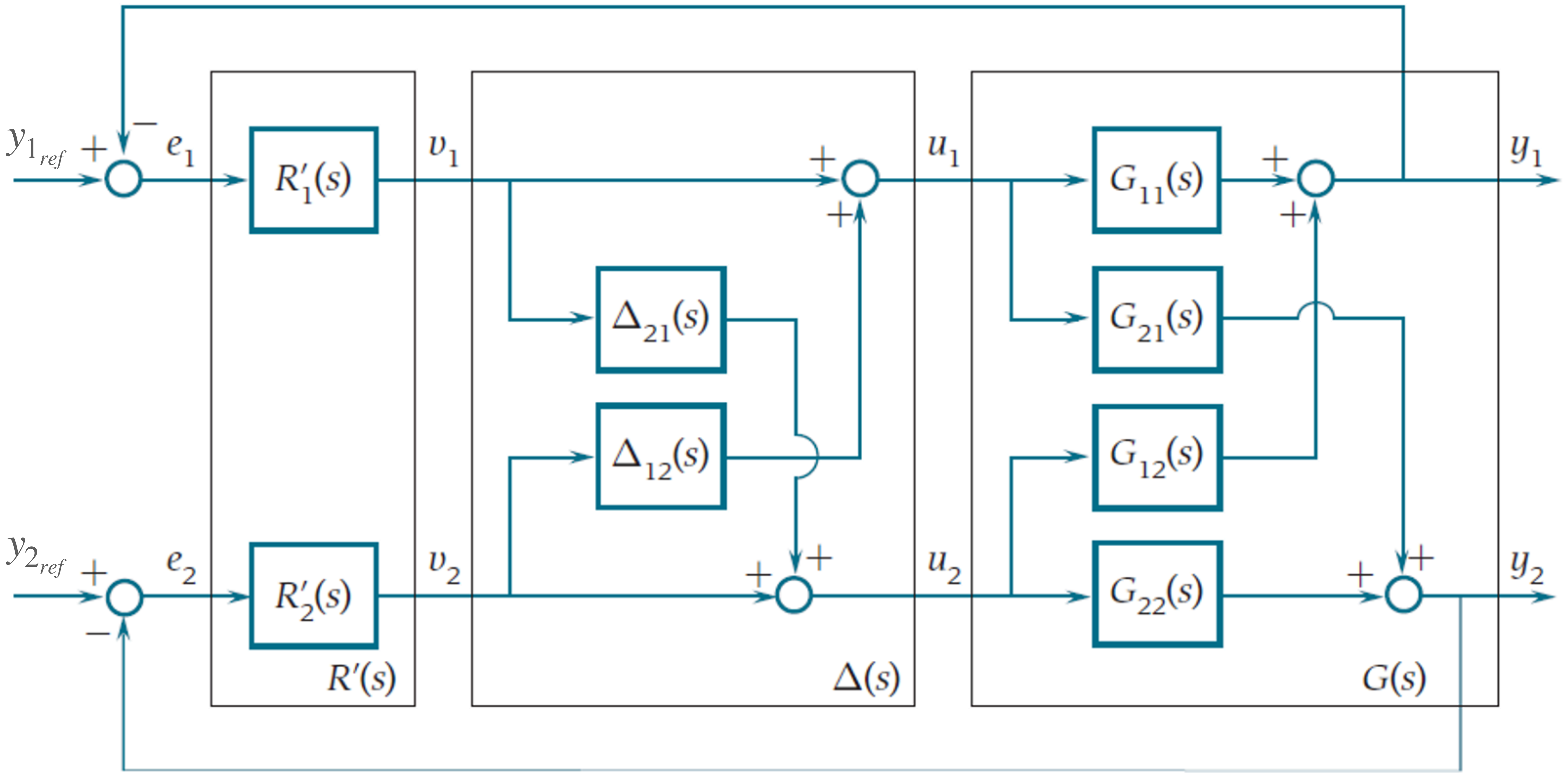
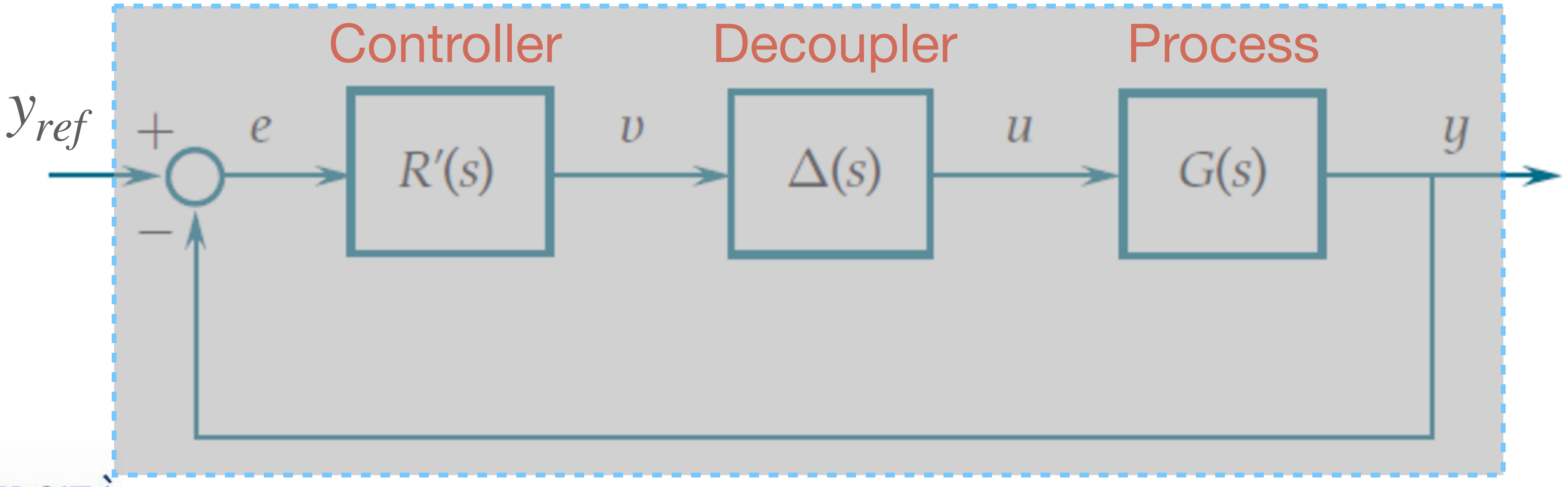
Decoupling Based Control Schemes: Forward Decoupling





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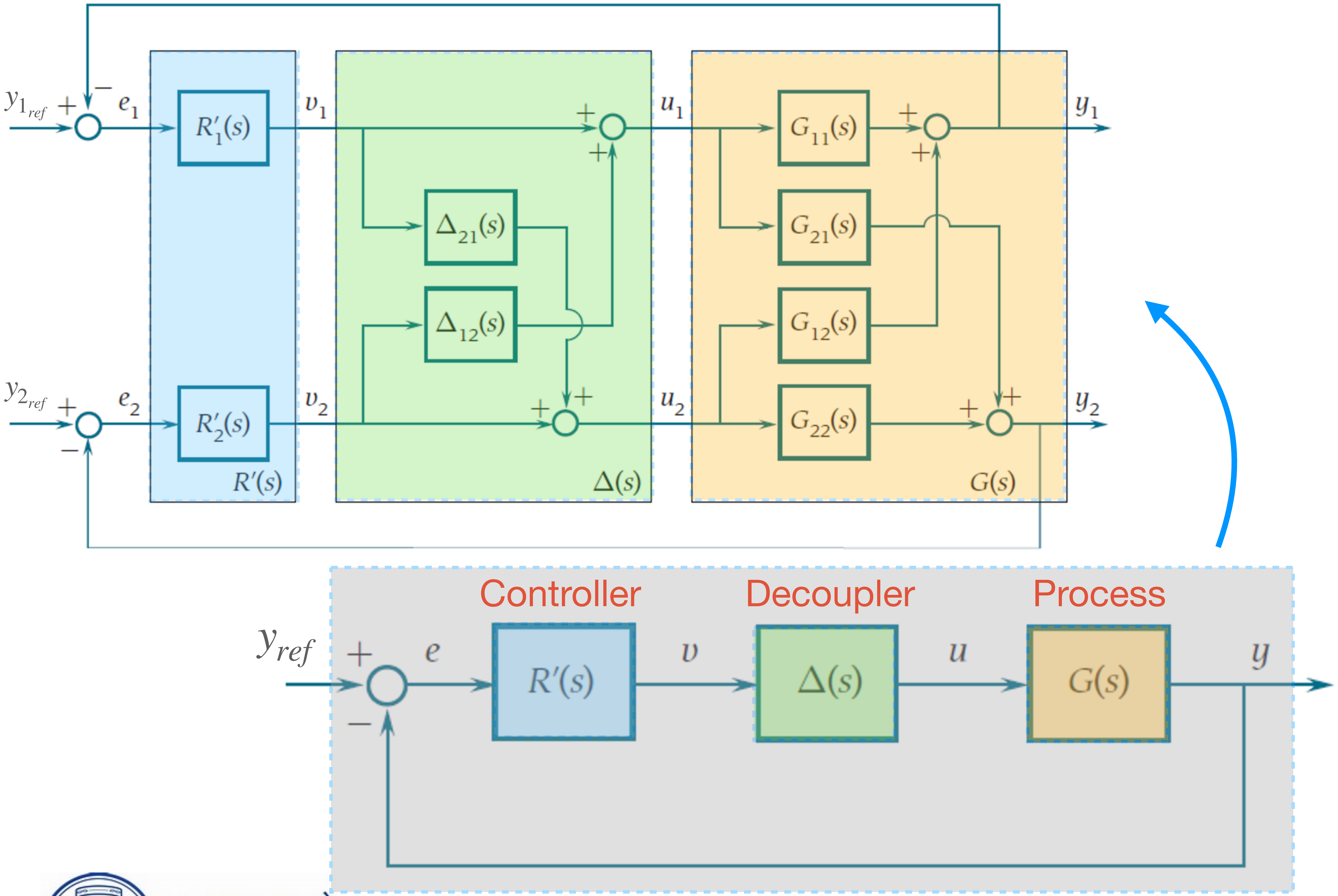

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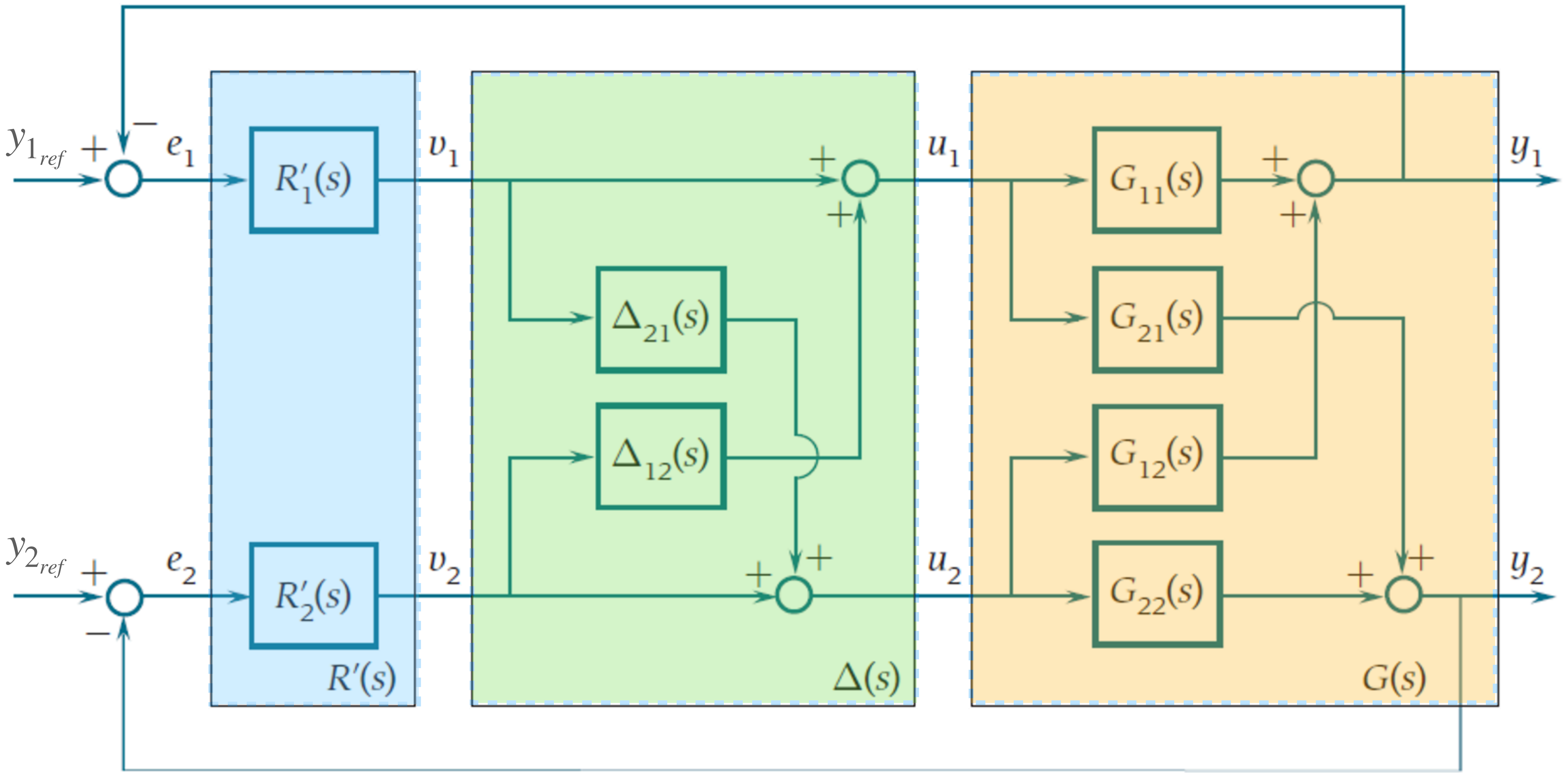
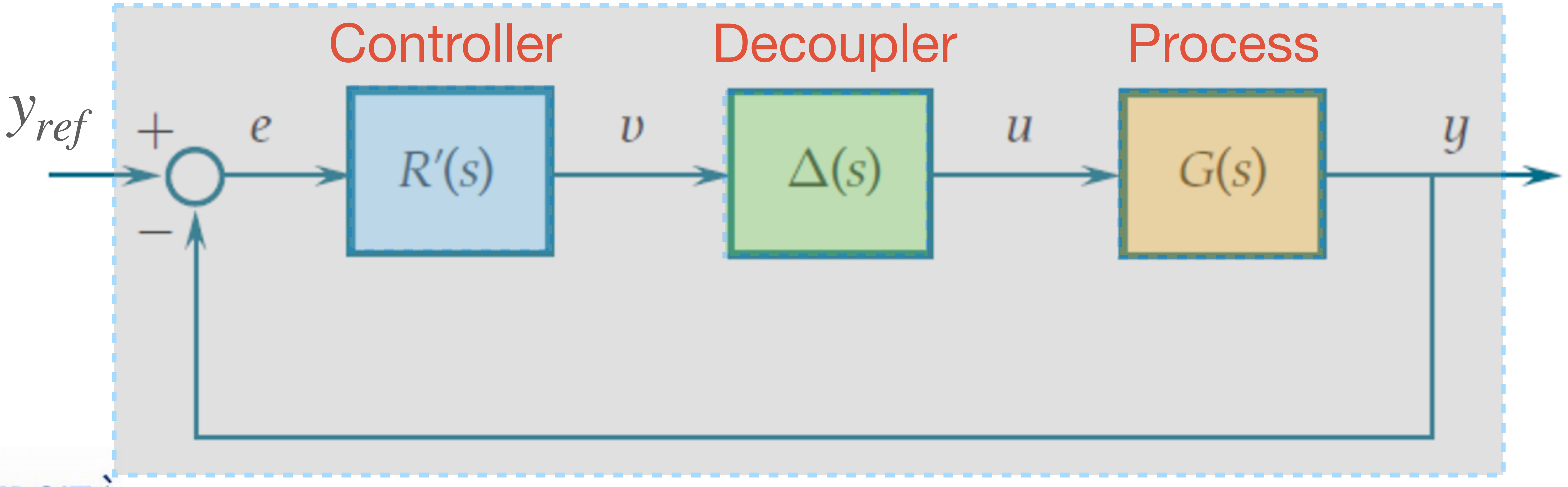
Decoupling Based Control Schemes: Forward Decoupling

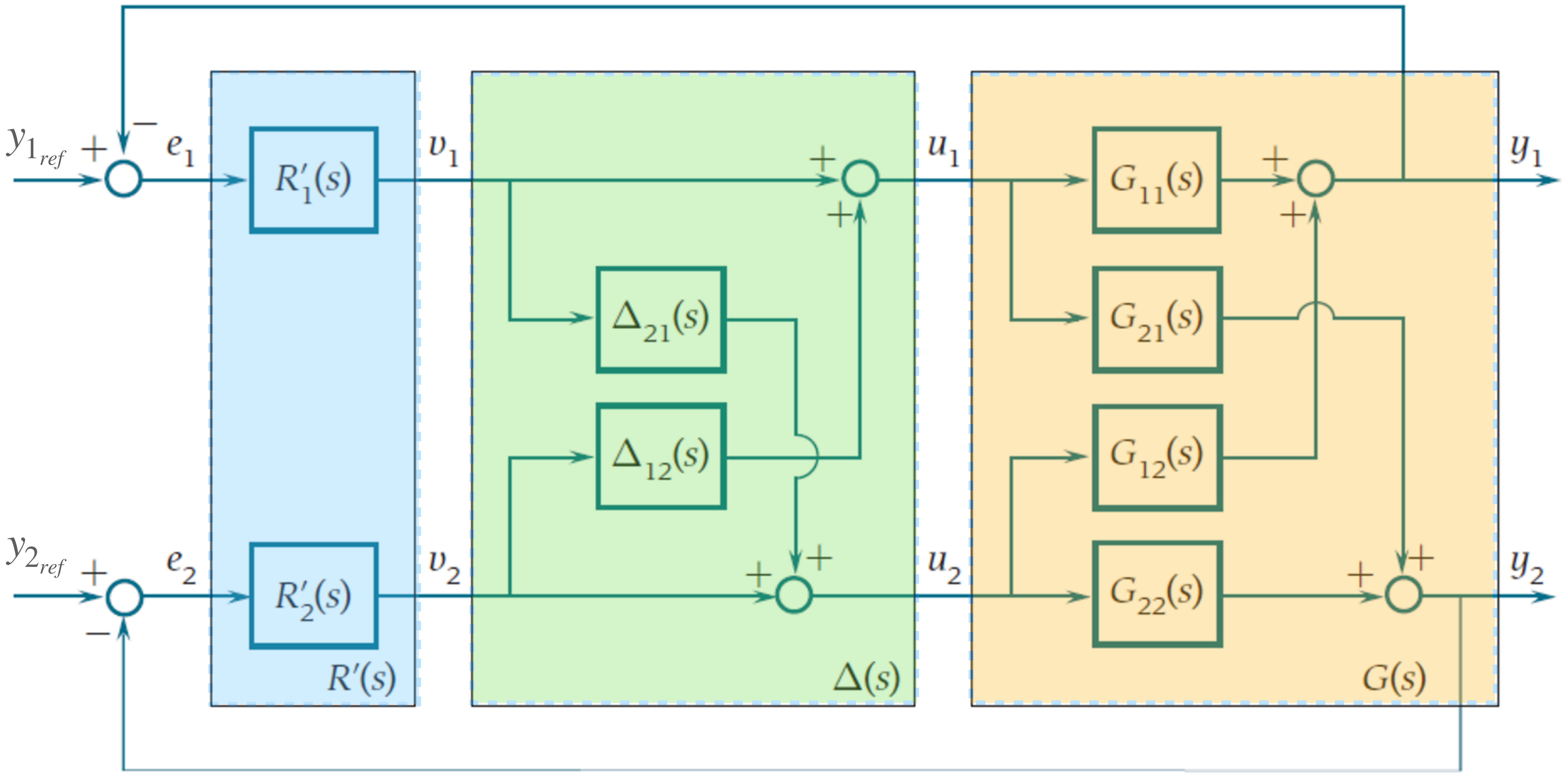


$$G(s)\Delta(s) = G_d(s)$$

$$\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} \Delta_{11}(s) & \Delta_{12}(s) \\ \Delta_{21}(s) & \Delta_{22}(s) \end{bmatrix} = \begin{bmatrix} G_{d_{11}}(s) & 0 \\ 0 & G_{d_{22}}(s) \end{bmatrix}$$



Decoupling Based Control Schemes: Forward Decoupling



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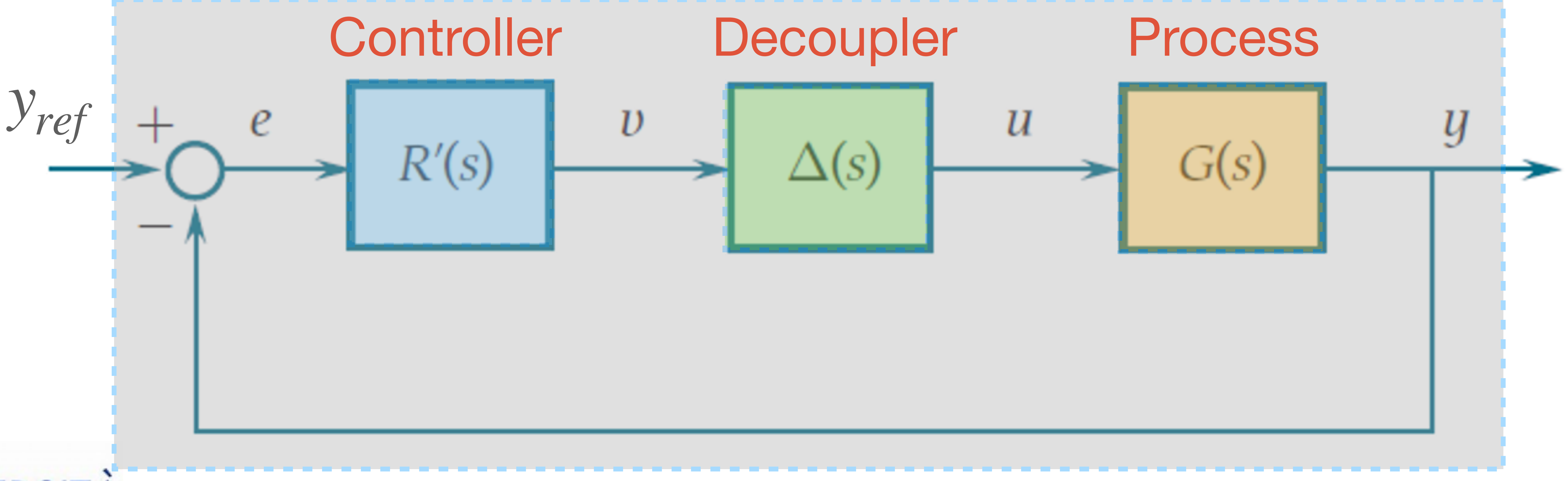
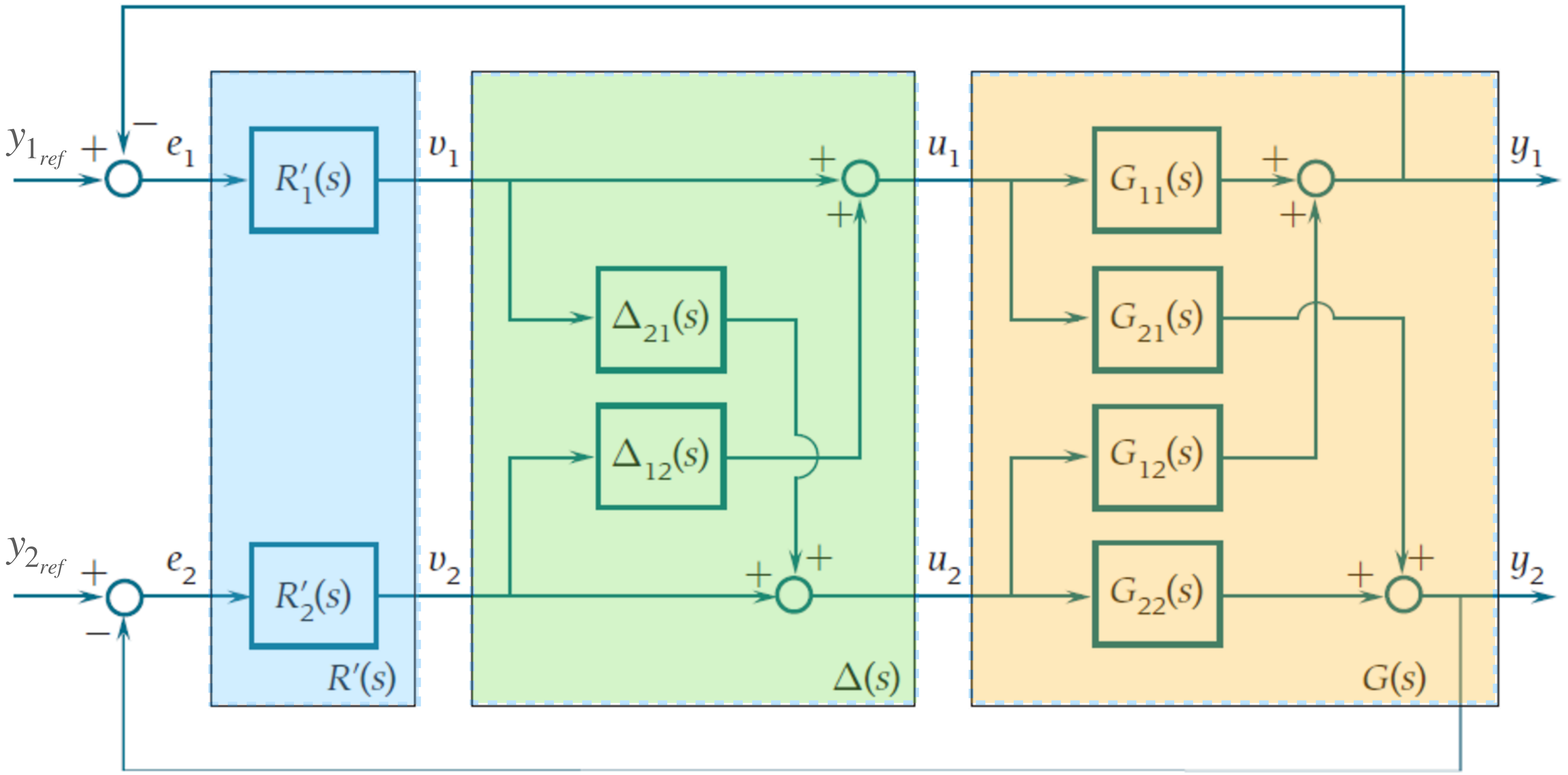
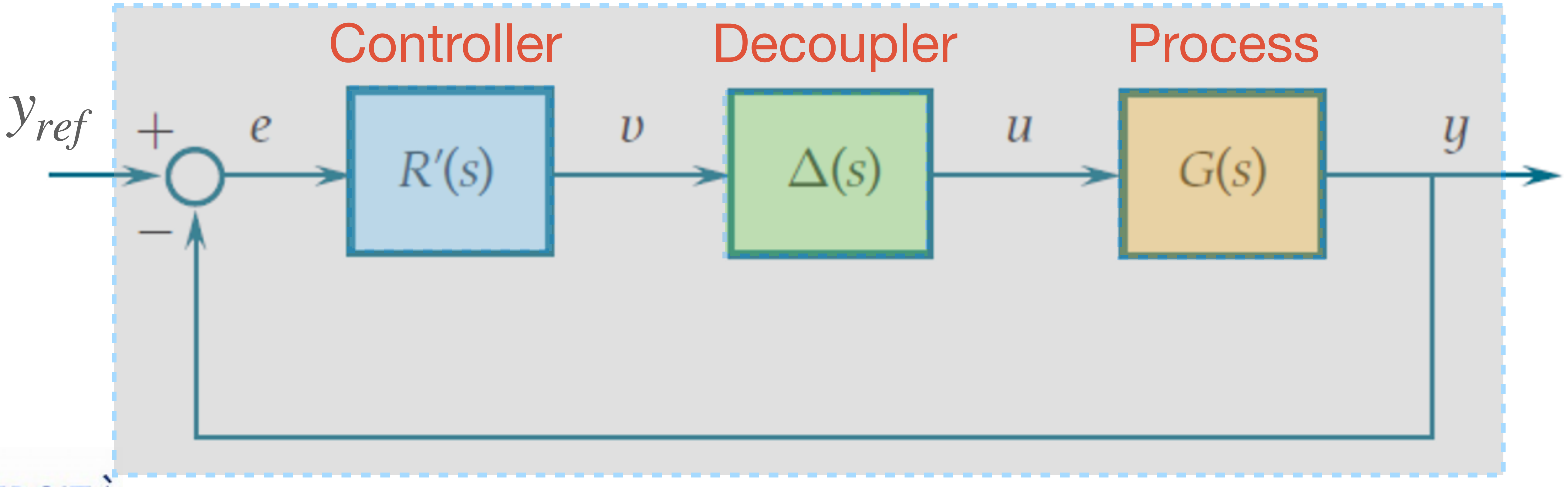
unknowns



Decoupling Based Control Schemes: Forward Decoupling





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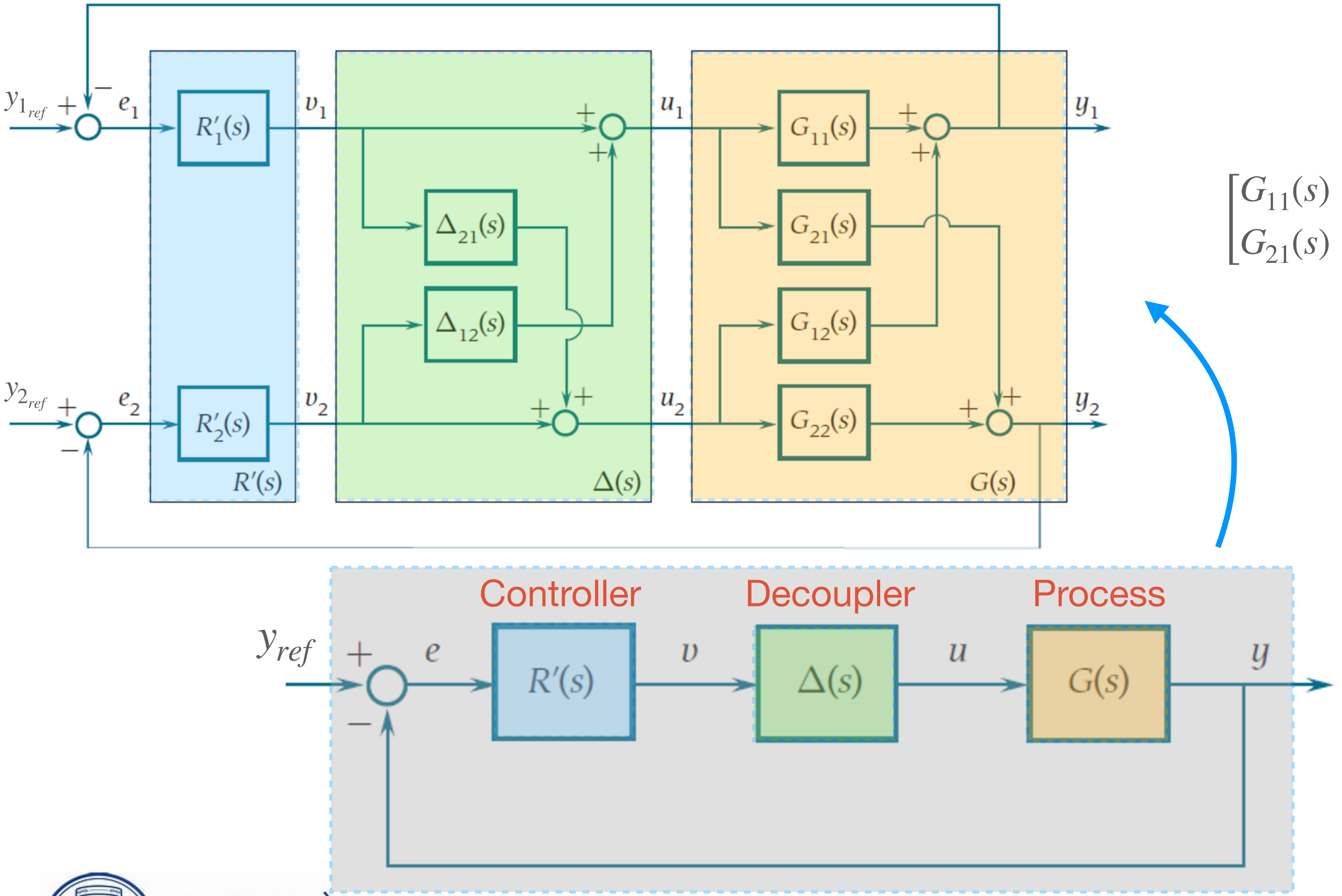
unknowns

$$\Delta_{11}(s) = \Delta_{22}(s) = 1$$

Decoupling Based Control Schemes: Forward Decoupling



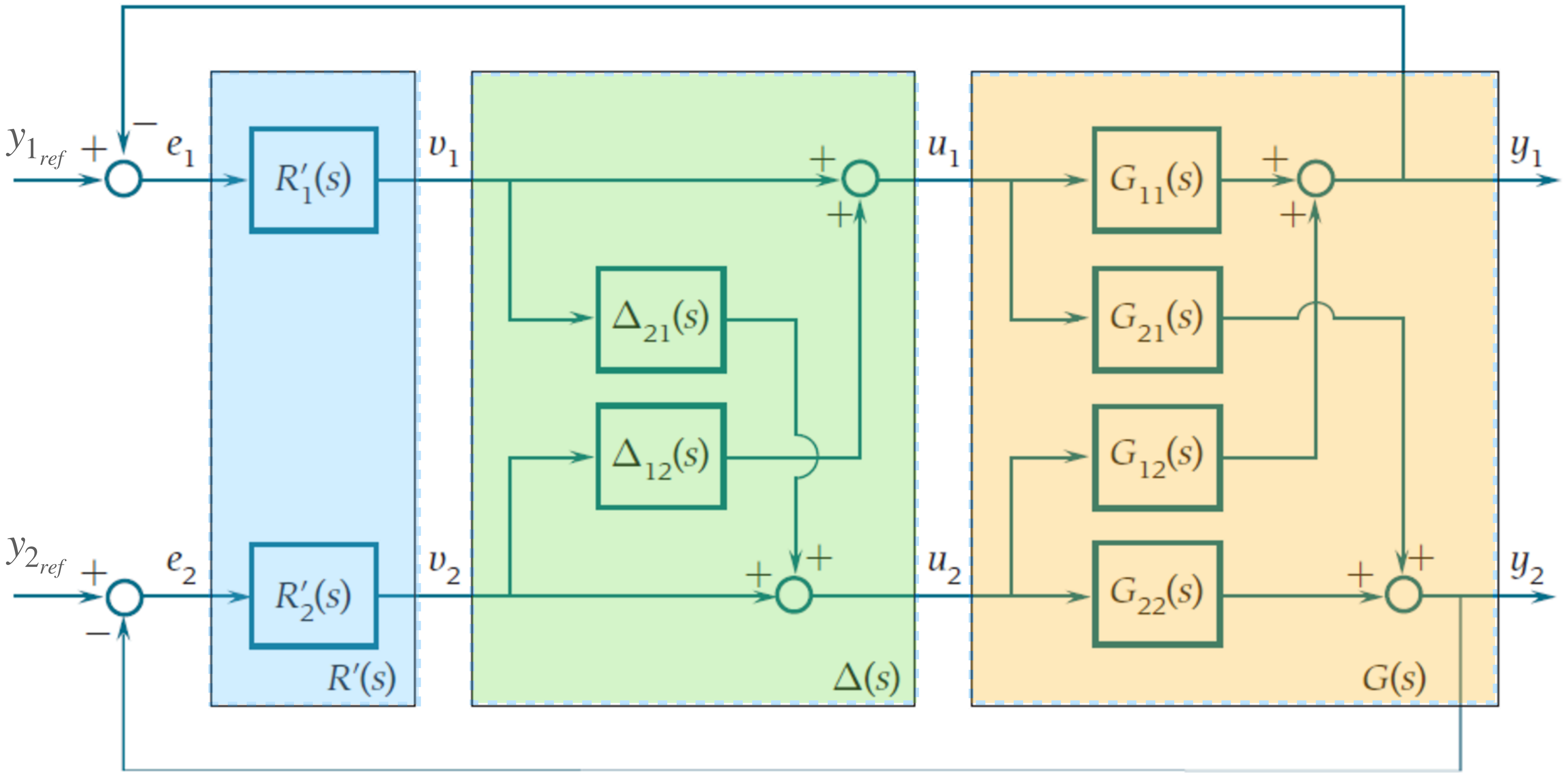
$$G(s)\Delta(s) = G_d(s)$$

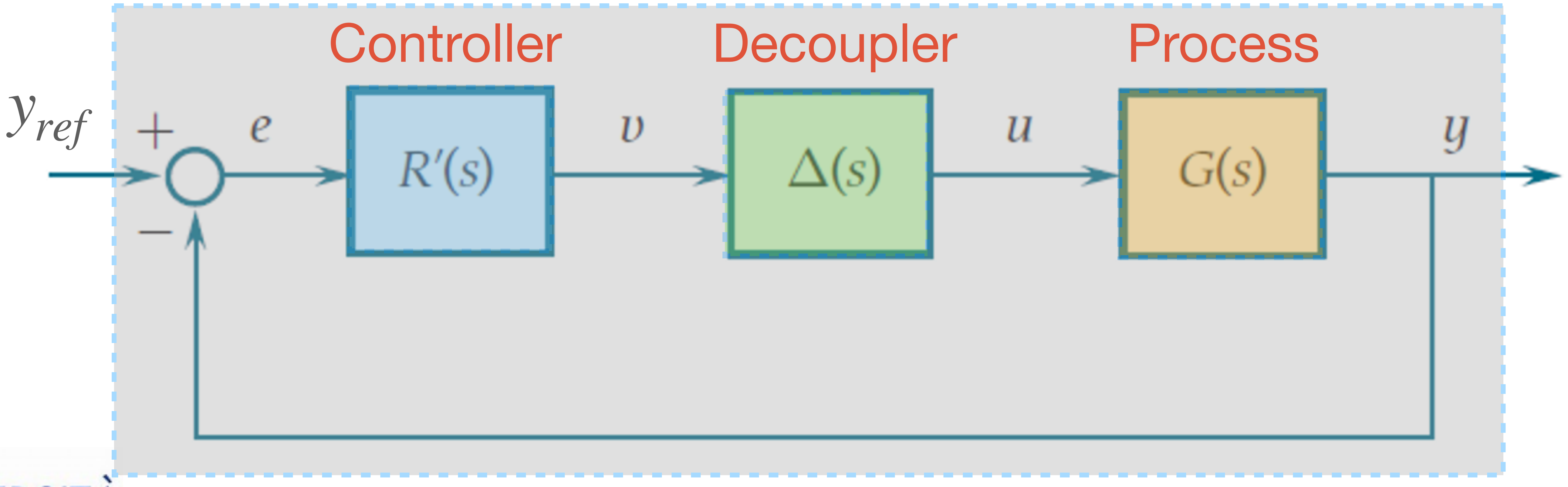
$$\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} 1 & \Delta_{12}(s) \\ \Delta_{21}(s) & 1 \end{bmatrix} = \begin{bmatrix} G_{d_{11}}(s) & 0 \\ 0 & G_{d_{22}}(s) \end{bmatrix}$$

unknowns

$$\Delta_{11}(s) = \Delta_{22}(s) = 1$$

Decoupling Based Control Schemes: Forward Decoupling





$$G(s)\Delta(s) = G_d(s)$$

$$\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} 1 & \Delta_{12}(s) \\ \Delta_{21}(s) & 1 \end{bmatrix} = \begin{bmatrix} G_{d_{11}}(s) & 0 \\ 0 & G_{d_{22}}(s) \end{bmatrix}$$

unknowns

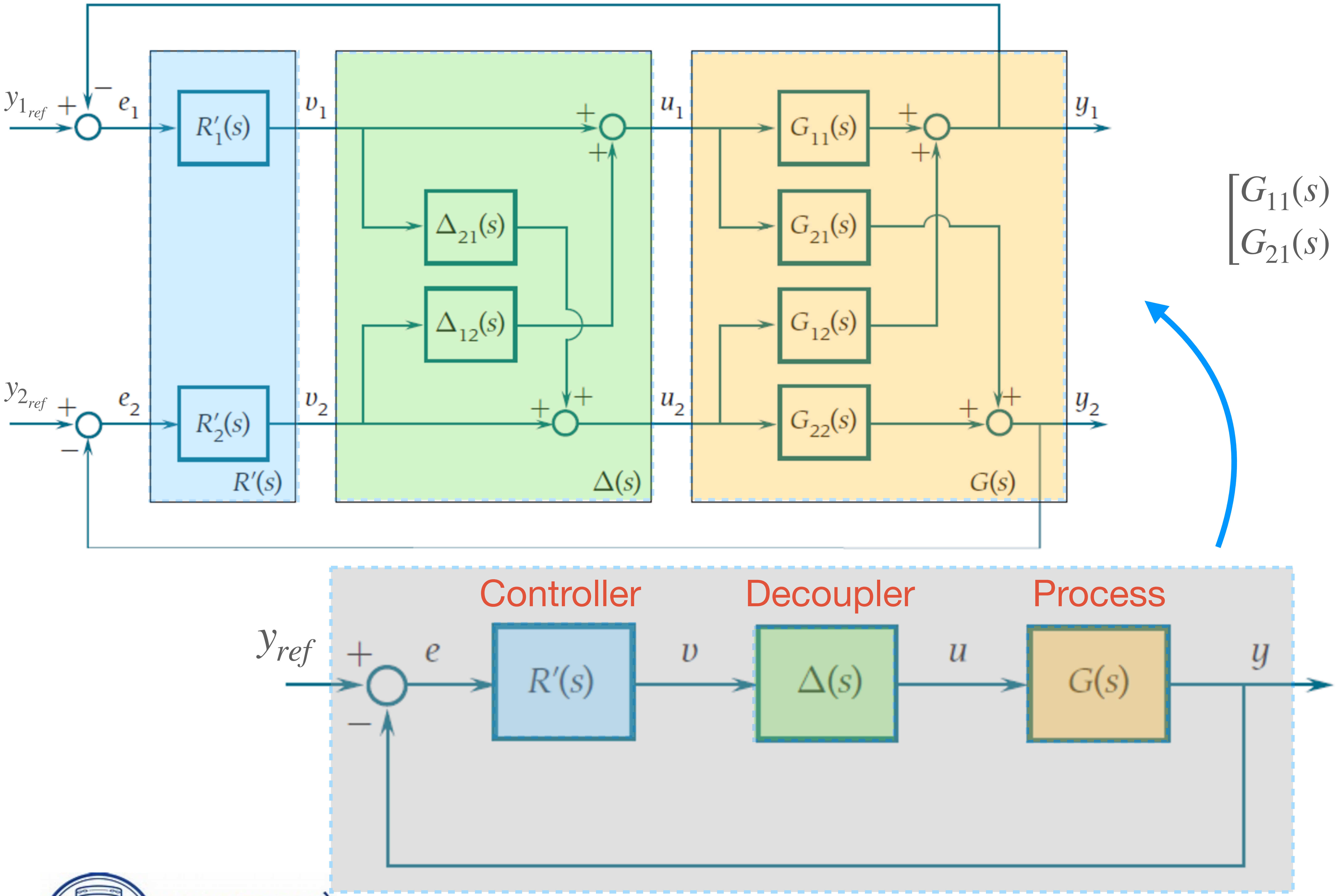
$$G_{11}(s) + G_{12}(s)\Delta_{21}(s) = G_{d_{11}}(s)$$

$$G_{11}(s)\Delta_{12}(s) + G_{12}(s) = 0$$

$$G_{21}(s) + G_{22}(s)\Delta_{21}(s) = 0$$

$$G_{21}(s)\Delta_{12}(s) + G_{22}(s) = G_{d_{22}}(s)$$

Decoupling Based Control Schemes: Forward Decoupling



$$G(s)\Delta(s) = G_d(s)$$

$$\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} 1 & \boxed{\Delta_{12}(s)} \\ \boxed{\Delta_{21}(s)} & 1 \end{bmatrix} = \begin{bmatrix} G_{d_{11}}(s) & 0 \\ 0 & G_{d_{22}}(s) \end{bmatrix}$$

unknowns

$$G_{11}(s) + G_{12}(s)\Delta_{21}(s) = G_{d_{11}}(s)$$

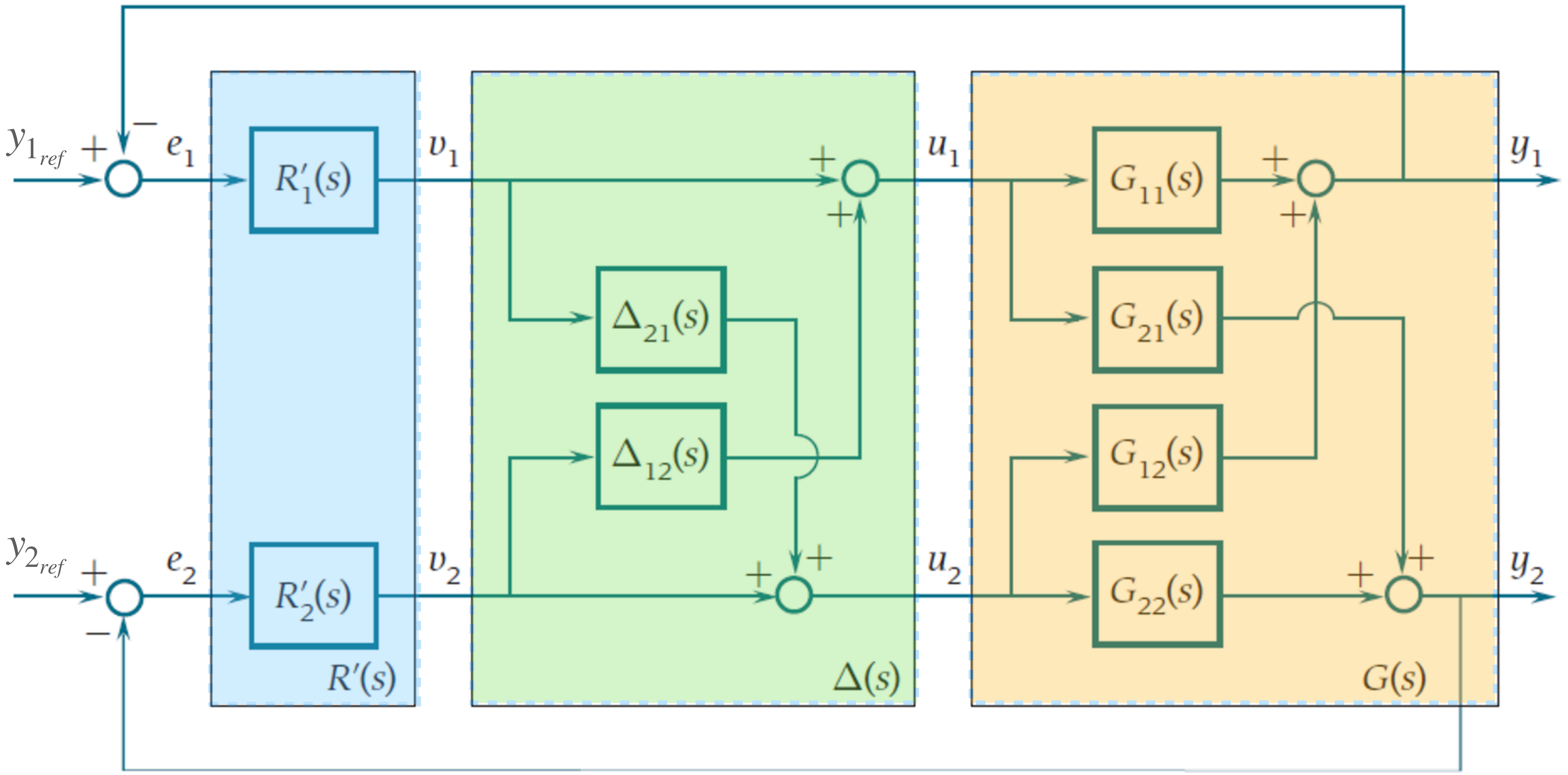
$$G_{11}(s)\Delta_{12}(s) + G_{12}(s) = 0$$

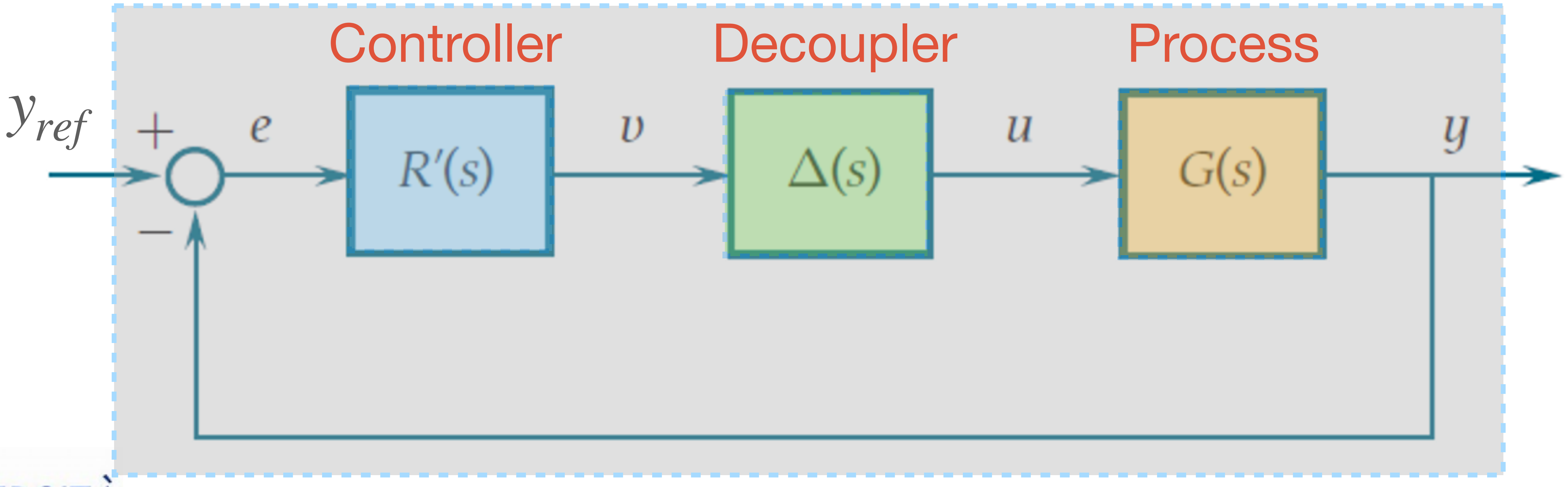
$$G_{21}(s) + G_{22}(s)\Delta_{21}(s) = 0$$

$$\rightarrow \quad \Delta_{12}(s), \quad \Delta_{21}(s)$$

$$G_{21}(s)\Delta_{12}(s) + G_{22}(s) = G_{d_{22}}(s)$$

Decoupling Based Control Schemes: Forward Decoupling





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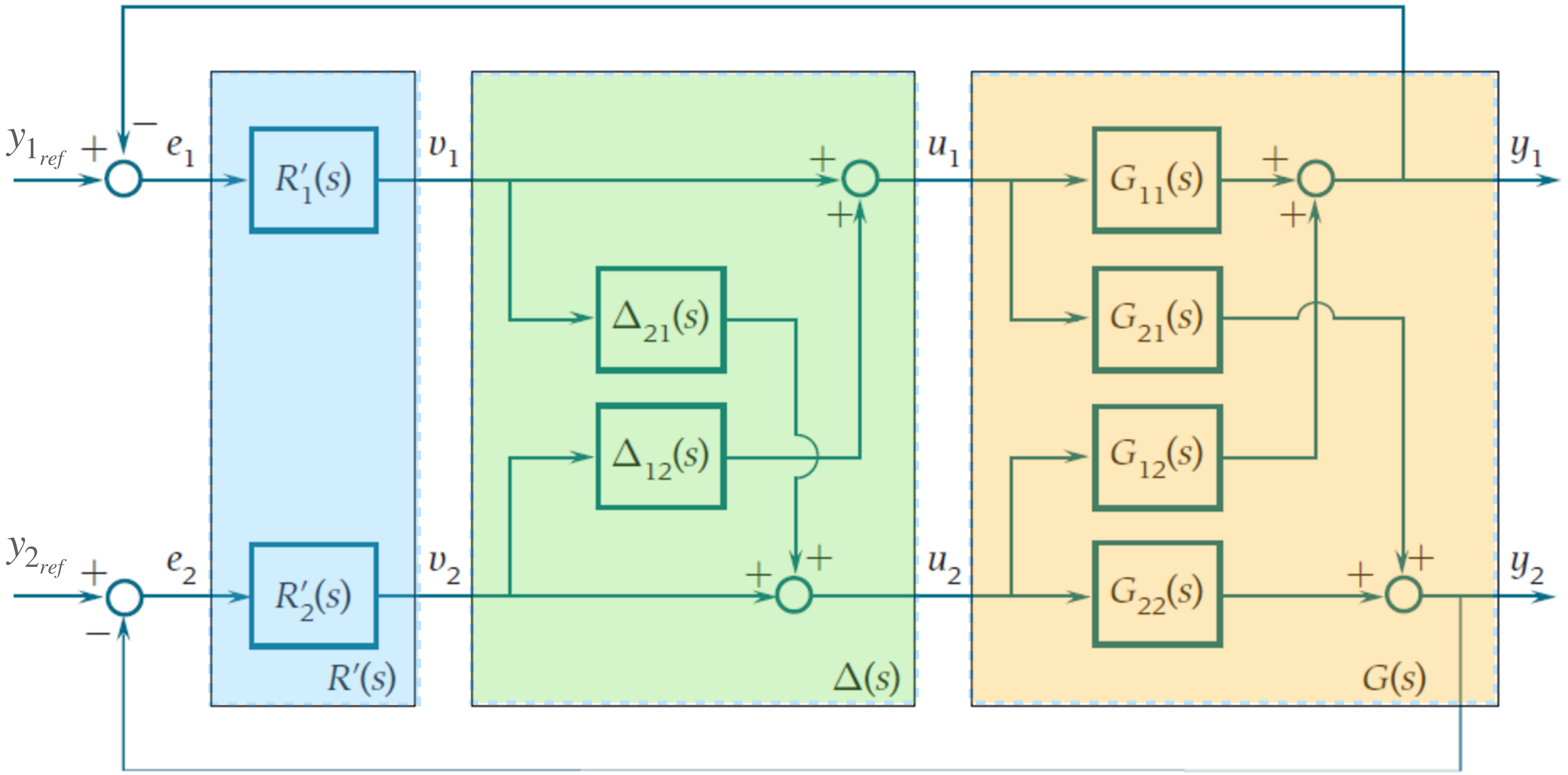
$$G_{21}(s)\Delta_{12}(s) + G_{22}(s) = G_{d_{22}}(s)$$

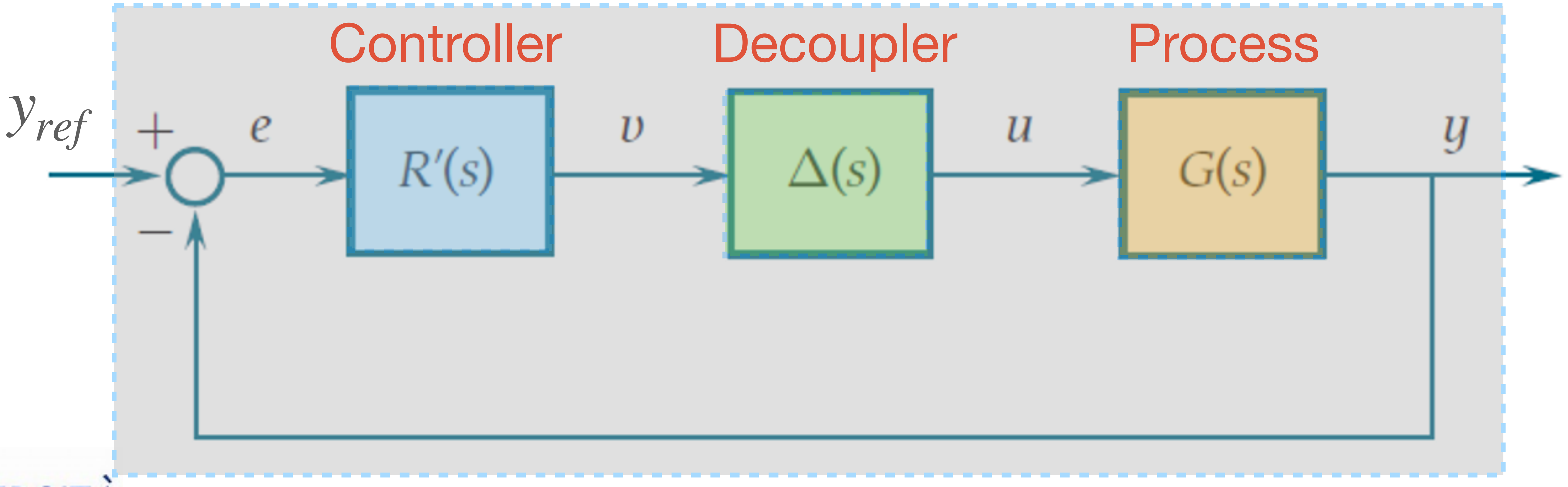
$\Delta_{12}(s)$

$\Delta_{21}(s)$

they must be causal!

Decoupling Based Control Schemes: Forward Decoupling





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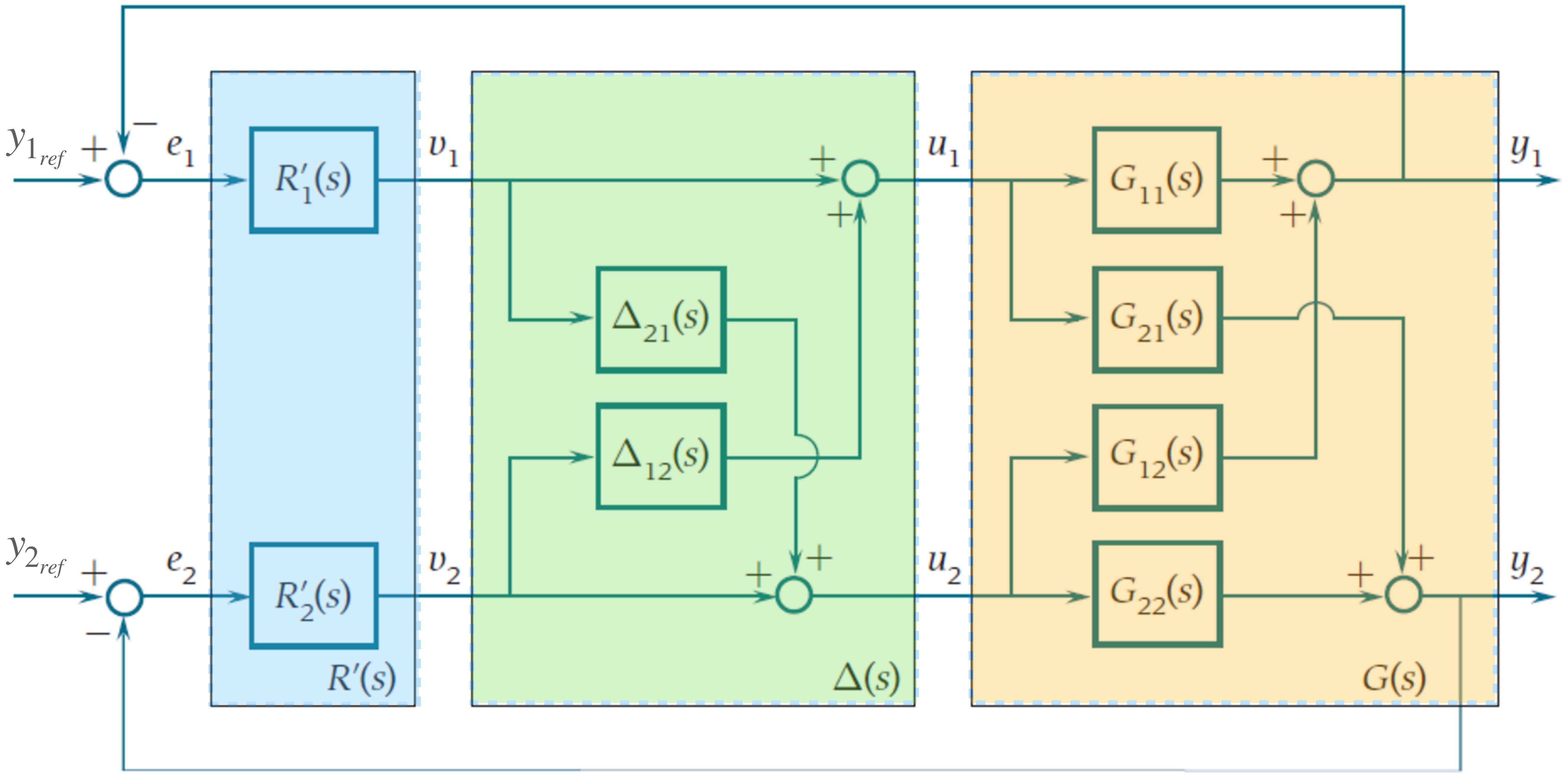
$\Delta_{12}(s)$

$\Delta_{21}(s)$

Decoupling Based Control Schemes: Forward Decoupling



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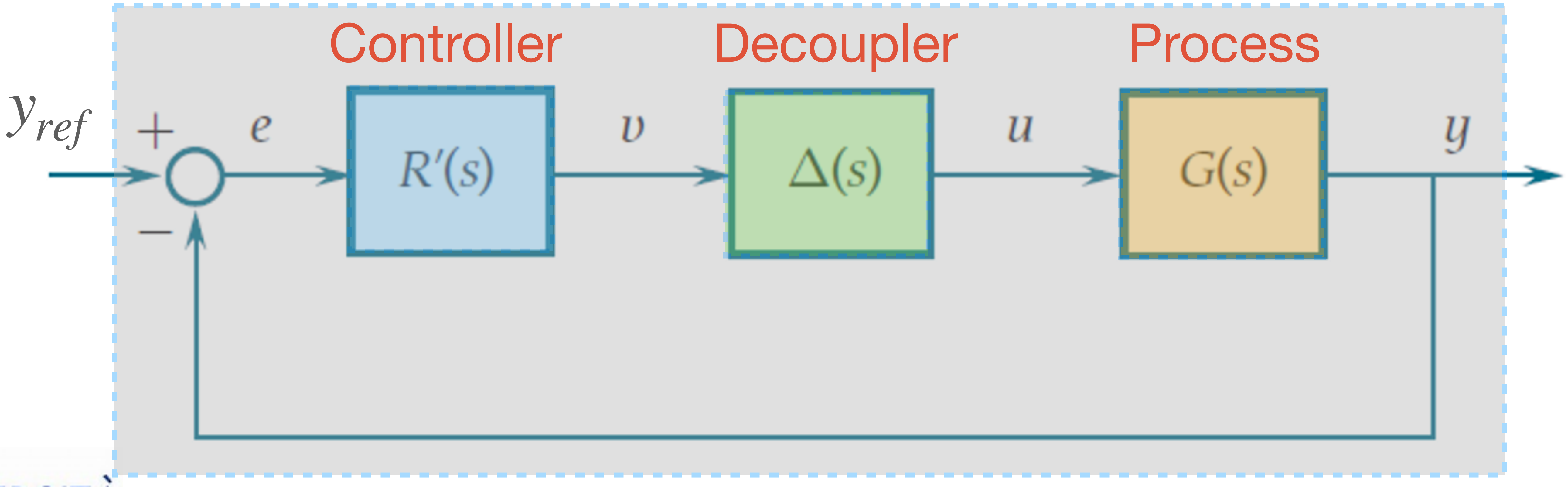
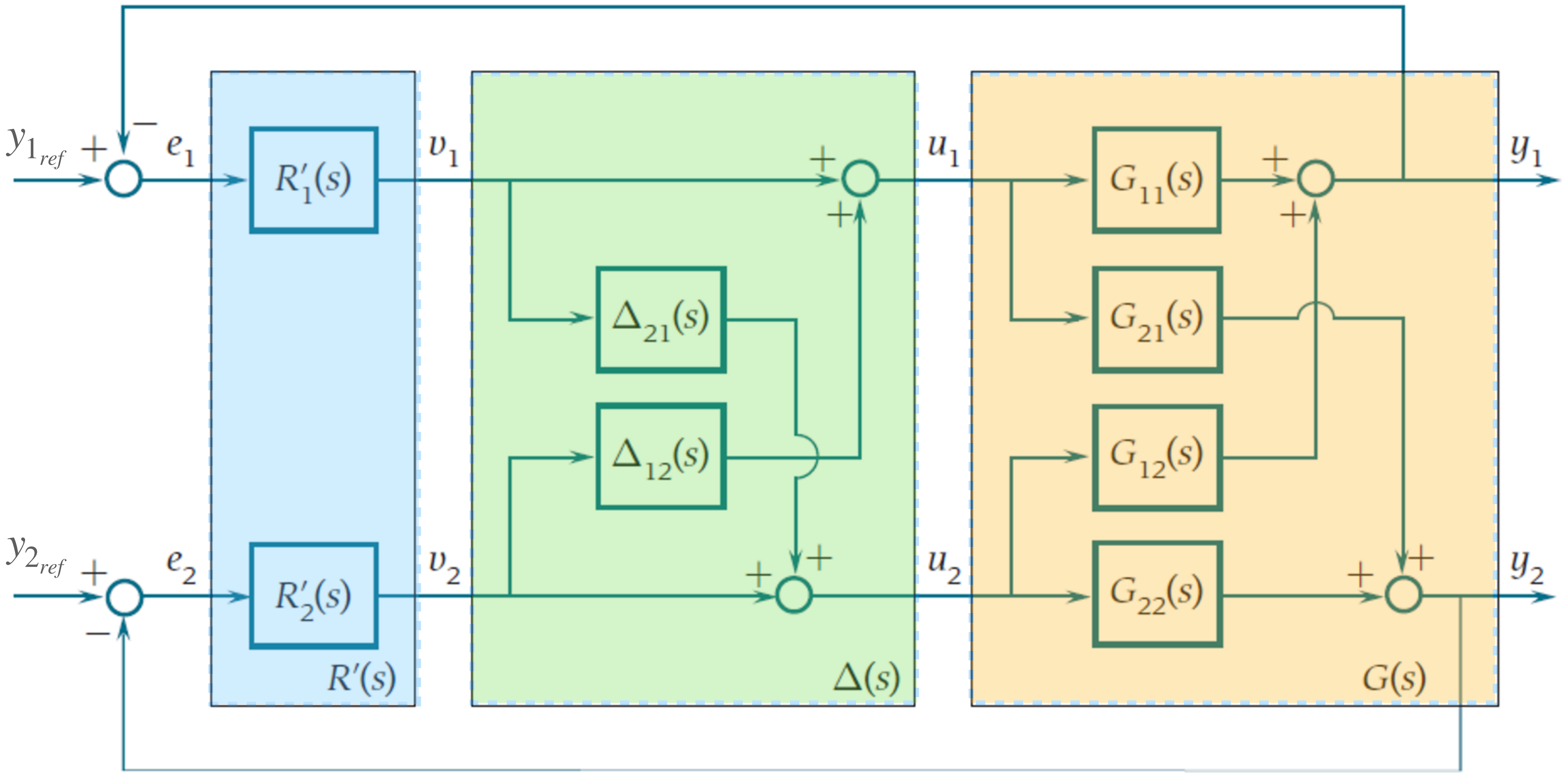
$\Delta_{12}(s)$

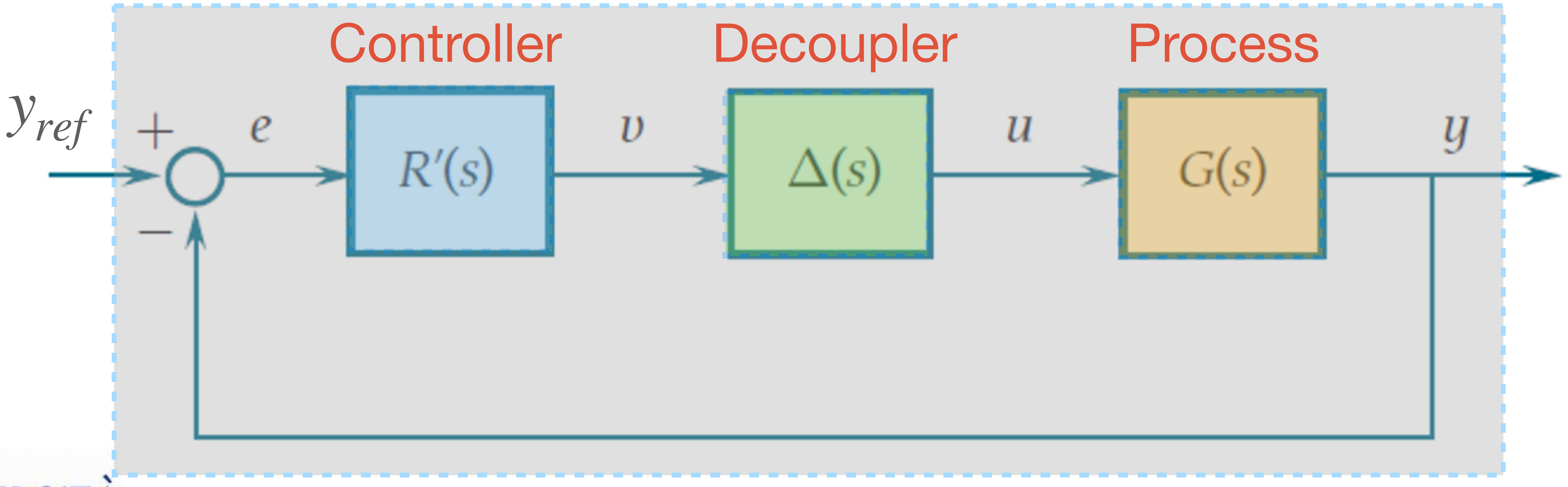
$\Delta_{21}(s)$



design: $R'(s) = \begin{bmatrix} R'_1(s) & 0 \\ 0 & R'_2(s) \end{bmatrix}$

Decoupling Based Control Schemes: Forward Decoupling





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$\Delta_{12}(s)$

$\Delta_{21}(s)$

$G_{d_{11}}(s)$

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$G_{d_{22}}(s)$

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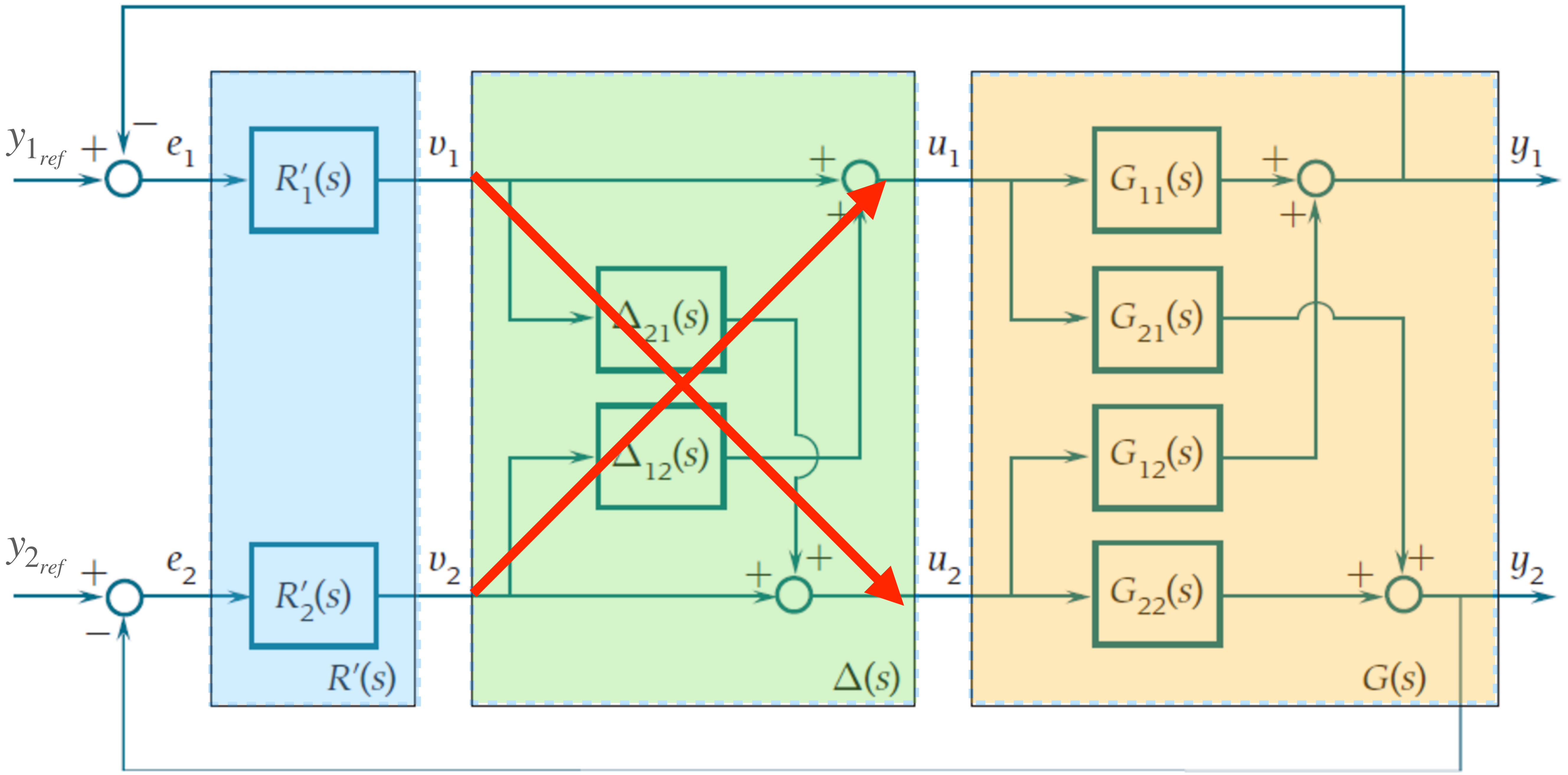

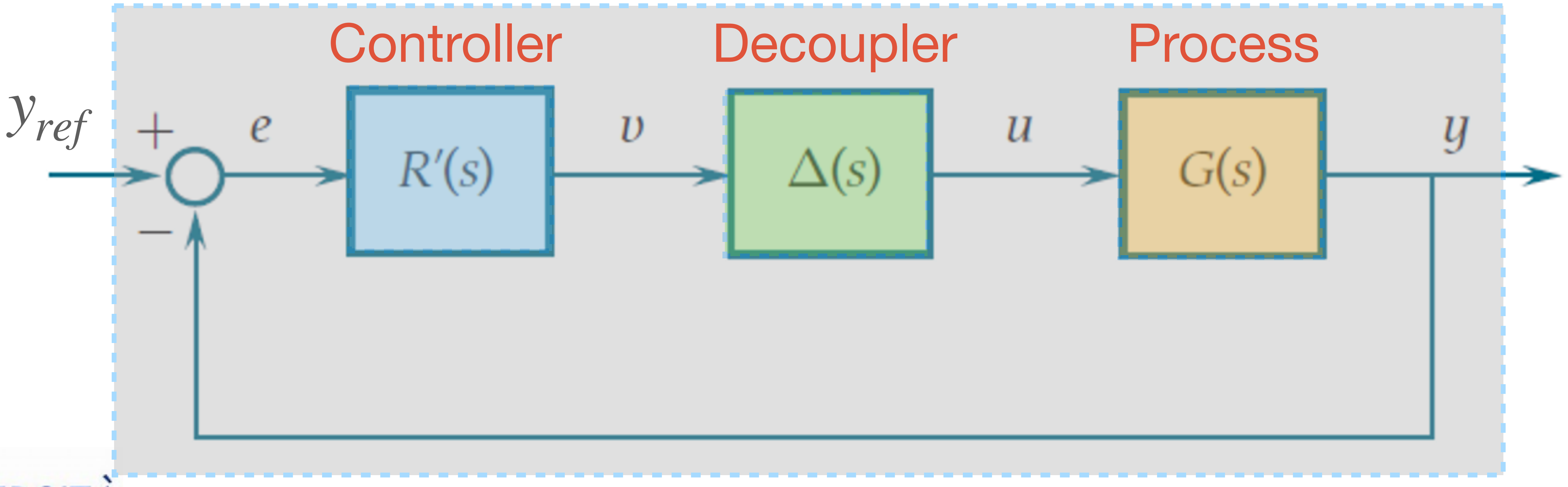

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$\Delta_{12}(s)$

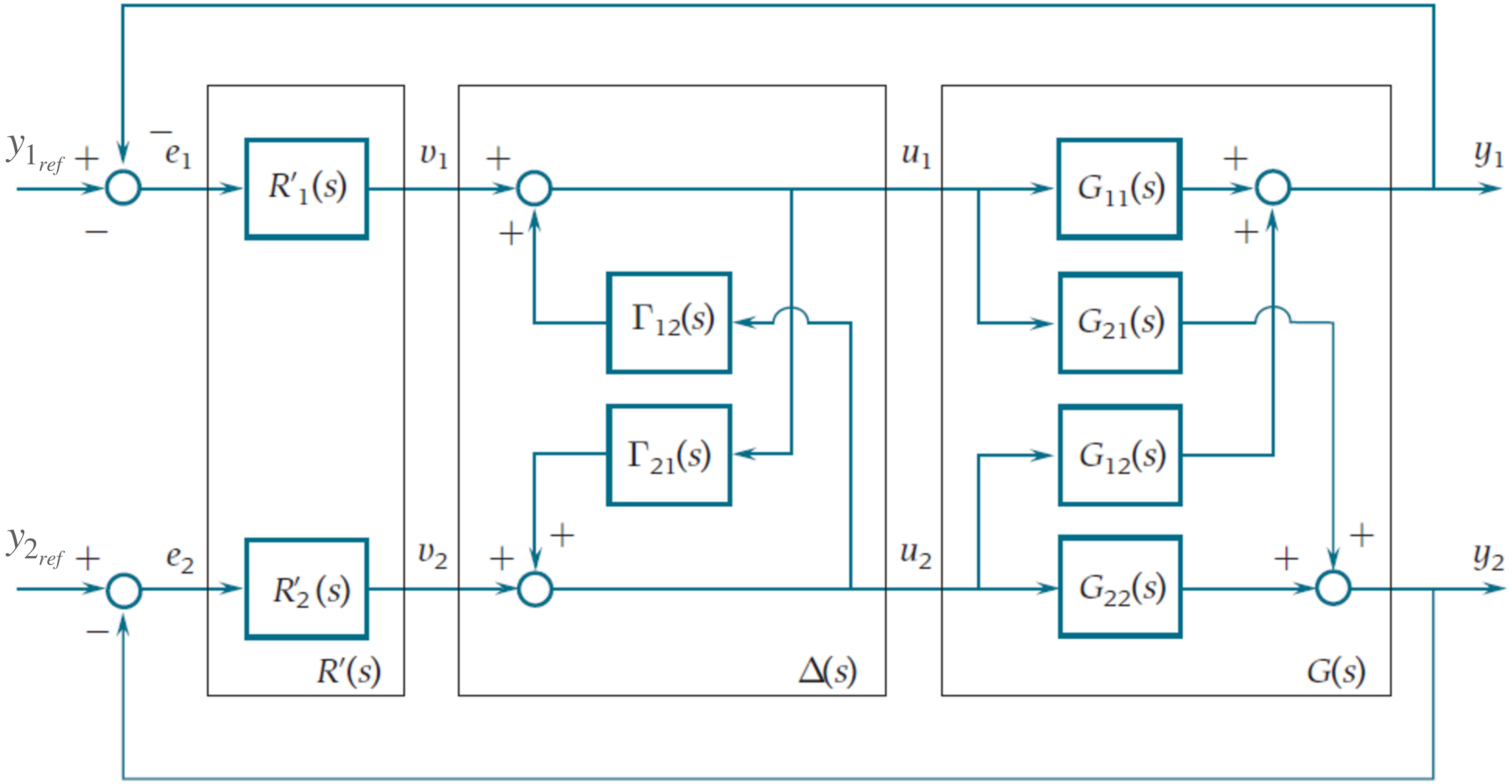
$\Delta_{21}(s)$

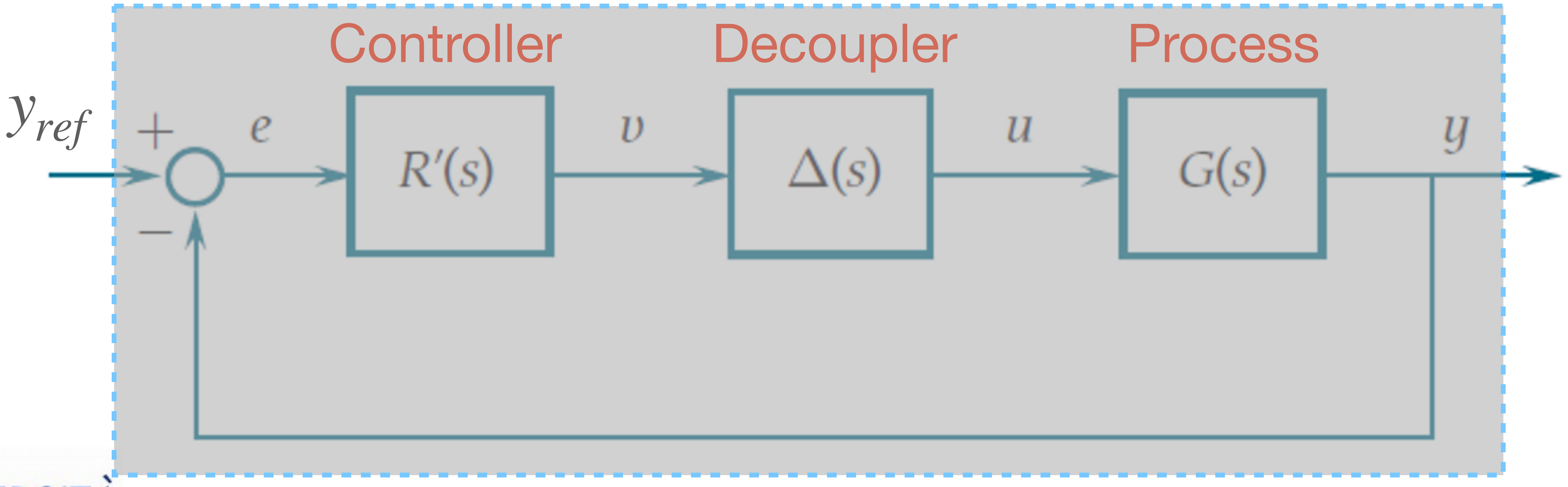
$G_{d_{11}}(s)$

design: $R'(s) = \begin{bmatrix} R'_1(s) & 0 \\ 0 & R'_2(s) \end{bmatrix}$

$G_{d_{22}}(s)$

Decoupling Based Control Schemes: Backward Decoupling





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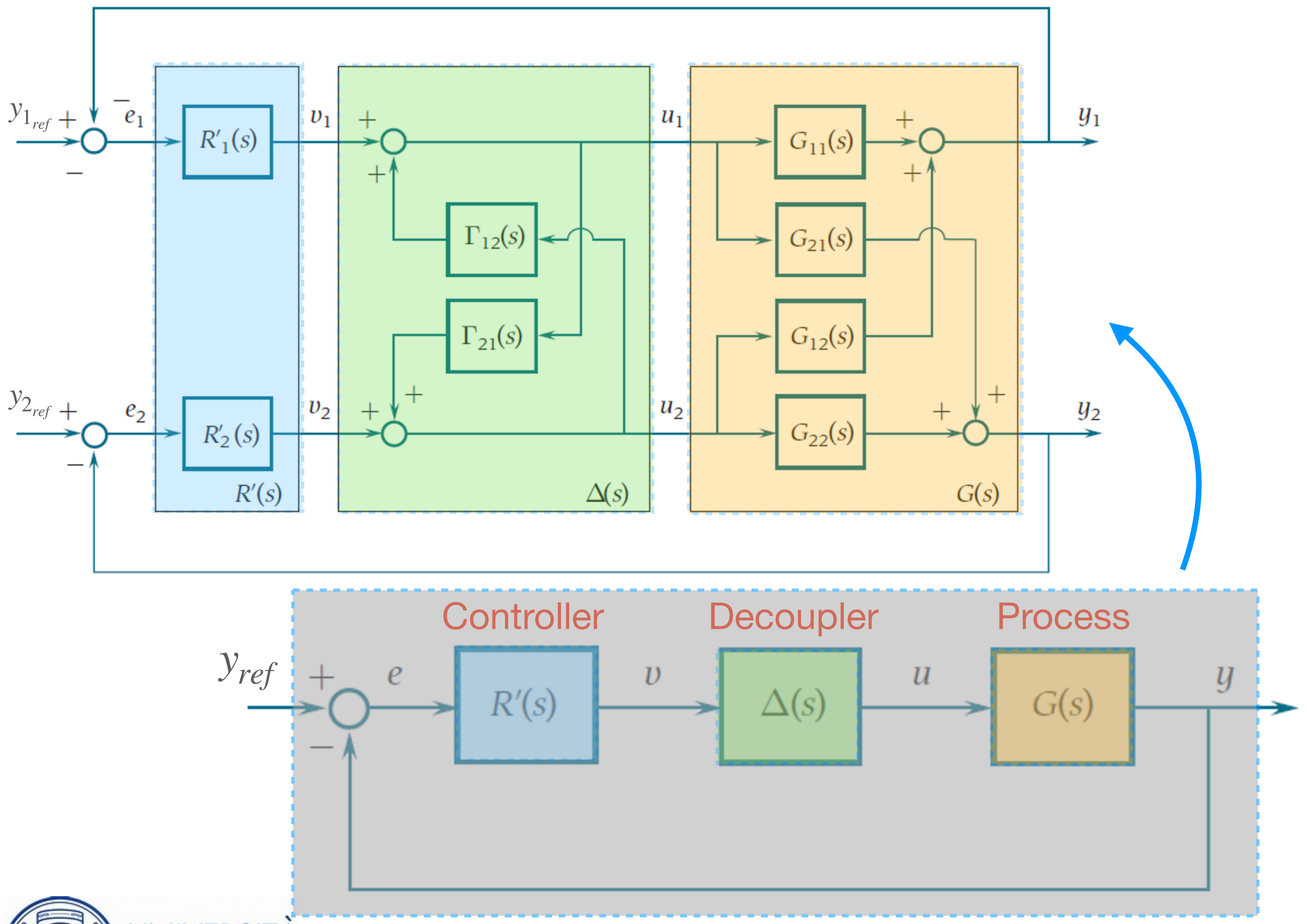
Decoupling Based Control Schemes: Backward Decoupling



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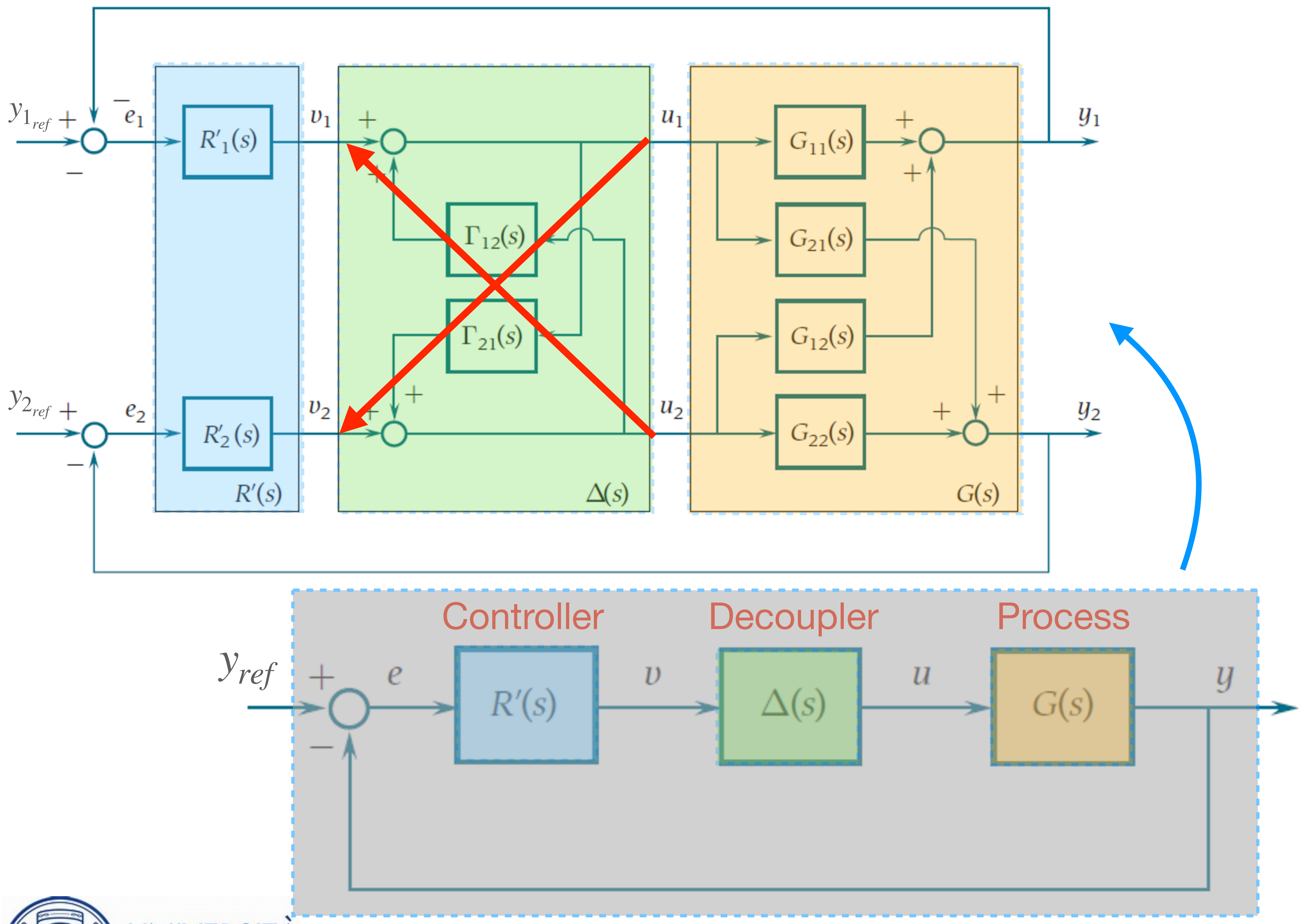
Decoupling Based Control Schemes: Backward Decoupling



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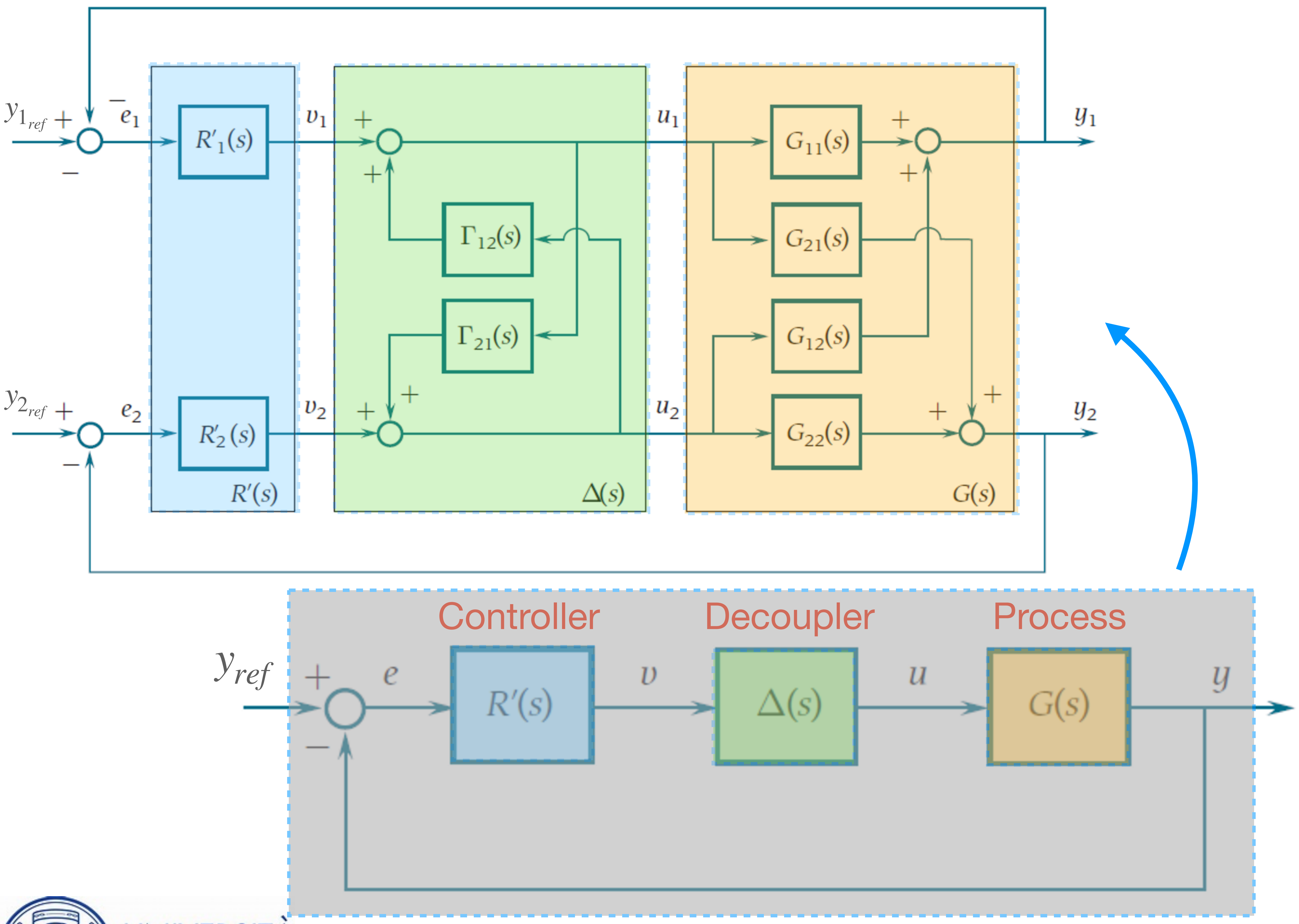
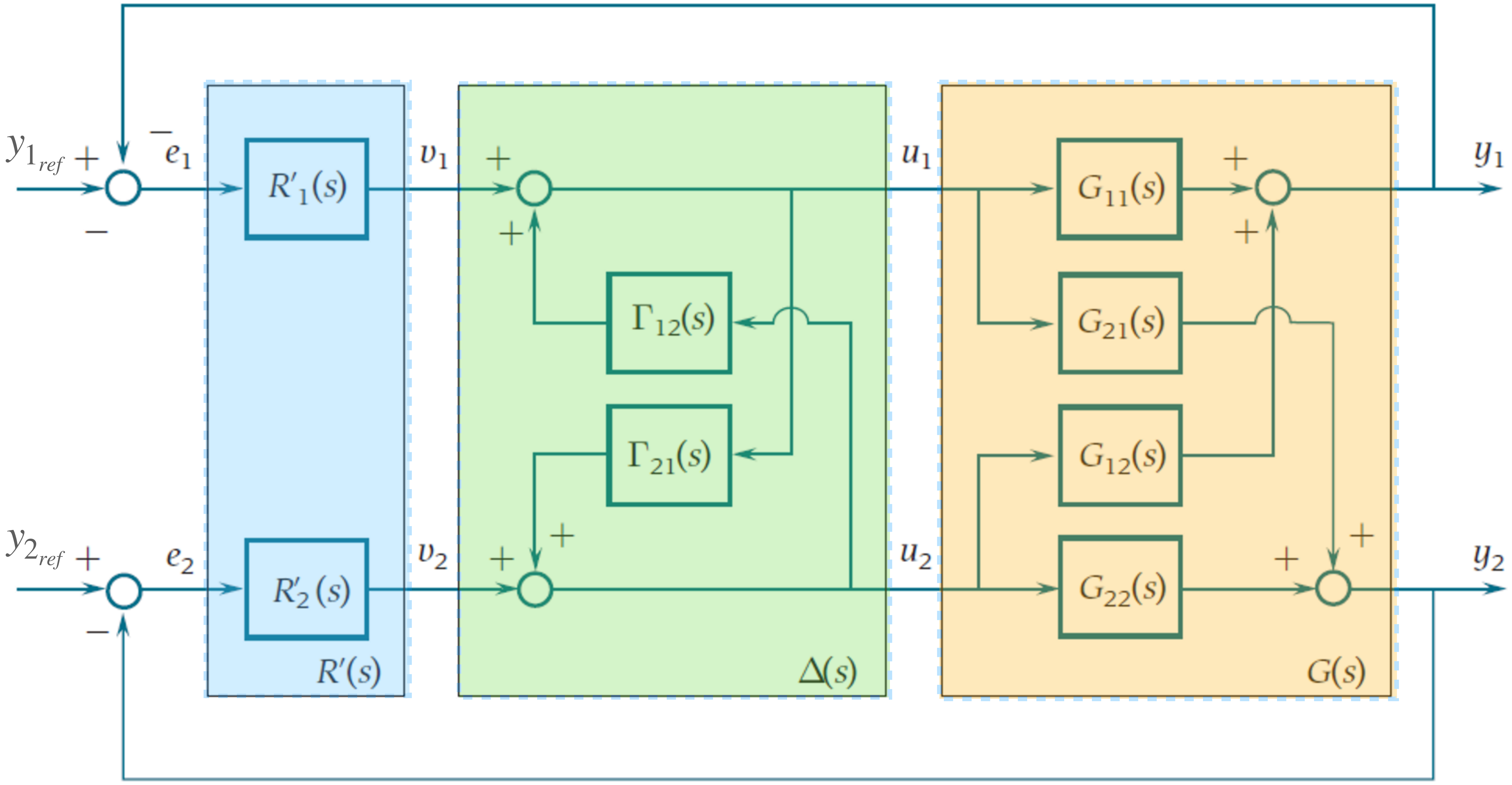
Decoupling Based Control Schemes: Backward Decoupling



$$G(s)\Delta(s) = \tilde{G}_d(s)$$

Decoupling Based Control Schemes: Backward Decoupling



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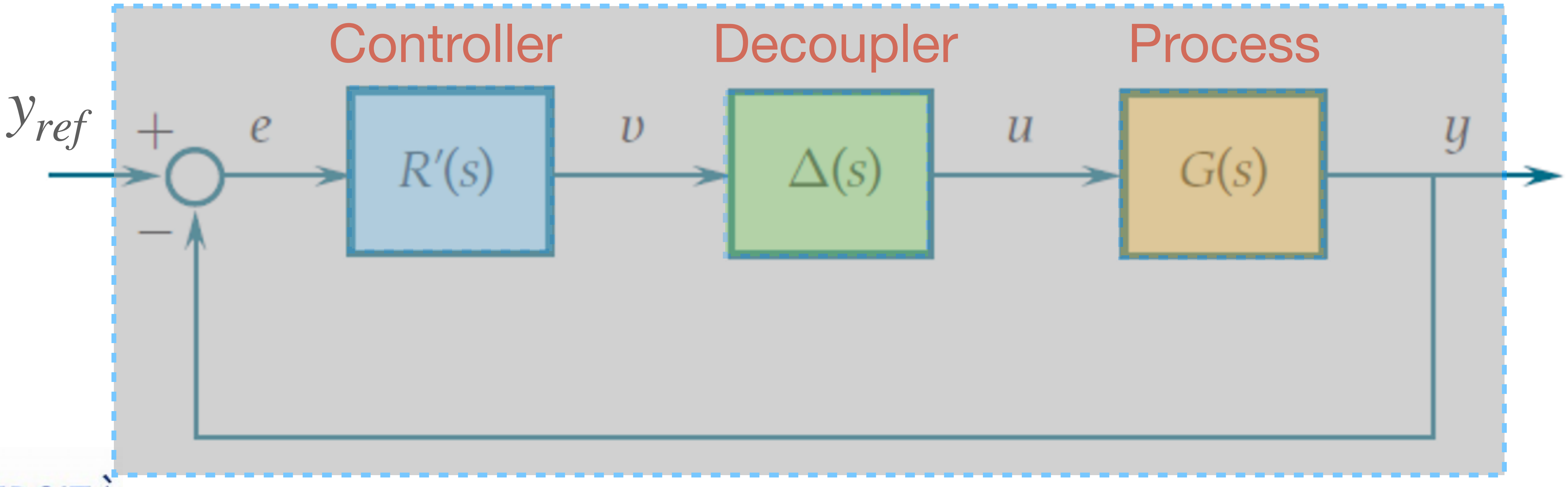
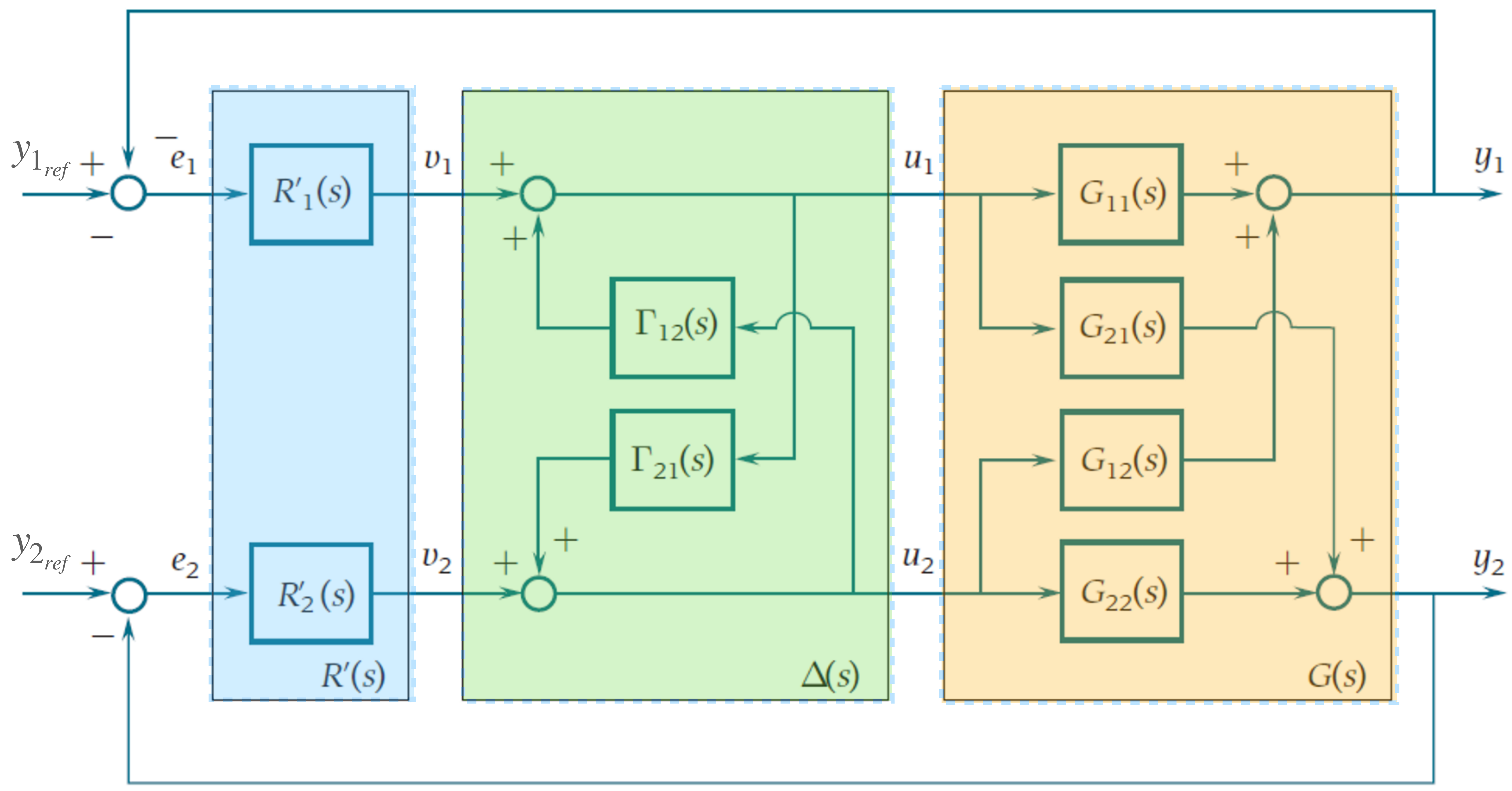


Decoupling Based Control Schemes: Backward Decoupling



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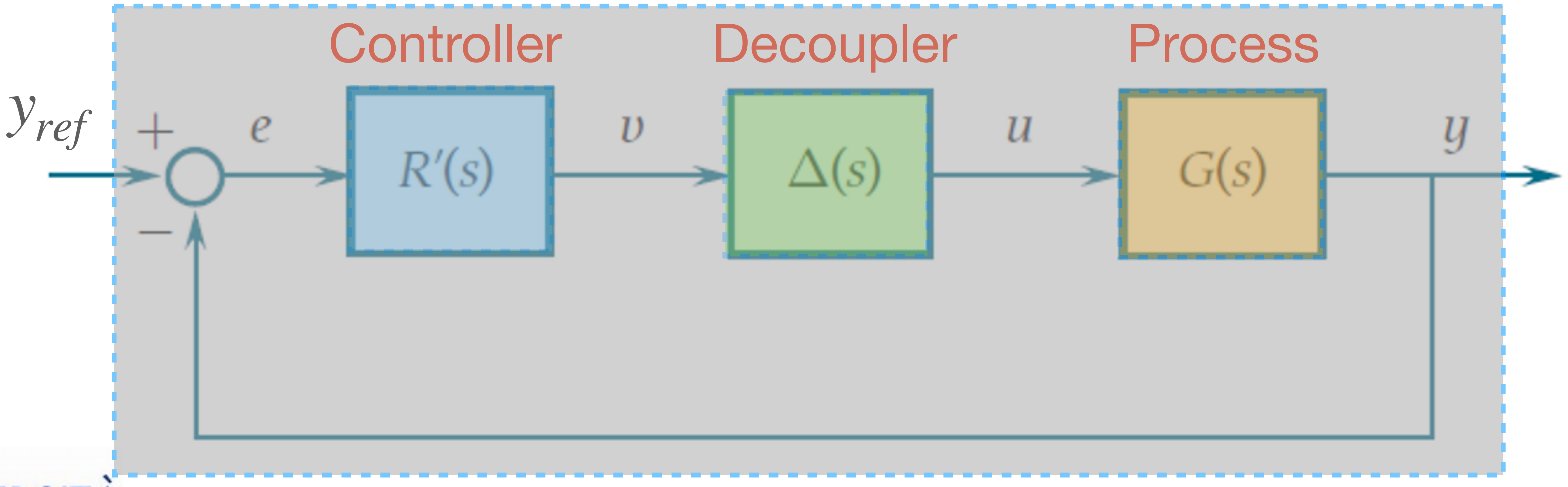
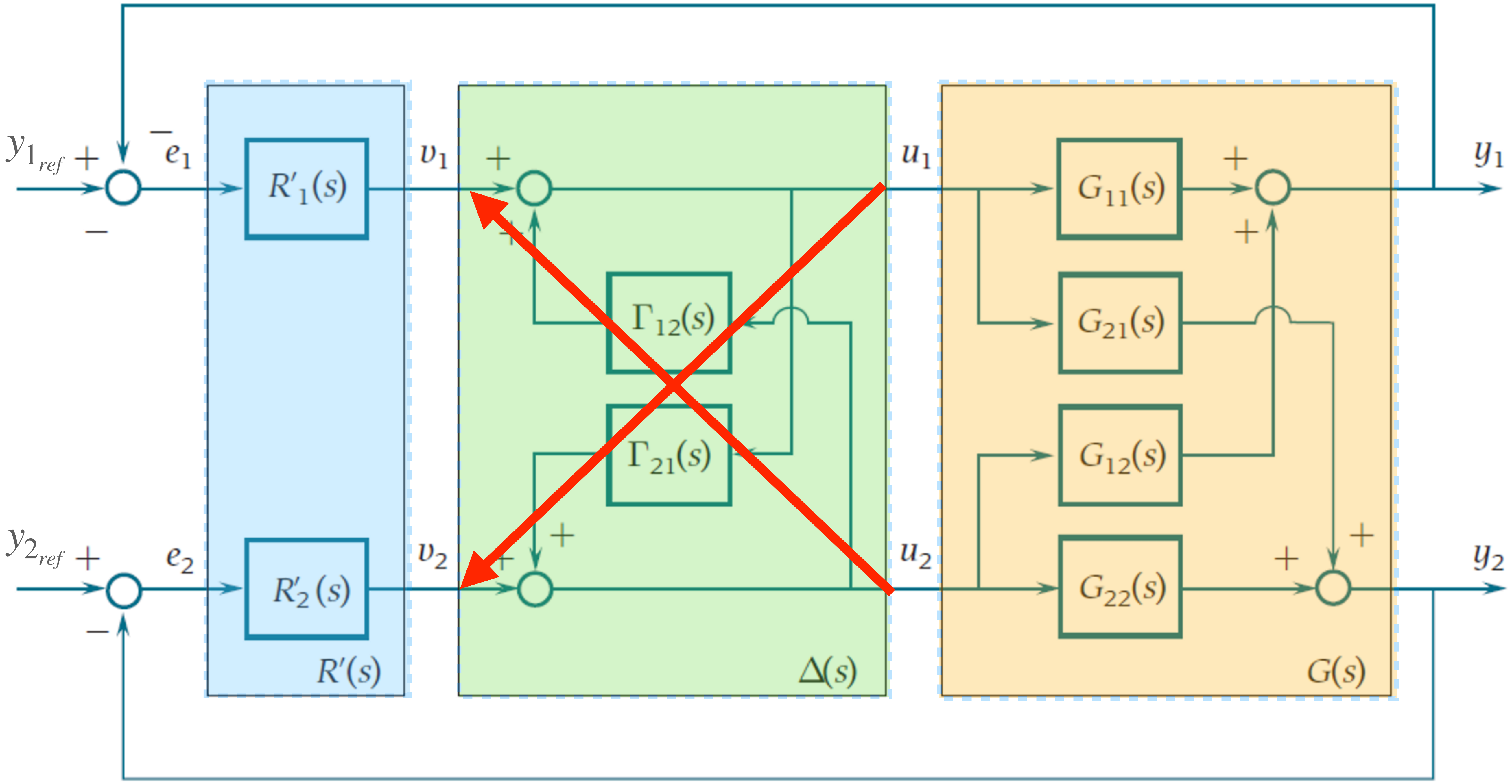


Decoupling Based Control Schemes: Backward Decoupling



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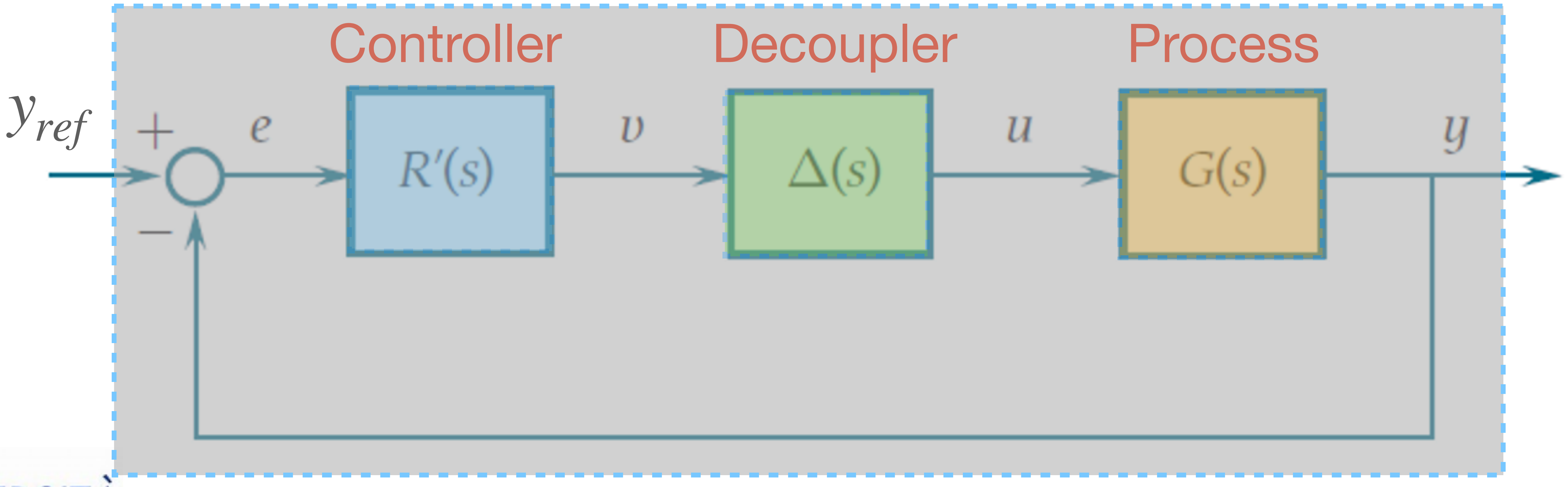
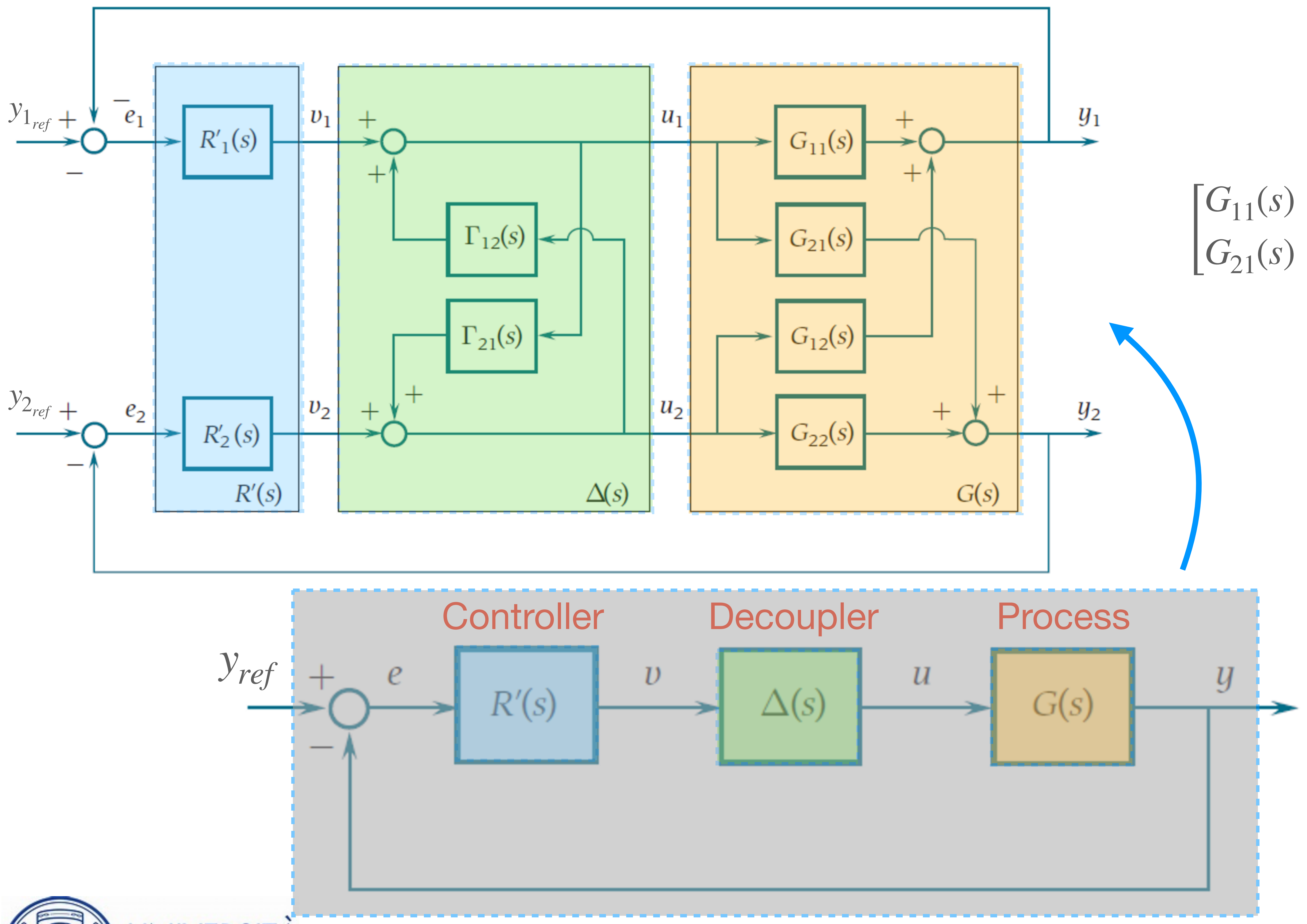
$$U(s) = \Gamma(s)U(s) + V(s)$$



Decoupling Based Control Schemes: Backward Decoupling



$$G(s)\Delta(s) = \tilde{G}_d(s)$$

$$\begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} \Delta_{11}(s) & \Delta_{12}(s) \\ \Delta_{21}(s) & \Delta_{22}(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & 0 \\ 0 & G_{22}(s) \end{bmatrix}$$

$$U(s) = \Gamma(s)U(s) + V(s)$$

$$U(s) - \Gamma(s)U(s) = V(s)$$

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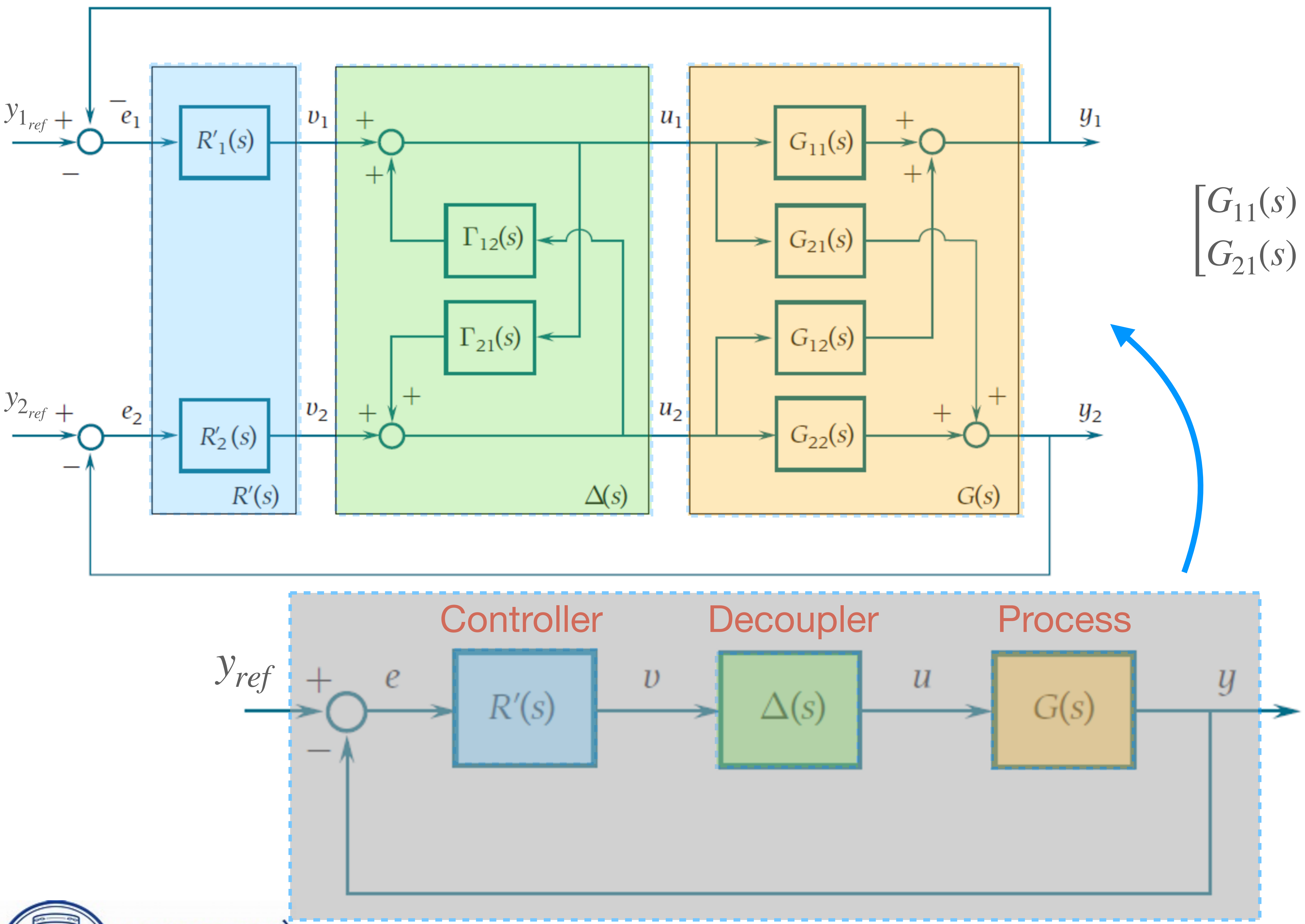

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$$U(s) - \Gamma(s)U(s) = V(s)$$

$$[I - \Gamma(s)]U(s) = V(s)$$

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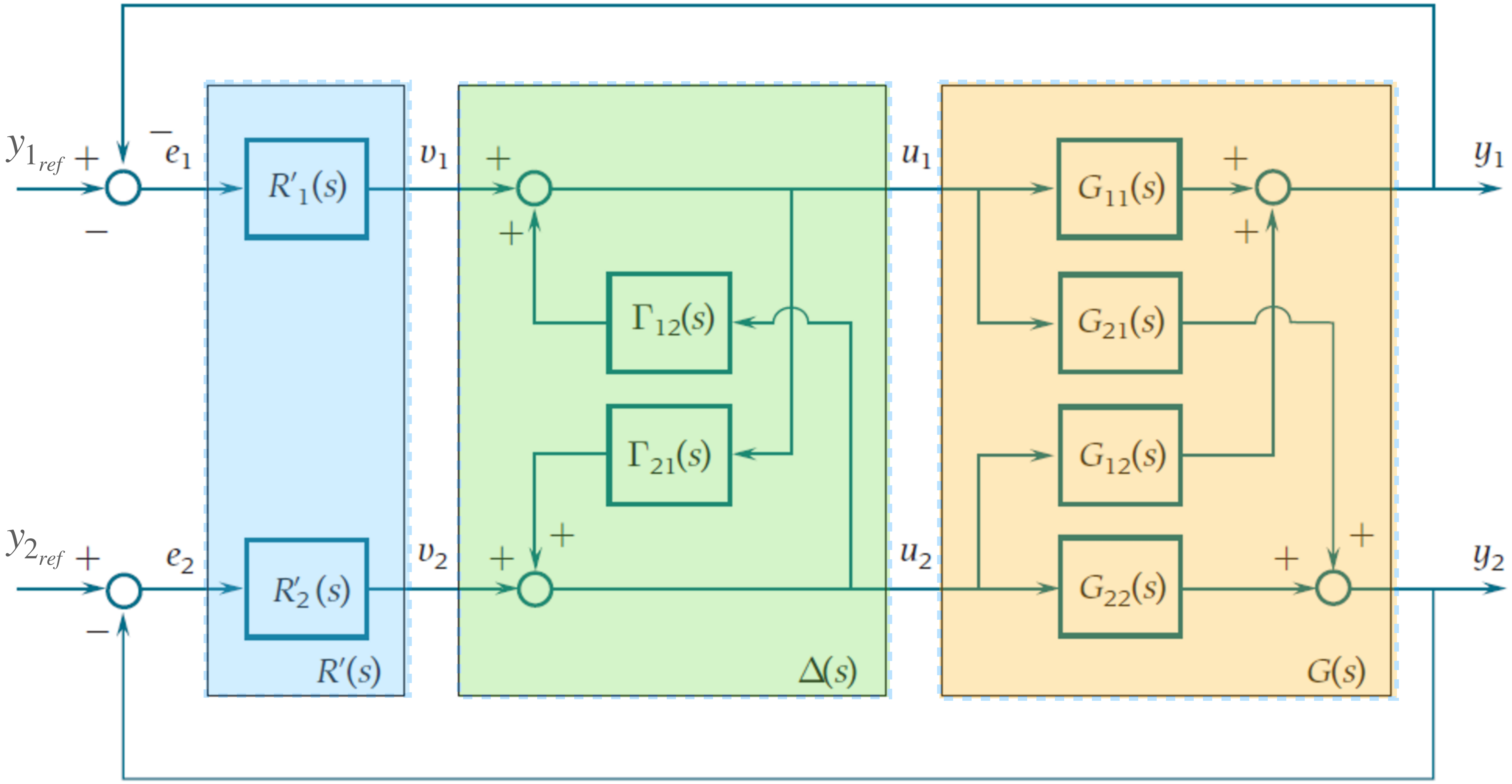

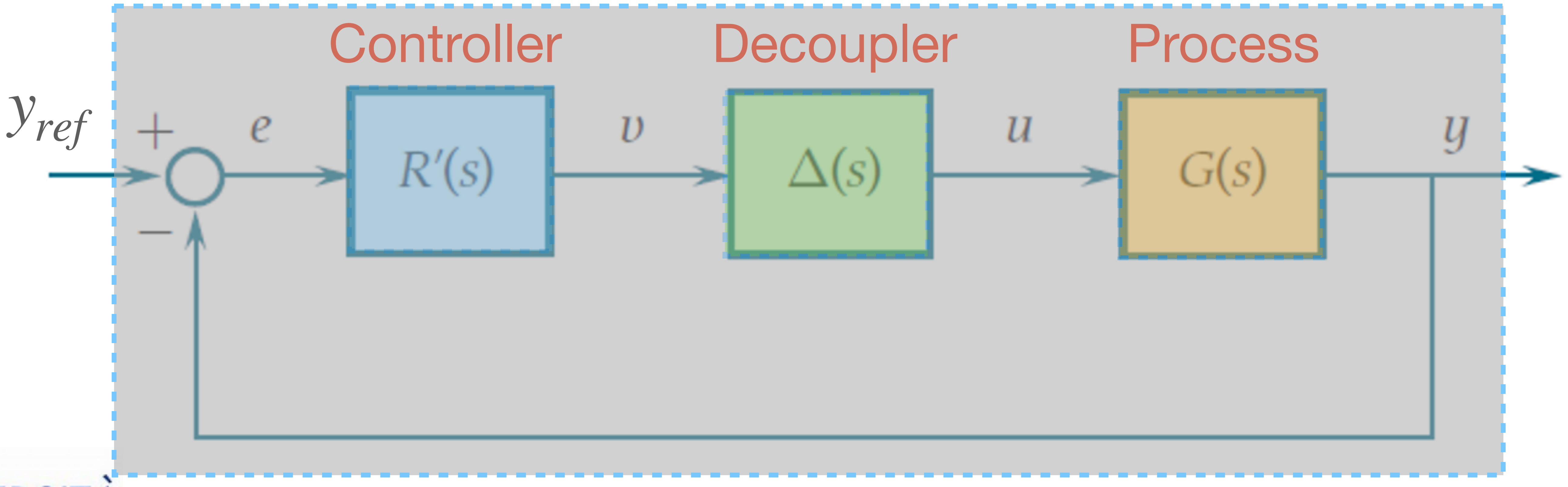

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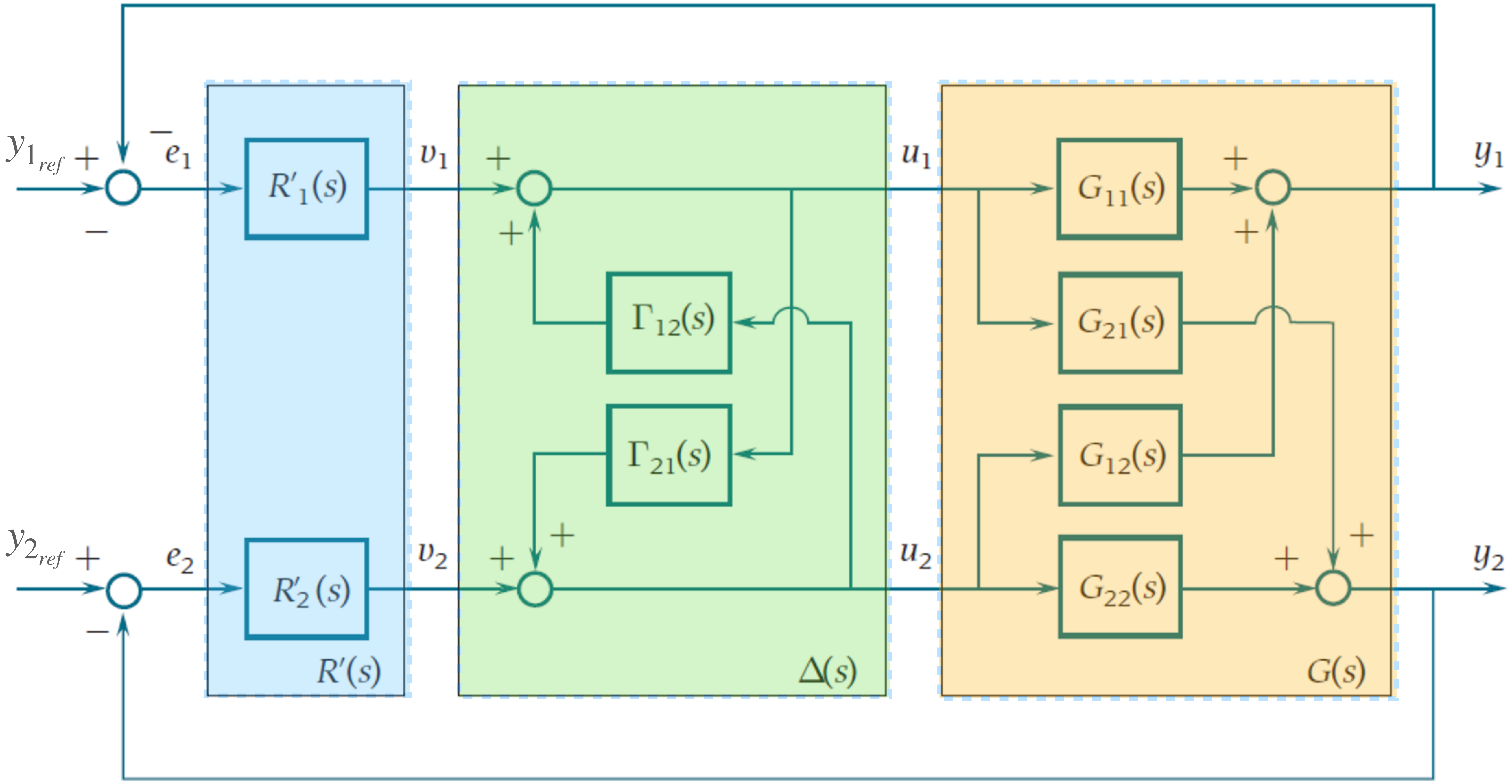

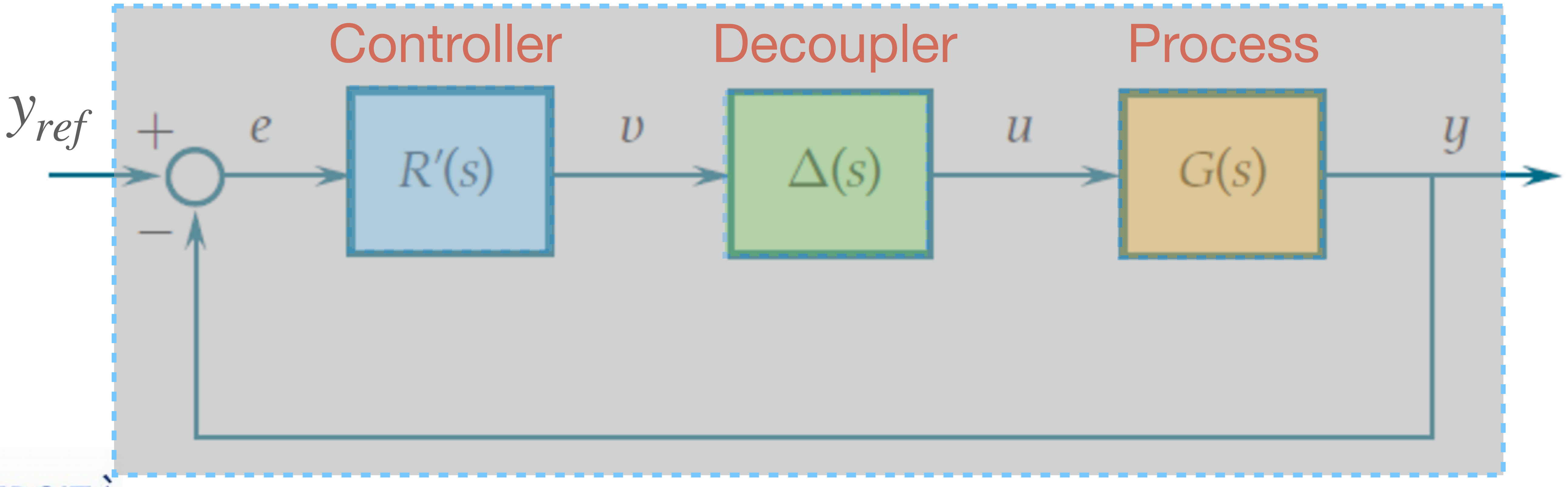

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?

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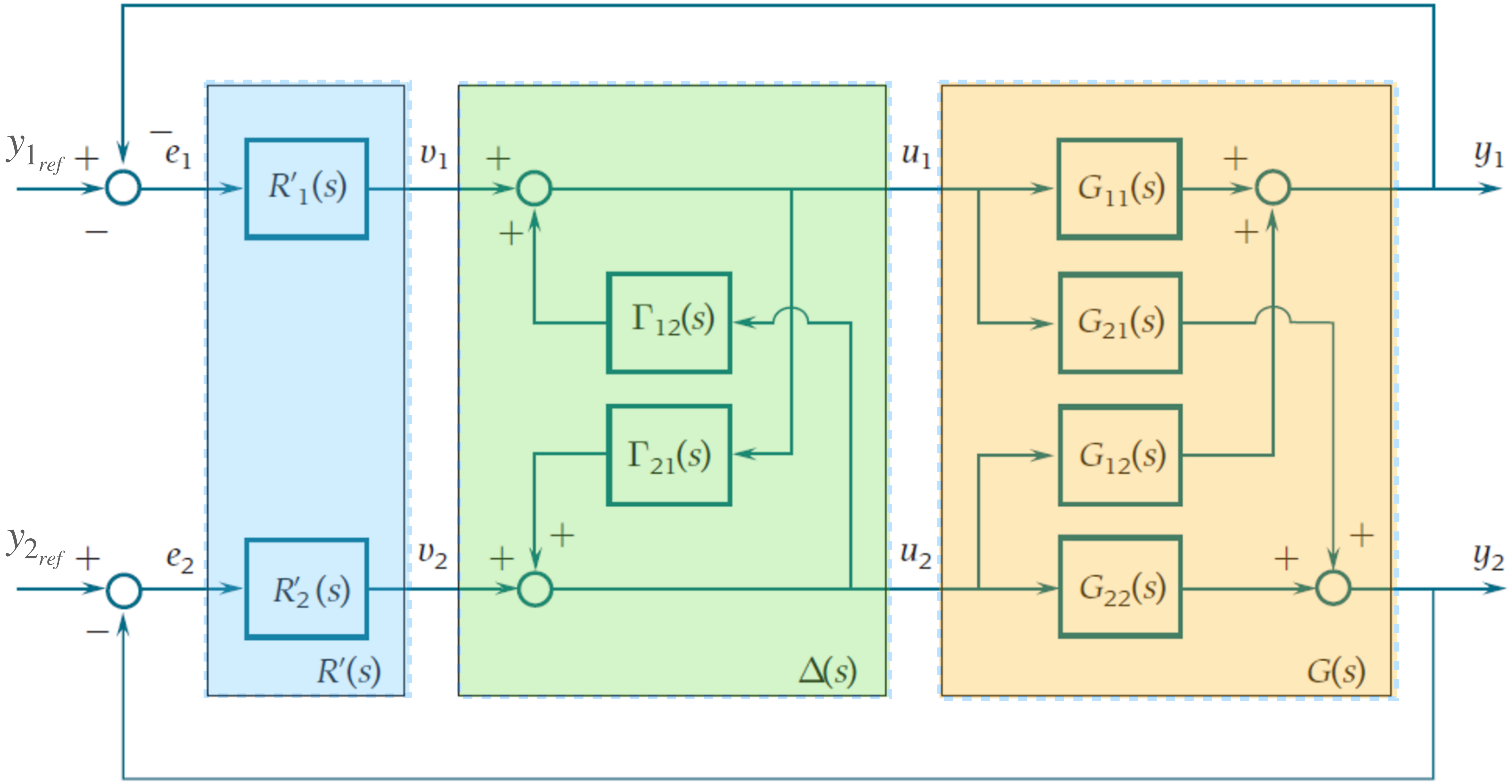

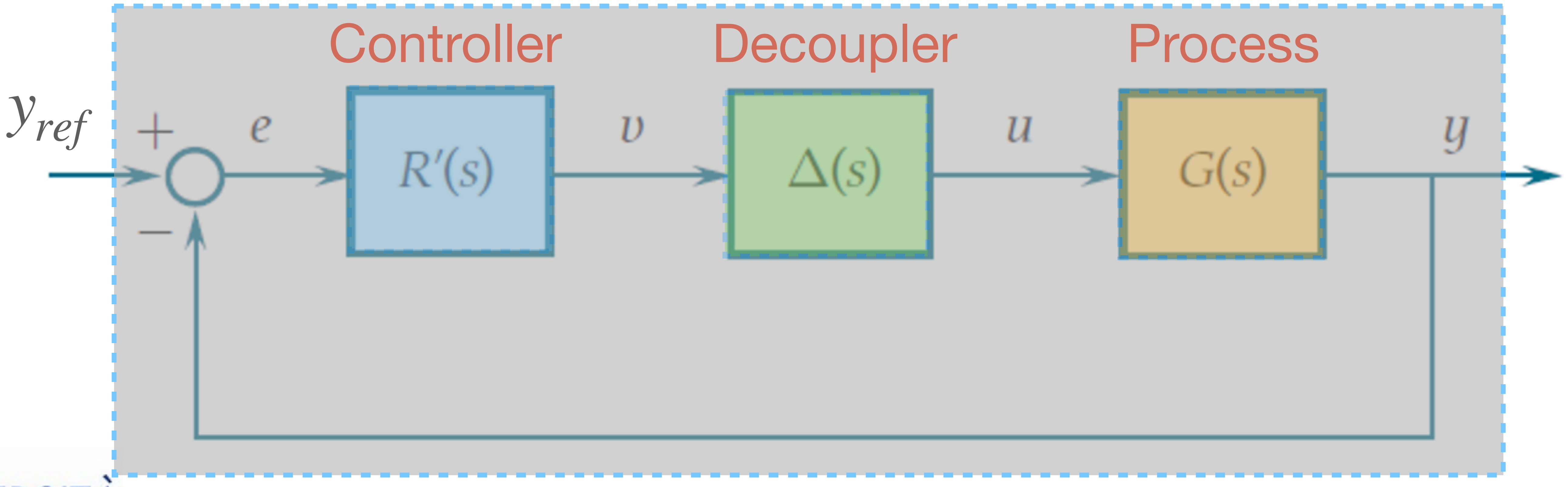

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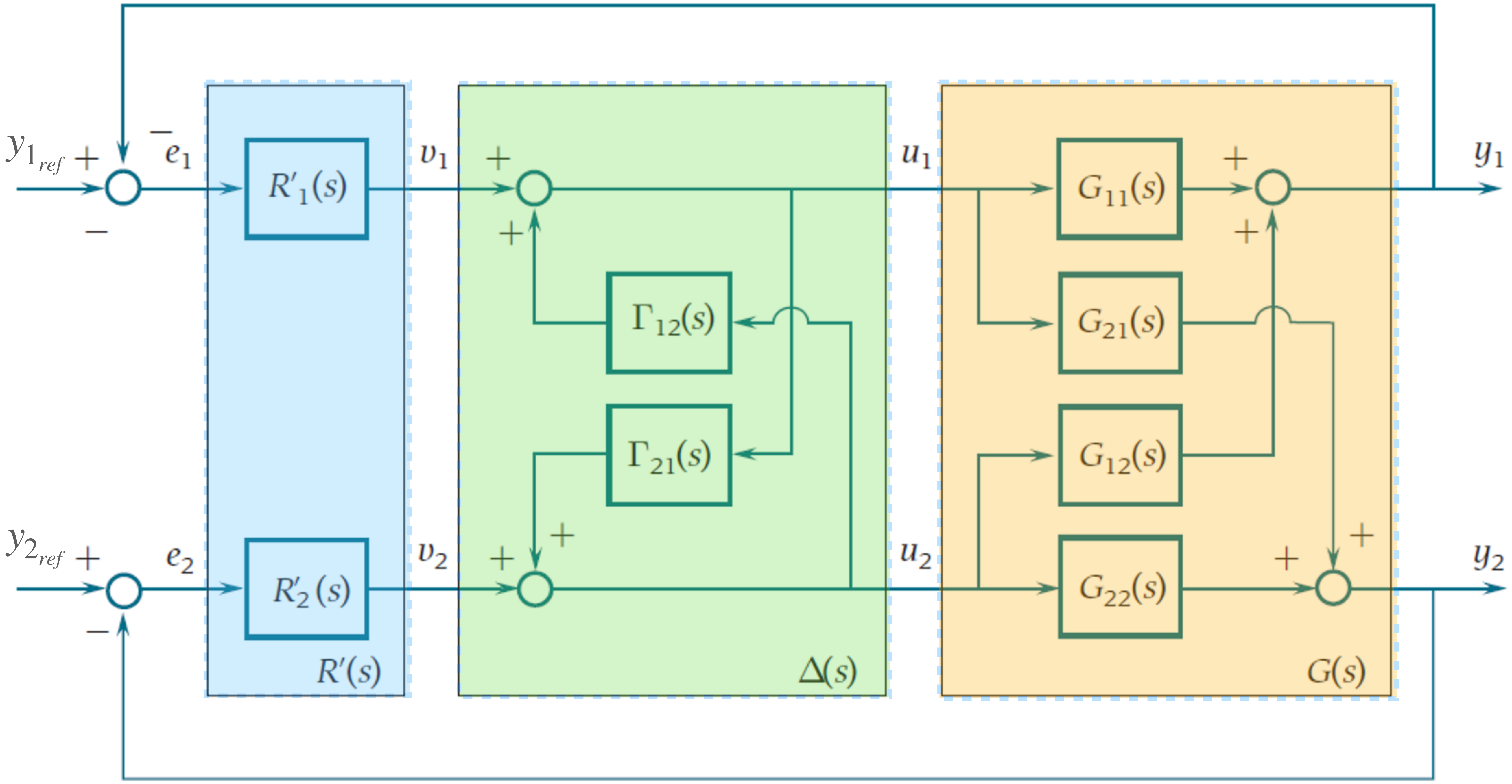

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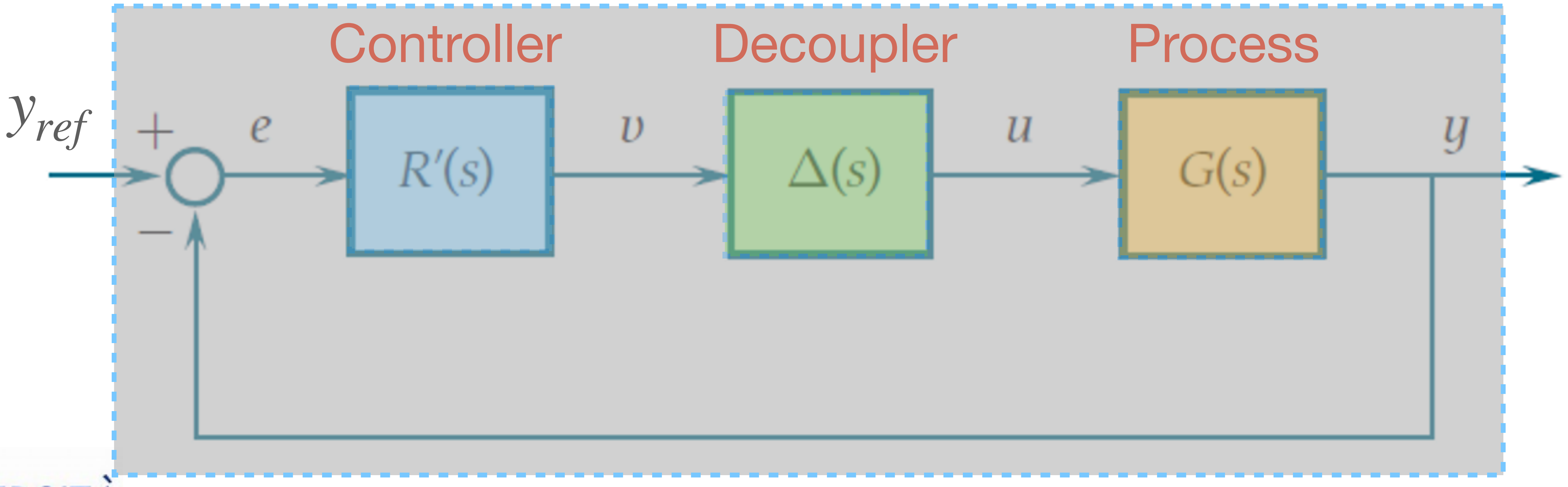

$$\Delta(s) = [I - \Gamma(s)]^{-1}$$

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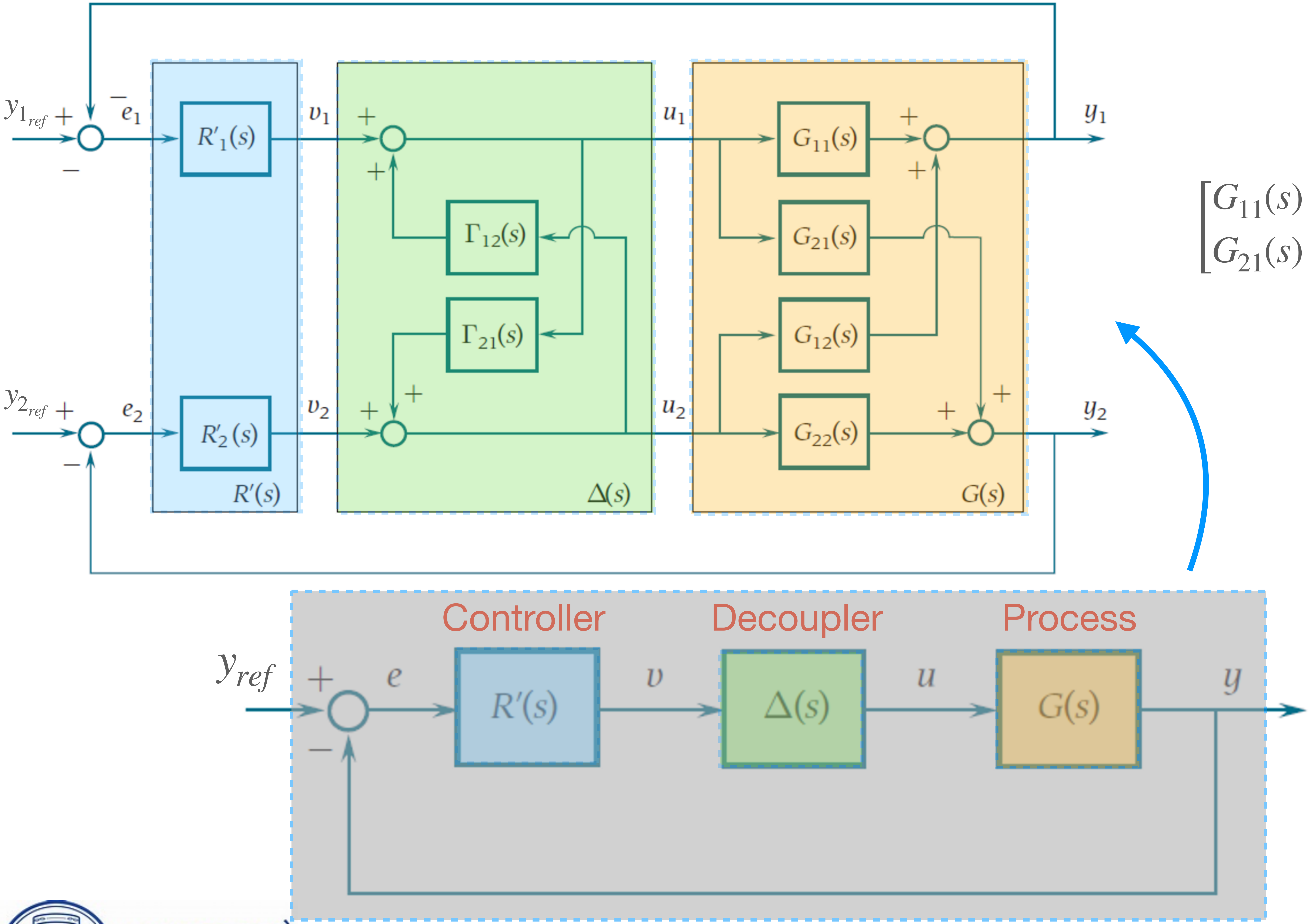

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$$G(s)[I - \Gamma(s)]^{-1} = \tilde{G}_d(s)$$

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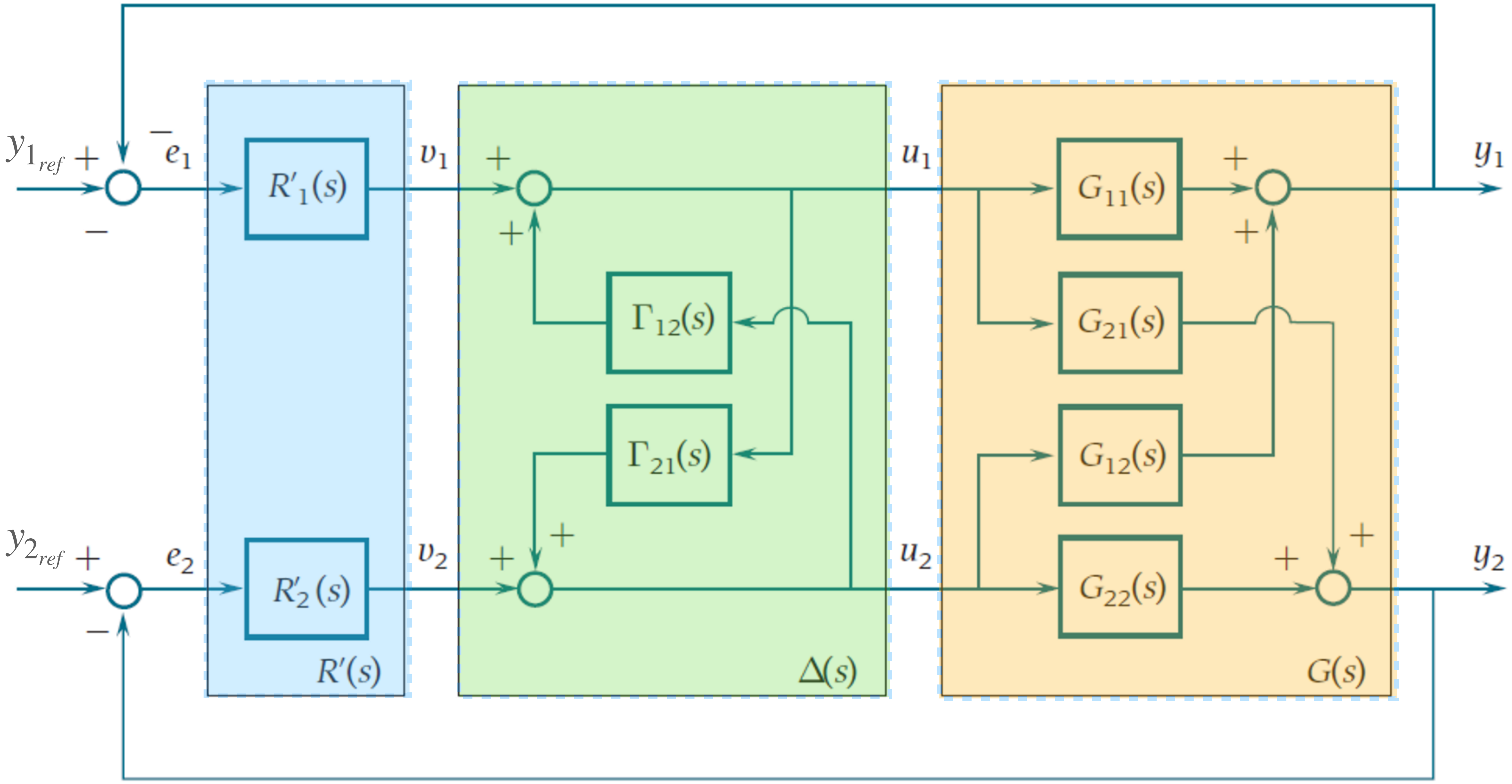

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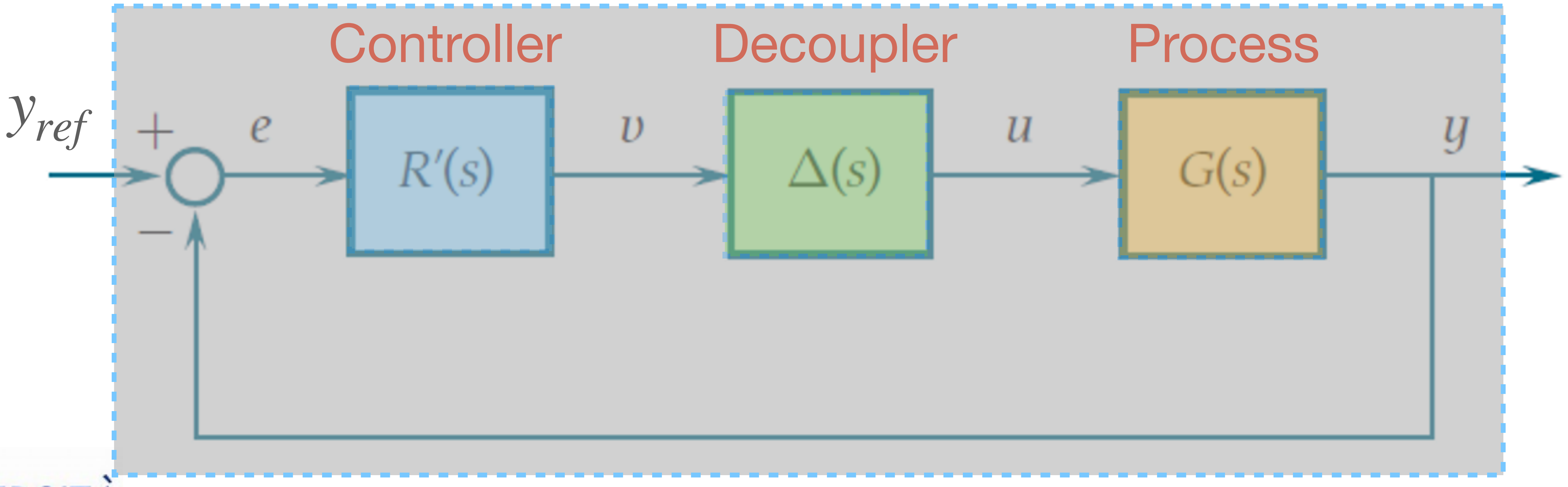

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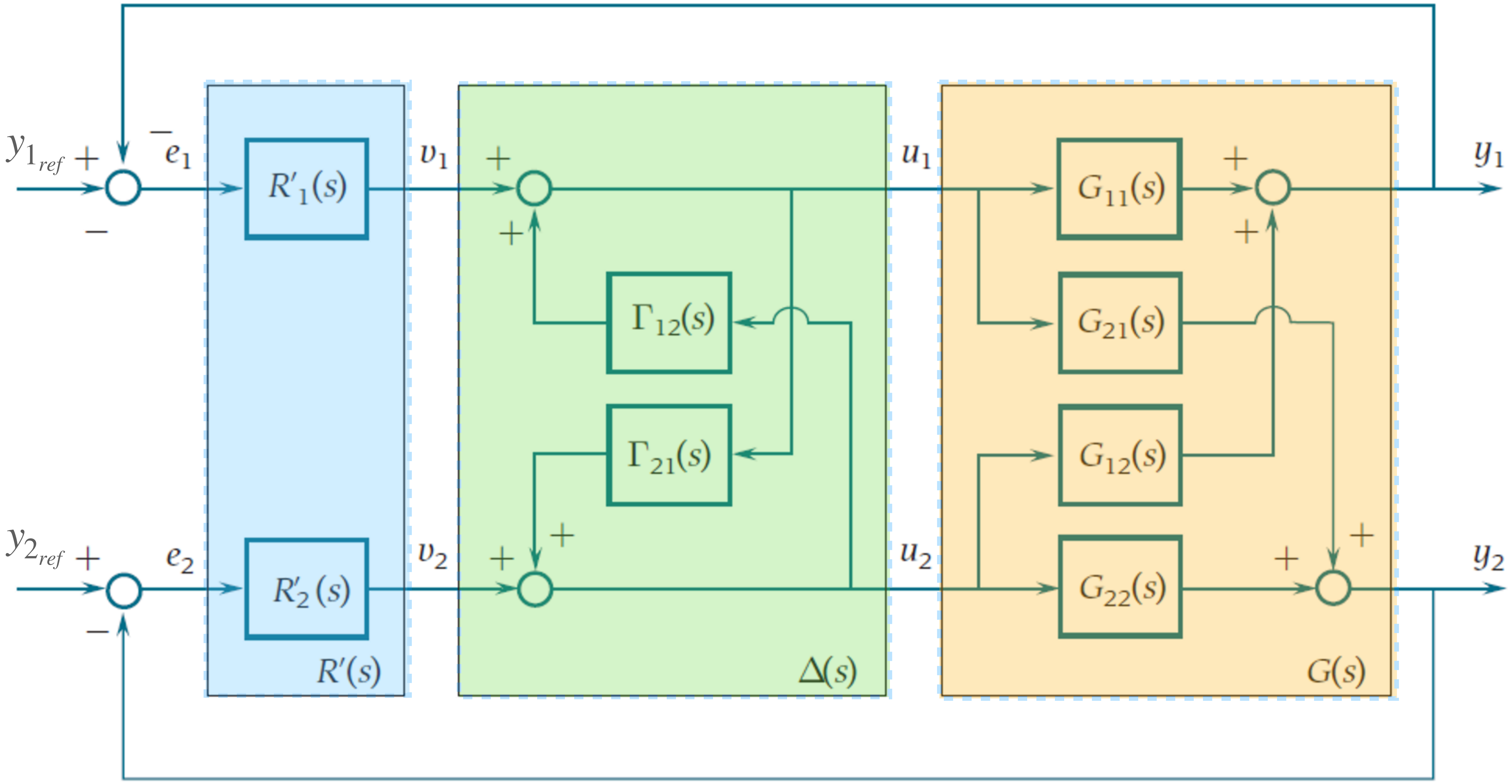

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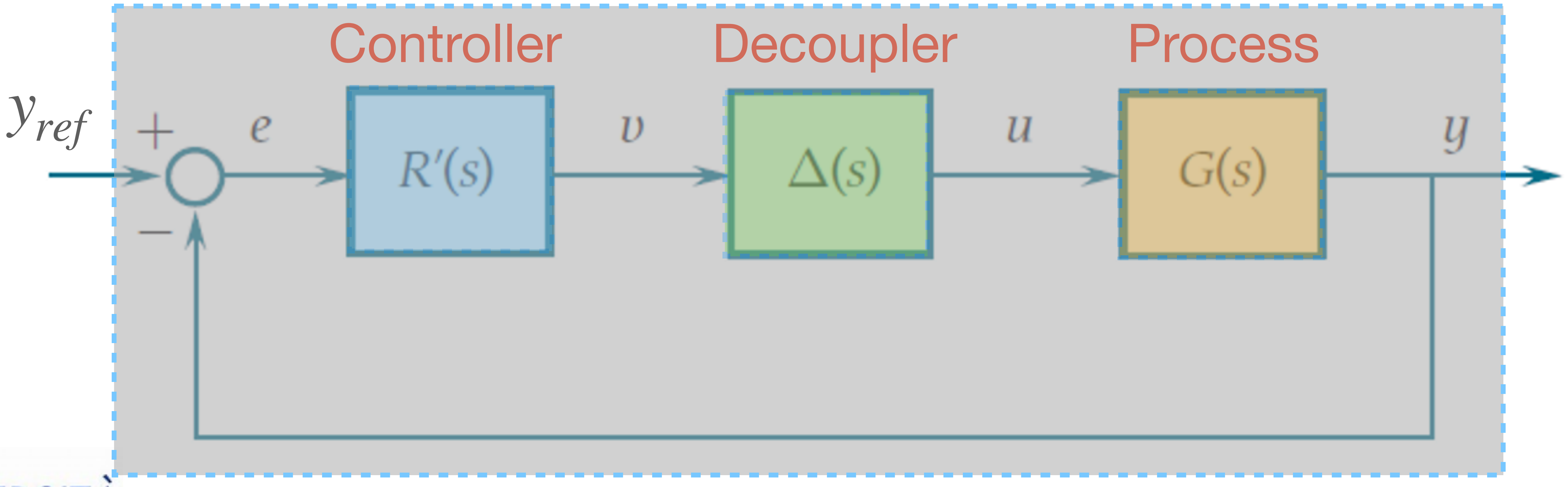

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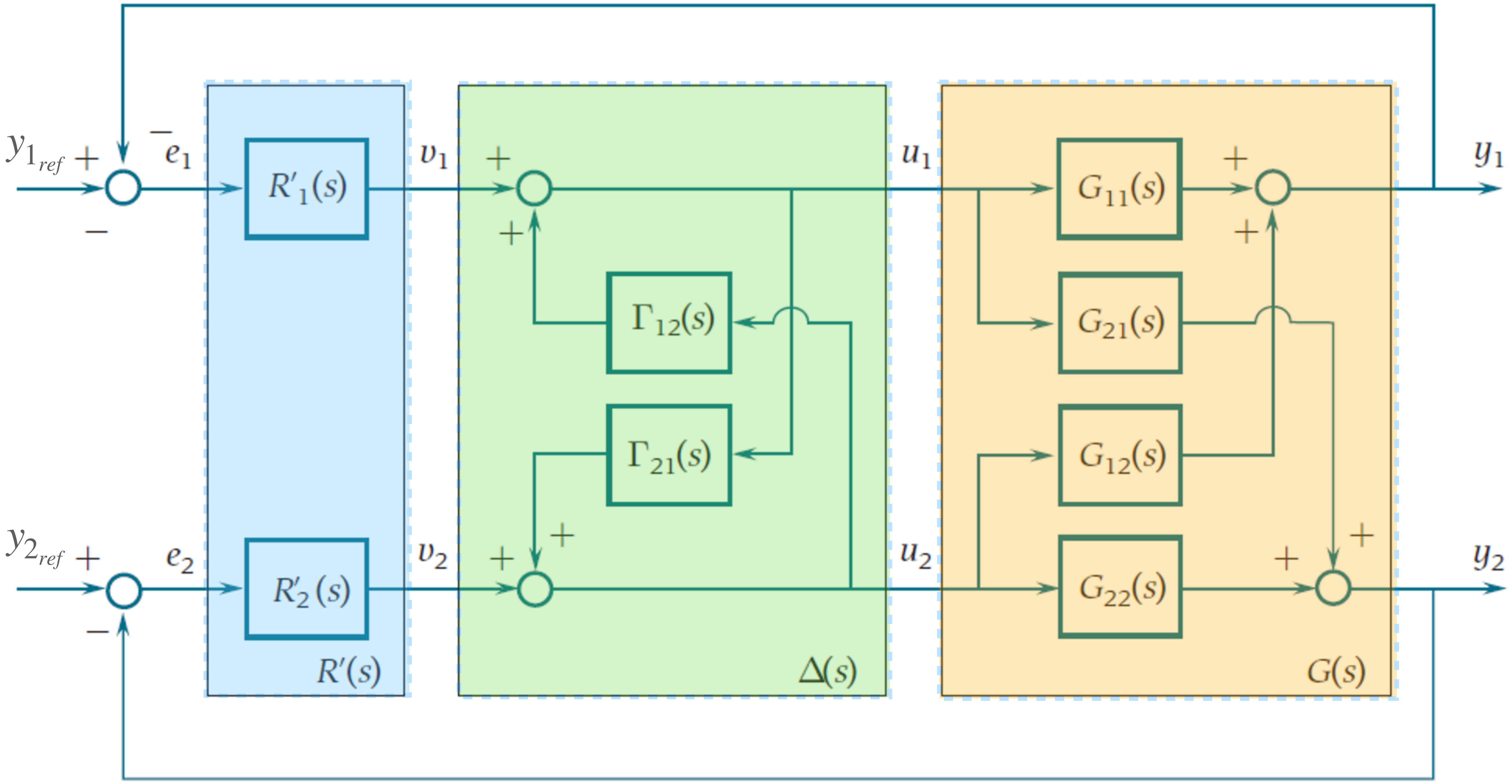

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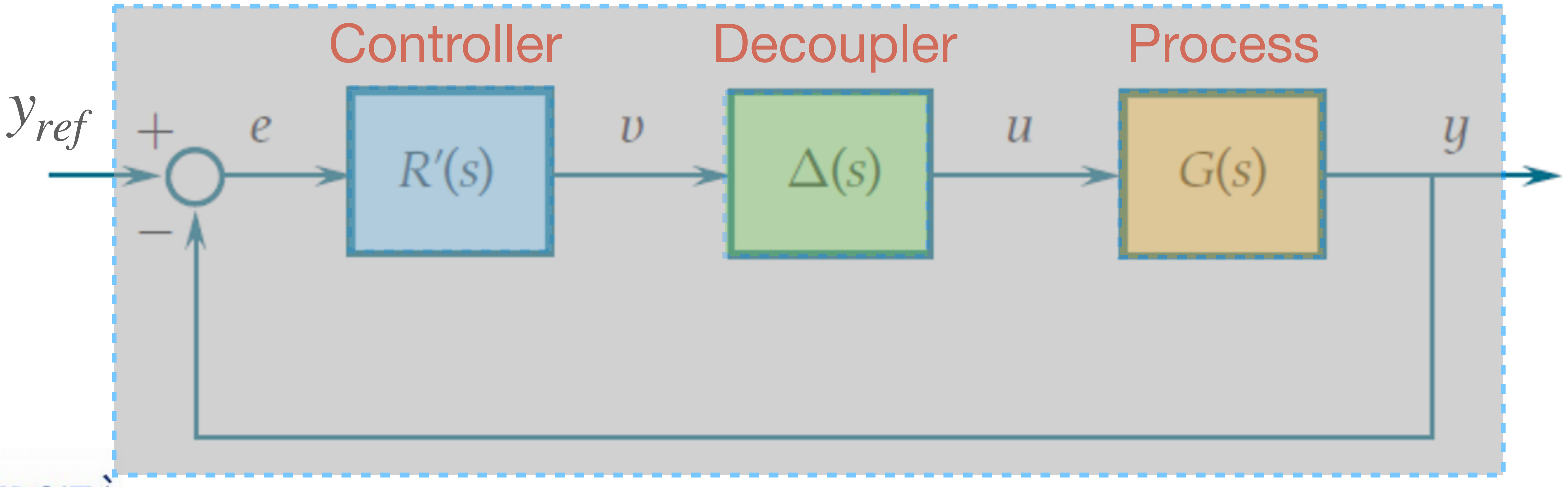

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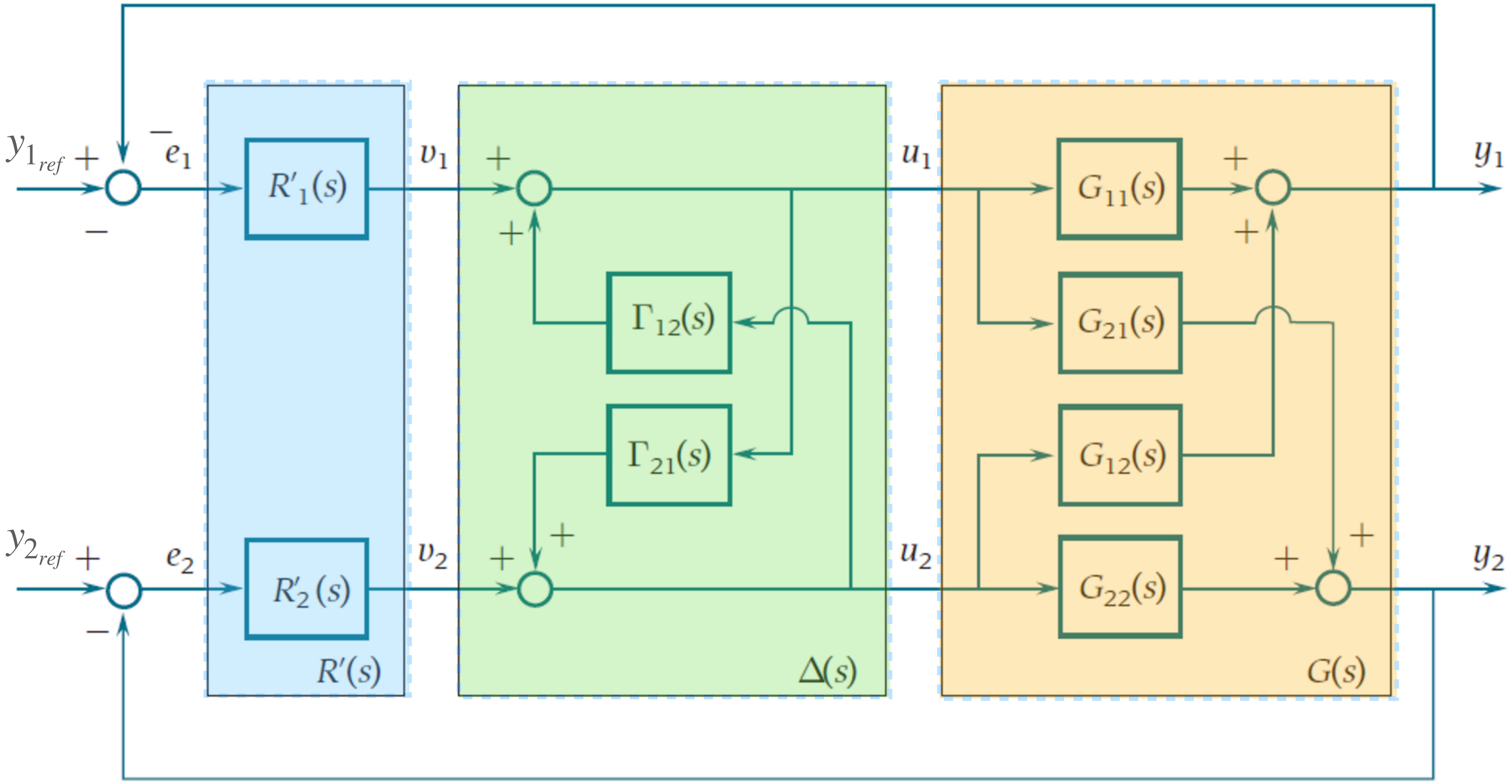

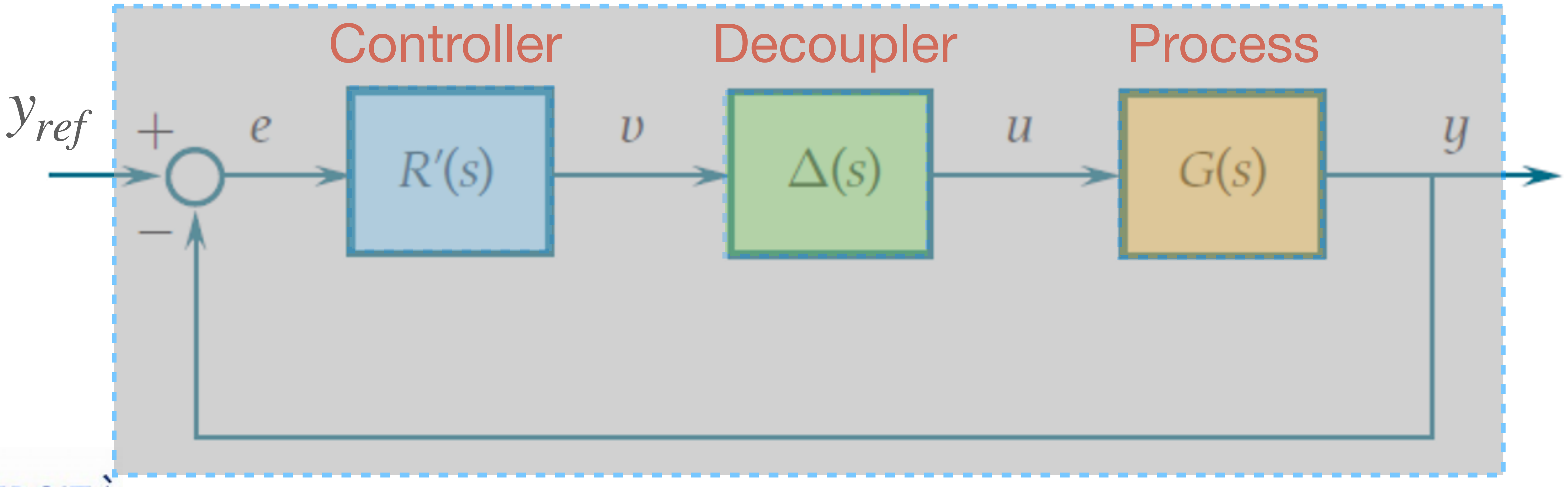

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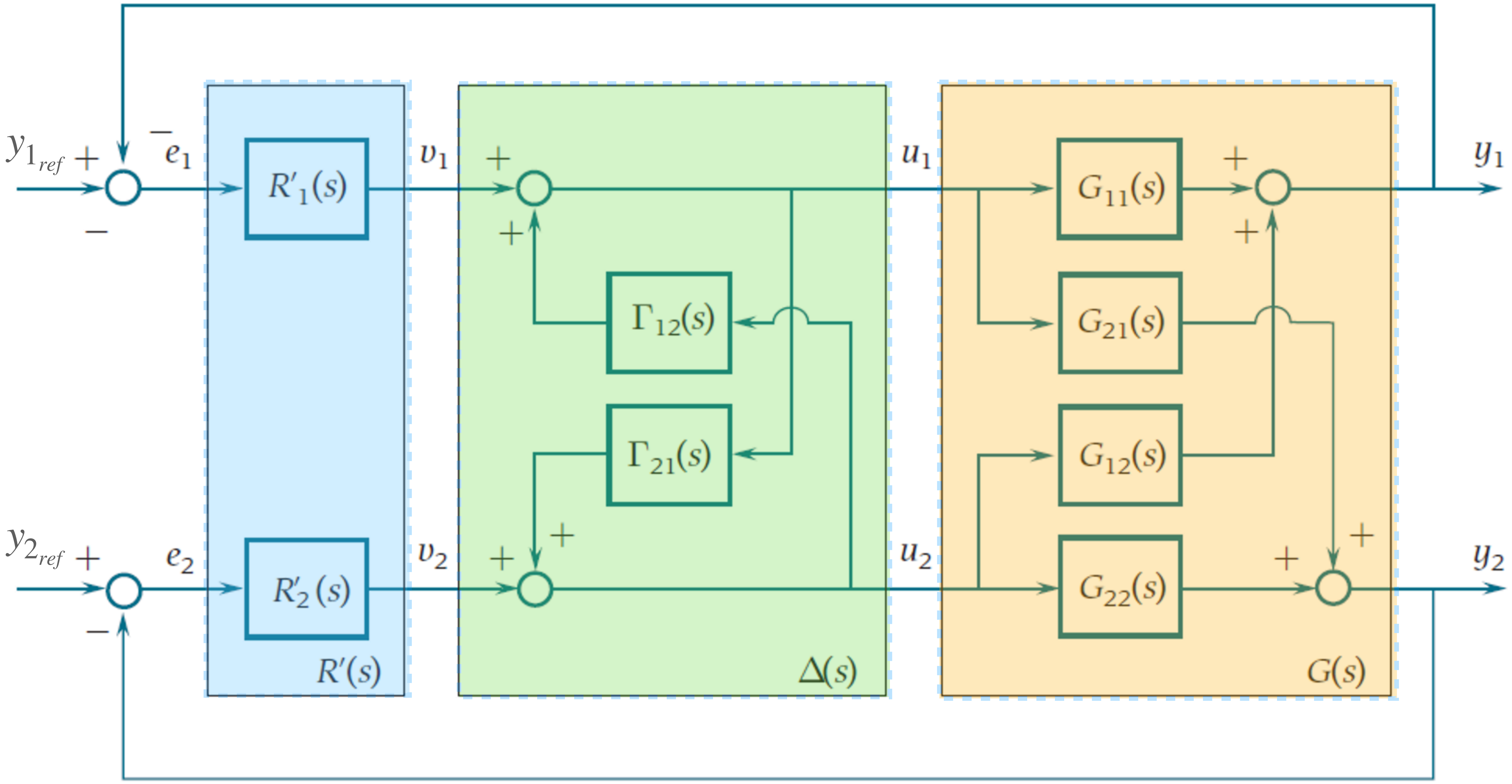

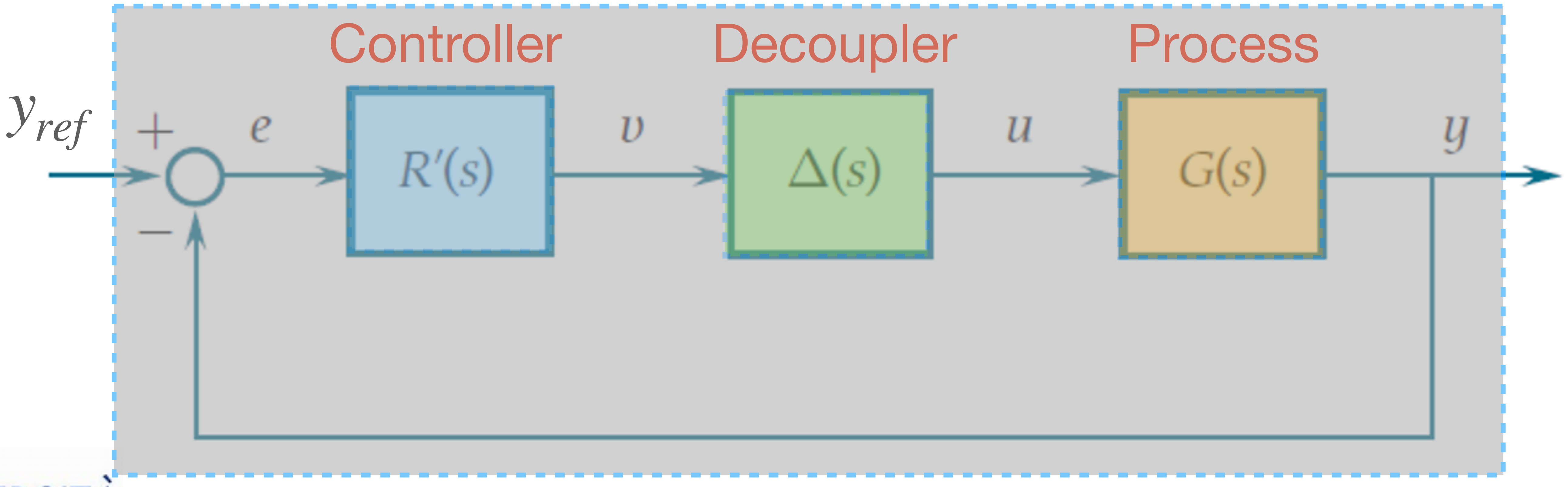

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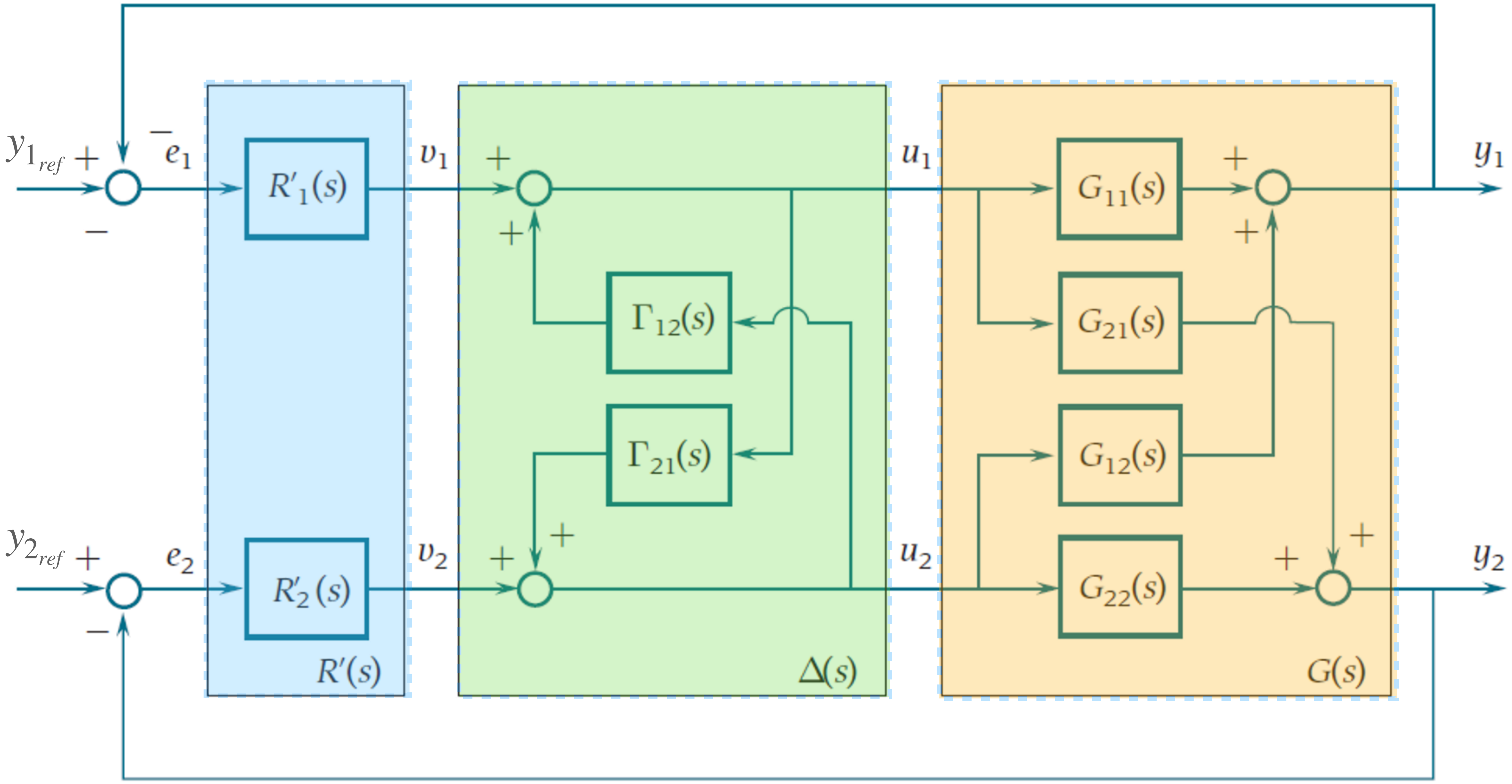

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In the 2×2 case:

$$\Gamma(s) = \tilde{G}_d^{-1}(s)[\tilde{G}_d(s) - G(s)]$$

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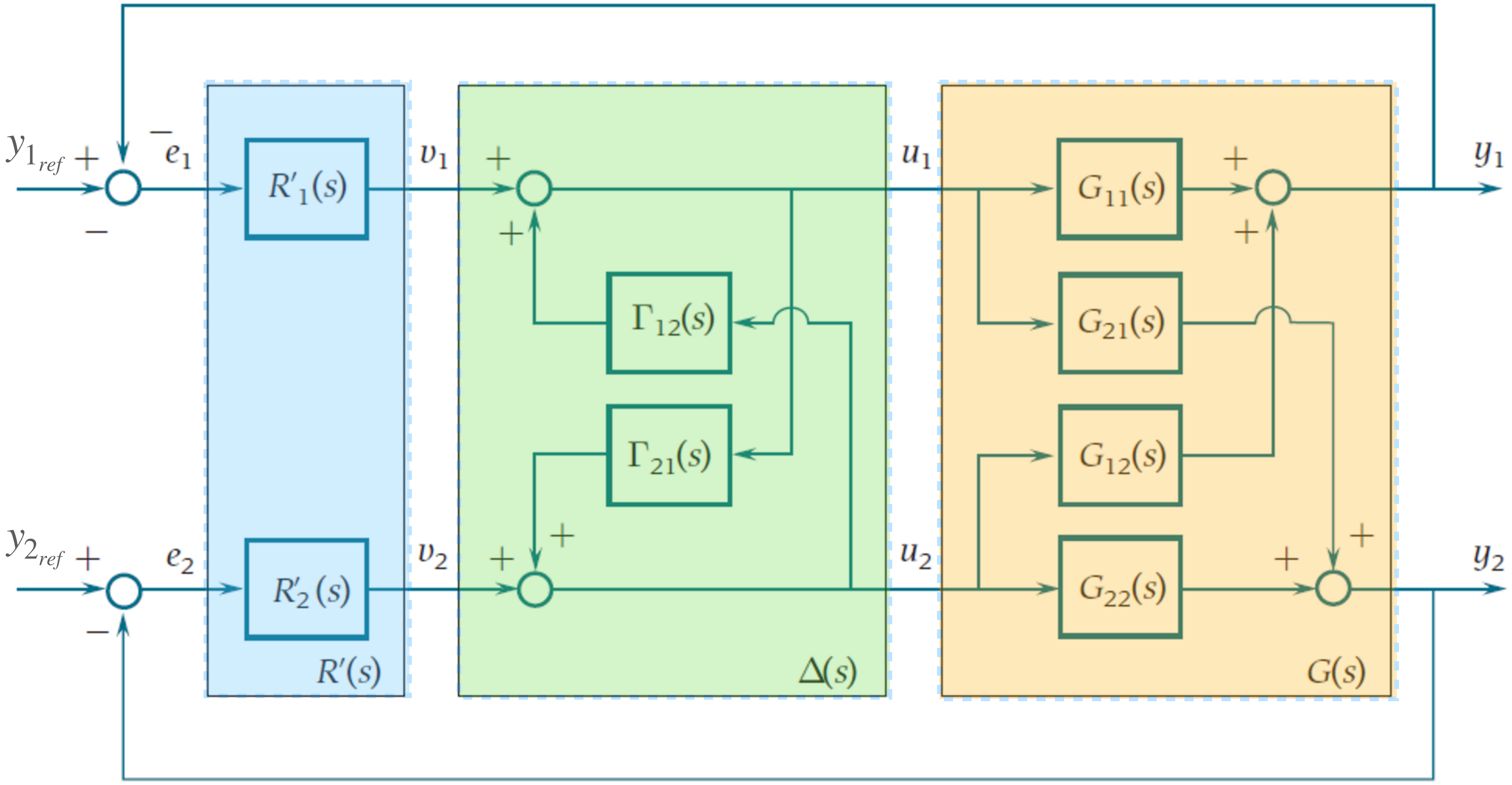

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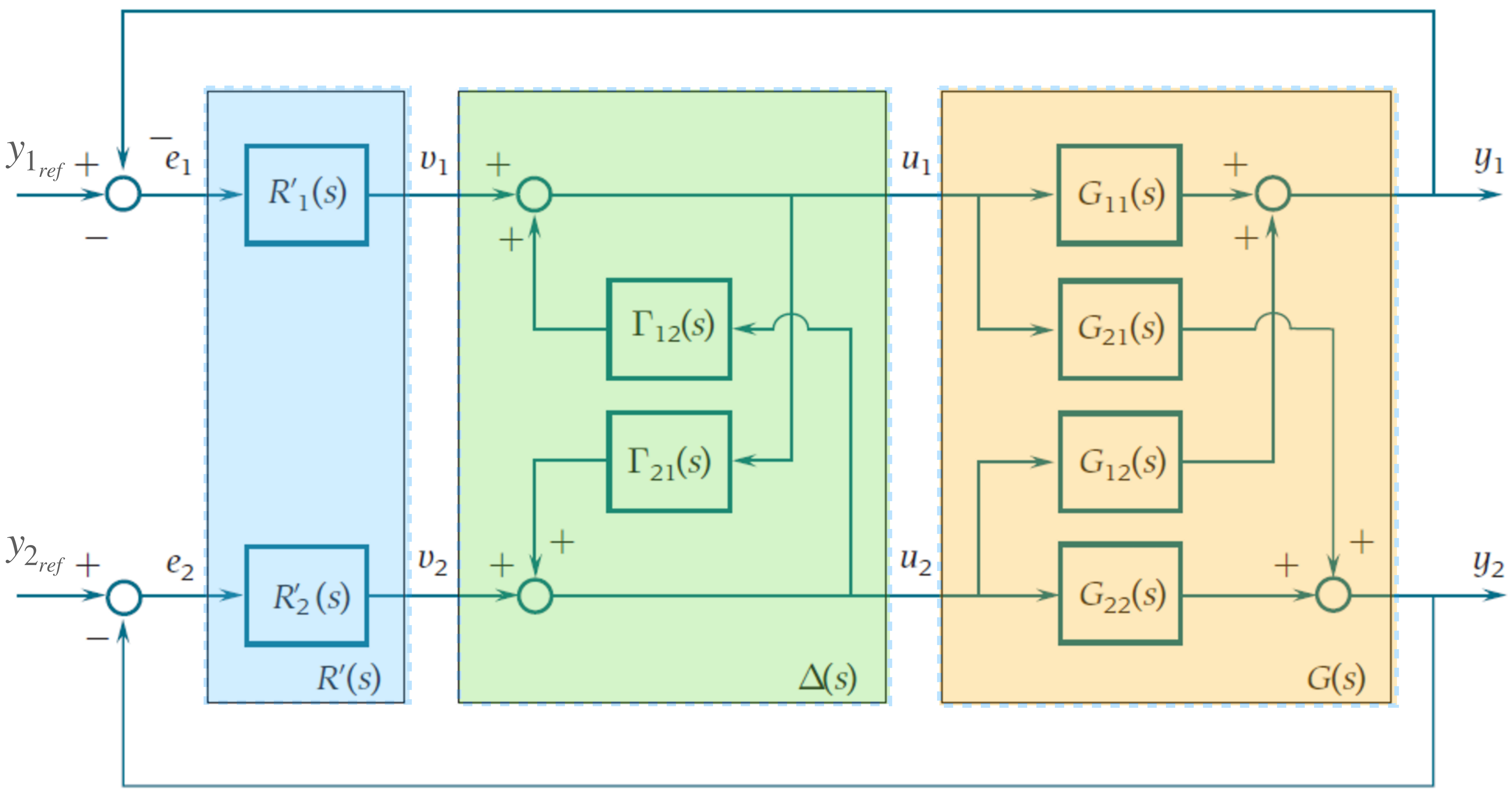

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$$\Gamma(s) = \begin{bmatrix} 0 & -\frac{G_{12}(s)}{G_{11}(s)} \\ -\frac{G_{21}(s)}{G_{22}(s)} & 0 \end{bmatrix}$$

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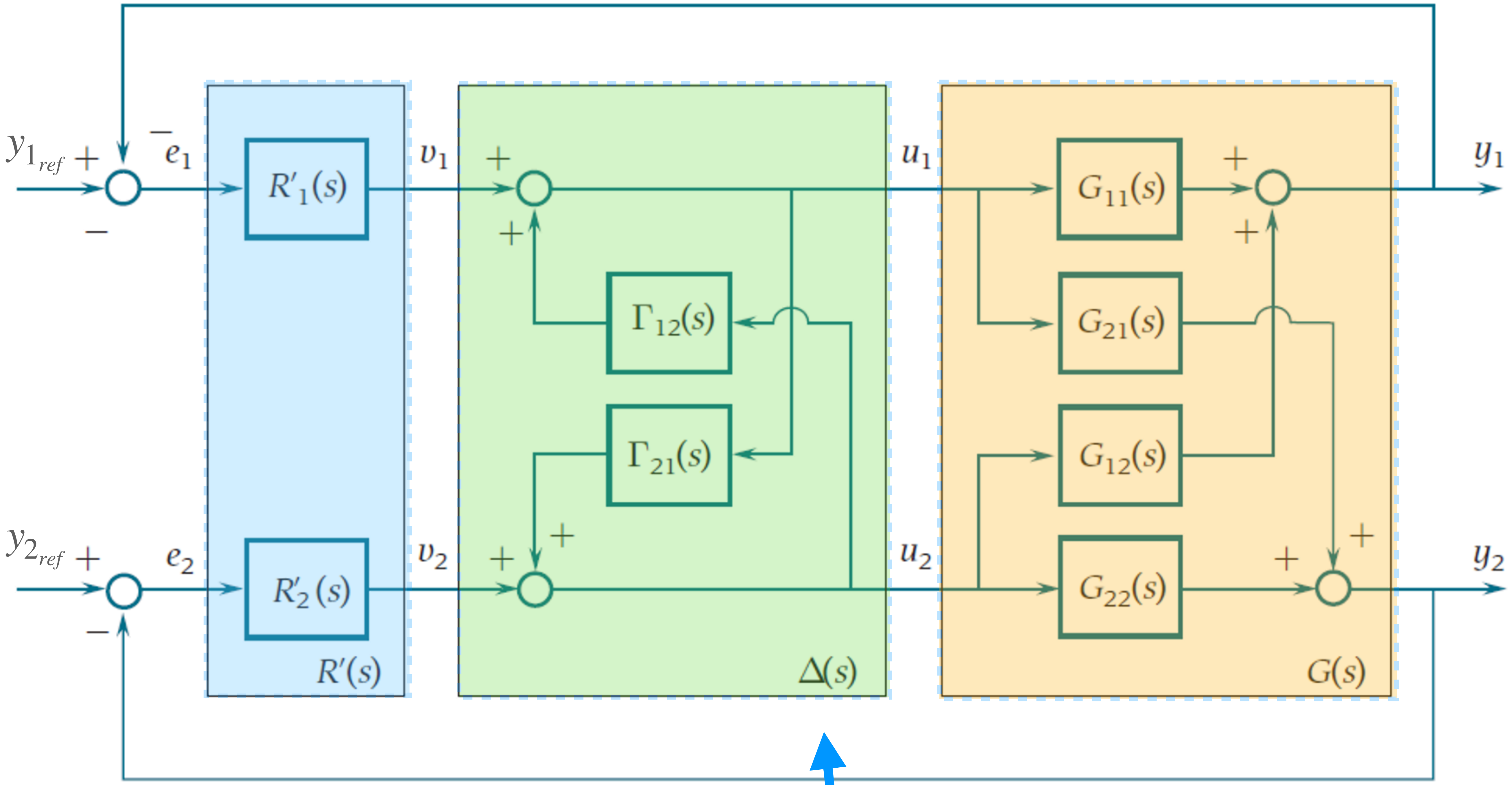

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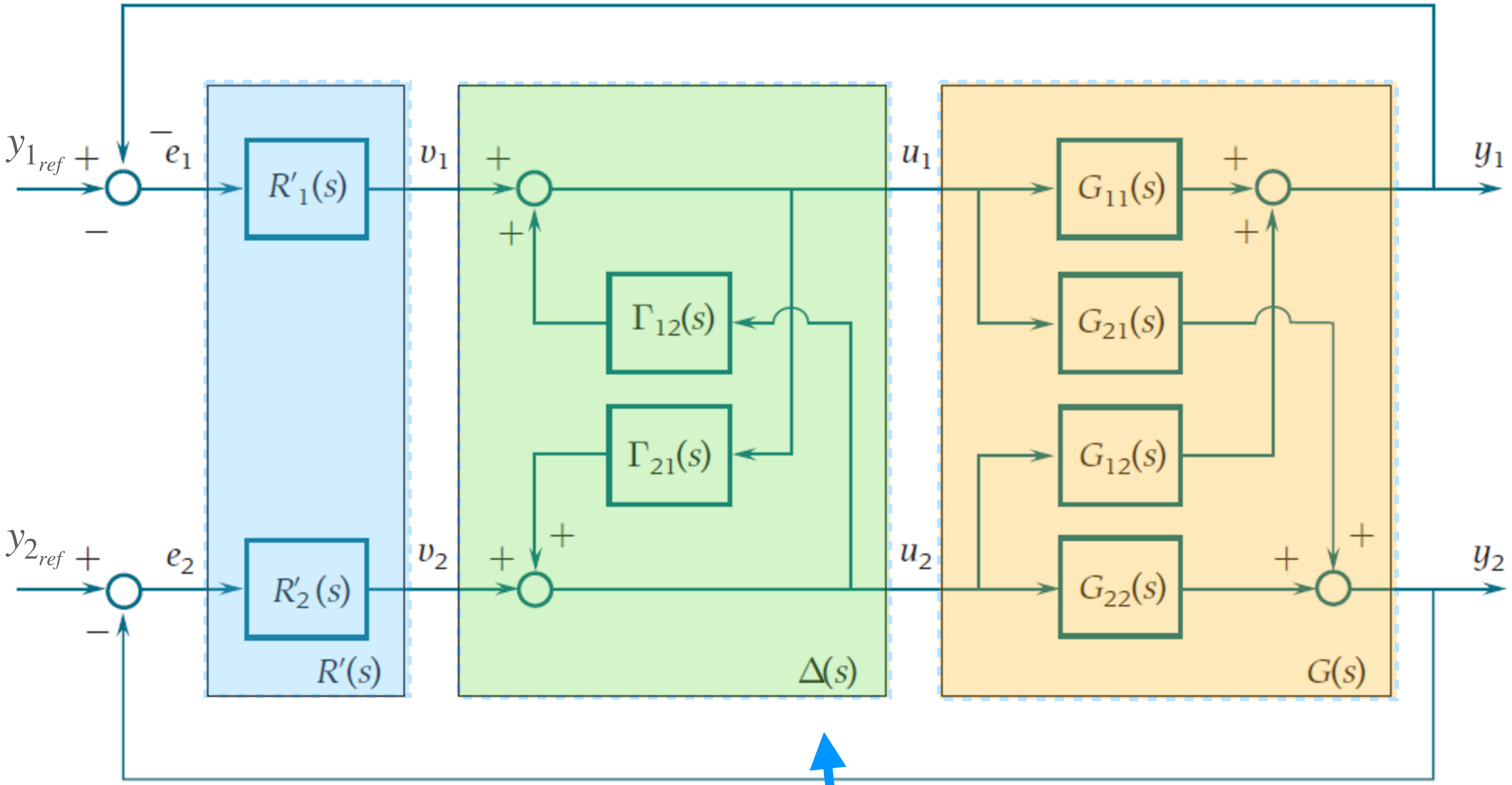

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$\Gamma(s)$ must be causal:
Approximations in the frequency domain of its component $\Gamma_{ij}(s)$ must be adopted if this is not true

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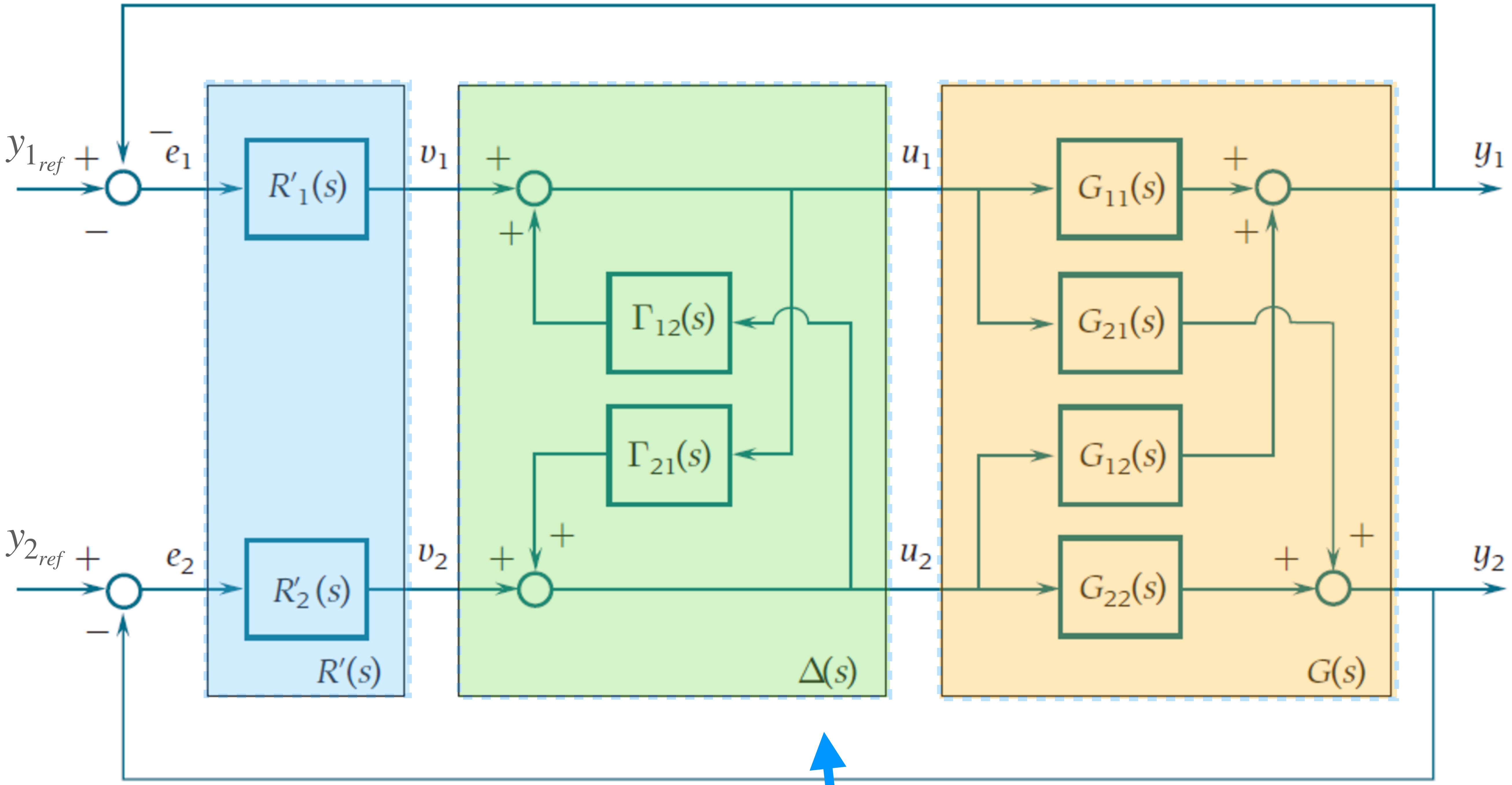

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To conclude:

$G_{11}(s)$

design: $R'(s) = \begin{bmatrix} R'_1(s) & 0 \\ 0 & R'_2(s) \end{bmatrix}$

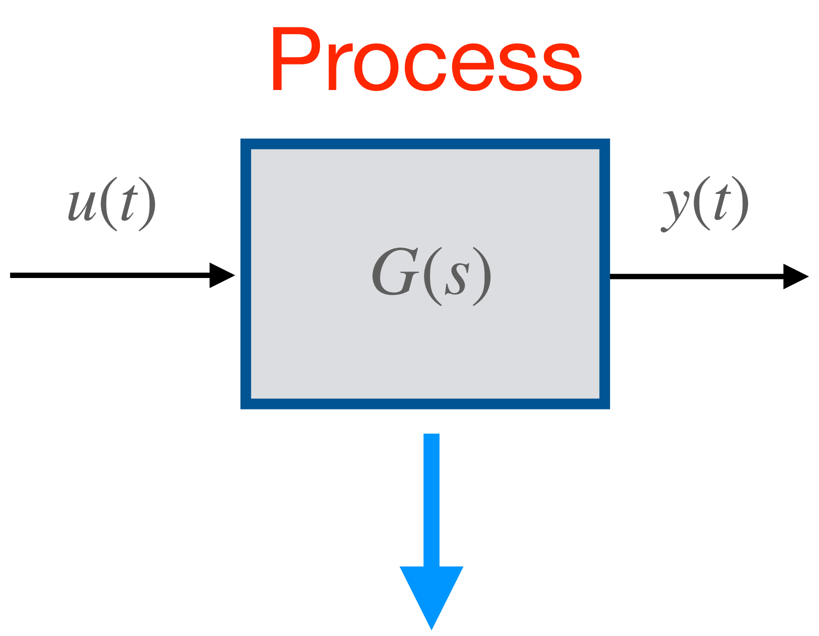
$G_{22}(s)$

$\Gamma(s)$ must be causal:
Approximations in the frequency domain of its component $\Gamma_{ij}(s)$ must be adopted if this is not true

$$\Delta(s) = [I - \Gamma(s)]^{-1}$$

$$\Gamma(s) = \begin{bmatrix} 0 & -\frac{G_{12}(s)}{G_{11}(s)} \\ -\frac{G_{21}(s)}{G_{22}(s)} & 0 \end{bmatrix}$$

Centralized vs. Decentralized MIMO Control Schemes



$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1m}(s) \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}(s) & G_{m2}(s) & \cdots & G_{mm}(s) \end{bmatrix}$$

$$u(t) \in \mathbb{R}^m$$

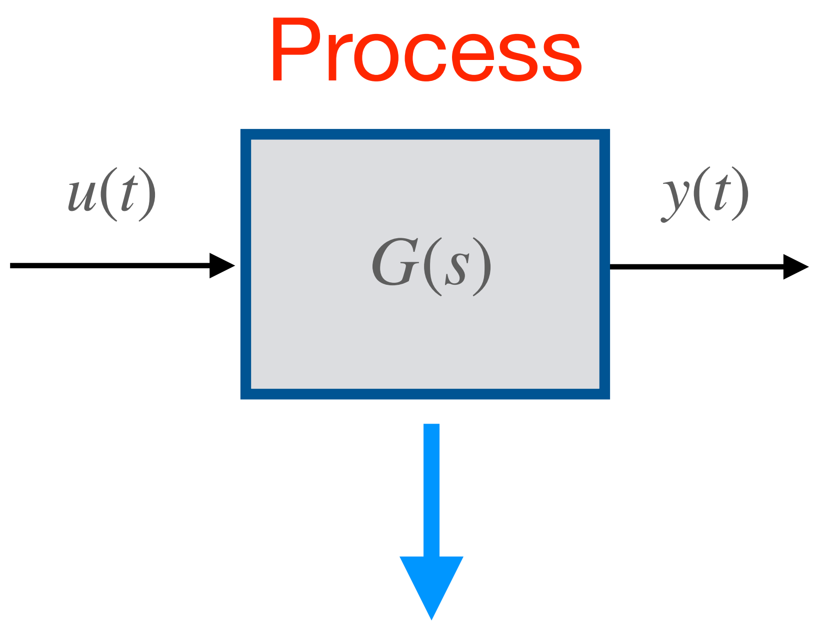
$$y(t) \in \mathbb{R}^m$$

$$G(s) \in \mathbb{R}^{m\times m}$$

Assumptions:

- $G(s) \in \mathbb{R}^{m\times m}$
- $G(s)$ full matrix

Centralized vs. Decentralized MIMO Control Schemes



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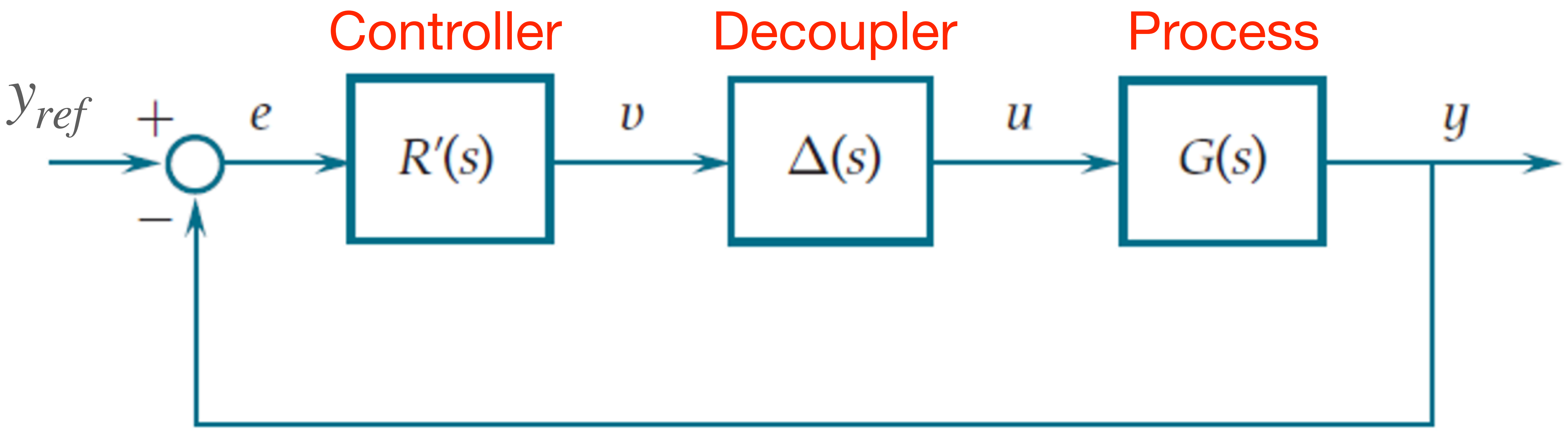
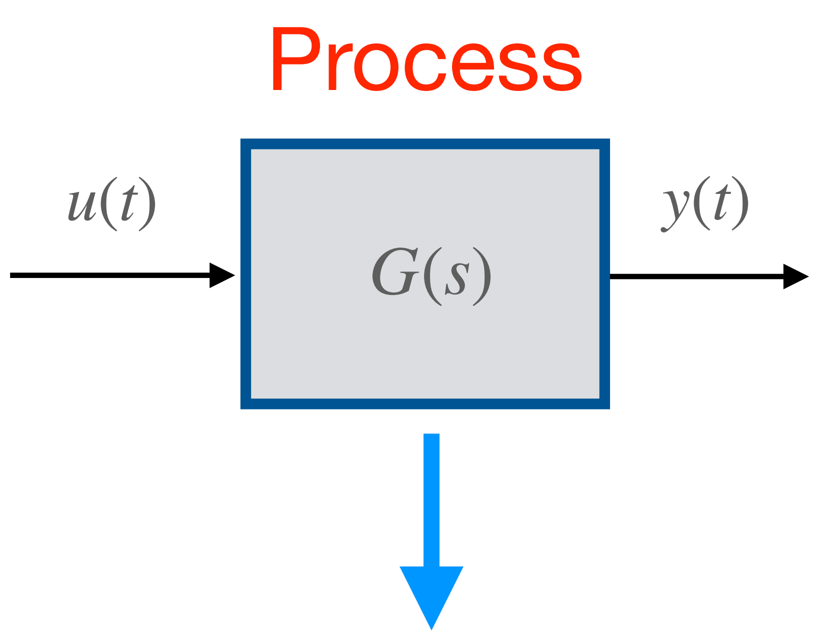


Centralized vs. Decentralized MIMO Control Schemes



$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1m}(s) \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}(s) & G_{m2}(s) & \cdots & G_{mm}(s) \end{bmatrix}$$

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Assumptions:

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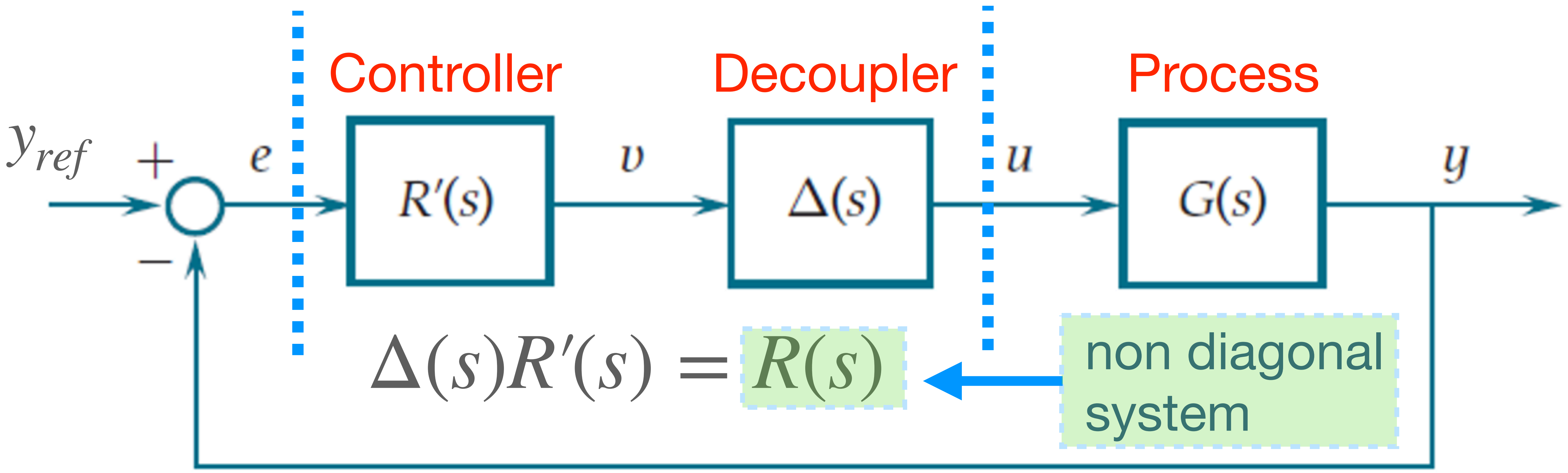
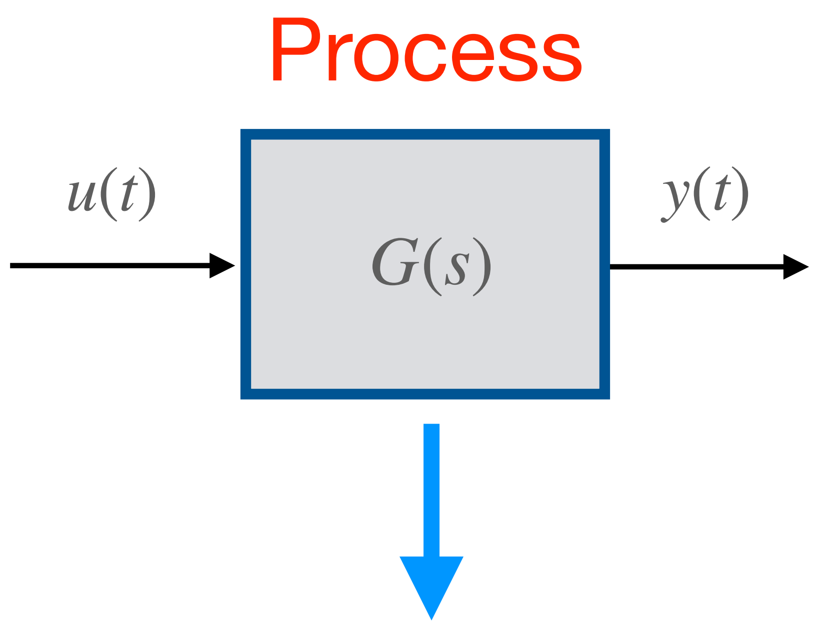


Centralized vs. Decentralized MIMO Control Schemes



$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1m}(s) \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}(s) & G_{m2}(s) & \cdots & G_{mm}(s) \end{bmatrix}$$

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Assumptions:

- $G(s) \in \mathbb{R}^{m \times m}$
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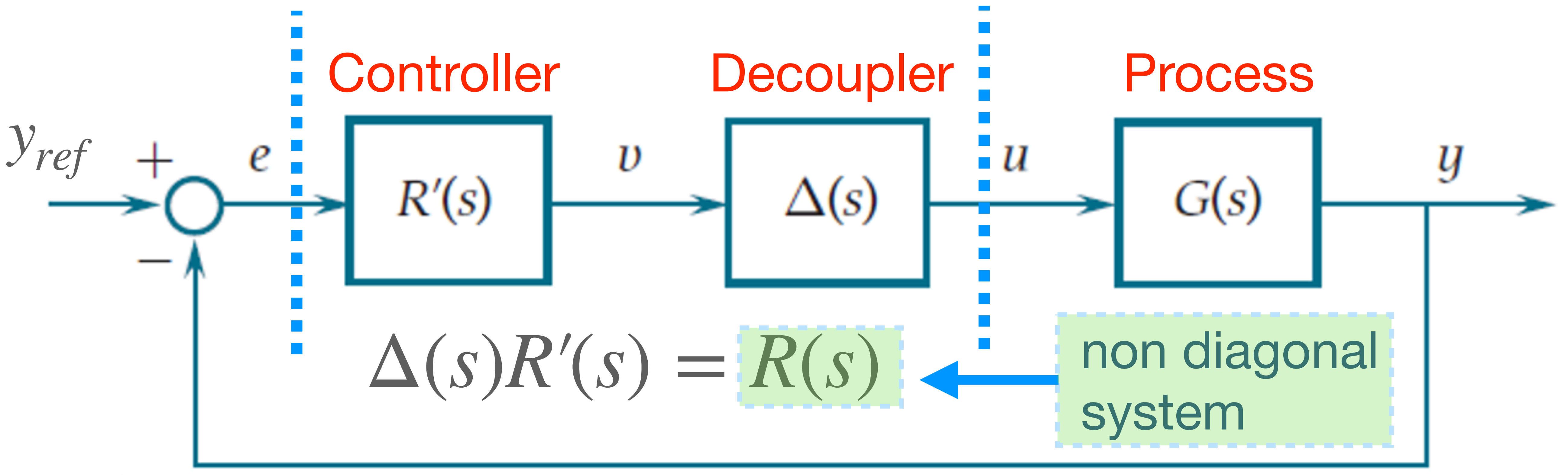
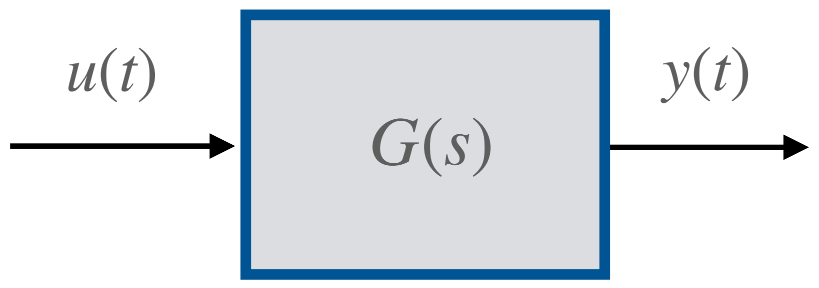


Any component $u_i\,,\ i = 1, \ \dots\ , m,$ of $u(t) \in \mathbb{R}^m$ depends, in general, on all the components of $e(t) \in \mathbb{R}^m$

Centralized vs. Decentralized MIMO Control Schemes

Process



$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1m}(s) \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}(s) & G_{m2}(s) & \cdots & G_{mm}(s) \end{bmatrix}$$

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^m$

$G(s) \in \mathbb{R}^{m\times m}$

Assumptions:

- $G(s) \in \mathbb{R}^{m\times m}$
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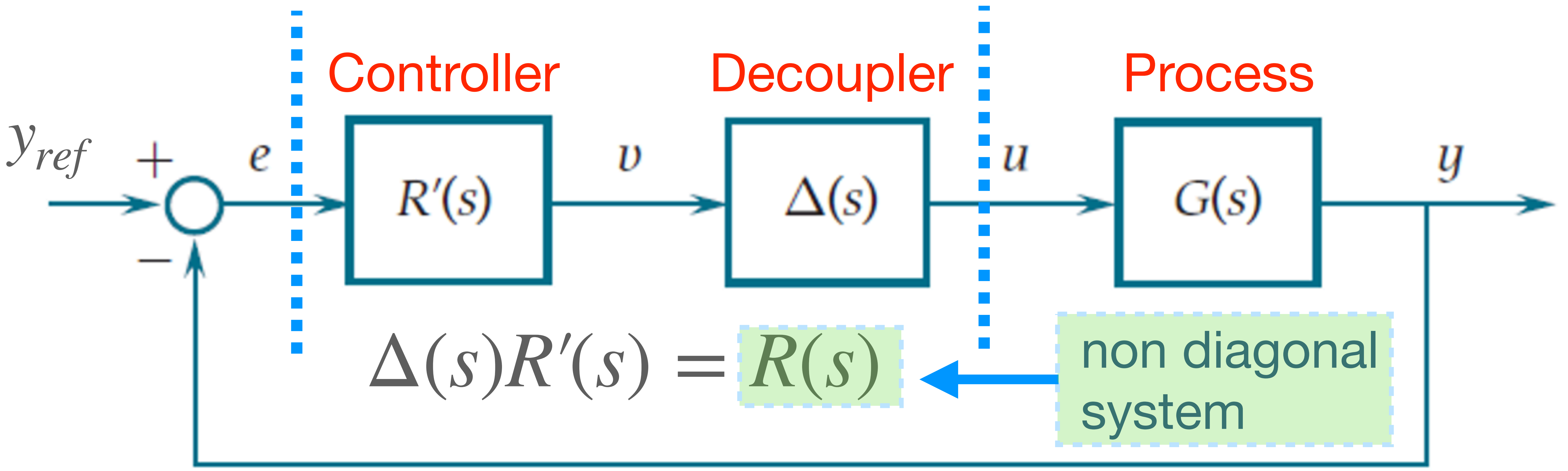
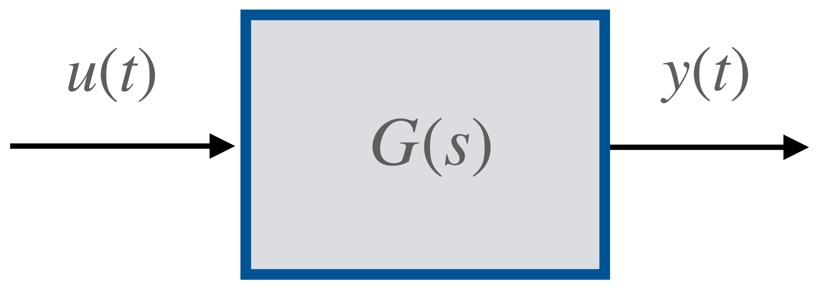


Any component u_i, $i = 1, \ldots, m$, of $u(t) \in \mathbb{R}^m$ depends, in general, on all the components of $e(t) \in \mathbb{R}^m$

Centralized Control

Centralized vs. Decentralized MIMO Control Schemes



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^m$

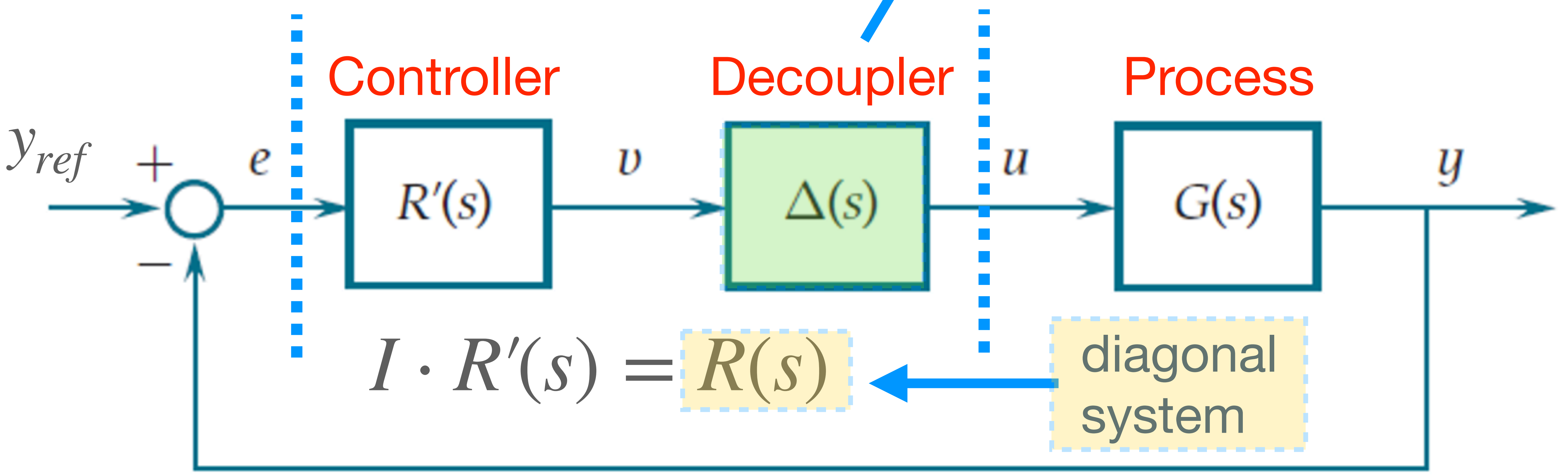
$G(s) \in \mathbb{R}^{m\times m}$

Assumptions:

- $G(s) \in \mathbb{R}^{m\times m}$
- $G(s)$ full matrix

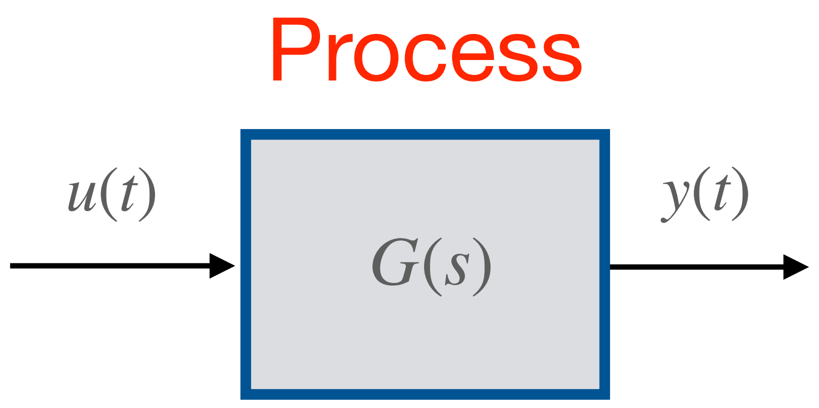
$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1m}(s) \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}(s) & G_{m2}(s) & \cdots & G_{mm}(s) \end{bmatrix}$$

$\Delta(s) = I$



$I \cdot R'(s) = R(s)$

Centralized vs. Decentralized MIMO Control Schemes



$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1m}(s) \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}(s) & G_{m2}(s) & \cdots & G_{mm}(s) \end{bmatrix}$$

$u(t) \in \mathbb{R}^m$

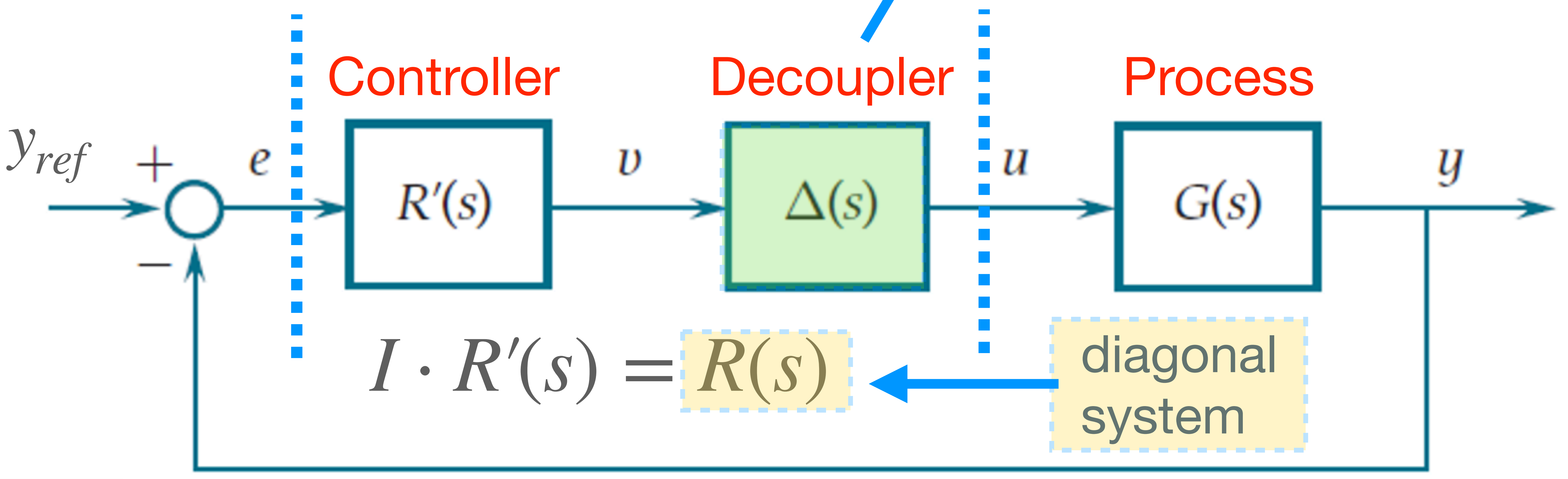
$y(t) \in \mathbb{R}^m$

$G(s) \in \mathbb{R}^{m \times m}$

Assumptions:

- $G(s) \in \mathbb{R}^{m \times m}$
- $G(s)$ full matrix

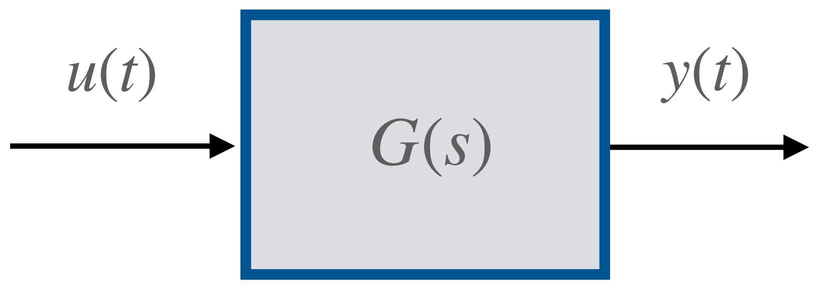
$\boxed{\Delta(s) = I}$



$I \cdot R'(s) = R(s)$

Any component u_i, $i = 1, \ldots, m$, of $u(t) \in \mathbb{R}^m$ depends only on the component e_i, $i = 1, \ldots, m$, of $e(t) \in \mathbb{R}^m$

Centralized vs. Decentralized MIMO Control Schemes



$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1m}(s) \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}(s) & G_{m2}(s) & \cdots & G_{mm}(s) \end{bmatrix}$$

$u(t) \in \mathbb{R}^m$

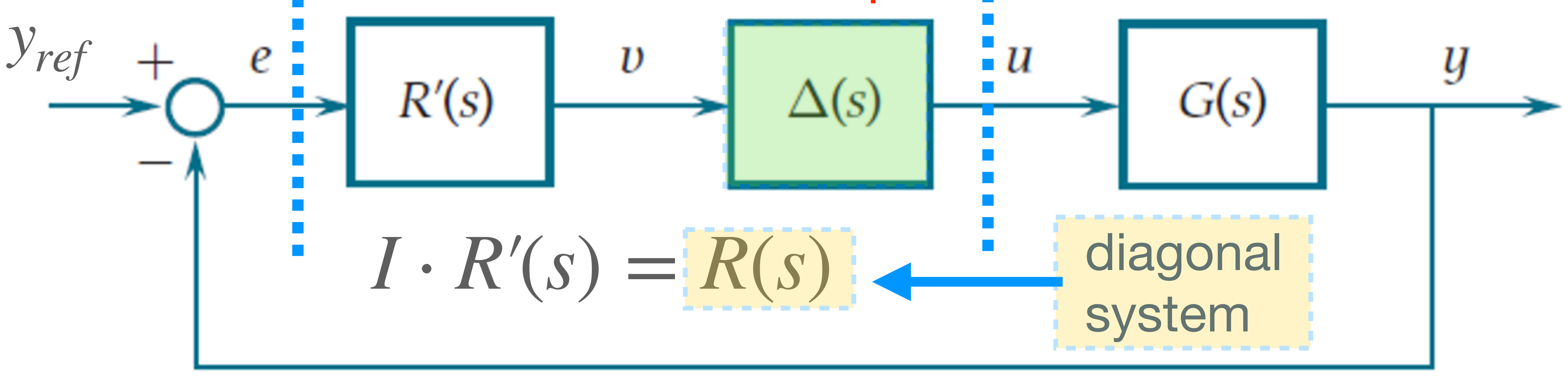
$y(t) \in \mathbb{R}^m$

$G(s) \in \mathbb{R}^{m \times m}$

Assumptions:

- $G(s) \in \mathbb{R}^{m \times m}$
- $G(s)$ full matrix

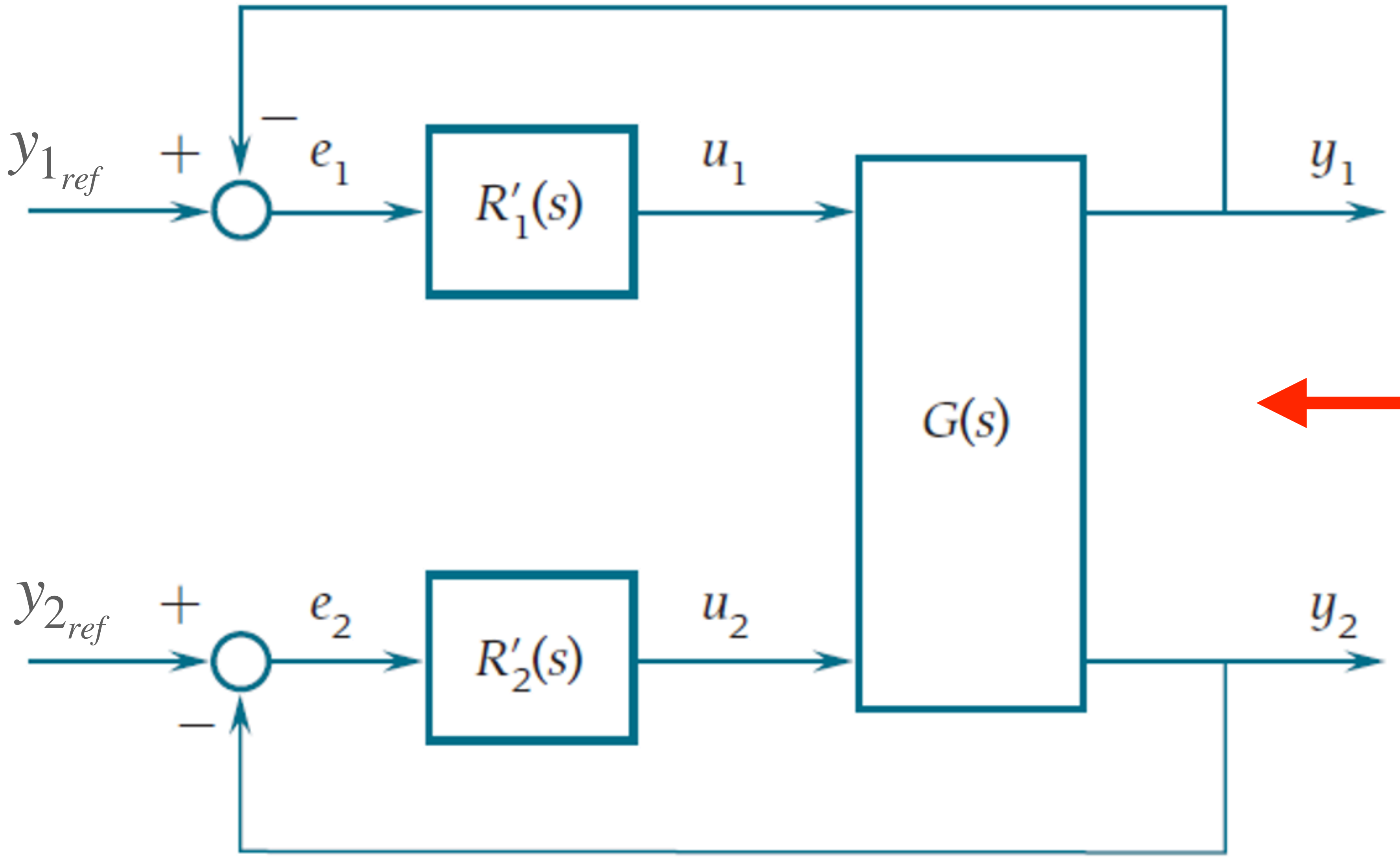
$\Delta(s) = I$



Any component $u_i,\ i = 1, \ \dots\ , m,$ of $u(t) \in \mathbb{R}^m$ depends only on the component $e_i,\ i = 1, \ \dots\ , m,$ of $e(t) \in \mathbb{R}^m$

Decentralized Control

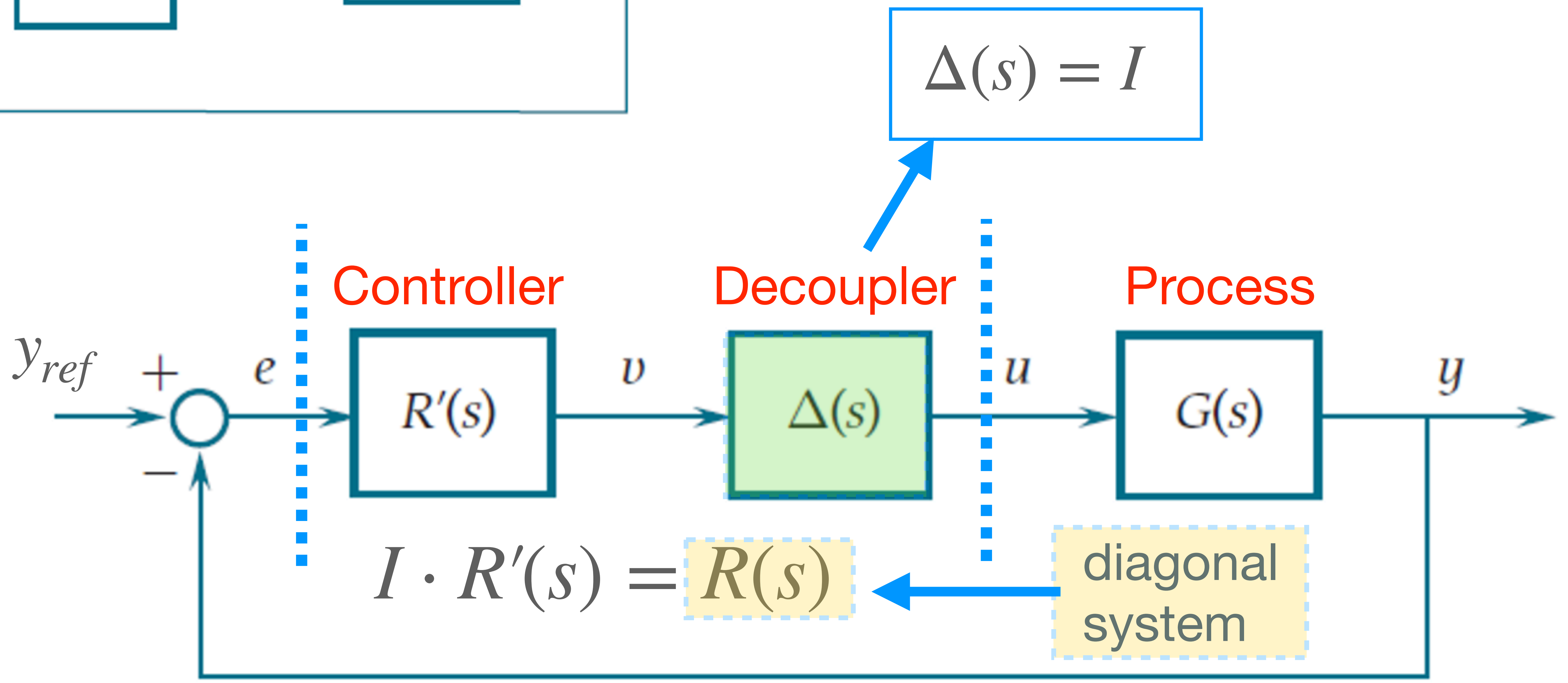
Centralized vs. Decentralized MIMO Control Schemes



In the 2×2 case

Assumptions:

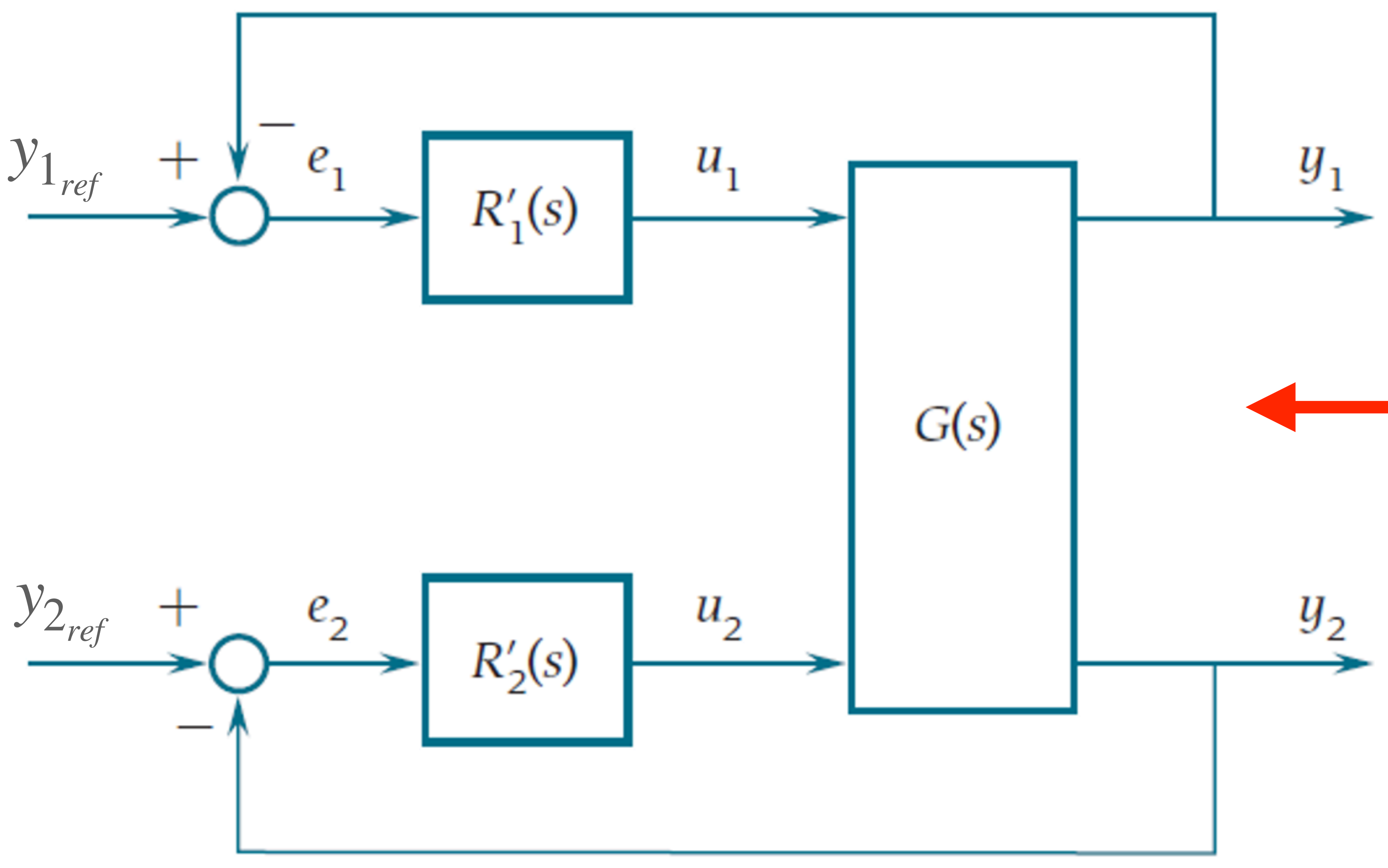
- $G(s) \in \mathbb{R}^{m \times m}$
- $G(s)$ full matrix



Any component $u_i\,,\ i = 1, \ \ldots\ , m,$ of $u(t) \in \mathbb{R}^m$ depends only on the component $e_i\,,\ i = 1, \ \ldots\ , m,$ of $e(t) \in \mathbb{R}^m$

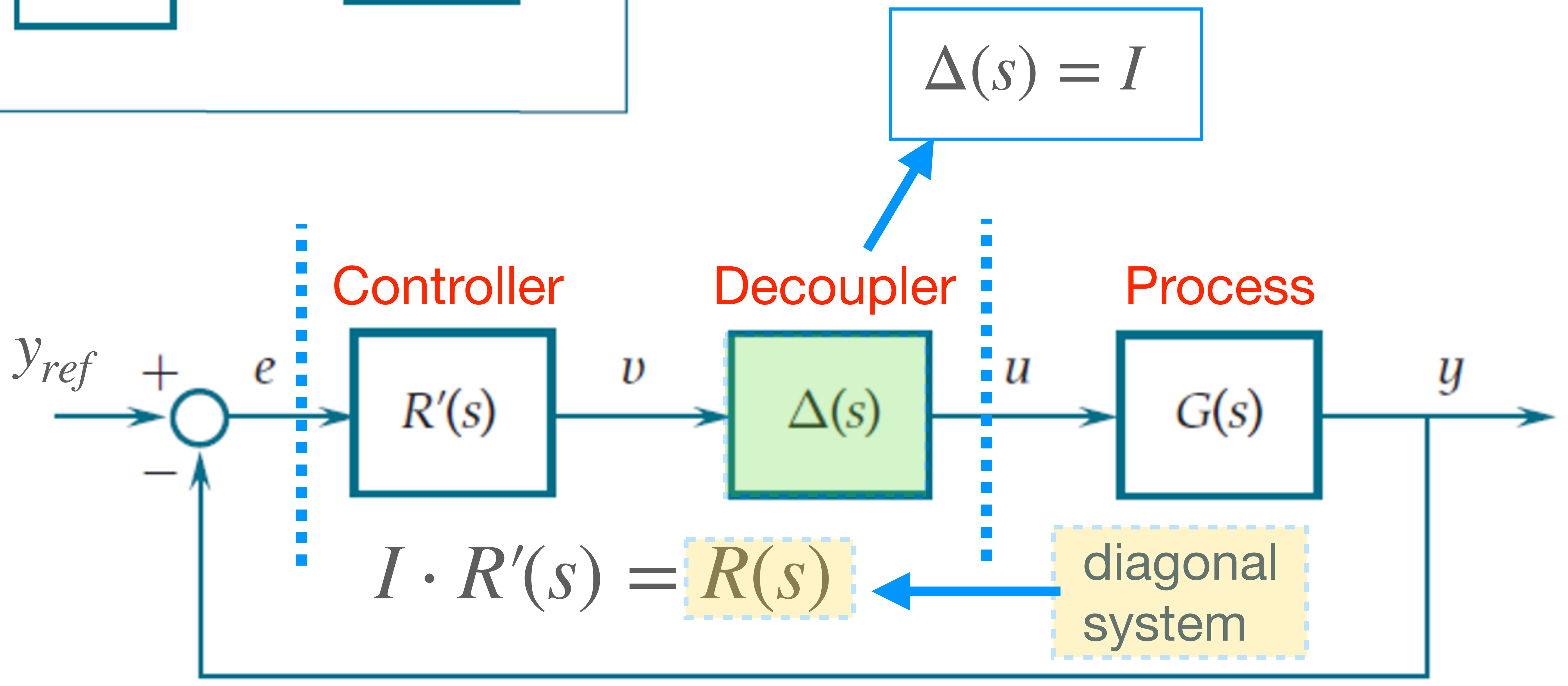
Decentralized Control

Decentralized MIMO Control Schemes: Heuristic Method



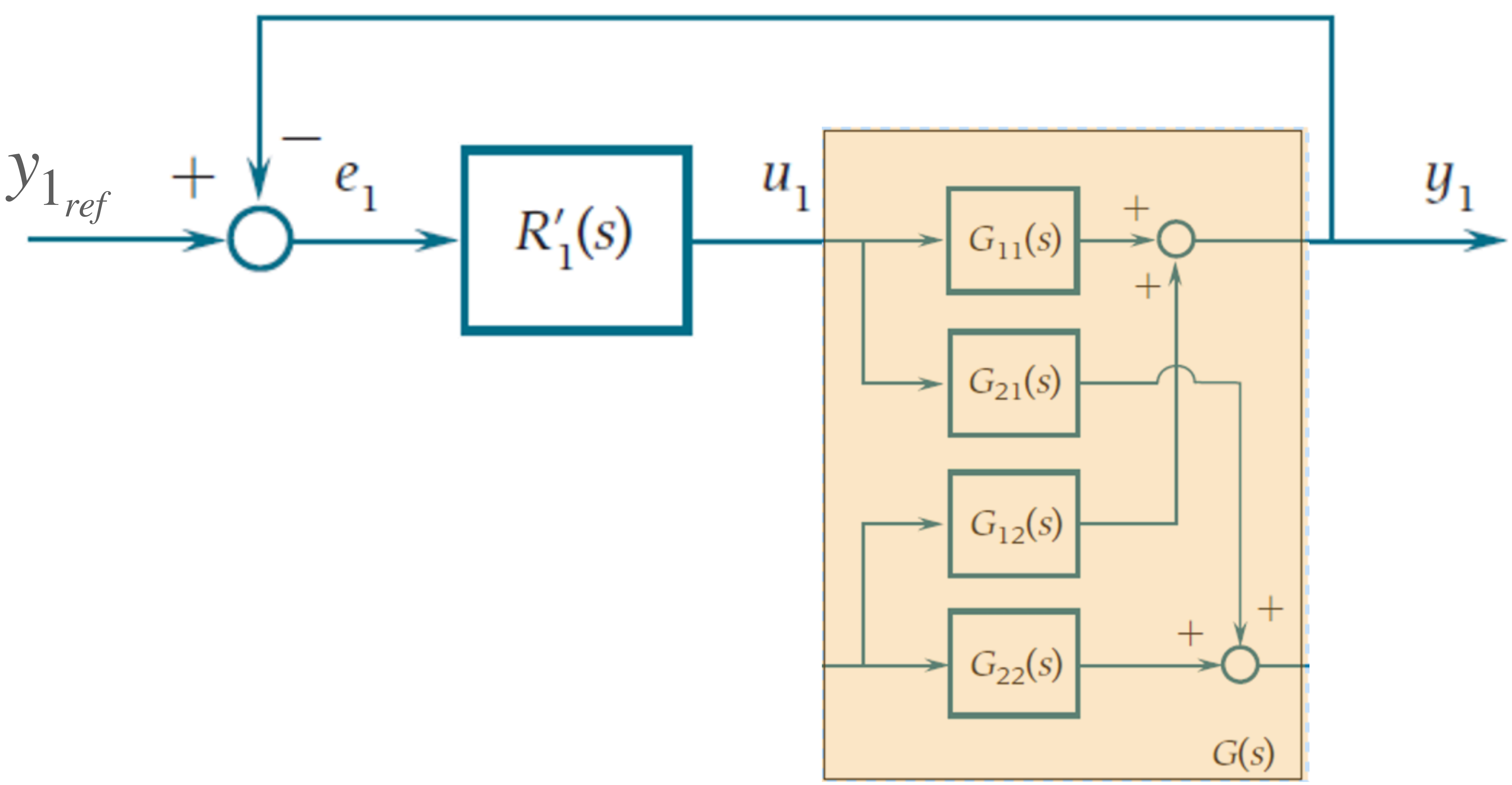
In the 2×2 case

Design $R'_j(s)$ taking into account that the controllers from $R'_1(s)$ to $R'_{j-1}(s)$ have already been inserted into the control system



Decentralized MIMO Control Schemes: Heuristic Method

In the 2×2 case



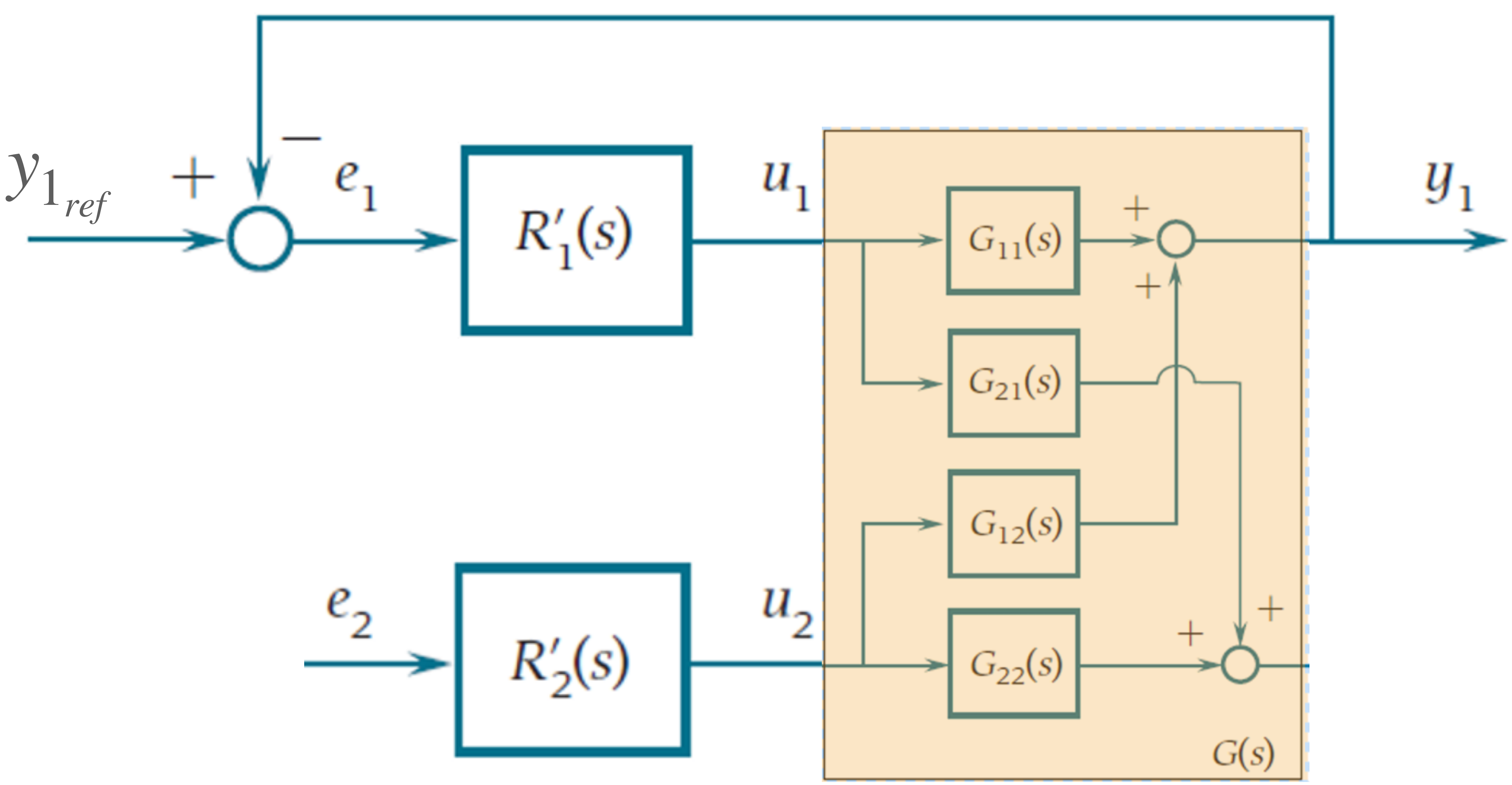
Design $R'_j(s)$ taking into account that the controllers from $R'_1(s)$ to $R'_{j-1}(s)$ have already been inserted into the control system

Step 1:

Design $R'_1(s)$ based on $G_{11}(s)$

Decentralized MIMO Control Schemes: Heuristic Method

In the 2×2 case



Design $R'_j(s)$ taking into account that the controllers from $R'_1(s)$ to $R'_{j-1}(s)$ have already been inserted into the control system

Step 1:

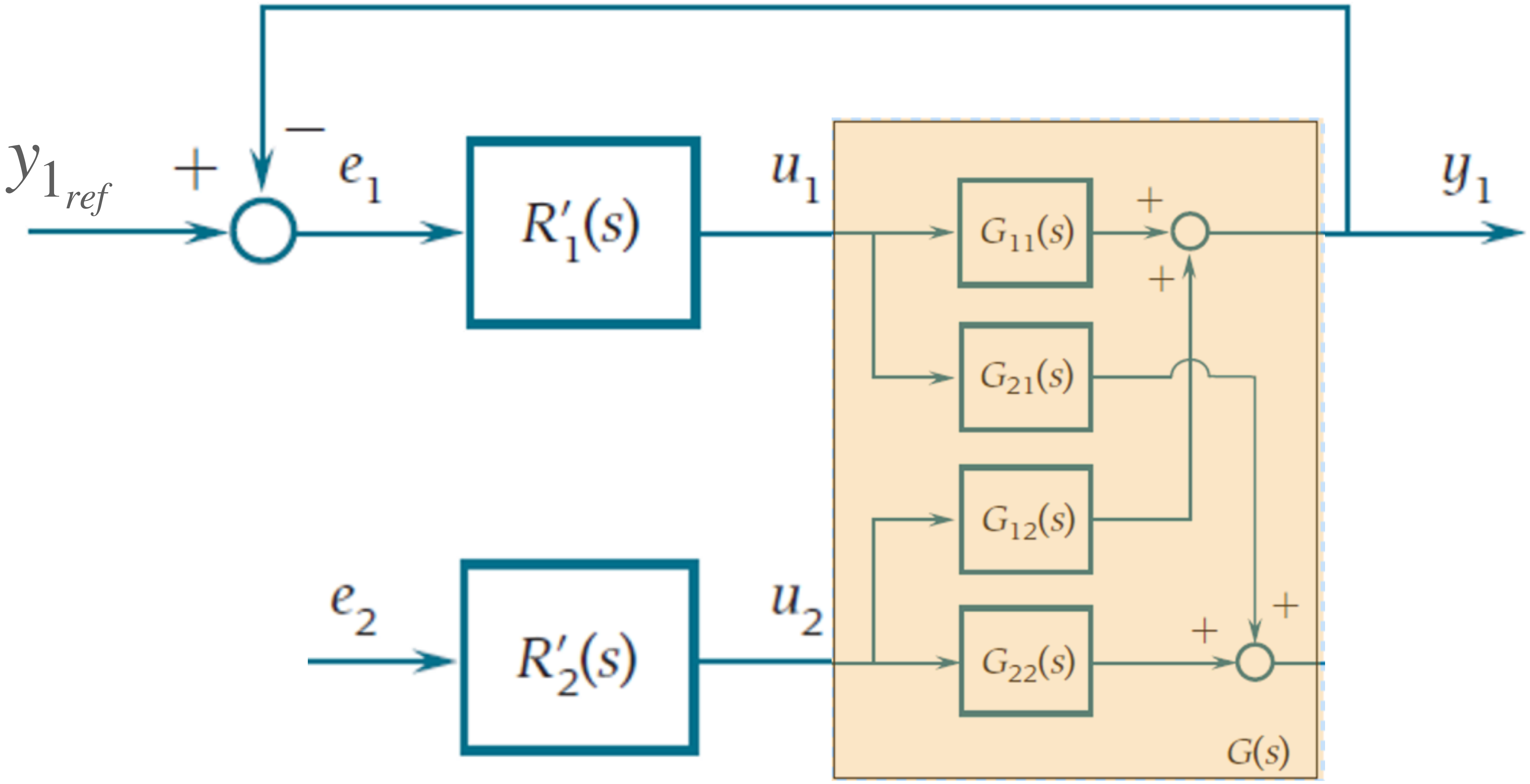
Design $R'_1(s)$ based on $G_{11}(s)$

Step 2:

Design $R'_2(s)$ based on $G'_{22}(s)$

Decentralized MIMO Control Schemes: Heuristic Method

In the 2×2 case



Design $R'_j(s)$ taking into account that the controllers from $R'_1(s)$ to $R'_{j-1}(s)$ have already been inserted into the control system

Step 1:

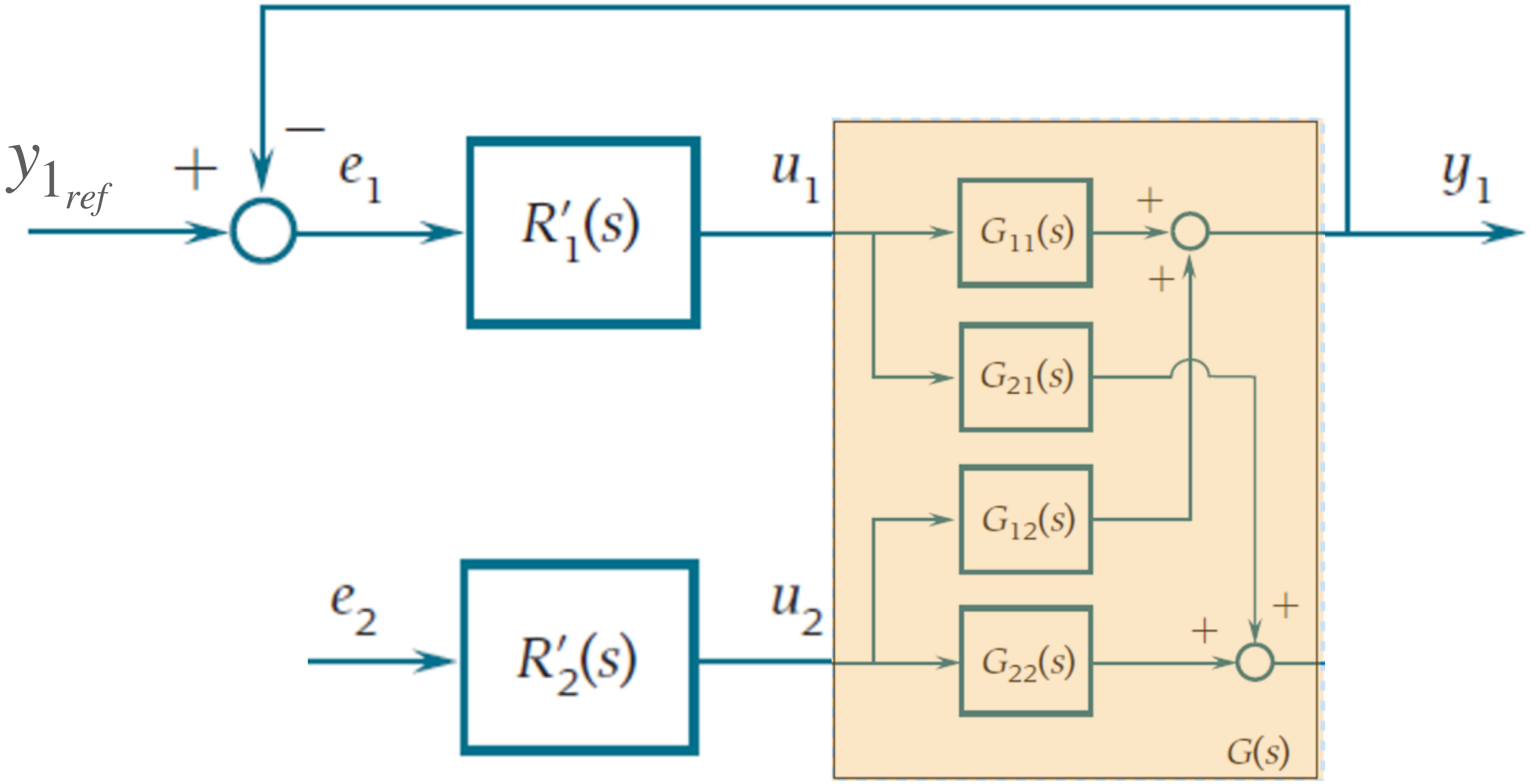
Design $R'_1(s)$ based on $G_{11}(s)$

Step 2:

Design $R'_2(s)$ based on $\boxed{G'_{22}(s)}$?

Decentralized MIMO Control Schemes: Heuristic Method

In the 2×2 case



Design $R_j'(s)$ taking into account that the controllers from $R_1'(s)$ to $R_{j-1}'(s)$ have already been inserted into the control system

Step 1:

Design $R_1'(s)$ based on $G_{11}(s)$

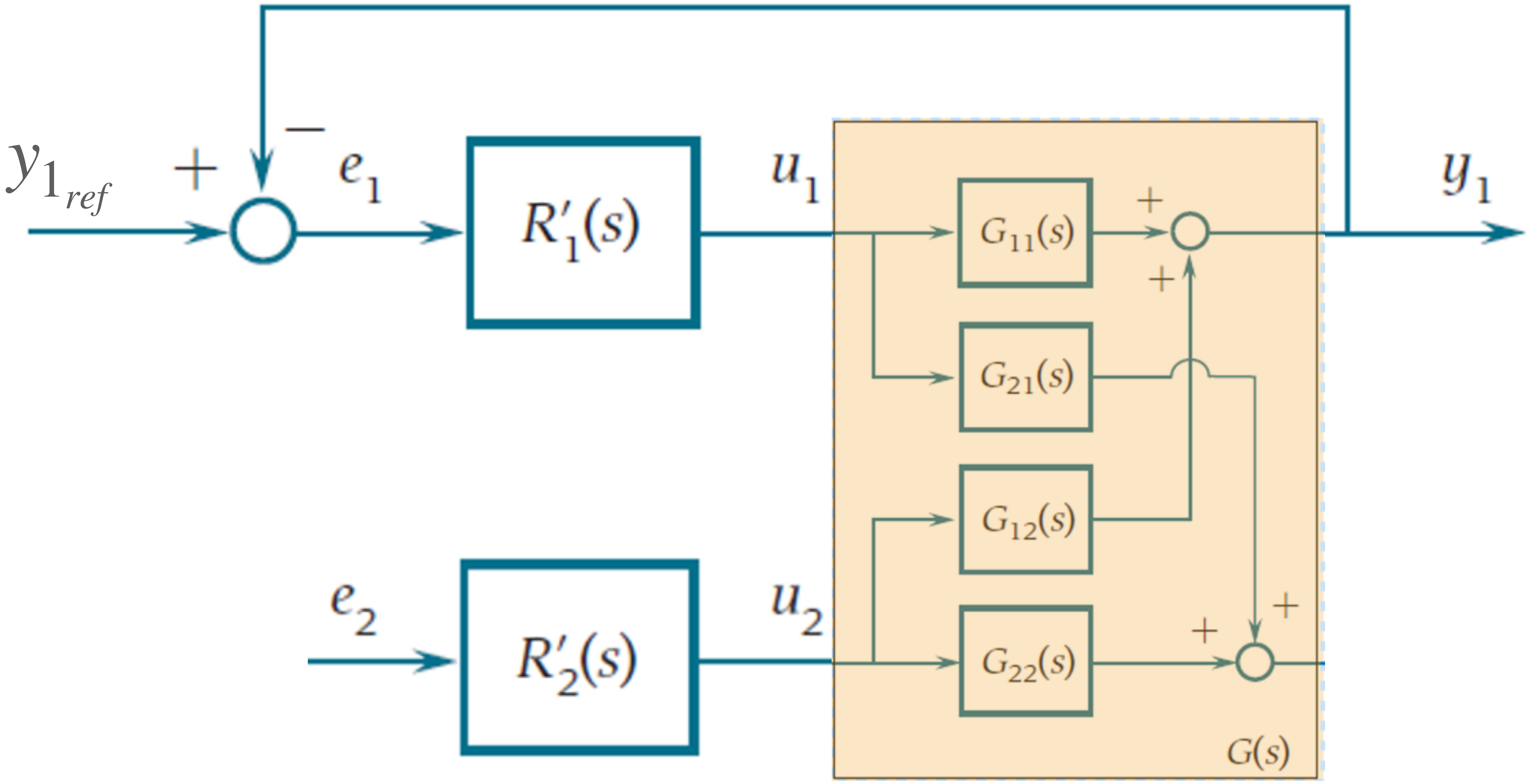
Step 2:

Design $R_2'(s)$ based on $G_{22}'(s)$?

$$G_{22}'(s) = G_{22}(s) - \frac{G_{12}(s)R_1'(s)G_{21}(s)}{1 + R_1'(s)G_{11}(s)}$$

Decentralized MIMO Control Schemes: Heuristic Method

In the 2×2 case



Design $R'_j(s)$ taking into account that the controllers from $R'_1(s)$ to $R'_{j-1}(s)$ have already been inserted into the control system

Step 1:

Design $R'_1(s)$ based on $G_{11}(s)$

Step 2:

Design $R'_2(s)$ based on $G'_{22}(s)$

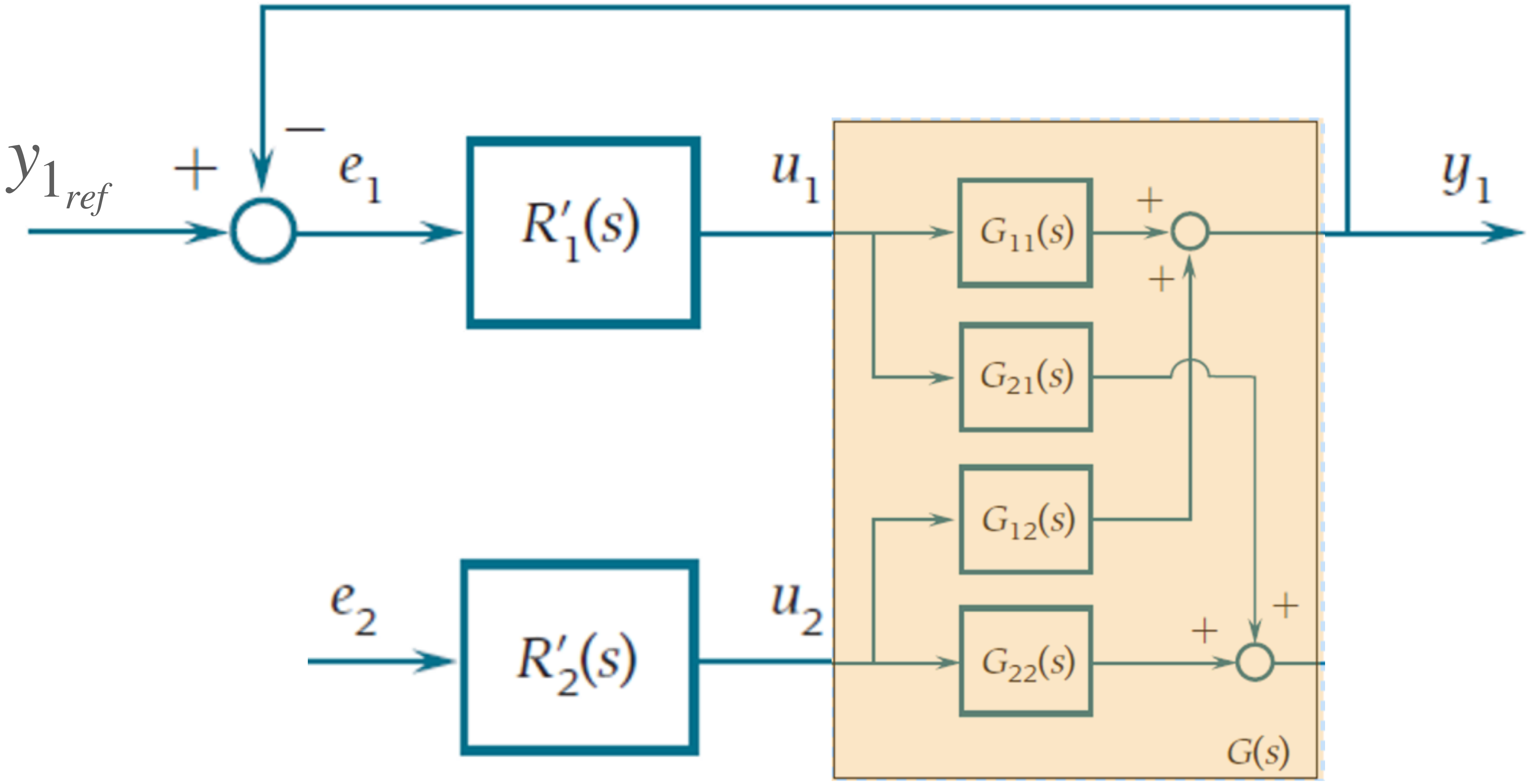
In general ($m \times m$ case):

Design $R'_j(s)$ based on $G'_{jj}(s)$

Decentralized MIMO Control Schemes: Heuristic Method

No performance and stability guarantees

In the 2×2 case



Design $R'_j(s)$ taking into account that the controllers from $R'_1(s)$ to $R'_{j-1}(s)$ have already been inserted into the control system

Step 1:

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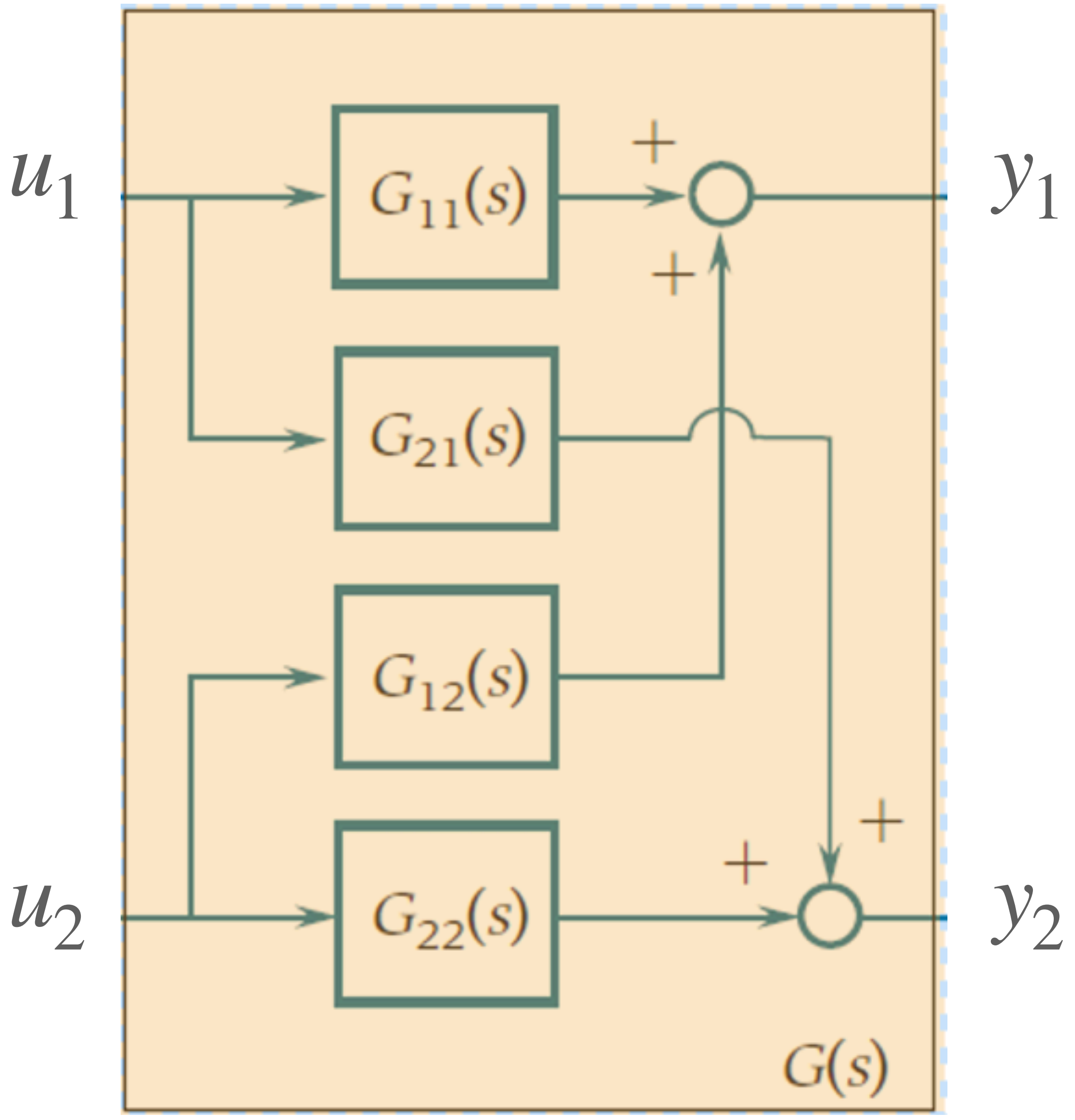
Step 2:

Design $R'_2(s)$ based on $G'_{22}(s)$

In general ($m \times m$ case):

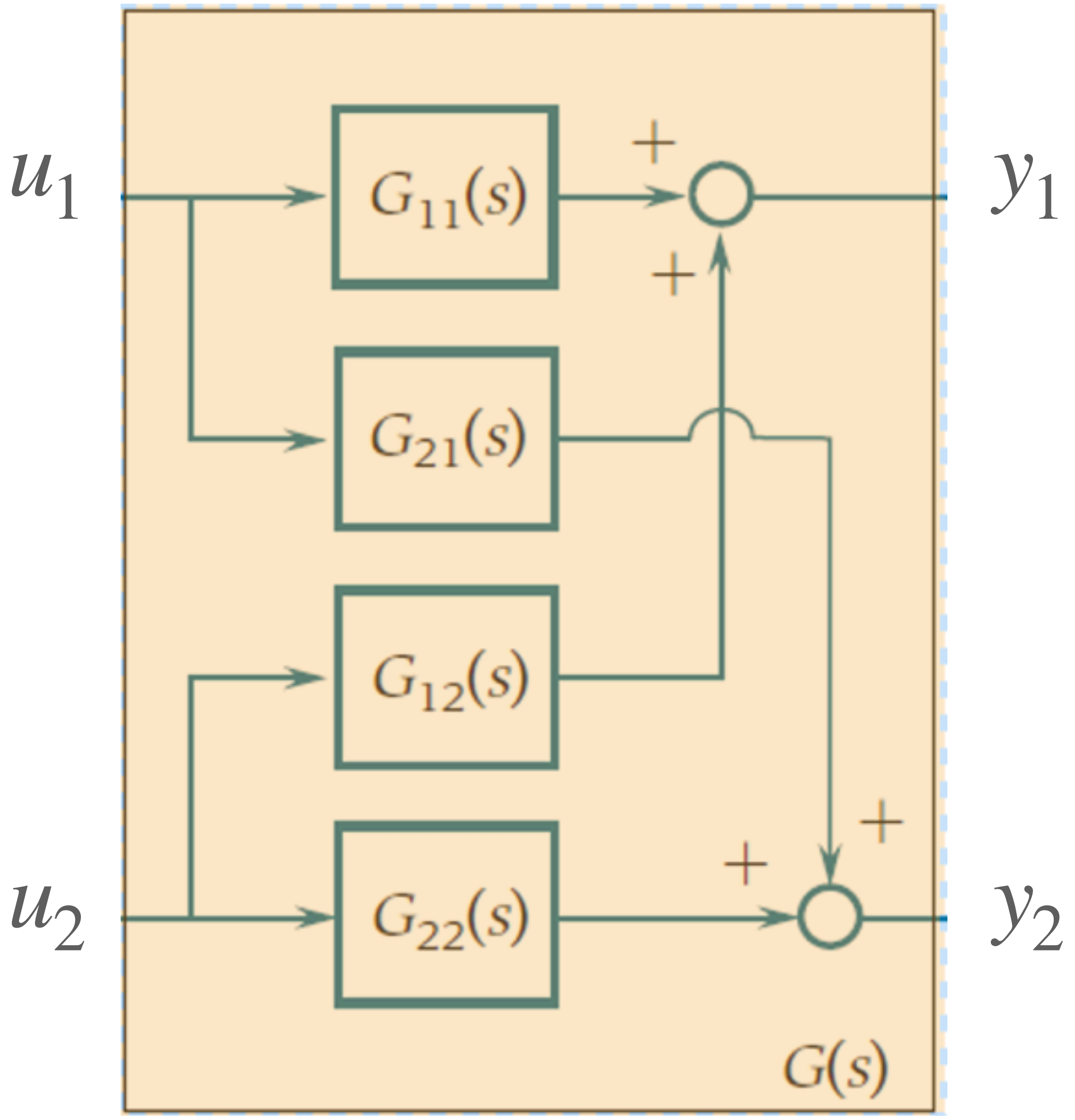
Design $R'_j(s)$ based on $G'_{jj}(s)$

Relative Gain Array



Aim: Design a decentralized control scheme

Relative Gain Array



Aim: Design a decentralized control scheme

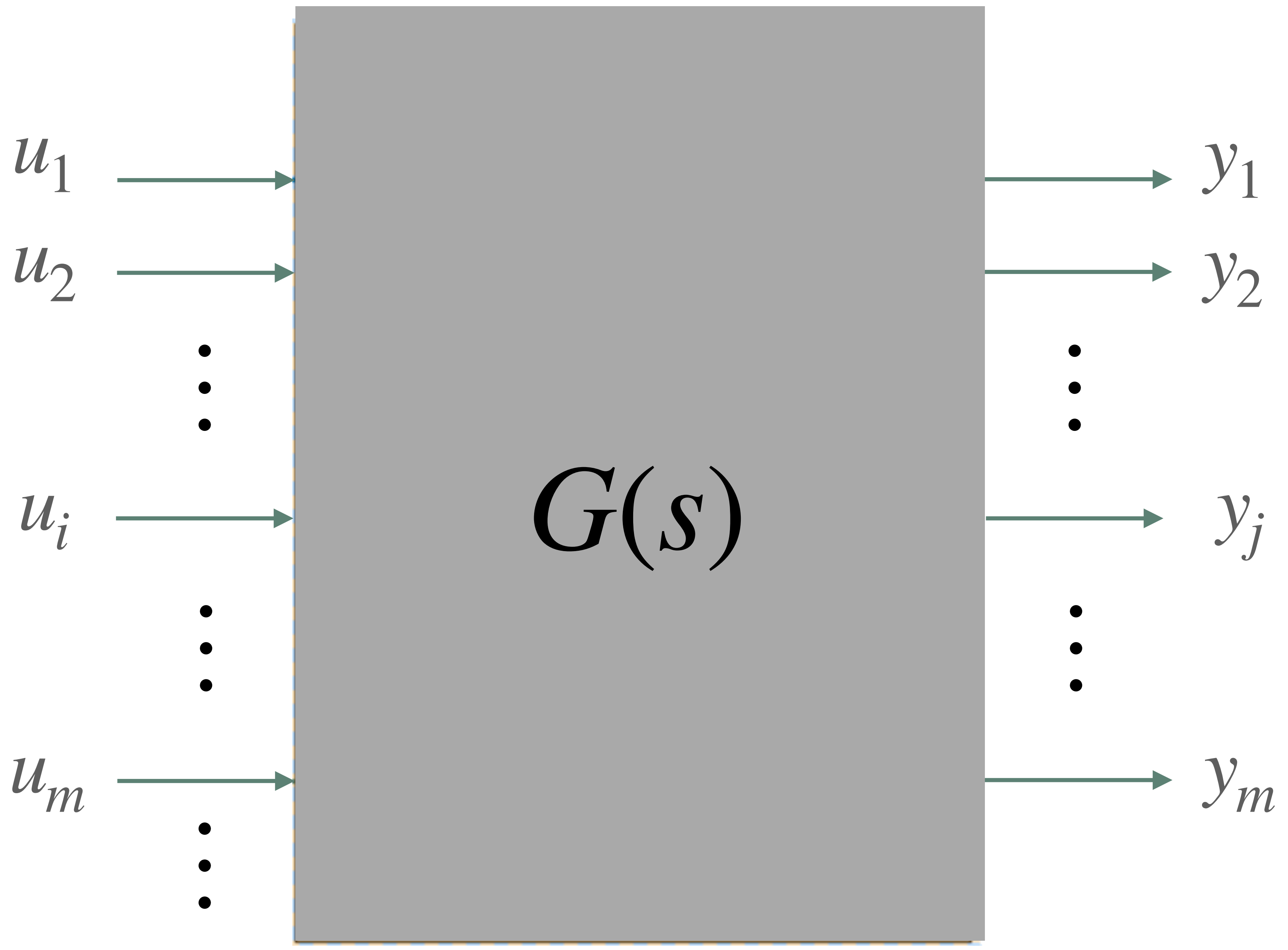
Question: Which pairings are the best?

$$\{u_1 \rightarrow y_1\} \cup \{u_2 \rightarrow y_2\}$$

$$\{u_1 \rightarrow y_2\} \cup \{u_2 \rightarrow y_1\}$$

?

Relative Gain Array



Aim: Design a decentralized control scheme

Question: Which pairings are the best?

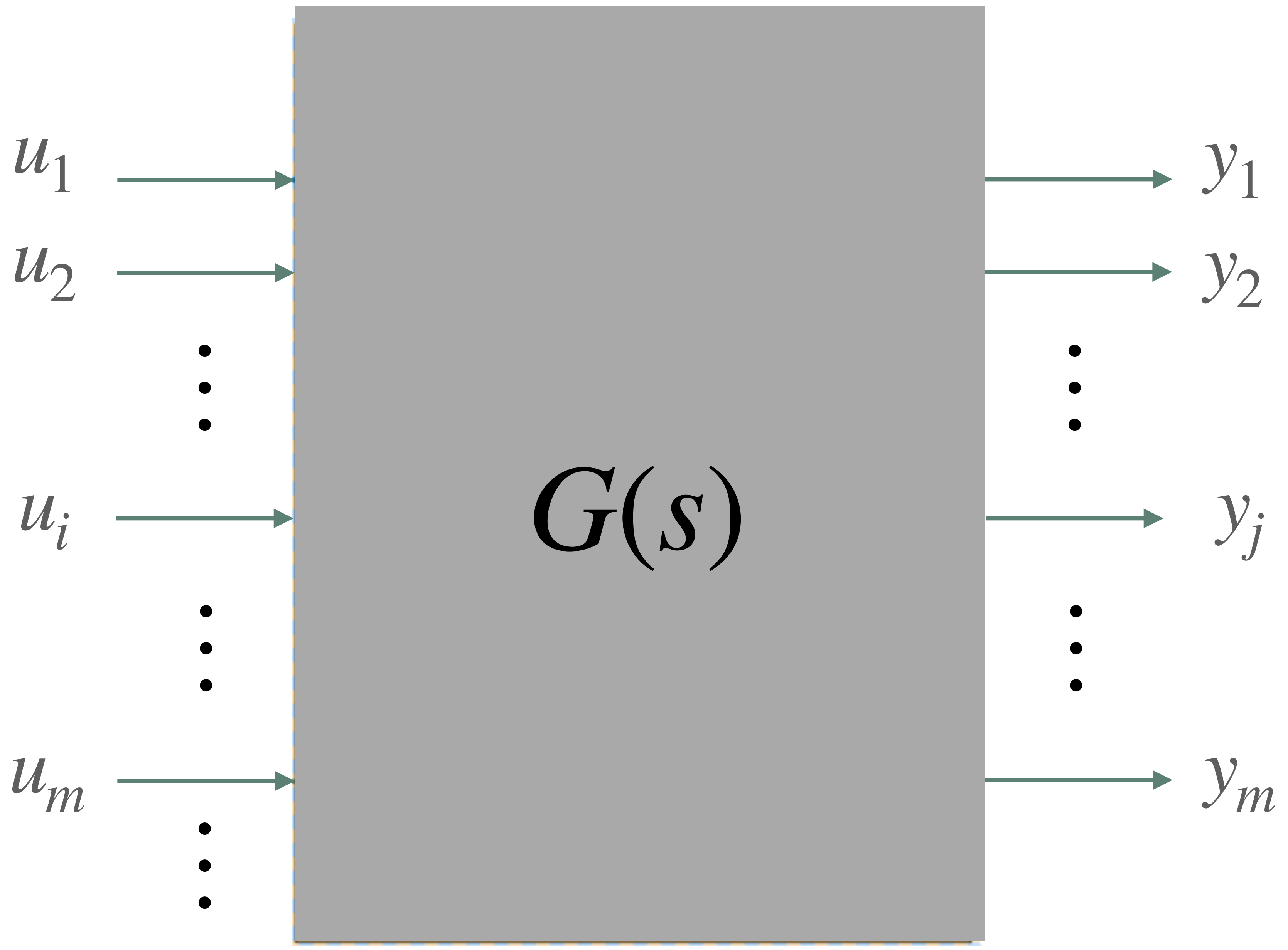
$$\{u_1 \rightarrow y_1\} \cup \{u_2 \rightarrow y_2\}$$

$$\{u_1 \rightarrow y_2\} \cup \{u_2 \rightarrow y_1\}$$

In the $m \times m$ case: Which pairings are the best?

$$\{u_i \rightarrow y_j\}, \quad i, j = 1, \dots, m \qquad ?$$

Relative Gain Array → Method to determine the best I/O pairings



Aim: Design a decentralized control scheme

Question: Which pairings are the best?

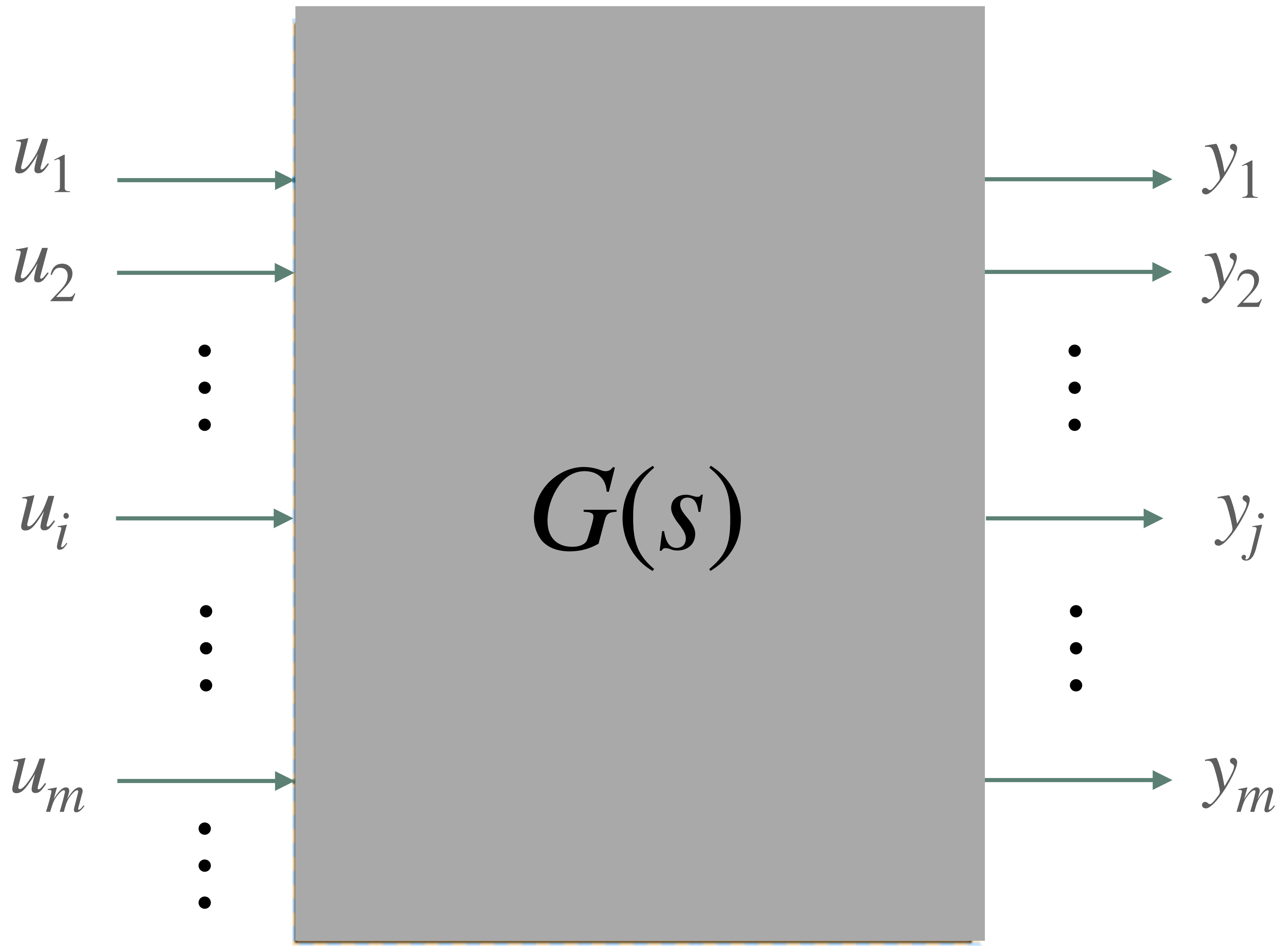
$$\{u_1 \rightarrow y_1\} \cup \{u_2 \rightarrow y_2\}$$

$$\{u_1 \rightarrow y_2\} \cup \{u_2 \rightarrow y_1\}$$

In the $m \times m$ case: Which pairings are the best?

$$\{u_i \rightarrow y_j\}, \quad i, j = 1, \ldots, m \qquad ?$$

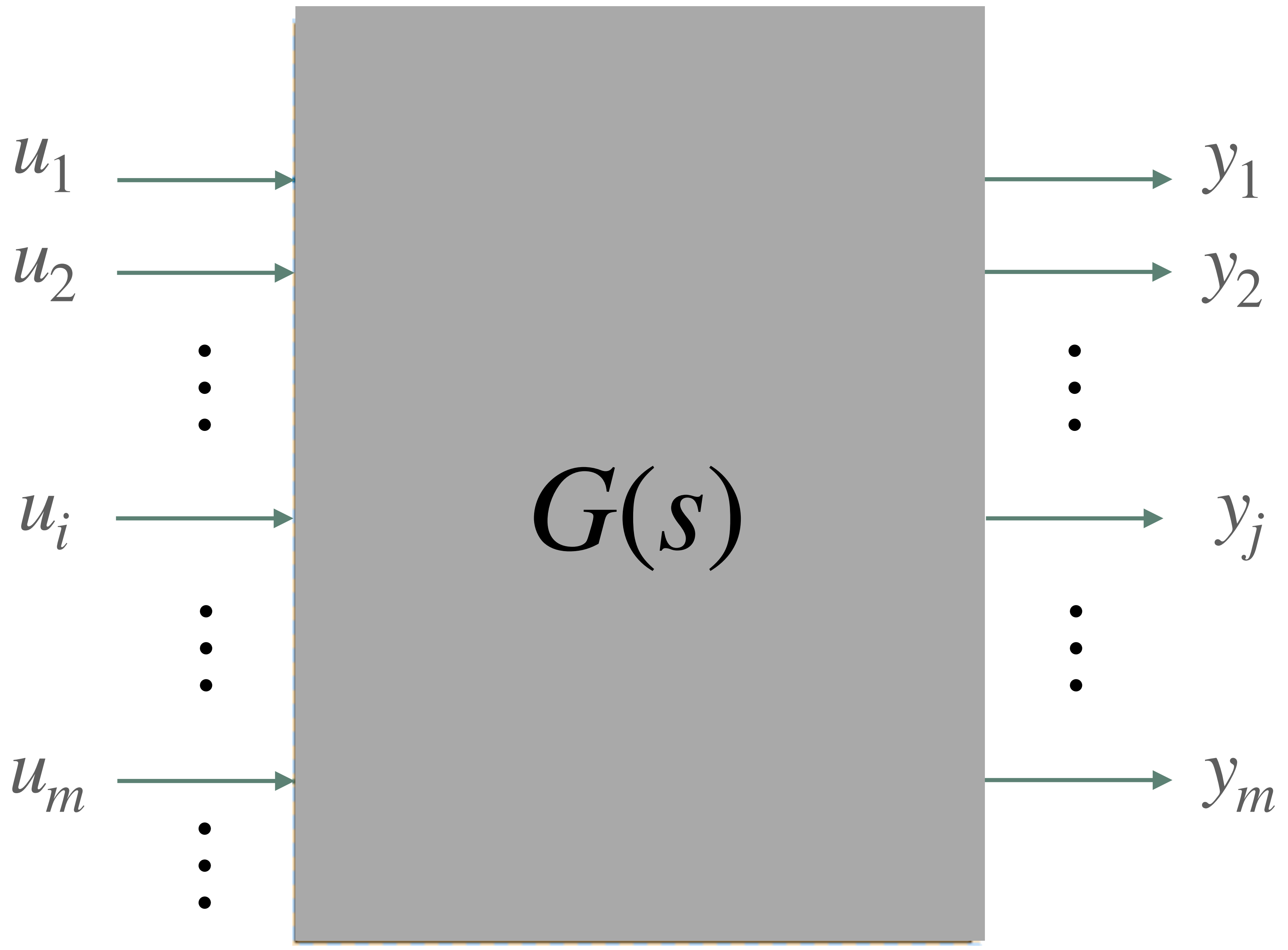
Relative Gain Array



Assumptions:

- Static case ($\omega = 0$)
- $G(s)$ As. Stable
- $\det\, G(0) \neq 0$
- System at the equilibrium at given constant values of the control variables

Relative Gain Array



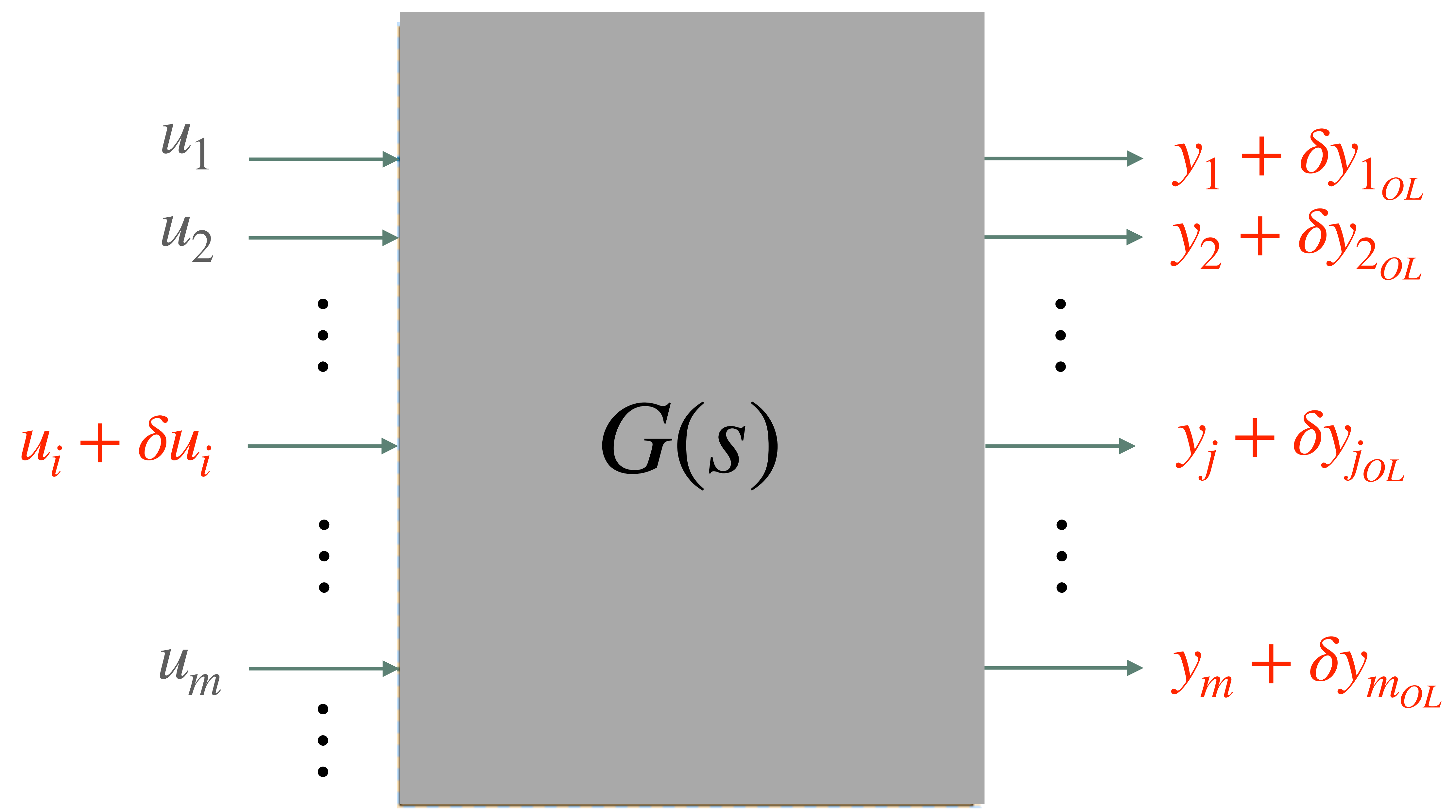
Assumptions:

- Static case ($\omega = 0$)
- $G(s)$ As. Stable
- $\det\ G(0) \neq 0$
- System at the equilibrium at given constant values of the control variables

Two tests are performed:

1) Open loop test
2) Closed loop test

Relative Gain Array



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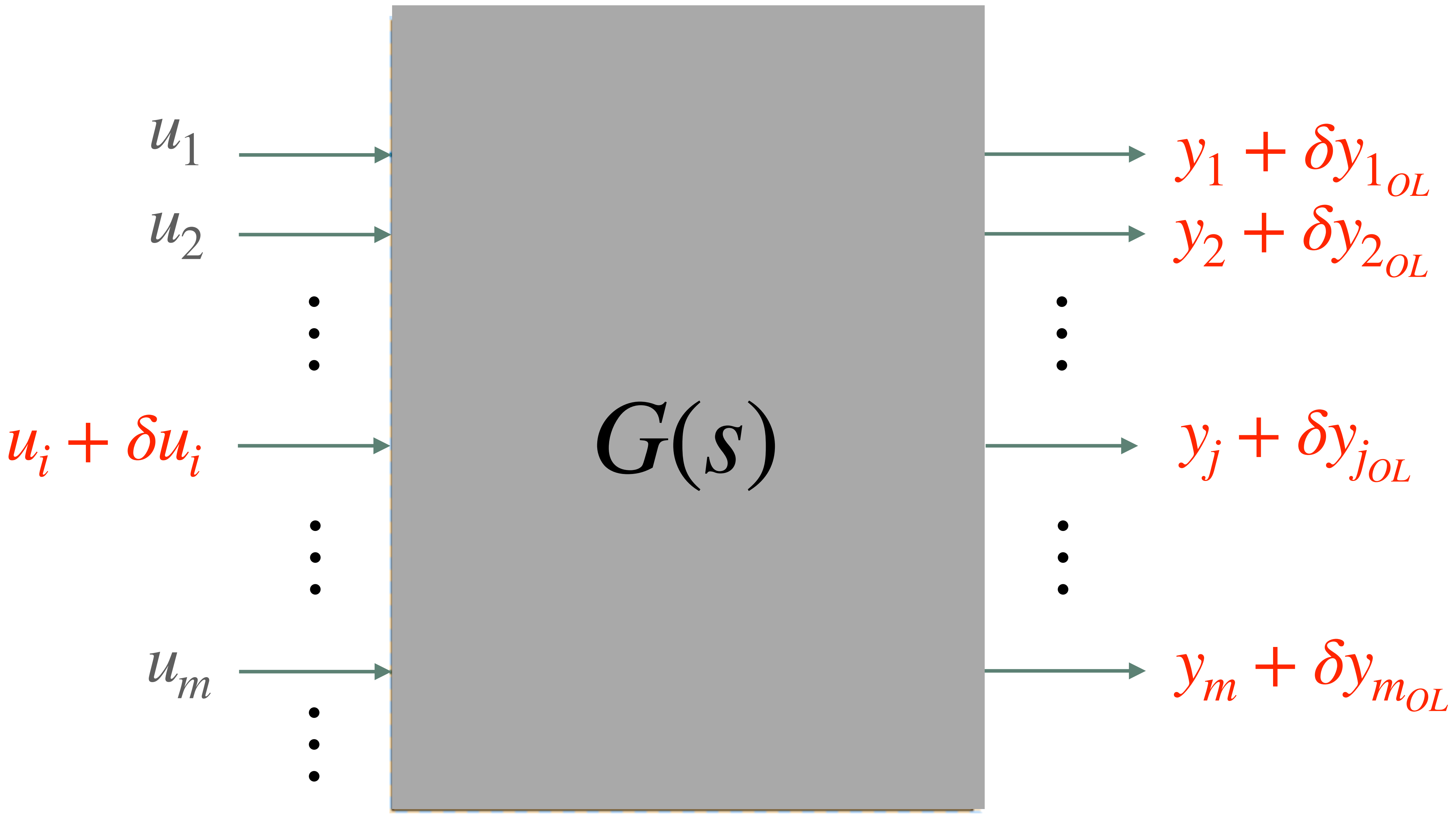
Relative Gain Array



Assumptions:

- Static case ($\omega = 0$)
- $G(s)$ As. Stable
- $\det\ G(0) \neq 0$
- System at the equilibrium at given constant values of the control variables

open loop gain

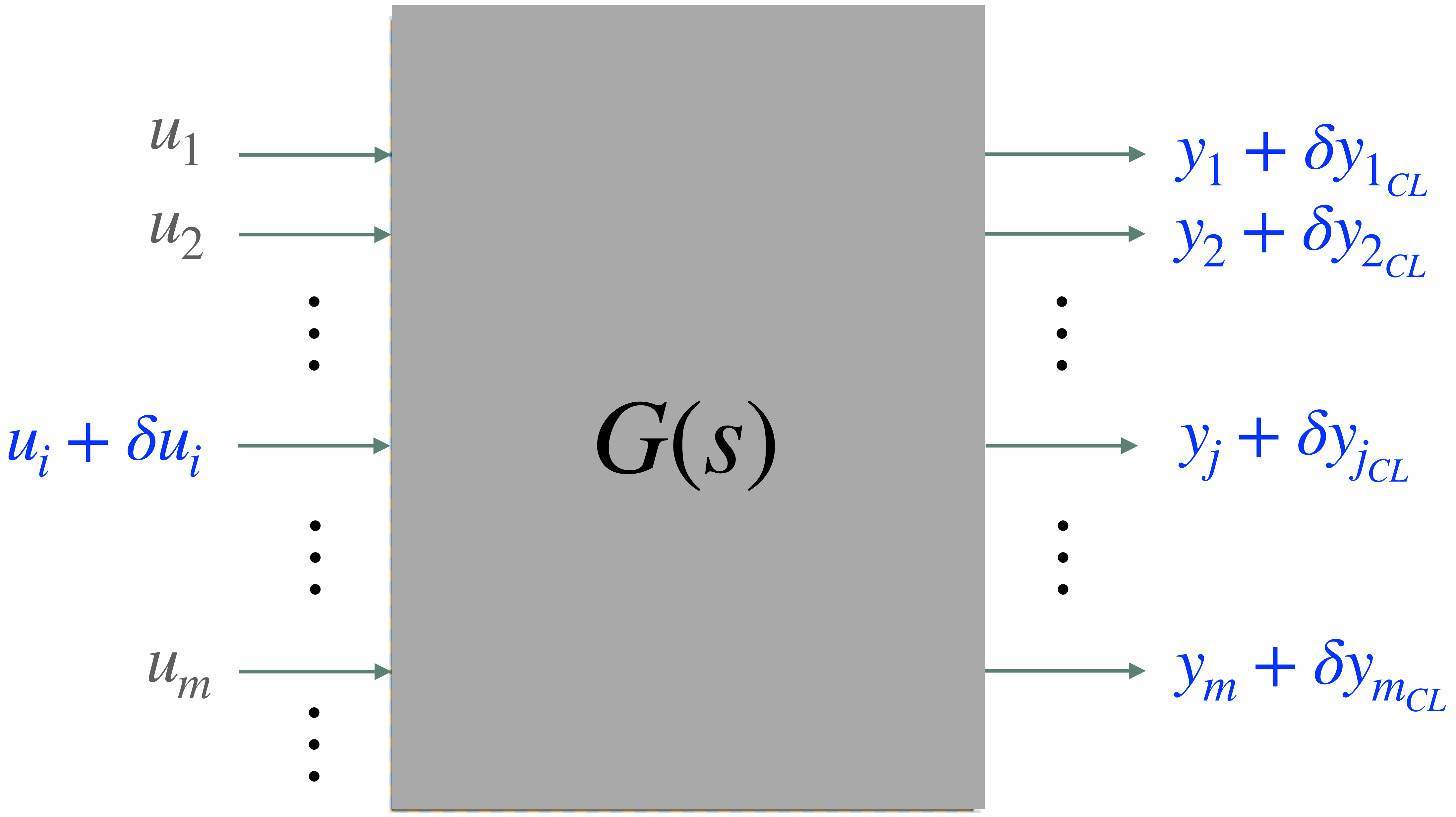
Two tests are performed:

1) Open loop test
2) Closed loop test

$$\frac{\delta y_{j_{OL}}}{\delta u_i} = G_{ji}(0) = g_{ji}$$

$$\forall \quad i, j = 1, \dots, m$$

Relative Gain Array



Assumptions:

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- $G(s)$ As. Stable
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Two tests are performed:

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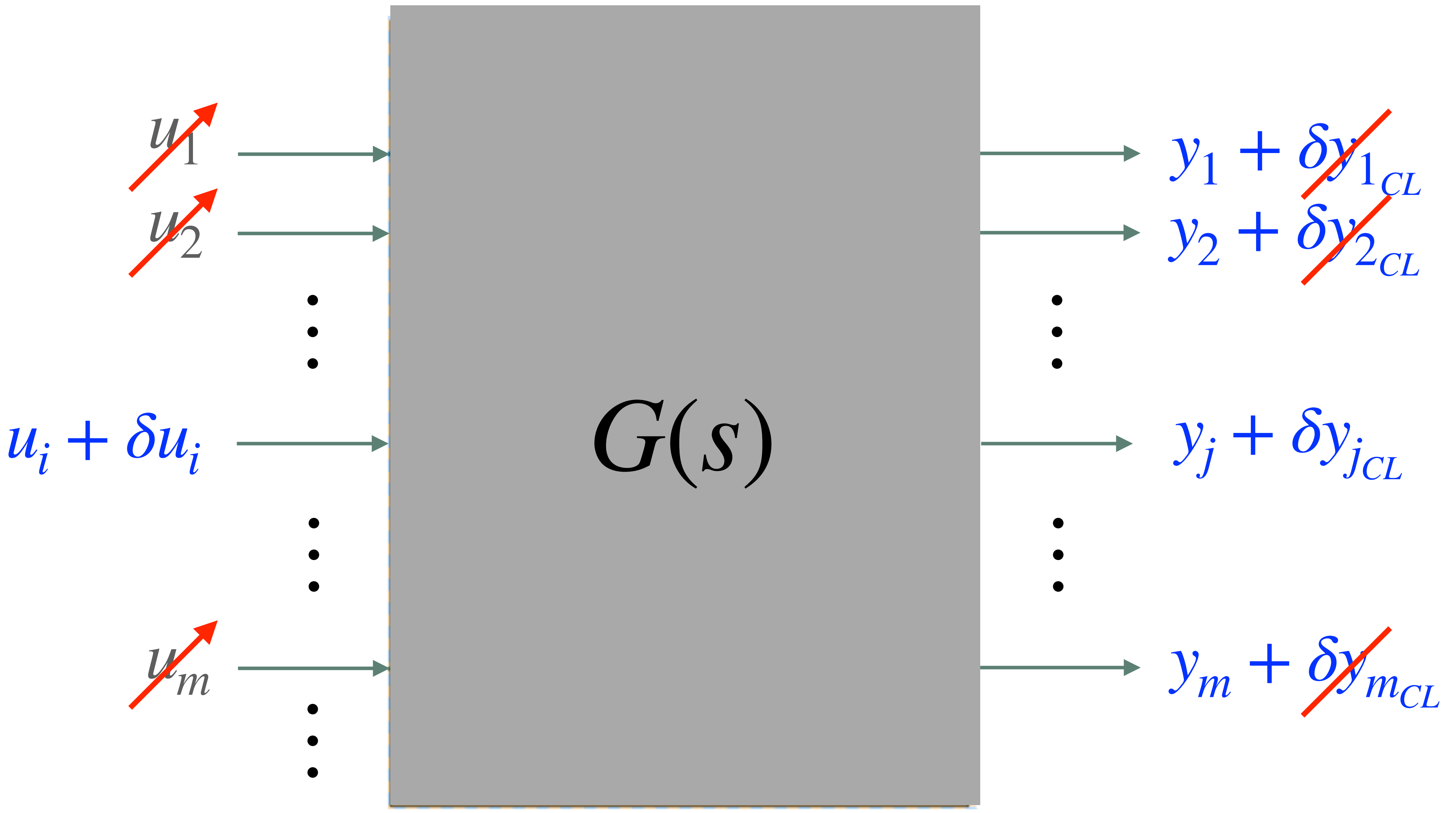
Relative Gain Array



Assumptions:

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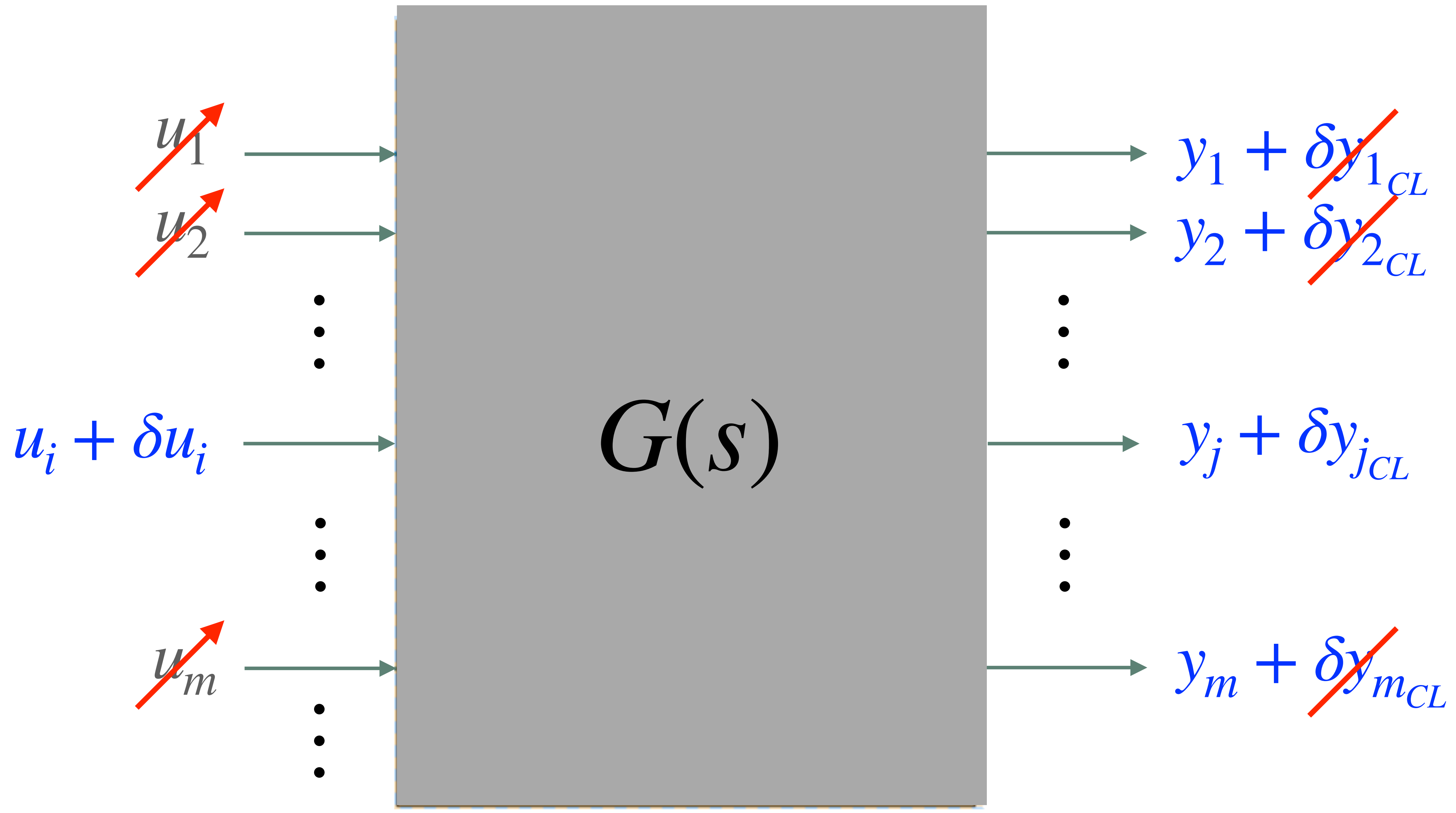
Relative Gain Array



Assumptions:

- Static case $(\omega = 0)$
- $G(s)$ As. Stable
- $\det\ G(0) \neq 0$
- System at the equilibrium at given constant values of the control variables

Two tests are performed:

1) Open loop test
2) Closed loop test

$$\frac{\delta y_{j_{CL}}}{\delta u_i} = \underbrace{h_{ji}}_{\text{closed loop gain}}$$

$$\forall \quad i, j = 1, \ldots, m$$

Relative Gain Array

$$\forall \quad i, j = 1, \ldots, m$$

$$\frac{\delta y_{j_{OL}}}{\delta u_i} = G_{ji}(0) = g_{ji} \qquad \frac{\delta y_{j_{CL}}}{\delta u_i} = h_{ji}$$

$$\Downarrow$$

$$\lambda_{ji} = \frac{g_{ji}}{h_{ji}}$$

Assumptions:

- Static case ($\omega = 0$)
- $G(s)$ As. Stable
- $\det\ G(0) \neq 0$
- System at the equilibrium at given constant values of the control variables

Two tests are performed:

1) Open loop test
2) Closed loop test

Relative Gain Array

$$\forall \quad i, j = 1, \dots, m$$

$$\frac{\delta y_{j_{OL}}}{\delta u_i} = G_{ji}(0) = g_{ji} \qquad \frac{\delta y_{j_{CL}}}{\delta u_i} = h_{ji}$$

$$\lambda_{ji} = \frac{g_{ji}}{h_{ji}}$$

$$\Lambda = \begin{bmatrix} \lambda_{11}(s) & \lambda_{12}(s) & \cdots & \lambda_{1m}(s) \\ \lambda_{21}(s) & \lambda_{22}(s) & \cdots & \lambda_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{m1}(s) & \lambda_{m2}(s) & \cdots & \lambda_{mm}(s) \end{bmatrix}$$

Assumptions:

- Static case ($\omega = 0$)
- $G(s)$ As. Stable
- $\det\ G(0) \neq 0$
- System at the equilibrium at given constant values of the control variables

Two tests are performed:

1) Open loop test
2) Closed loop test

Relative Gain Array

$$\forall \quad i, j = 1, \ldots, m$$

$$\frac{\delta y_{j_{OL}}}{\delta u_i} = G_{ji}(0) = g_{ji} \qquad \frac{\delta y_{j_{CL}}}{\delta u_i} = h_{ji}$$

$$\lambda_{ji} = \frac{g_{ji}}{h_{ji}}$$

$$\Lambda = \begin{bmatrix} \lambda_{11}(s) & \lambda_{12}(s) & \cdots & \lambda_{1m}(s) \\ \lambda_{21}(s) & \lambda_{22}(s) & \cdots & \lambda_{2m}(s) \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{m1}(s) & \lambda_{m2}(s) & \cdots & \lambda_{mm}(s) \end{bmatrix}$$

Relative Gain Array (RGA)

Assumptions:

- Static case ($\omega = 0$)
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Relative Gain Array (RGA) →

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Two tests are performed:

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$\boxed{\text{The best pairings correspond to } \lambda_{ji} > 0 \text{ and close to } 1}$

Relative Gain Array

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Relative Gain Array (RGA)

Alternative approach:
- Compute

$$\Lambda = G(0) \circ \left[G(0)^{-1}\right]^T$$

Relative Gain Array

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Relative Gain Array (RGA)

Alternative approach:

- Compute

$$\Lambda = G(0) \circ \left[G(0)^{-1}\right]^T$$

Properties of the RGA:

- $\sum_{i=1}^{m} \lambda_{ji} = 1, \quad$ for $j = 1, \ldots, m$
- $\sum_{j=1}^{m} \lambda_{ji} = 1, \quad$ for $i = 1, \ldots, m$
- elements independent of the measurement units used to model $G(s)$
- Λ diagonal if $G(s)$ is triangular or diagonal

Relative Gain Array

Example 1:

Given

$$G(s) = \begin{bmatrix} \dfrac{5}{10s+1} & \dfrac{1}{5s+1} & \dfrac{0.5}{8s+1} \\ \dfrac{2}{7s+1} & \dfrac{4}{9s+1} & \dfrac{1}{6s+1} \\ \dfrac{1}{4s+1} & \dfrac{0.5}{3s+1} & \dfrac{3}{5s+1} \end{bmatrix}$$

compute the RGA Λ and determine the best input-output pairings

Relative Gain Array

Example 1:

Given

$$G(s) = \begin{bmatrix} \dfrac{5}{10s+1} & \dfrac{1}{5s+1} & \dfrac{0.5}{8s+1} \\ \dfrac{2}{7s+1} & \dfrac{4}{9s+1} & \dfrac{1}{6s+1} \\ \dfrac{1}{4s+1} & \dfrac{0.5}{3s+1} & \dfrac{3}{5s+1} \end{bmatrix}$$

compute the RGA Λ and determine the best input-output pairings

By using the formula:

$$\Lambda = G(0) \circ \left[G(0)^{-1}\right]^T$$

/MATLAB Drive/RGA_example.m

```
%% RGA Example 3x3 System with Automatic Coupling Selection
clear; clc;

%% Define Laplace variable
s = tf('s');

%% Define the 3x3 transfer function matrix G(s)
G = [ 5/(10*s+1)   1/(5*s+1)     0.5/(8*s+1);
      2/(7*s+1)    4/(9*s+1)     1/(6*s+1);
      1/(4*s+1)    0.5/(3*s+1)   3/(5*s+1) ];

%% Compute DC gain
G0 = dcgain(G);

%% Compute RGA at steady state
RGA = G0 .* transpose(inv(G0));

%% Display RGA numerically
disp('RGA matrix at steady state:');
disp(RGA);
```

Relative Gain Array

Example 1:

Given

$$G(s) = \begin{bmatrix} \frac{5}{10s+1} & \frac{1}{5s+1} & \frac{0.5}{8s+1} \\ \frac{2}{7s+1} & \frac{4}{9s+1} & \frac{1}{6s+1} \\ \frac{1}{4s+1} & \frac{0.5}{3s+1} & \frac{3}{5s+1} \end{bmatrix}$$

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%% Compute DC gain
G0 = dcgain(G);

%% Compute RGA at steady state
RGA = G0 .* transpose(inv(G0));

%% Display RGA numerically
disp('RGA matrix at steady state:');
disp(RGA);
```

Command Window

New to MATLAB? See resources for Getting Started.

```
RGA matrix at steady state:
    1.1275   -0.0980   -0.0294
   -0.1078    1.1373   -0.0294
   -0.0196   -0.0392    1.0588

Suggested input-output pairings based on RGA:
  Output y3  ↔  Input u3   (RGA = 1.059)
  Output y1  ↔  Input u1   (RGA = 1.127)
  Output y2  ↔  Input u2   (RGA = 1.137)
```

Best pairings:

$$\{u_1 \rightarrow y_1\} \cup \{u_2 \rightarrow y_2\} \cup \{u_3 \rightarrow y_3\}$$

Relative Gain Array

Example 2:

Given

$$G(s) = \begin{bmatrix} \frac{2}{1+s} & \frac{1}{1+s} \\ \frac{-1}{1+s} & \frac{2}{1+s} \end{bmatrix}$$

compute the RGA Λ and determine the best input-output pairings

Relative Gain Array

Example 2:

Given

$$G(s) = \begin{bmatrix} \dfrac{2}{1+s} & \dfrac{1}{1+s} \\ \dfrac{-1}{1+s} & \dfrac{2}{1+s} \end{bmatrix}$$

compute the RGA Λ and determine the best input-output pairings

By using the formula:

$$\Lambda = G(0) \circ \left[G(0)^{-1}\right]^T$$

/MATLAB Drive/RGA_example_2.m

```
%% RGA Example 2x2 System with Automatic Coupling Selection
clear; clc;

%% Define Laplace variable
s = tf('s');

%% Define 2x2 transfer function matrix G(s) with given static gains
G = [ 2/(1+s)   1/(1+s);
     -1/(1+s)   2/(1+s) ];

%% Compute DC gain
G0 = dcgain(G);

disp('DC gain G(0):');
disp(G0);

%% Compute RGA
RGA = G0 .* transpose(inv(G0));
disp('RGA matrix:');
disp(RGA);
```

Relative Gain Array

Example 2:

Given

$$G(s) = \begin{bmatrix} \dfrac{2}{1+s} & \dfrac{1}{1+s} \\ \dfrac{-1}{1+s} & \dfrac{2}{1+s} \end{bmatrix}$$

compute the RGA Λ and determine the best input-output pairings

Best pairings:

$$\{u_1 \rightarrow y_1\} \cup \{u_2 \rightarrow y_2\}$$

By using the formula:

$$\Lambda = G(0) \circ \left[G(0)^{-1}\right]^T$$

/MATLAB Drive/RGA_example_2.m

```
%% RGA Example 2x2 System with Automatic Coupling Selection
clear; clc;

%% Define Laplace variable
s = tf('s');

%% Define 2x2 transfer function matrix G(s) with given static gains
G = [ 2/(1+s)   1/(1+s);
     -1/(1+s)   2/(1+s) ];

%% Compute DC gain
G0 = dcgain(G);

disp('DC gain G(0):');
disp(G0);

%% Compute RGA
RGA = G0 .* transpose(inv(G0));
disp('RGA matrix:');
disp(RGA);
```

Command Window

New to MATLAB? See resources for Getting Started.

```
RGA matrix:
    0.8000    0.2000
    0.2000    0.8000

Suggested input-output pairings:
  Output y1 ↔ Input u1 (RGA = 0.800)
  Output y2 ↔ Input u2 (RGA = 0.800)
```