

- I want to take the BASIC EXAM (maximum grade is 24/30)
 I want to take the ADVANCED EXAM (maximum grade is 30/30 cum laude)

Total time is 1 hour.

BASIC EXAM

1. Given the Cauchy problem

$$\begin{cases} y'(t) = 2ty(t) & \text{for } t > 0 \\ y(0) = 3 \end{cases},$$

compute two steps of the Implicit Euler Method, with $\Delta t = 1$, in order to approximate $y(2)$. Report the intermediate computations.

2. Describe two possible criteria for stopping iterative solvers for linear systems.

Starting from $x^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, compute two iterations of the Jacobi method applied to the system $Ax = b$, where

$$A = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 4 & 2 \\ -1 & 2 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} -9 \\ -18 \\ -15 \end{bmatrix}.$$

Report intermediate computations.

ADVANCED EXAM

3. Write the pseudo-code of the Gaussian Elimination Method (without pivoting) used to solve linear systems. Describe (with full justification) its computational cost. Show how the Gaussian Elimination Method (without pivoting) works when solving the system $Ax = b$, with

$$A = \begin{bmatrix} -4 & -2 & -3 \\ -16 & -10 & -13 \\ -4 & 0 & -4 \end{bmatrix}, \quad b = \begin{bmatrix} -2 \\ -14 \\ -4 \end{bmatrix}.$$

4. State and prove the theorem on the existence and uniqueness of the Lagrange interpolation polynomial of a given function. State, without a proof, an error estimate for the interpolation error.