

- I want to take the BASIC EXAM
 I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.
- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

BASIC EXAM

1. Apply the Gaussian elimination method (GEM) algorithm to solve

$$\begin{bmatrix} 3 & 6 & 3 \\ 1 & 4 & 3 \\ 2 & 6 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 6 \\ 0 \end{bmatrix}$$

showing the intermediate computations.

2. Write the pseudocode of the explicit Euler method to solve a Cauchy problem of the form :

$$\begin{cases} y'(t) = f(t, y(t)) \text{ for } t > 0 \\ y(0) = y_0. \end{cases}$$

Then, apply it to the case

$$\begin{cases} y'(t) = -t(y(t))^2 \text{ for } t > 0 \\ y(0) = 2. \end{cases}$$

and compute two steps of the method, with $\Delta t = 1/2$, in order to approximate $y(1)$. Report the intermediate computations.

ADVANCED EXAM

3. Explain what is the *LU* factorization of a matrix, without pivoting, and give its pseudocode. Then, compute the LU factorization, without pivoting, of

$$\begin{bmatrix} 3 & 6 & 3 \\ 1 & 4 & 3 \\ 2 & 6 & 7 \end{bmatrix}$$

Report the intermediate computations. Why pivoting is necessary in the general case?

4. Write the pseudocode of the Newton method and state the error estimate that expresses its convergence.