

## EXERCISE 10

- Write a Matlab function `[x, mu] = power(A, maxit, tol)` that implements the power method to approximate the dominant eigenvalue of a matrix (use a random vector as initial guess). Test it on the matrix

$$A = Q * \text{diag}(1:10) * \text{inv}(Q), \text{ with } Q = \text{orth}(\text{randn}(10,10))$$

Note that the spectrum of  $A$  is  $\{1, \dots, 10\}$ . Modify the function to be able to plot the relative error  $|\lambda_1 - \mu_k| / |\lambda_1|$  and also  $(|\lambda_2 / \lambda_1|)^k$ ,  $k = 1, 2, \dots$ . What do you observe?

- Same as above, but test on the nonsymmetric matrix obtained using `Q = randn(10,10)`. Compare the results with the previous case.
- Write another function, `inverse_power`, that implements the inverse power method (use “backslash” to solve the linear system), and test it on the matrix used in the previous point to approximate  $\lambda_9 = 2$  using

$$\mu = 1.55, 1.65, 1.75, 1.85, 1.95.$$

Plot the number of iterations required to converge vs. the value  $|\lambda_9 - \mu| / |\lambda_{10} - \mu|$  and comment.