General Information

Prof. Antonella Ferrara https://scholar.google.com/citations?user=r5JuMskAAAAJ&hl=en

Course Teaching Material: KIRO UNIPV https://elearning.unipv.it/

- 504462 - PROCESS CONTROL 2025-26 - PROF.SSA FERRARA ANTONELLA

Lecture Time-table: http://www-3.unipv.it/ingserv/orario2526/1sem/insegnamenti/ProCont.html

Exams: https://studentionline.unipv.it/esse3/Home.do https://kirotesting.unipv.it/

Introduction

- The course and its objectives
- Kiro UNIPV
- Teaching material
- The preparatory course
- The exam
- Office hours
- The Intelligent Robotics Lab
- Theses and research
- The role of Ph.D. Students and PostDocs

Introduction

- Program of the course:

Advanced SISO control schemes:
Pre-filters and parallel compensators, two degrees of freedom control schemes, compensation of measurable disturbances, systems with delays and Smith Predictor, Padé approximation, decoupling in the frequency domain, control of open loop unstable systems.

Advanced MIMO control schemes:
Decoupling based control schemes, decentralized control, relative gain array.

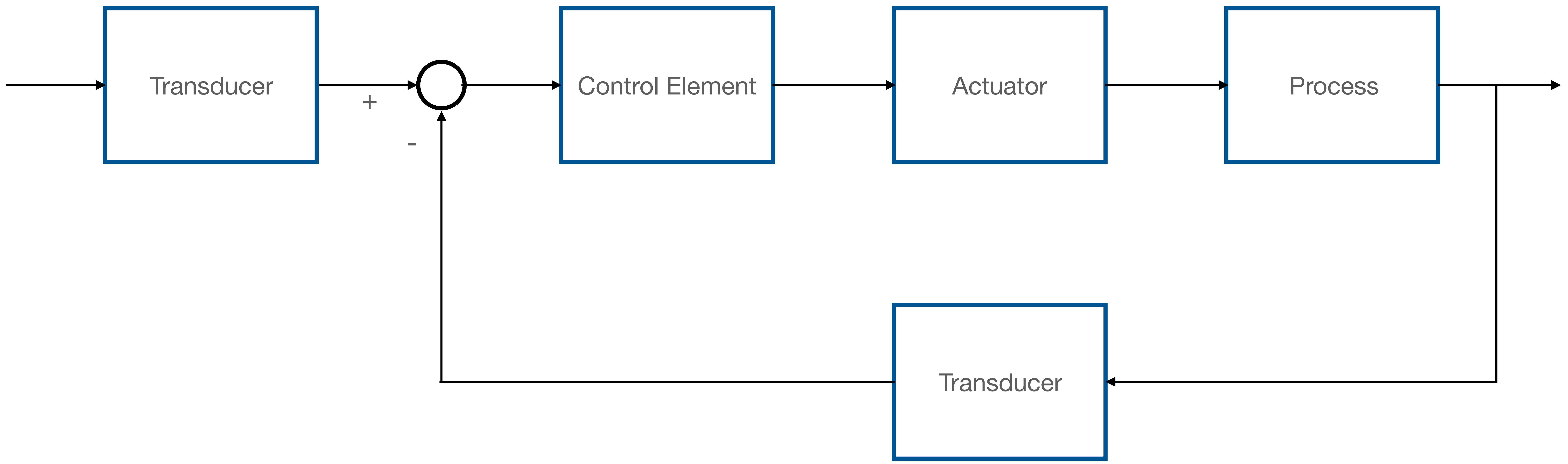
PID controllers:
Features and properties. Rules for the empirical calibration. Wind-up and anti wind-up schemes.

Digital control:
Discrete-time systems. The concept of equilibrium for discrete-time systems. Stability of linear time-invariant discrete-time systems. Jury test. Digital control schemes. Zeta transform and its properties. Transfer functions in the z domain. Sampling and aliasing. Choice of the sampling time. Zero-order-Hold. Discretization of continuous-time controllers. Bilinear transformation, Euler and Tustin methods.

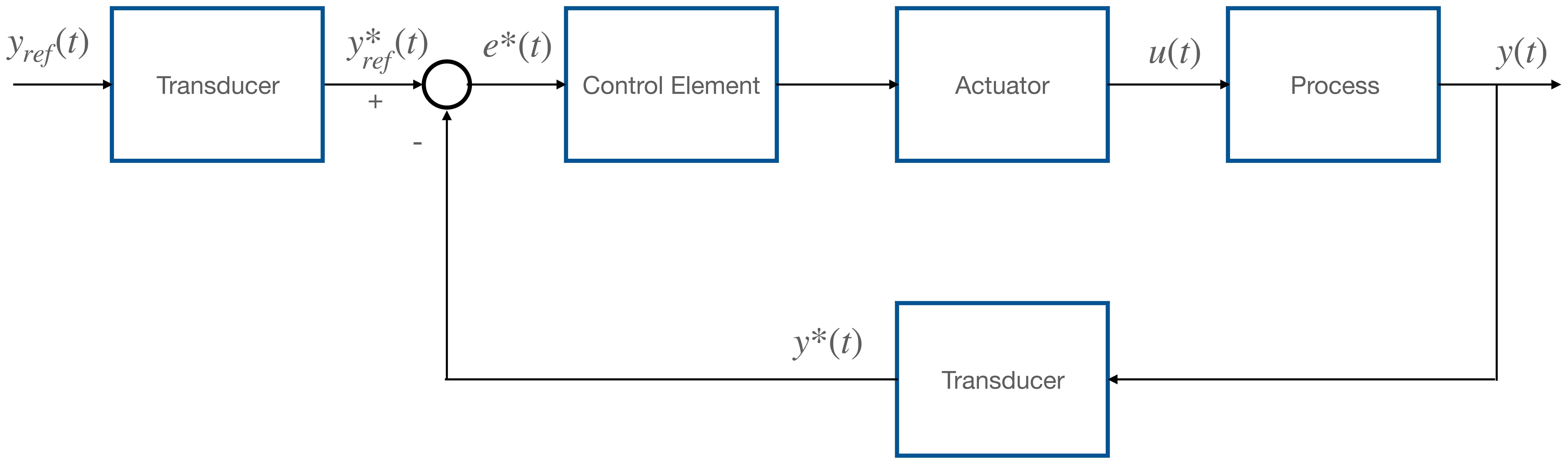
Introduction

- Some of the figures in these slides, kindly provided by McGraw-Hill, are those of the Textbook:

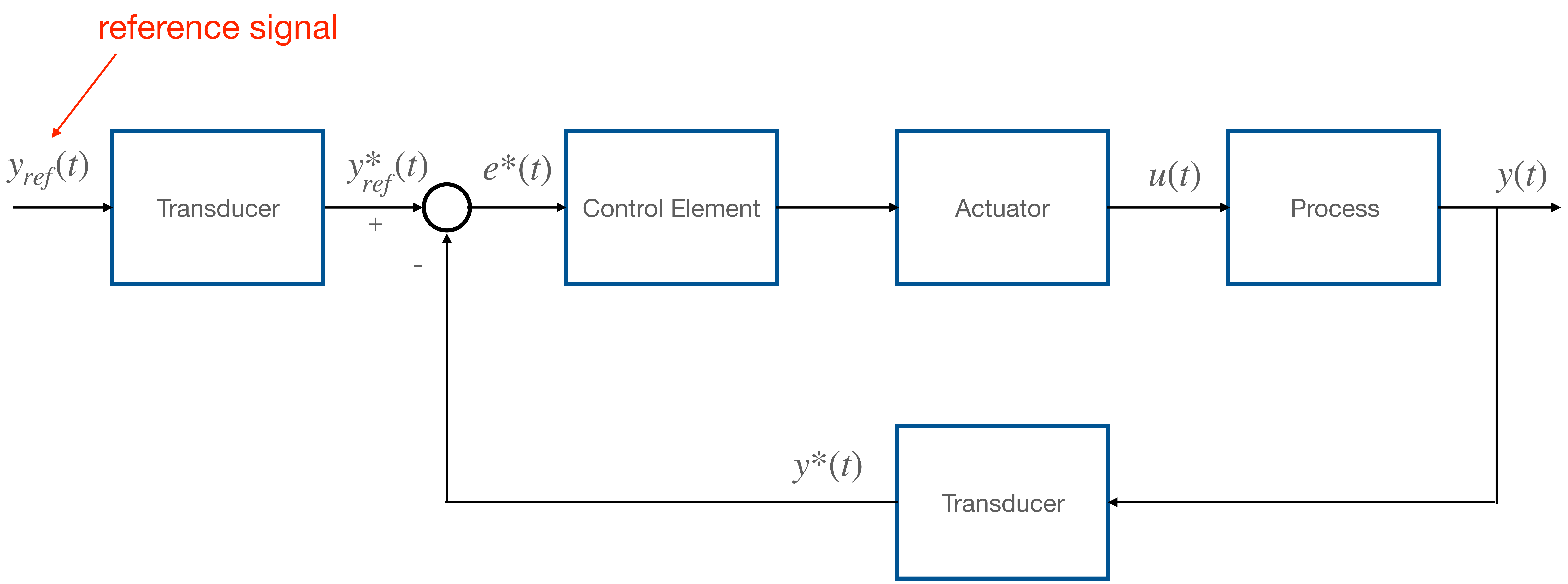
Basic Control Scheme



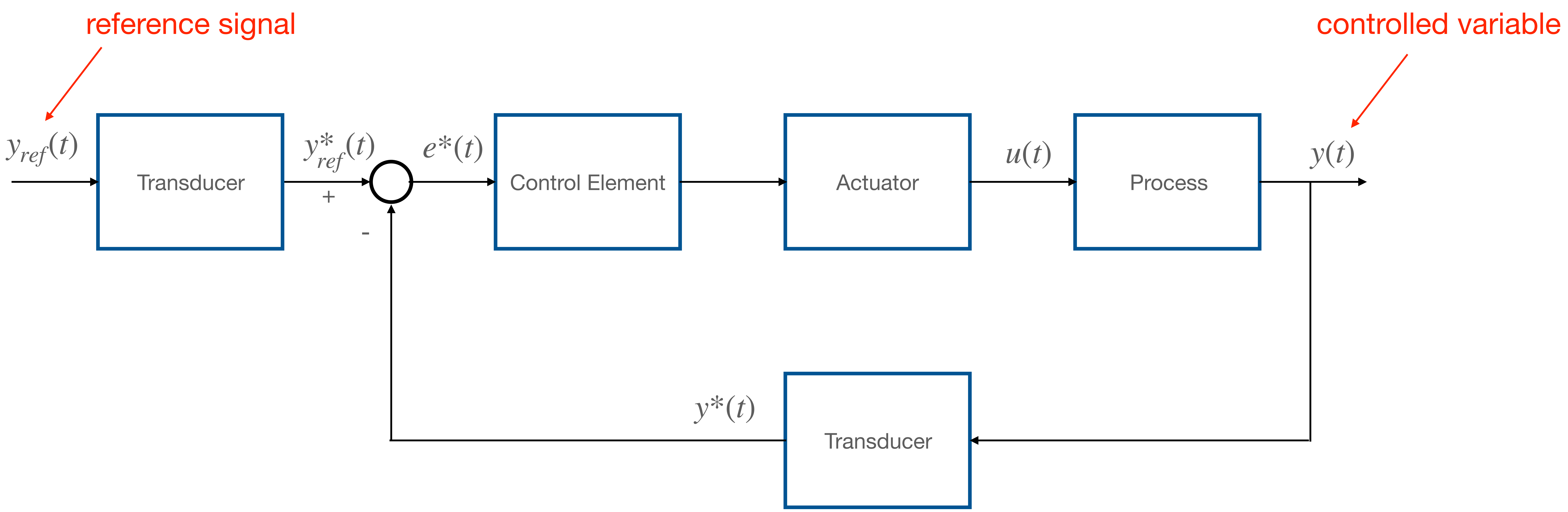
Basic Control Scheme



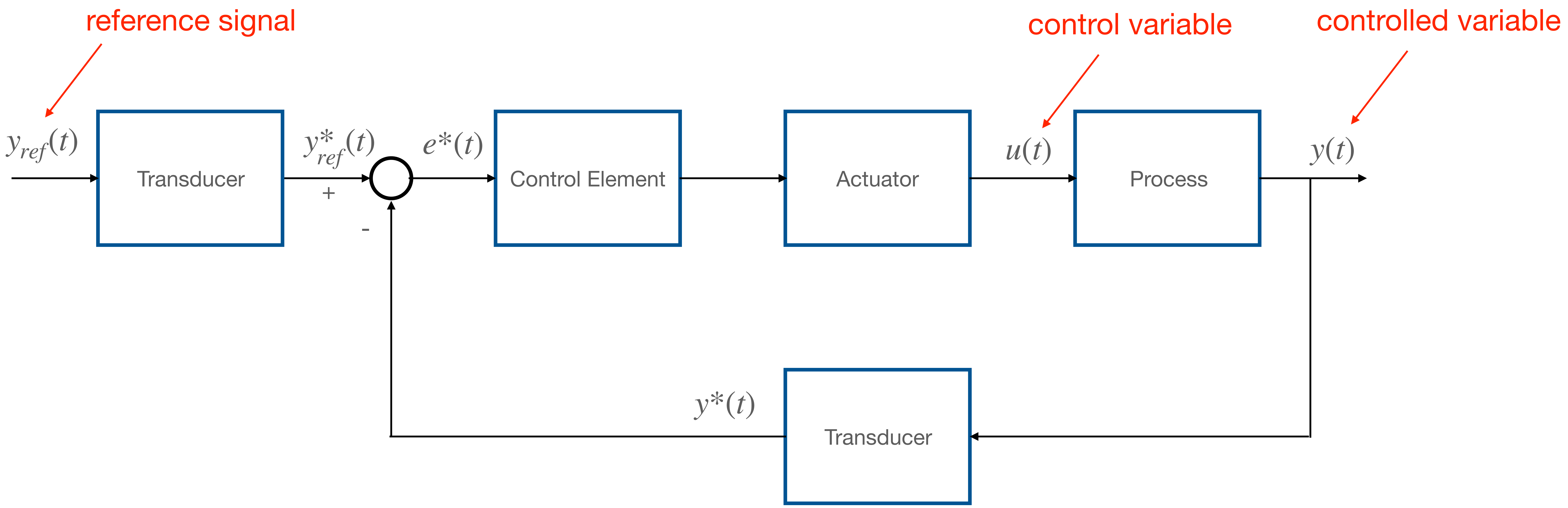
Basic Control Scheme



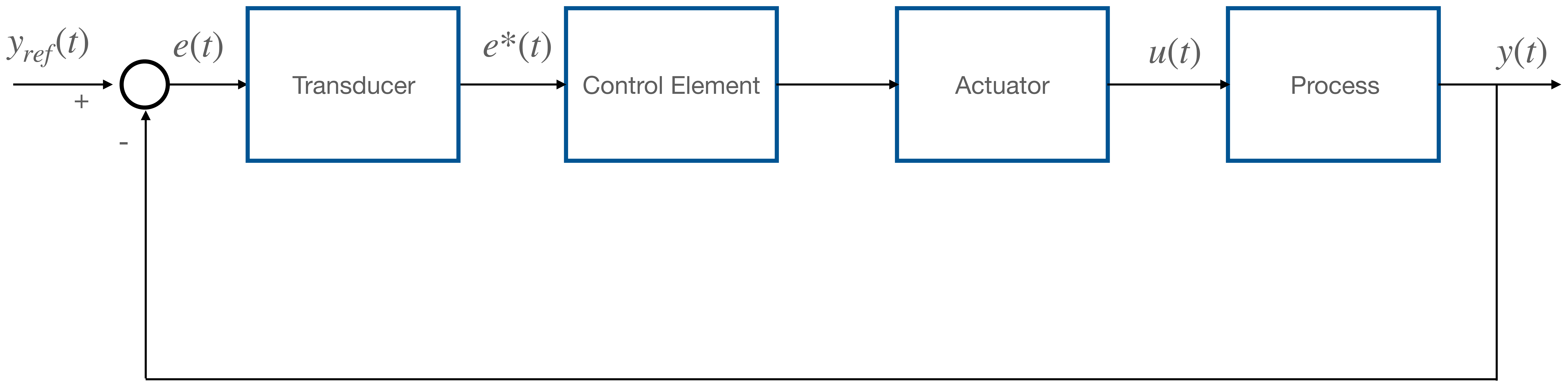
Basic Control Scheme



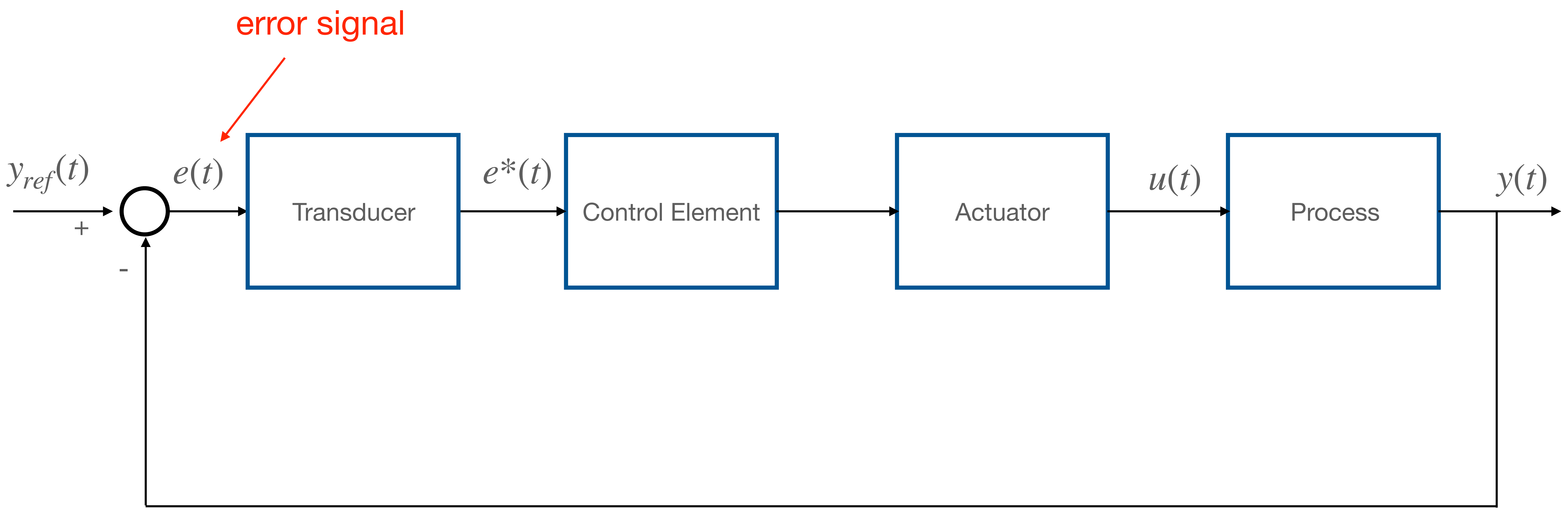
Basic Control Scheme



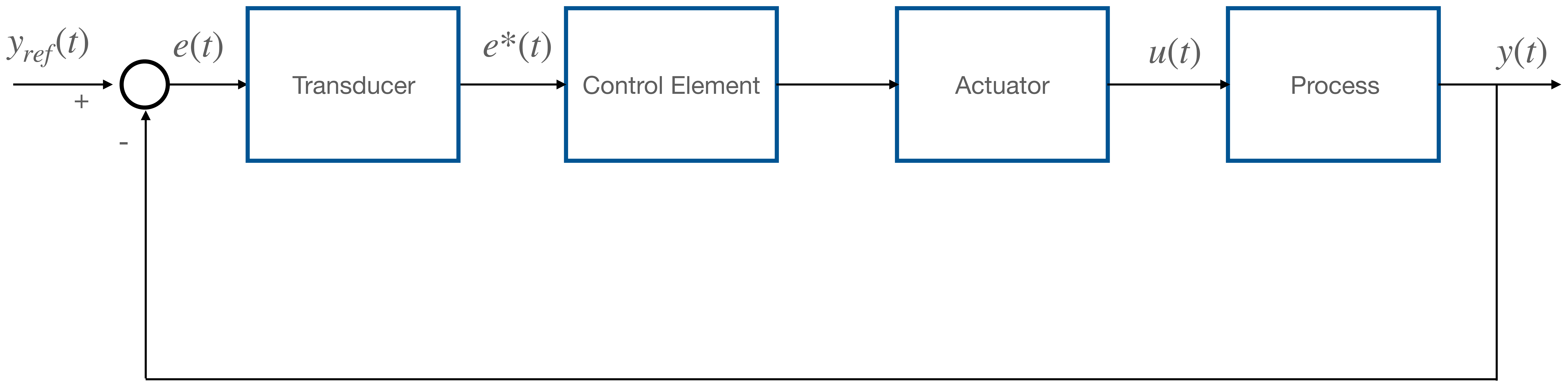
Basic Control Scheme



Basic Control Scheme

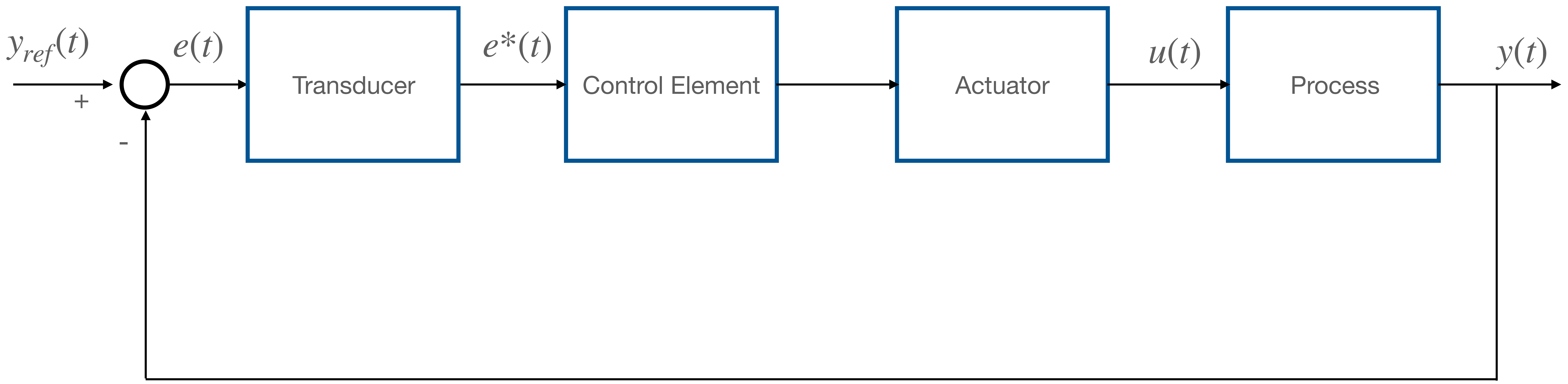


Basic Control Scheme



Assumption: Linear Time-Invariant (LTI) Dynamic Systems

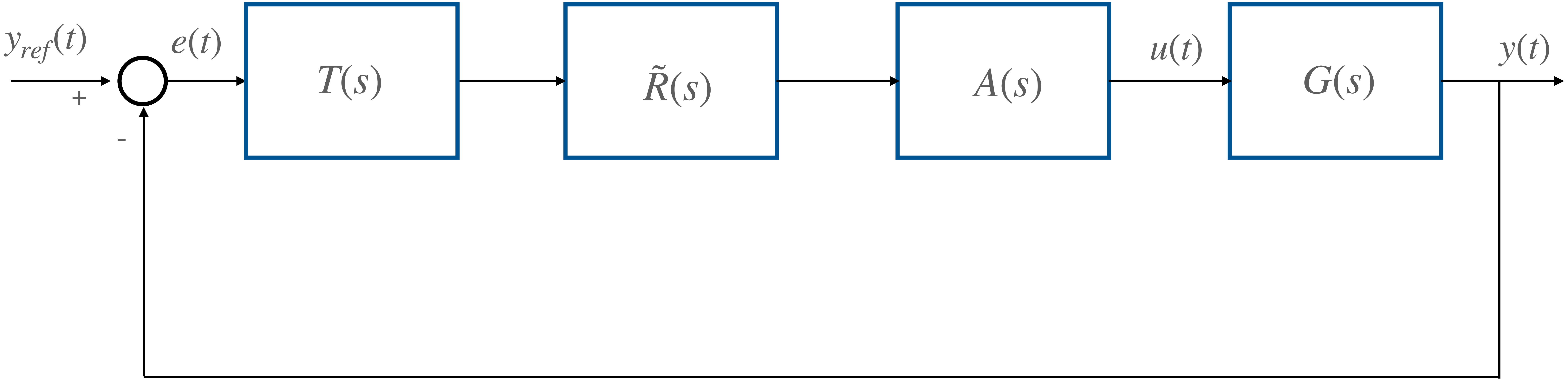
Basic Control Scheme



Assumption: Linear Time-Invariant (LTI) Dynamic Systems

They can be modelled via Transfer Functions

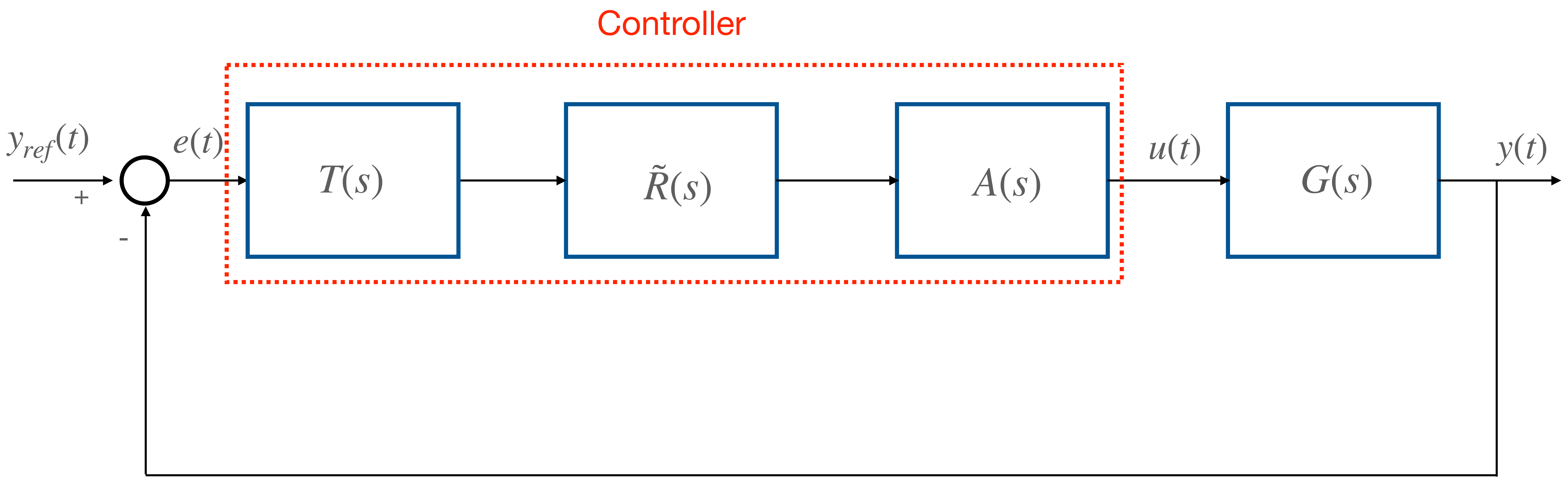
Basic Control Scheme



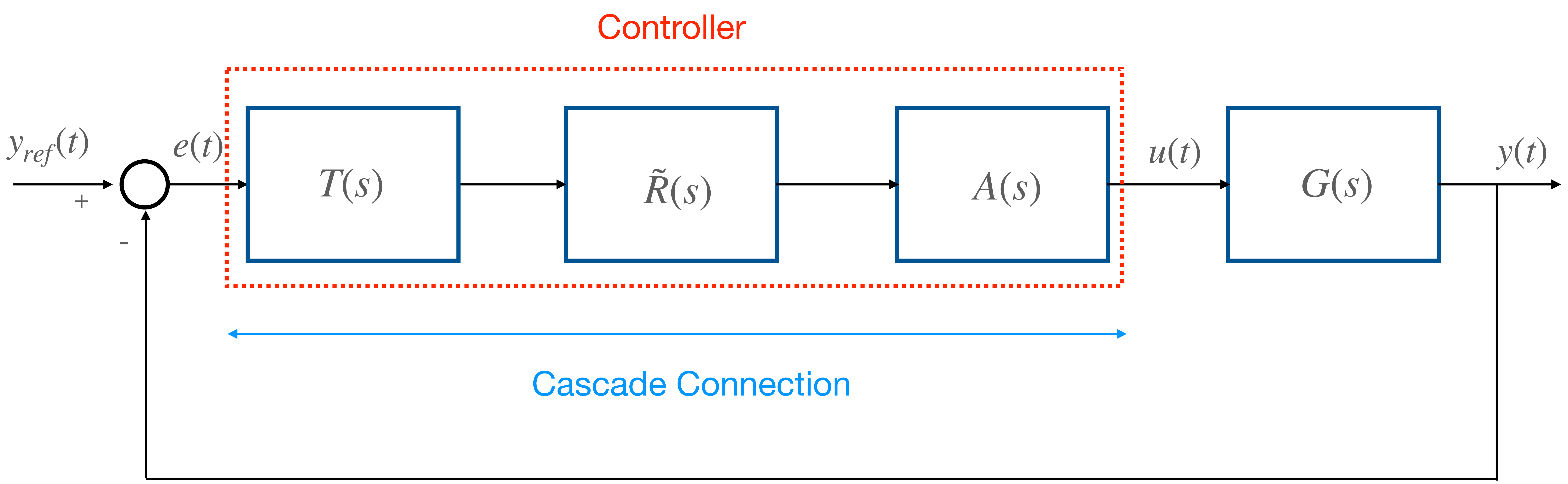
Assumption: Linear Time-Invariant (LTI) Dynamic Systems

They can be modelled via Transfer Functions

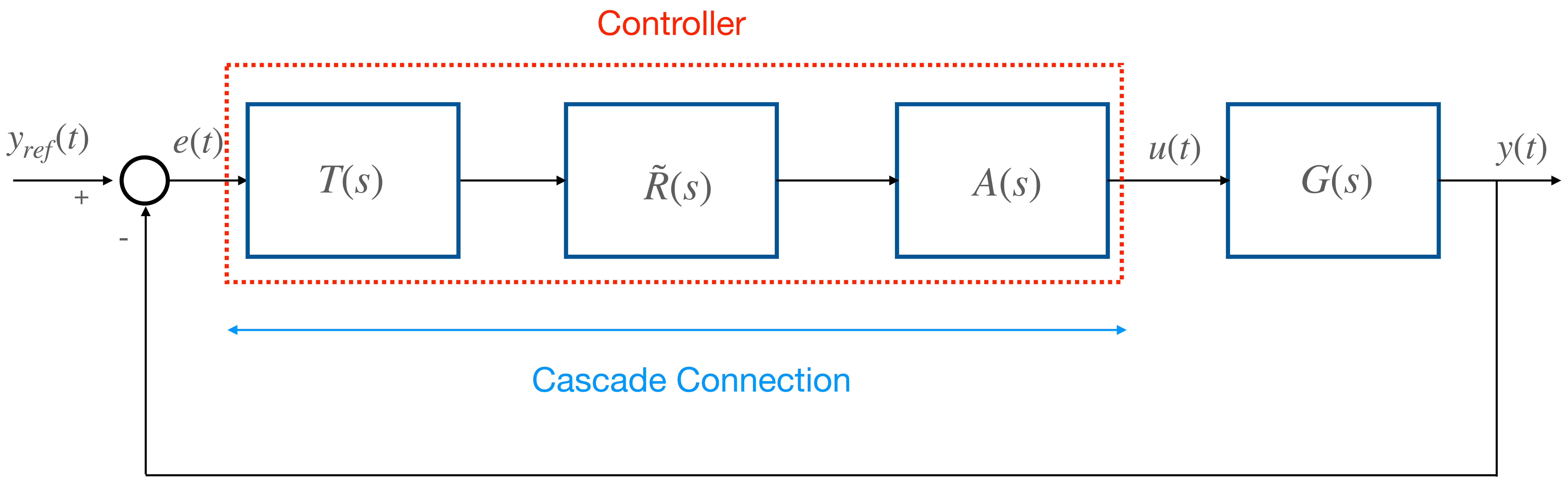
Basic Control Scheme



Basic Control Scheme

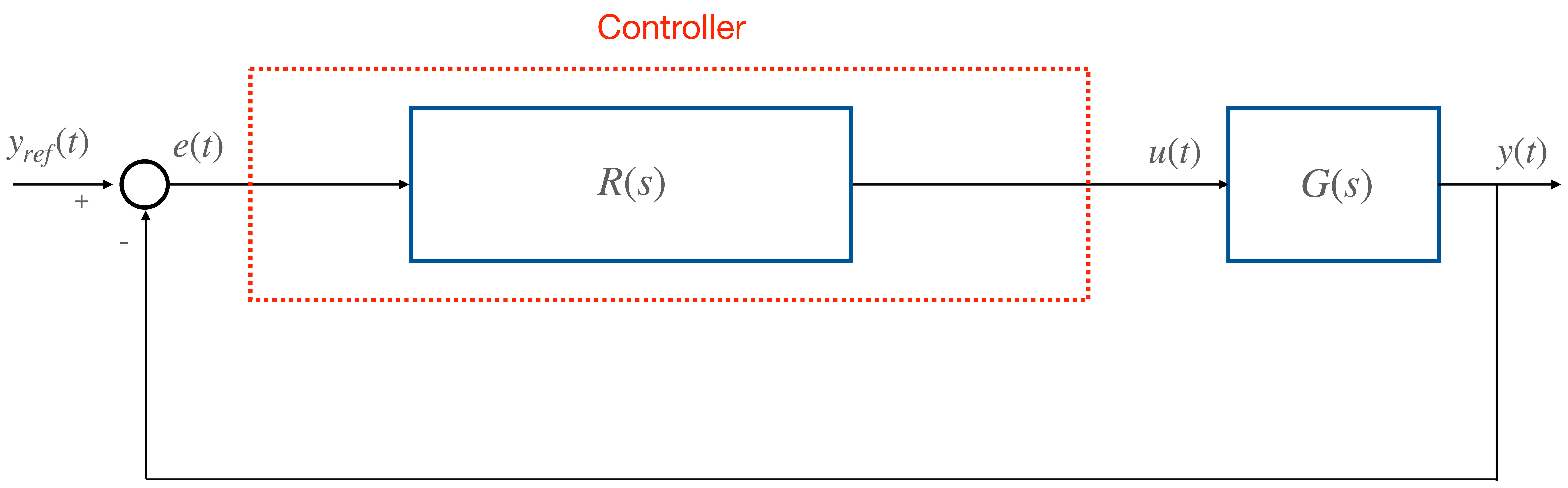


Basic Control Scheme



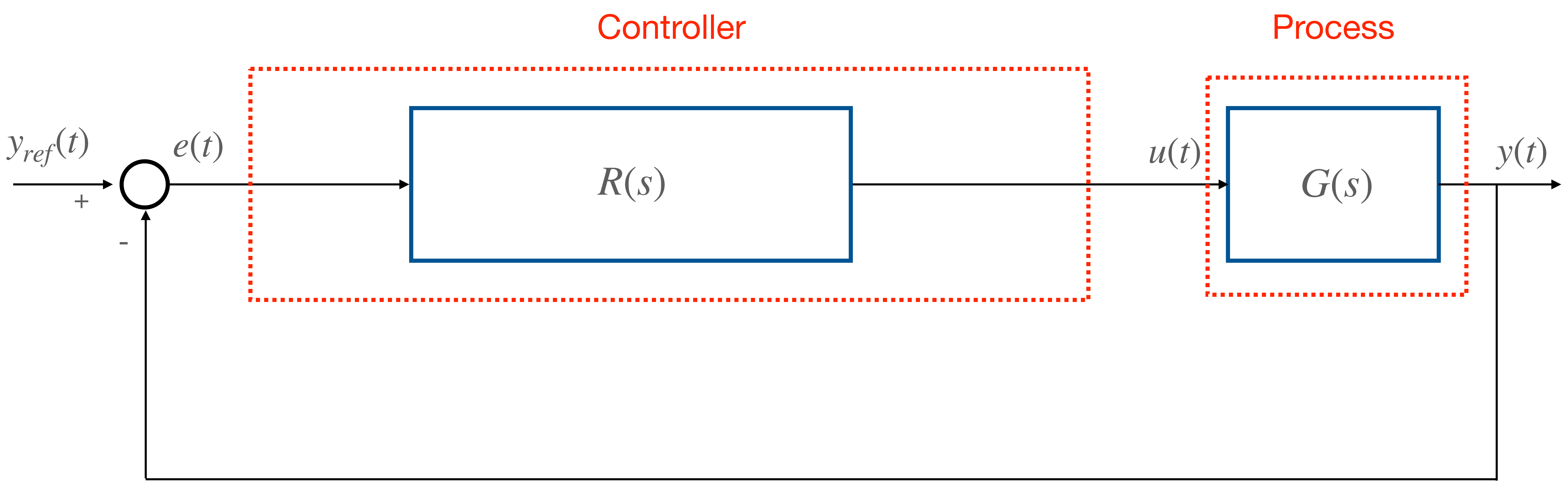
Block Algebra Rule: $T(s) \cdot \tilde{R}(s) \cdot A(s) = R(s)$

Basic Control Scheme

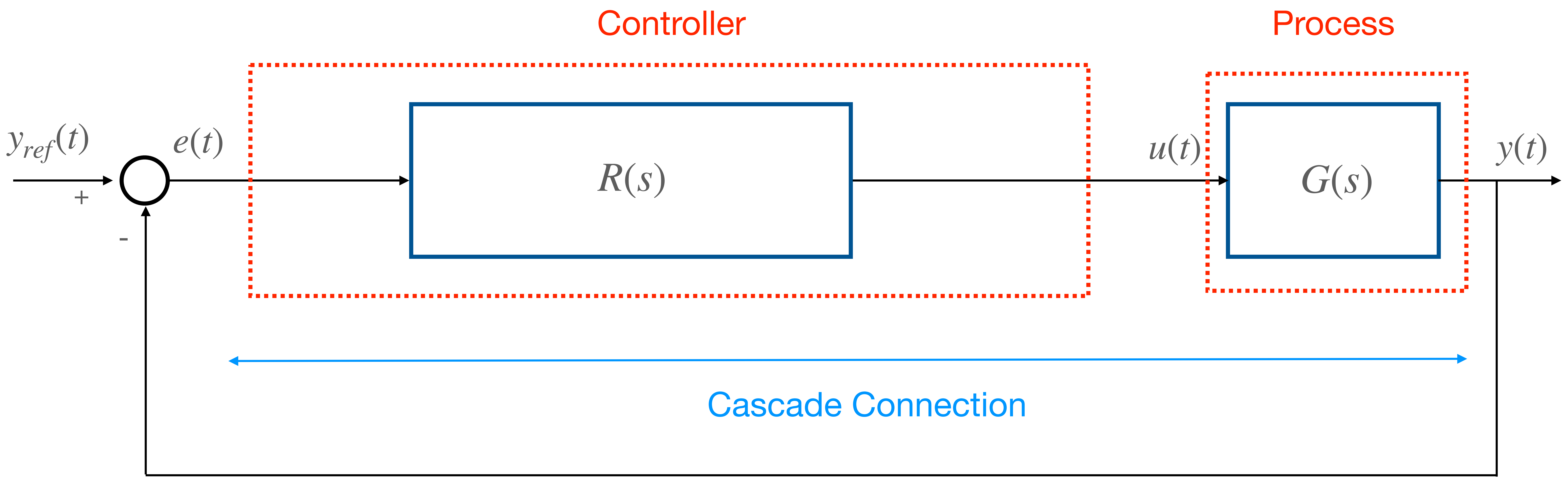


Block Algebra Rule: $T(s) \cdot \tilde{R}(s) \cdot A(s) = R(s)$

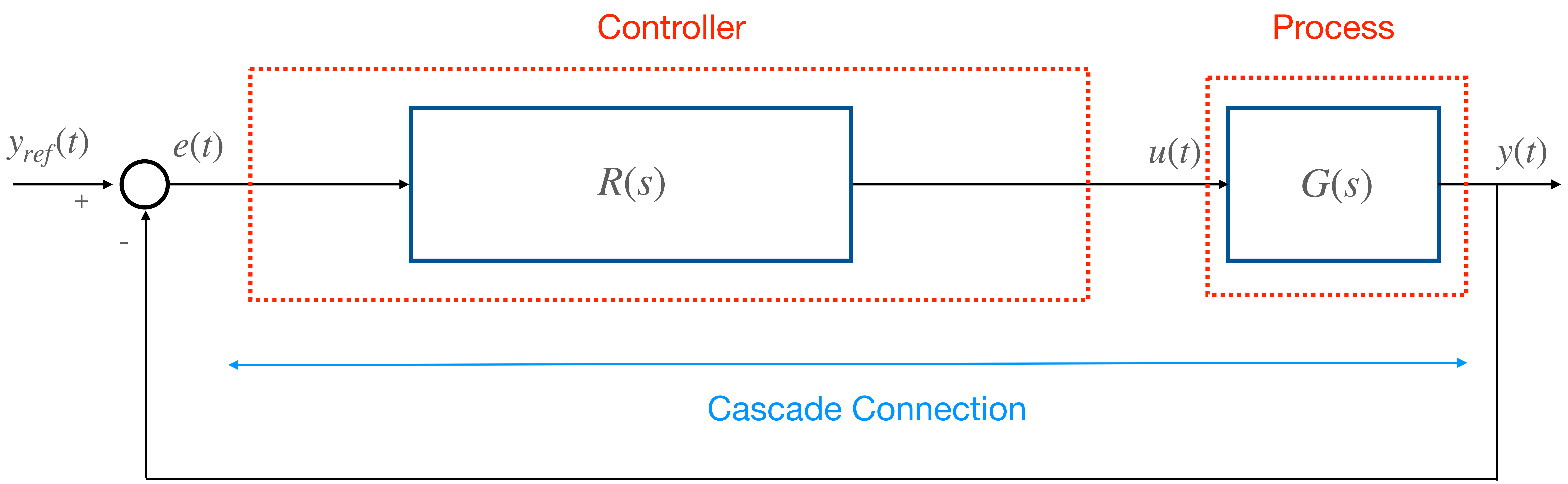
Basic Control Scheme



Basic Control Scheme

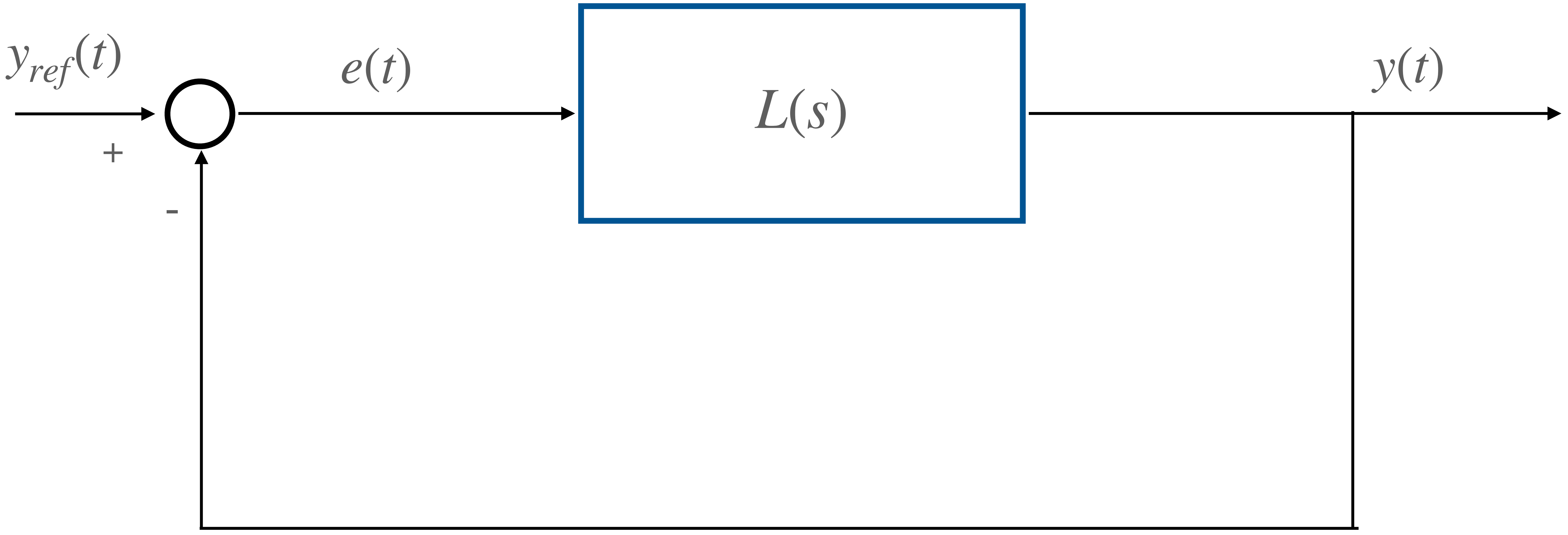


Basic Control Scheme



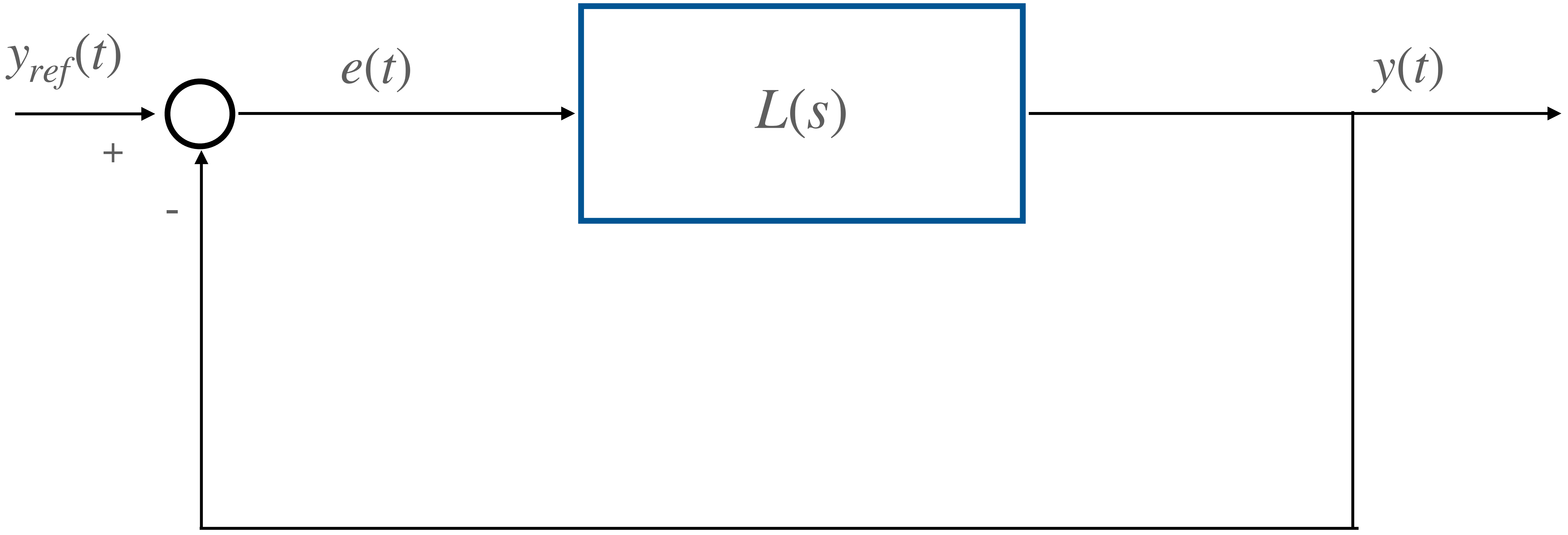
Block Algebra Rule: $R(s) \cdot G(s) = L(s)$

Basic Control Scheme



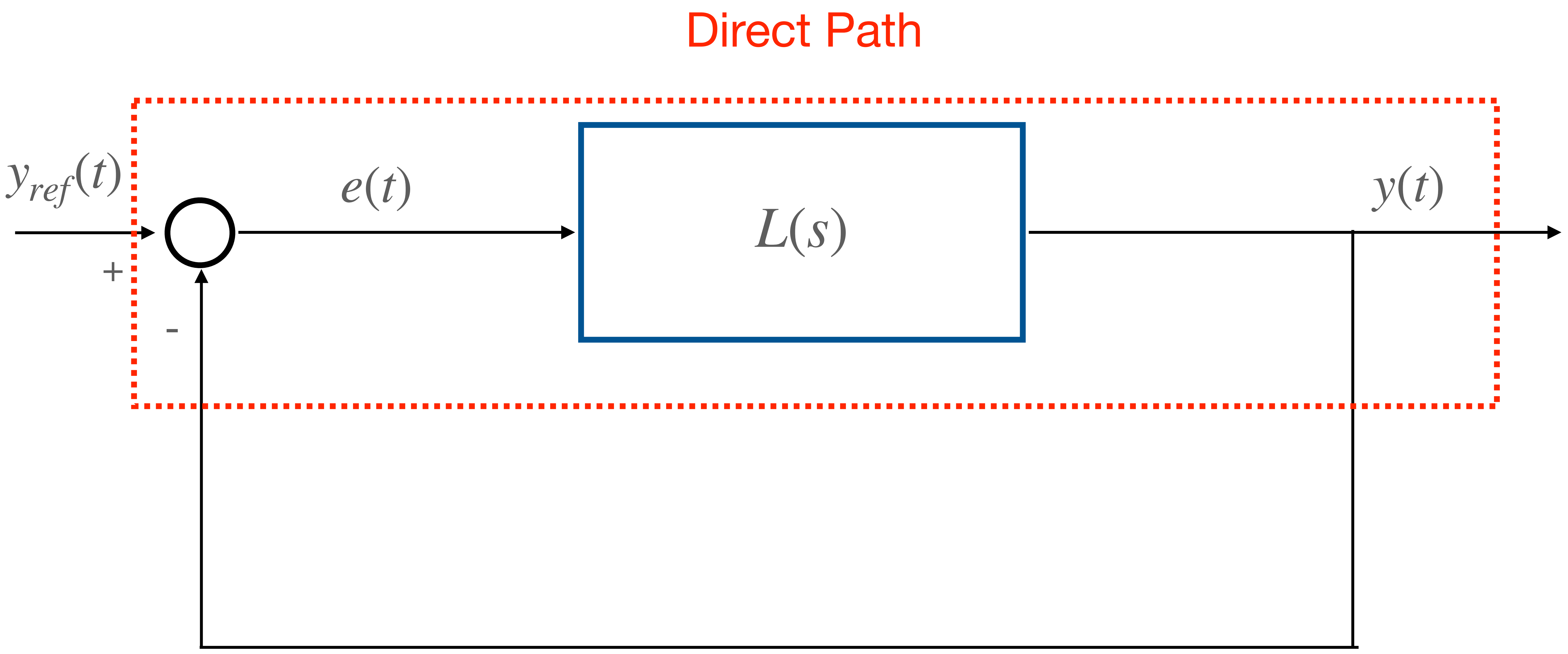
Block Algebra Rule: $R(s) \cdot G(s) = L(s)$

Unitary Feedback Control Scheme



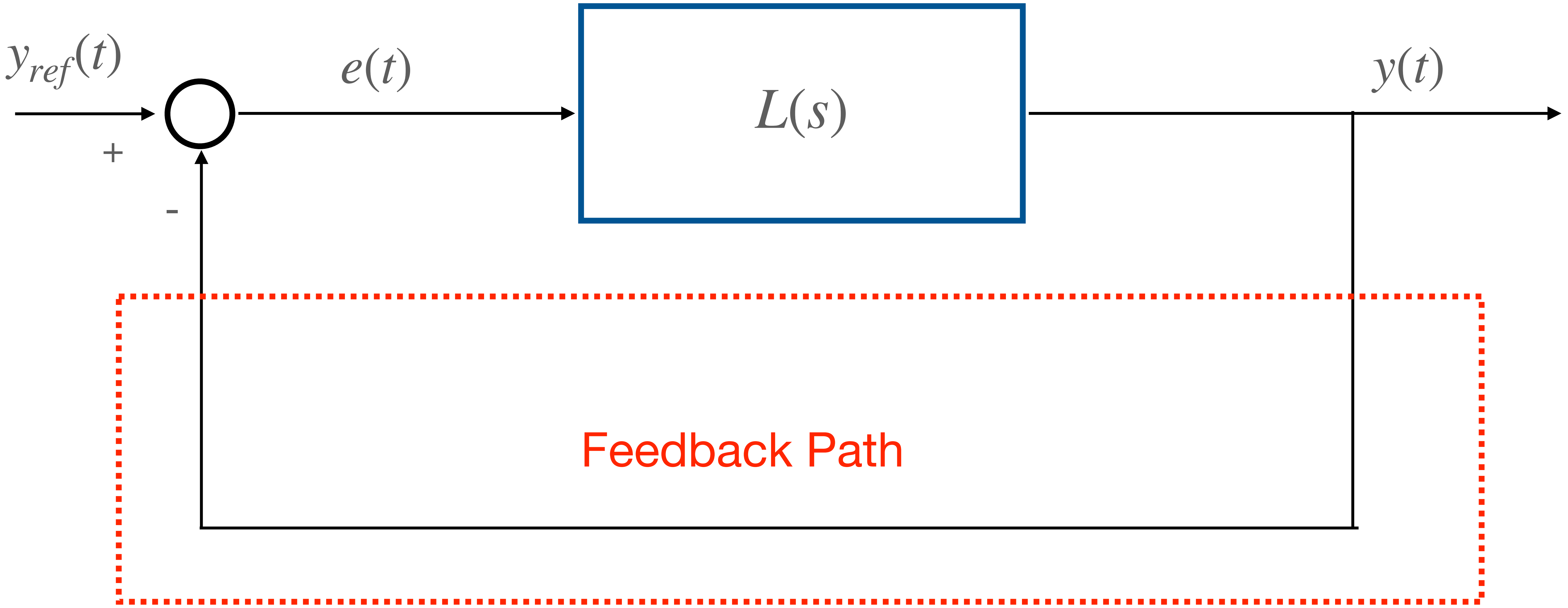
Block Algebra Rule: $R(s) \cdot G(s) = L(s)$

Unitary Feedback Control Scheme



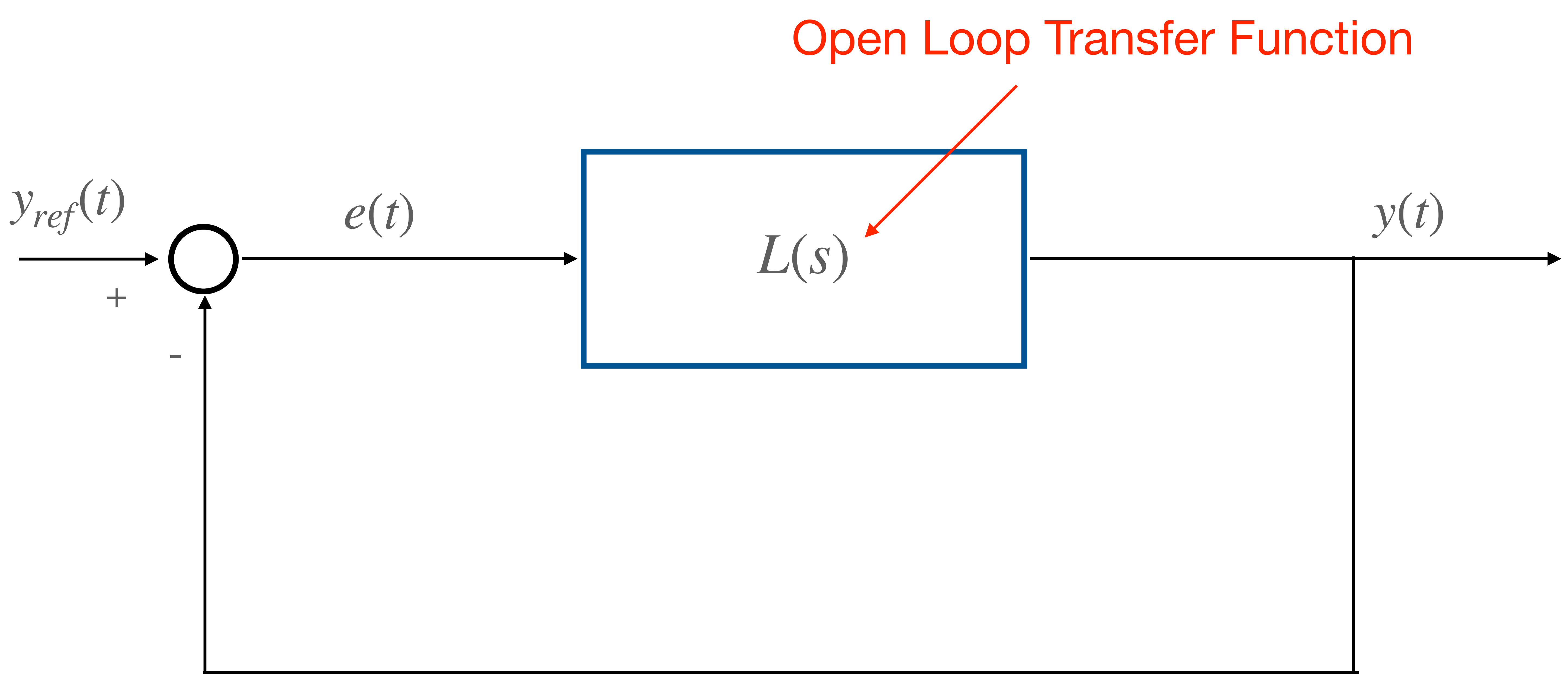
Block Algebra Rule: $R(s) \cdot G(s) = L(s)$

Unitary Feedback Control Scheme



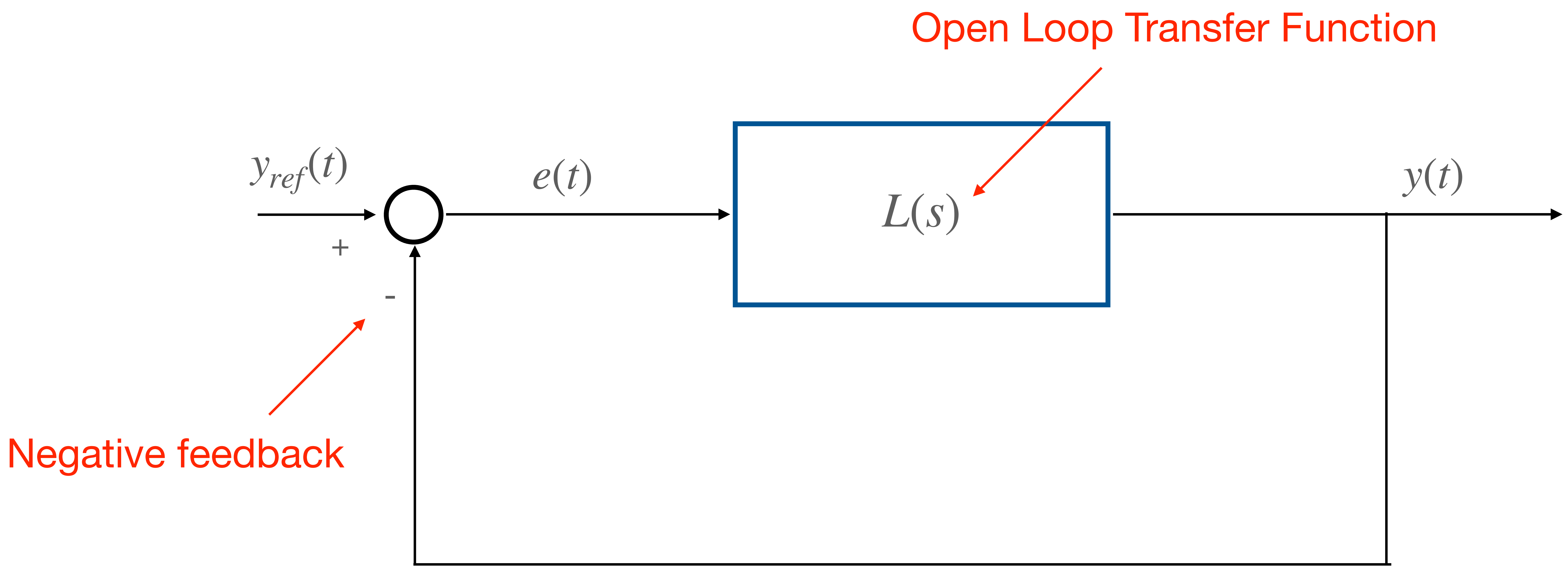
Block Algebra Rule: $R(s) \cdot G(s) = L(s)$

Unitary Feedback Control Scheme



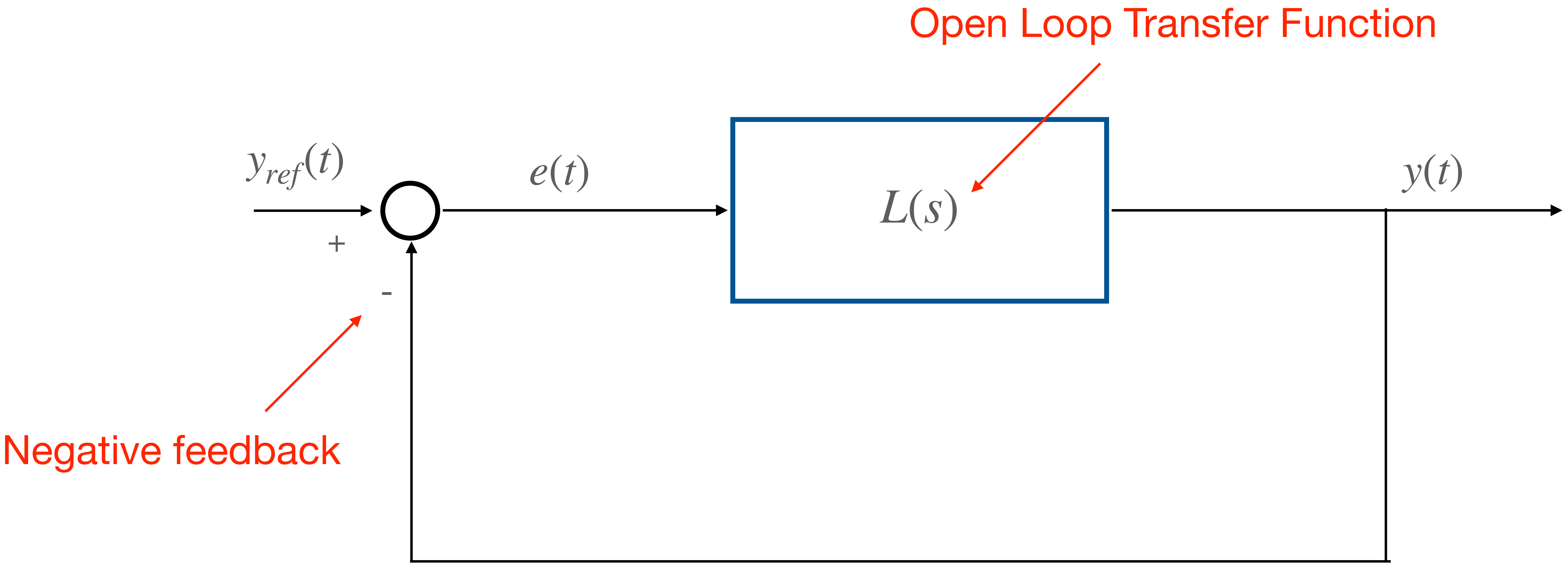
Block Algebra Rule: $R(s) \cdot G(s) = L(s)$

Unitary Feedback Control Scheme



Block Algebra Rule: $R(s) \cdot G(s) = L(s)$

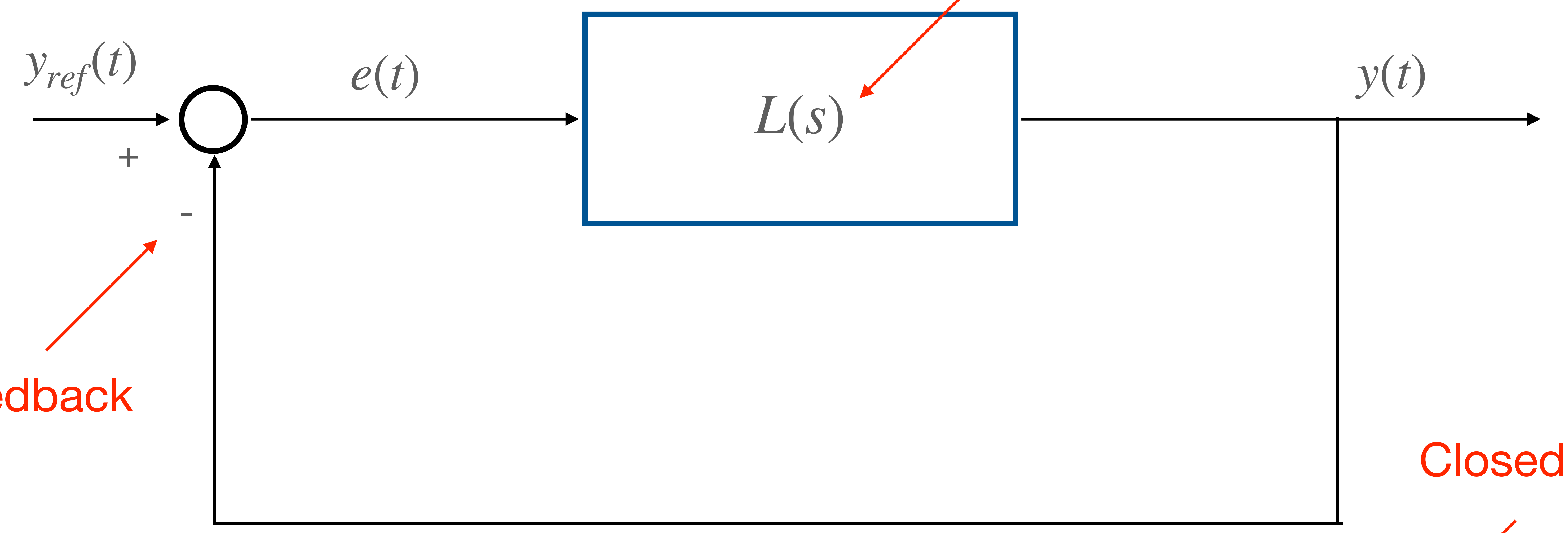
Unitary Feedback Control Scheme



Block Algebra Rule: $$F(s) = \frac{R(s) \cdot G(s)}{1 + R(s) \cdot G(s)} = \frac{L(s)}{1 + L(s)}$$

Unitary Feedback Control Scheme

Open Loop Transfer Function



Negative feedback

Closed Loop Transfer Function

Block Algebra Rule: $$F(s) = \frac{R(s) \cdot G(s)}{1 + R(s) \cdot G(s)} = \frac{L(s)}{1 + L(s)}$$

Limitations of the Basic Unitary Feedback Control Scheme

- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)
- Possible difficulties in enforcing steady-state precision with step reference signal via the classical approach

Limitations of the Basic Unitary Feedback Control Scheme

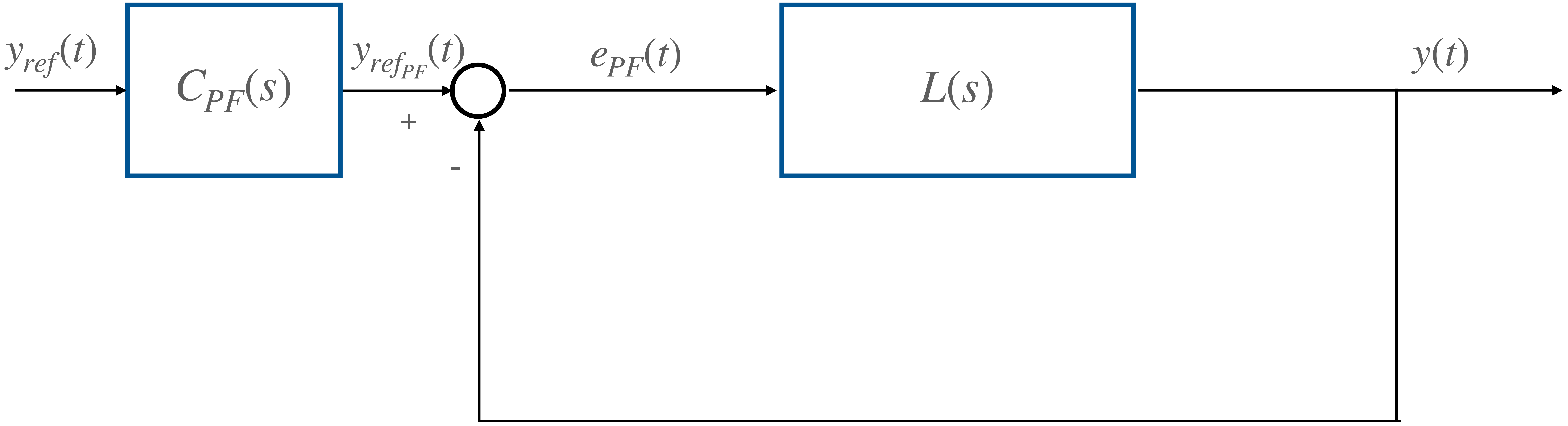
- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)
- Possible difficulties in enforcing steady-state precision with step reference signal via the classical approach (by introducing integrators in the direct path)

Limitations of the Basic Unitary Feedback Control Scheme

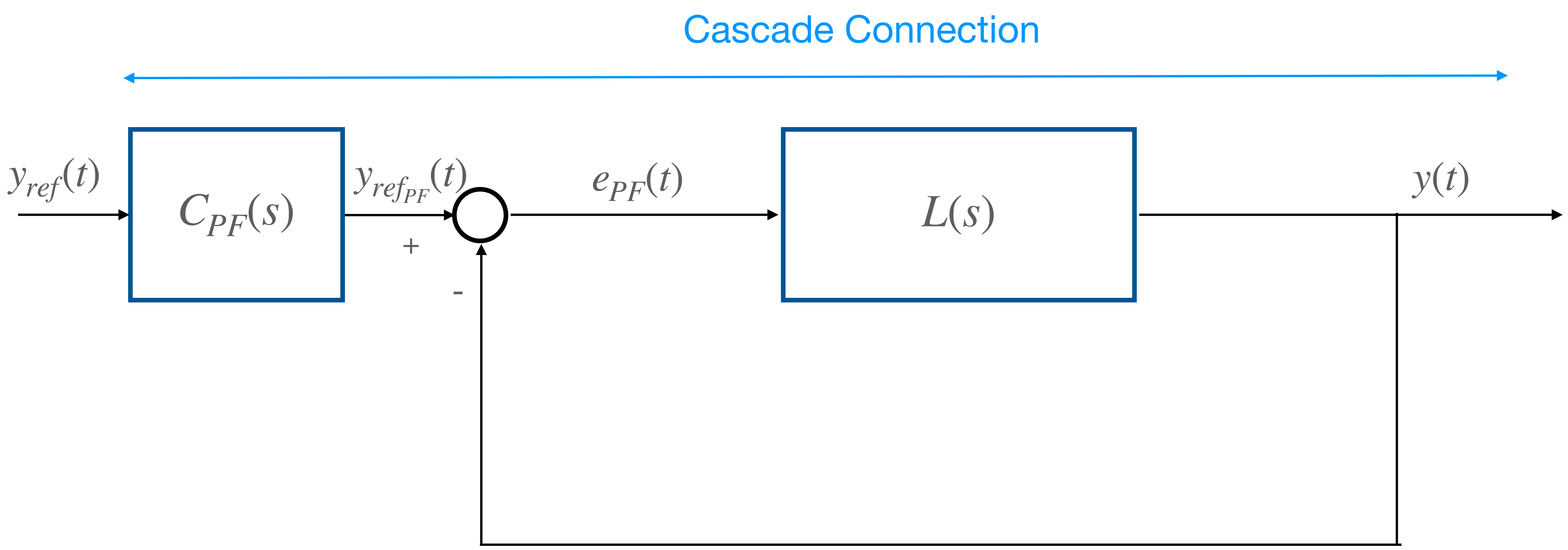
- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)
- Possible difficulties in enforcing steady-state precision with step reference signal via the classical approach (by introducing integrators in the direct path)

Possible solution: add a Pre-Filter to the basic unitary feedback control scheme

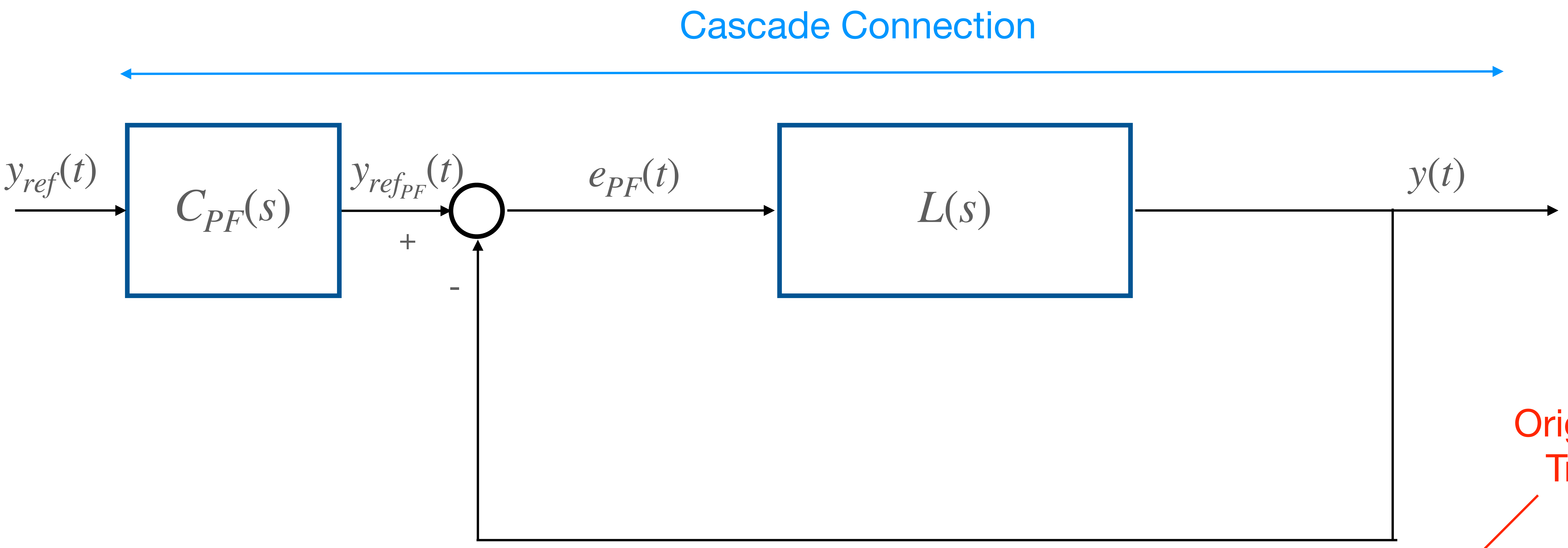
Pre-filter Based Control Scheme



Pre-filter Based Control Scheme



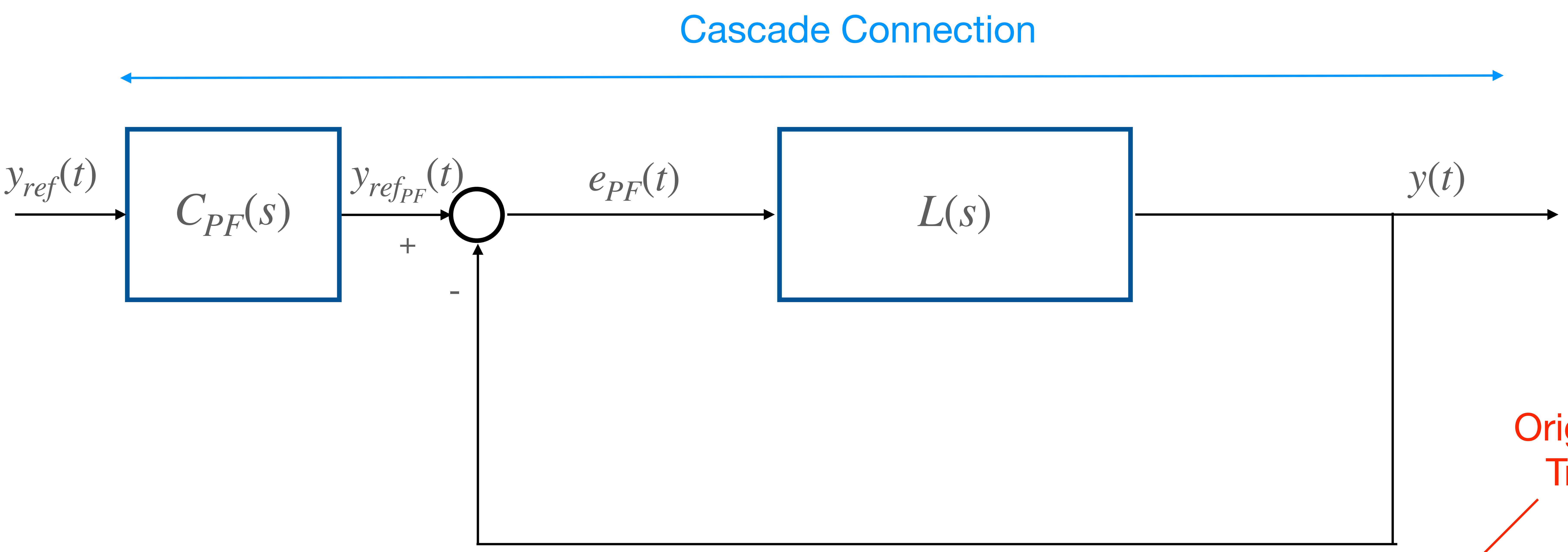
Pre-filter Based Control Scheme



Original Closed Loop Transfer Function

Block Algebra Rule: $$F(s) = \frac{R(s) \cdot G(s)}{1 + R(s) \cdot G(s)} = \frac{L(s)}{1 + L(s)}$$

Pre-filter Based Control Scheme



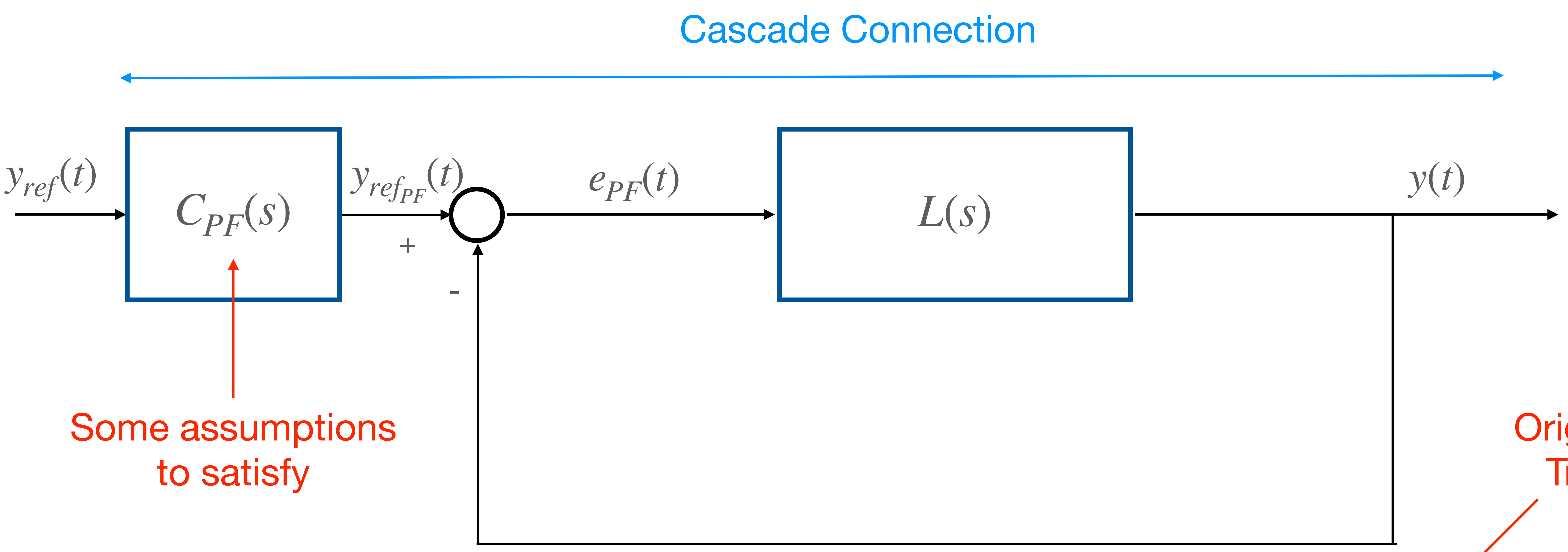
Original Closed Loop Transfer Function

New Closed Loop Transfer Function

Block Algebra Rule: $$F(s) = \frac{R(s) \cdot G(s)}{1 + R(s) \cdot G(s)} = \frac{L(s)}{1 + L(s)}$$

$$F_{PF}(s) = C_{PF}(s) \cdot F(s) = C_{PF}(s) \cdot \frac{L(s)}{1 + L(s)}$$

Pre-filter Based Control Scheme



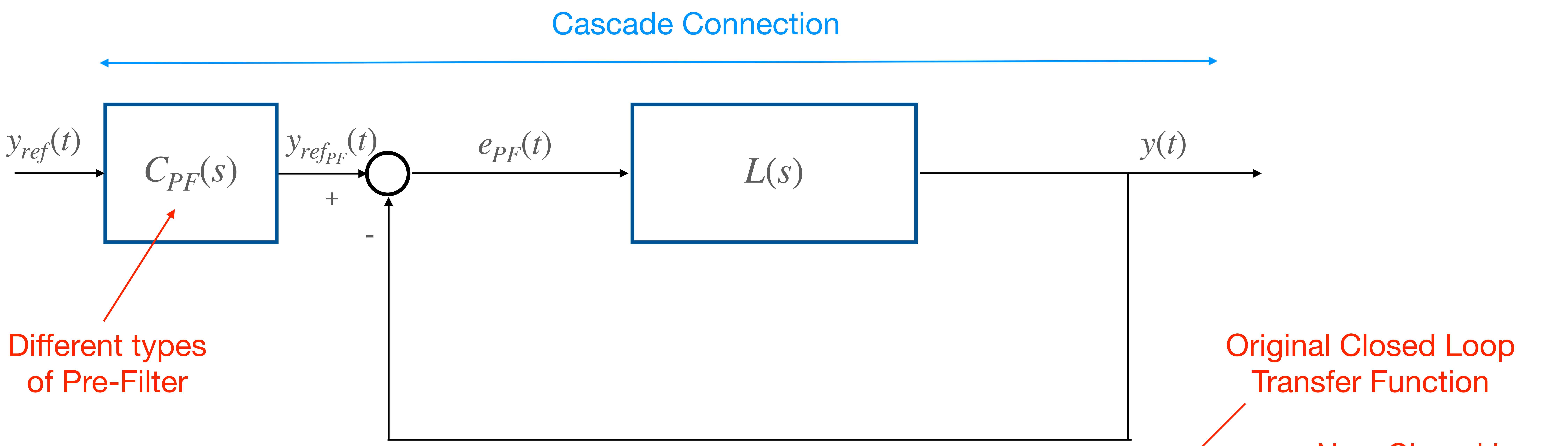
Original Closed Loop Transfer Function

New Closed Loop Transfer Function

Block Algebra Rule: $$F(s) = \frac{R(s) \cdot G(s)}{1 + R(s) \cdot G(s)} = \frac{L(s)}{1 + L(s)}$$

$$F_{PF}(s) = C_{PF}(s) \cdot F(s) = C_{PF}(s) \cdot \frac{L(s)}{1 + L(s)}$$

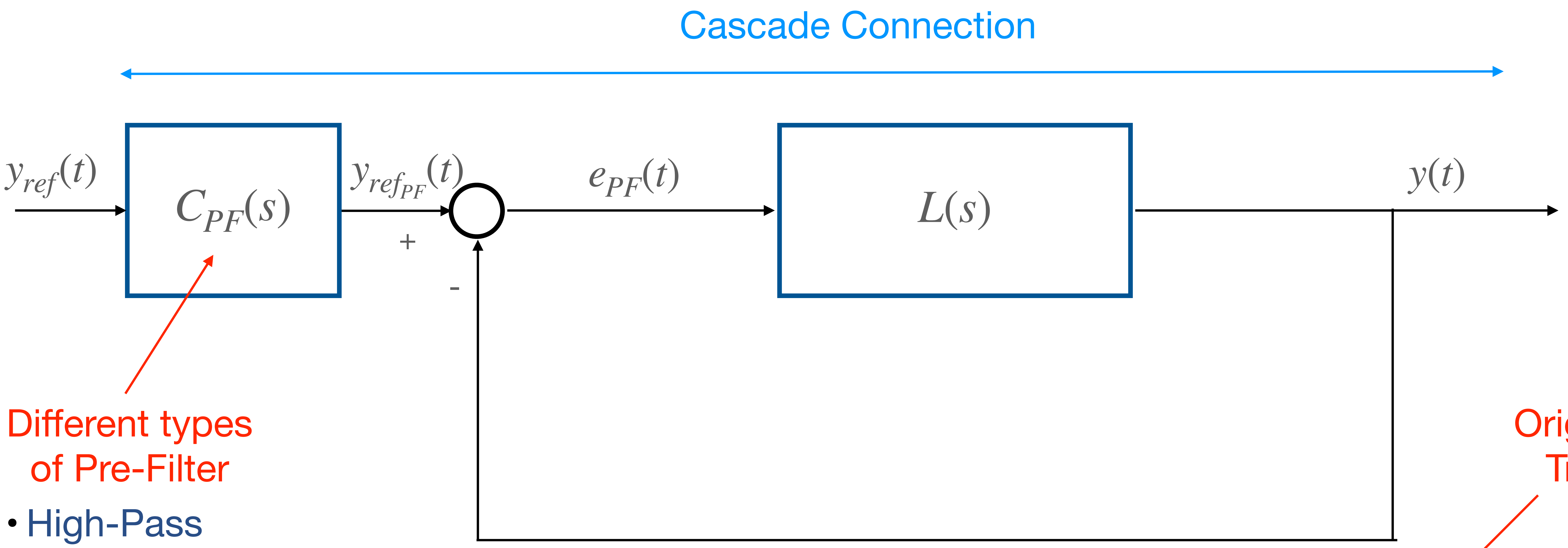
Pre-filter Based Control Scheme



Block Algebra Rule: $$F(s) = \frac{R(s) \cdot G(s)}{1 + R(s) \cdot G(s)} = \frac{L(s)}{1 + L(s)}$$

$$F_{PF}(s) = C_{PF}(s) \cdot F(s) = C_{PF}(s) \cdot \frac{L(s)}{1 + L(s)}$$

Pre-filter Based Control Scheme



Different types of Pre-Filter

- High-Pass
- Low-Pass
- For steady-state compensation

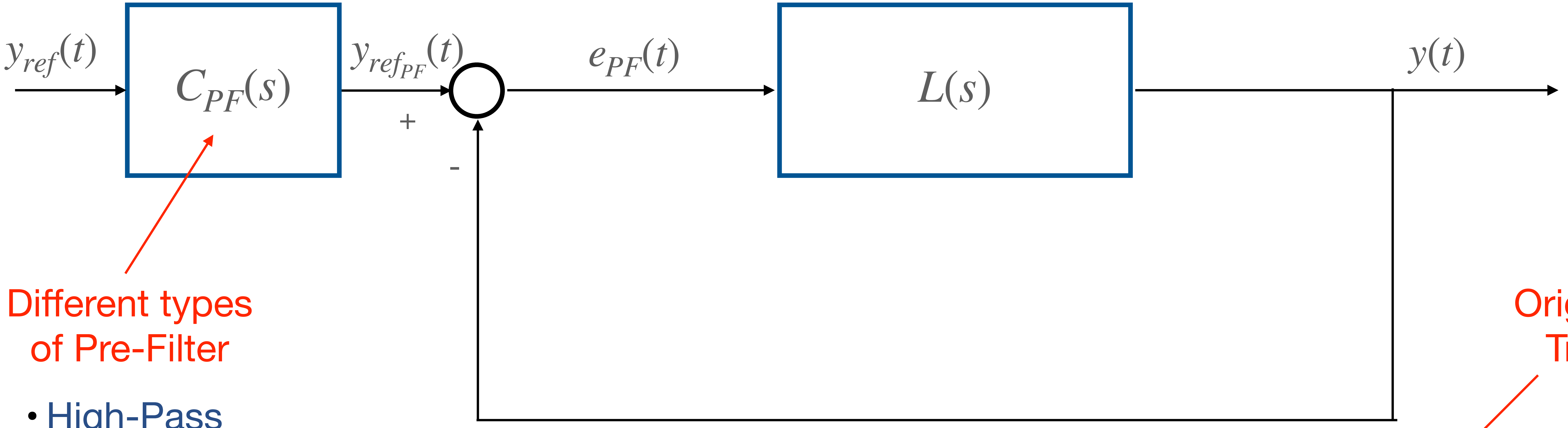
Original Closed Loop Transfer Function

New Closed Loop Transfer Function

Block Algebra Rule: $$F(s) = \frac{R(s)\cdot G(s)}{1+R(s)\cdot G(s)} = \frac{L(s)}{1+L(s)}$$

$$F_{PF}(s) = C_{PF}(s)\cdot F(s) = C_{PF}(s)\cdot \frac{L(s)}{1+L(s)}$$

Pre-filter Based Control Scheme: Case 1



Different types of Pre-Filter

- High-Pass

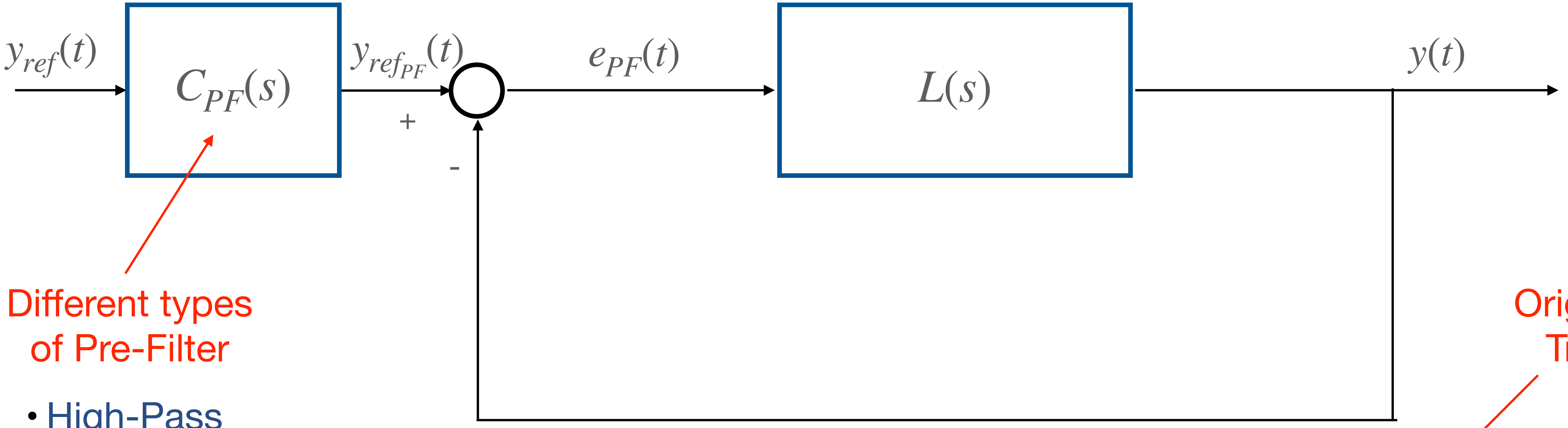
Original Closed Loop Transfer Function

New Closed Loop Transfer Function

Block Algebra Rule: $$F(s) = \frac{R(s)\cdot G(s)}{1 + R(s)\cdot G(s)} = \frac{L(s)}{1+L(s)}$$

$$F_{PF}(s) = C_{PF}(s)\cdot F(s) = C_{PF}(s)\cdot \frac{L(s)}{1+L(s)}$$

Pre-filter Based Control Scheme: Case 1



Different types of Pre-Filter

- High-Pass

Original Closed Loop Transfer Function

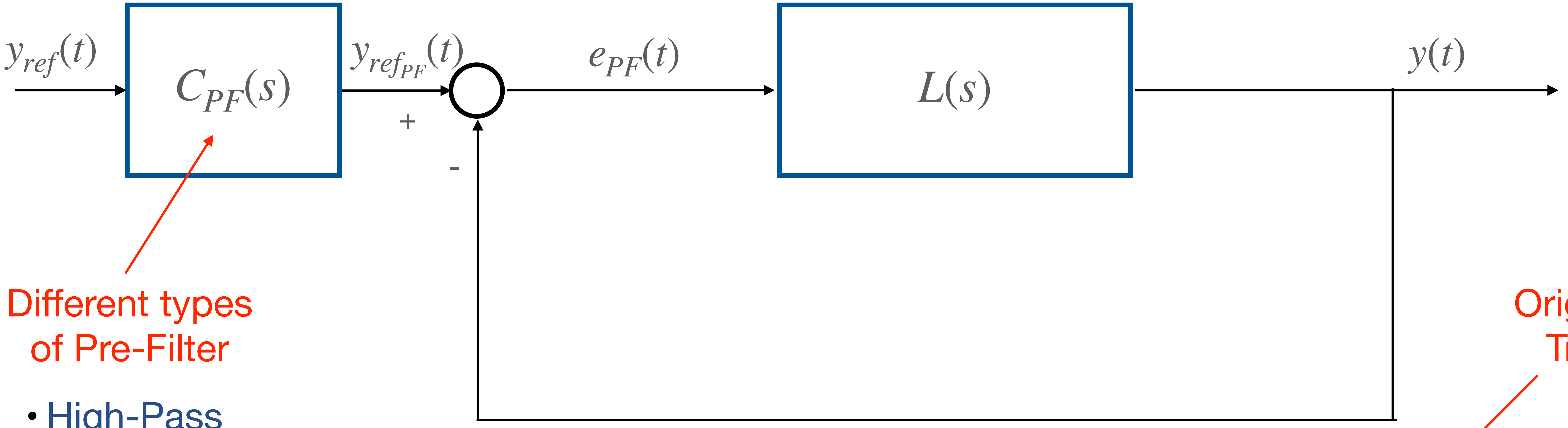
New Closed Loop Transfer Function

Block Algebra Rule: $$F(s) = \frac{R(s) \cdot G(s)}{1 + R(s) \cdot G(s)} = \frac{L(s)}{1 + L(s)}$$

$$F_{PF}(s) = C_{PF}(s) \cdot F(s) = C_{PF}(s) \cdot \frac{L(s)}{1 + L(s)} = \frac{\mu(1 + \tau' s)}{(1 + \tau s)} \cdot \frac{L(s)}{1 + L(s)}$$

UNIVERSITÀ DI PAVIA

Pre-filter Based Control Scheme: Case 1



Different types of Pre-Filter

- High-Pass

Original Closed Loop Transfer Function

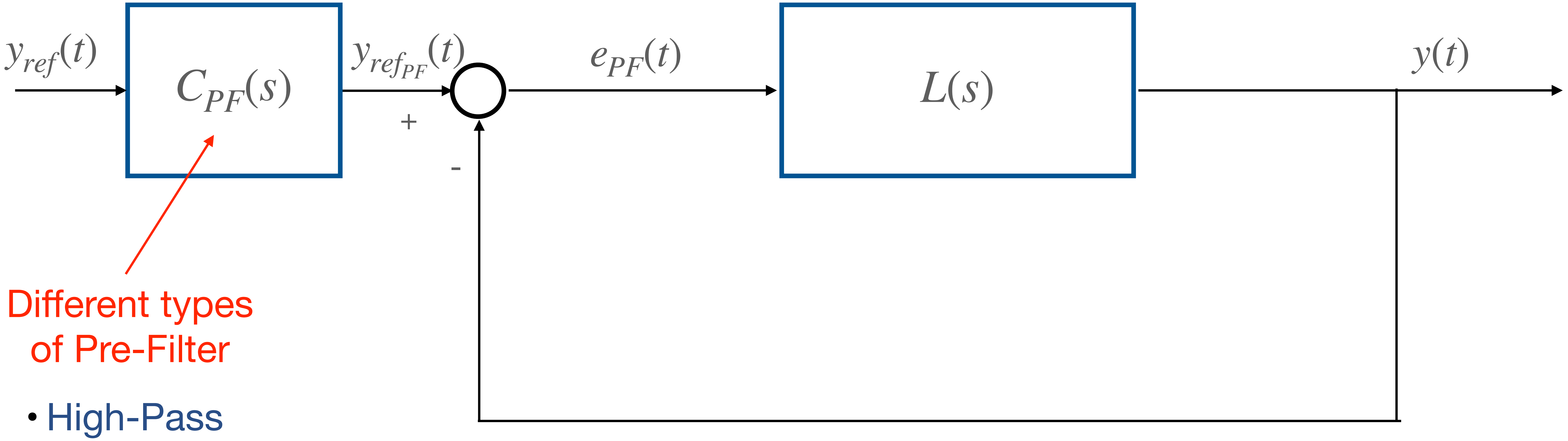
Block Algebra Rule: $$F(s) = \frac{R(s) \cdot G(s)}{1 + R(s) \cdot G(s)} = \frac{L(s)}{1 + L(s)}$$

New Closed Loop Transfer Function

The zero precedes the pole on the ω axis

$$F_{PF}(s) = C_{PF}(s) \cdot F(s) = C_{PF}(s) \cdot \frac{L(s)}{1 + L(s)} = \frac{\mu(1 + \tau' s)}{(1 + \tau s)} \cdot \frac{L(s)}{1 + L(s)}$$

Pre-filter Based Control Scheme: Case 1



Different types of Pre-Filter

- High-Pass

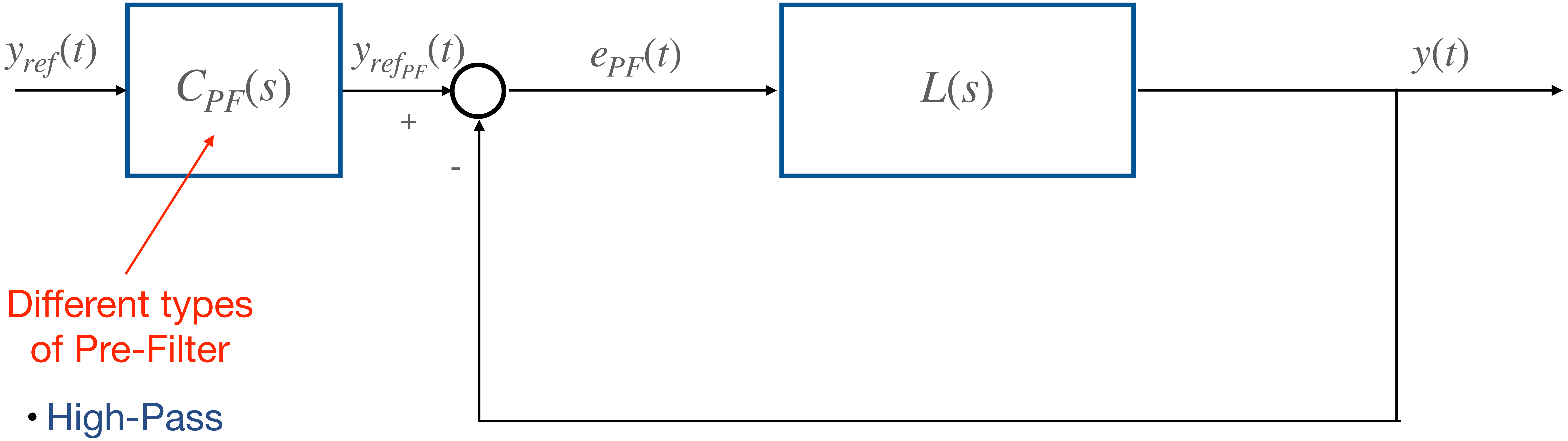
Assumptions:

- $C_{PF}(s)$ As. Stable
- Proper
- Unitary gain

The zero precedes the pole on the ω axis

$$F_{PF}(s) = C_{PF}(s) \cdot F(s) = C_{PF}(s) \cdot \frac{L(s)}{1+L(s)} = \frac{\mu(1+\tau' s)}{(1+\tau s)} \cdot \frac{L(s)}{1+L(s)}$$

Pre-filter Based Control Scheme: Case 1



Different types of Pre-Filter

- High-Pass

Assumptions:

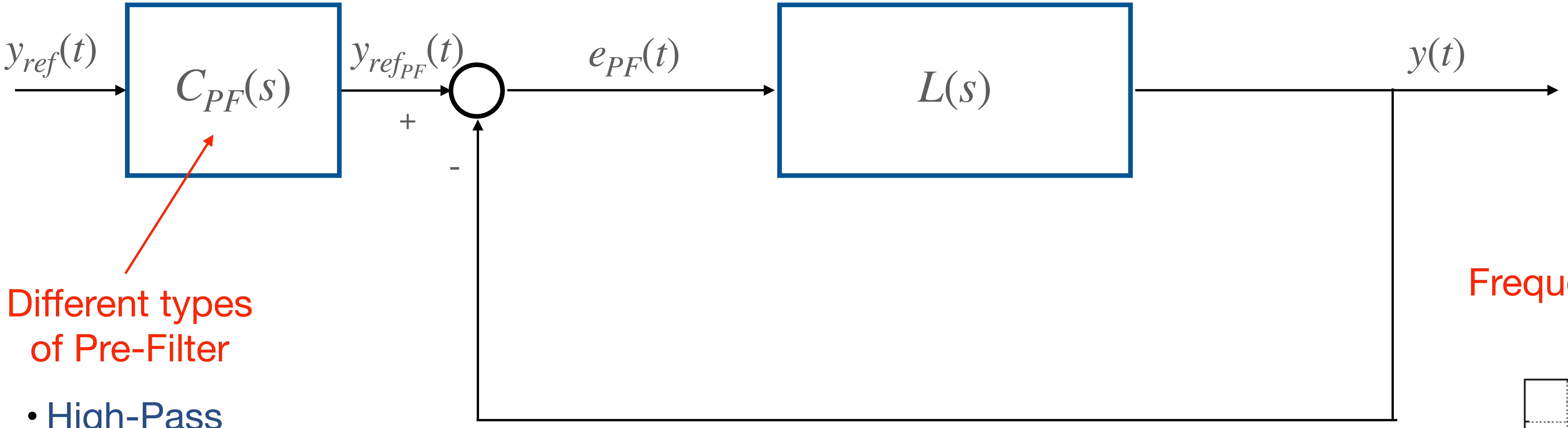
- $C_{PF}(s)$ As. Stable
- Proper
- Unitary gain

$$\lim_{s\to 0} \frac{\mu(1+\tau' s)}{1+\tau s} = \mu = 1$$

The zero precedes the pole on the ω axis

$$F_{PF}(s) = C_{PF}(s)\cdot F(s) = C_{PF}(s)\cdot \frac{L(s)}{1+L(s)} = \frac{\mu(1+\tau' s)}{(1+\tau s)}\cdot \frac{L(s)}{1+L(s)}$$

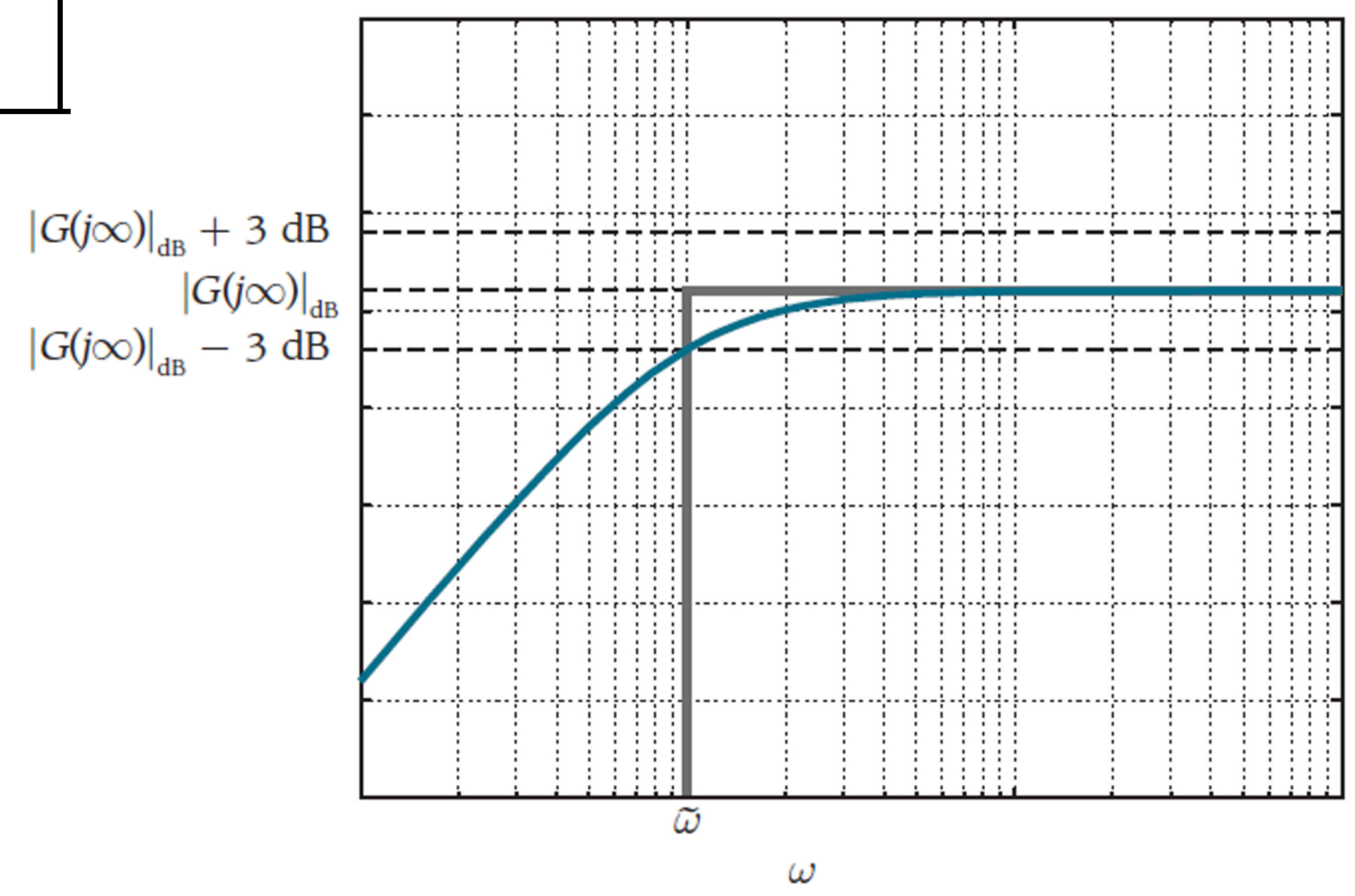
Pre-filter Based Control Scheme: Case 1



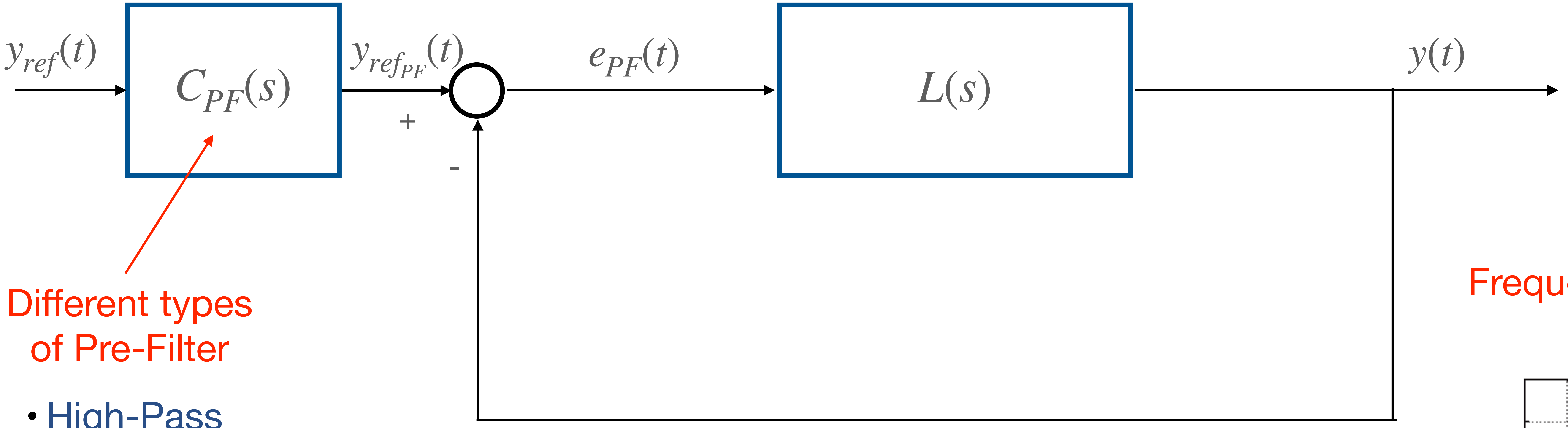
Different types of Pre-Filter

- High-Pass

Frequency response of HP filter (magnitude)



Pre-filter Based Control Scheme: Case 1

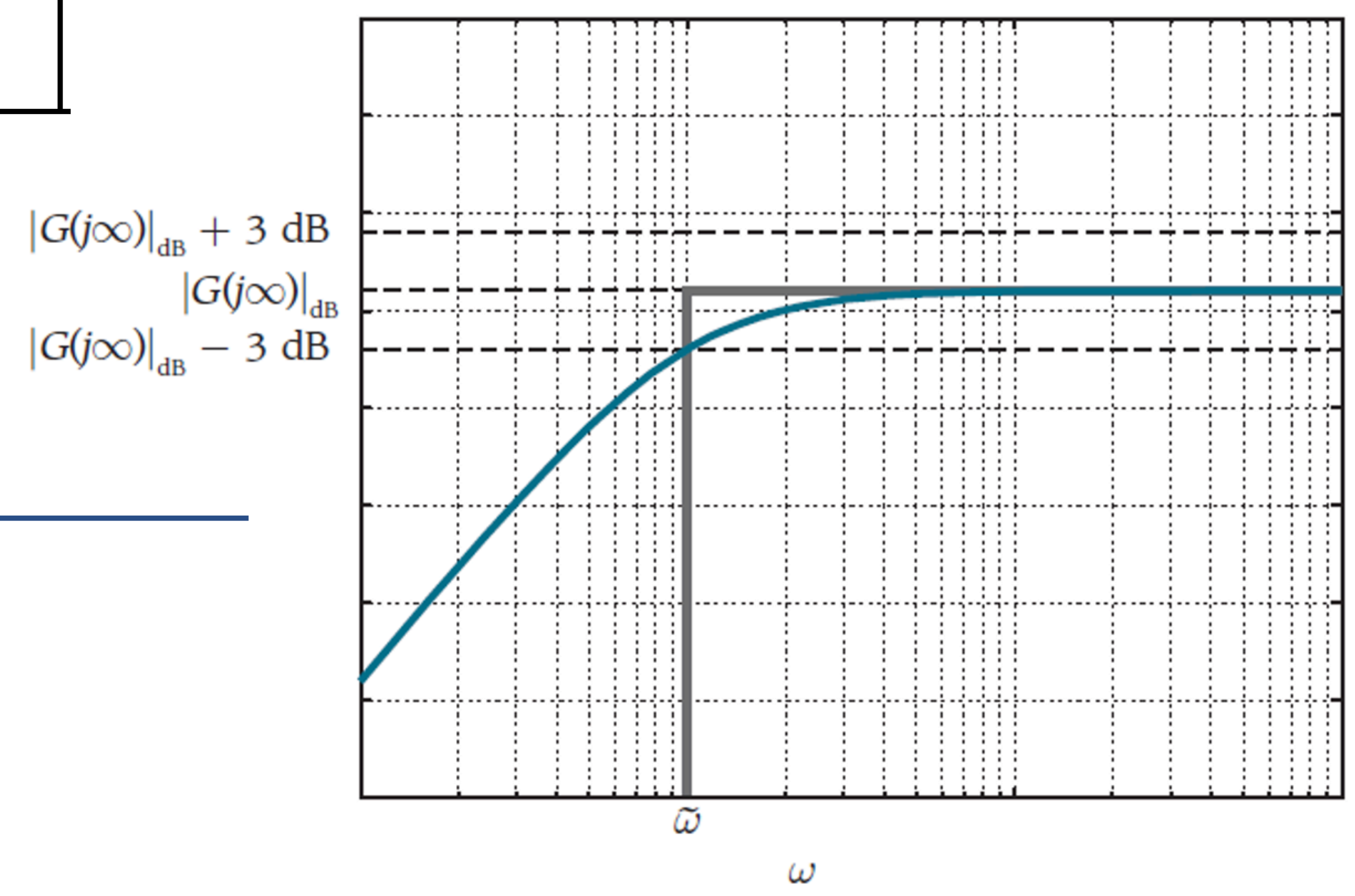


Different types of Pre-Filter

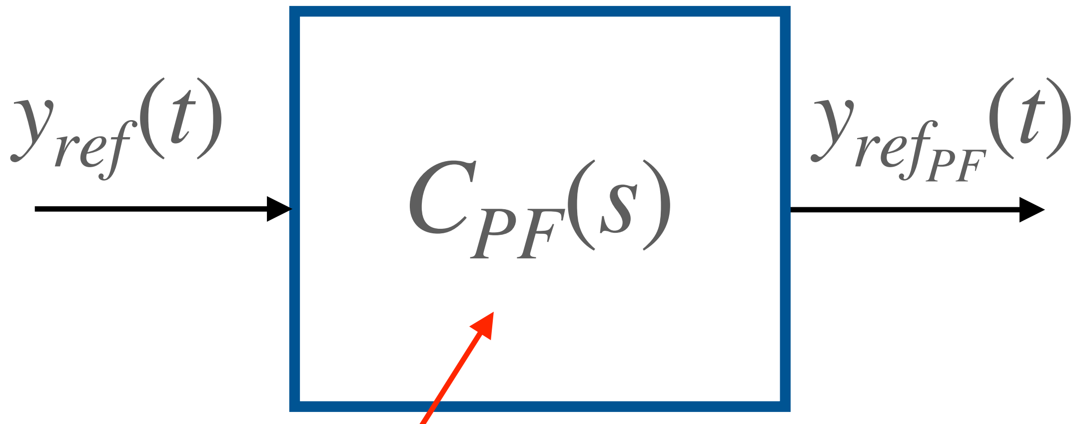
- High-Pass

An HP Pre-filter can be used to to enlarge the closed-loop bandwidth $\omega_{BW_{CL}}$ a posteriori

Frequency response of HP filter (magnitude)



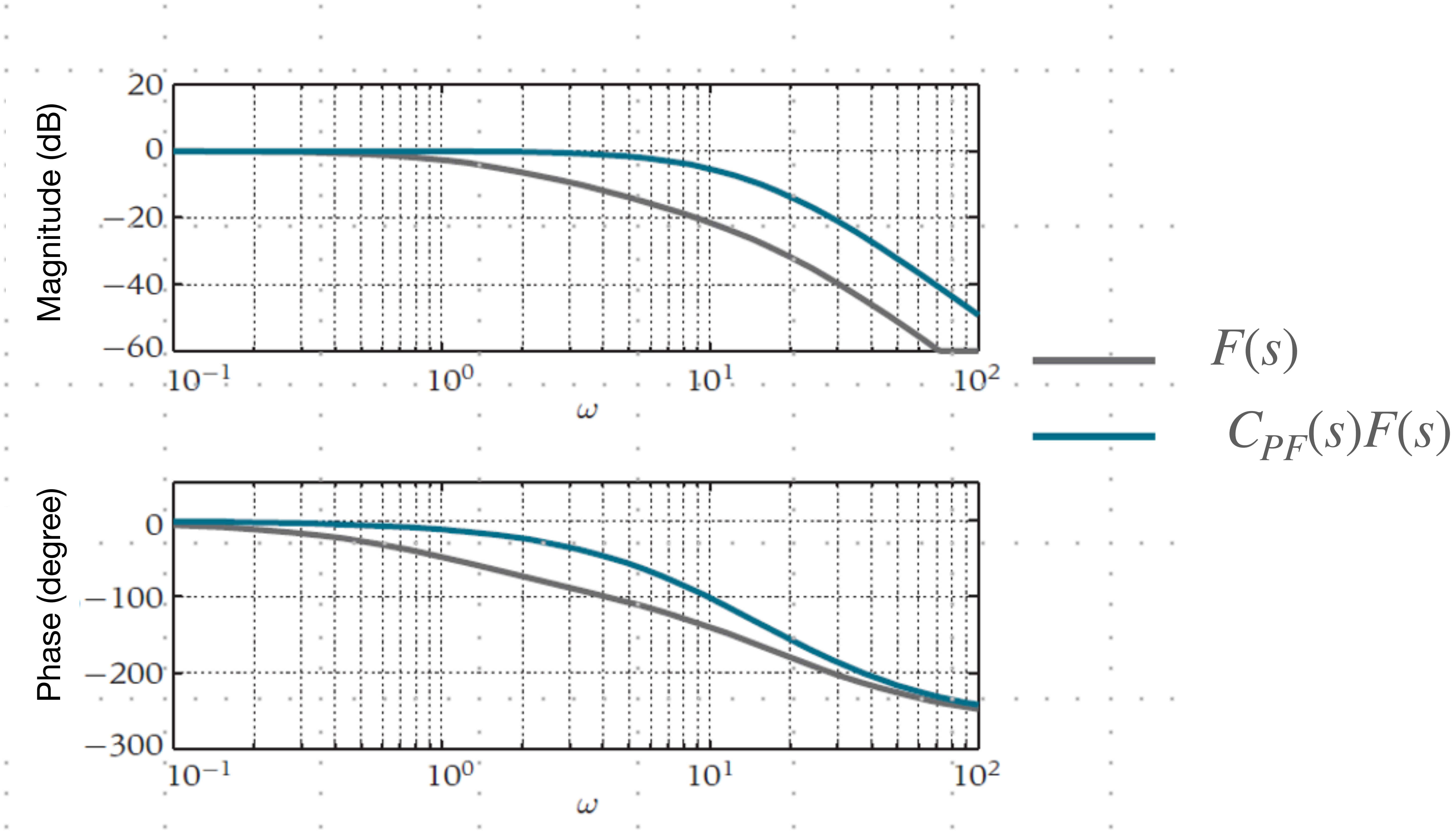
Pre-filter Based Control Scheme: Case 1



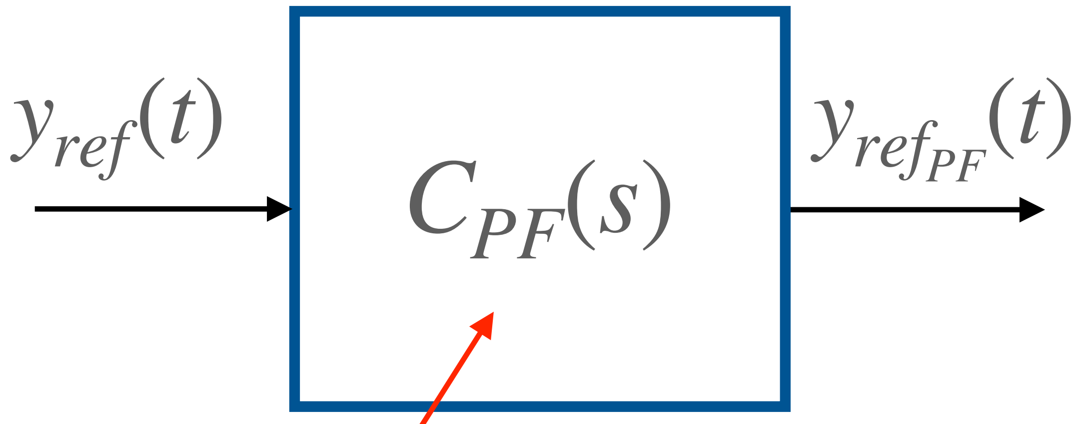
Different types of Pre-Filter

- High-Pass

Frequency domain



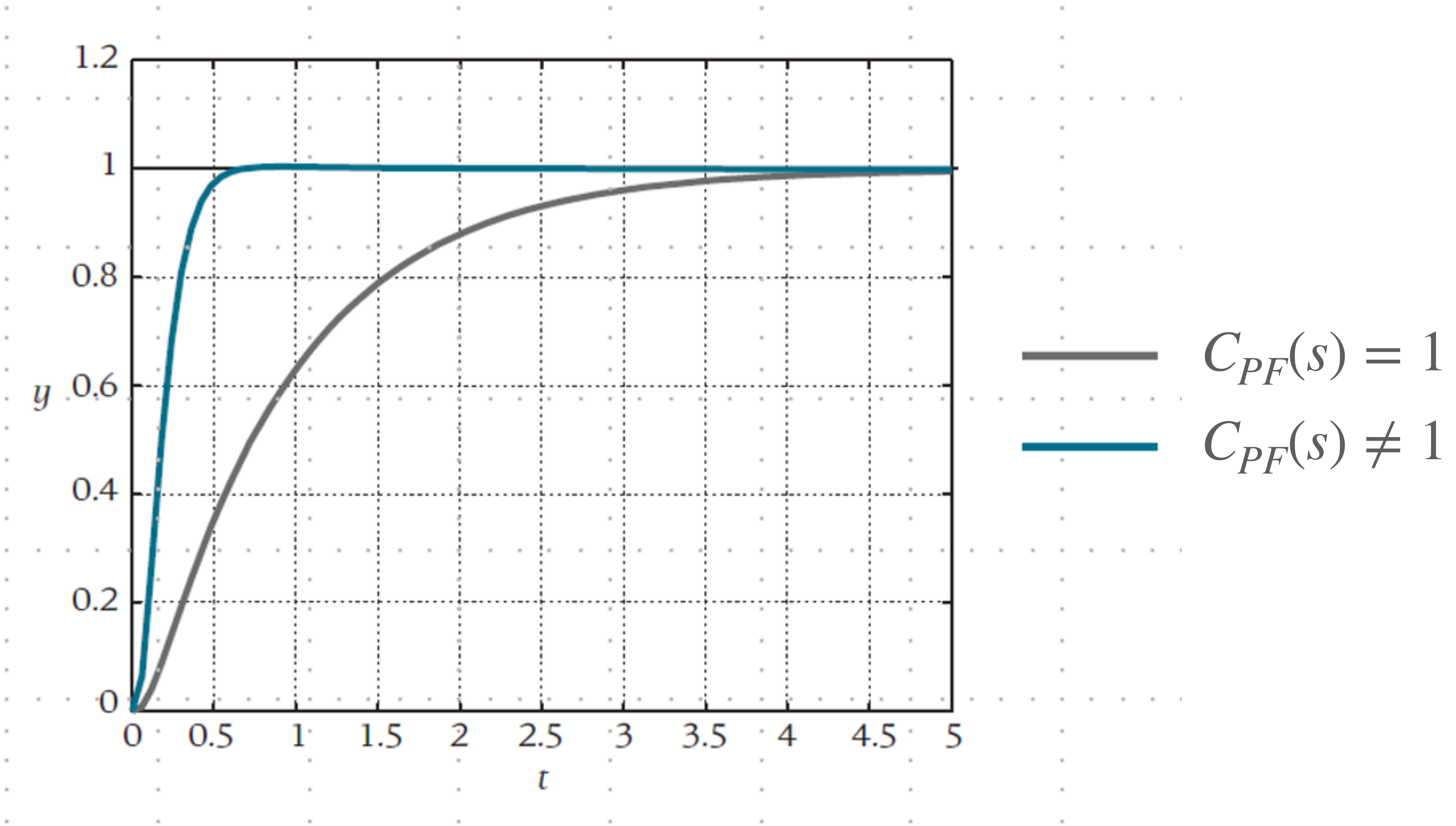
Pre-filter Based Control Scheme: Case 1



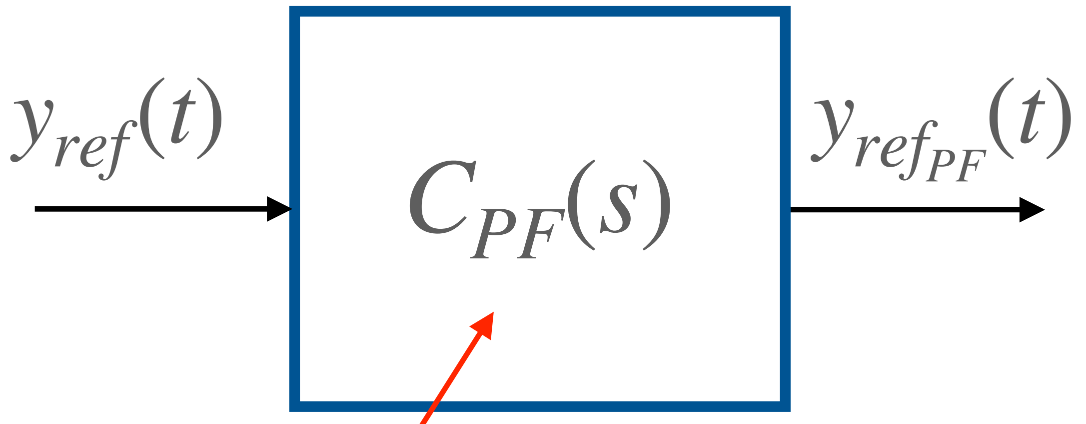
Different types of Pre-Filter

- High-Pass

Time domain



Pre-filter Based Control Scheme: Case 1

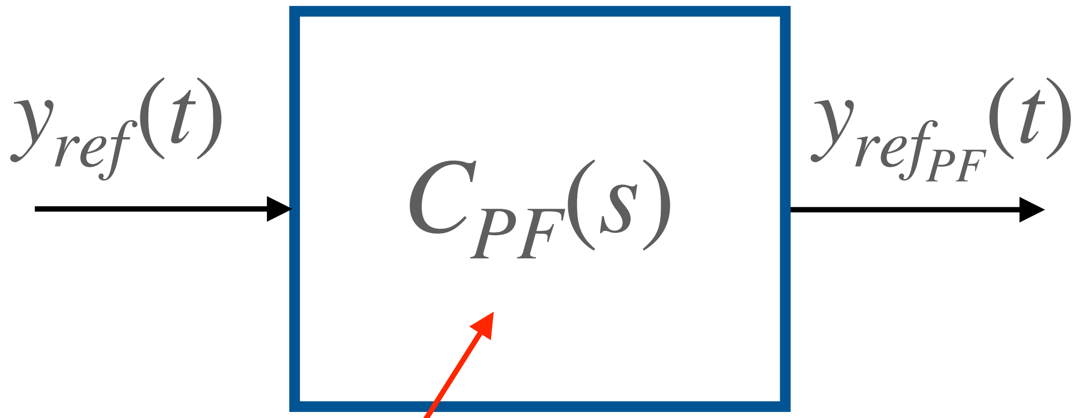


Different types of Pre-Filter

- High-Pass

- Limited bandwidth in case of measurement disturbances

Pre-filter Based Control Scheme: Case 1

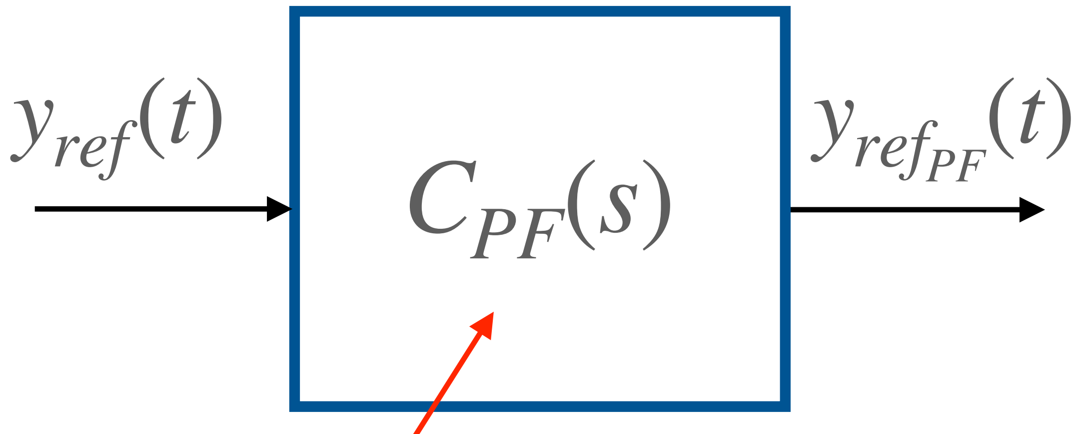


Different types of Pre-Filter

- High-Pass

- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays

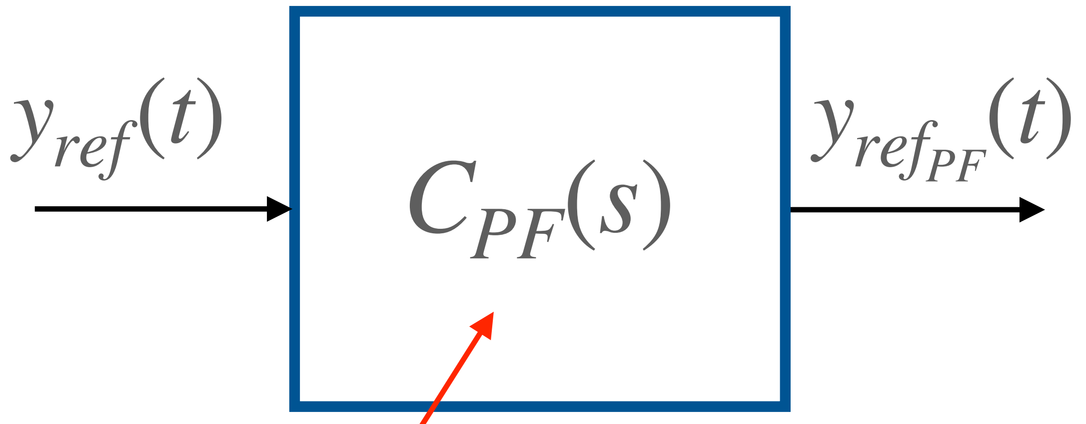
Pre-filter Based Control Scheme: Case 1



Different types of Pre-Filter

- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)

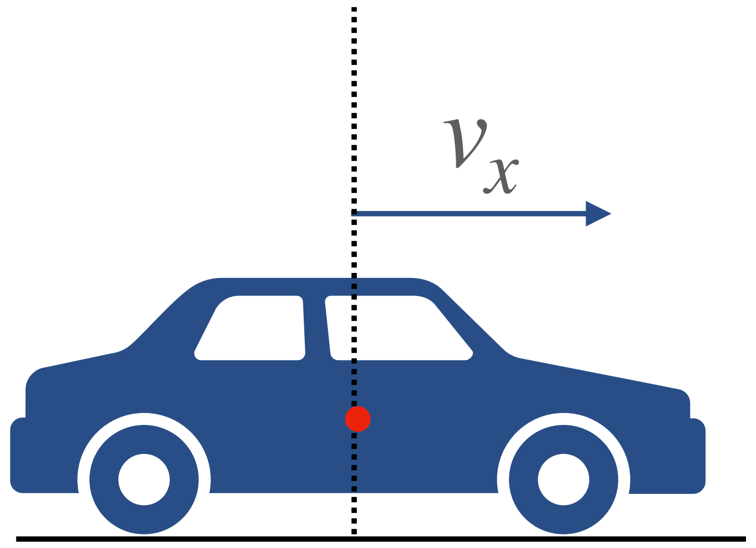
Pre-filter Based Control Scheme: Case 1



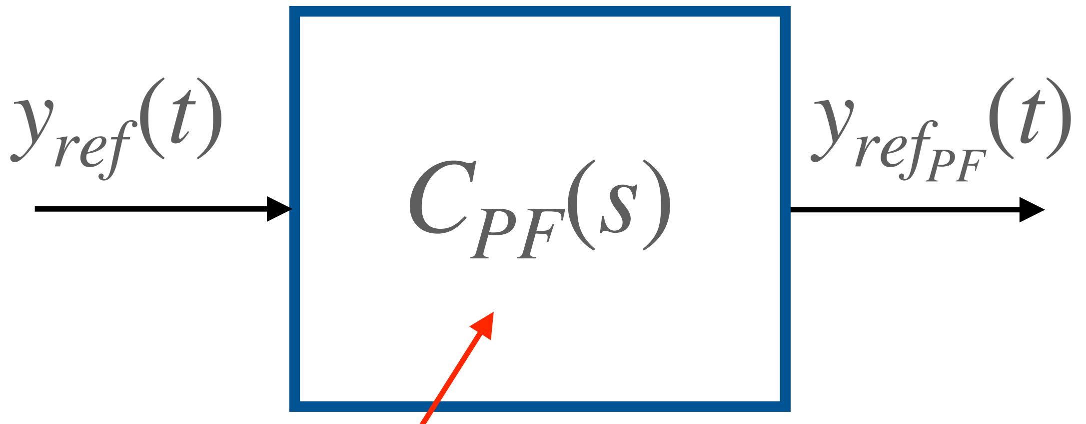
Different types of Pre-Filter

- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)

Example:



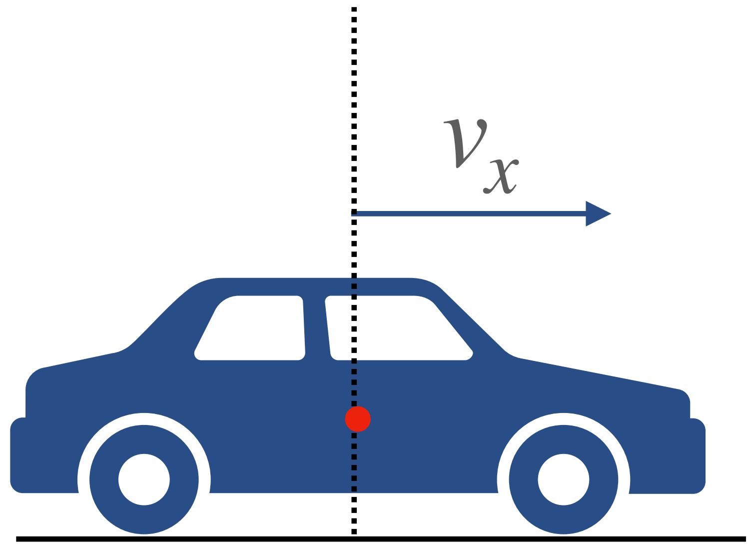
Pre-filter Based Control Scheme: Case 1



Different types of Pre-Filter

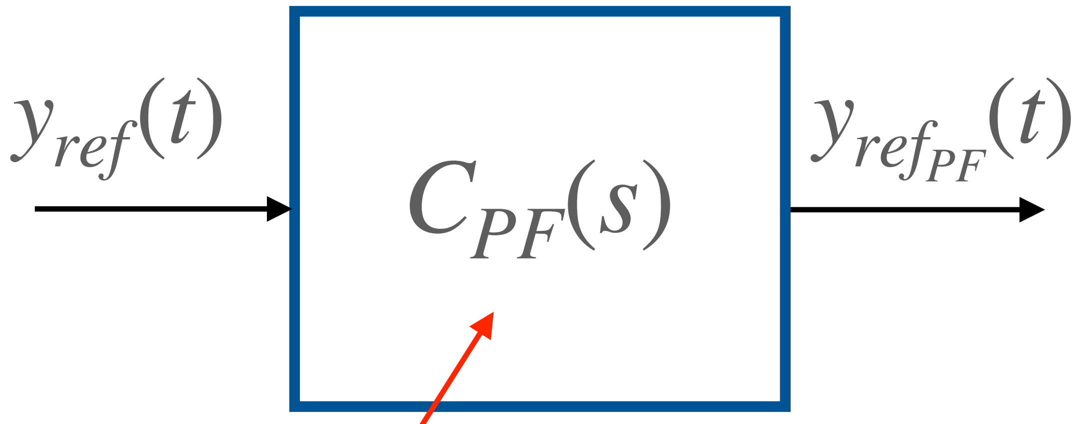
- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)

Example:



$$\lambda := \frac{\omega r - v_x}{\max(\omega r, v_x)}$$

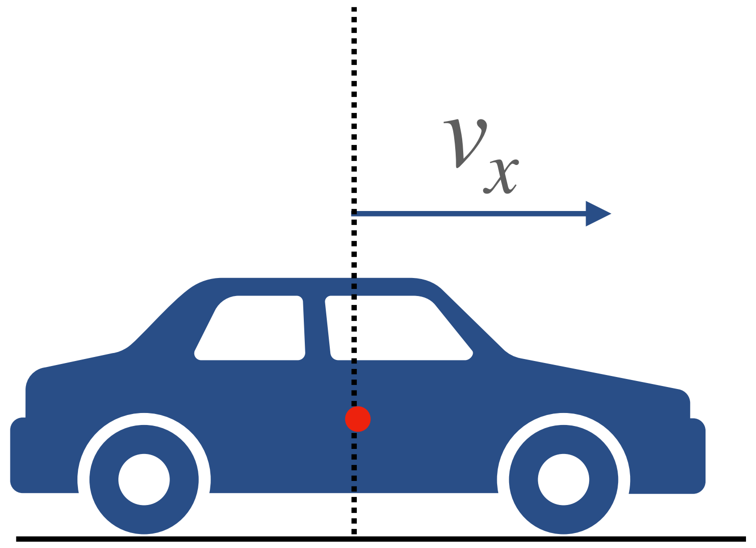
Pre-filter Based Control Scheme: Case 1



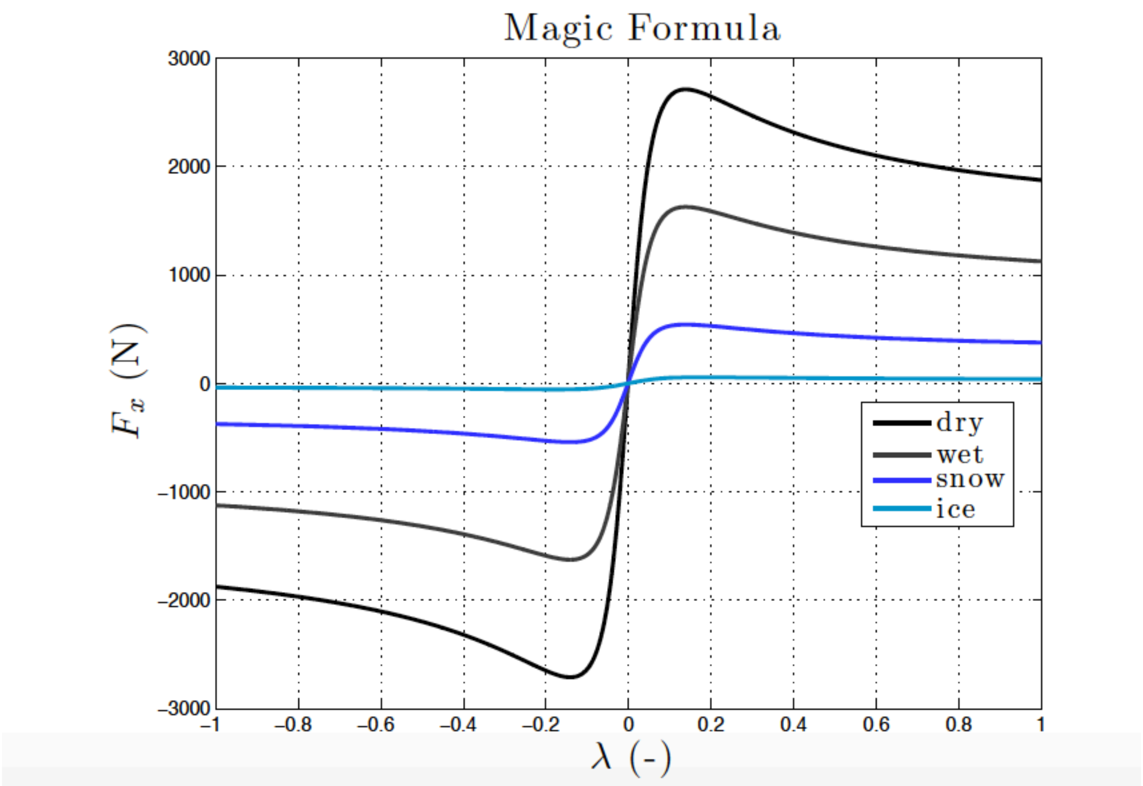
Different types of Pre-Filter

- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)

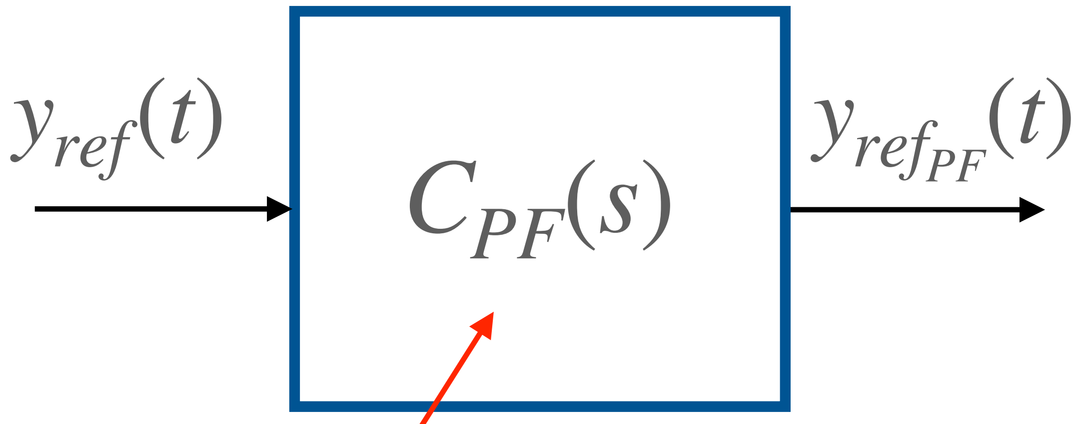
Example:



$$\lambda := \frac{\omega r - v_x}{\max(\omega r, v_x)}$$



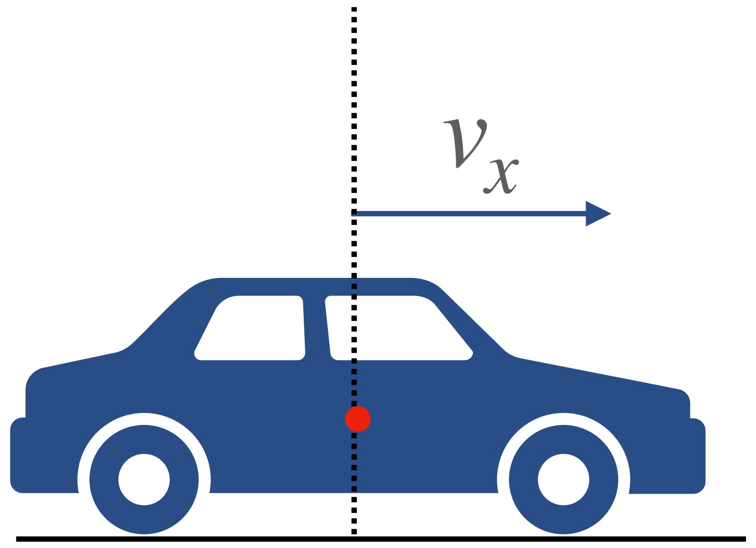
Pre-filter Based Control Scheme: Case 1



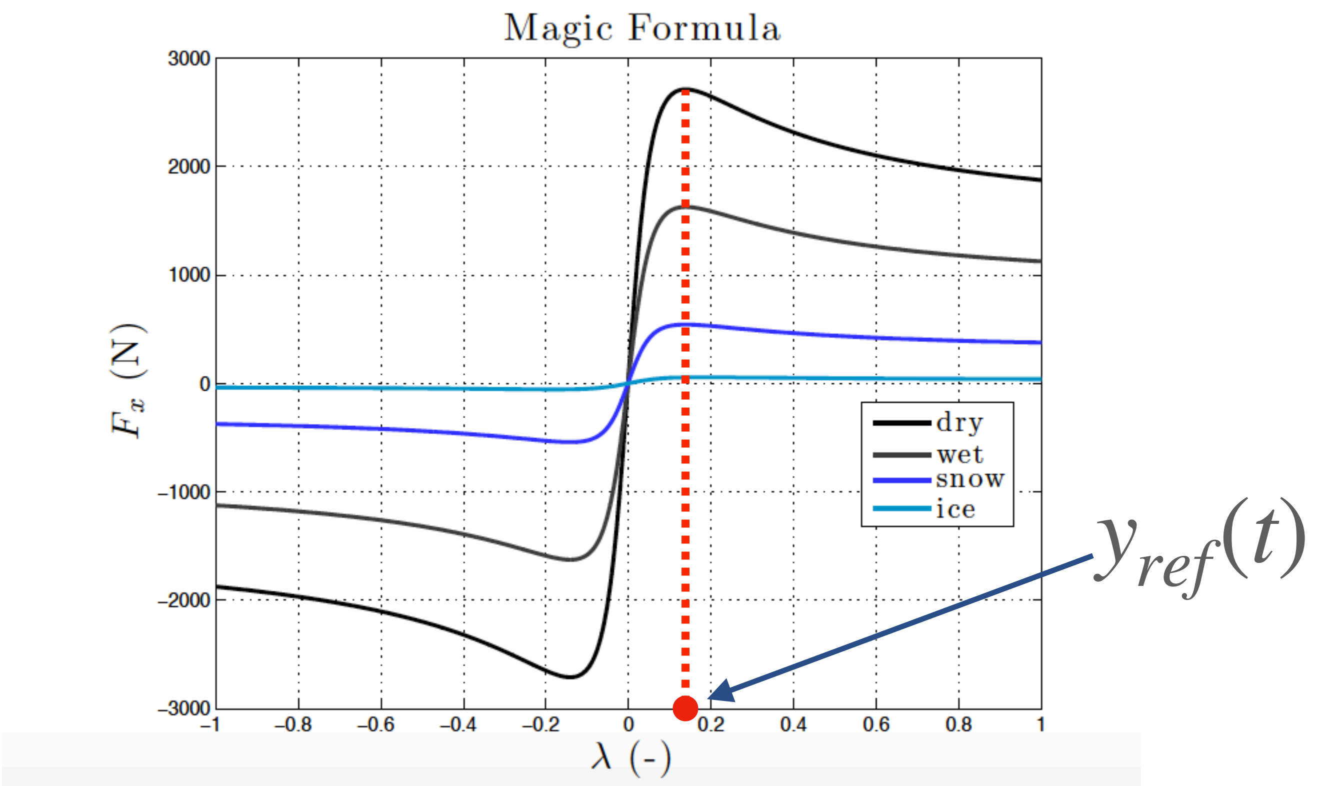
Different types of Pre-Filter

- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No "control moderation" in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)

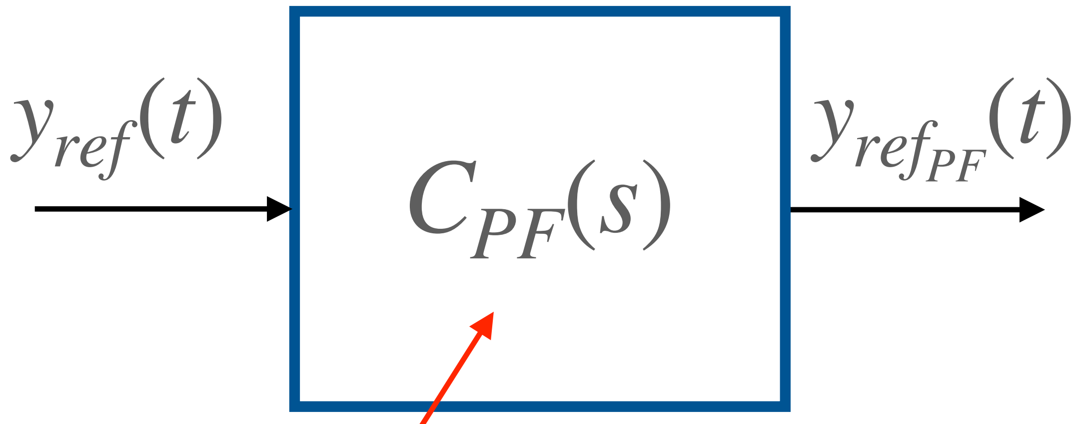
Example:



$$\lambda := \frac{\omega r - v_x}{\max(\omega r, v_x)}$$



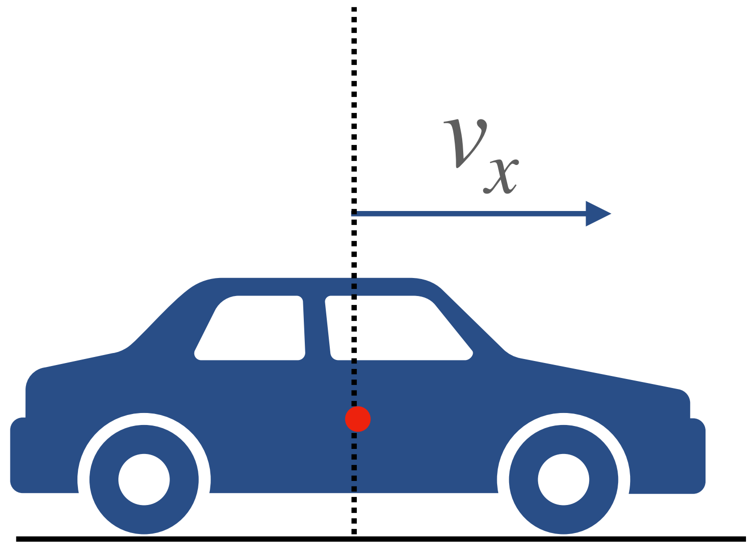
Pre-filter Based Control Scheme: Case 1



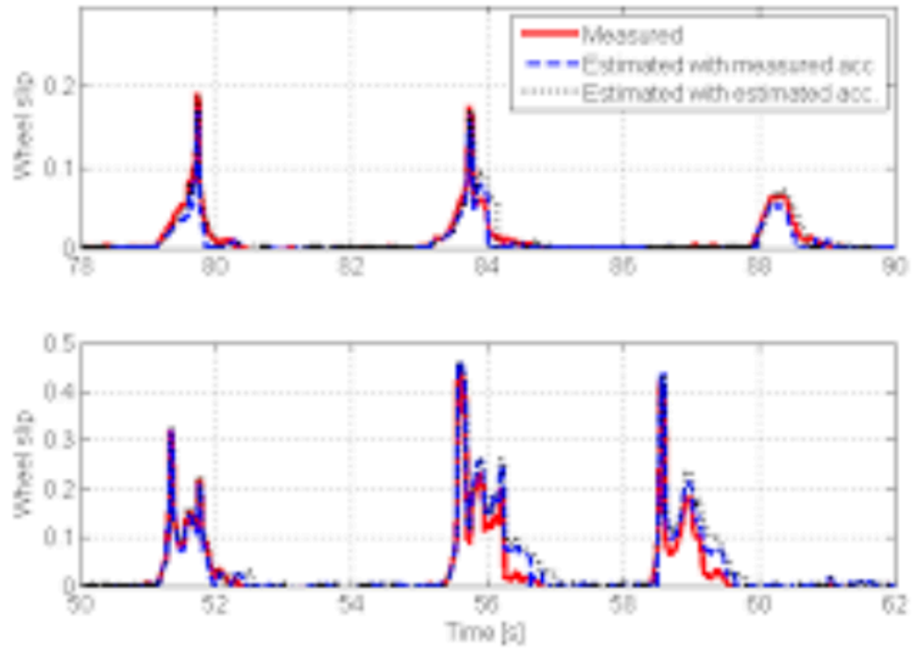
Different types of Pre-Filter

- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)

Example:

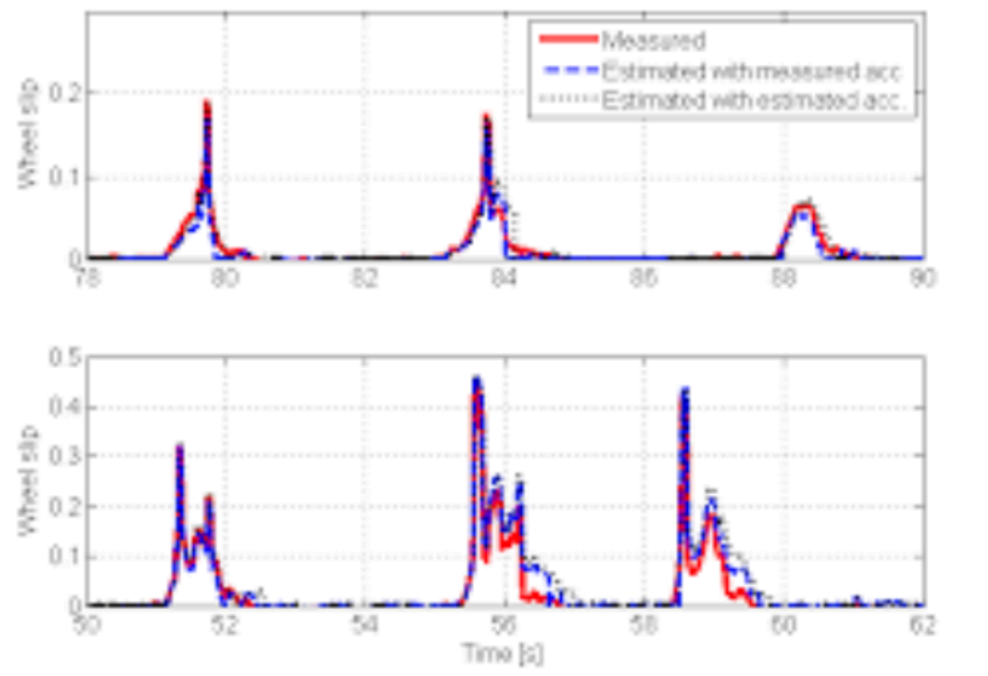


$$\lambda := \frac{\omega r - v_x}{\max(\omega r, v_x)}$$

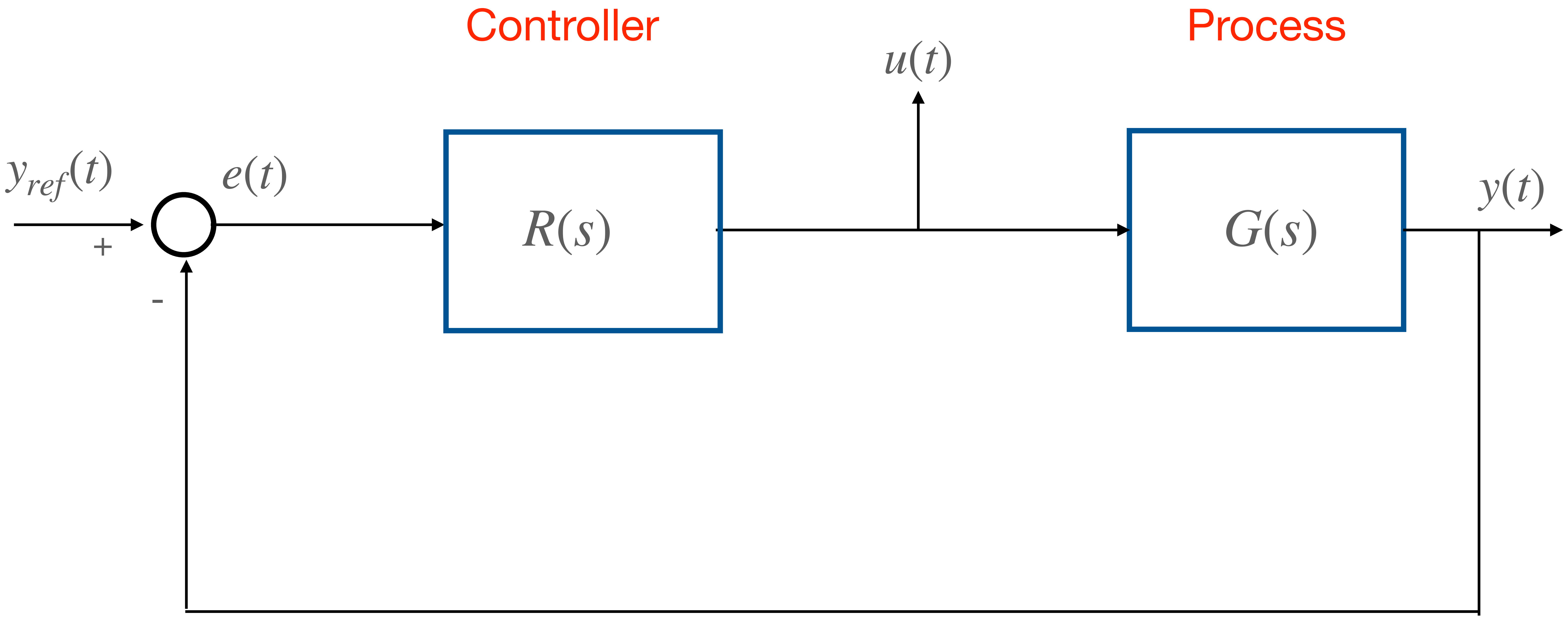


WHEEL SLIP

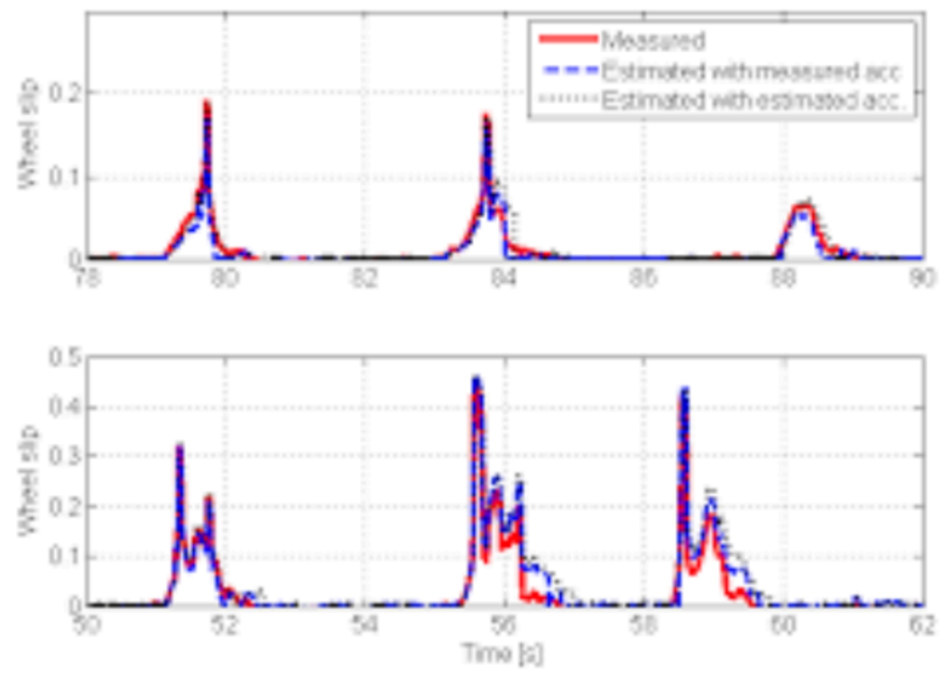
Basic Control Scheme



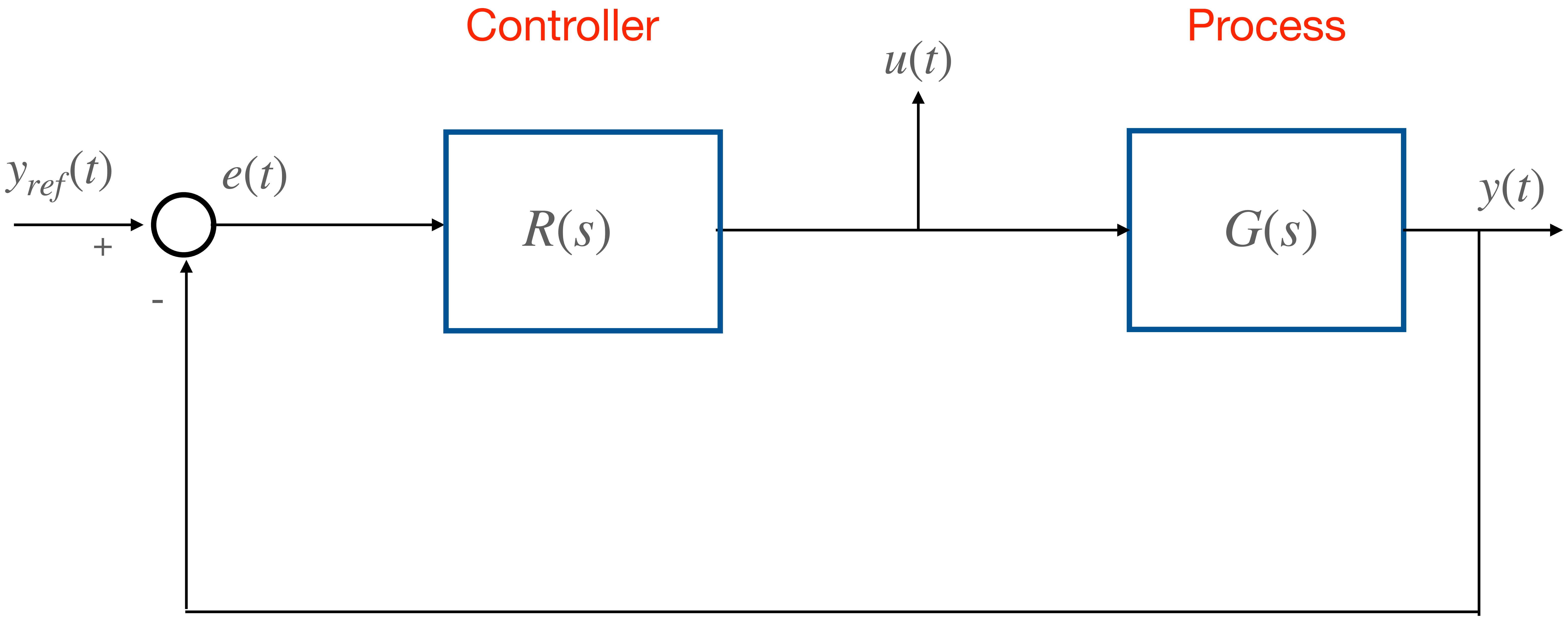
WHEEL SLIP



Basic Control Scheme

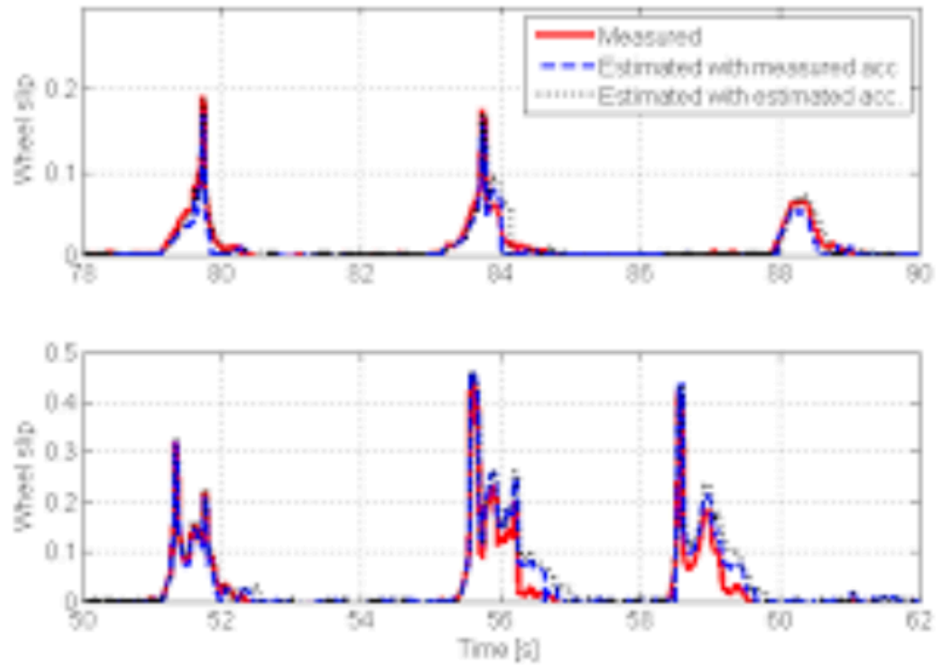


WHEEL SLIP

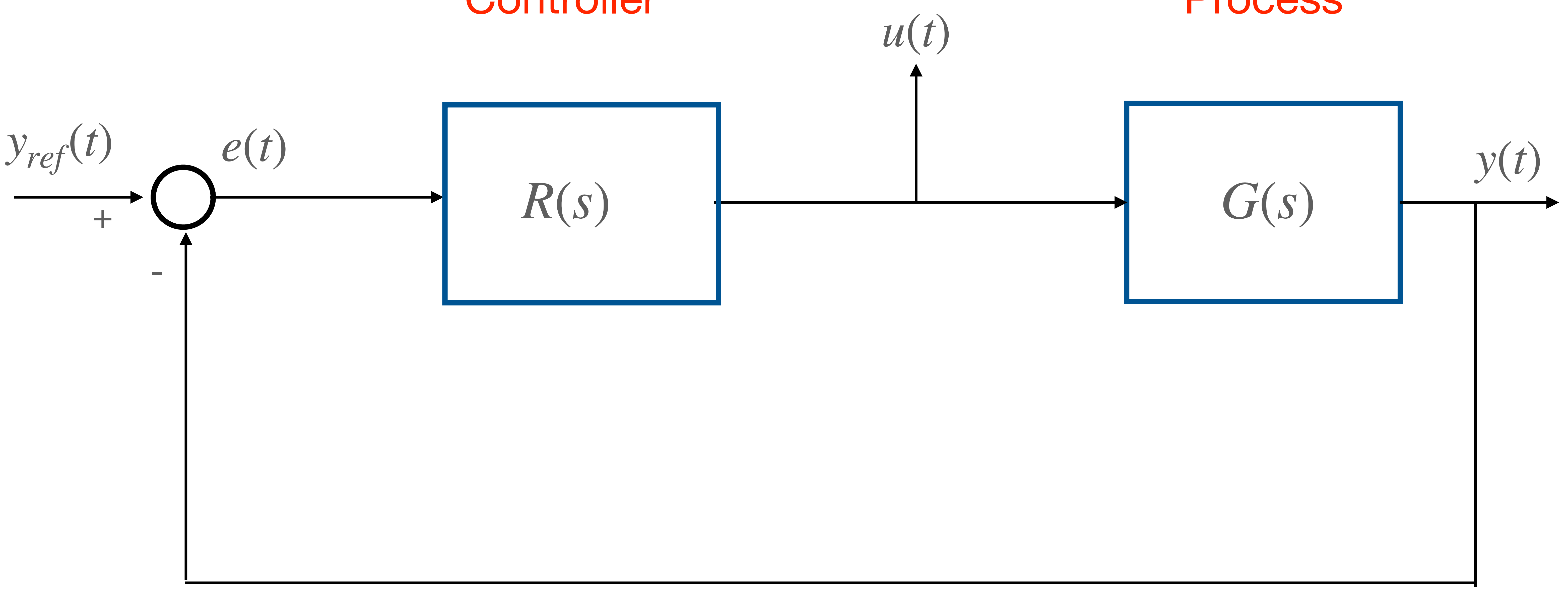


Block Algebra Rule: $y_{ref} \rightarrow u : Q(s) = \dfrac{R(s)}{1 + R(s)G(s)}$

Basic Control Scheme

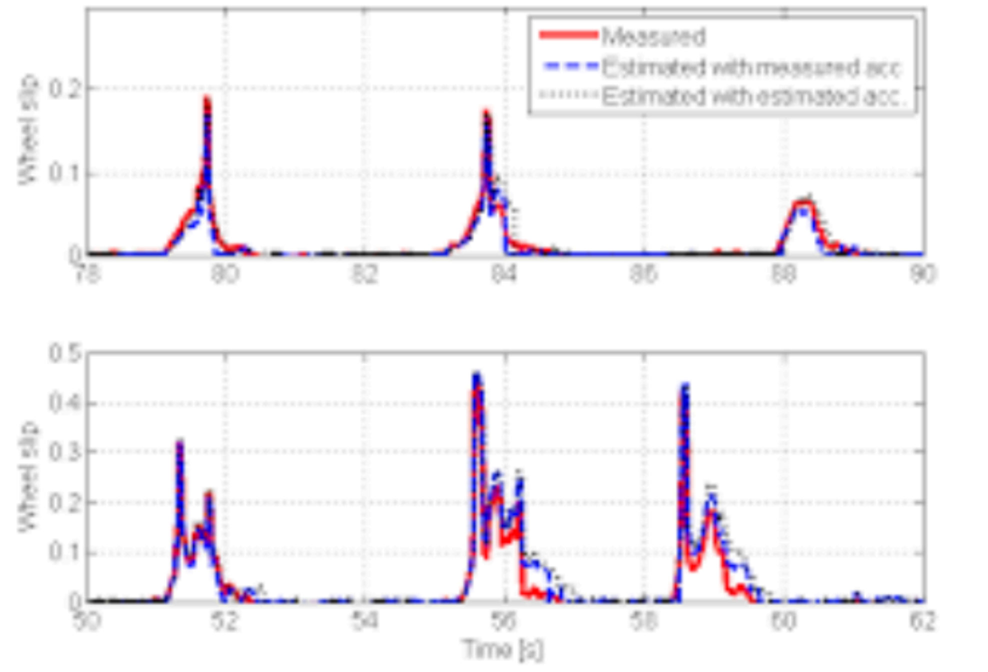


WHEEL SLIP

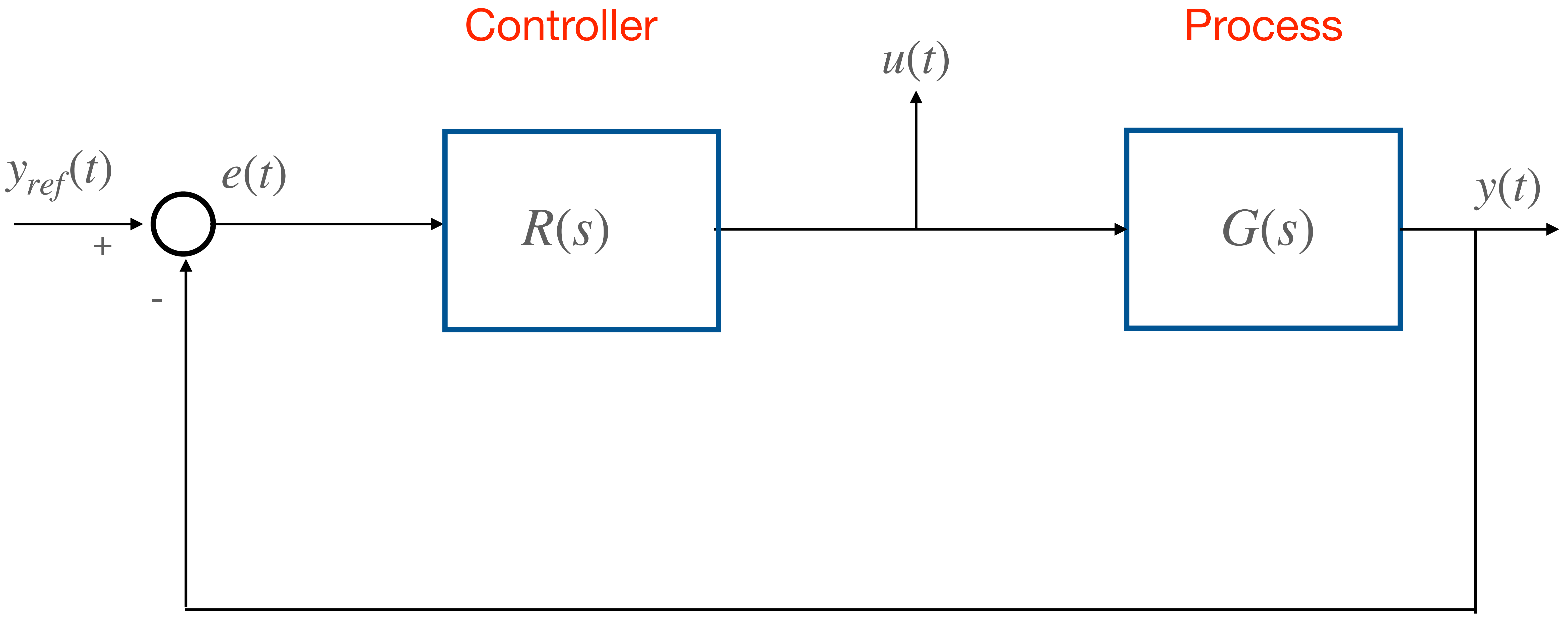


Block Algebra Rule: $y_{ref} \rightarrow u : Q(s) = \dfrac{R(s)}{1 + R(s)G(s)}$ Control Sensitivity Function

Basic Control Scheme



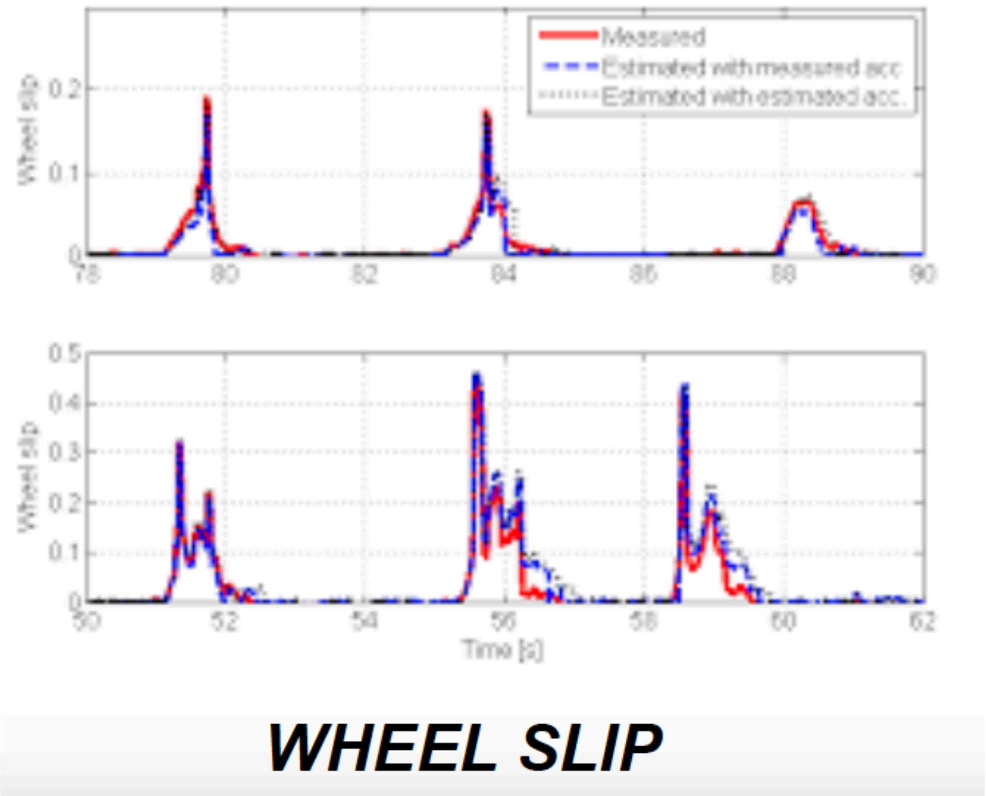
WHEEL SLIP

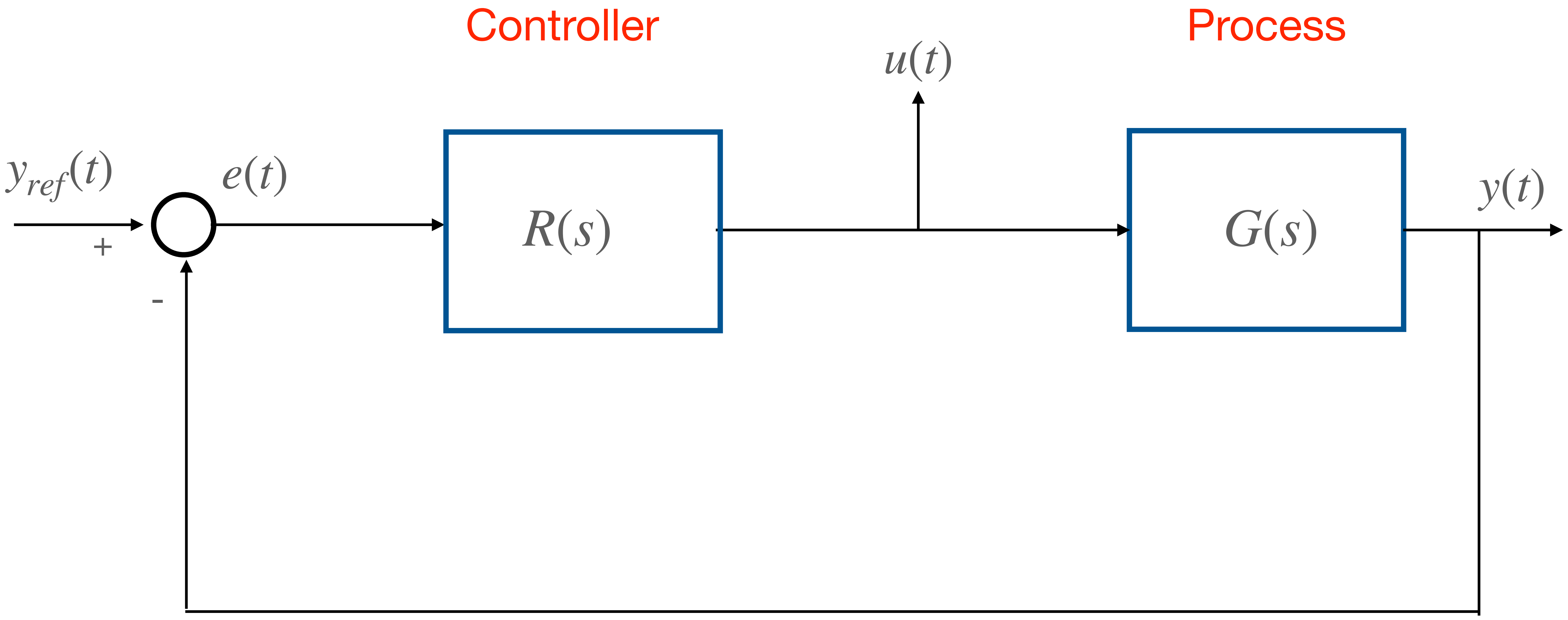


Block Algebra Rule: $y_{ref} \to u : Q(s) = \dfrac{R(s)}{1 + R(s)G(s)}$ Control Sensitivity Function

Recommendation: $|Q(j\omega)|$ sufficiently low $\forall \omega \in \mathbb{R}_{+\cup\{0\}}$

Basic Control Scheme



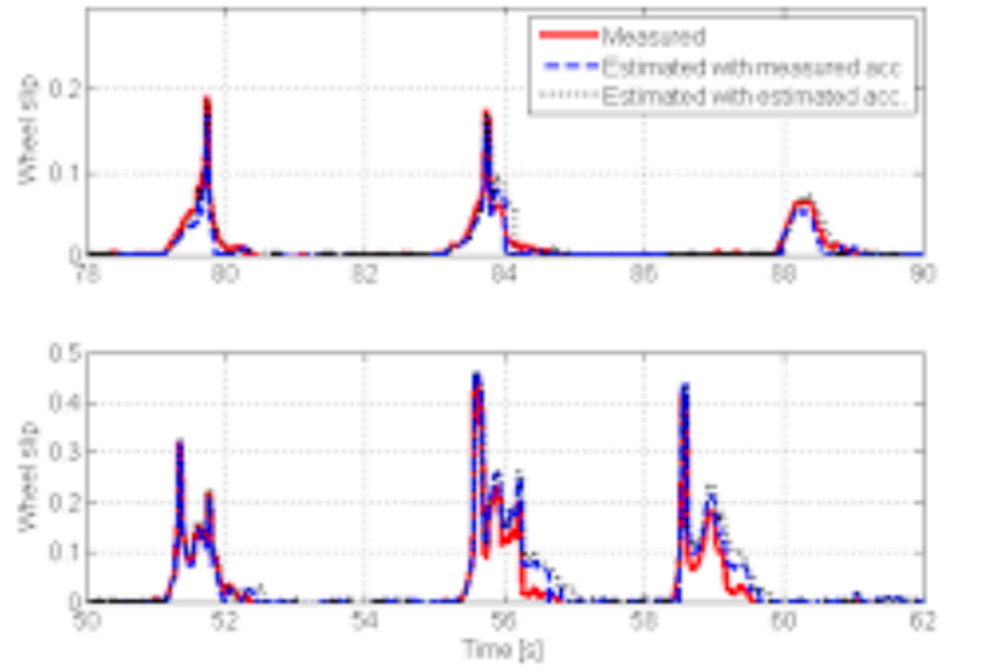


Block Algebra Rule: $y_{ref} \to u : Q(s) = \dfrac{R(s)}{1 + R(s)G(s)}$ Control Sensitivity Function

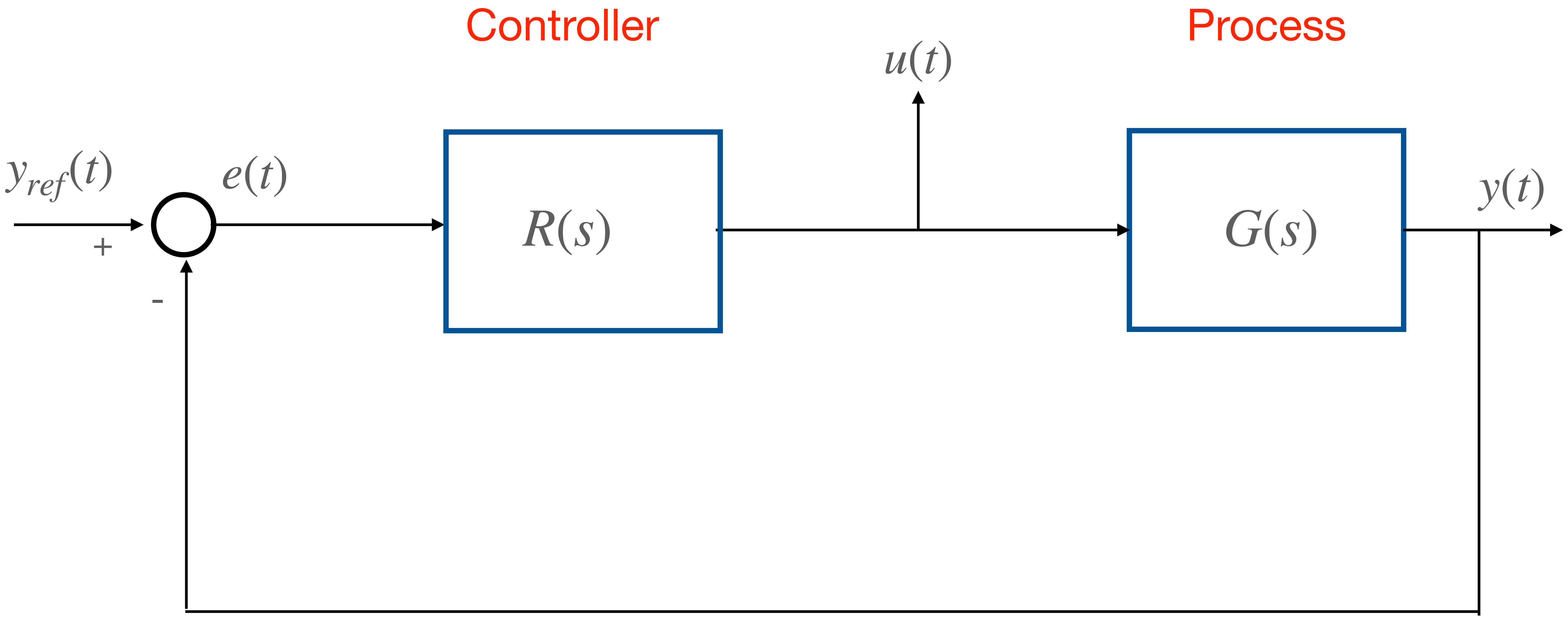
Recommendation: $|Q(j\omega)|$ sufficiently low $\forall \omega \in \mathbb{R}_{+\cup\{0\}}$

LF: $|Q(j\omega)| \to \left|\dfrac{1}{G(j\omega)}\right|$

Basic Control Scheme



WHEEL SLIP



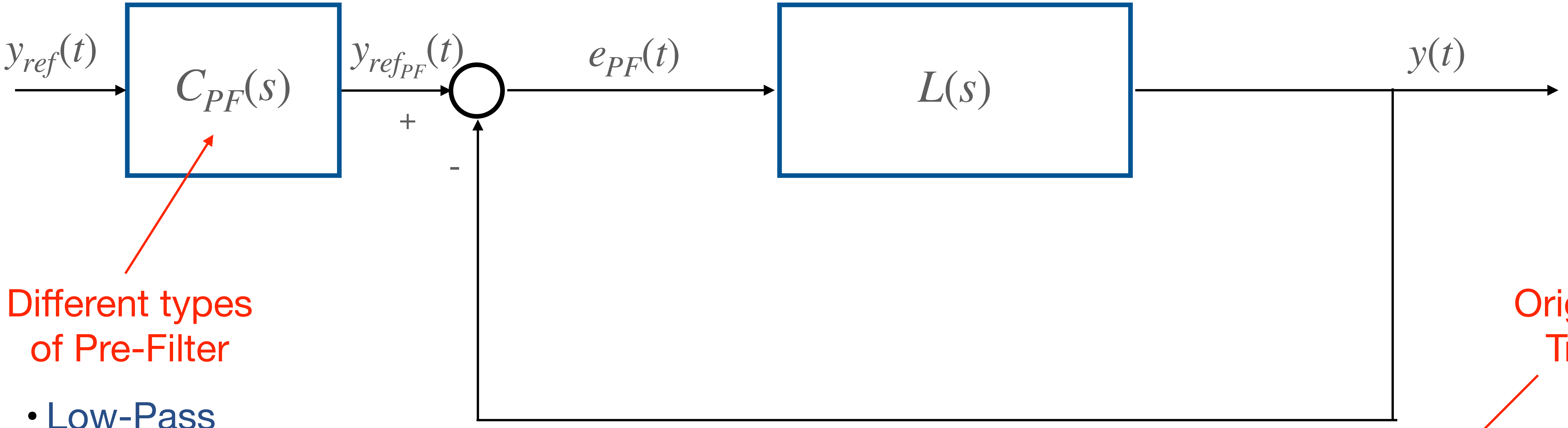
Block Algebra Rule: $y_{ref} \to u : Q(s) = \dfrac{R(s)}{1 + R(s)G(s)}$ Control Sensitivity Function

Recommendation: $|Q(j\omega)|$ sufficiently low $\forall \omega \in \mathbb{R}_{+\cup\{0\}}$

LF: $|Q(j\omega)| \to \left|\dfrac{1}{G(j\omega)}\right|$

HF: $|Q(j\omega)| \to |R(j\omega)|$

Pre-filter Based Control Scheme: Case 2



Different types of Pre-Filter

- Low-Pass

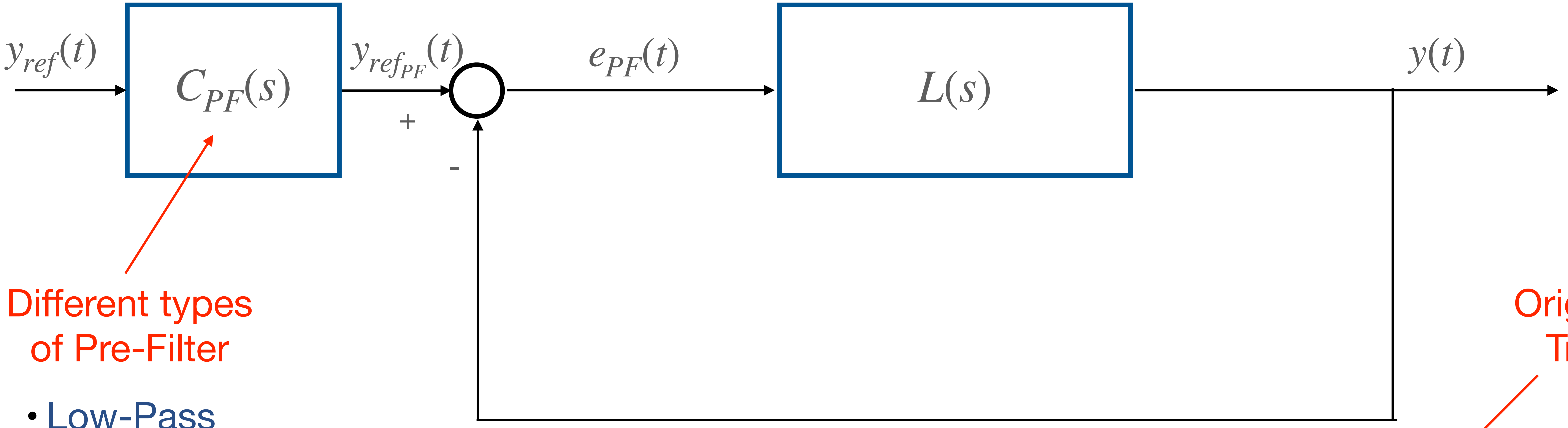
Original Closed Loop Transfer Function

New Closed Loop Transfer Function

Block Algebra Rule: $$F(s) = \frac{R(s) \cdot G(s)}{1 + R(s) \cdot G(s)} = \frac{L(s)}{1 + L(s)}$$

$$F_{PF}(s) = C_{PF}(s) \cdot F(s) = C_{PF}(s) \cdot \frac{L(s)}{1 + L(s)} = \frac{\mu(1 + \tau' s)}{(1 + \tau s)} \cdot \frac{L(s)}{1 + L(s)}$$

Pre-filter Based Control Scheme: Case 2



Different types of Pre-Filter

- Low-Pass

Original Closed Loop Transfer Function

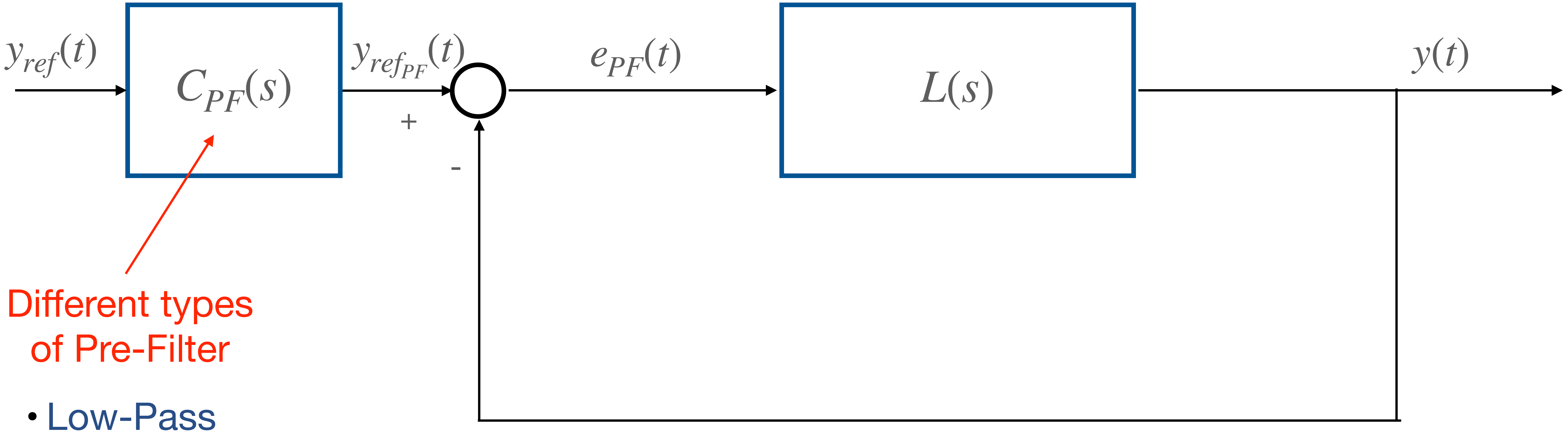
Block Algebra Rule: $F(s) = \dfrac{R(s) \cdot G(s)}{1 + R(s) \cdot G(s)} = \dfrac{L(s)}{1 + L(s)}$

New Closed Loop Transfer Function

The pole precedes the zero on the ω axis

$$F_{PF}(s) = C_{PF}(s) \cdot F(s) = C_{PF}(s) \cdot \frac{L(s)}{1 + L(s)} = \frac{\mu(1 + \tau' s)}{(1 + \tau s)} \cdot \frac{L(s)}{1 + L(s)}$$

Pre-filter Based Control Scheme: Case 2



Different types of Pre-Filter

- Low-Pass

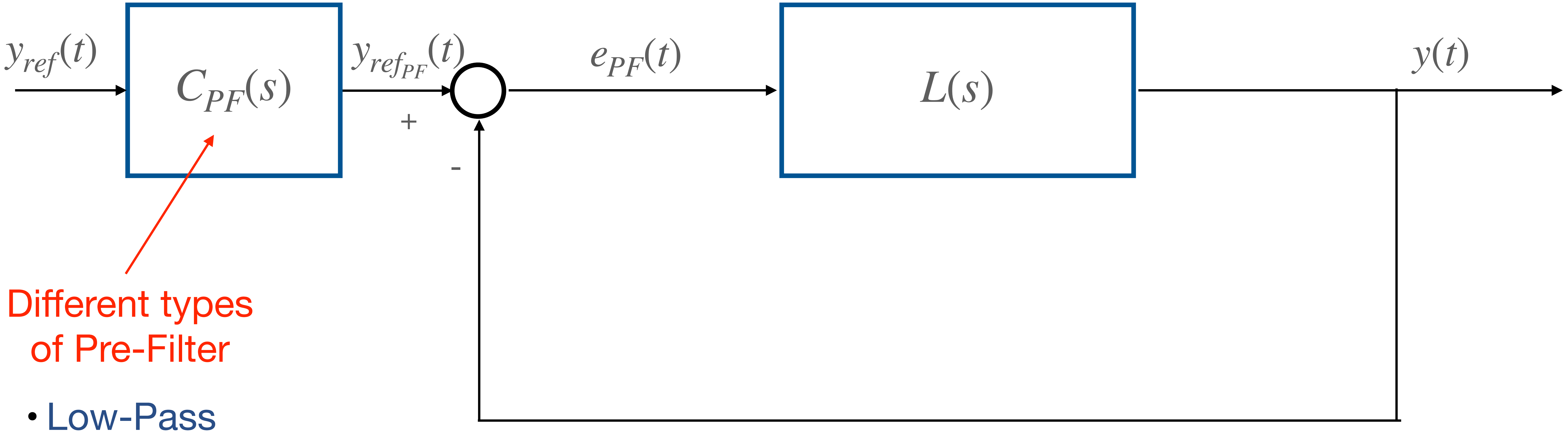
Assumptions:

- $C_{PF}(s)$ As. Stable
- Proper
- Unitary gain

The pole precedes the zero on the ω axis

$$F_{PF}(s) = C_{PF}(s) \cdot F(s) = C_{PF}(s) \cdot \frac{L(s)}{1+L(s)} = \frac{\mu(1+\tau' s)}{(1+\tau s)} \cdot \frac{L(s)}{1+L(s)}$$

Pre-filter Based Control Scheme: Case 2



Different types of Pre-Filter

- Low-Pass

Assumptions:

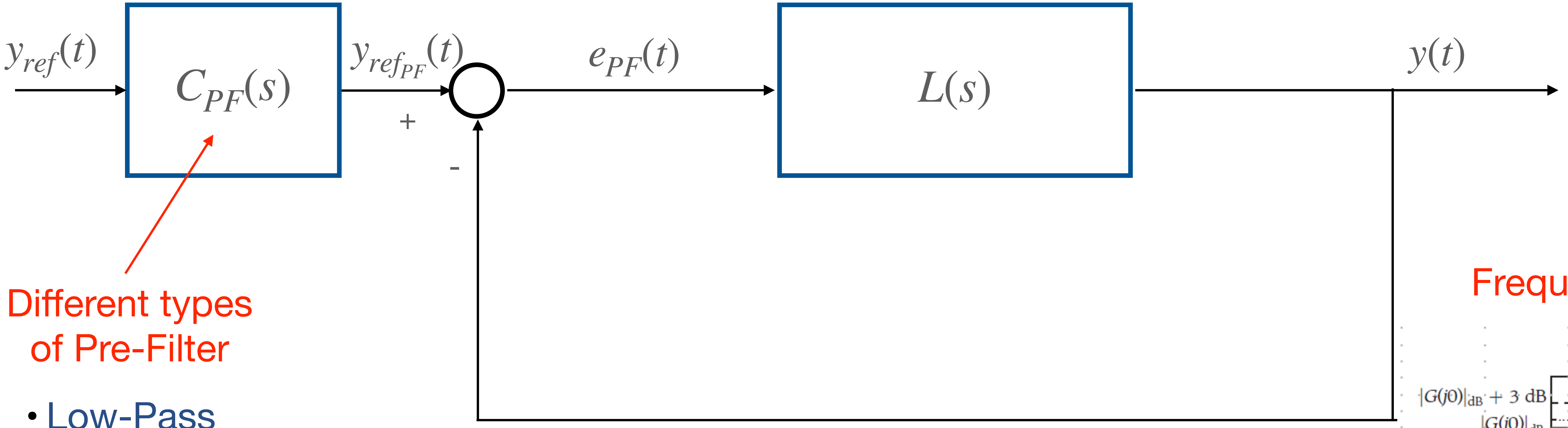
- $C_{PF}(s)$ As. Stable
- Proper
- Unitary gain

$$\lim_{s \to 0} \frac{\mu(1+\tau' s)}{1+\tau s} = \mu = 1$$

The pole precedes the zero on the ω axis

$$F_{PF}(s) = C_{PF}(s) \cdot F(s) = C_{PF}(s) \cdot \frac{L(s)}{1+L(s)} = \frac{\mu(1+\tau' s)}{(1+\tau s)} \cdot \frac{L(s)}{1+L(s)}$$

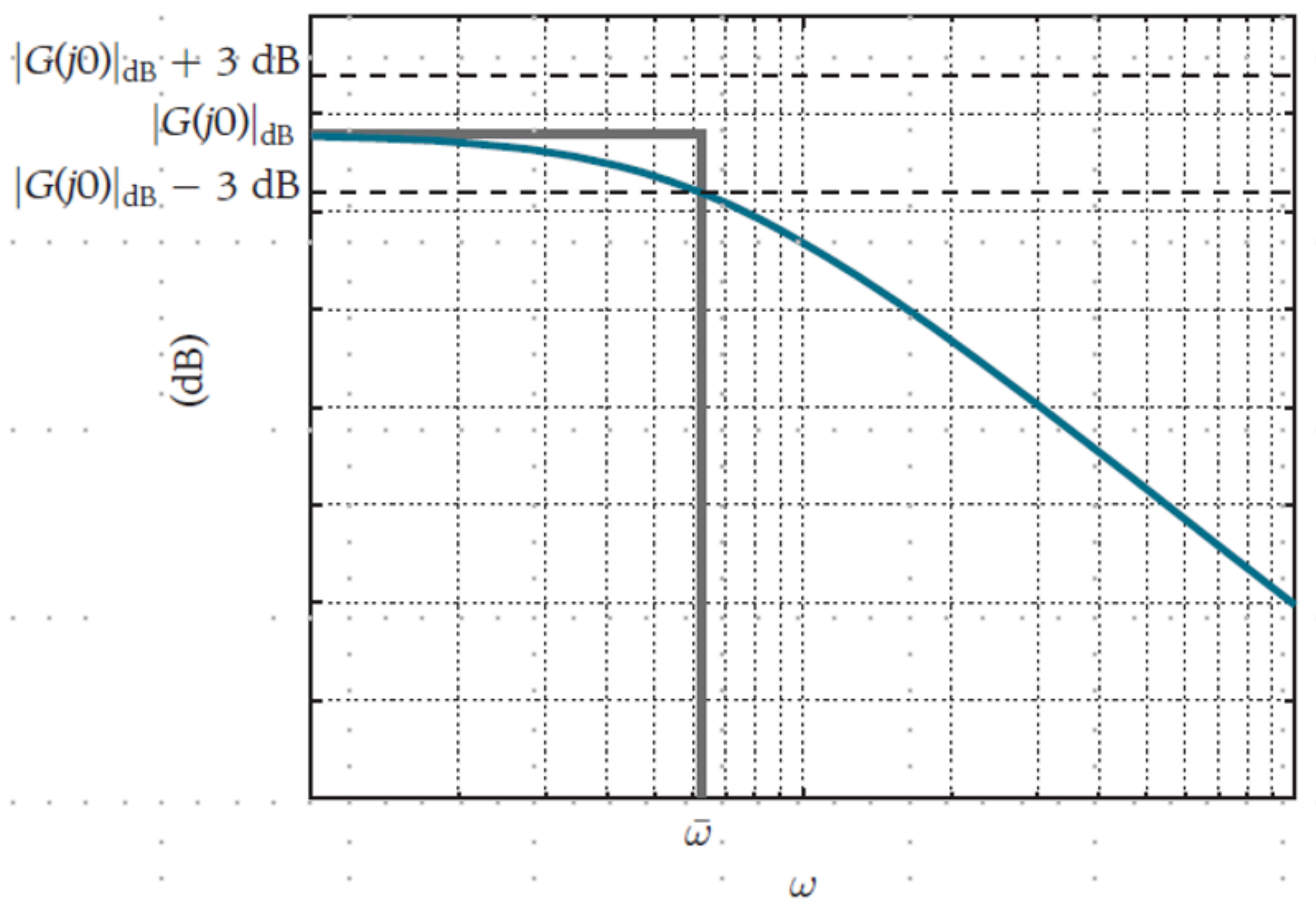
Pre-filter Based Control Scheme: Case 2



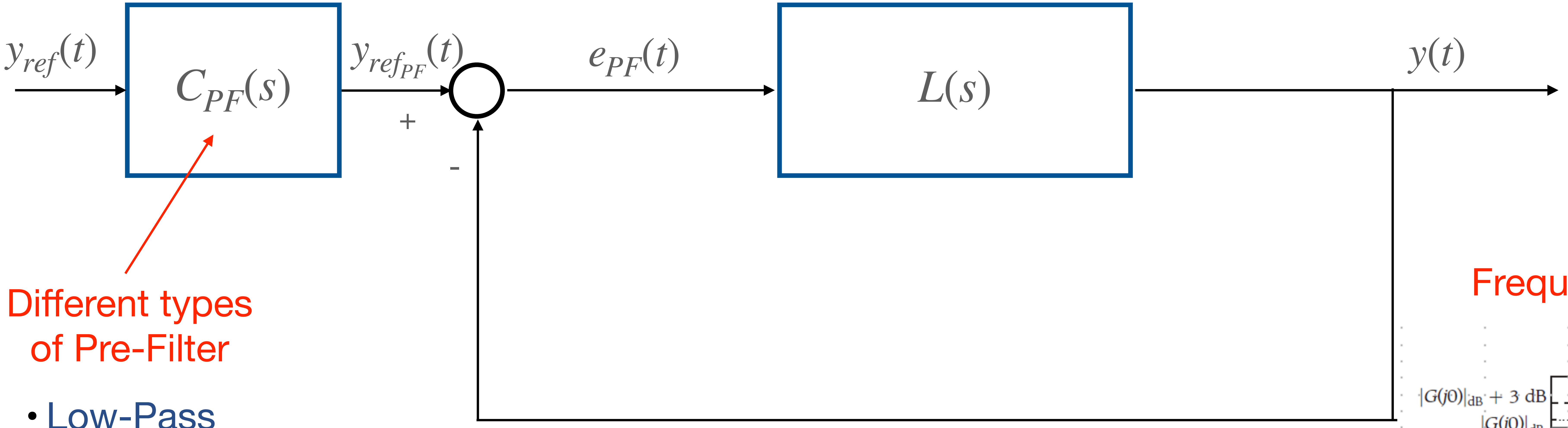
Different types of Pre-Filter

- Low-Pass

Frequency response of LP filter (magnitude)



Pre-filter Based Control Scheme: Case 2

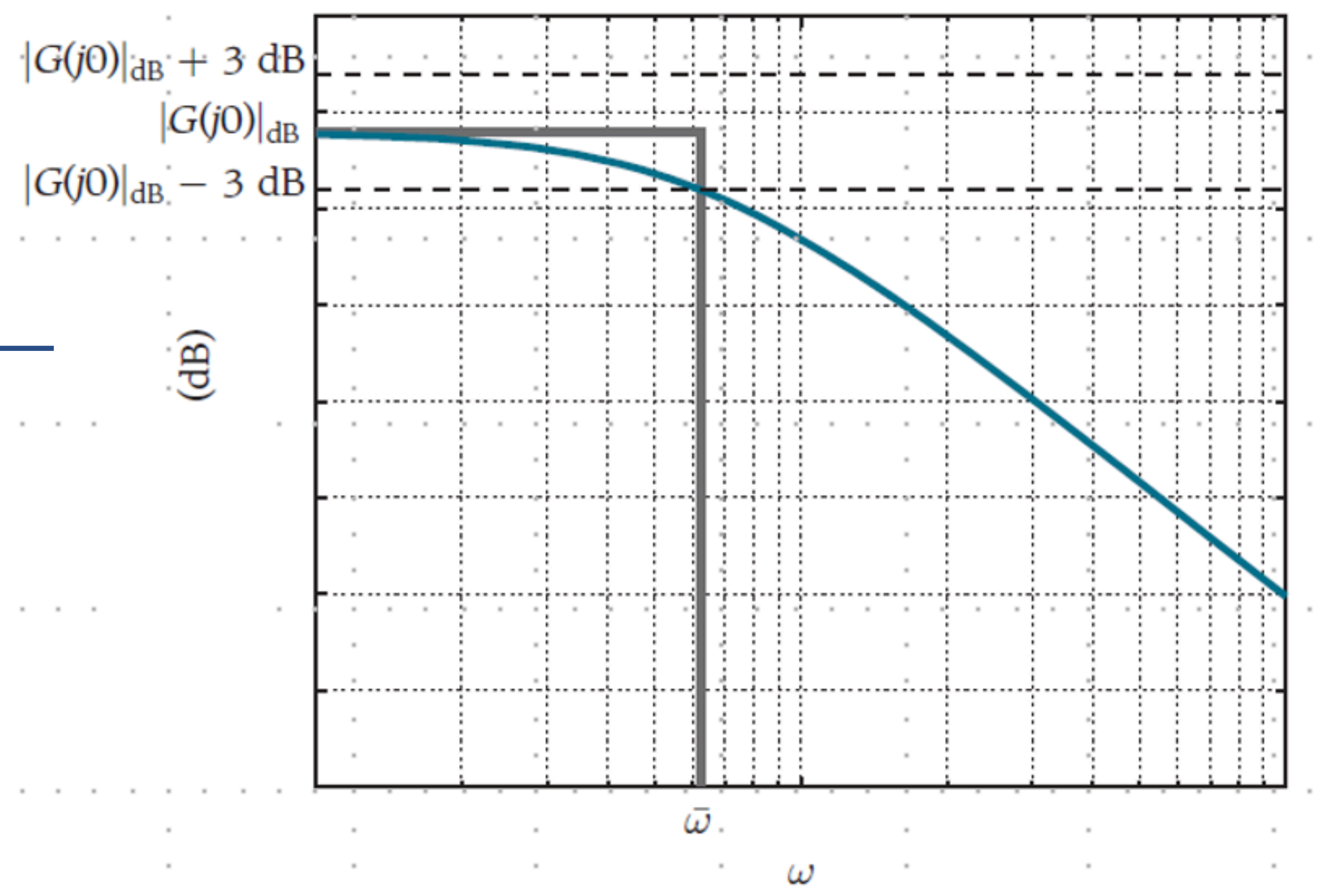


Different types of Pre-Filter

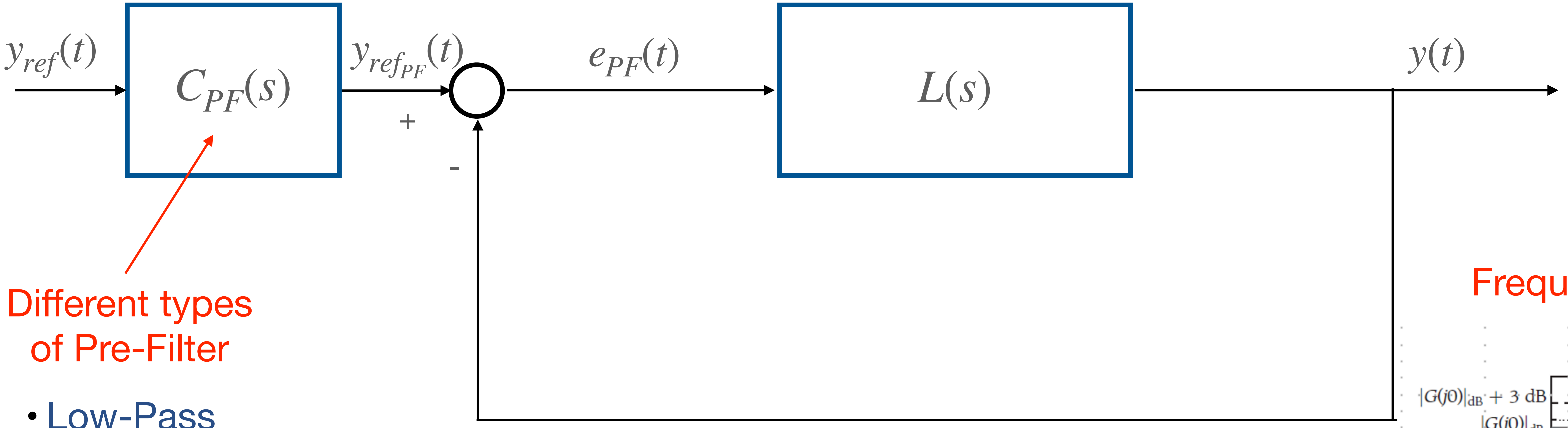
- Low-Pass

A LP Pre-filter is used to to filter the reference signal to alleviate control peaking phenomena

Frequency response of LP filter (magnitude)



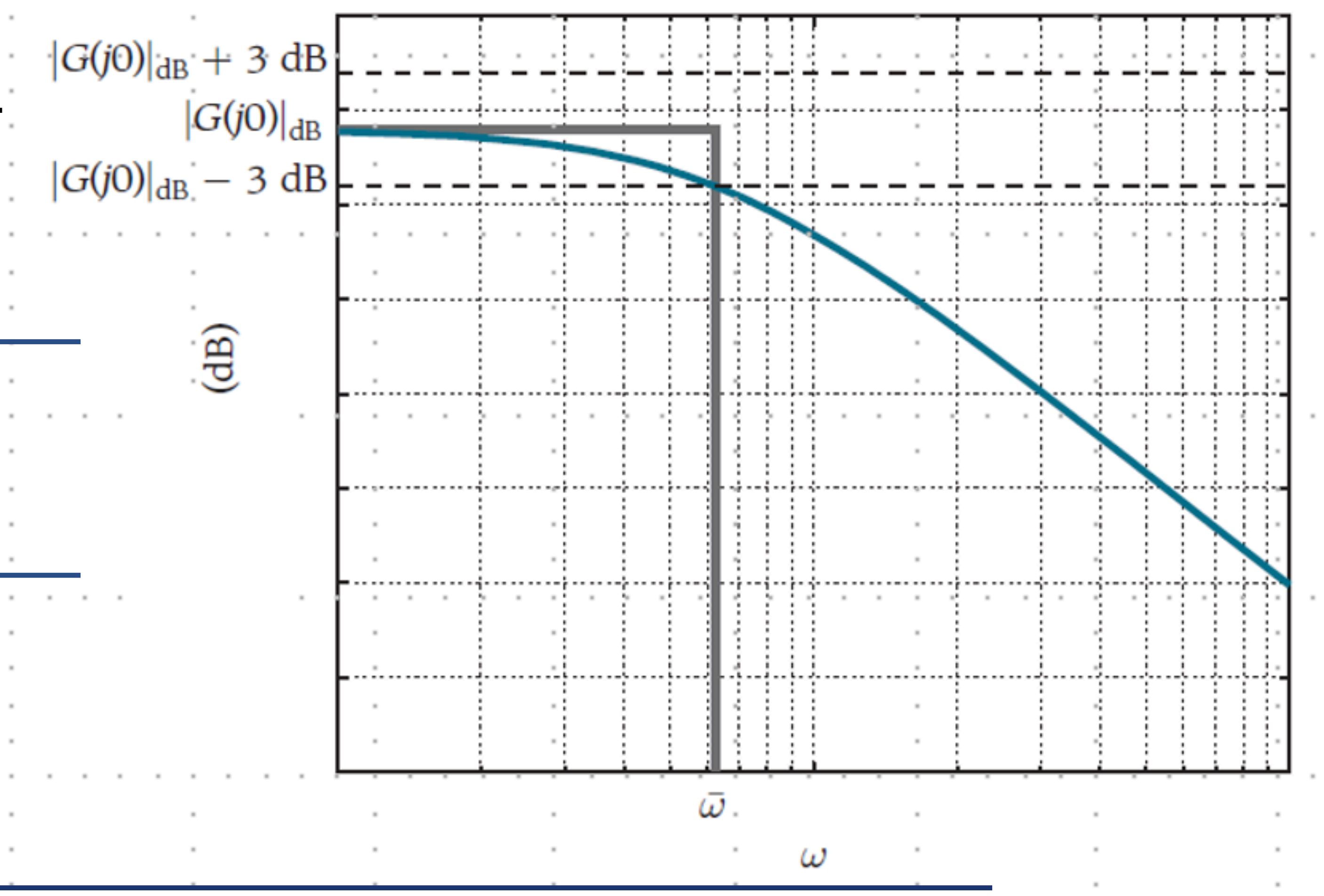
Pre-filter Based Control Scheme: Case 2



Different types of Pre-Filter

- Low-Pass

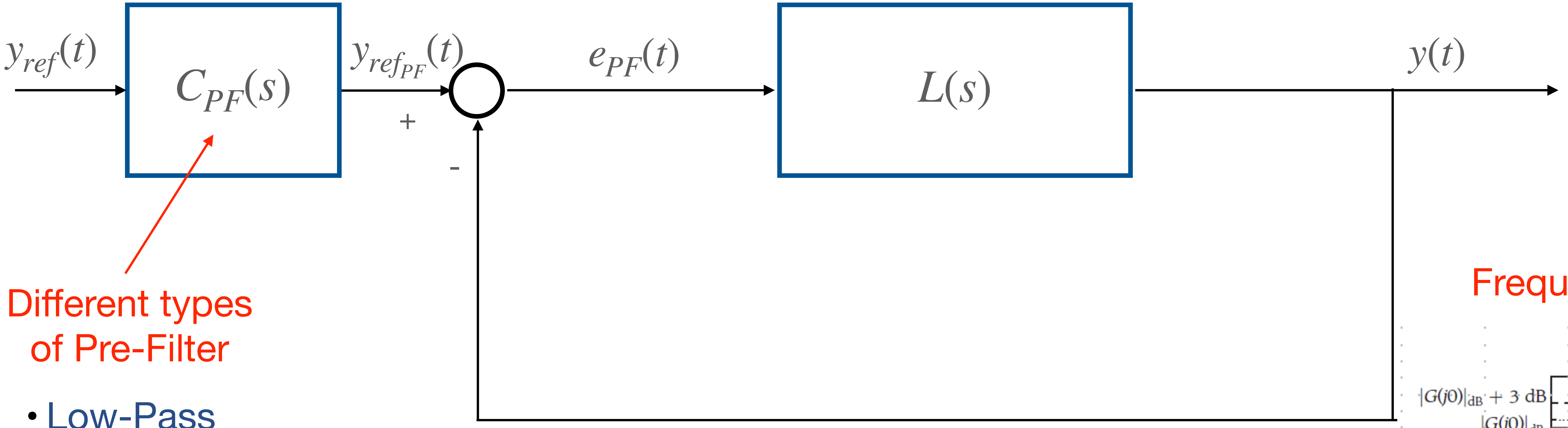
Frequency response of LP filter (magnitude)



A LP Pre-filter is used to to filter the reference signal to alleviate control peaking phenomena

Care must be taken not to reduce the closed-loop bandwidth $\omega_{BW_{CL}}$

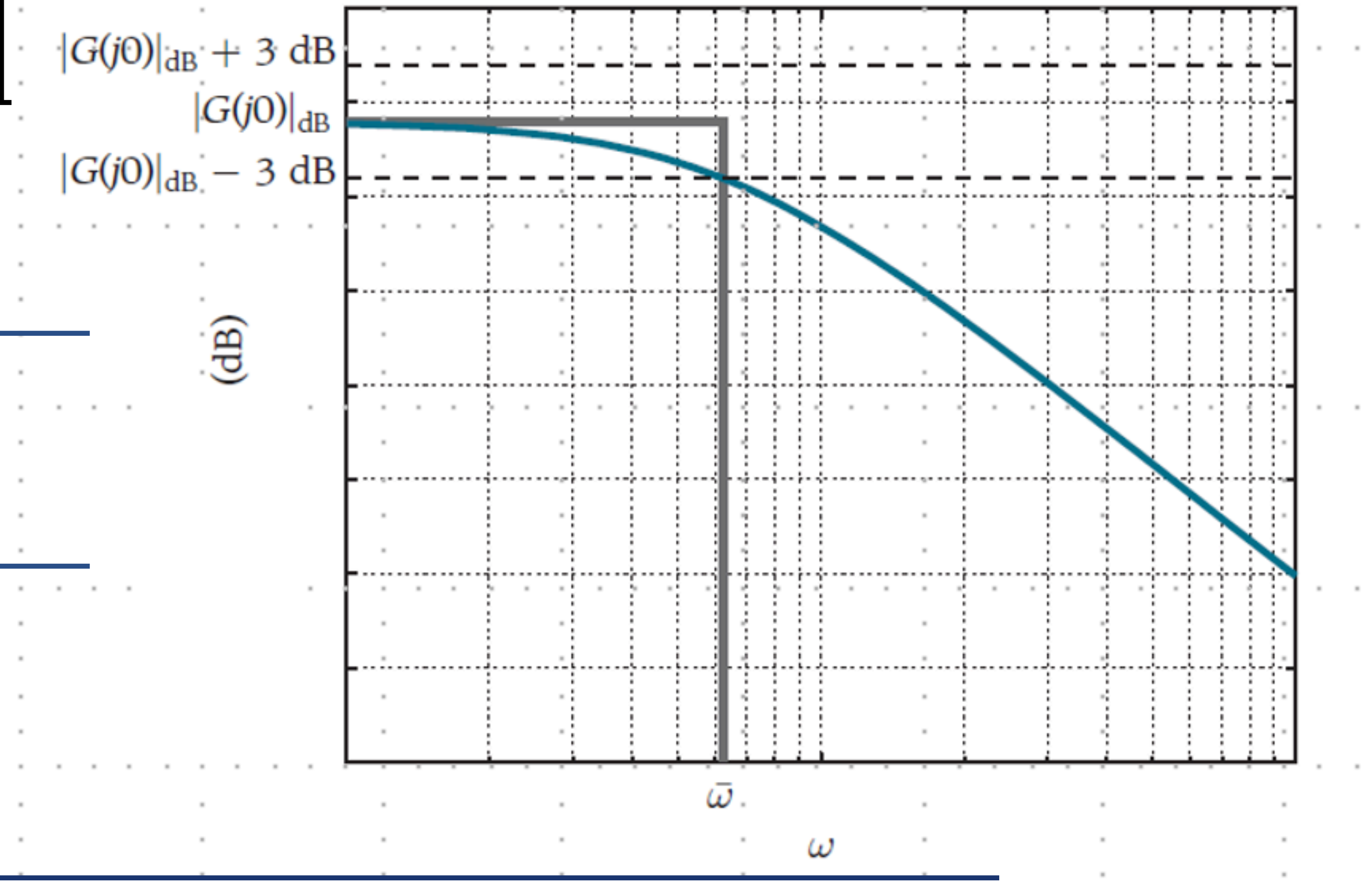
Pre-filter Based Control Scheme: Case 2



Different types of Pre-Filter

- Low-Pass

Frequency response of LP filter (magnitude)

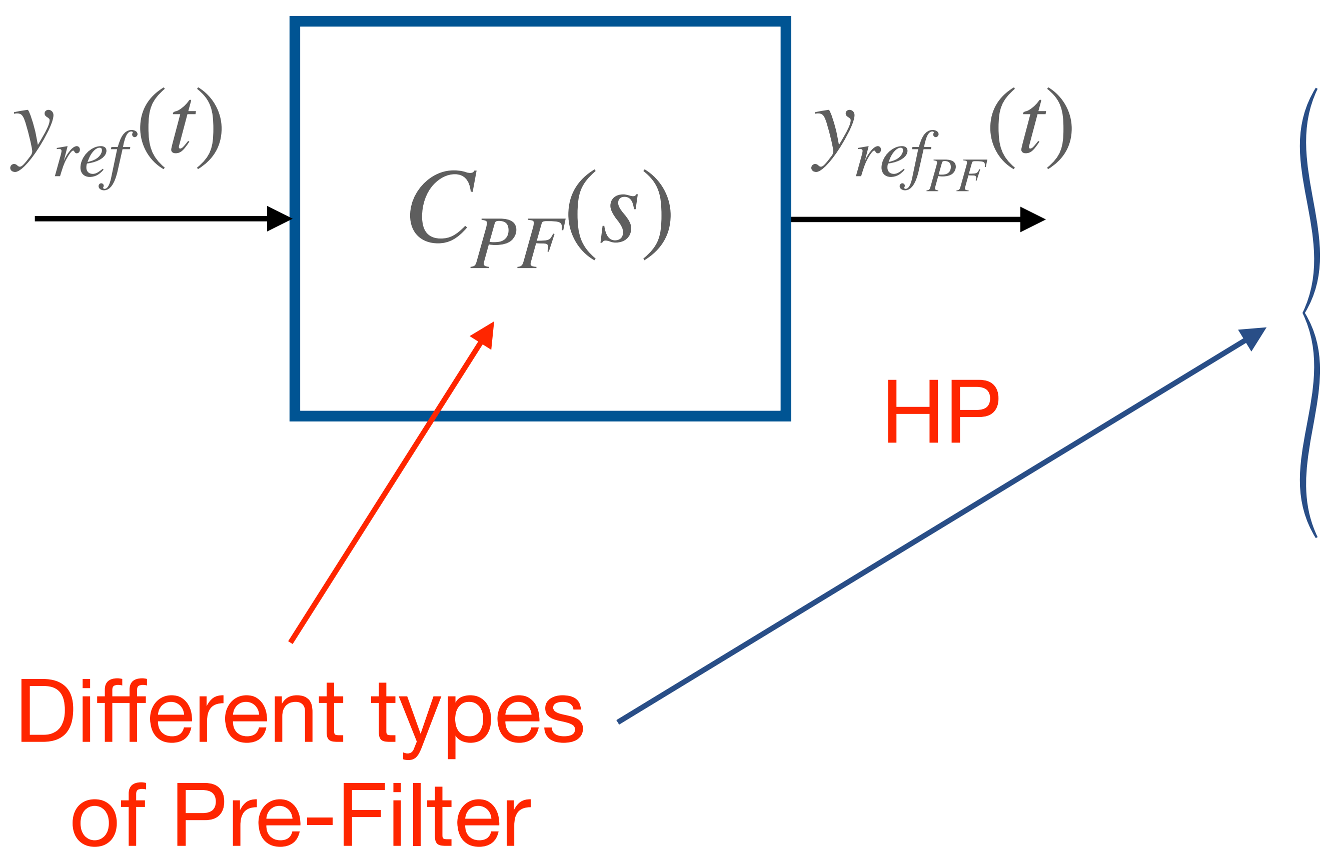


A LP Pre-filter is used to to filter the reference signal to alleviate control peaking phenomena

Care must be taken not to reduce the closed-loop bandwidth $\omega_{BW_{CL}}$

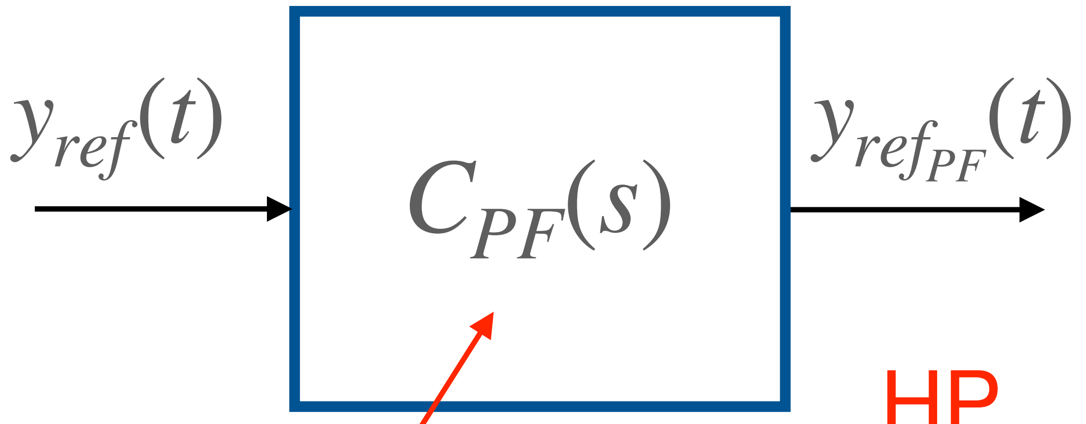
$$\boxed{\omega_{BW_{PF}} \geq \omega_{BW_{CL}}}$$

Pre-filter Based Control Scheme: Case 3



- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No "control moderation" in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)

Pre-filter Based Control Scheme: Case 3



Different types of Pre-Filter

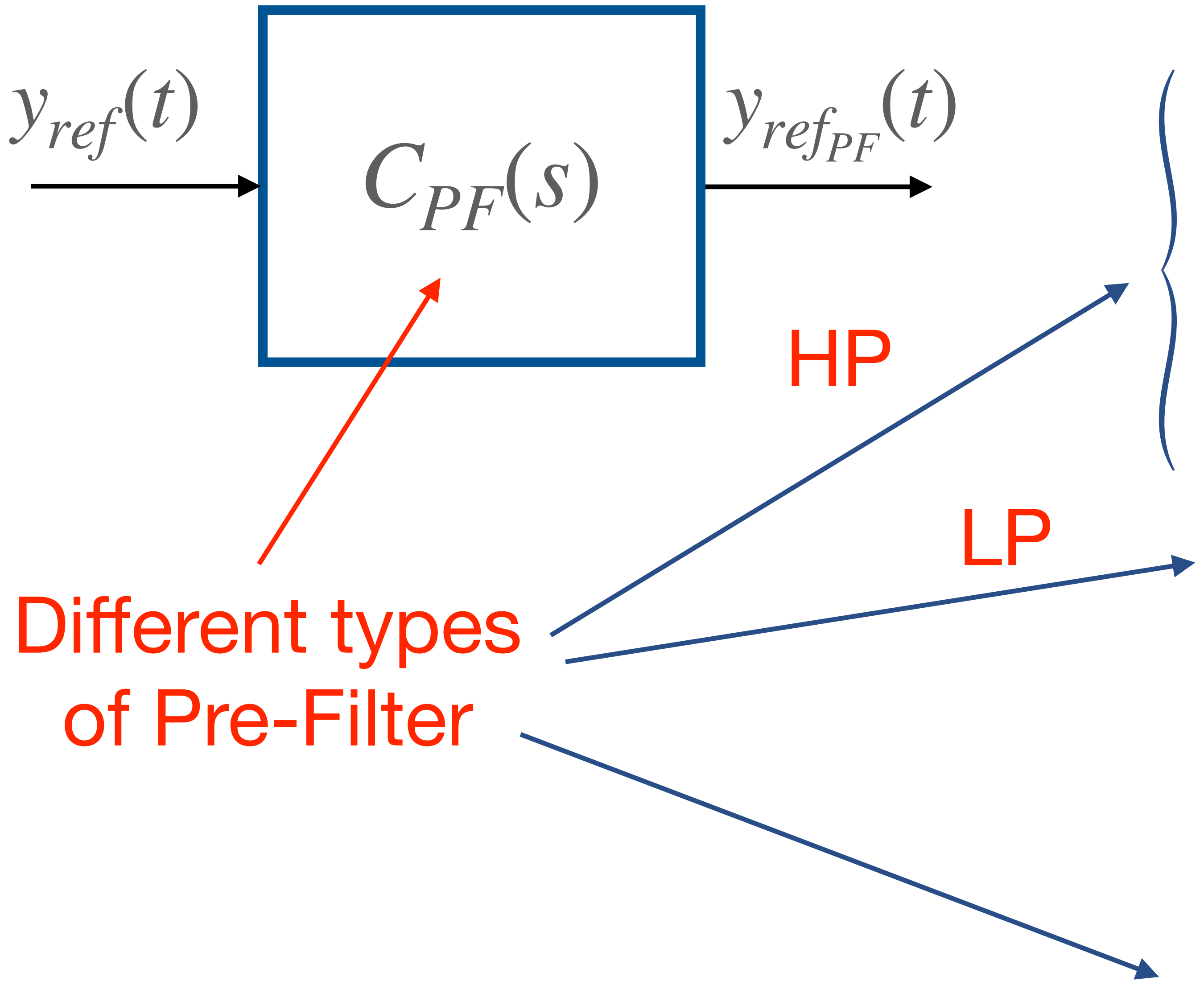
HP

- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays

LP

- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)

Pre-filter Based Control Scheme: Case 3



- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)
- Possible difficulties in enforcing steady-state precision with step reference signal via the classical approach

Pre-filter Based Control Scheme: Case 3

$$y_{ref}(t) \longrightarrow \boxed{C_{PF}(s)} \longrightarrow y_{ref_{PF}}(t)$$

Different types of Pre-Filter

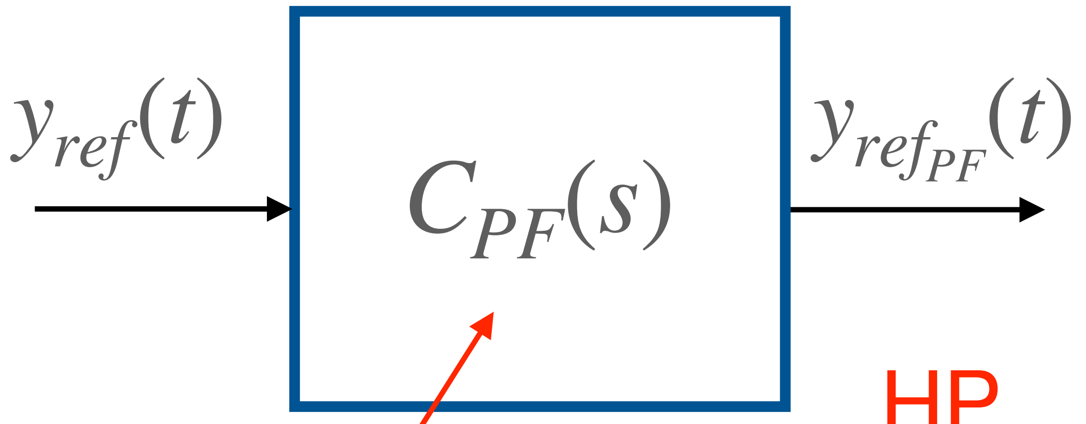
HP

- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays

LP

- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)
- Possible difficulties in enforcing steady-state precision with step reference signal via the classical approach (by introducing integrators in the direct path)

Pre-filter Based Control Scheme: Case 3



Different types of Pre-Filter

- For steady-state compensation

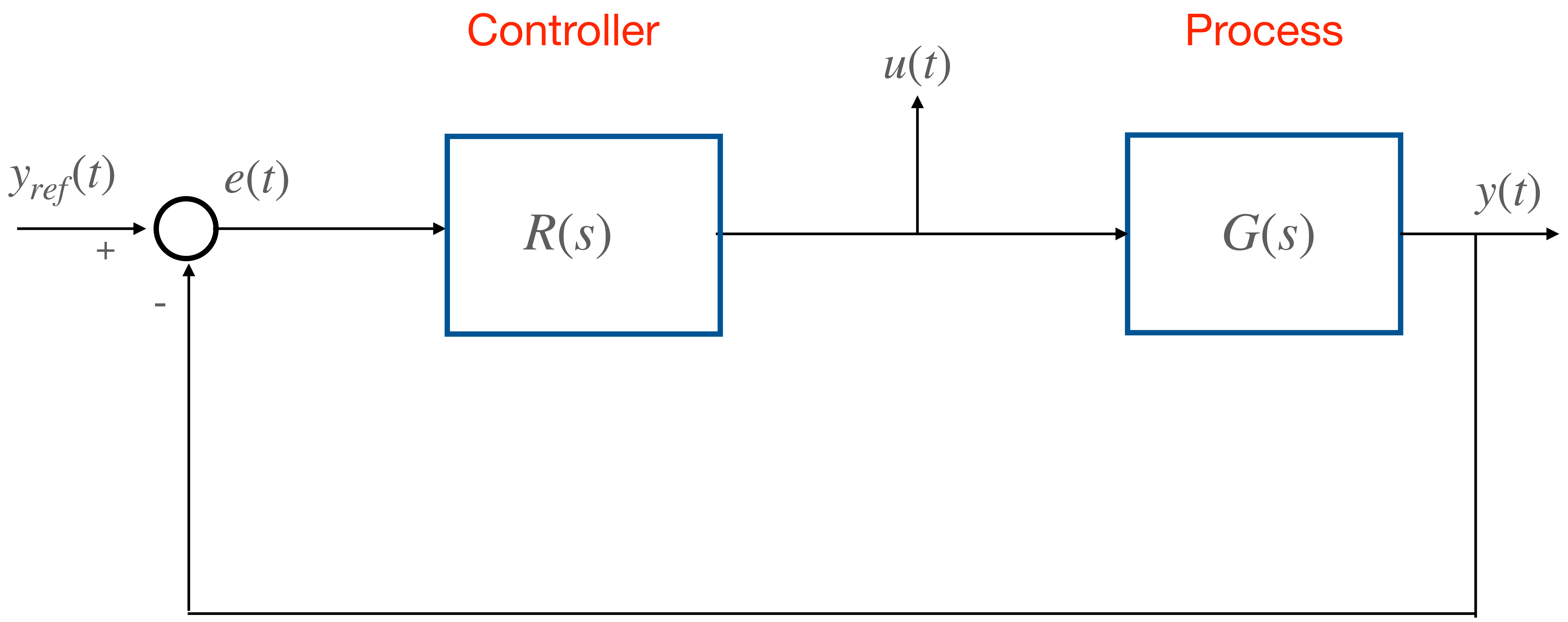
HP

- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays

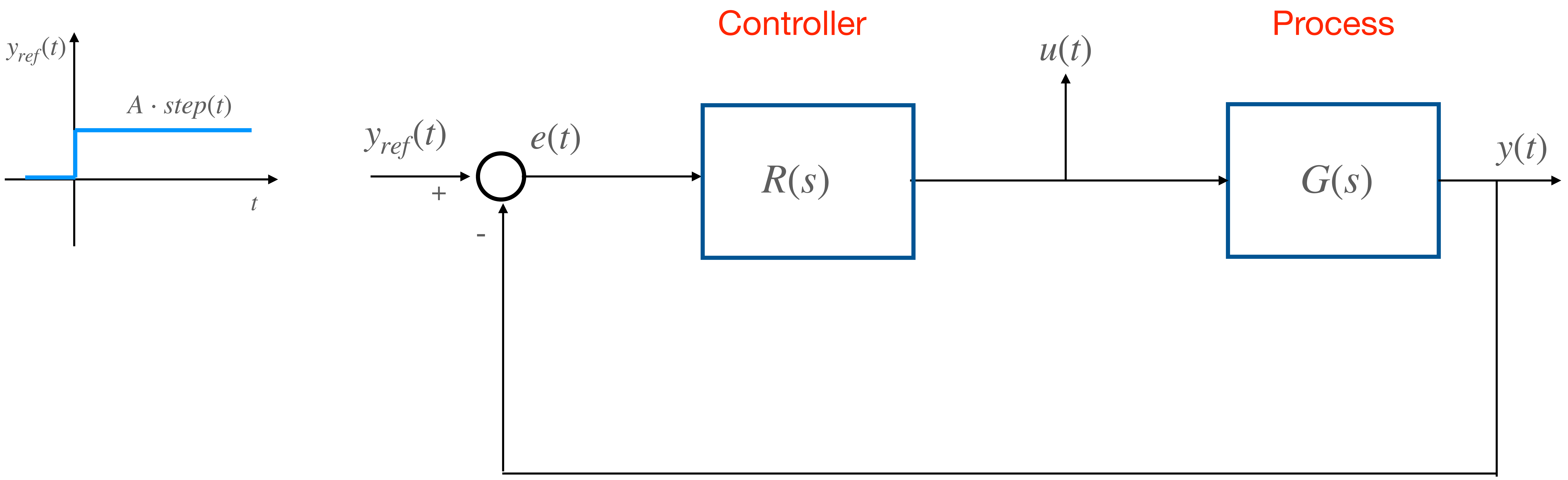
LP

- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)
- Possible difficulties in enforcing steady-state precision with step reference signal via the classical approach (by introducing integrators in the direct path)

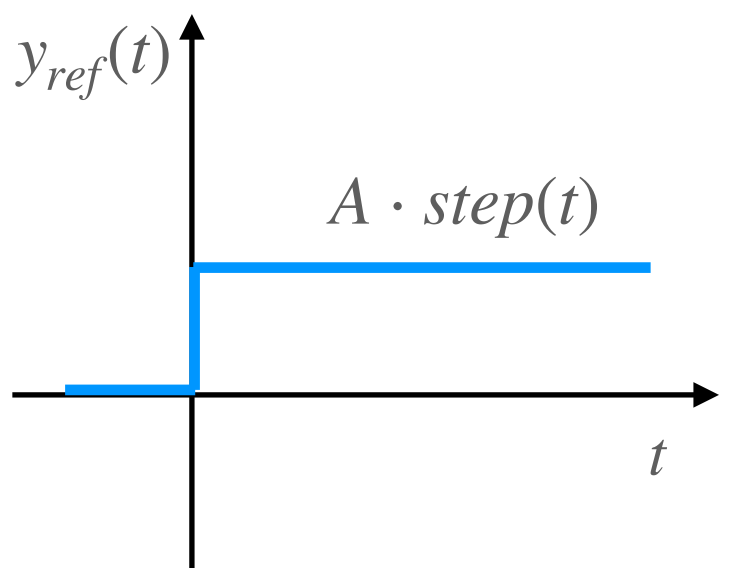
Basic Control Scheme

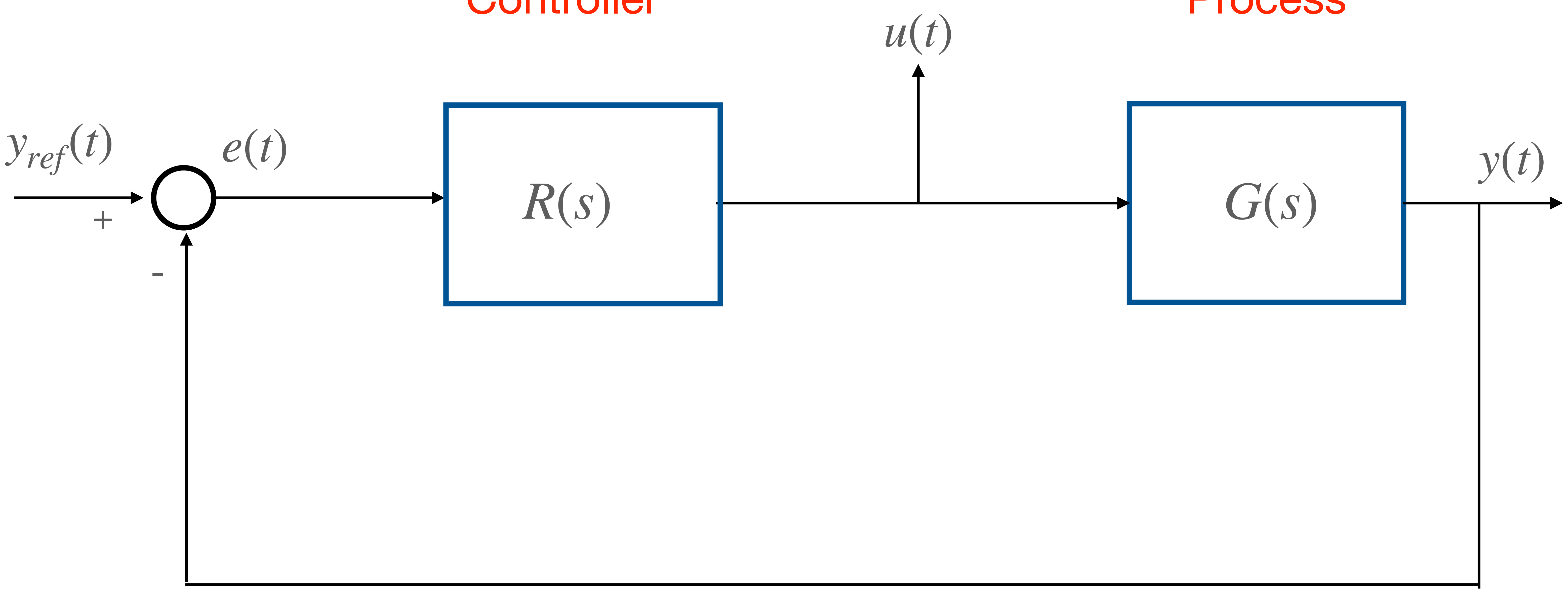


Basic Control Scheme



Basic Control Scheme

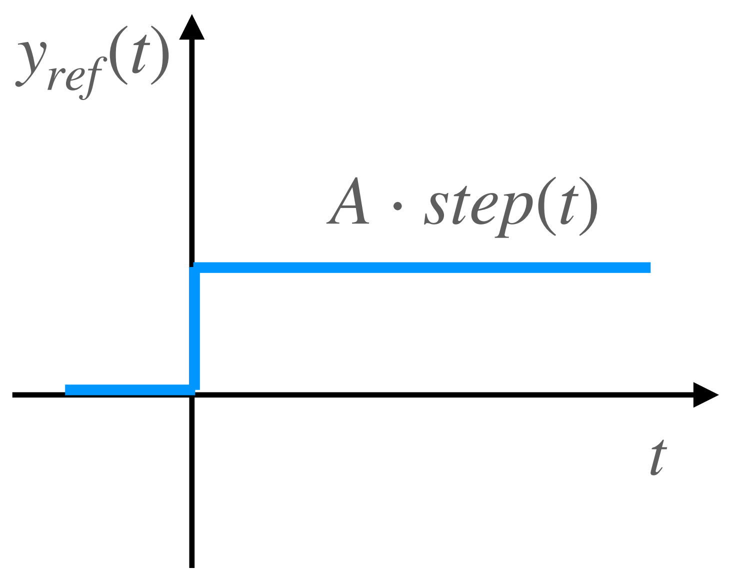


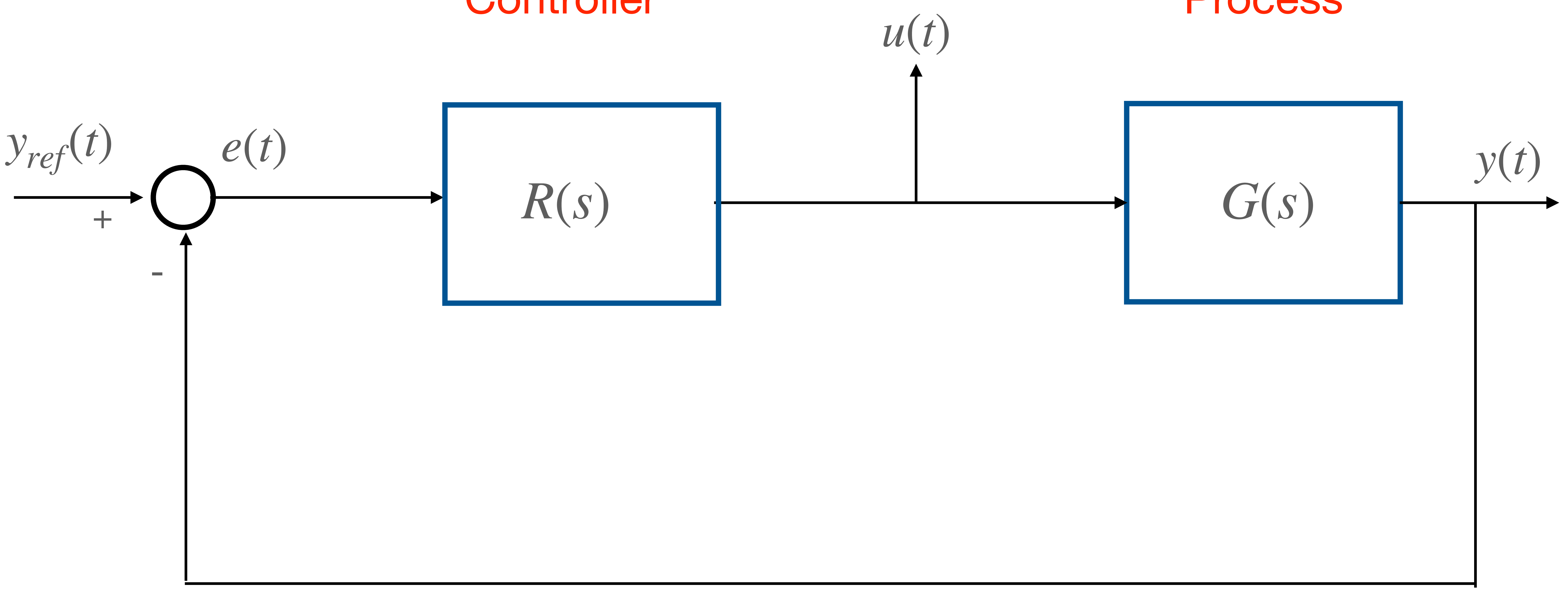


Block Algebra Rule: $y_{ref} \to y : F(s) = \dfrac{R(s)G(s)}{1 + R(s)G(s)} = \dfrac{L(s)}{1 + L(s)}$

$$y \approx y_{ref} : \quad F(0) = 1$$

Basic Control Scheme

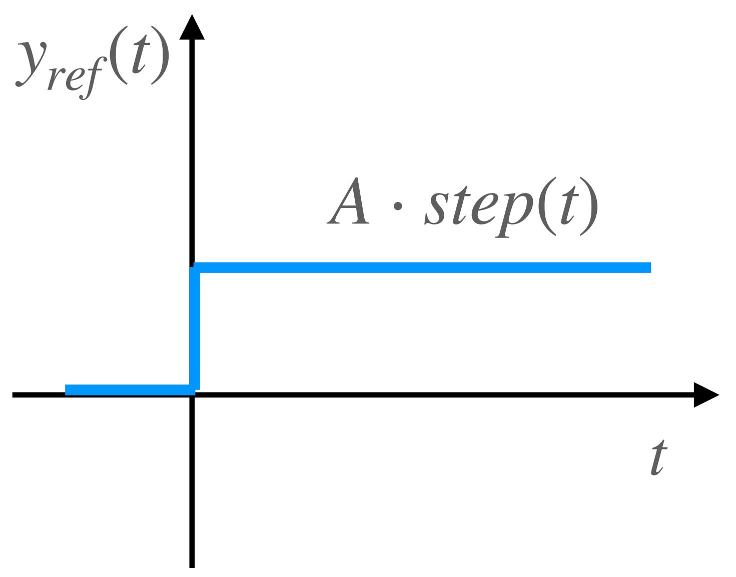


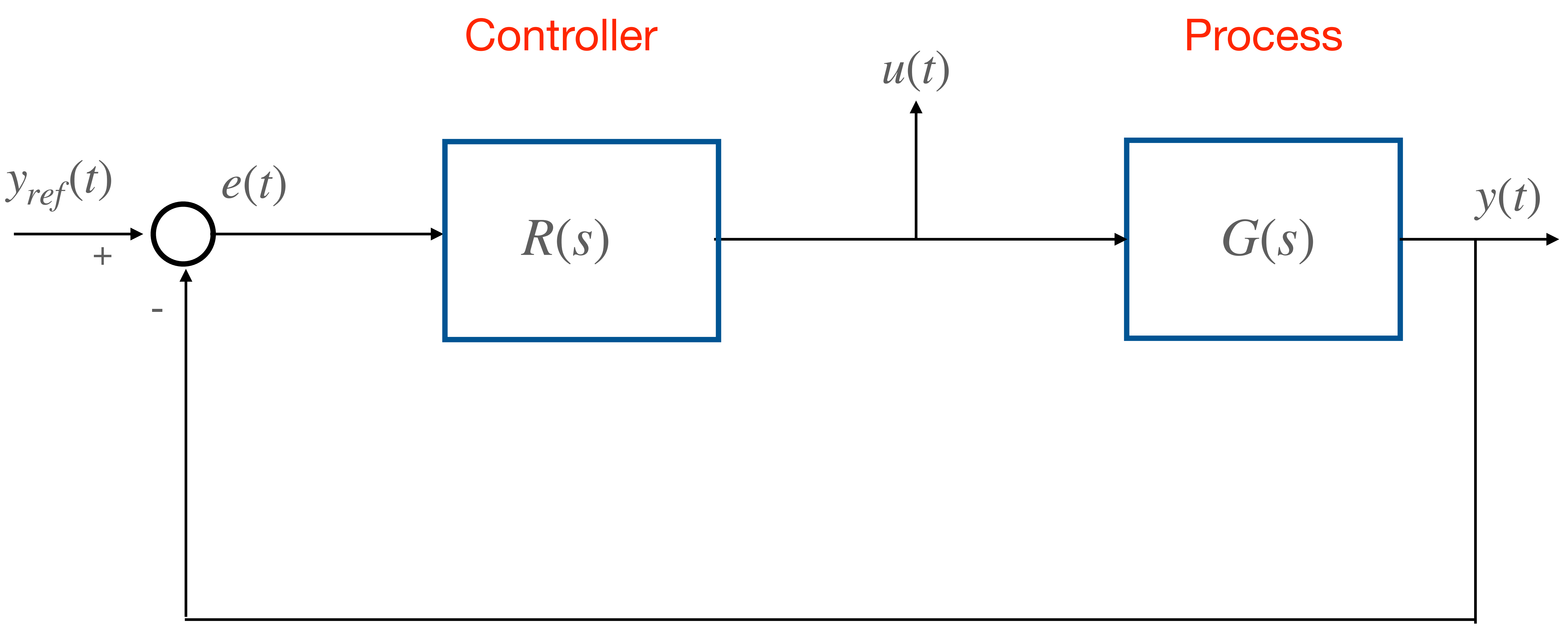


Block Algebra Rule: $y_{ref} \rightarrow y : F(s) = \dfrac{R(s)G(s)}{1 + R(s)G(s)} = \dfrac{L(s)}{1 + L(s)}$

$y \approx y_{ref} : \quad F(0) = 1$ ⟶ Impossible if $L(s)$ is not at least a TYPE 1 transfer function

Basic Control Scheme: Example





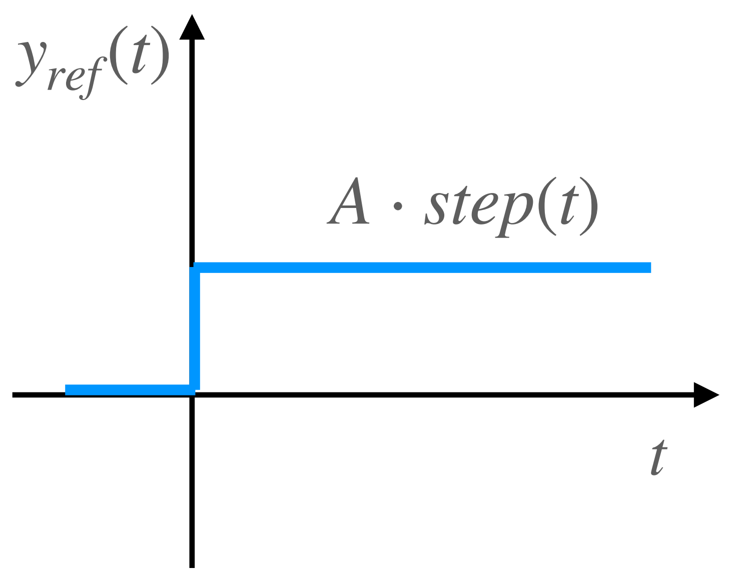
$$L(s) = \frac{100}{s+10} \rightarrow F(s) = \frac{\frac{100}{s+10}}{1 + \frac{100}{s+10}} = \frac{100}{s+110}$$

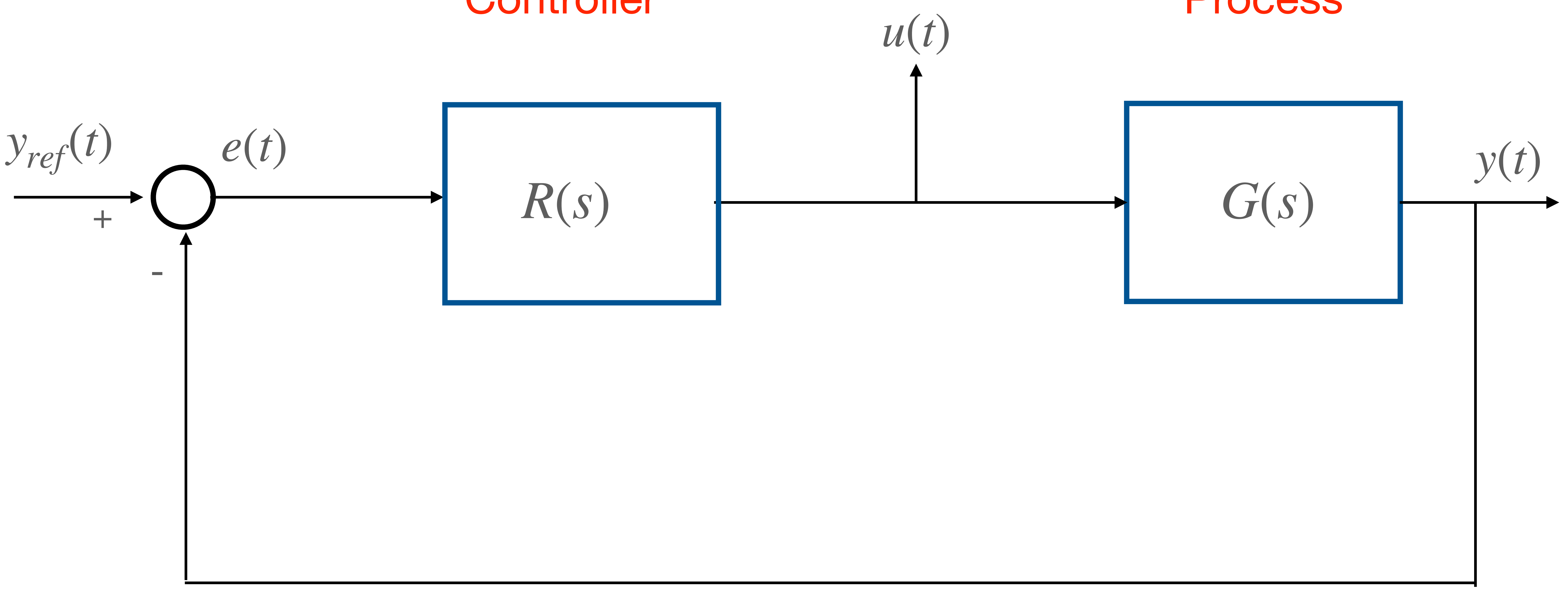
$$y_{ref} \rightarrow y : F(s) = \frac{R(s)G(s)}{1 + R(s)G(s)} = \frac{L(s)}{1 + L(s)}$$

$$y \approx y_{ref} : \quad F(0) = 1$$

$\longrightarrow$ Impossible if $L(s)$ is not at least a TYPE 1 transfer function

Basic Control Scheme: Example





$$L(s) = \frac{100}{s+10} \rightarrow F(s) = \frac{\frac{100}{s+10}}{1+\frac{100}{s+10}} = \frac{100}{s+110}$$

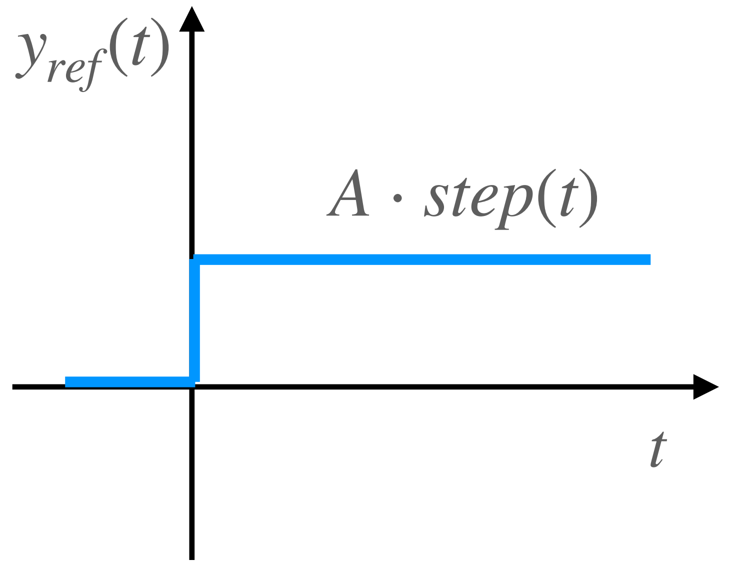
$$lim_{s \to 0} F(s) = \frac{100}{110} = \frac{10}{11} \neq 1$$

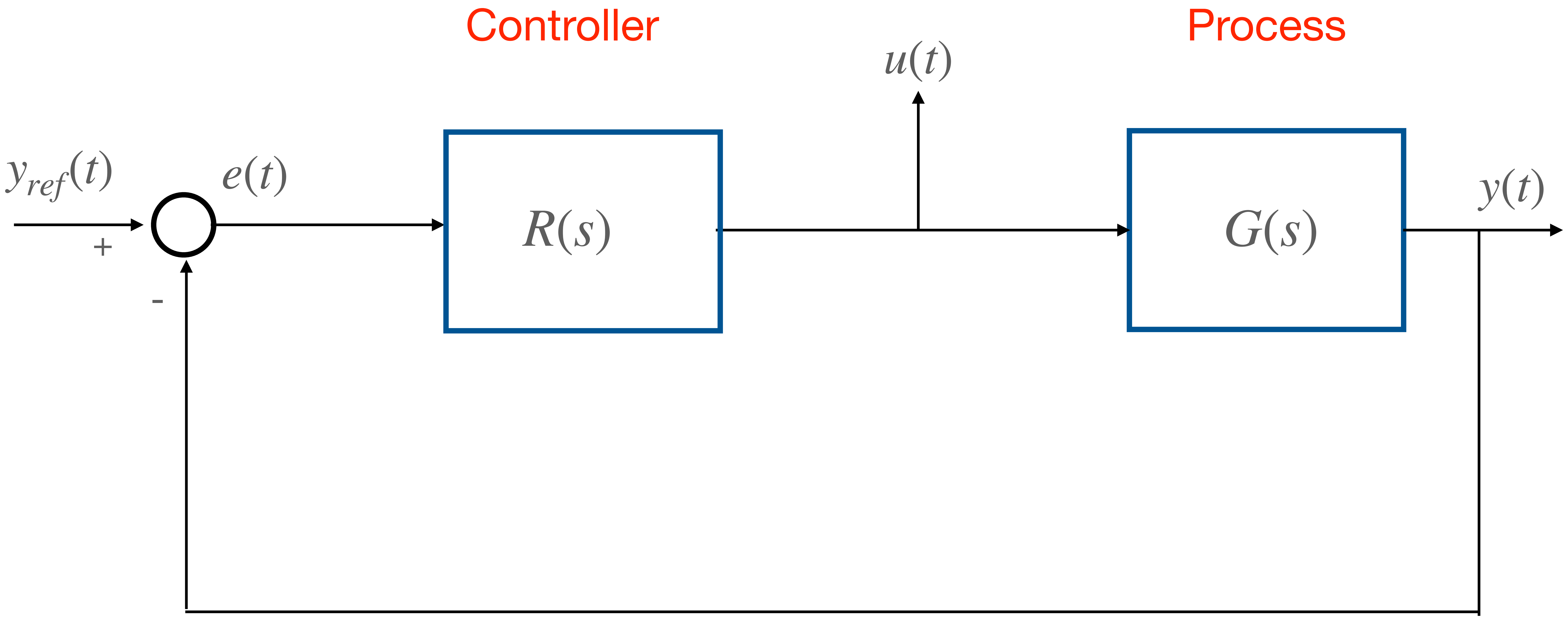
$$y_{ref} \rightarrow y : F(s) = \frac{R(s)G(s)}{1+R(s)G(s)} = \frac{L(s)}{1+L(s)}$$

$$y \approx y_{ref} : \quad F(0) = 1$$

$\longrightarrow$ Impossible if $L(s)$ is not at least a TYPE 1 transfer function

Basic Control Scheme: Example





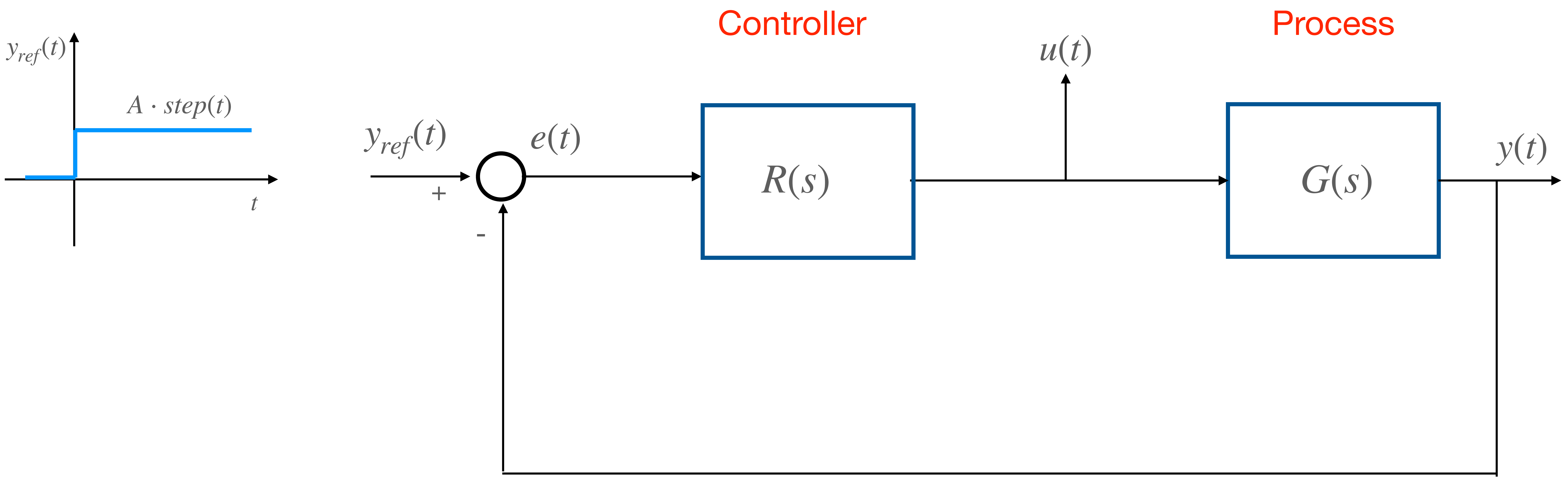
$$L(s) = \frac{100}{s(s+10)} \rightarrow F(s) = \frac{\frac{100}{s(s+10)}}{1 + \frac{100}{s(s+10)}} = \frac{100}{s(s+10)+100}$$

$$y_{ref} \rightarrow y : F(s) = \frac{R(s)G(s)}{1+R(s)G(s)} = \frac{L(s)}{1+L(s)}$$

$$y \approx y_{ref} : \quad F(0) = 1$$

Impossible if $L(s)$ is not at least a TYPE 1 transfer function

Basic Control Scheme: Example



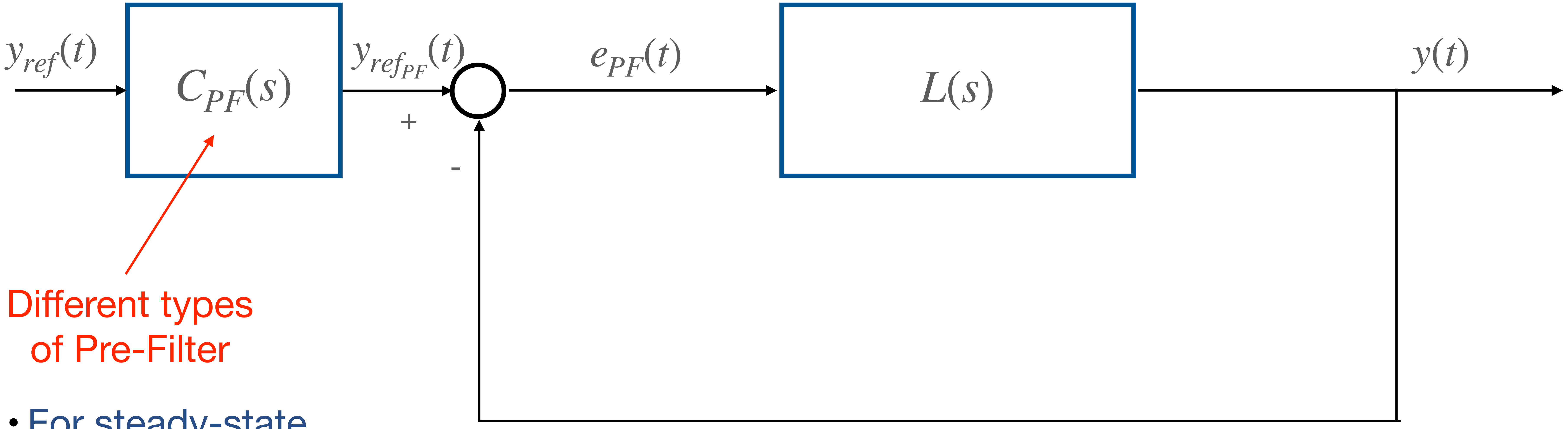
$$L(s) = \frac{100}{s(s+10)} \rightarrow F(s) = \frac{\frac{100}{s(s+10)}}{1 + \frac{100}{s(s+10)}} = \frac{100}{s(s+10)+100}$$

$$lim_{s \to 0} F(s) = \frac{100}{100} = 1$$

$$y_{ref} \rightarrow y : F(s) = \frac{R(s)G(s)}{1+R(s)G(s)} = \frac{L(s)}{1+L(s)}$$

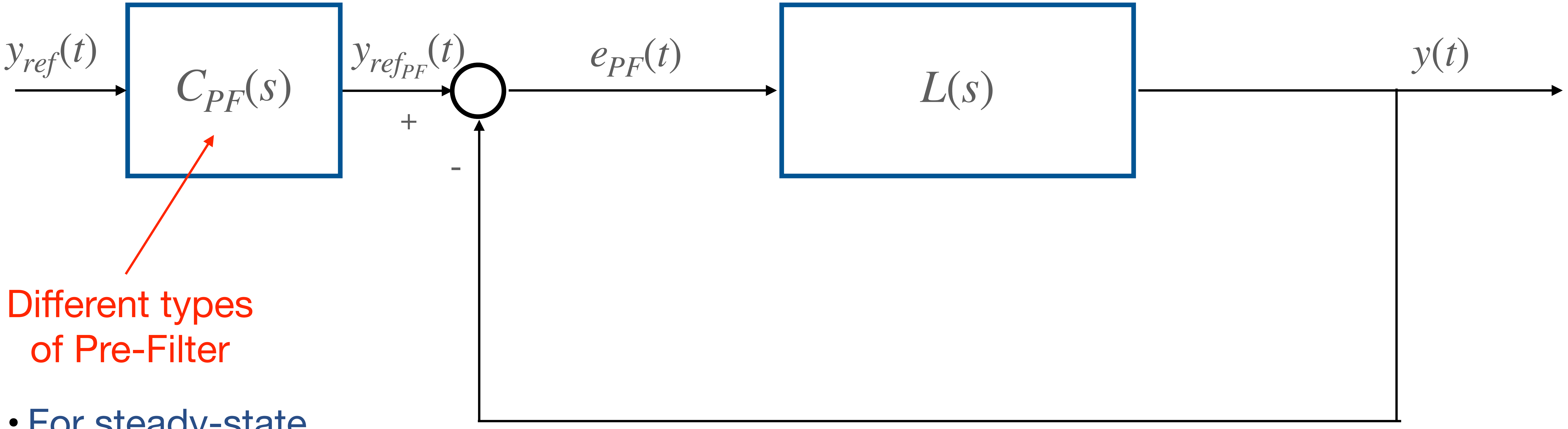
$y \approx y_{ref}$: $F(0) = 1$ $\longrightarrow$ Impossible if $L(s)$ is not at least a TYPE 1 transfer function

Pre-filter Based Control Scheme: Case 3



- For steady-state compensation

Pre-filter Based Control Scheme: Case 3



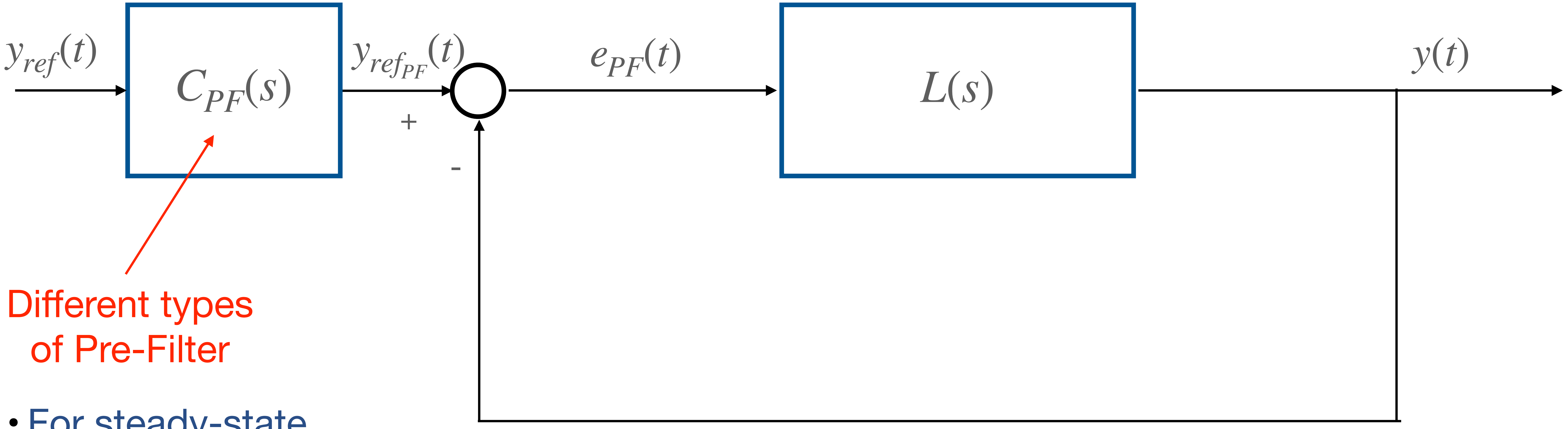
Assumptions:

- $C_{PF}(s)$ As. Stable
- Proper
- ~~Unitary gain~~

Different types of Pre-Filter

- For steady-state compensation

Pre-filter Based Control Scheme: Case 3



Different types of Pre-Filter

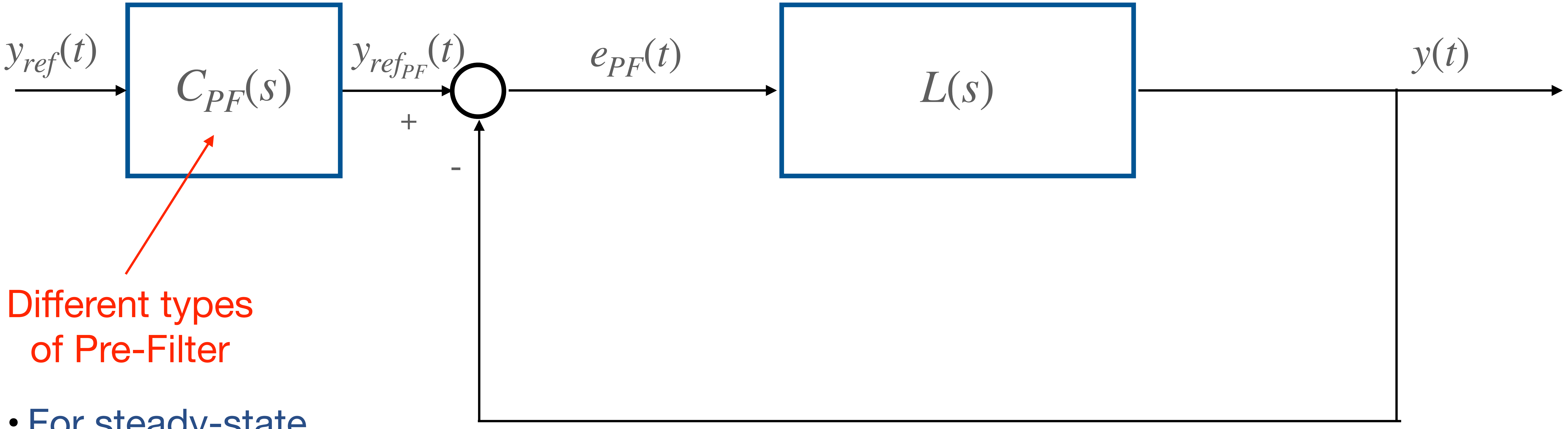
- For steady-state compensation

Assumptions:

- $C_{PF}(s)$ As. Stable
- Proper
- ~~Unitary gain~~

Block Algebra Rule: $y_{ref} \rightarrow y : C_{PF}(s) \cdot \dfrac{L(s)}{1+L(s)} = C_{PF}(s) \cdot F(s)$

Pre-filter Based Control Scheme: Case 3



Different types of Pre-Filter

- For steady-state compensation

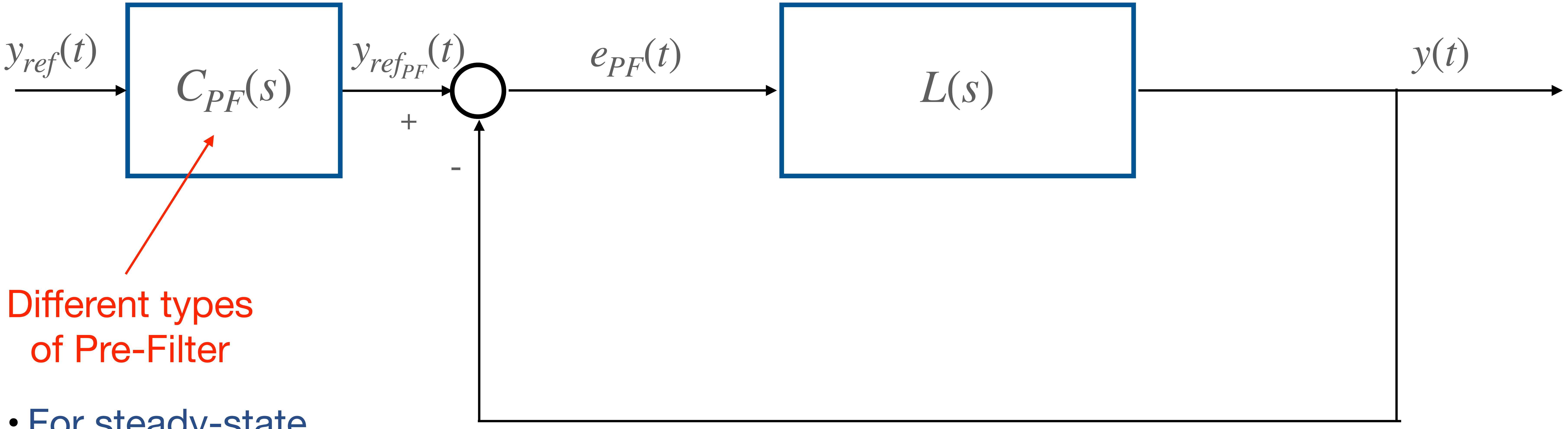
Assumptions:

- $C_{PF}(s)$ As. Stable
- Proper
- ~~Unitary gain~~

Block Algebra Rule: $y_{ref} \rightarrow y : C_{PF}(s) \cdot \dfrac{L(s)}{1+L(s)} = C_{PF}(s) \cdot F(s)$

$$y \approx y_{ref} : \qquad C_{PF}(0) \cdot F(0) = 1$$

Pre-filter Based Control Scheme: Case 3



Different types of Pre-Filter

- For steady-state compensation

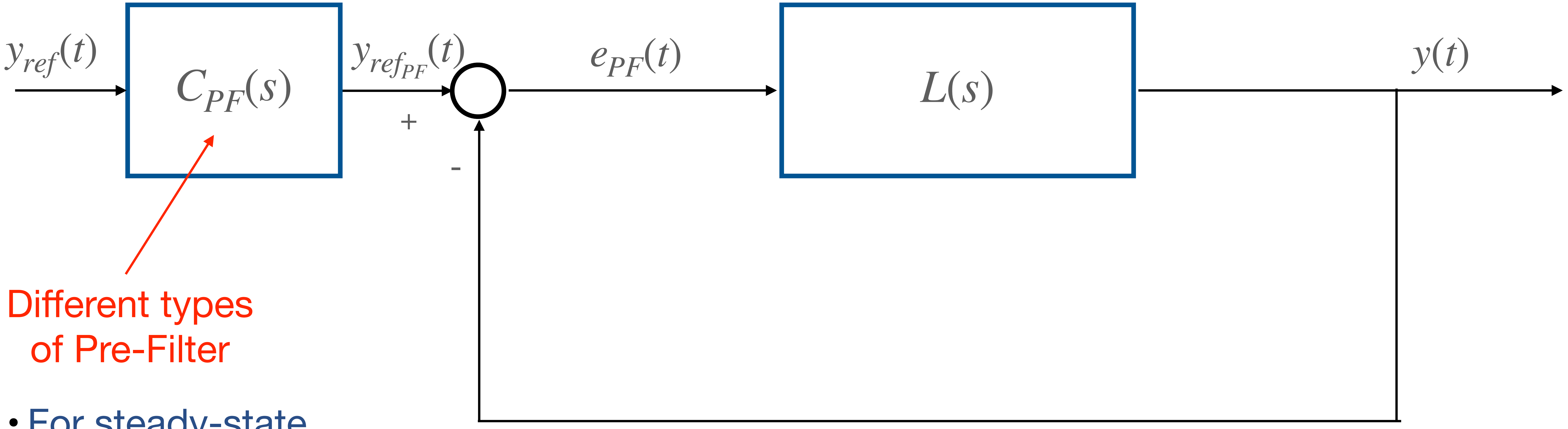
Assumptions:

- $C_{PF}(s)$ As. Stable
- Proper
- ~~Unitary gain~~

Block Algebra Rule: $y_{ref} \rightarrow y : C_{PF}(s) \cdot \dfrac{L(s)}{1+L(s)} = C_{PF}(s) \cdot F(s)$

$y \approx y_{ref} : \quad C_{PF}(0) \cdot F(0) = 1 \longrightarrow \boxed{C_{PF}(0) = \dfrac{1}{F(0)}}$

Pre-filter Based Control Scheme: Case 3



Assumptions:

- $C_{PF}(s)$ As. Stable
- Proper
- ~~Unitary gain~~

Different types of Pre-Filter

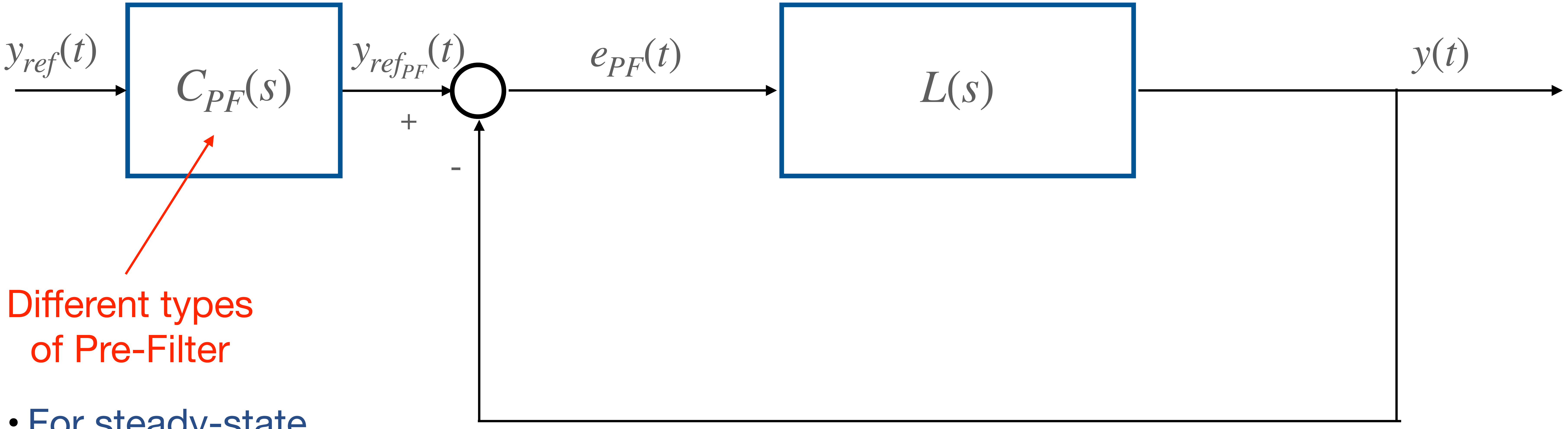
- For steady-state compensation

The filter could be just a constant gain

Block Algebra Rule: $y_{ref} \rightarrow y : C_{PF}(s) \cdot \dfrac{L(s)}{1 + L(s)} = C_{PF}(s) \cdot F(s)$

$y \approx y_{ref} : \quad C_{PF}(0) \cdot F(0) = 1 \longrightarrow \boxed{C_{PF}(0) = \dfrac{1}{F(0)}}$

Pre-filter Based Control Scheme: Case 3



Assumptions:

- $C_{PF}(s)$ As. Stable
- Proper
- ~~Unitary gain~~

Different types of Pre-Filter

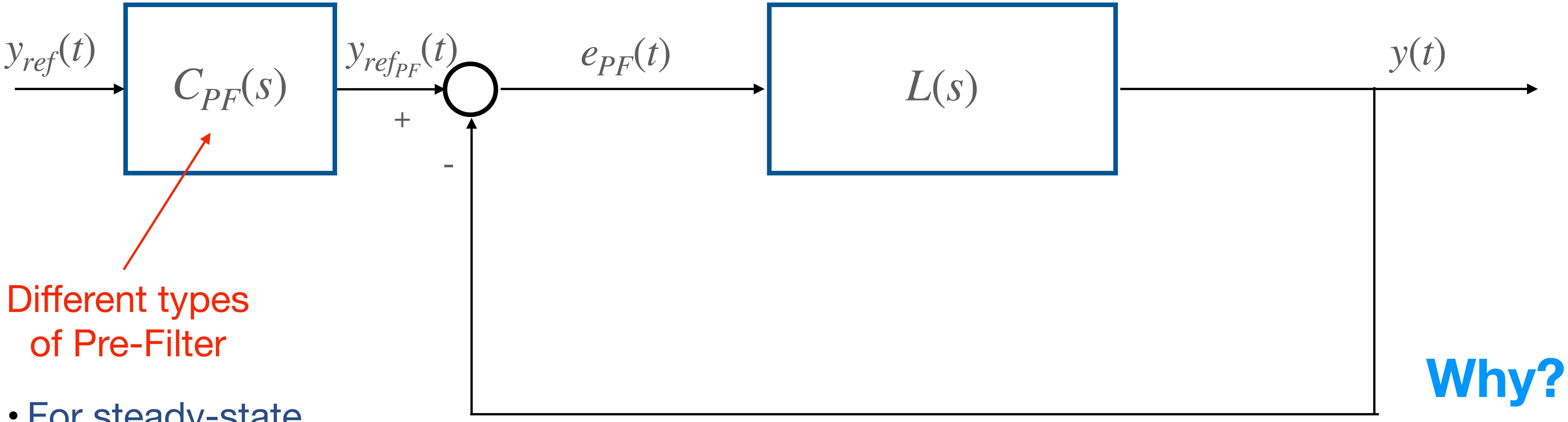
- For steady-state compensation

Warning:
Use only when strictly necessary!

Block Algebra Rule: $y_{ref} \rightarrow y : C_{PF}(s) \cdot \dfrac{L(s)}{1+L(s)} = C_{PF}(s) \cdot F(s)$

$y \approx y_{ref}$: $C_{PF}(0) \cdot F(0) = 1 \longrightarrow \boxed{C_{PF}(0) = \dfrac{1}{F(0)}}$

Pre-filter Based Control Scheme: Case 3



Assumptions:

- $C_{PF}(s)$ As. Stable
- Proper
- ~~Unitary gain~~

Different types of Pre-Filter

- For steady-state compensation

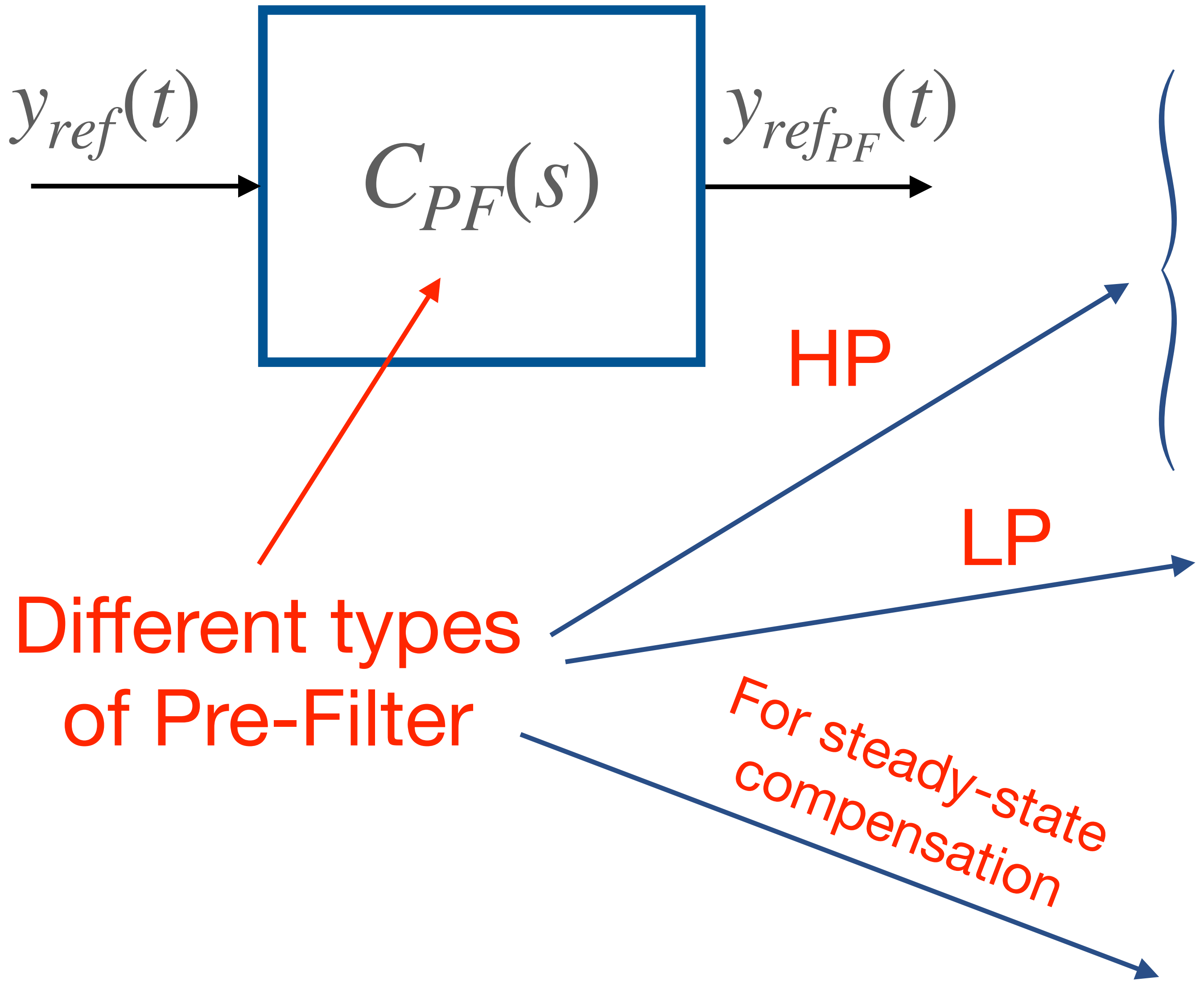
Why?

Warning:
Use only when strictly necessary!

Block Algebra Rule: $y_{ref} \rightarrow y : C_{PF}(s) \cdot \dfrac{L(s)}{1+L(s)} = C_{PF}(s) \cdot F(s)$

$y \approx y_{ref} : \quad C_{PF}(0) \cdot F(0) = 1 \longrightarrow \boxed{C_{PF}(0) = \dfrac{1}{F(0)}}$

Pre-filter Based Control Scheme: Summary

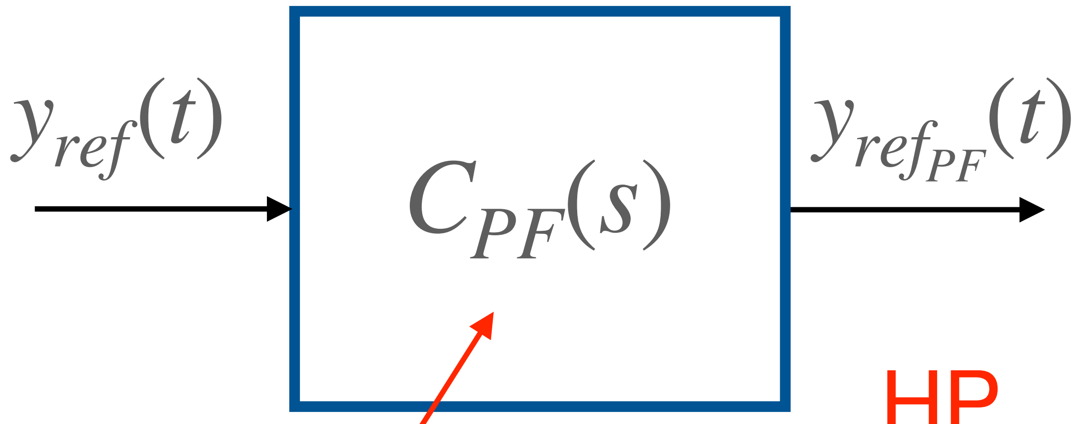


- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)
- Possible difficulties in enforcing steady-state precision with step reference signal via the classical approach (by introducing integrators in the direct path)

Pre-filter Based Control Scheme: Summary

Assumptions:

- $C_{PF}(s)$ As. Stable
- Proper
- Unitary gain



Different types of Pre-Filter

HP

- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays

LP

- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)

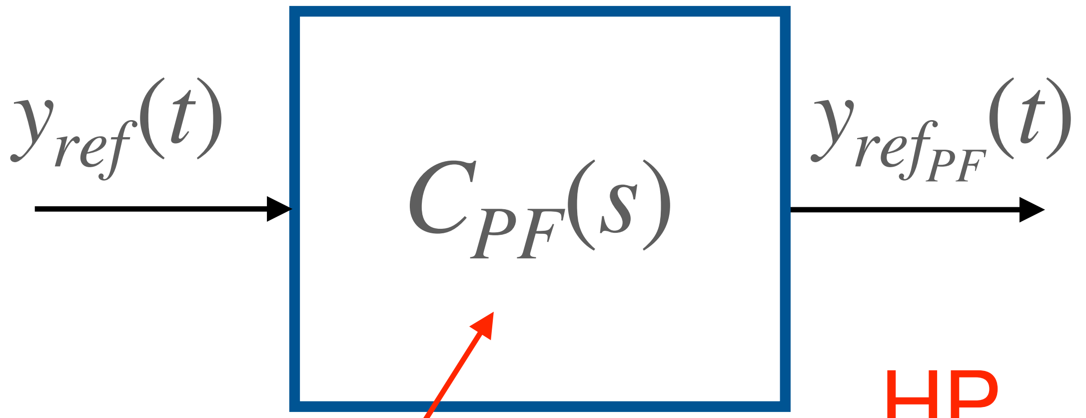
For steady-state compensation

- Possible difficulties in enforcing steady-state precision with step reference signal via the classical approach (by introducing integrators in the direct path)

Pre-filter Based Control Scheme: Summary

Assumptions:

- $C_{PF}(s)$ As. Stable
- Proper
- Unitary gain



HP

LP

Different types of Pre-Filter

For steady-state compensation

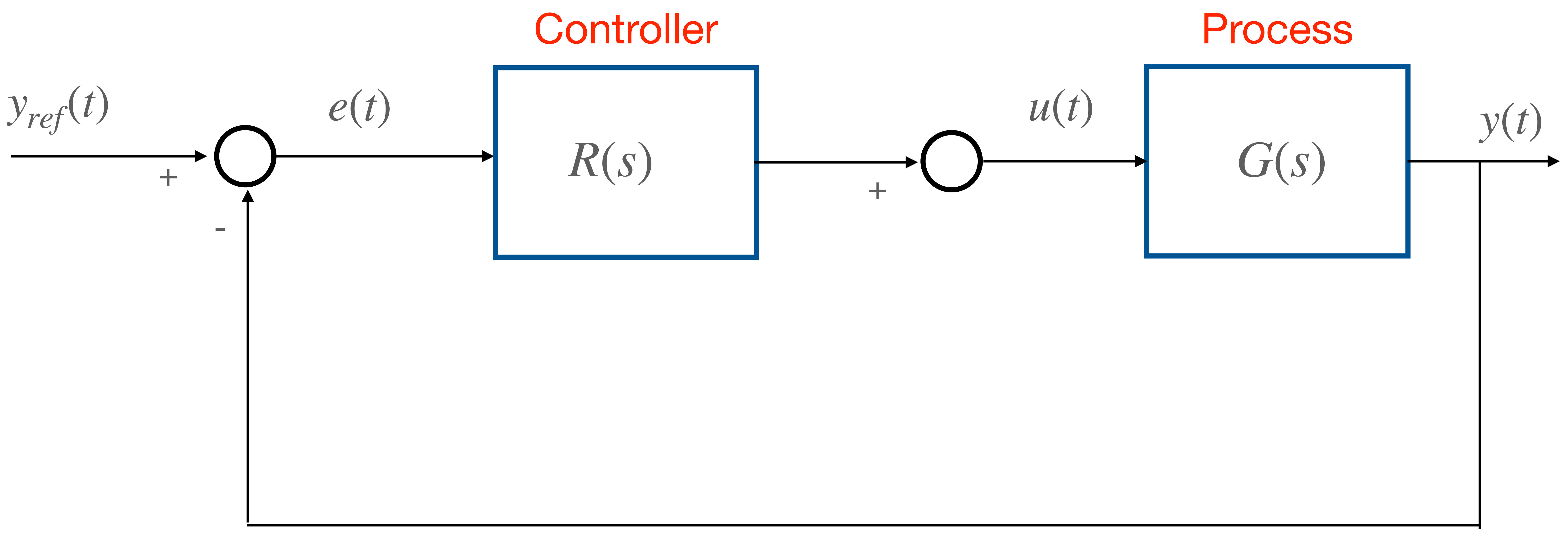
- Limited bandwidth in case of measurement disturbances
- Limited bandwidth in case of model uncertainties
- Limited bandwidth in case of delays
- No “control moderation” in case of reference signal characterized by sharp transitions in the waveform (abrupt transitions and near-vertical edges in the waveform)

- Possible difficulties in enforcing steady-state precision with step reference signal via the classical approach (by introducing integrators in the direct path)

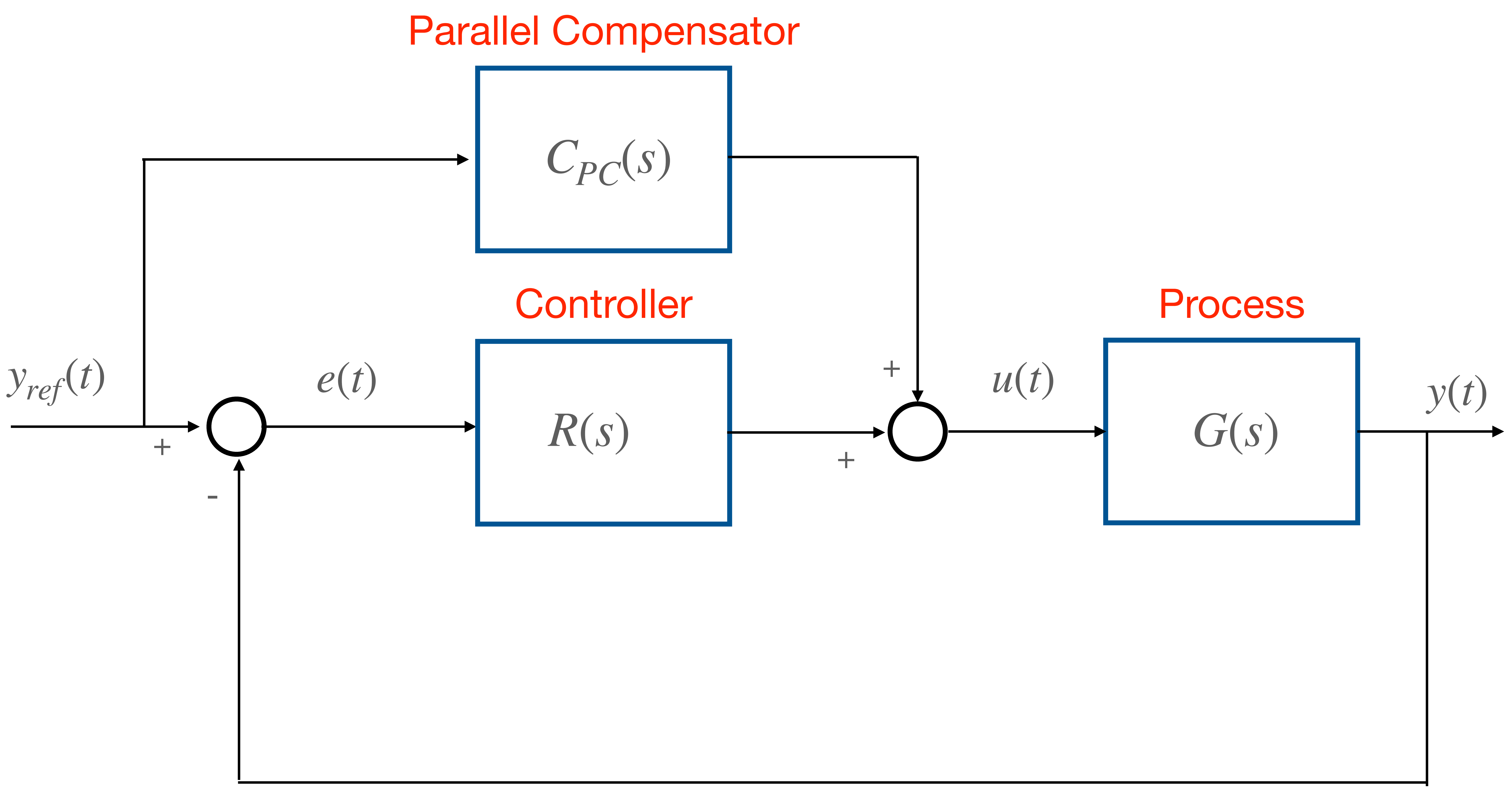
Assumptions:

- $C_{PF}(s)$ As. Stable
- Proper

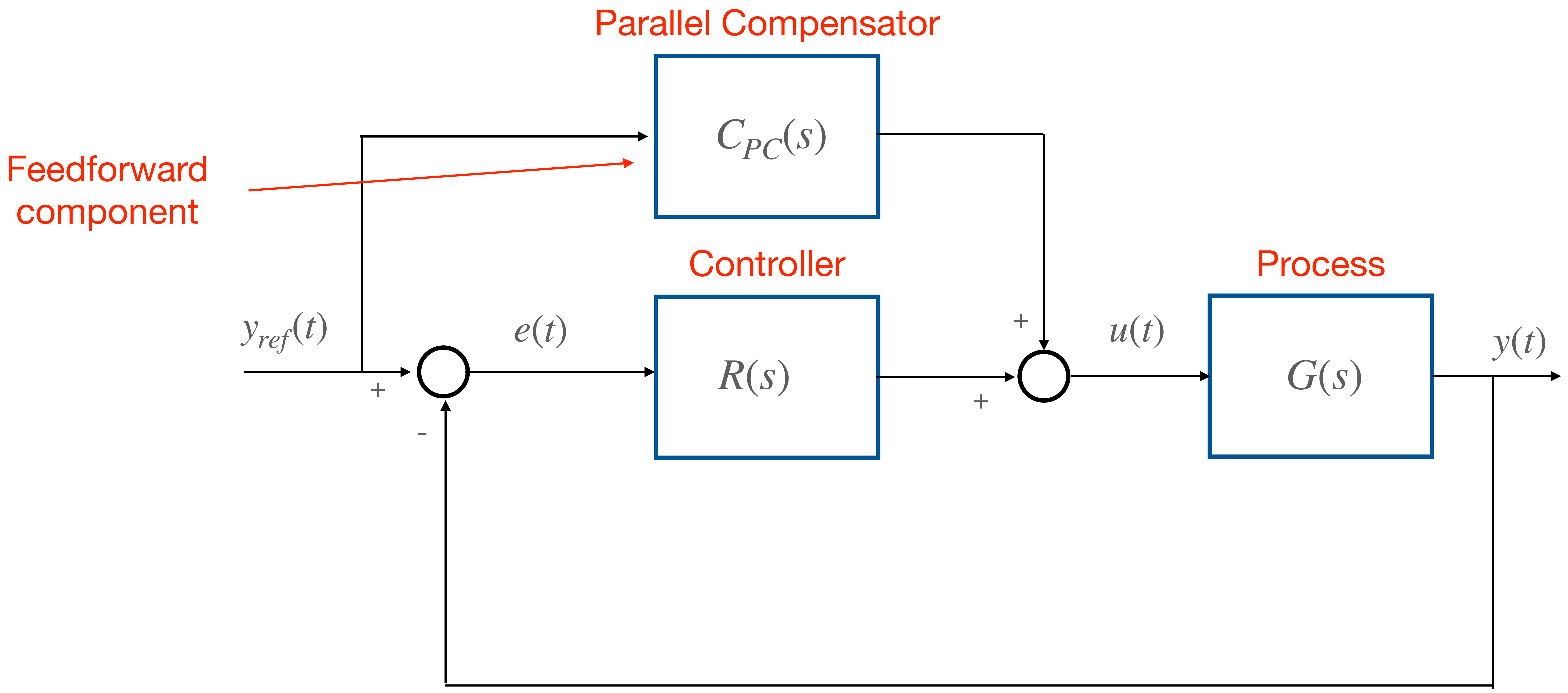
Parallel-compensator Based Control Scheme



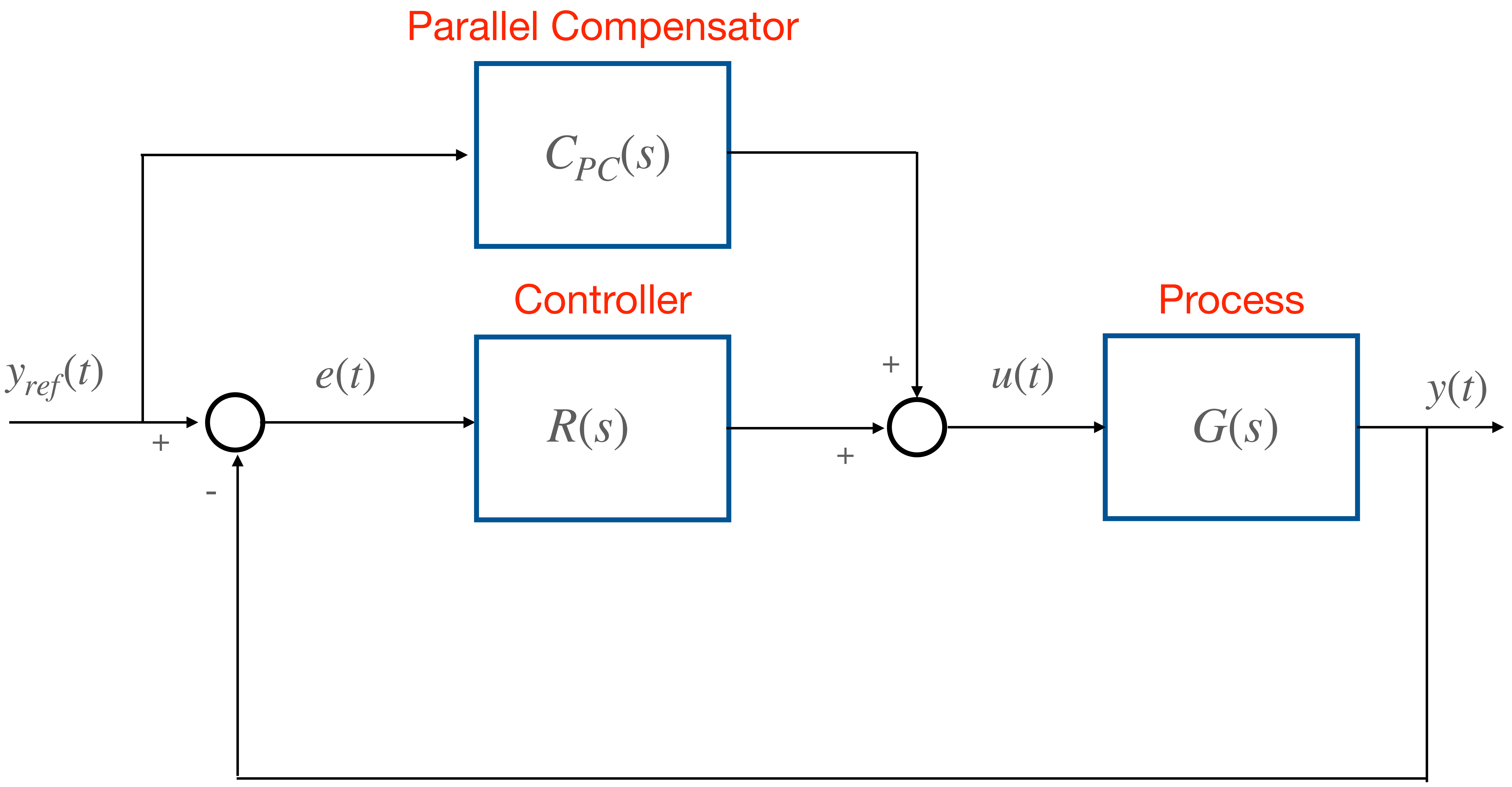
Parallel-compensator Based Control Scheme



Parallel-compensator Based Control Scheme

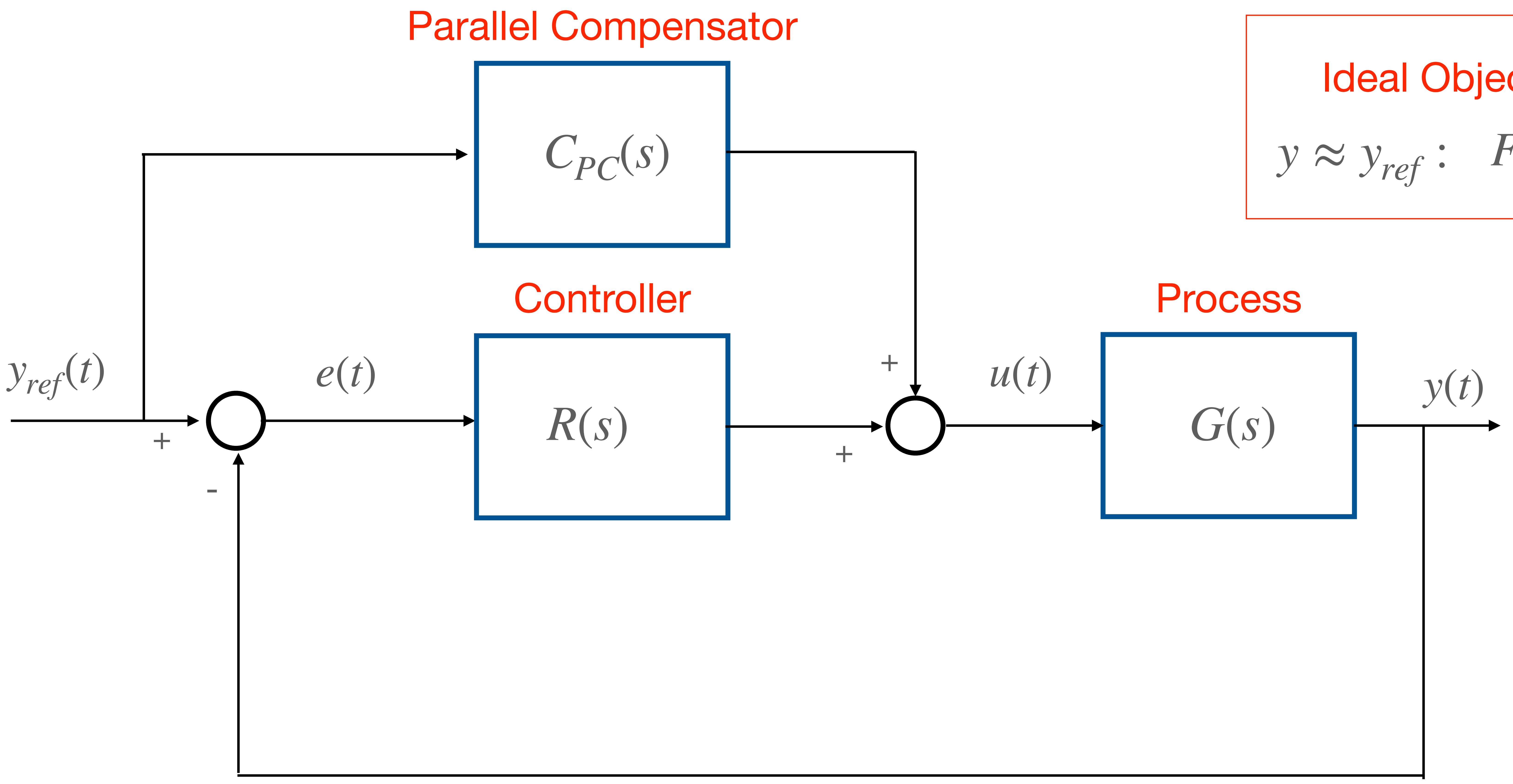


Parallel-compensator Based Control Scheme



$$y_{ref} \rightarrow y : F_{PC}(s) = \frac{C_{PC}(s)G(s) + R(s)G(s)}{1 + R(s)G(s)}$$

Parallel-compensator Based Control Scheme

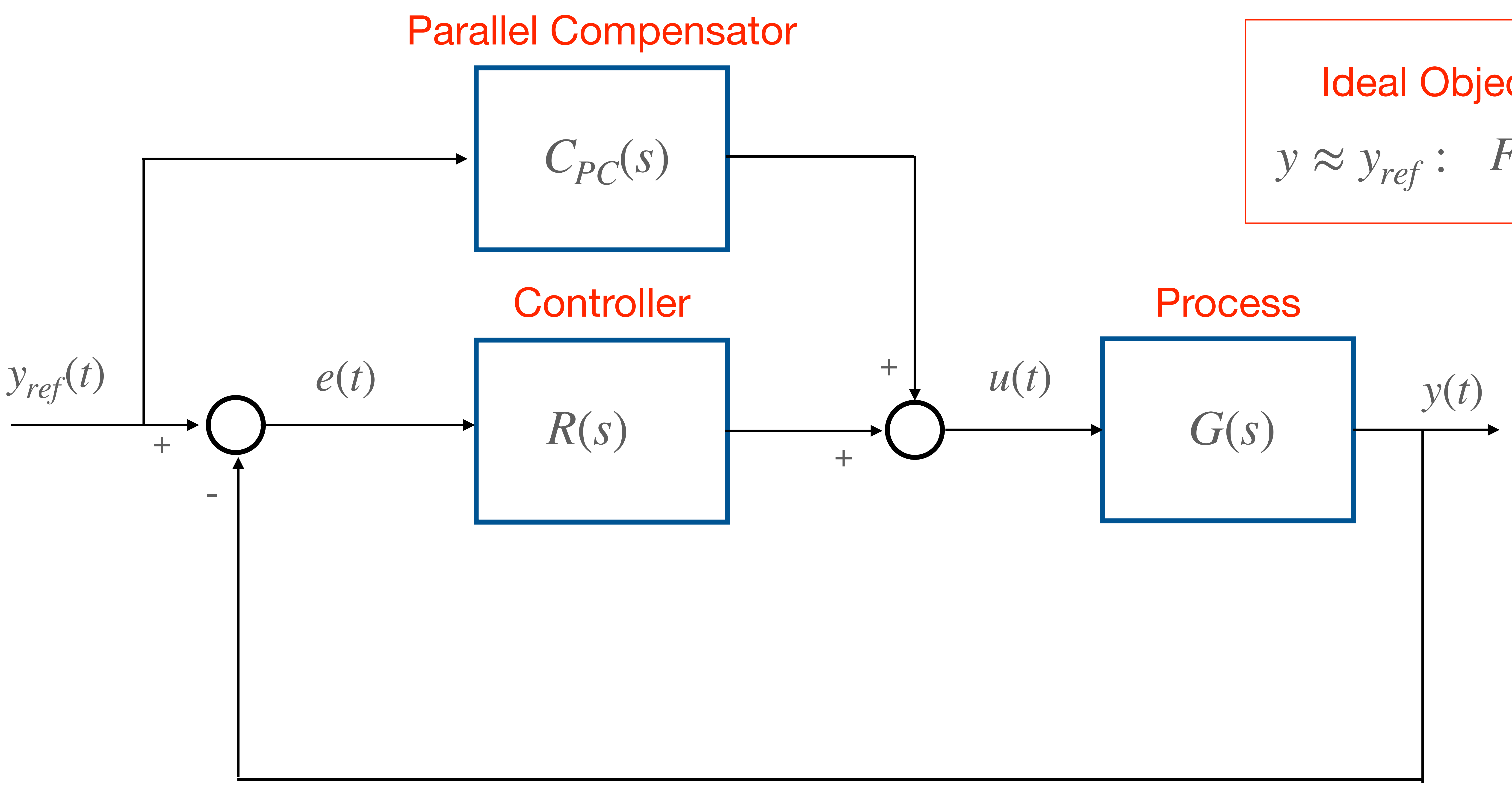


Ideal Objective

$y \approx y_{ref}: \quad F_{PC}(s) \approx 1$

$$y_{ref} \rightarrow y : F_{PC}(s) = \frac{C_{PC}(s)G(s) + R(s)G(s)}{1 + R(s)G(s)}$$

Parallel-compensator Based Control Scheme

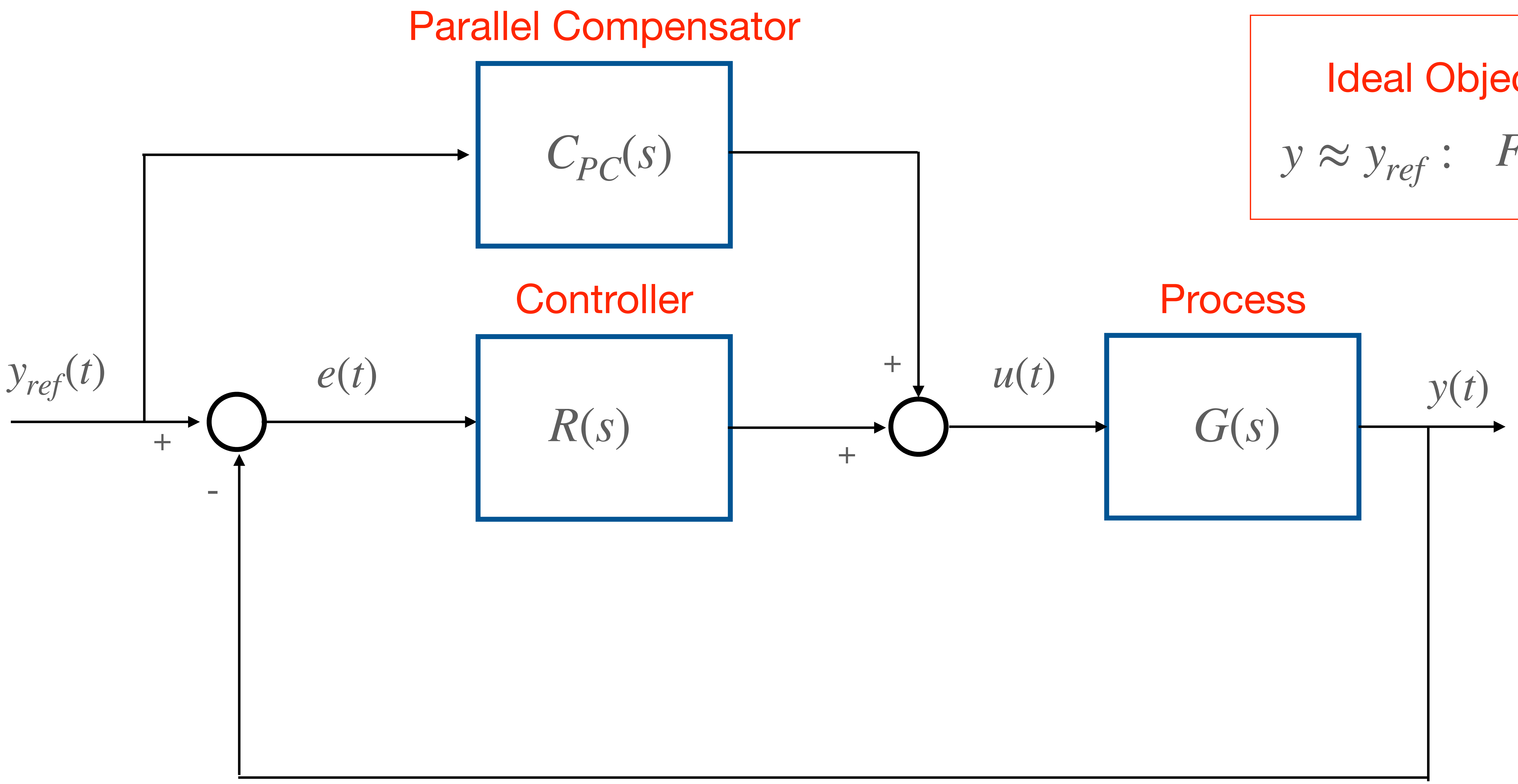


Ideal Objective

$y \approx y_{ref}$: $F_{PC}(s) \approx 1$

$$y_{ref} \to y : F_{PC}(s) = \frac{C_{PC}(s)G(s) + R(s)G(s)}{1 + R(s)G(s)} \longrightarrow C_{PC}(s)G(s) = 1$$

Parallel-compensator Based Control Scheme



Ideal Objective

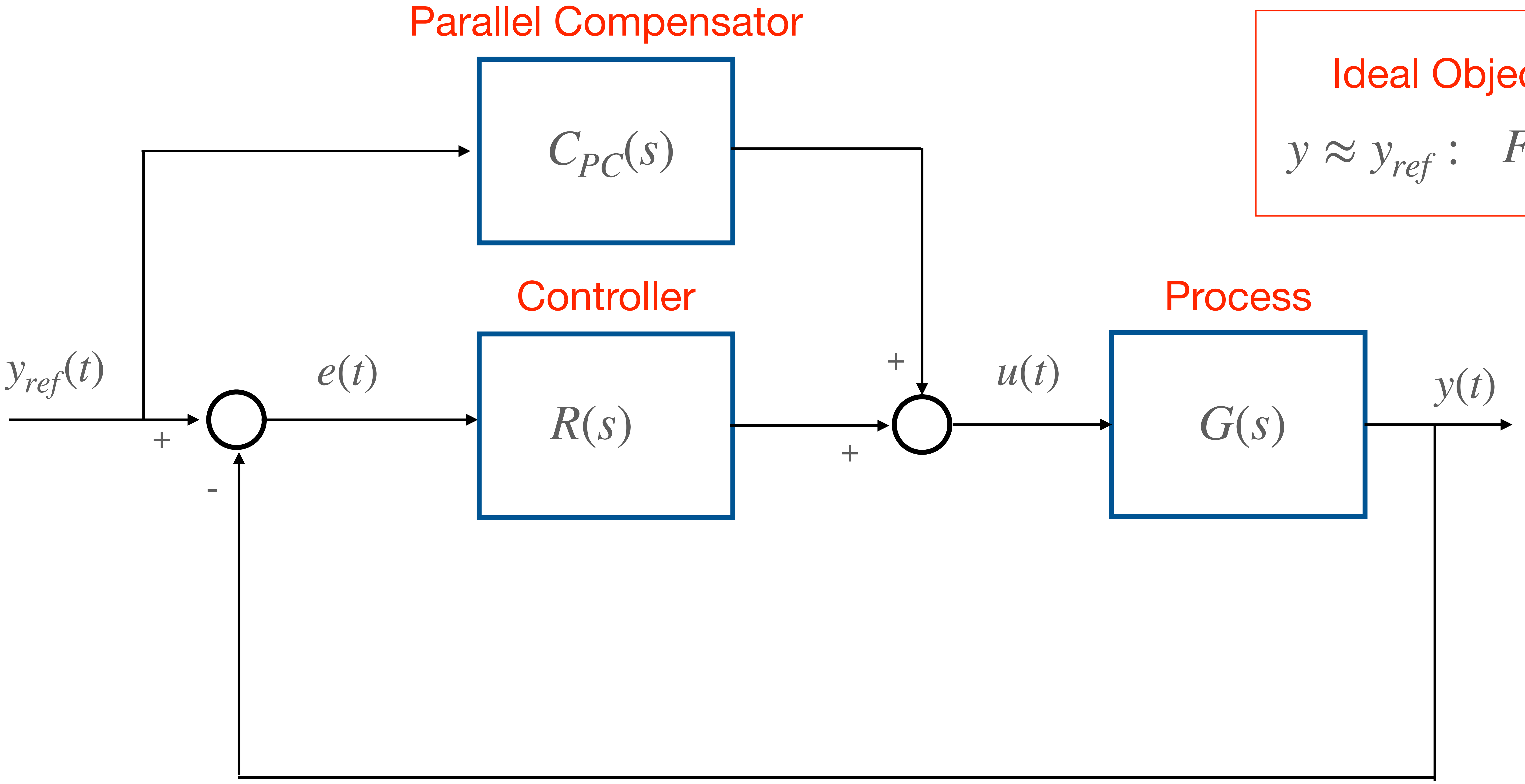
$y \approx y_{ref}: \quad F_{PC}(s) \approx 1$

$$y_{ref} \rightarrow y : F_{PC}(s) = \frac{C_{PC}(s)G(s) + R(s)G(s)}{1 + R(s)G(s)} \longrightarrow C_{PC}(s)G(s) = 1 \longrightarrow C_{PC}(s) = \frac{1}{G(s)}$$

Ideal Design

$$C_{PC}(s) = \frac{1}{G(s)}$$

Parallel-compensator Based Control Scheme



Ideal Objective

$y \approx y_{ref}$: $F_{PC}(s) \approx 1$

Warning:

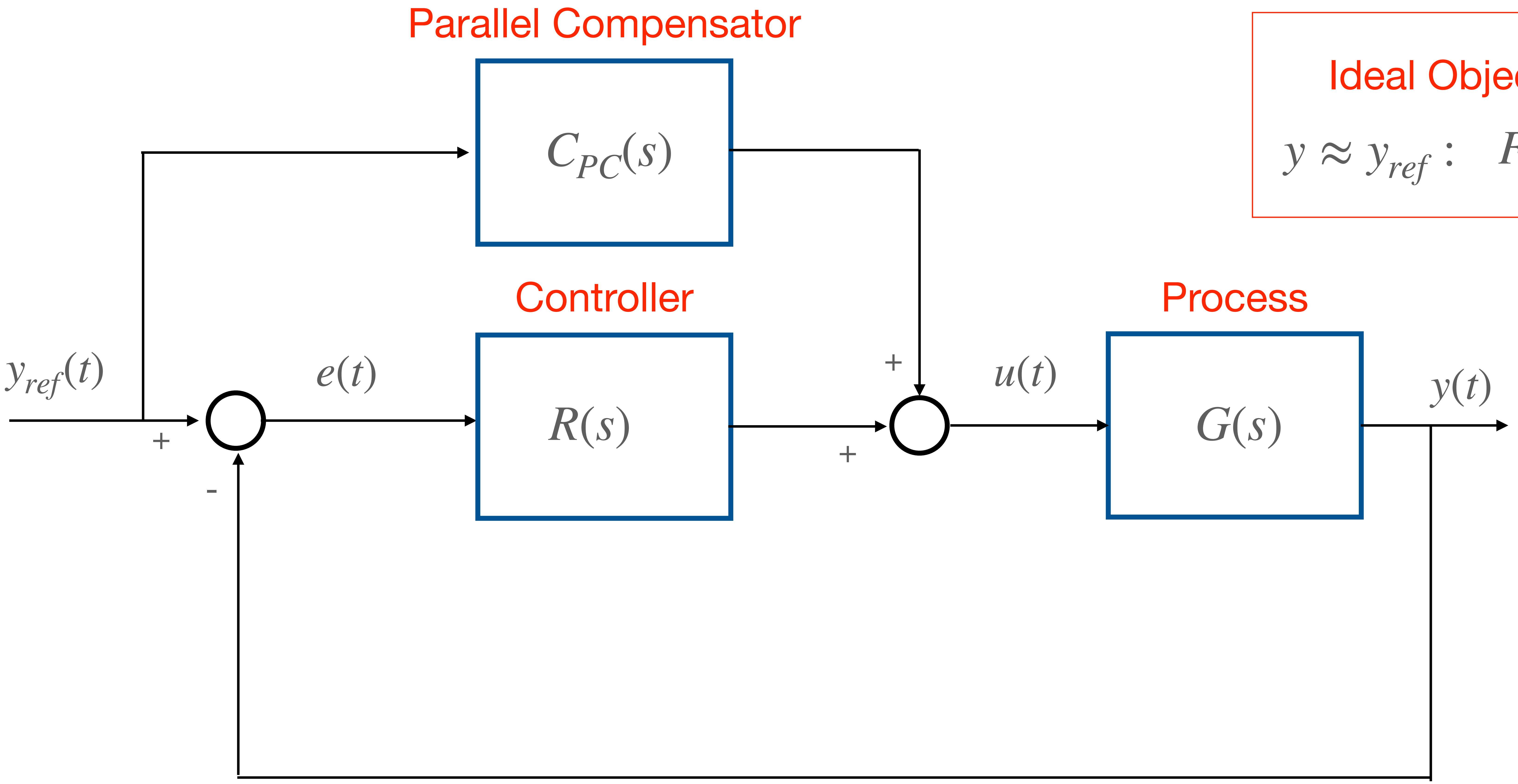
- $C_{PC}(s)$ As. Stable
- Causal

$$y_{ref} \to y : F_{PC}(s) = \frac{C_{PC}(s)G(s) + R(s)G(s)}{1 + R(s)G(s)} \longrightarrow C_{PC}(s)G(s) = 1 \longrightarrow$$

Ideal Design

$$C_{PC}(s) = \frac{1}{G(s)}$$

Parallel-compensator Based Control Scheme



Ideal Objective

$$y \approx y_{ref} : \quad F_{PC}(s) \approx 1$$

Warning:

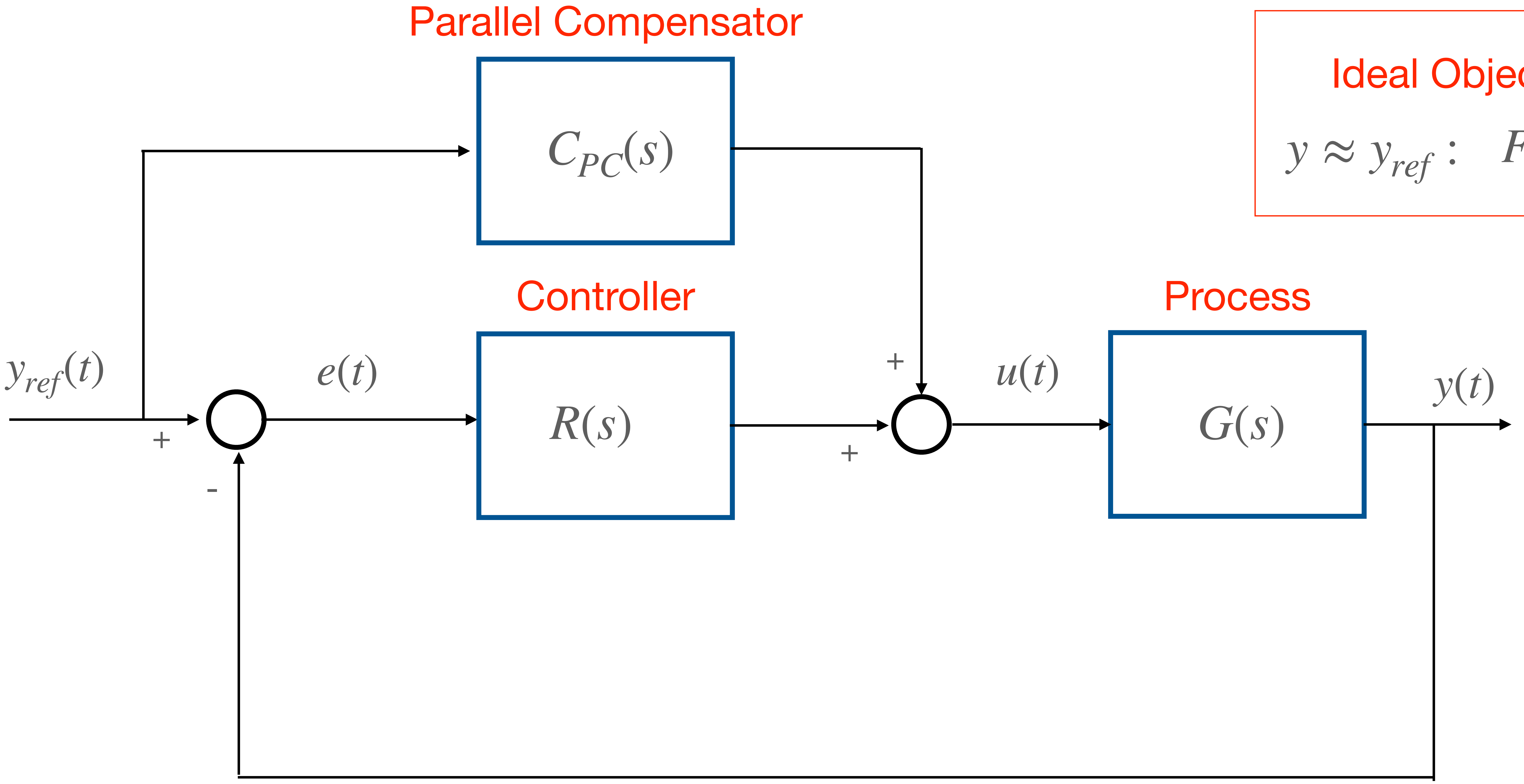
- $C_{PC}(s)$ $\boxed{\text{As. Stable}}$ → not guaranteed
- ~~Causal~~

$$y_{ref} \to y : F_{PC}(s) = \frac{C_{PC}(s)G(s) + R(s)G(s)}{1 + R(s)G(s)} \longrightarrow C_{PC}(s)G(s) = 1 \longrightarrow$$

Ideal Design

$$C_{PC}(s) = \frac{1}{G(s)}$$

Parallel-compensator Based Control Scheme



Ideal Objective

$y \approx y_{ref}: \quad F_{PC}(s) \approx 1$

not guaranteed

Warning:

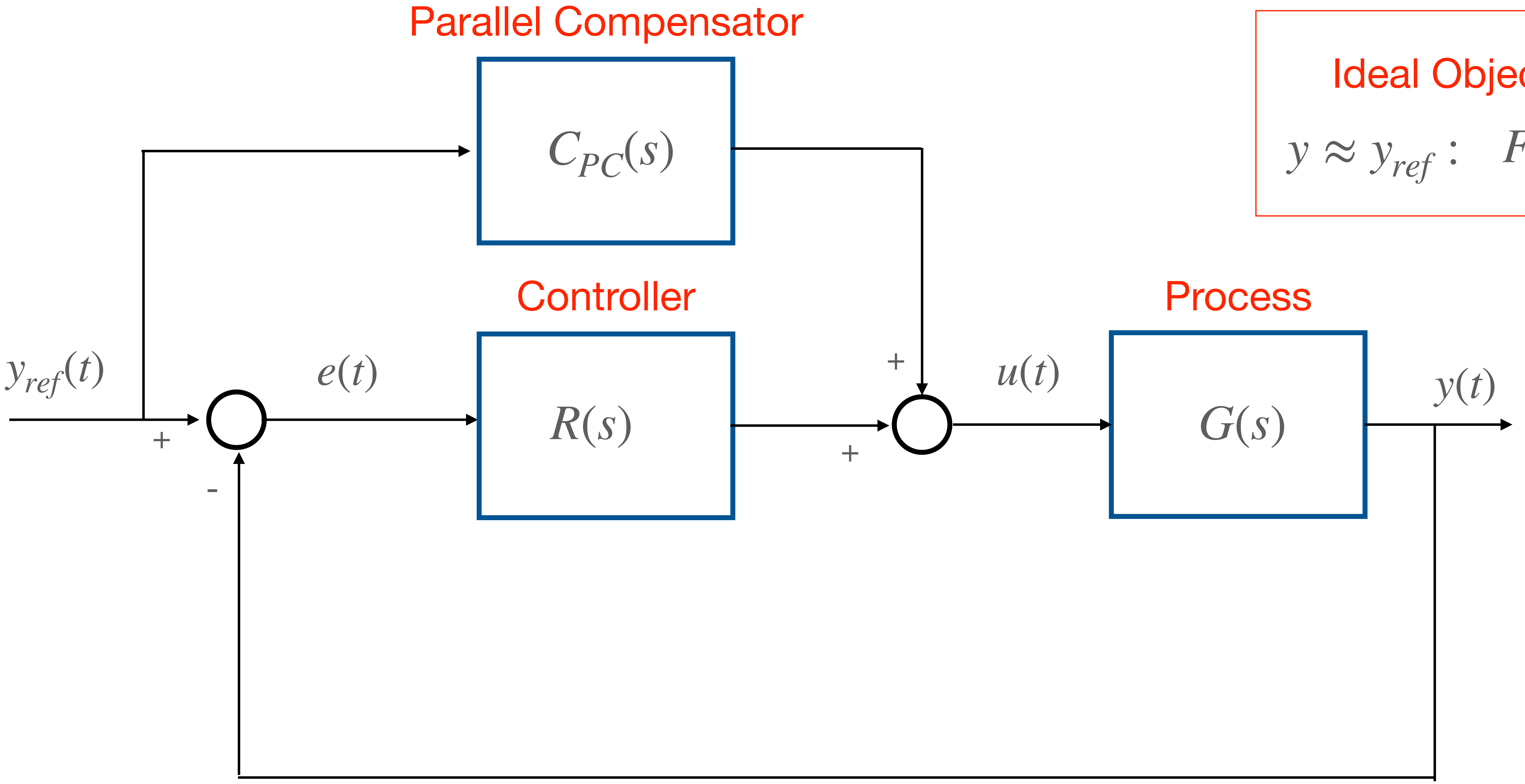
- $C_{PC}(s)$ As. Stable
- ~~Causal~~

Actual Design

$$C_{PC}(j\omega) = \frac{1}{G(j\omega)}$$

in the frequency range where $y_{ref}(t)$ has significant harmonics

Parallel-compensator Based Control Scheme



Ideal Objective

$y \approx y_{ref}$: $F_{PC}(s) \approx 1$

not guaranteed

Warning:

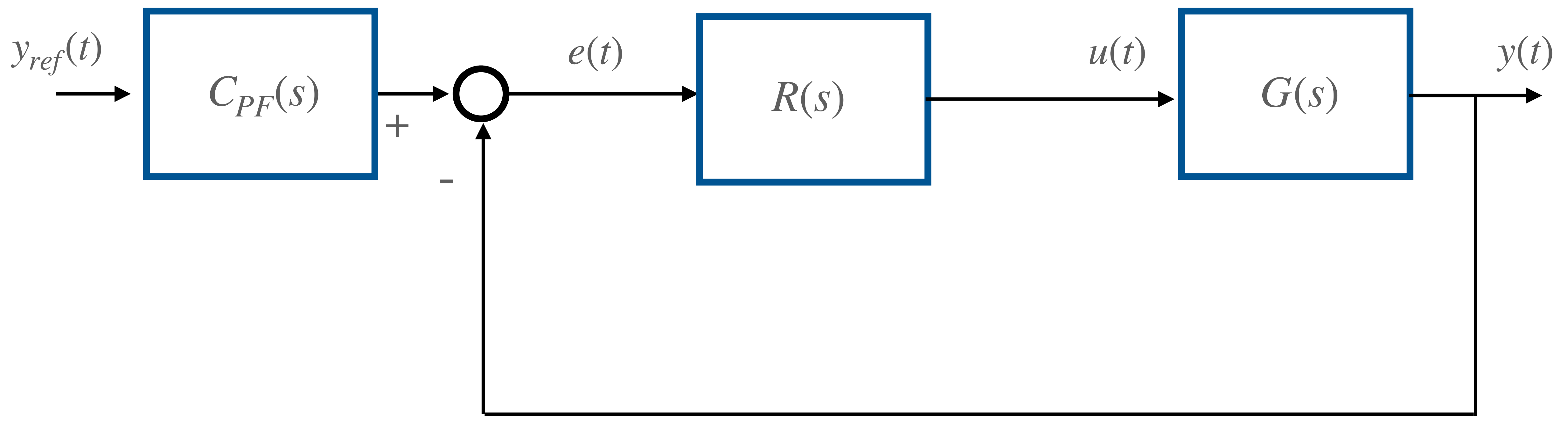
- $C_{PC}(s)$ $\boxed{\text{As. Stable}}$
- ~~Causal~~

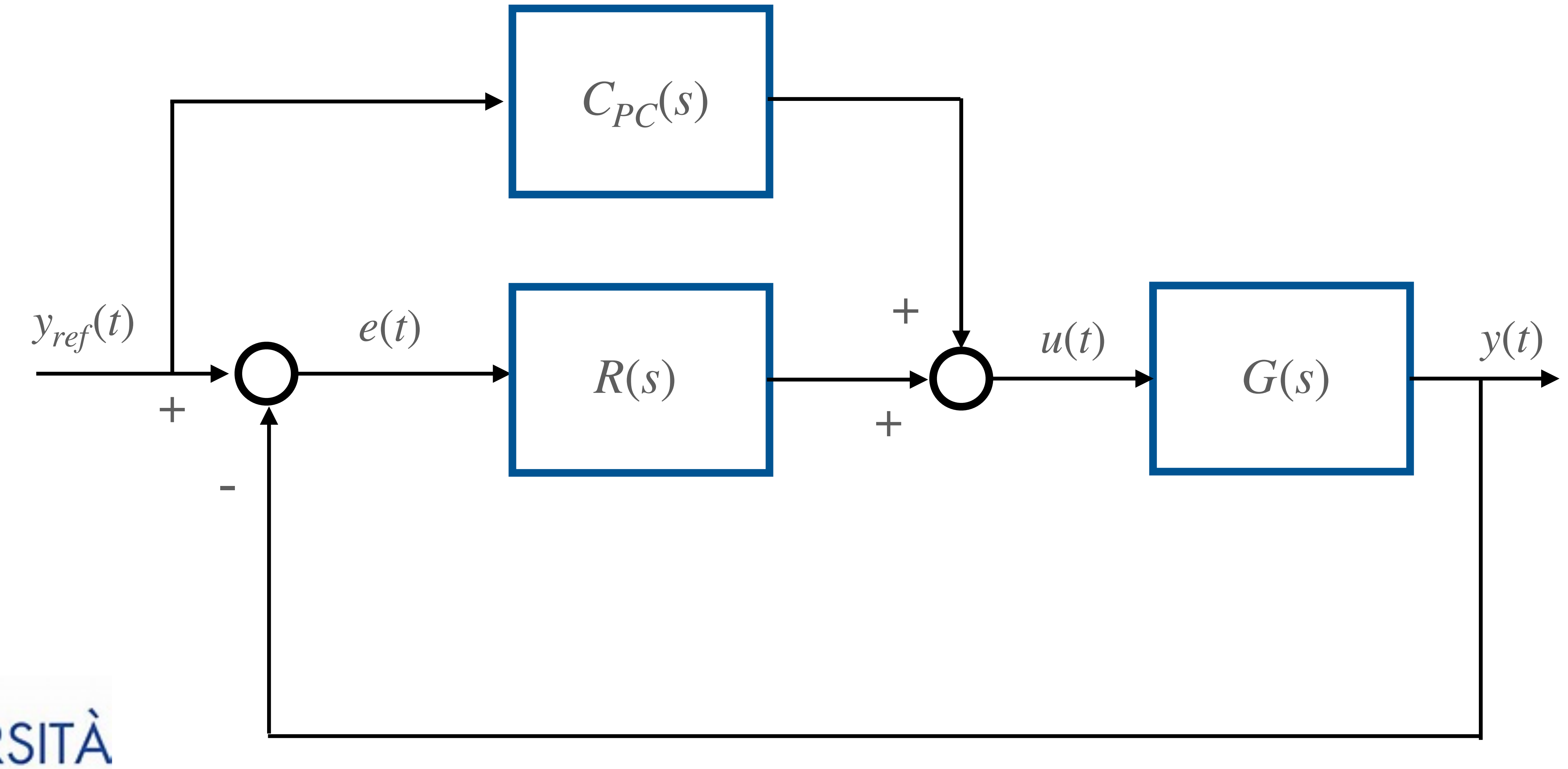
Actual Design

$$C_{PC}(j\omega) = \frac{1}{G(j\omega)}$$

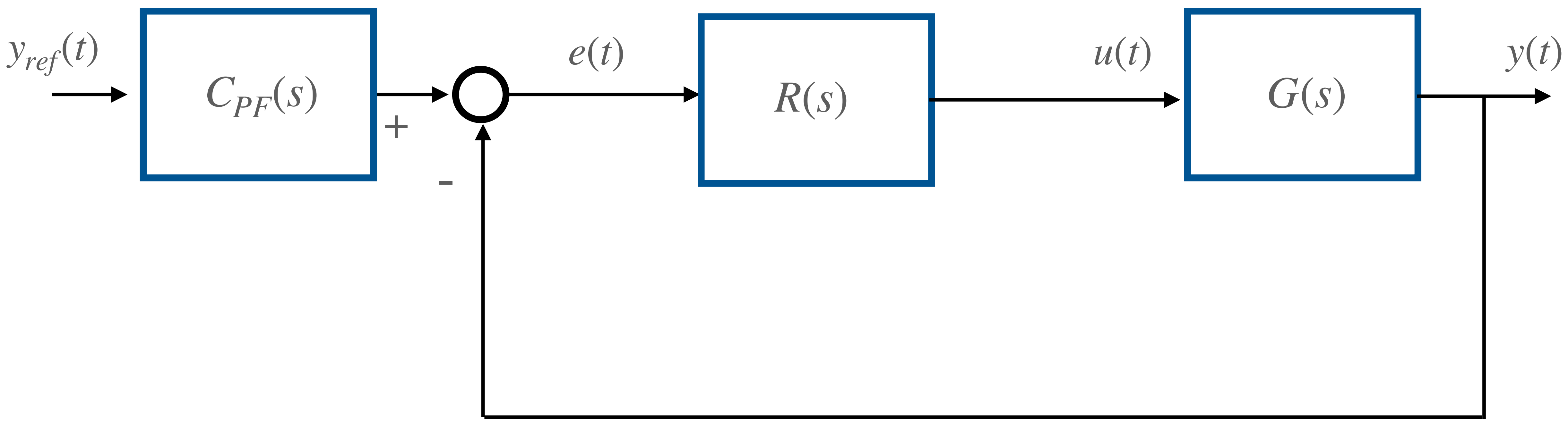
$\forall\, \omega \in [0, \omega_{BW_{CL}})$ ← in the frequency range where $y_{ref}(t)$ has significant harmonics ←

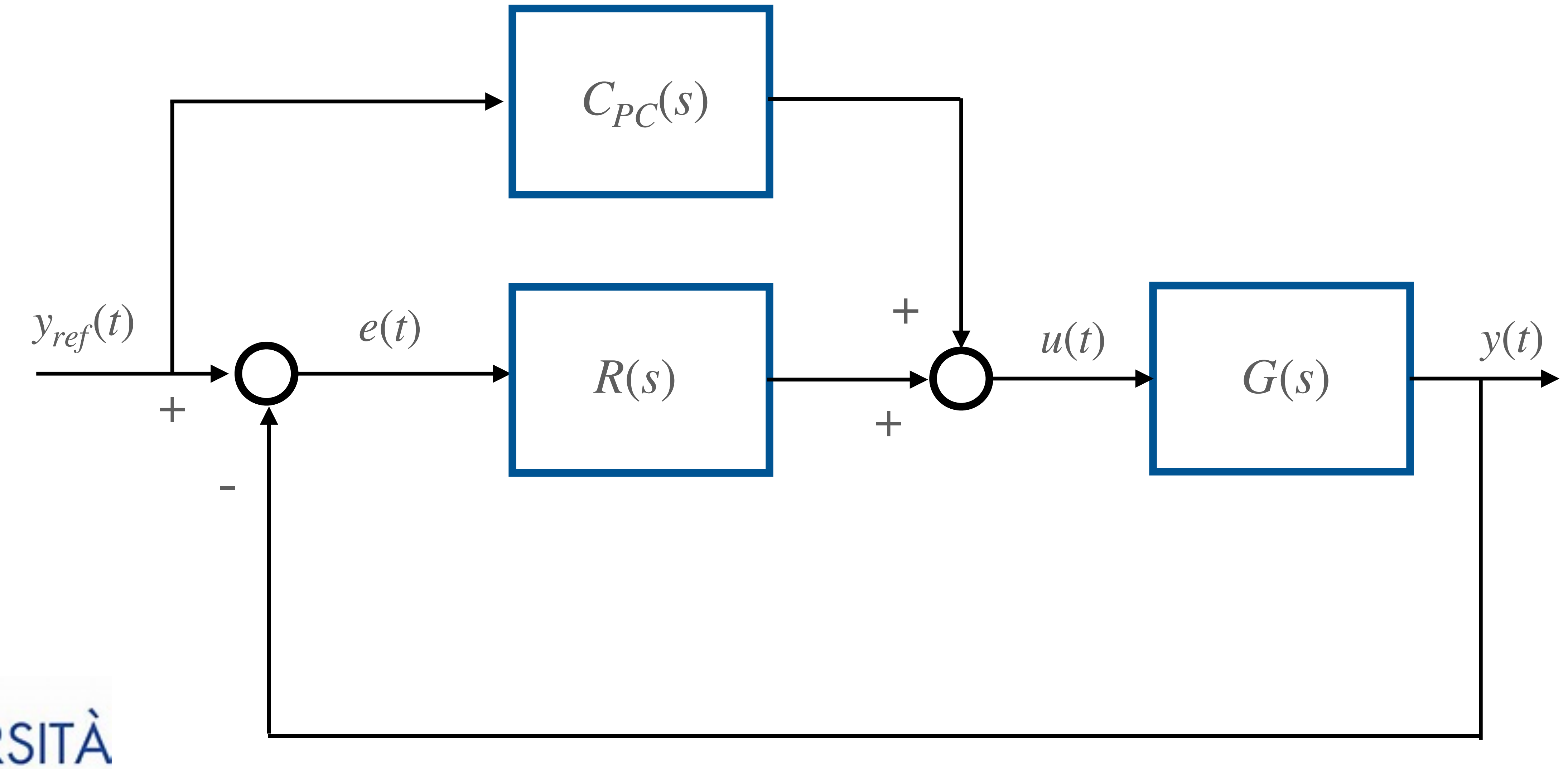
Prefilter-based and Parallel-compensator Based Control Schemes: 2DoF Control Schemes



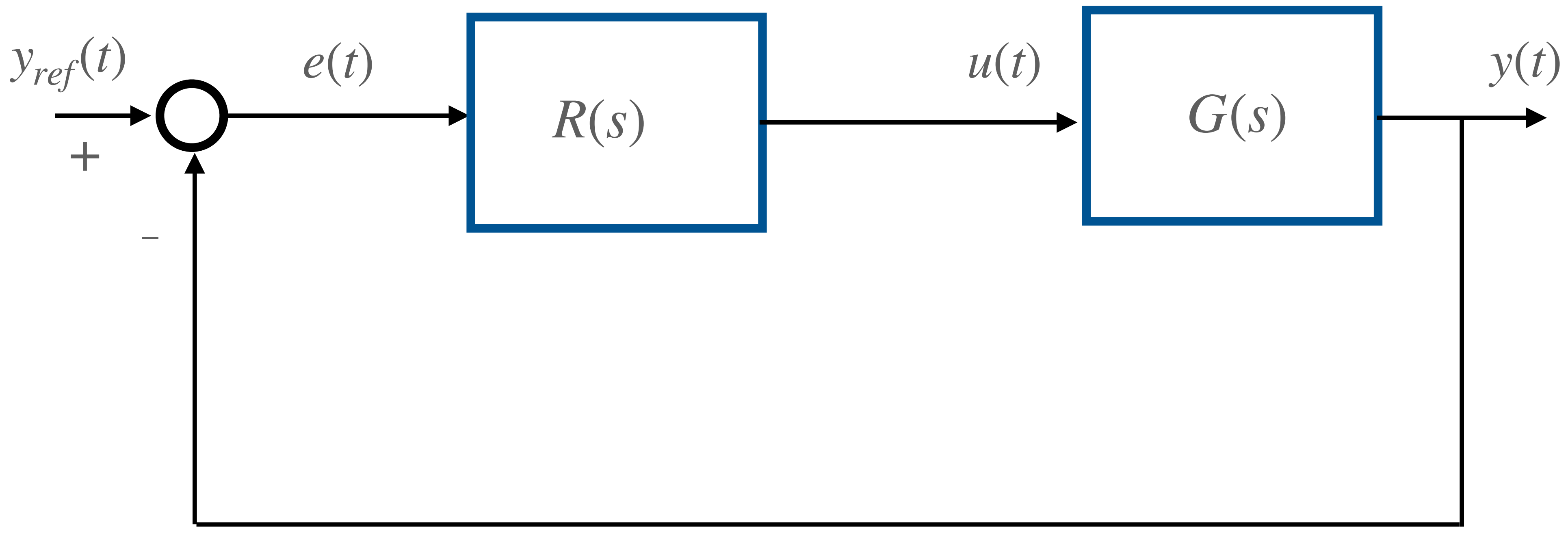


Prefilter-based and Parallel-compensator Based Control Schemes: 2DoF Control Schemes

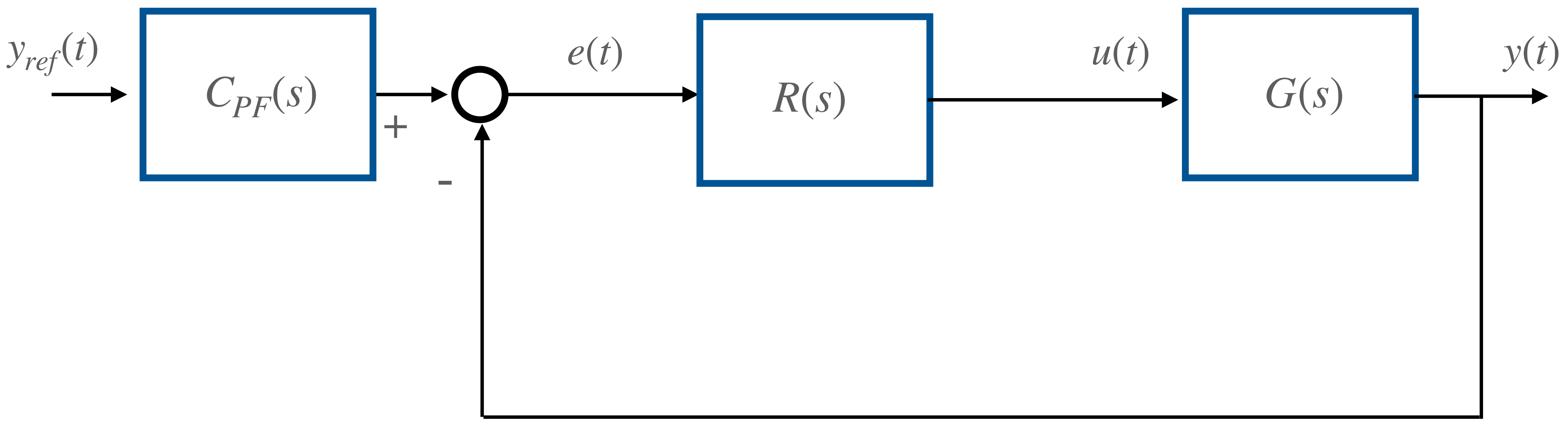


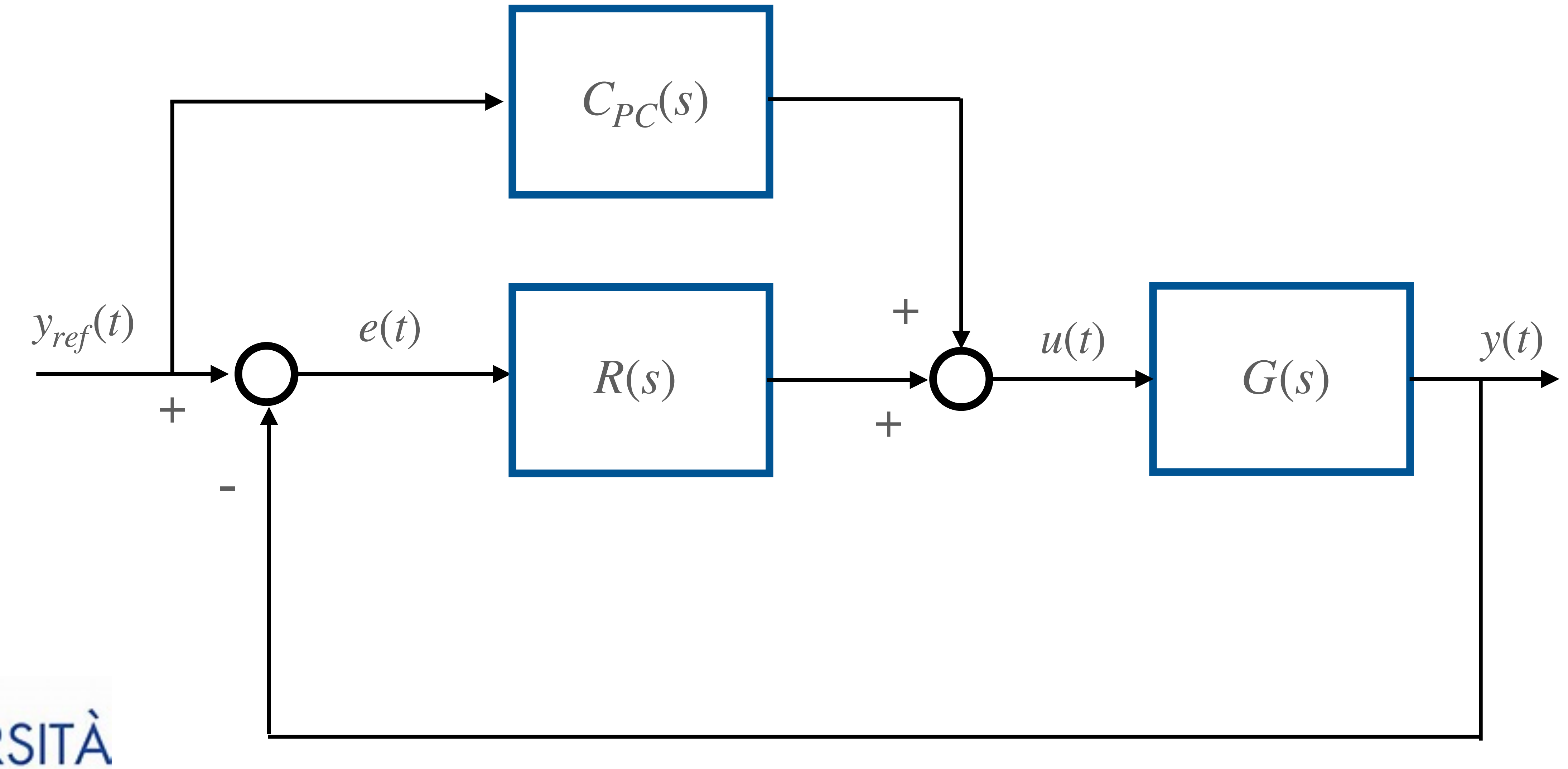


1DoF scheme



Prefilter-based and Parallel-compensator Based Control Schemes: 2DoF Control Schemes

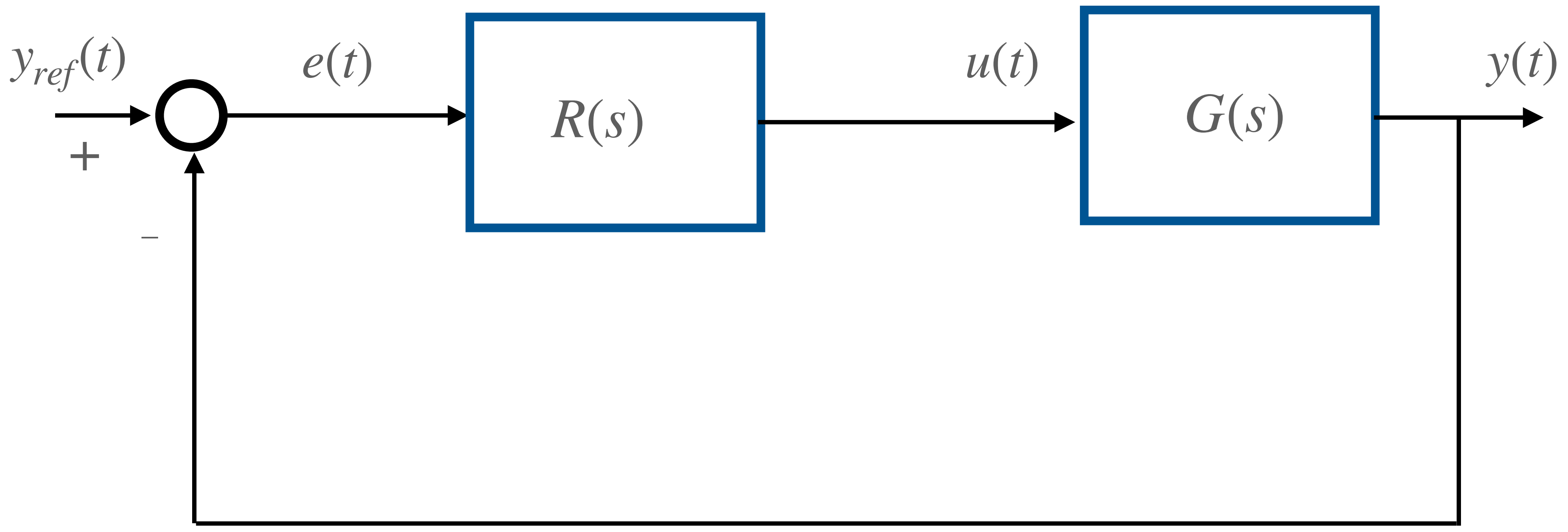




1DoF scheme

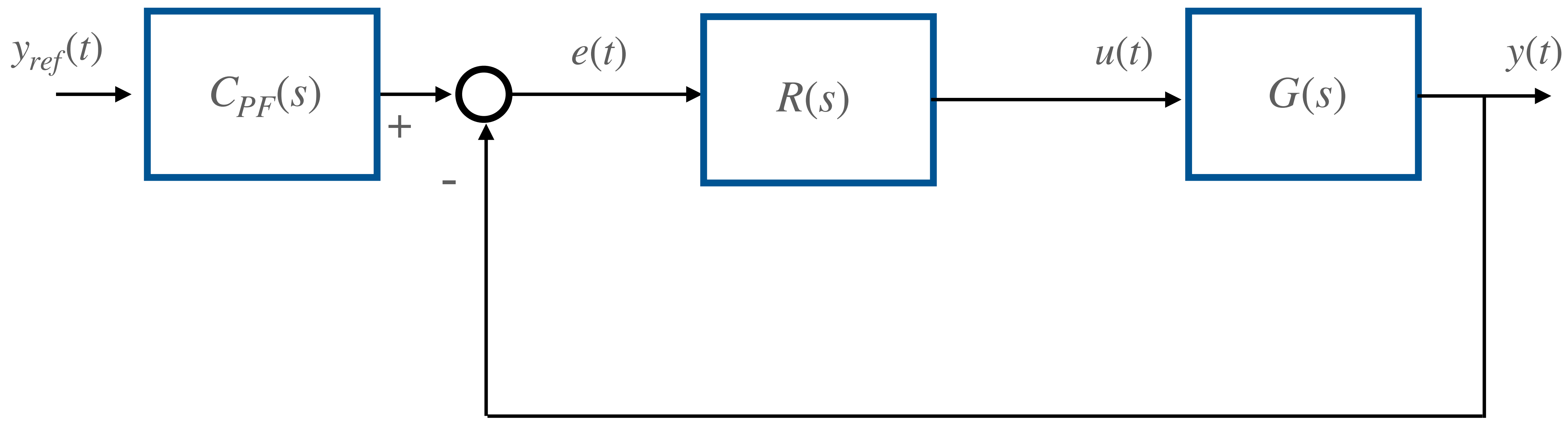
$$y_{ref} \to u : \frac{R(s)}{1 + R(s)Gs)}$$

$$y \to u : -\frac{R(s)}{1 + R(s)Gs)}$$



UNIVERSITÀ DI PAVIA

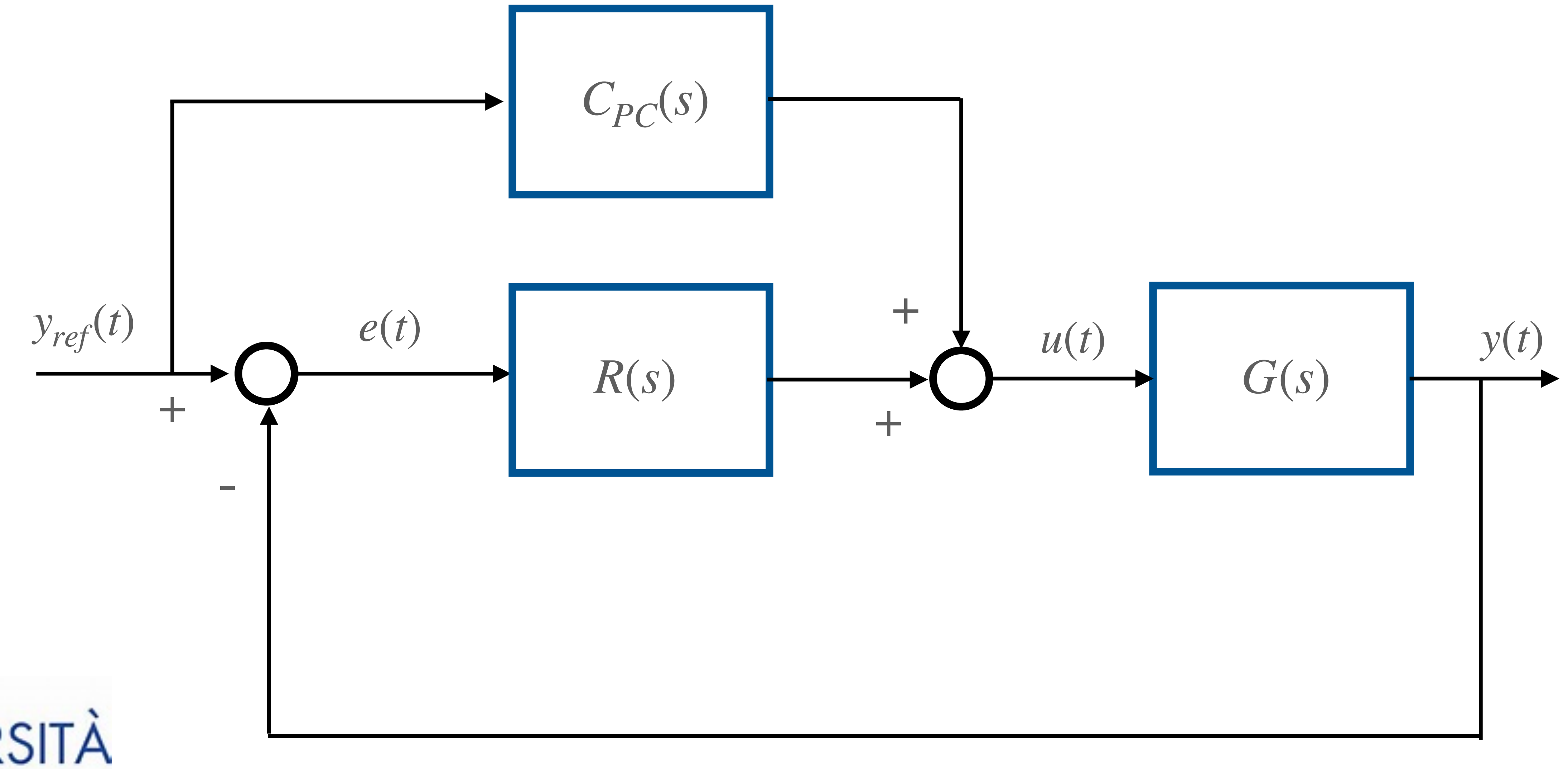
Prefilter-based and Parallel-compensator Based Control Schemes: 2DoF Control Schemes



2DoF scheme

$$y_{ref} \to u : \frac{C_{PF}(s)R(s)}{1 + R(s)Gs)}$$

$$y \to u : -\frac{R(s)}{1 + R(s)Gs)}$$

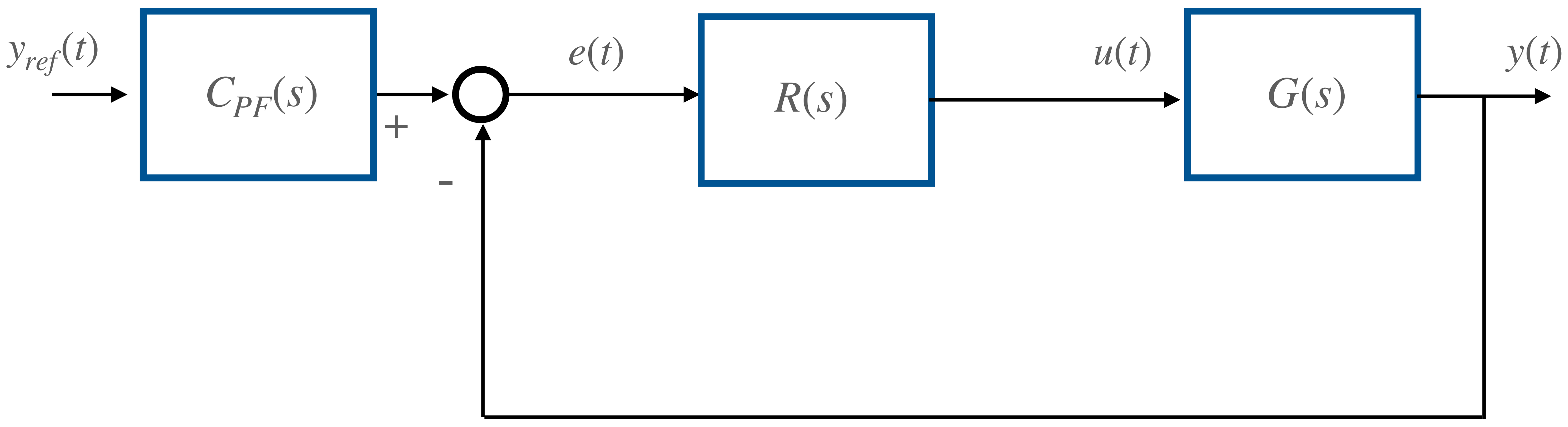


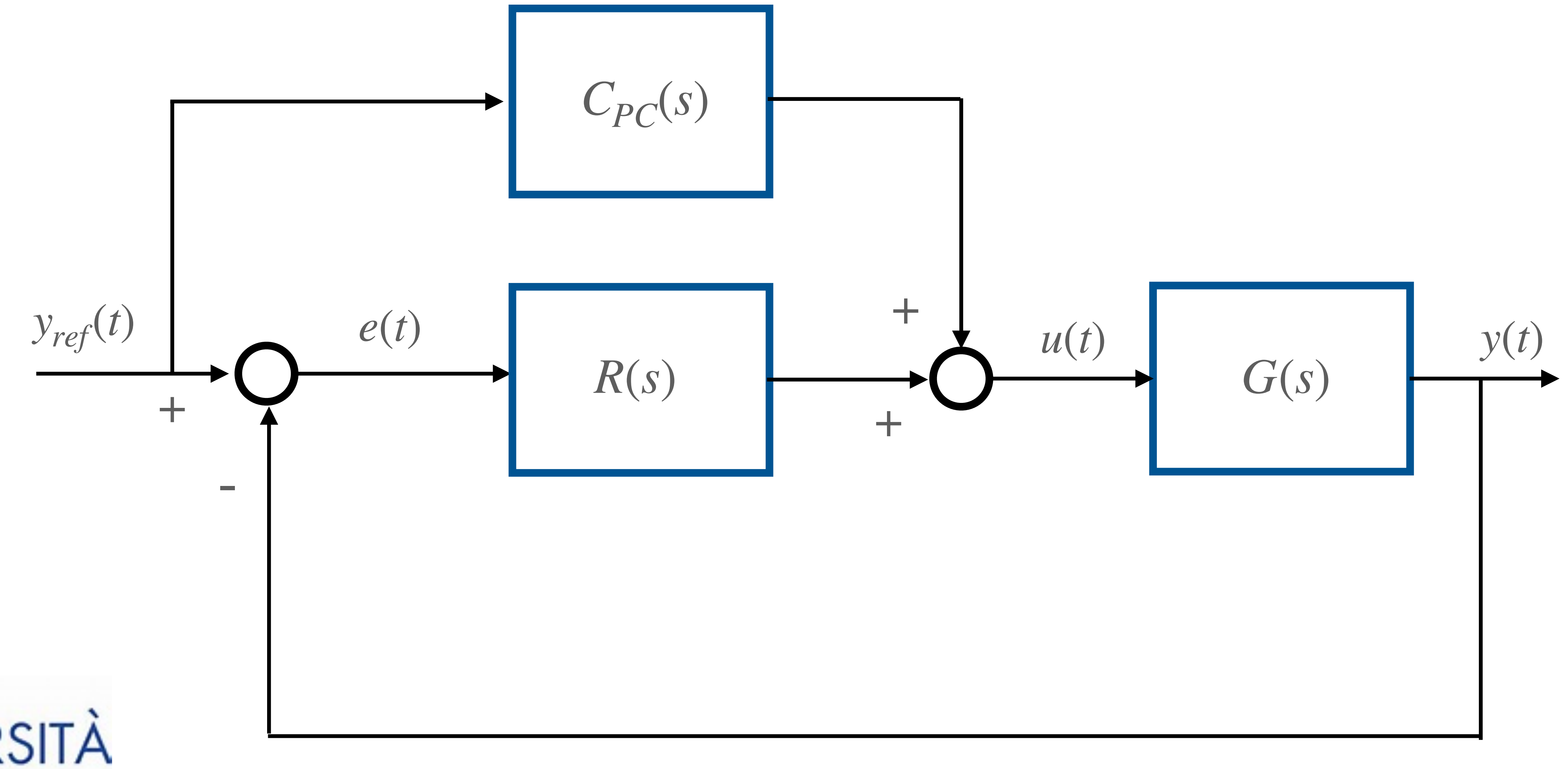
2DoF scheme

$$y_{ref} \to u : \frac{C_{PC}(s) + R(s)}{1 + R(s)Gs)}$$

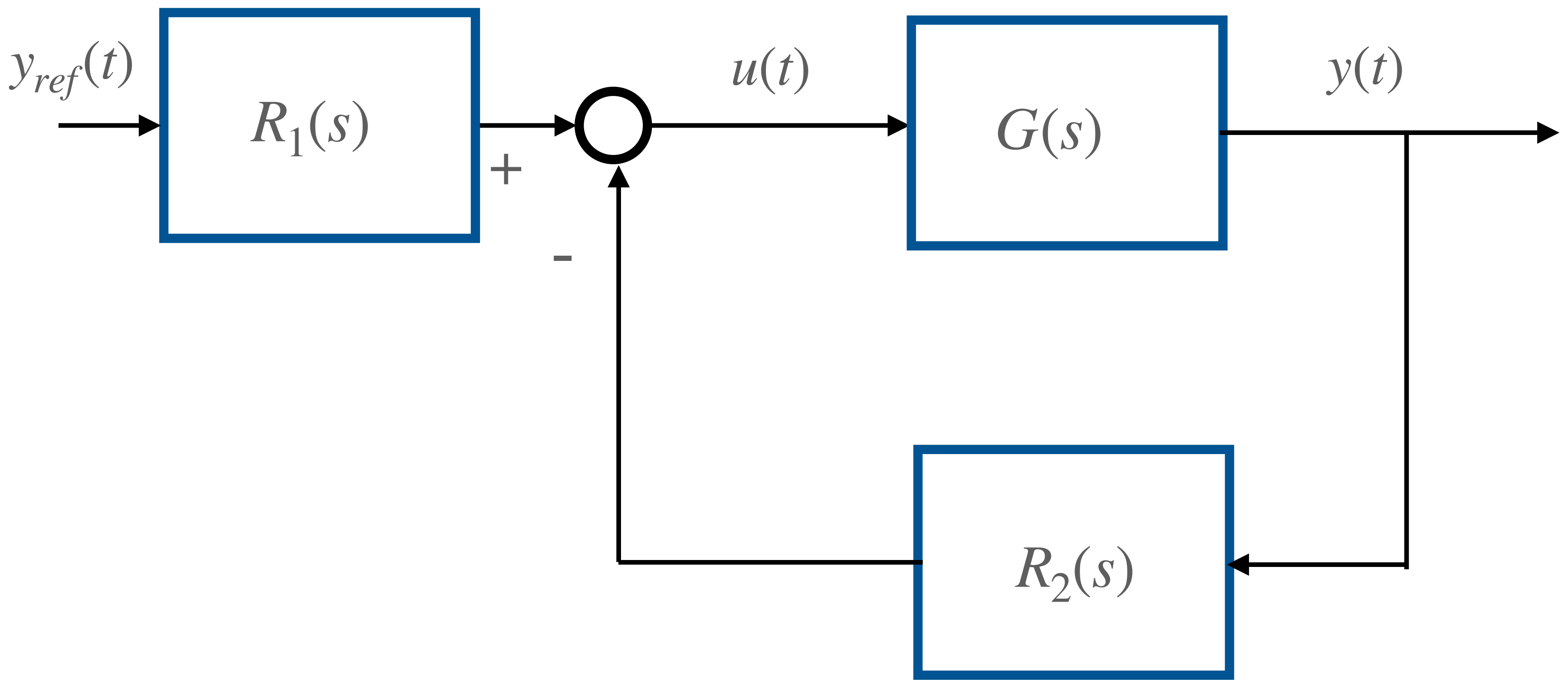
$$y \to u : -\frac{R(s)}{1 + R(s)Gs)}$$

Prefilter-based and Parallel-compensator Based Control Schemes: 2DoF Control Schemes

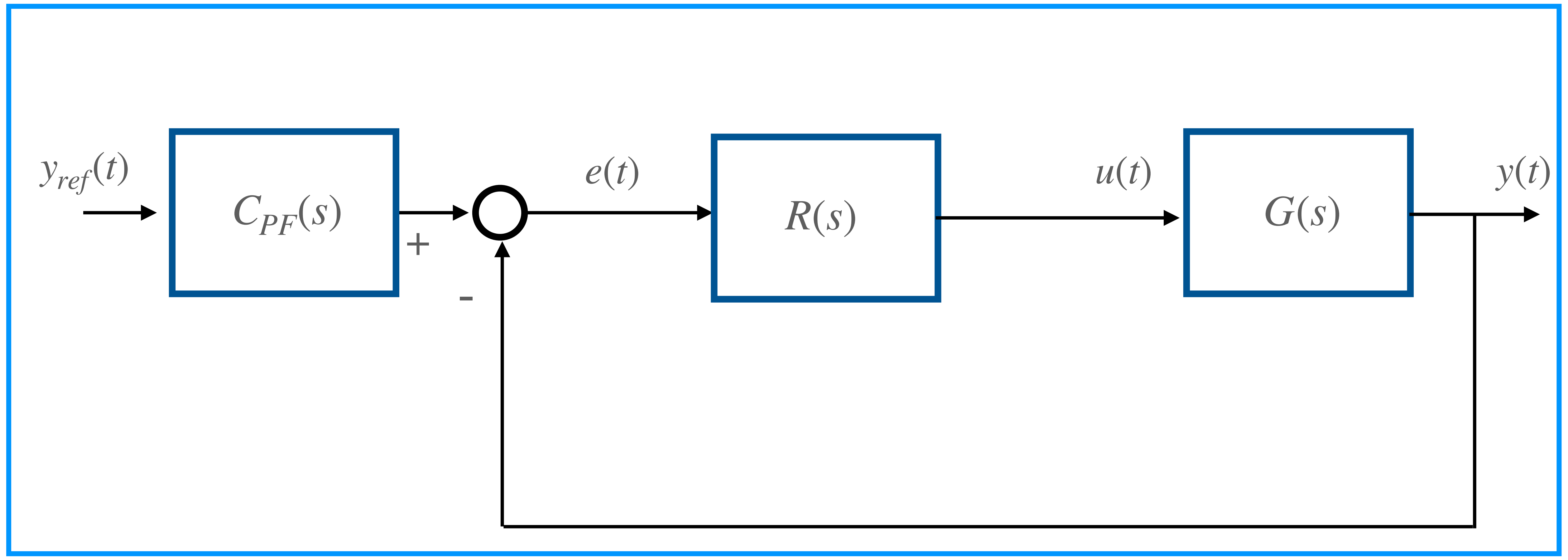


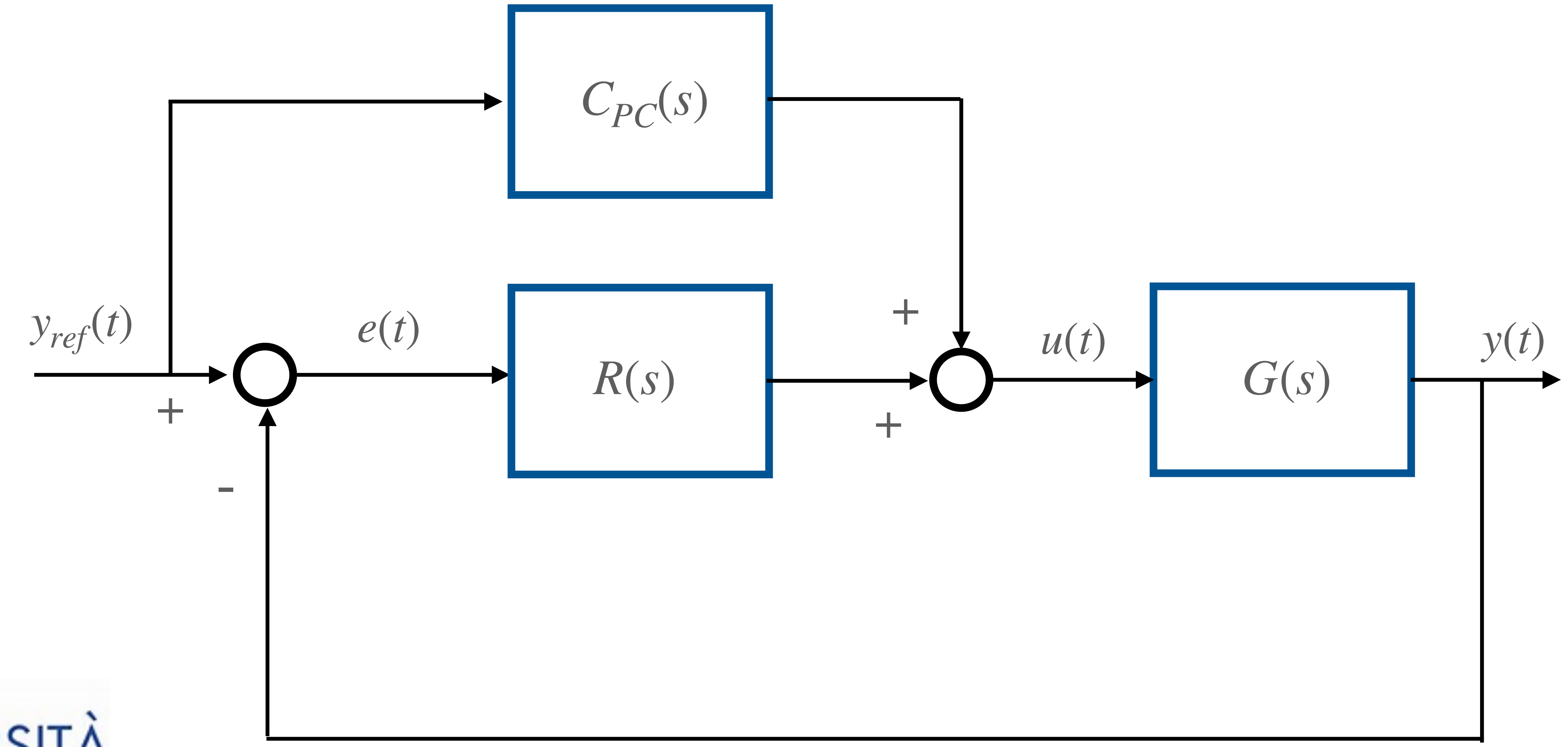


General 2DoF scheme

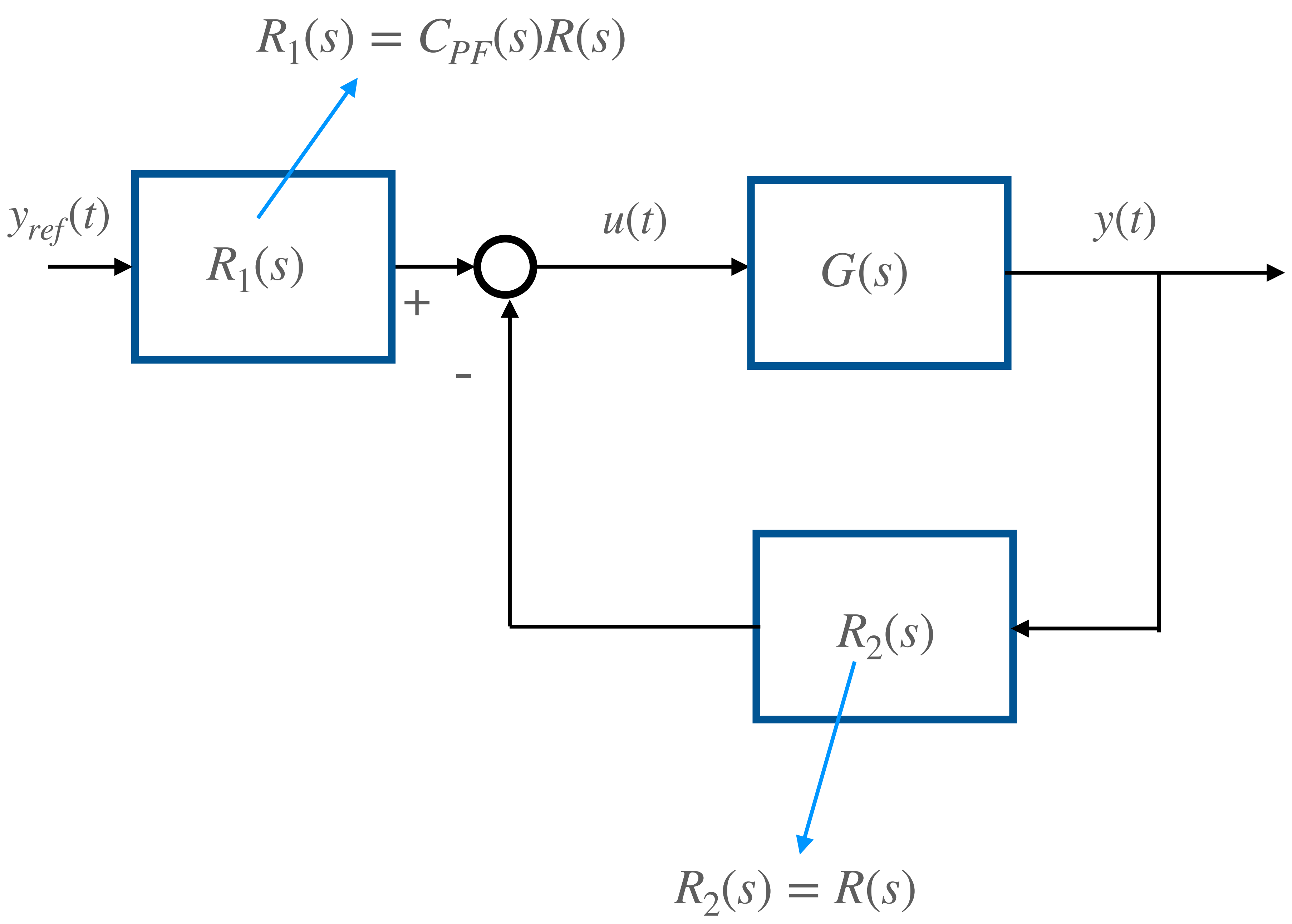


Prefilter-based and Parallel-compensator Based Control Schemes: 2DoF Control Schemes

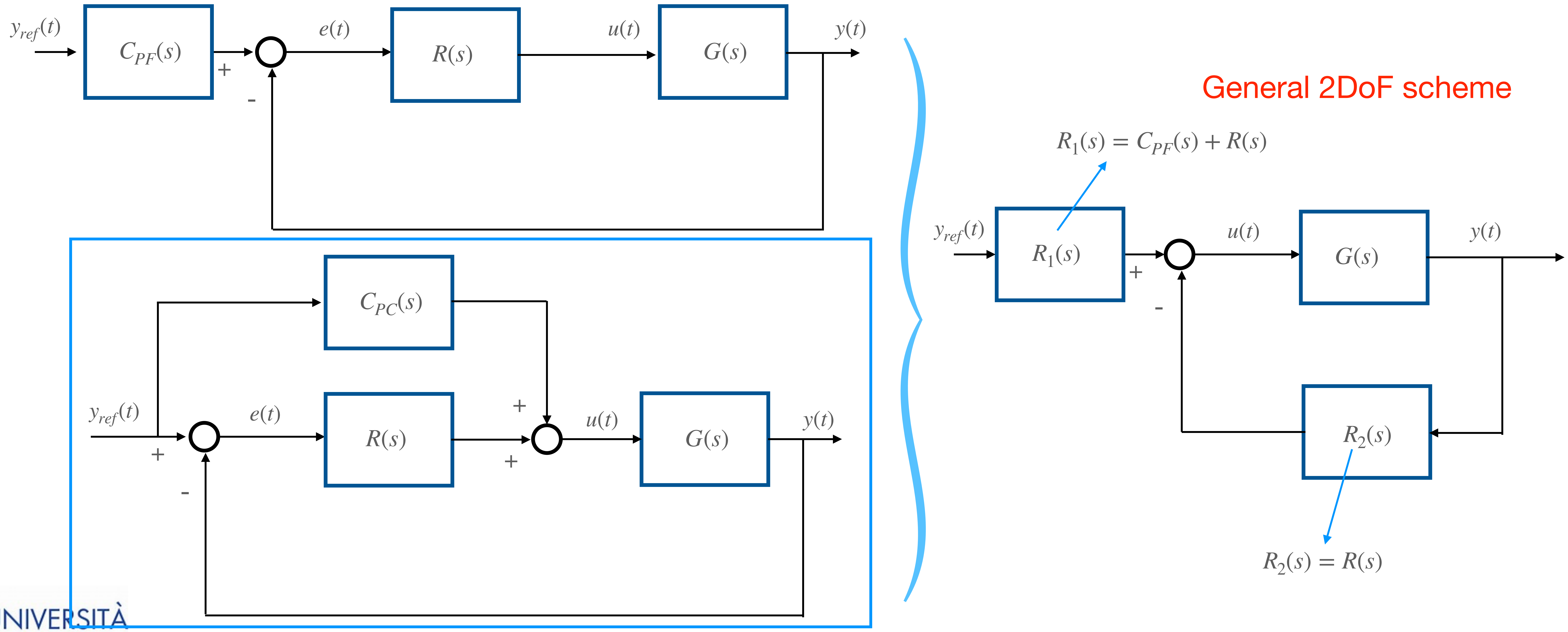




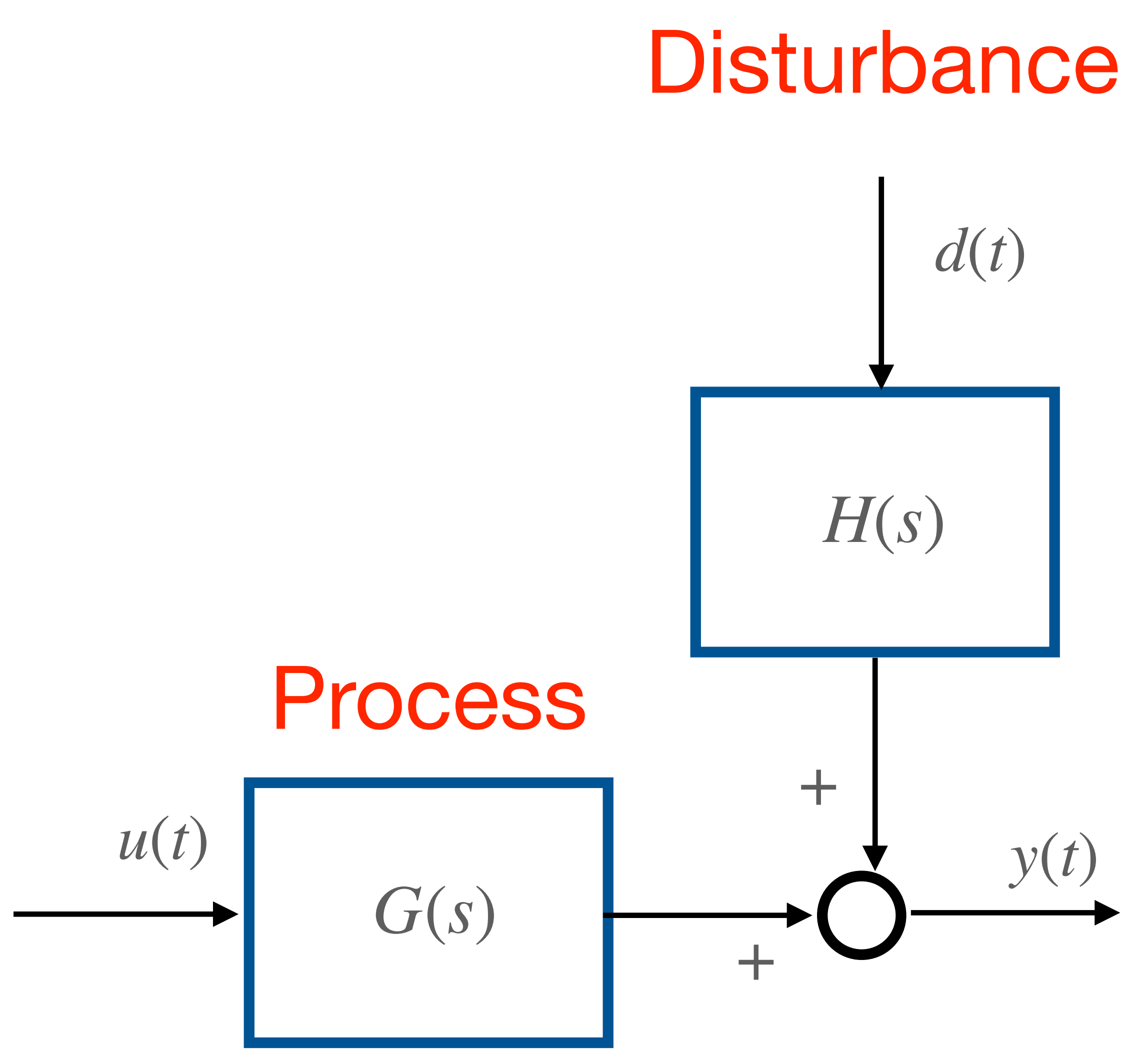
General 2DoF scheme



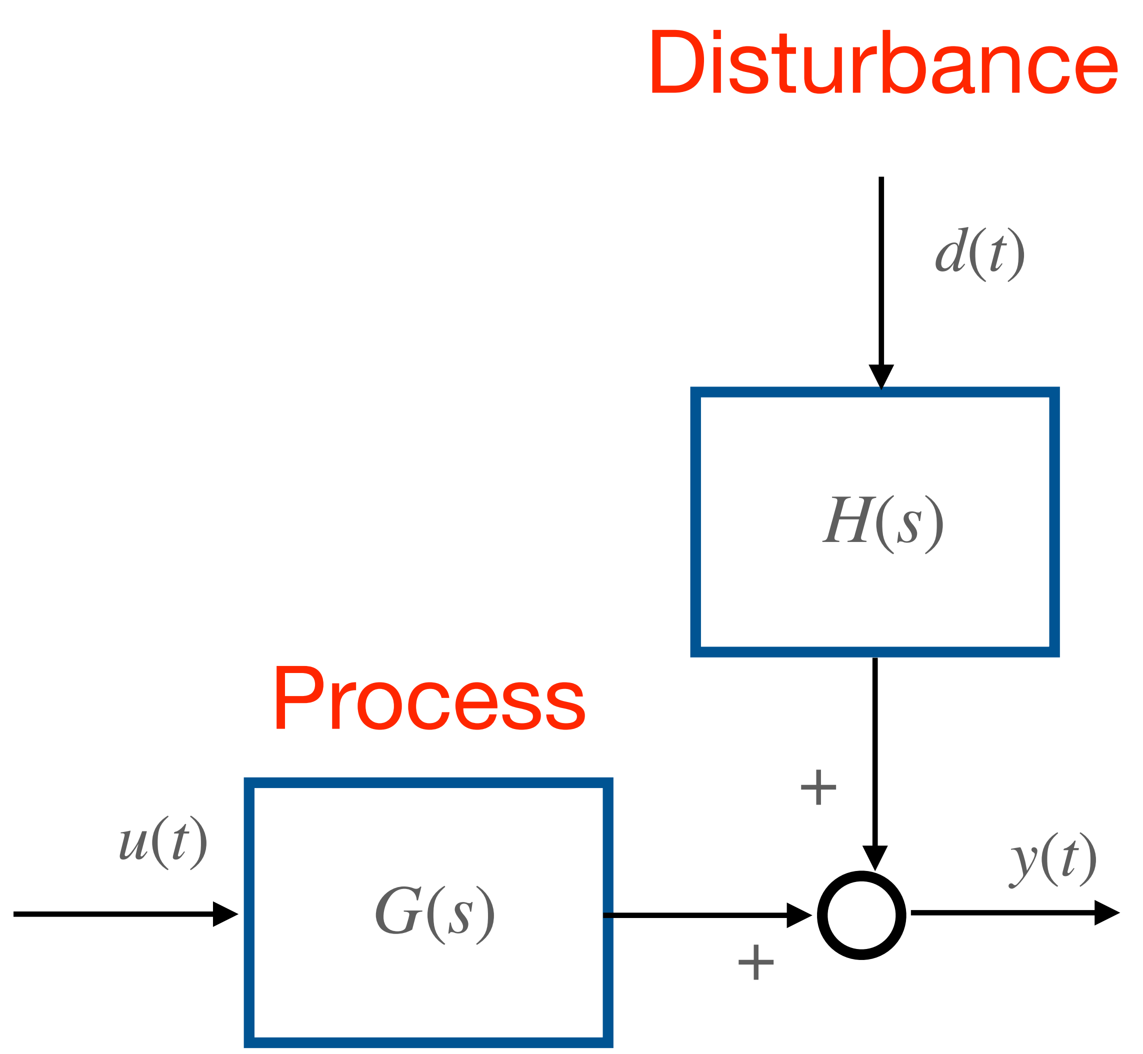
Prefilter-based and Parallel-compensator Based Control Schemes: 2DoF Control Schemes



Control Scheme with Measurable Disturbance Compensation



Control Scheme with Measurable Disturbance Compensation

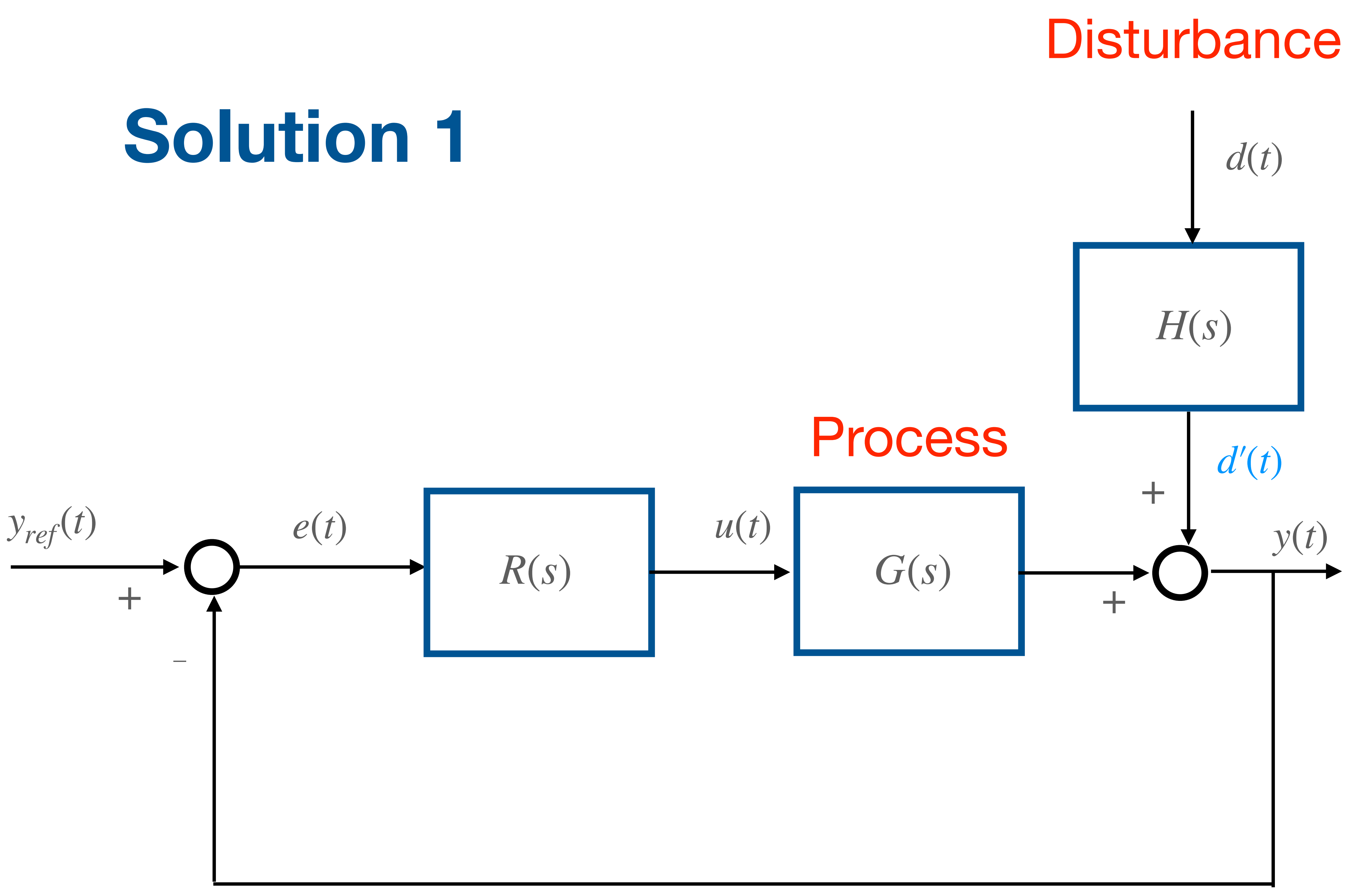


Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

Control Scheme with Measurable Disturbance Compensation

Solution 1

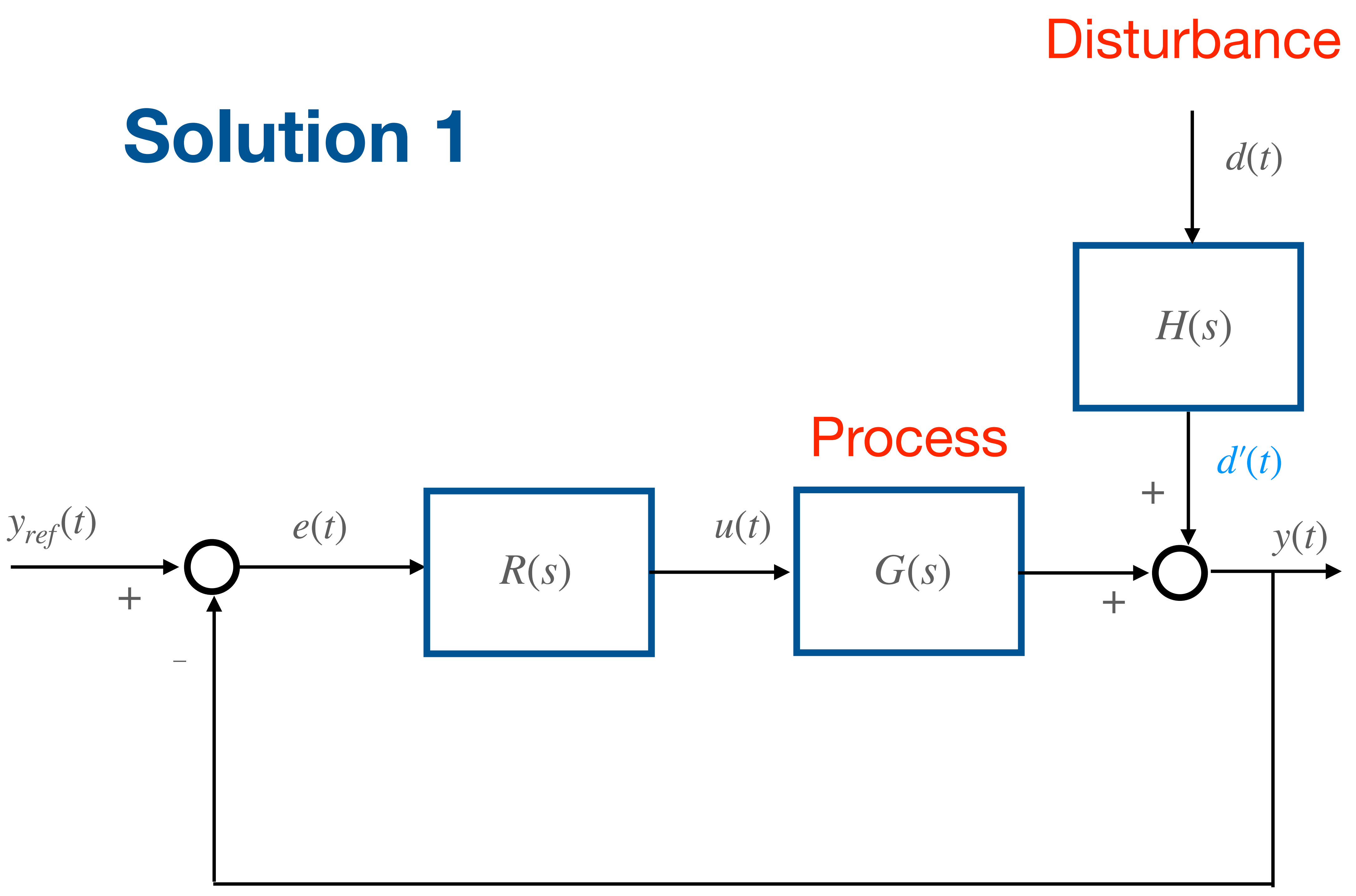


Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

Control Scheme with Measurable Disturbance Compensation

Solution 1



Assumptions:

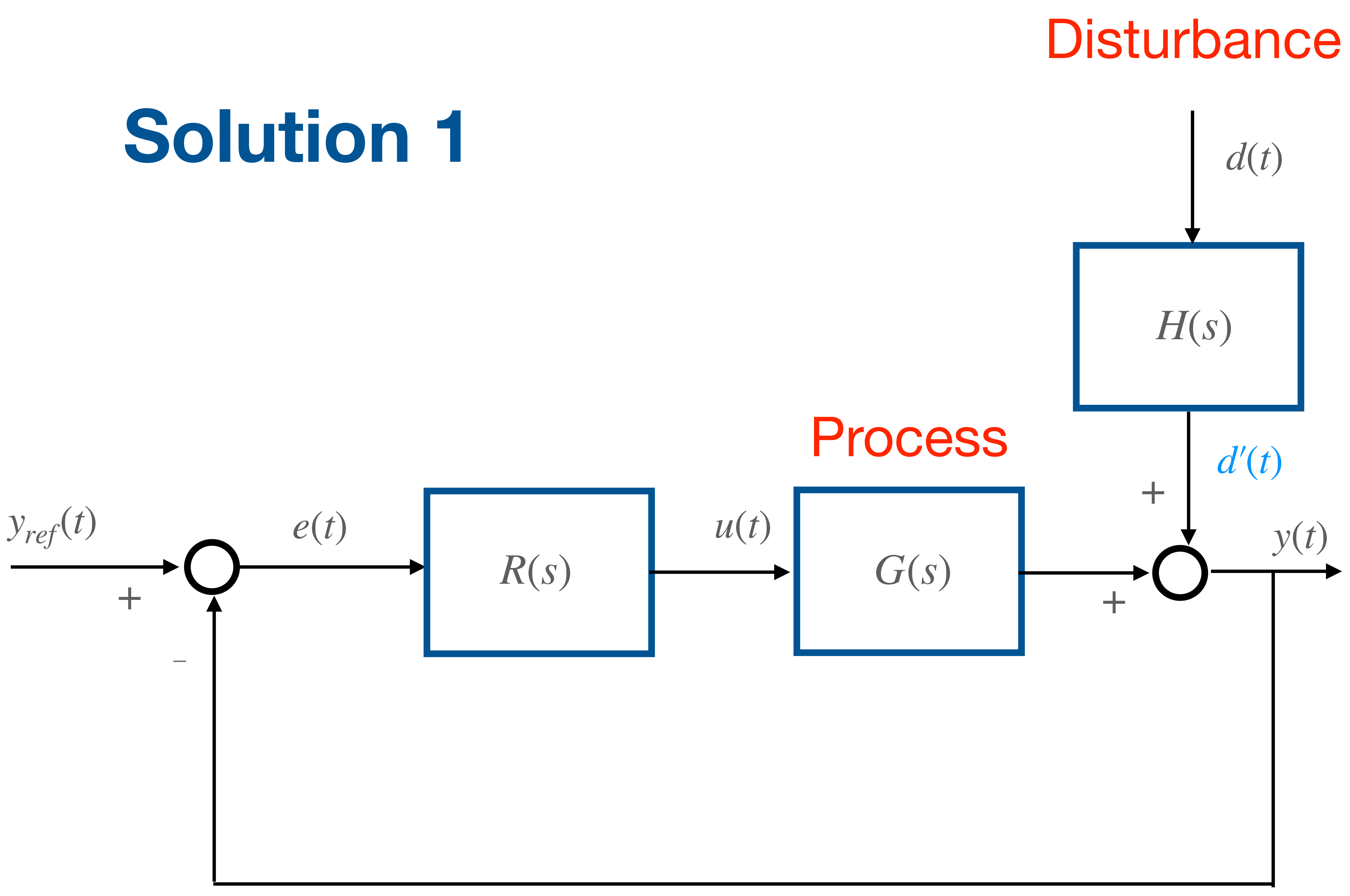
- $d(t)$ measurable
- $H(s)$ known (possibly 1)

Sensitivity Function

$$d' \rightarrow y: \quad S(s) = \frac{1}{1 + R(s)G(s)}$$

Control Scheme with Measurable Disturbance Compensation

Solution 1



Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

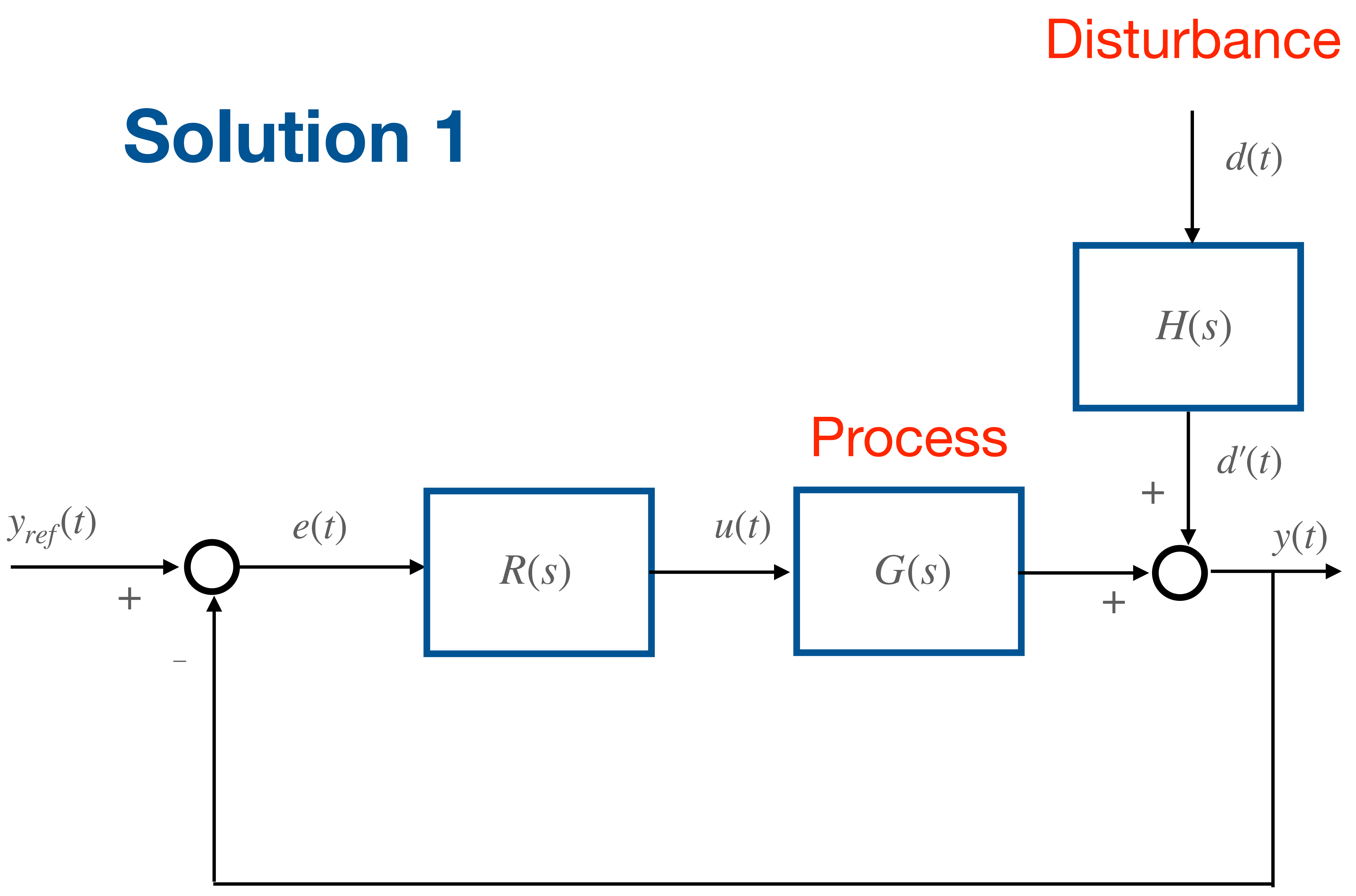
Sensitivity Function

$$d' \rightarrow y : \quad S(s) = \frac{1}{1 + R(s)G(s)}$$

Recommendation: $|S(j\omega)|$ sufficiently low $\forall \omega$ in the frequency range where $d'(t)$ has significant harmonics

Control Scheme with Measurable Disturbance Compensation

Solution 1



Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

Sensitivity Function

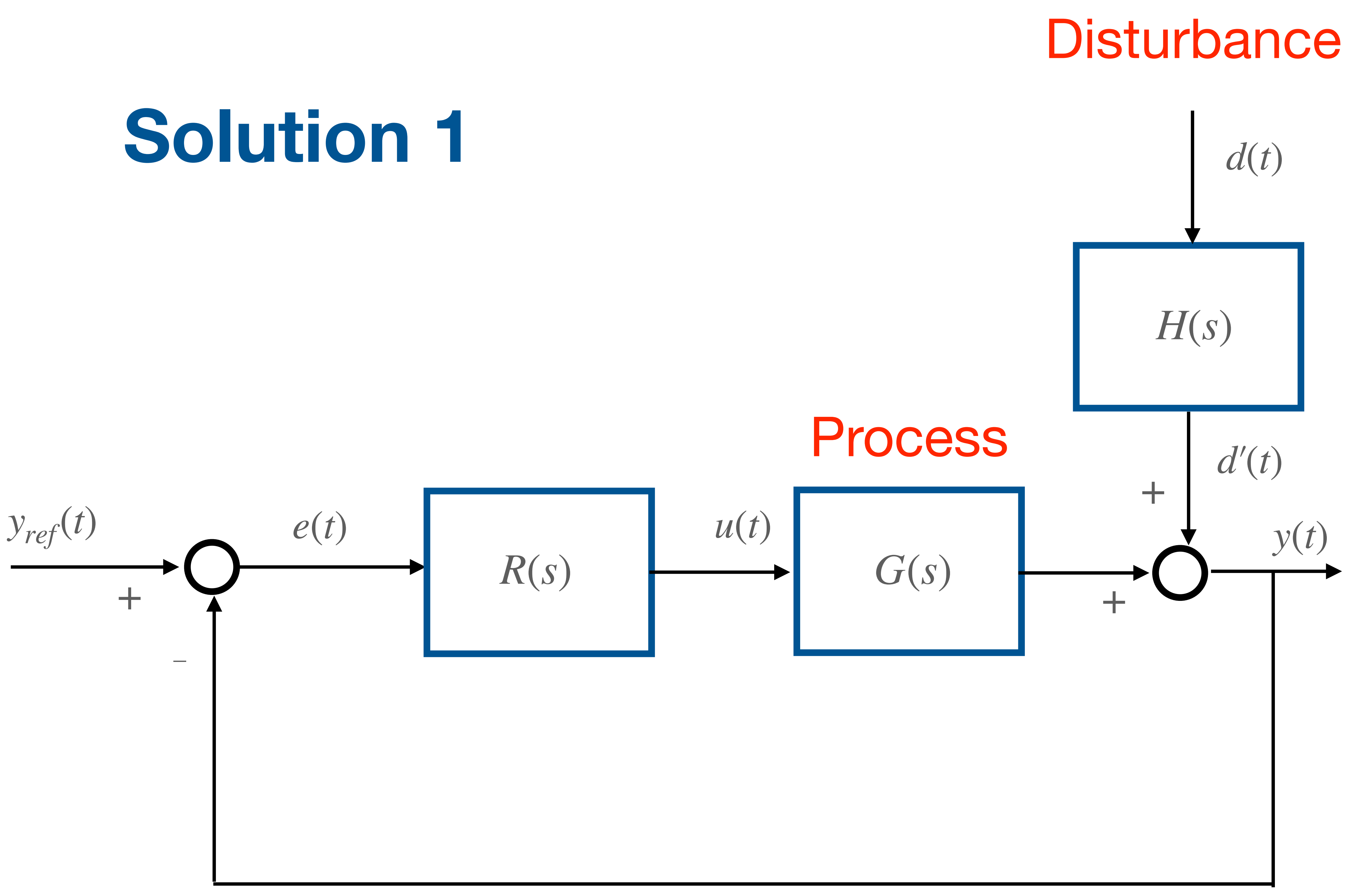
$$d' \to y : \quad S(s) = \frac{1}{1 + R(s)G(s)}$$

Recommendation: $|S(j\omega)|$ sufficiently low $\forall \omega$ in the frequency range where $d'(t)$ has significant harmonics

$$\omega_{d'} \in [0, \omega_{BW_{CL}})$$

Control Scheme with Measurable Disturbance Compensation

Solution 1



Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

Sensitivity Function

$$d' \rightarrow y: \quad S(s) = \frac{1}{1 + R(s)G(s)}$$

Recommendation: $|S(j\omega)|$ sufficiently low $\forall \omega$ in the frequency range where $d'(t)$ has significant harmonics

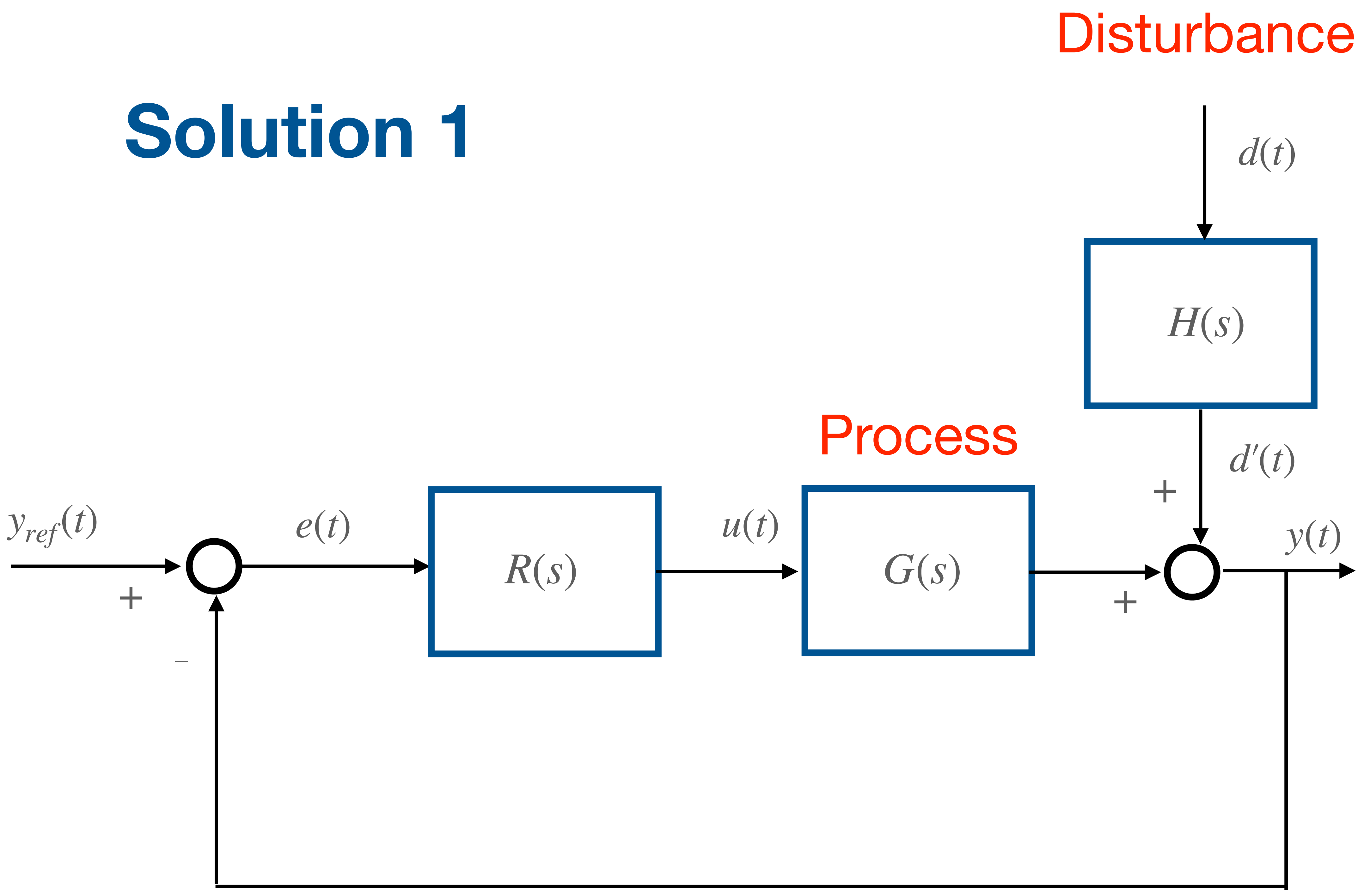
$$\omega_{d'} \in [0, \omega_{BW_{CL}})$$

$|S(j\omega)|$

- LF: $|S(j\omega)| \rightarrow \left| \dfrac{1}{L(j\omega)} \right|$
- HF: $|S(j\omega)| \rightarrow 1$

Control Scheme with Measurable Disturbance Compensation

Solution 1



Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

Sensitivity Function

$$d' \rightarrow y: \quad S(s) = \frac{1}{1 + R(s)G(s)}$$

Recommendation: $|S(j\omega)|$ sufficiently low $\forall \omega$ in the frequency range where $d'(t)$ has significant harmonics

Recommendation: $|L(j\omega)|$ sufficiently high $\forall\, \omega \in [0, \omega_{BW_{CL}})$

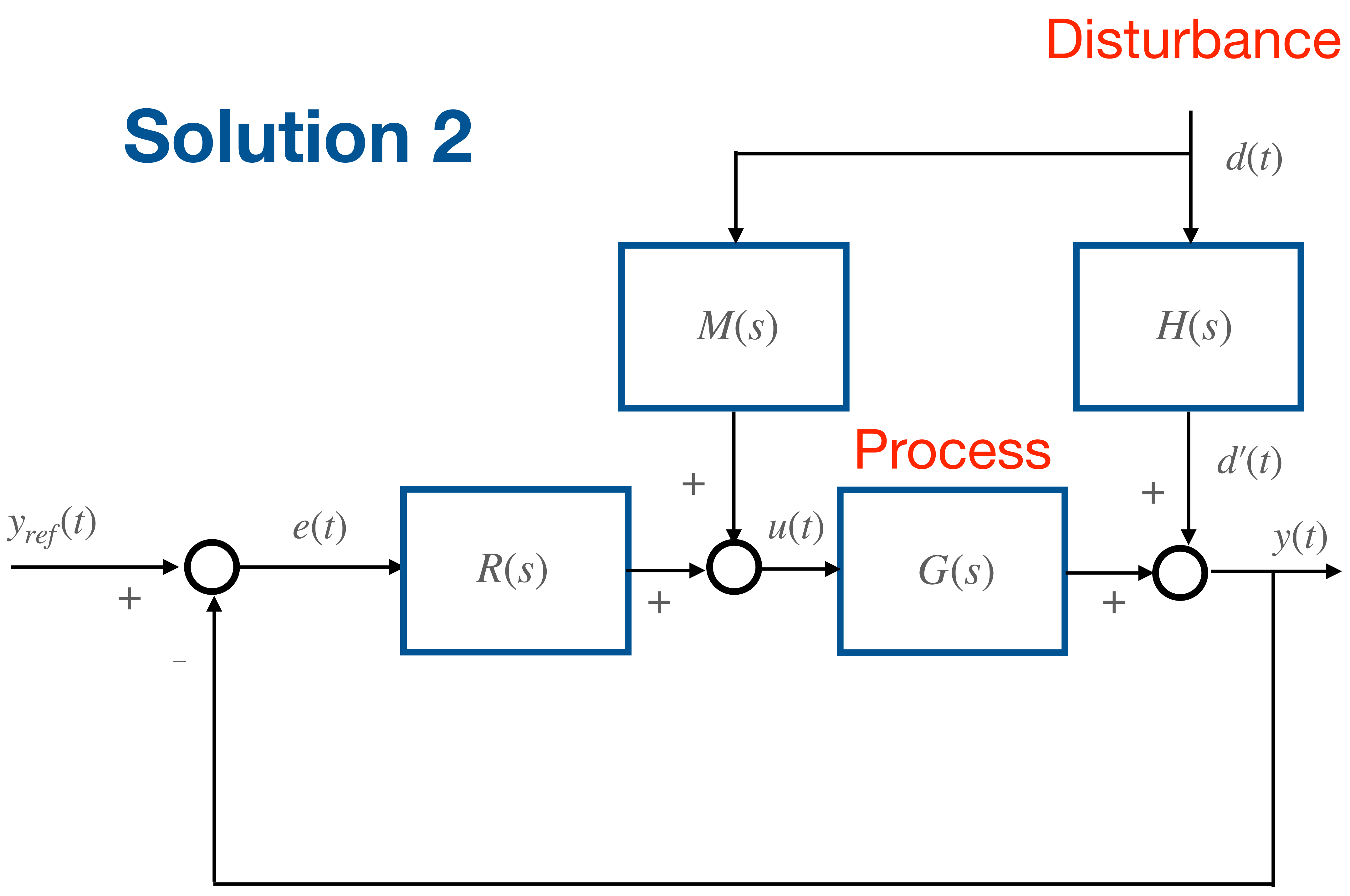
$|S(j\omega)|$

LF: $|S(j\omega)| \rightarrow \left| \frac{1}{L(j\omega)} \right|$

HF: $|S(j\omega)| \rightarrow 1$

Control Scheme with Measurable Disturbance Compensation

Solution 2

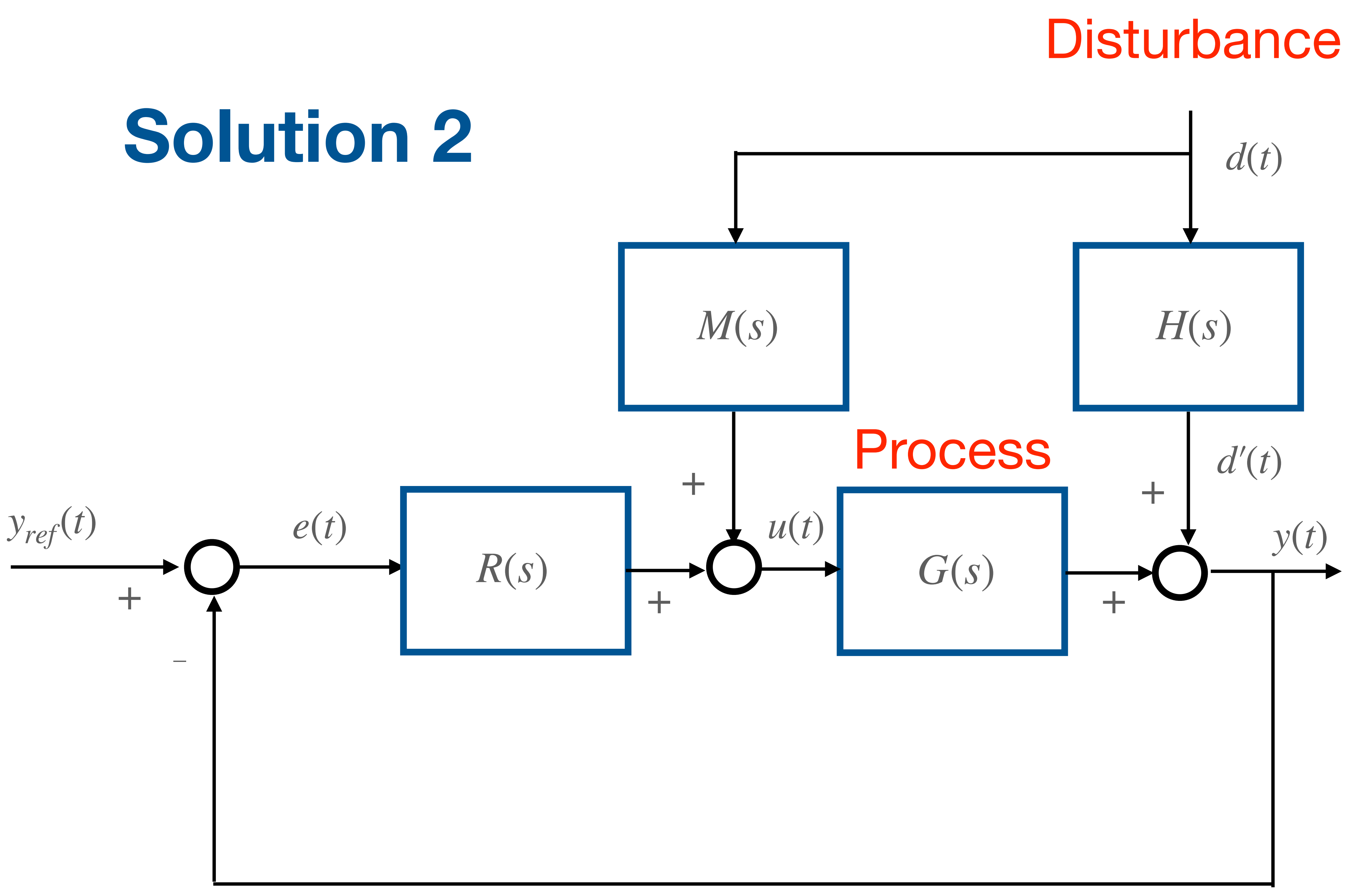


Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

Control Scheme with Measurable Disturbance Compensation

Solution 2



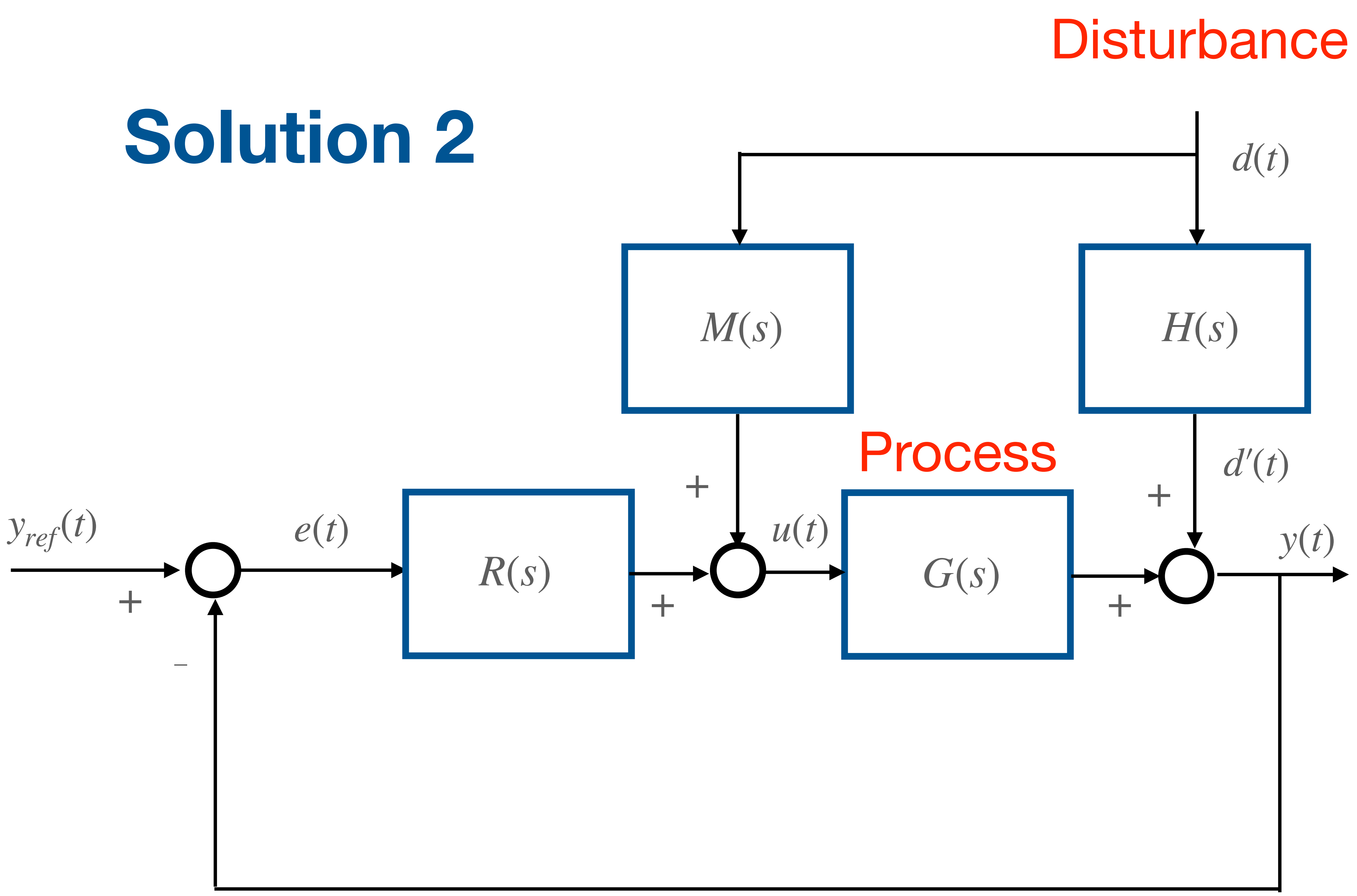
Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

$$d \rightarrow y : \quad \frac{H(s) + M(s)G(s)}{1 + R(s)G(s)}$$

Control Scheme with Measurable Disturbance Compensation

Solution 2



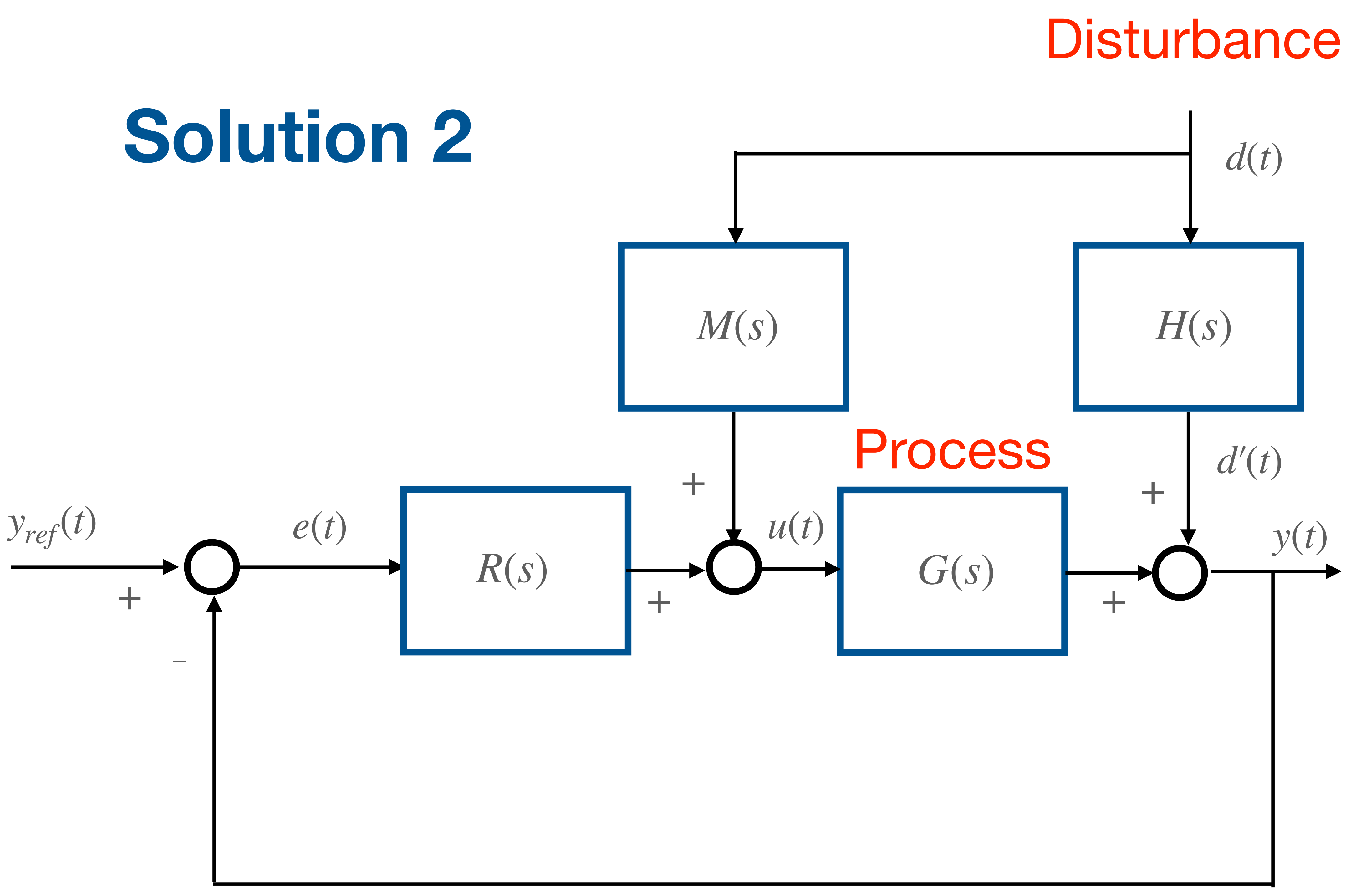
Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

$$d \rightarrow y : \quad \frac{H(s) + M(s)G(s)}{1 + R(s)G(s)} \approx 0$$

Control Scheme with Measurable Disturbance Compensation

Solution 2



Assumptions:

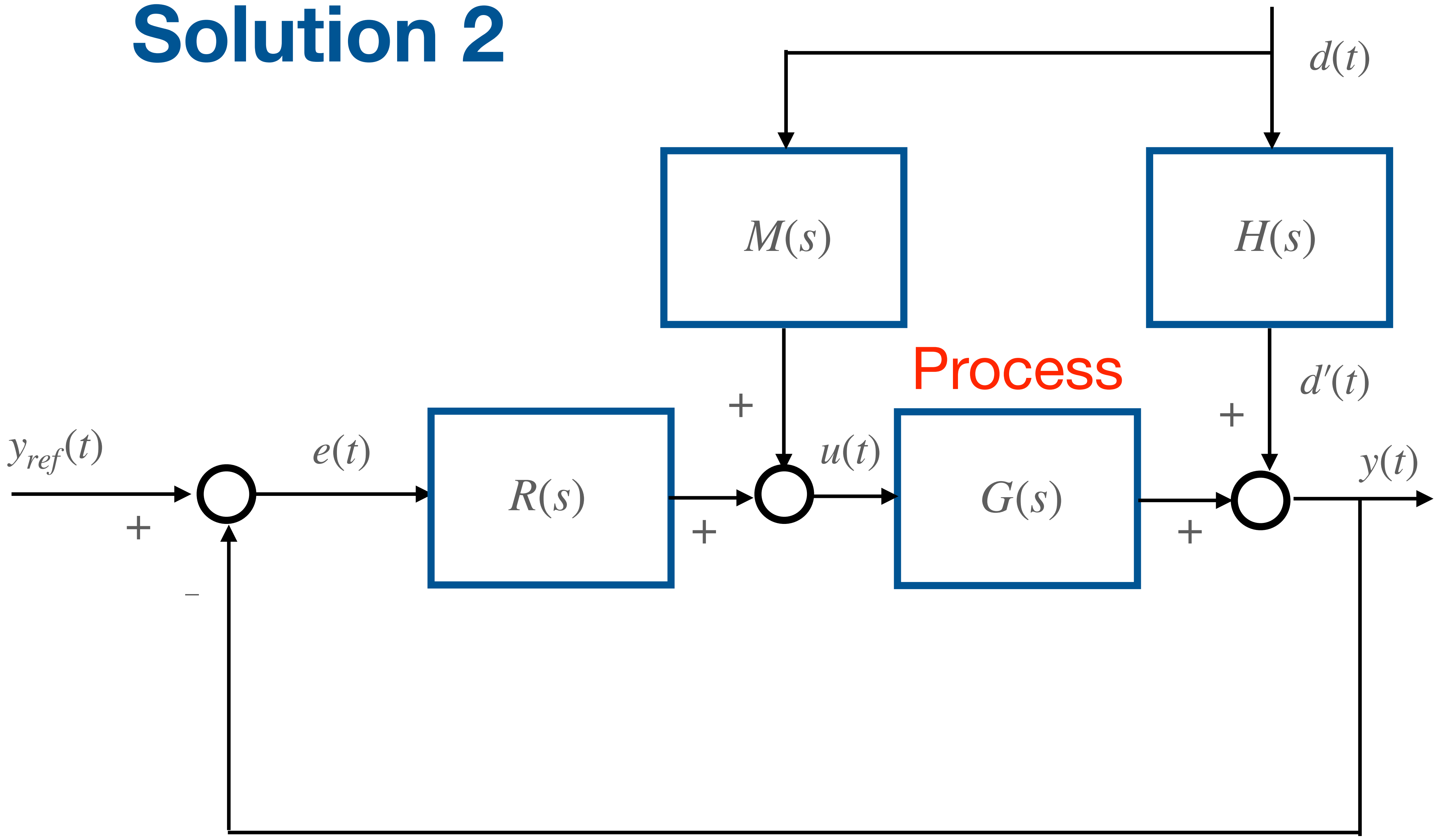
- $d(t)$ measurable
- $H(s)$ known (possibly 1)

$$d \rightarrow y : \quad \frac{H(s) + M(s)G(s)}{1 + R(s)G(s)} \approx 0$$

$$M(s) = -\frac{H(s)}{G(s)}$$

Control Scheme with Measurable Disturbance Compensation

Solution 2



Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

$$d \rightarrow y : \quad \frac{H(s) + M(s)G(s)}{1 + R(s)G(s)} \approx 0$$

Ideal Design

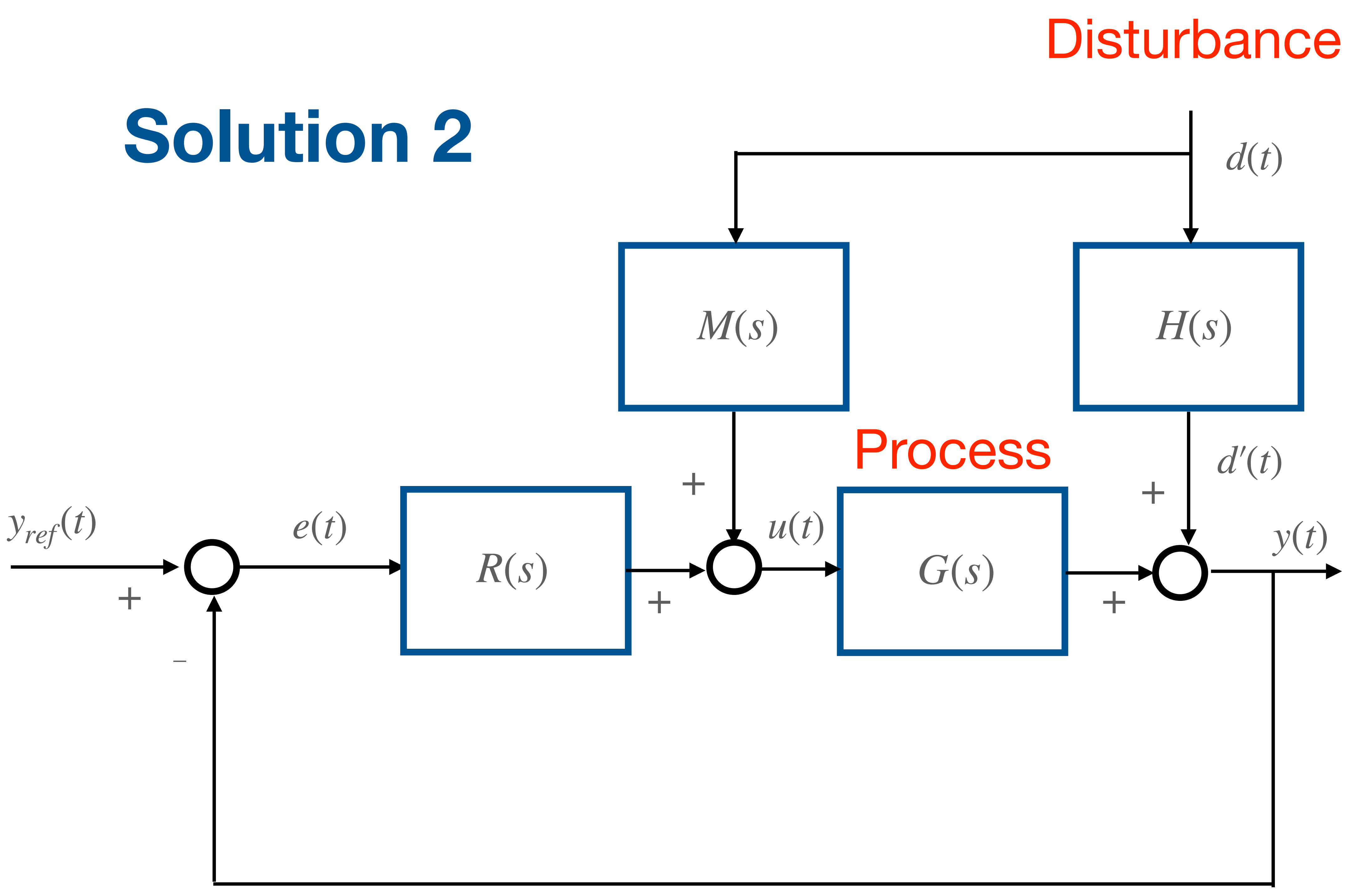
$$M(s) = -\frac{H(s)}{G(s)}$$

Warning:

- $M(s)$ As. Stable
- Causal

Control Scheme with Measurable Disturbance Compensation

Solution 2



Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

$$d \rightarrow y : \quad \frac{H(s) + M(s)G(s)}{1 + R(s)G(s)} \approx 0$$

Ideal Design

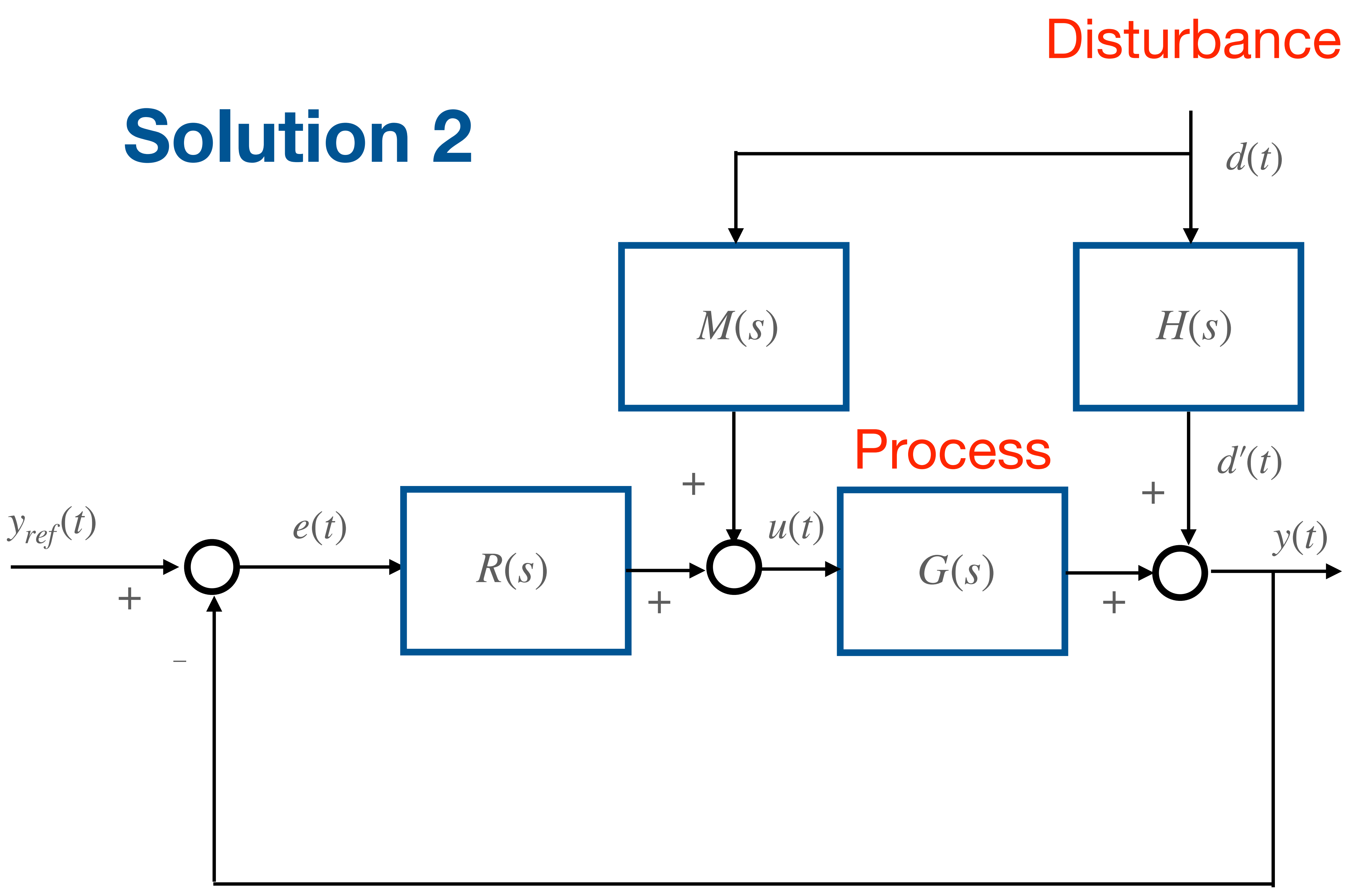
$$M(s) = -\frac{H(s)}{G(s)}$$

Warning:

- $M(s)$ $\boxed{\text{As. Stable}}$ not guaranteed
- ~~Causal~~

Control Scheme with Measurable Disturbance Compensation

Solution 2



Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

$$d \rightarrow y: \quad \frac{H(s) + M(s)G(s)}{1 + R(s)G(s)} \approx 0$$

Actual Design

$$M(j\omega) = -\frac{H(j\omega)}{G(j\omega)}$$

in the frequency range where $d(t)$ has significant harmonics

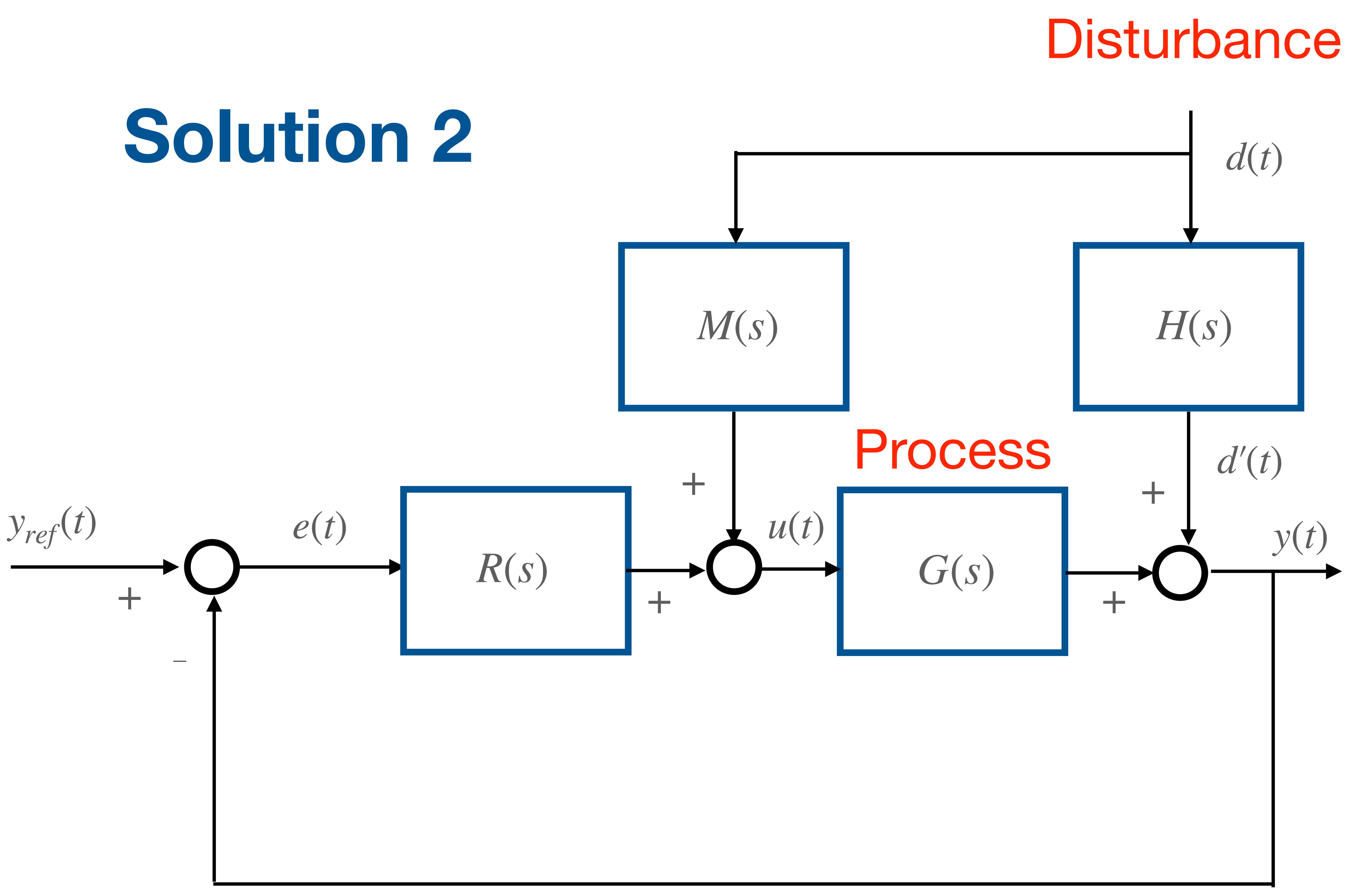
Warning:

- $M(s)$ As. Stable
- ~~Causal~~

not guaranteed

Control Scheme with Measurable Disturbance Compensation

Solution 2



Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

$$d \to y : \quad \frac{H(s) + M(s)G(s)}{1 + R(s)G(s)} \approx 0$$

Actual Design

$$M(j\omega) = -\frac{H(j\omega)}{G(j\omega)}$$

Warning:

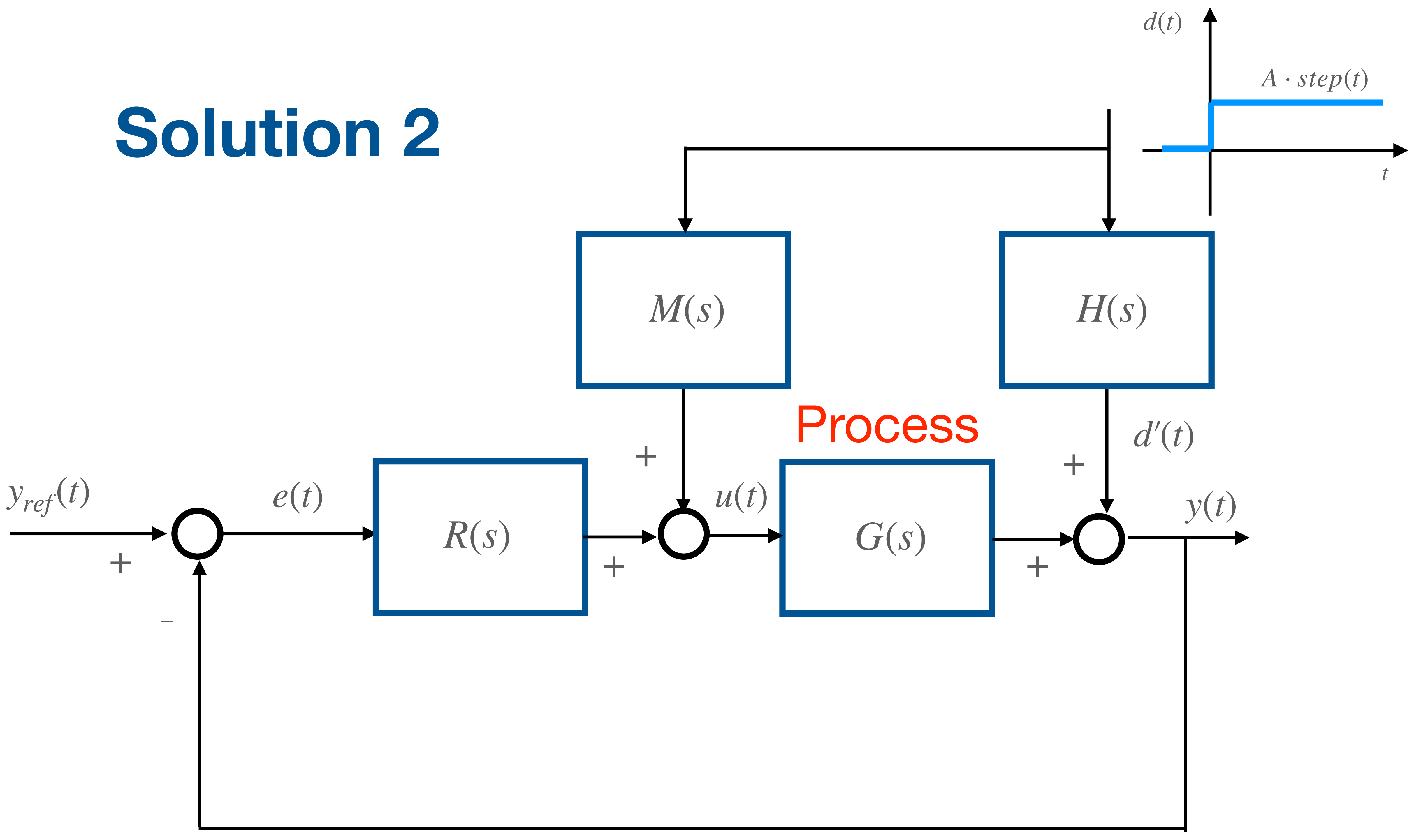
- $M(s)$ $\boxed{\text{As. Stable}}$ not guaranteed
- ~~Causal~~

Example 1:

$$d(t) = sin(\hat{\omega}\, t) \longrightarrow M(s) \text{ s.t. } M(j\omega)\Big|_{\omega=\hat{\omega}} = -\frac{H(j\hat{\omega})}{G(j\hat{\omega})}$$

Control Scheme with Measurable Disturbance Compensation

Solution 2



Assumptions:

- $d(t)$ measurable
- $H(s)$ known (possibly 1)

$$d \rightarrow y : \frac{H(s) + M(s)G(s)}{1 + R(s)G(s)} \approx 0$$

Actual Design

$$M(j\omega) = -\frac{H(j\omega)}{G(j\omega)}$$

Warning:

- $M(s)$ $\boxed{\text{As. Stable}}$ not guaranteed
- ~~Causal~~

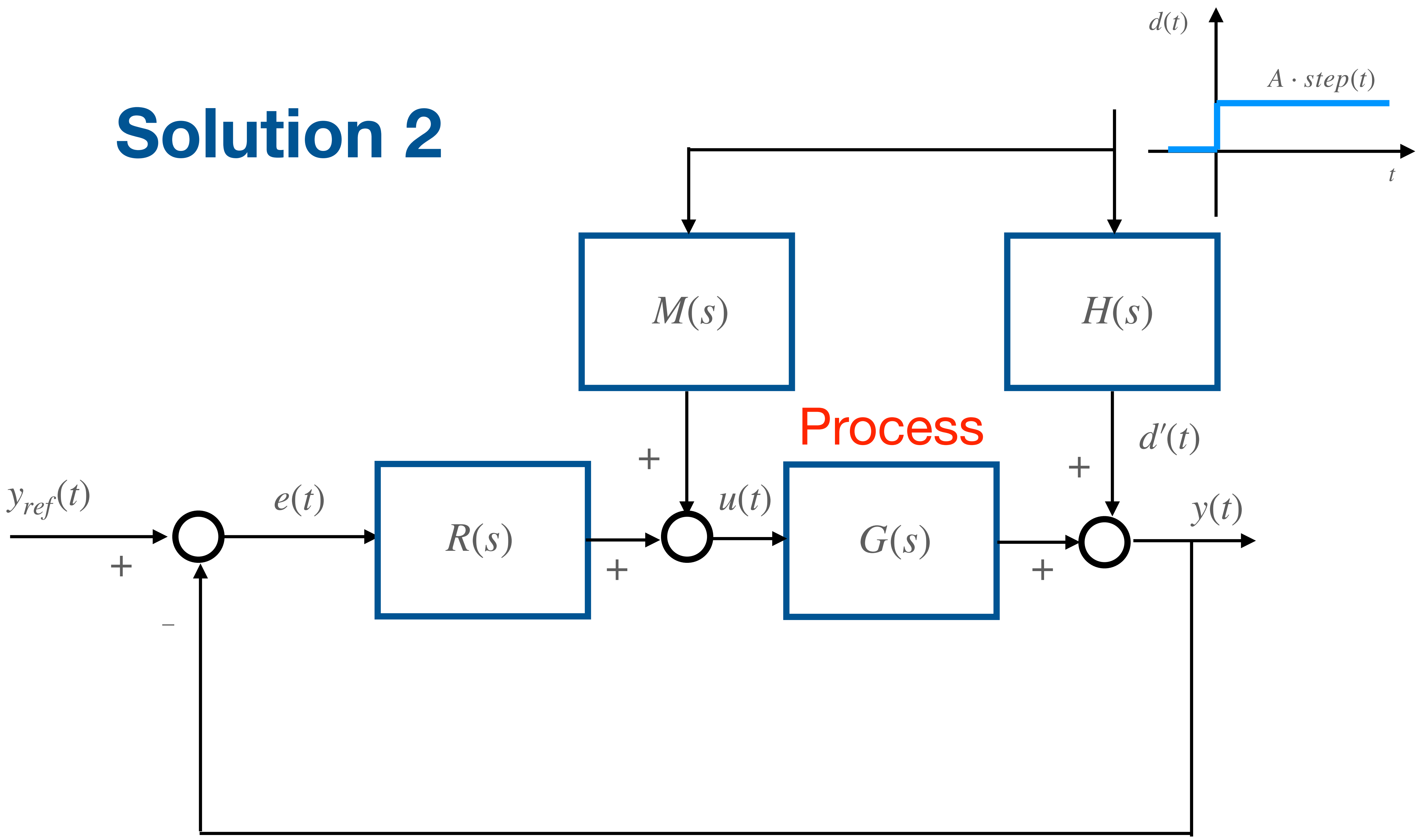
Example 2:

static case $\hat{\omega} = 0$ → $M(s)$ s.t.

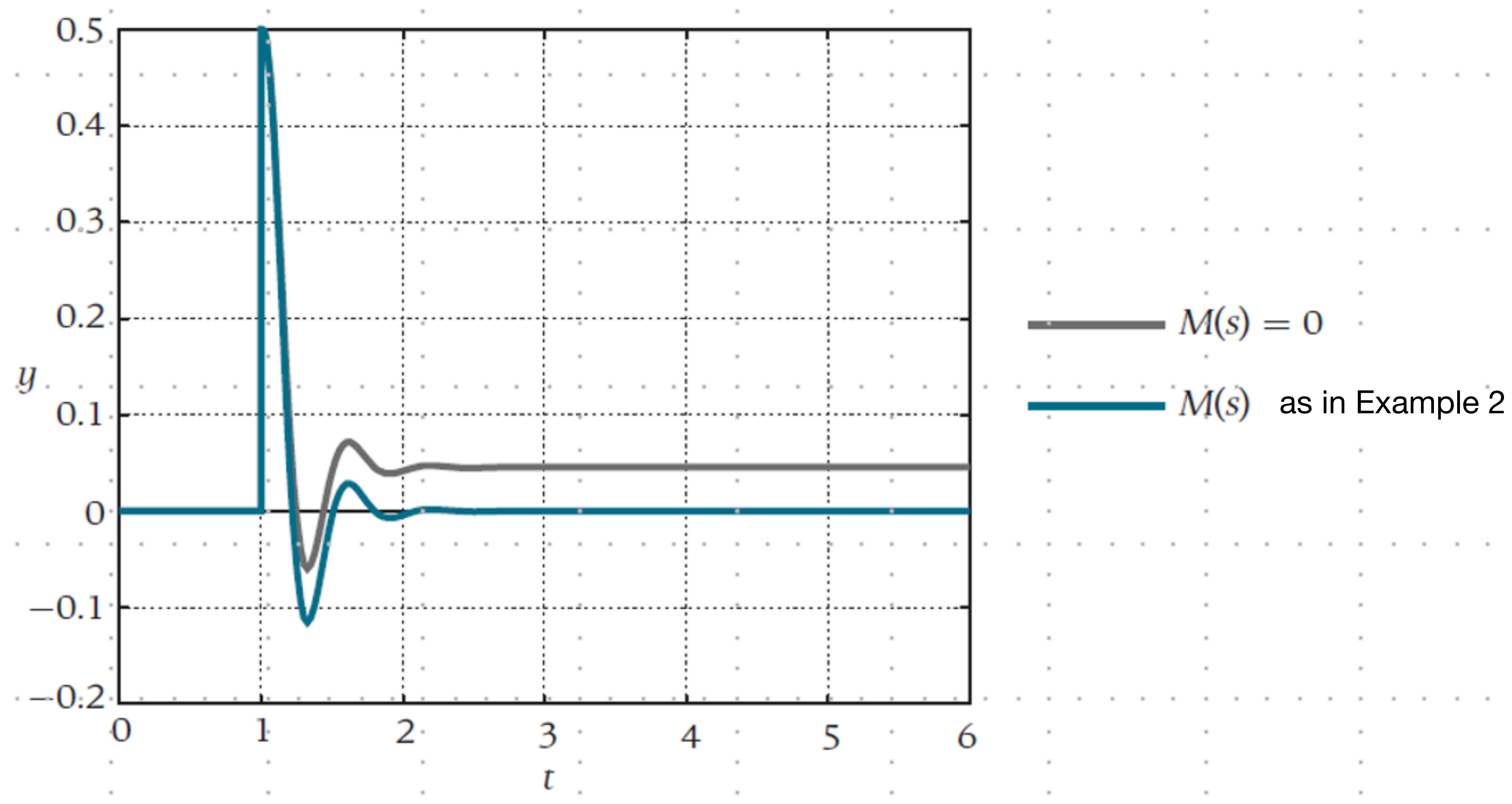
$$M(j\omega)\Big|_{\omega=0} = -\frac{H(0)}{G(0)}$$

Control Scheme with Measurable Disturbance Compensation

Solution 2



Response to a step disturbance



Example 2:

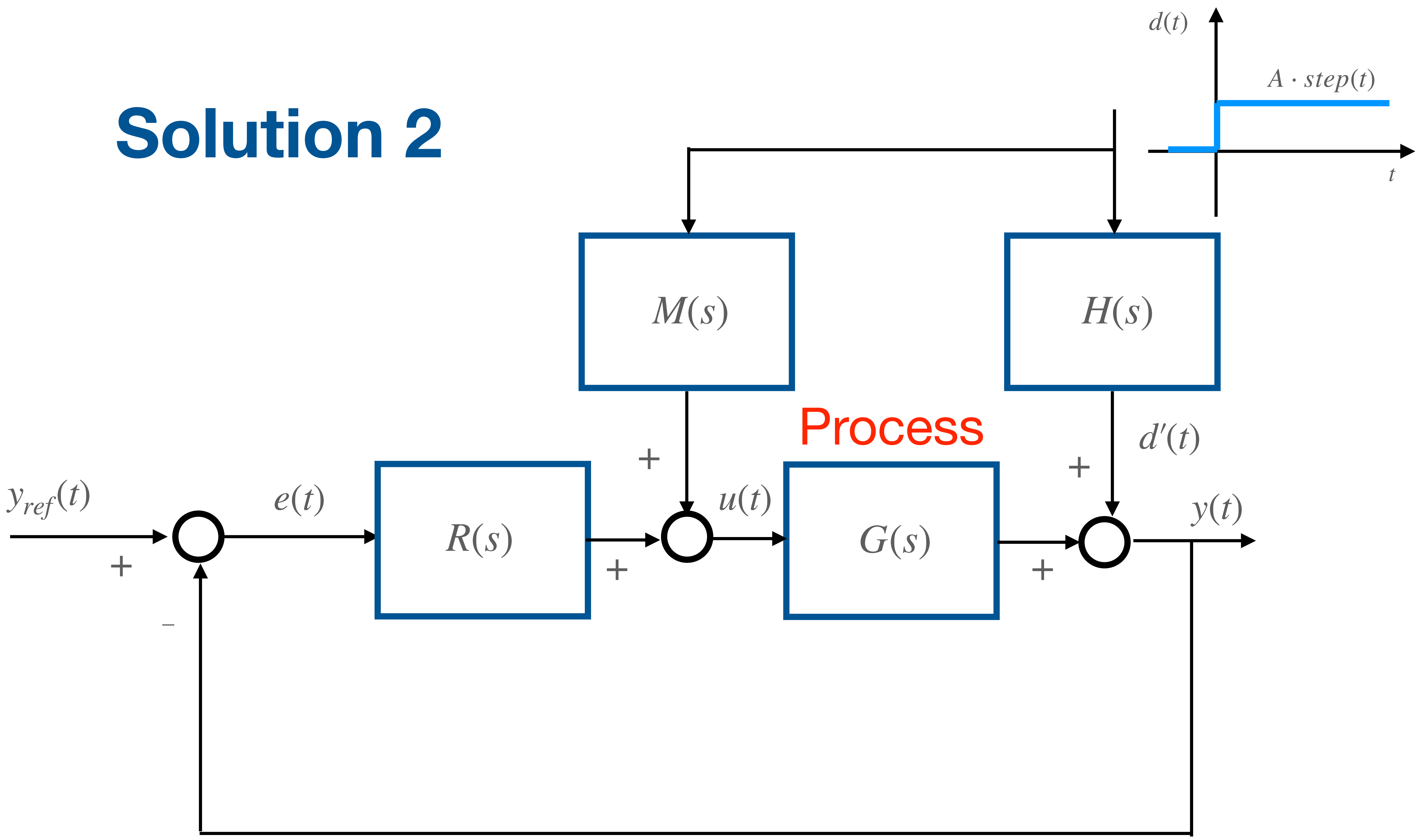
static case

$\hat{\omega} = 0$ → $M(s)$ s.t.

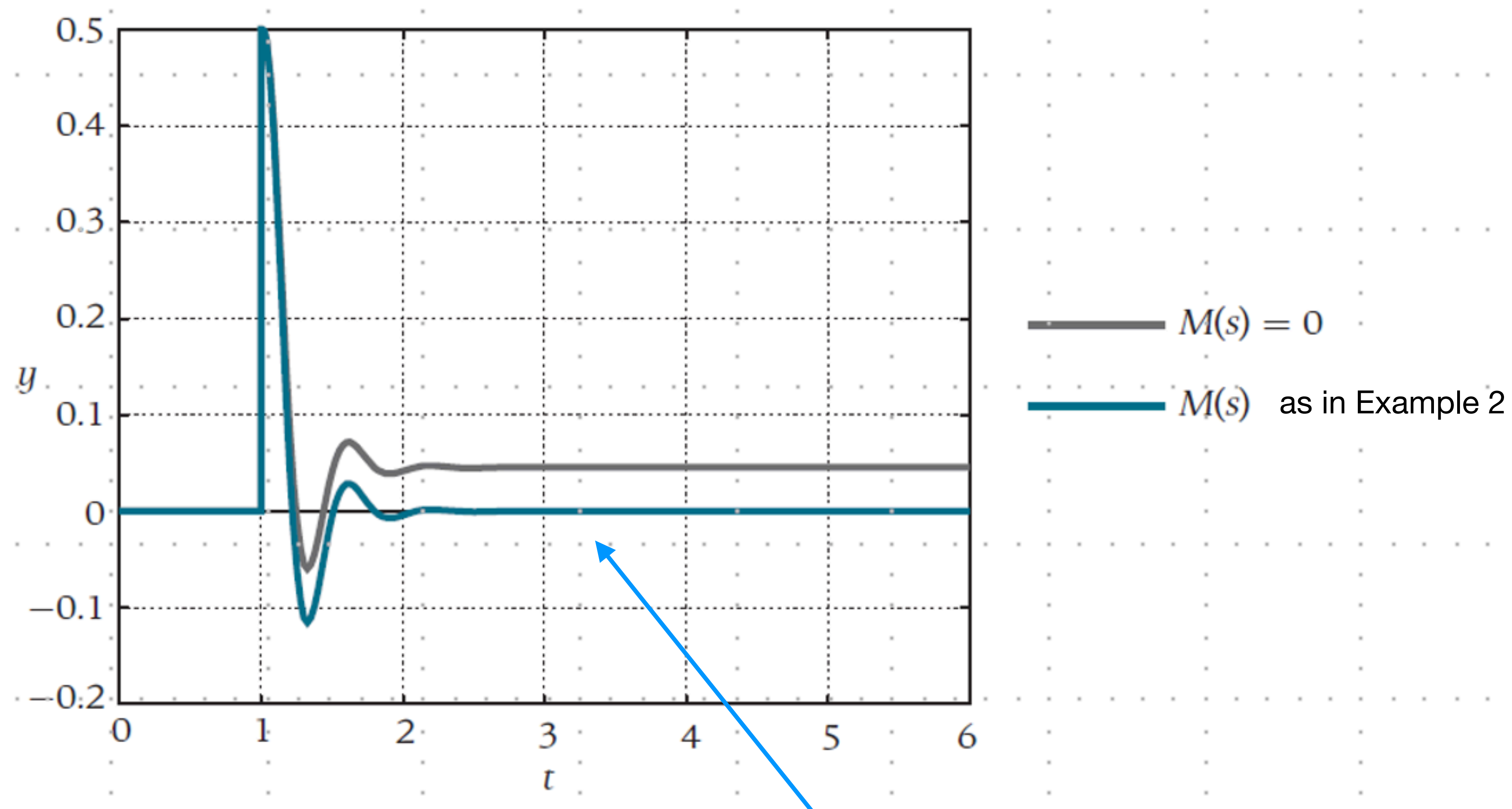
$$M(j\omega)\Big|_{\omega=0} = -\frac{H(0)}{G(0)}$$

Control Scheme with Measurable Disturbance Compensation

Solution 2



Response to a step disturbance



the disturbance is rejected in steady-state

Example 2:

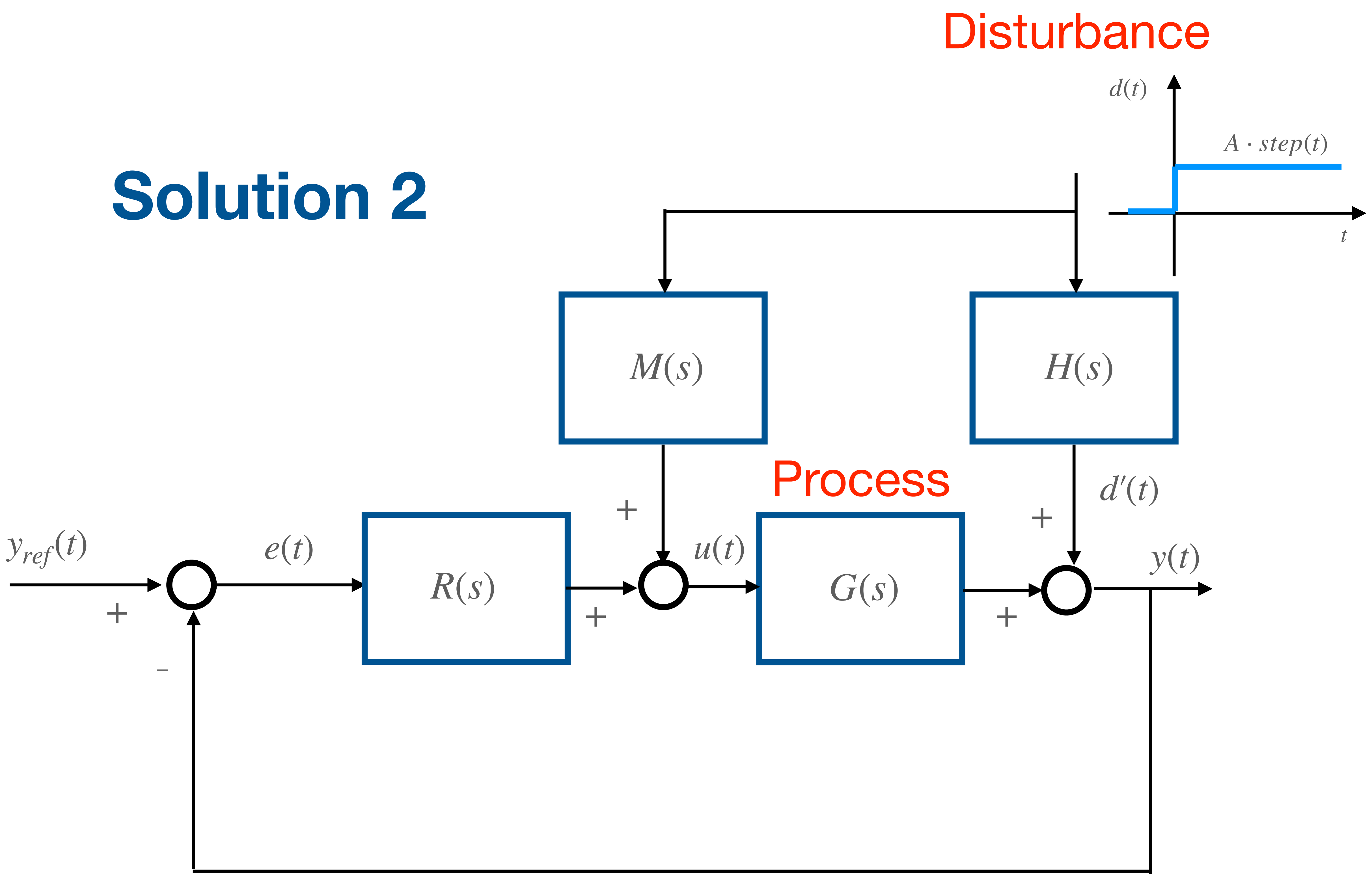
static case

$\hat{\omega} = 0$ → $M(s)$ s.t.

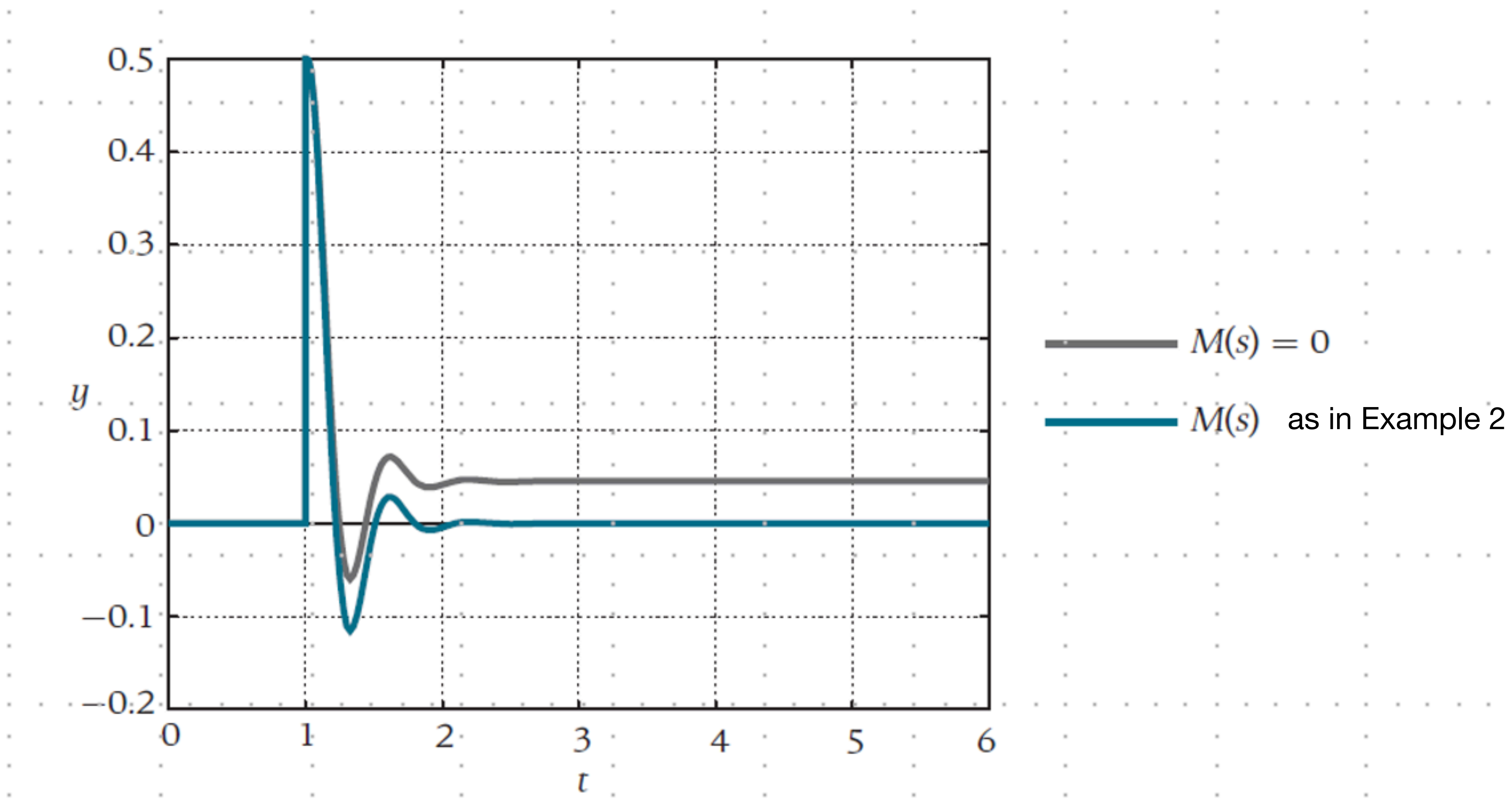
$$M(j\omega)\Big|_{\omega=0} = -\frac{H(0)}{G(0)}$$

Control Scheme with Measurable Disturbance Compensation

Solution 2



Response to a step disturbance



Example 2:

static case

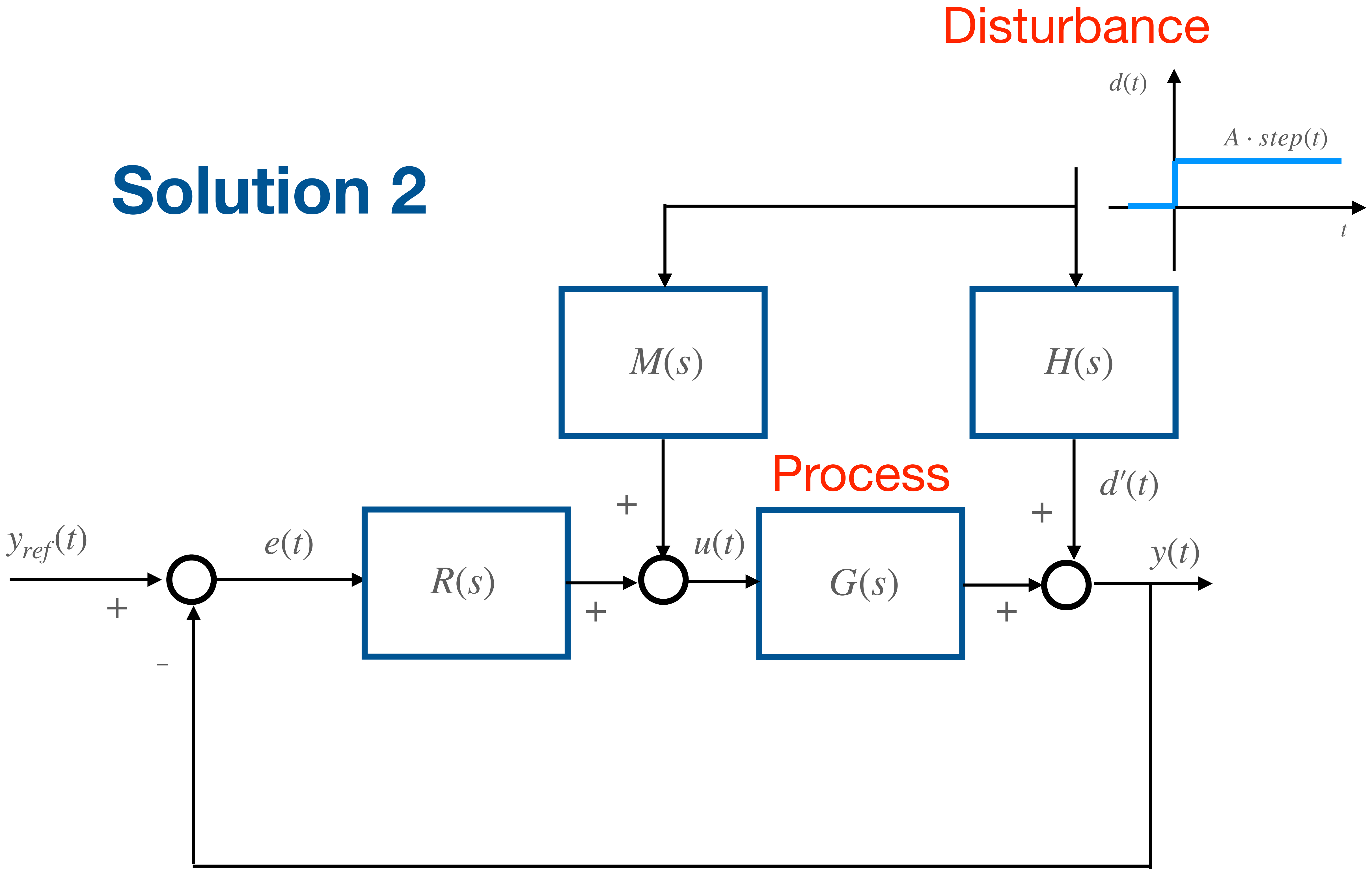
$\hat{\omega} = 0$ → $M(s)$ s.t.

$$M(j\omega)\Big|_{\omega=0} = -\frac{H(0)}{G(0)}$$

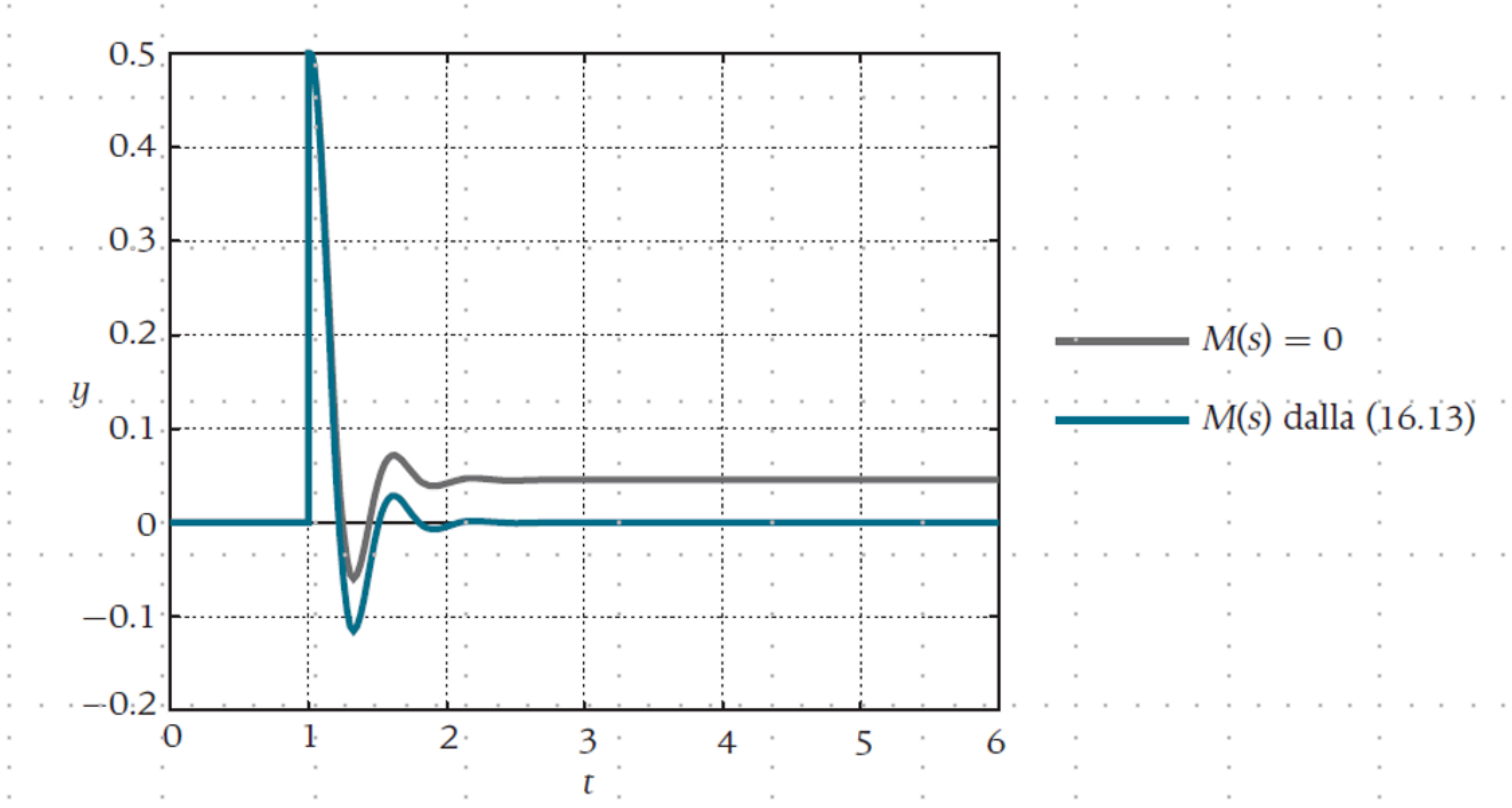
Is this static compensation robust?

Control Scheme with Measurable Disturbance Compensation

Solution 2



Response to a step disturbance



Example 2:
static case
$\hat{\omega} = 0$

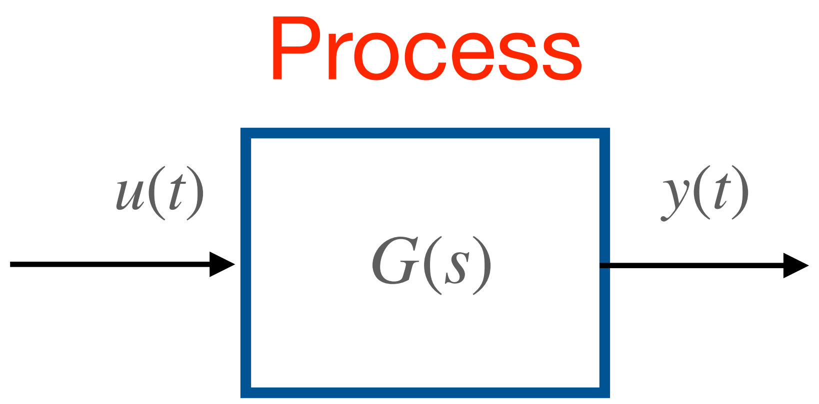
→ $M(s)$ s.t.

$$M(j\omega)\Big|_{\omega=0} = -\frac{H(0)}{G(0)}$$

Is this static compensation robust?

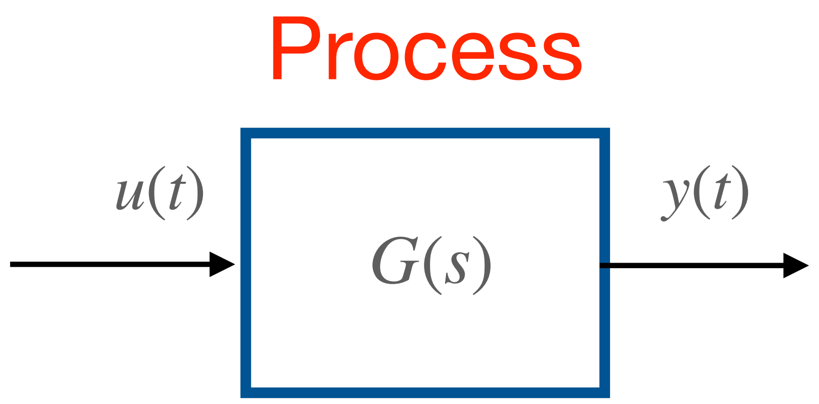
Is there a better alternative?

Control of LTI Systems with Delays



$$G(s) = e^{-\tau s}\,\frac{N(s)}{D(s)} = e^{-\tau s}\,G'(s)$$

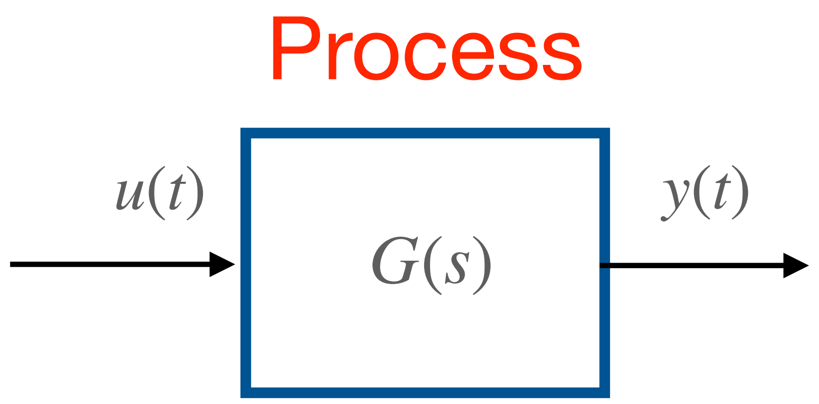
Control of LTI Systems with Delays



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}\,G'(s)$$

Image credit to: https://www.gramconveyor.com/how-to-install-conveyor-belt/

Control of LTI Systems with Delays



$$G(s) = e^{-\tau s} \frac{N(s)}{D(s)} = e^{-\tau s} G'(s)$$

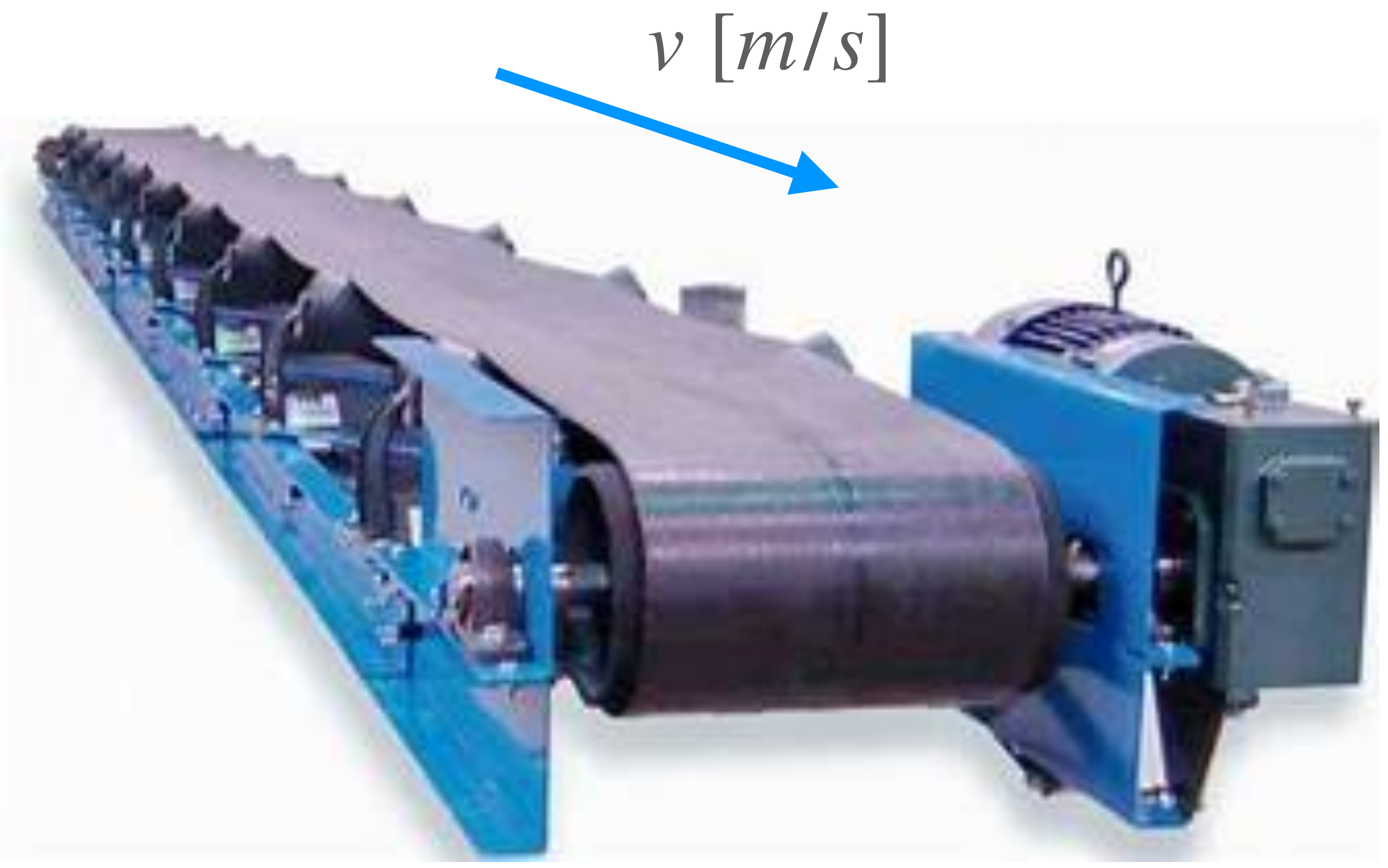
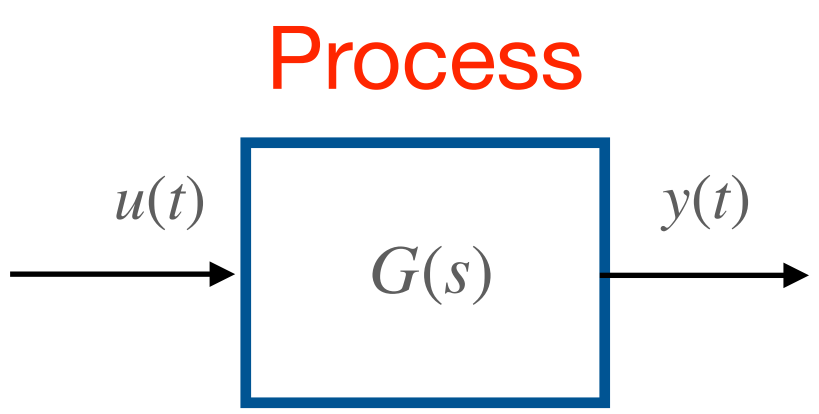


Image credit to: https://www.gramconveyor.com/how-to-install-conveyor-belt/

Control of LTI Systems with Delays



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}G'(s)$$

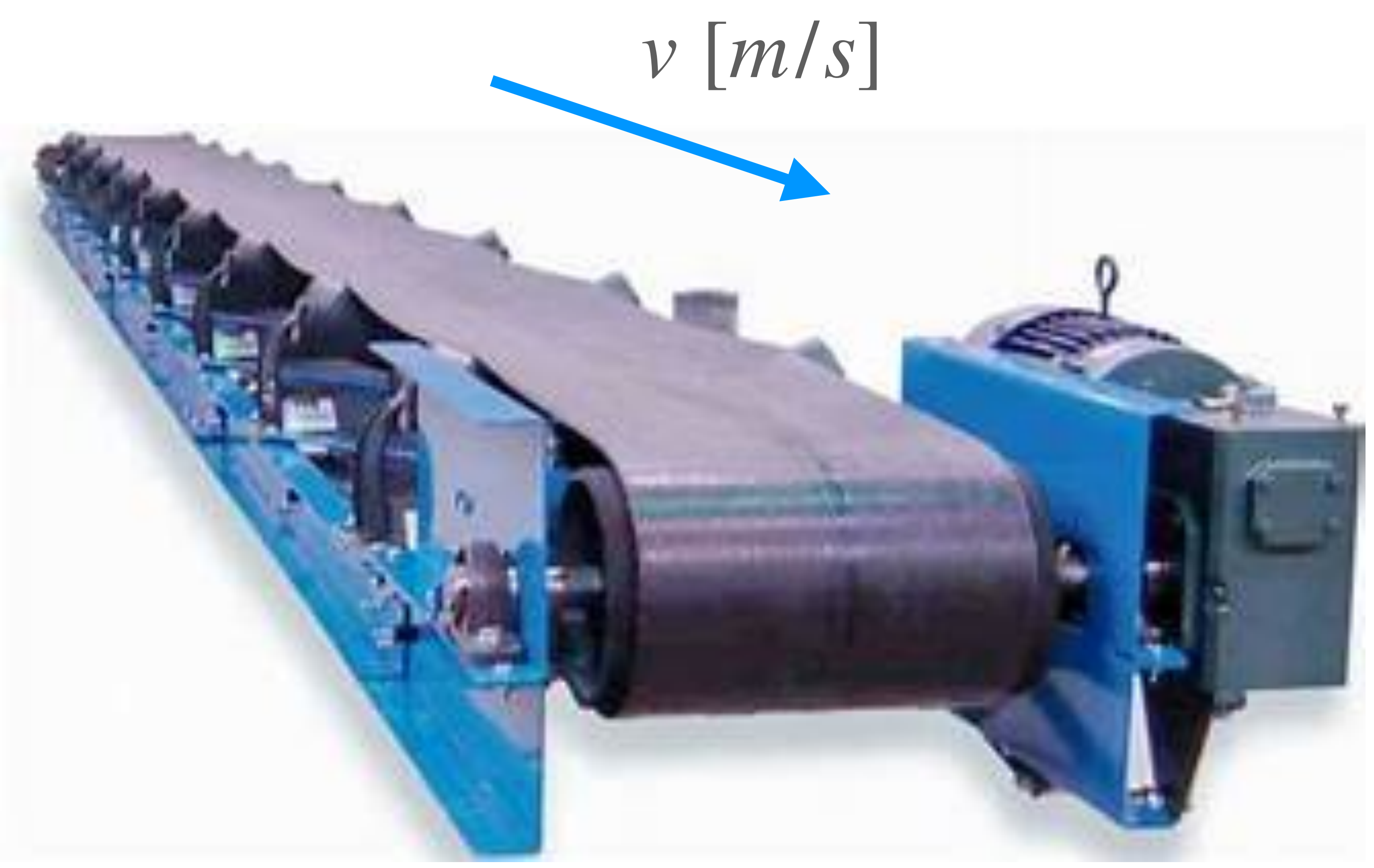
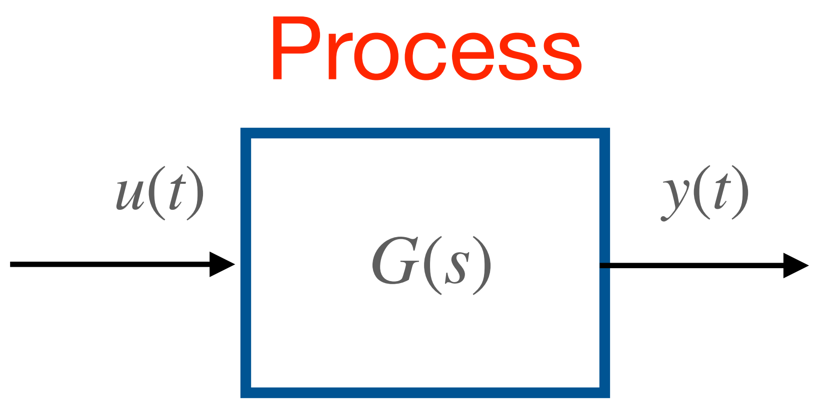


Image credit to: https://www.gramconveyor.com/how-to-install-conveyor-belt/

$$q_{OUT}(t) = q_{IN}(t - \tau) \qquad \tau = \frac{L}{v}$$

Control of LTI Systems with Delays



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}\,G'(s)$$

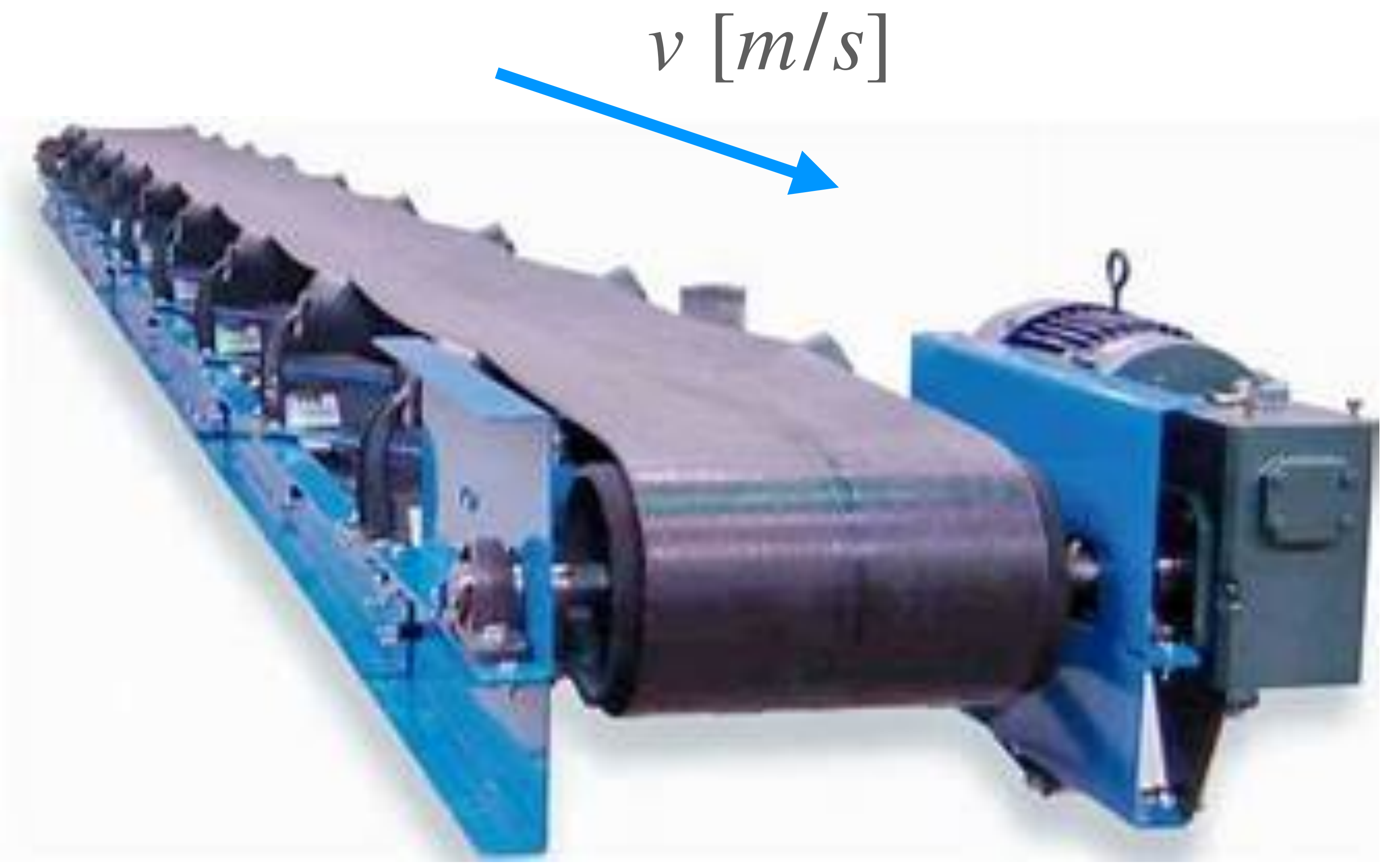


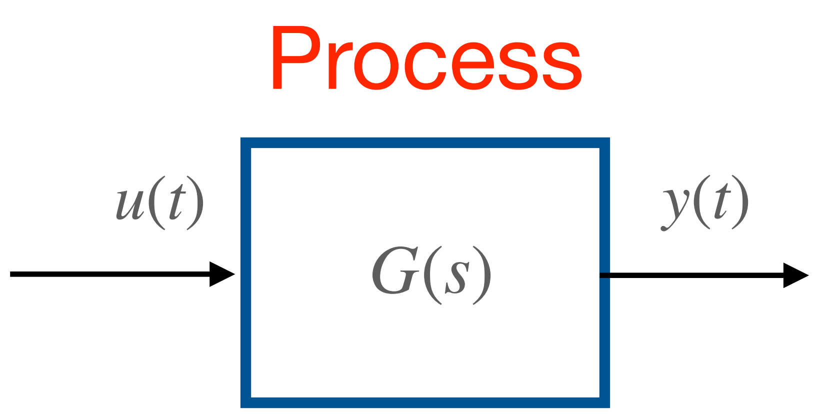
Image credit to: https://www.gramconveyor.com/how-to-install-conveyor-belt/

$$q_{OUT}(t) = q_{IN}(t-\tau) \qquad\qquad \tau = \frac{L}{v}$$

$\mathscr{L}$

$$Q_{OUT}(s) = e^{-\tau s}Q_{IN}(s)$$

Control of LTI Systems with Delays



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}\,G'(s)$$

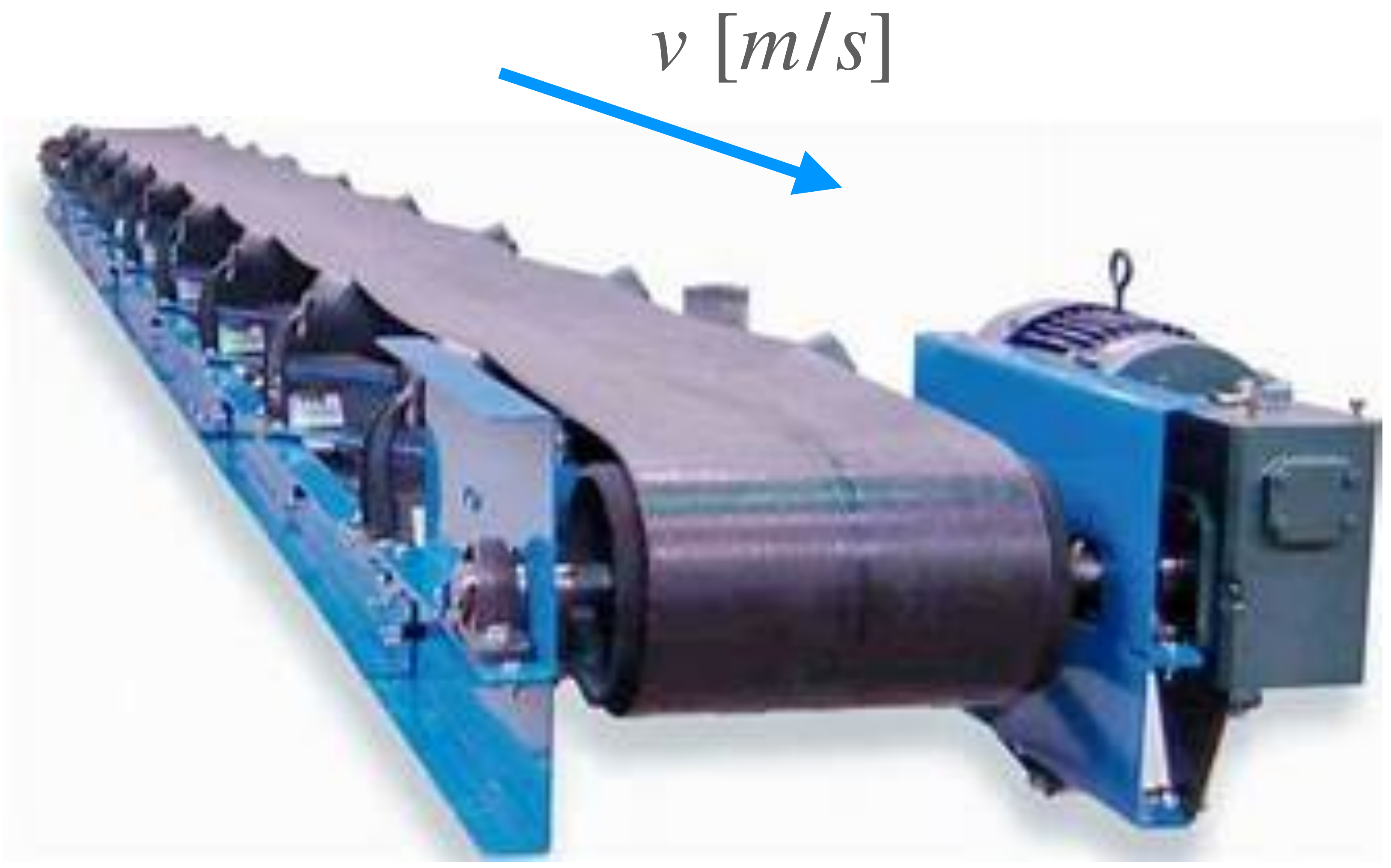


Image credit to: https://www.gramconveyor.com/how-to-install-conveyor-belt/

$$q_{OUT}(t) = q_{IN}(t-\tau) \qquad \tau = \frac{L}{v}$$

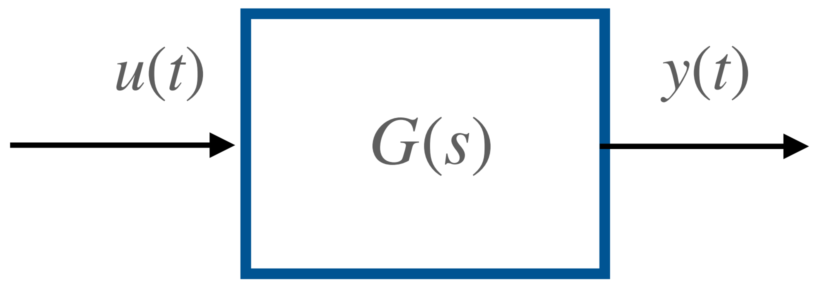
$$\mathscr{L}$$

$$Q_{OUT}(s) = e^{-\tau s}Q_{IN}(s)$$

$$G(s) = e^{-\tau s}$$

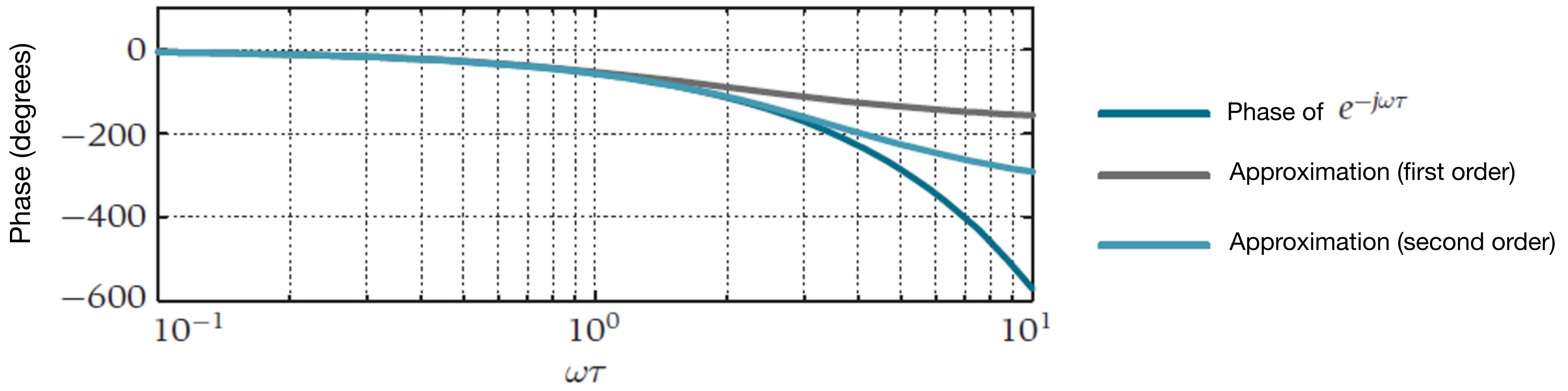
Control of LTI Systems with Delays

Process



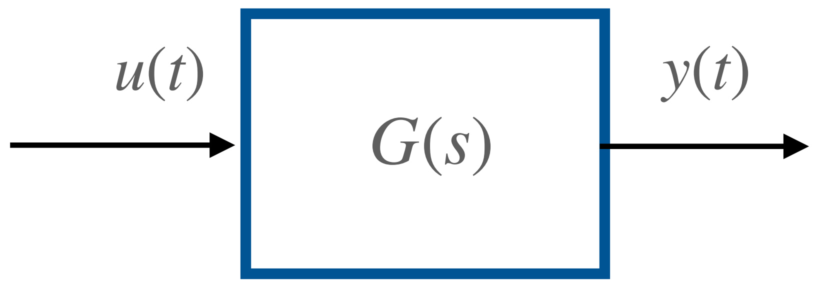
$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}\,G'(s)$$

Phase Lag due to the delay term



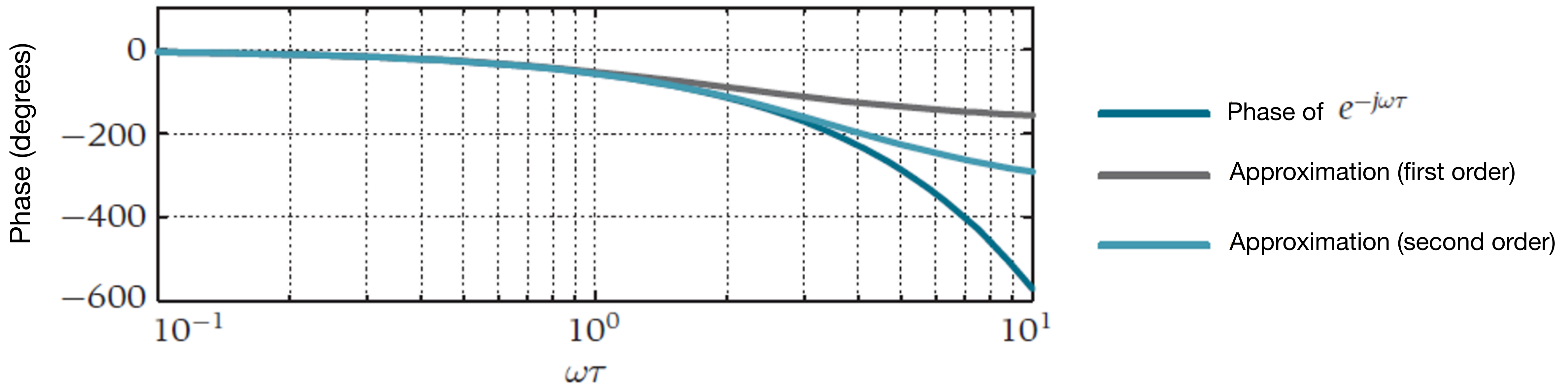
Control of LTI Systems with Delays

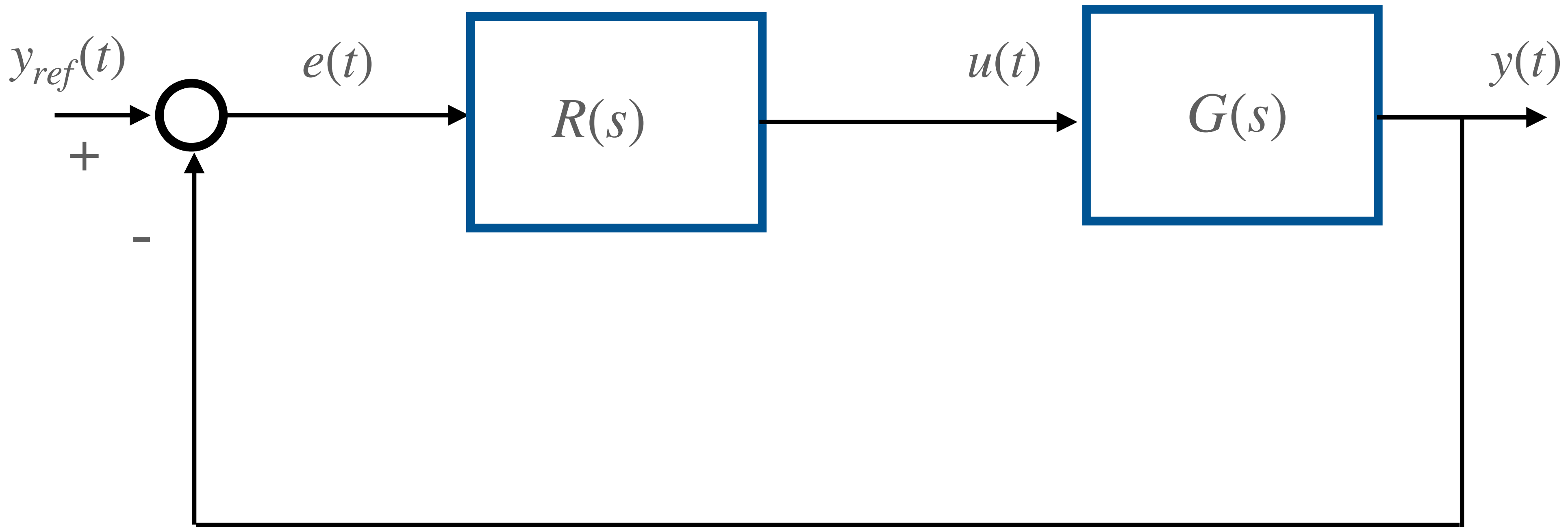
Process



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}G'(s)$$

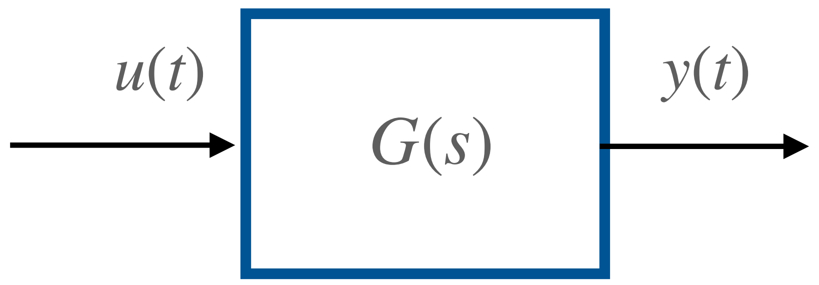
Phase Lag due to the delay term





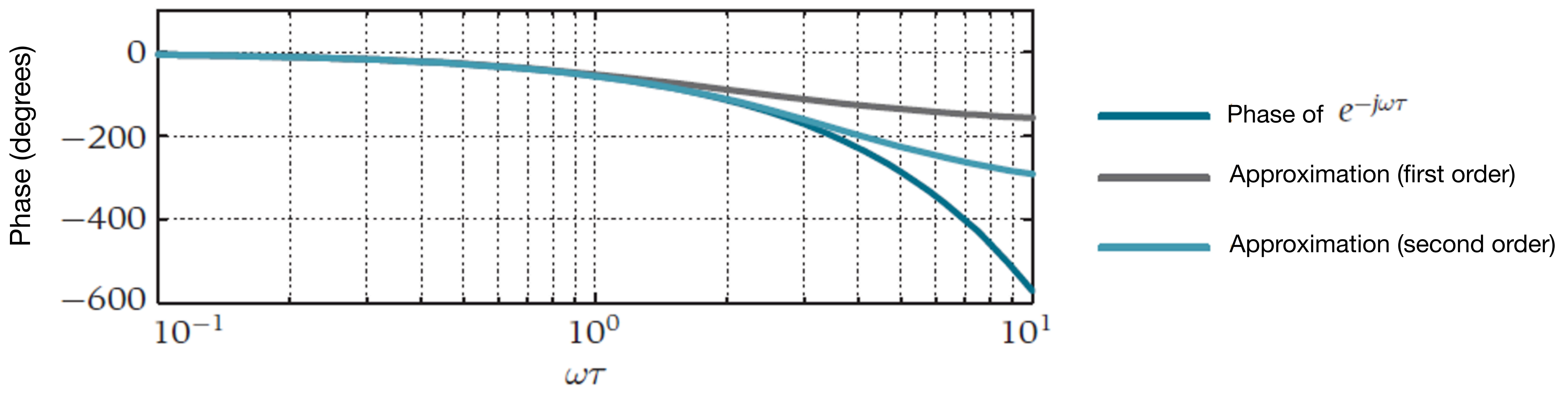
Control of LTI Systems with Delays

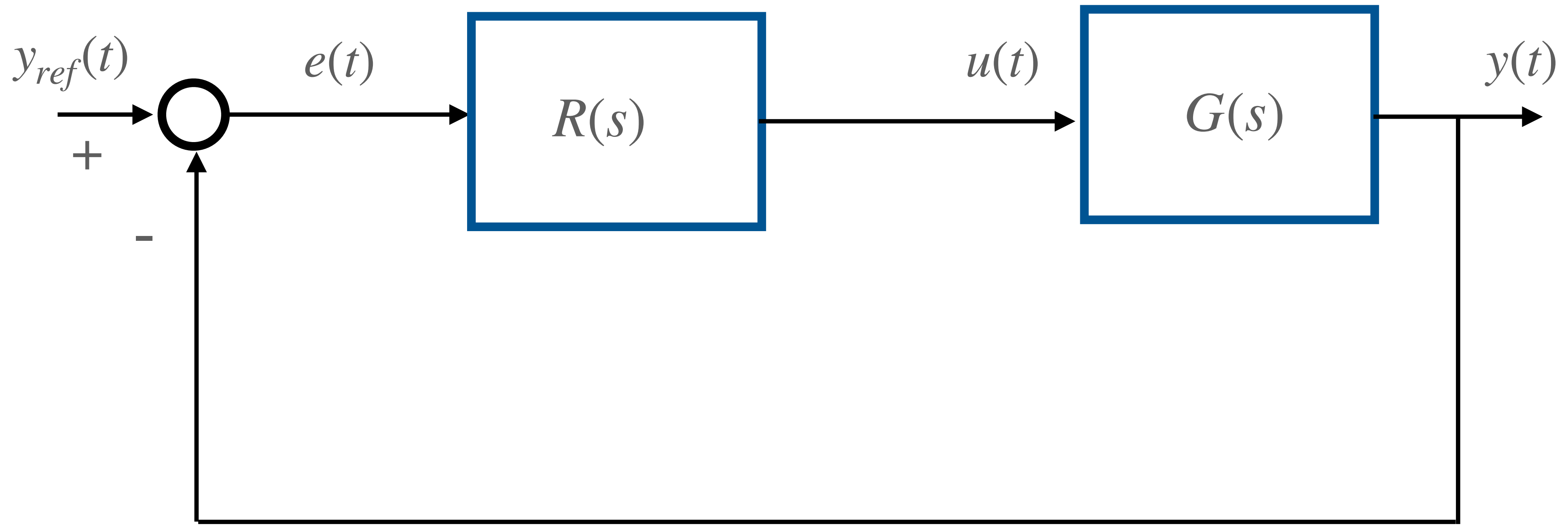
Process



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}\,G'(s)$$

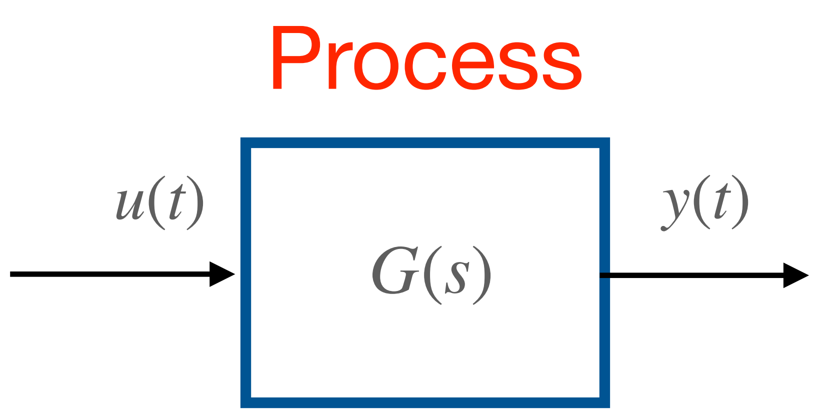
Phase Lag due to the delay term





Recommendation: $\omega_{BW_{CL}}$ sufficiently narrow

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme



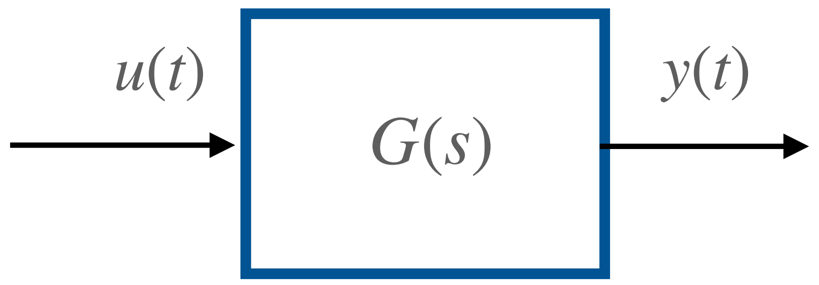
$$G(s) = e^{-\tau s} \frac{N(s)}{D(s)} = e^{-\tau s} G'(s)$$

Assumptions:

- $G'(s)$ As. Stable
- τ known

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

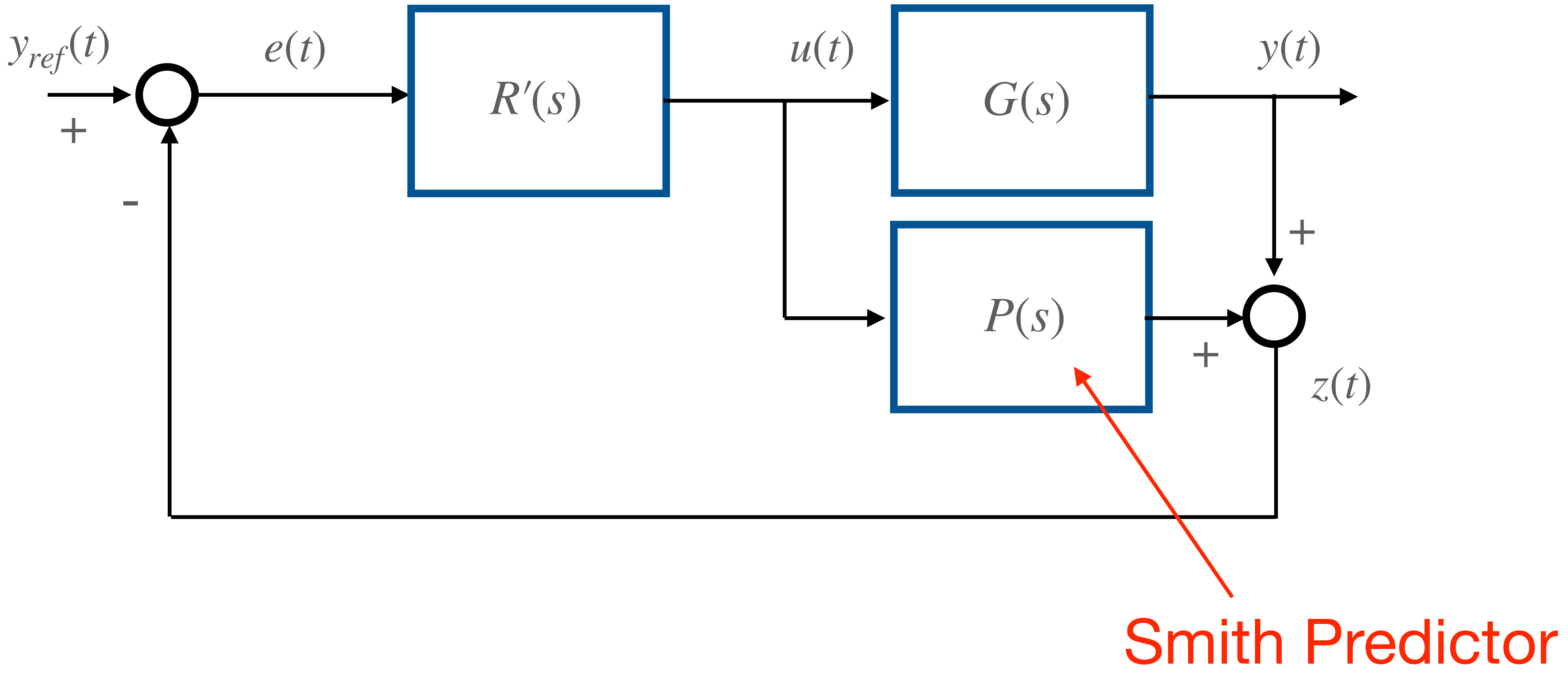
Process



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}\,G'(s)$$

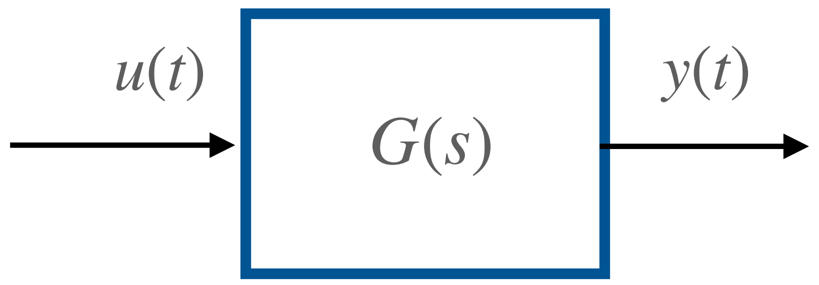
Assumptions:

- $G'(s)$ As. Stable
- τ known



Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

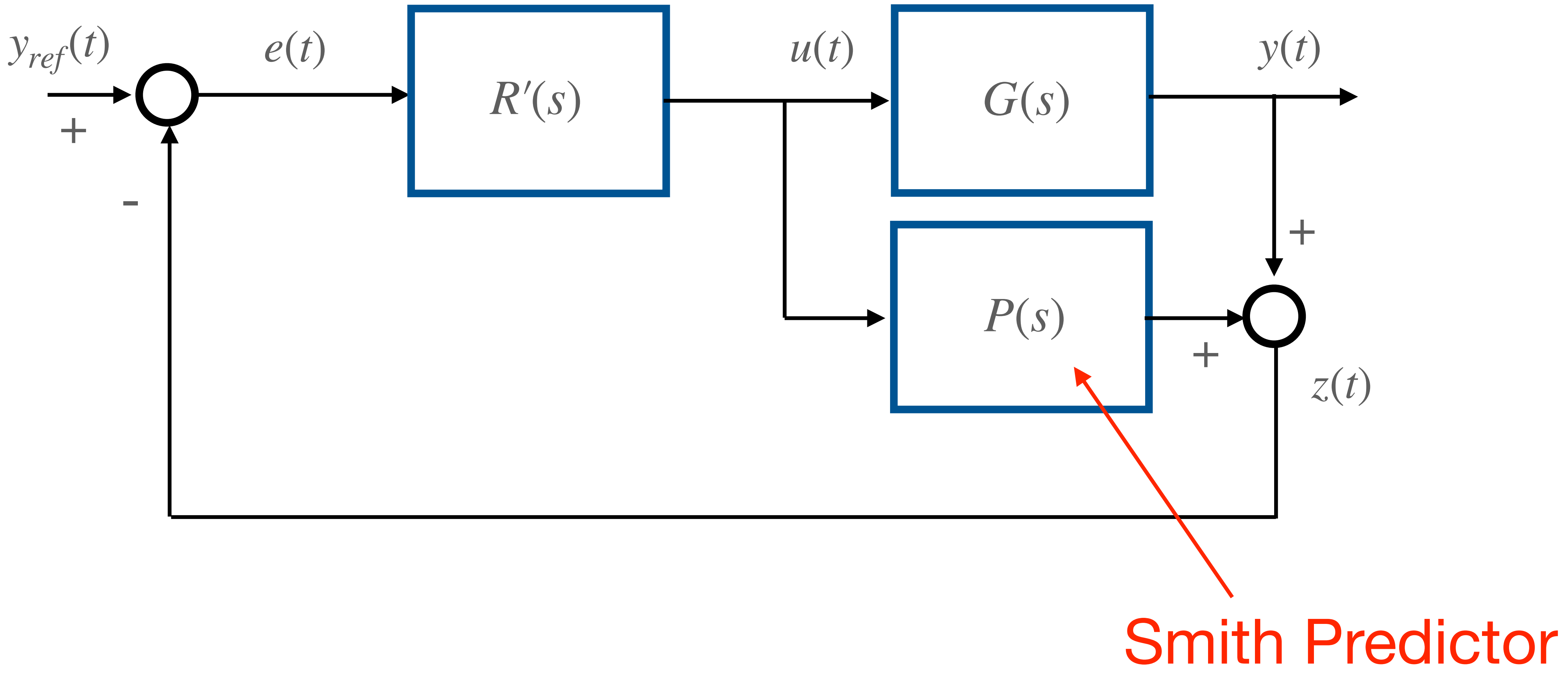
Process



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}\, G'(s)$$

Assumptions:

- $G'(s)$ As. Stable
- τ known



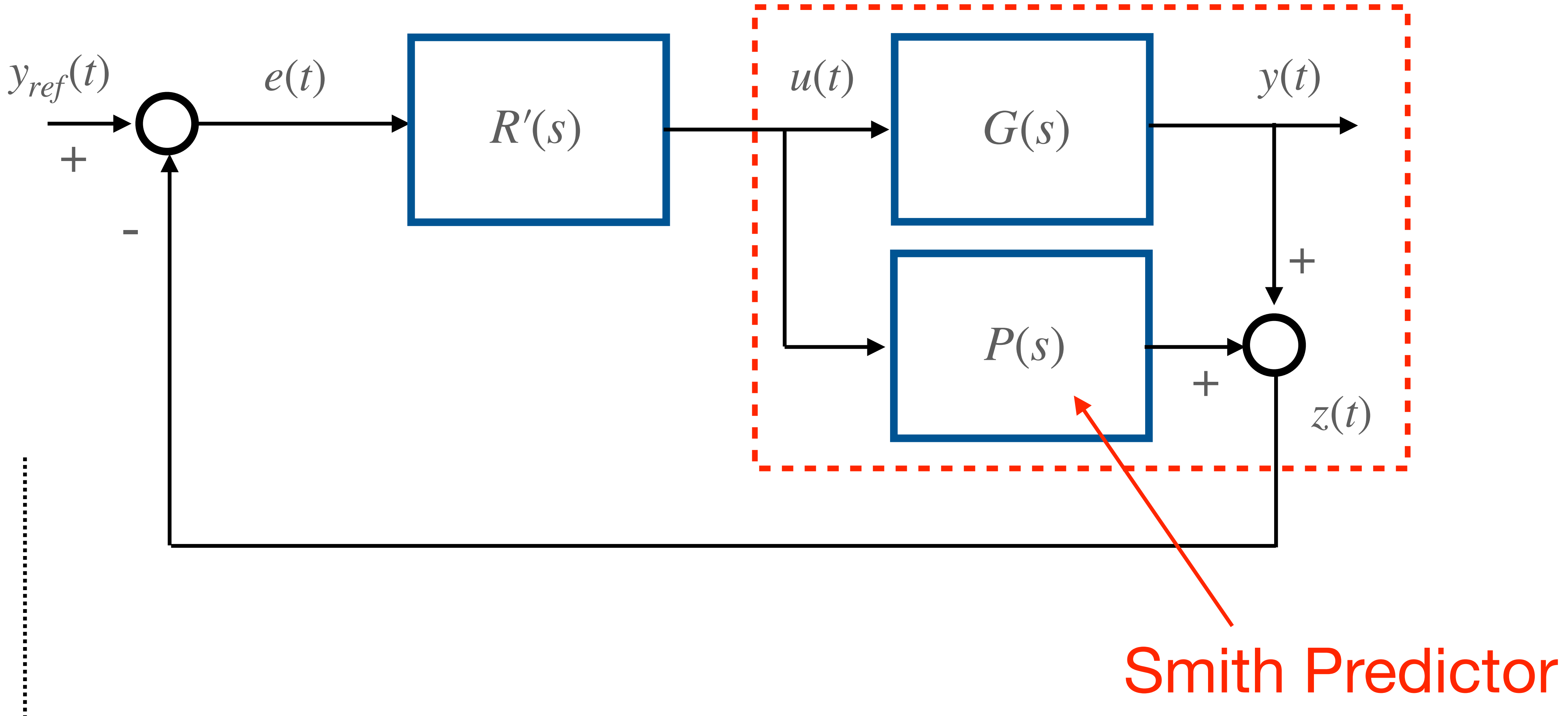
$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

$$u \to z: \quad G(s) + P(s) = e^{-\tau s}\, G'(s) + (1 - e^{-\tau s})\, G'(s)$$

$$= e^{-\tau s}\, G'(s) + G'(s) - e^{-\tau s}\, G'(s)$$

$$= G'(s)$$



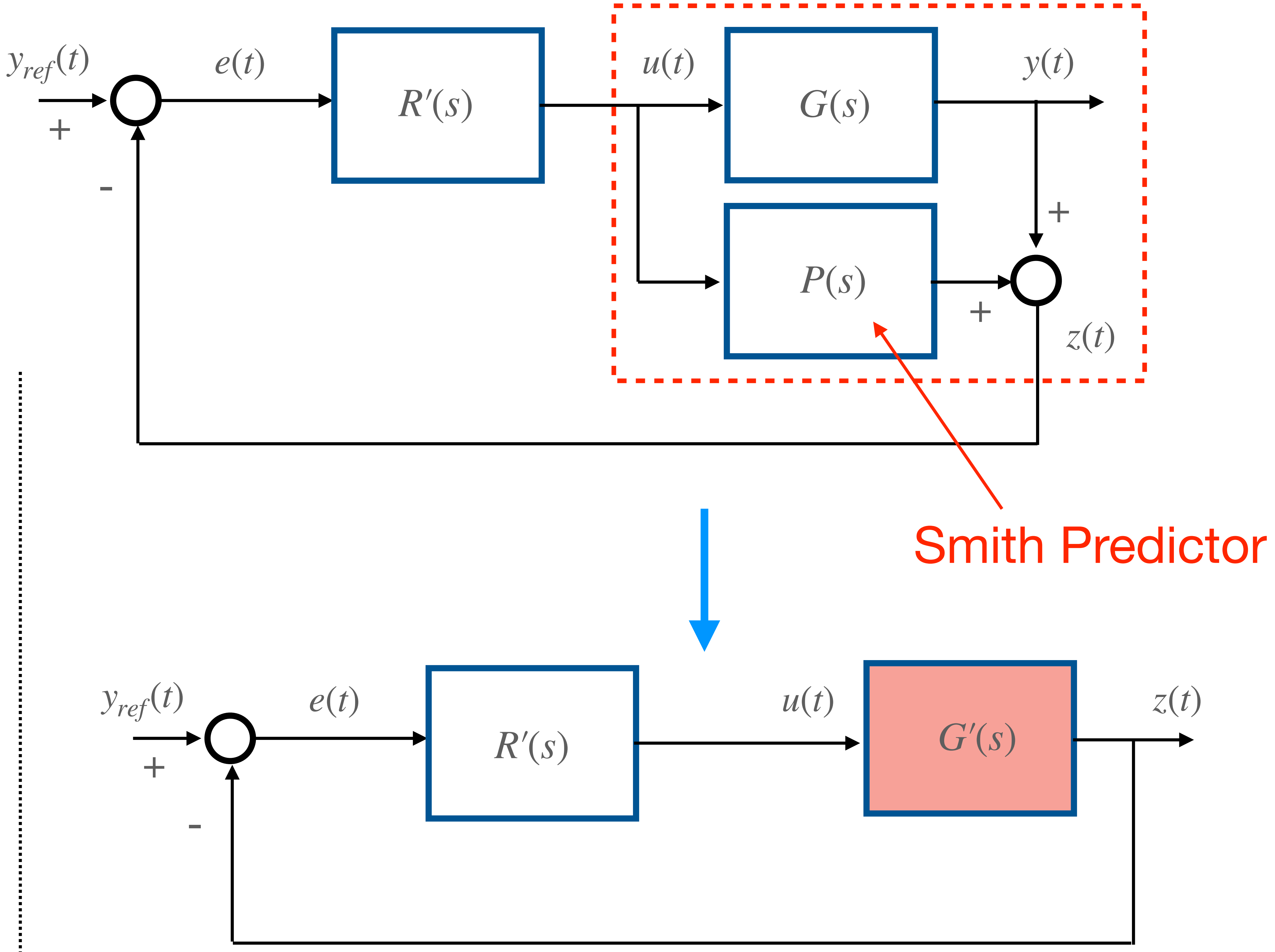
$$P(s) = (1 - e^{-\tau s})\, \frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

$$u \rightarrow z: \quad G(s) + P(s) = e^{-\tau s}\, G'(s) + (1 - e^{-\tau s})\, G'(s)$$

$$= e^{-\tau s}\, G'(s) + G'(s) - e^{-\tau s}\, G'(s)$$

$$= G'(s)$$



Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

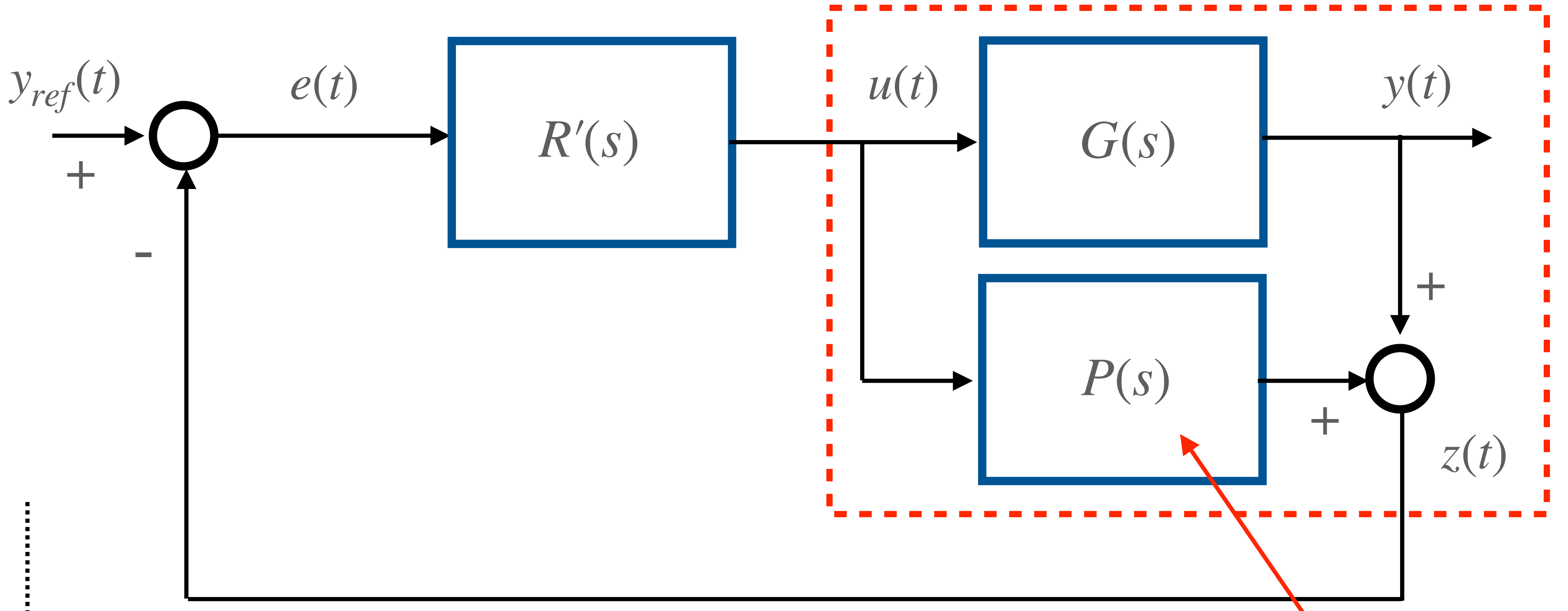
$$u \rightarrow z: \quad G(s) + P(s) = e^{-\tau s}\, G'(s) + (1 - e^{-\tau s})\, G'(s)$$

$$= e^{-\tau s}\, G'(s) + G'(s) - e^{-\tau s}\, G'(s)$$

$$= G'(s)$$

Warning:

$$\boxed{z(t) \neq y(t)}$$

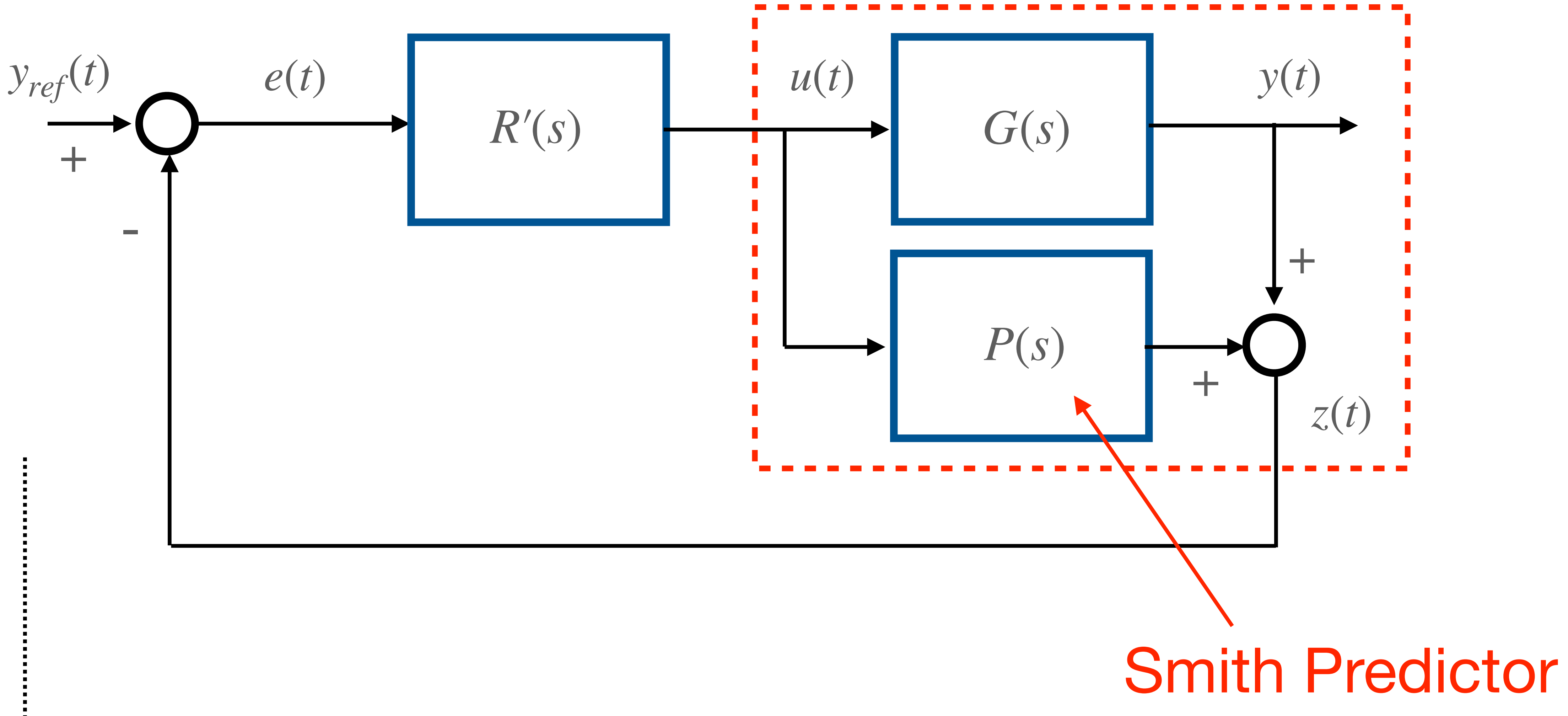


Smith Predictor

$$P(s) = (1 - e^{-\tau s})\,\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

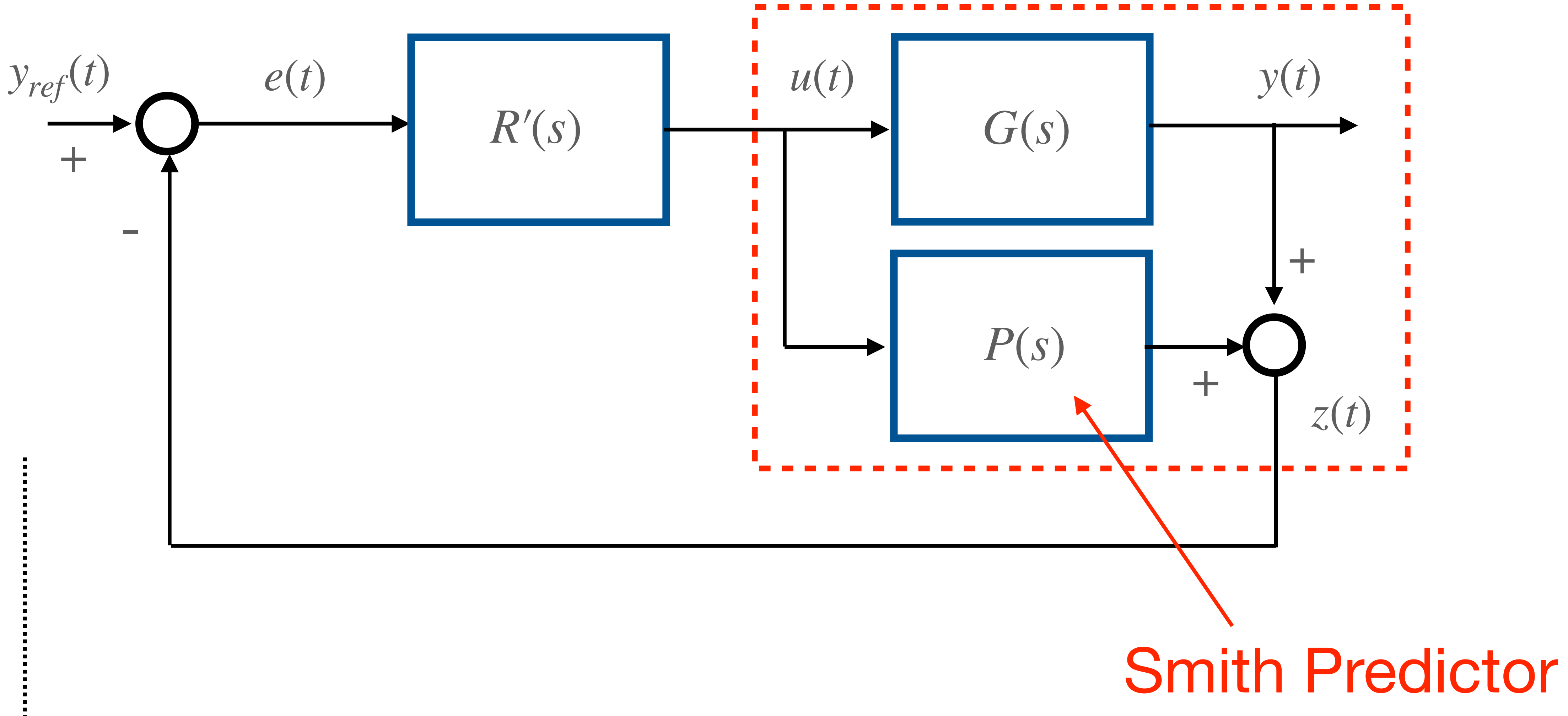
$$\boxed{z(t) \neq y(t)} \longrightarrow z(t)\ ?$$



$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

$$\boxed{z(t) \neq y(t)} \rightarrow z(t)\ ?$$



In open loop:

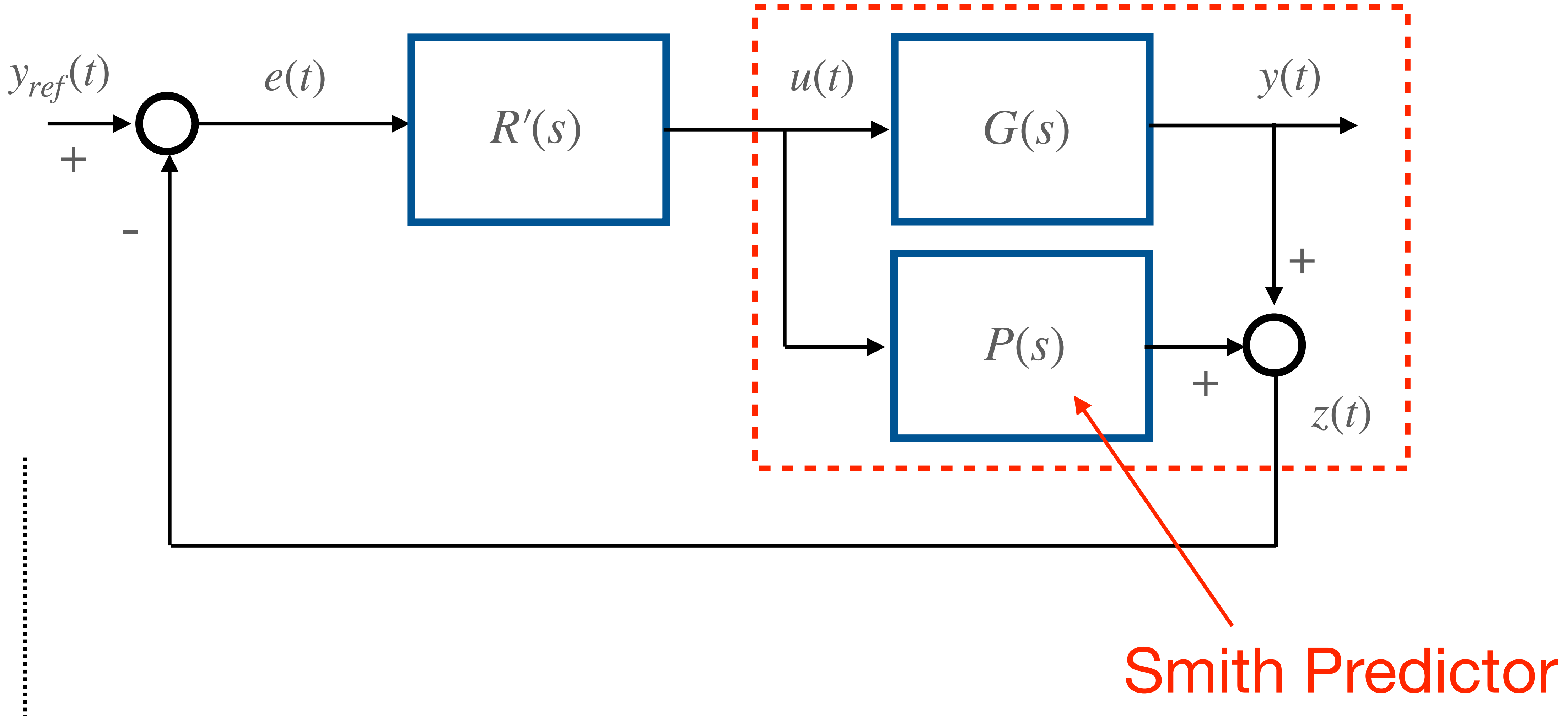
$$u \rightarrow z: \quad G'(s)$$

$$u \rightarrow y: \quad e^{-\tau s}\, G'(s)$$

$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

$\boxed{z(t) \neq y(t)}$ → $z(t)$?



In open loop:

$$u \to z: \quad G'(s)$$

$$u \to y: \quad e^{-\tau s}\, G'(s)$$

$\mathscr{L}$

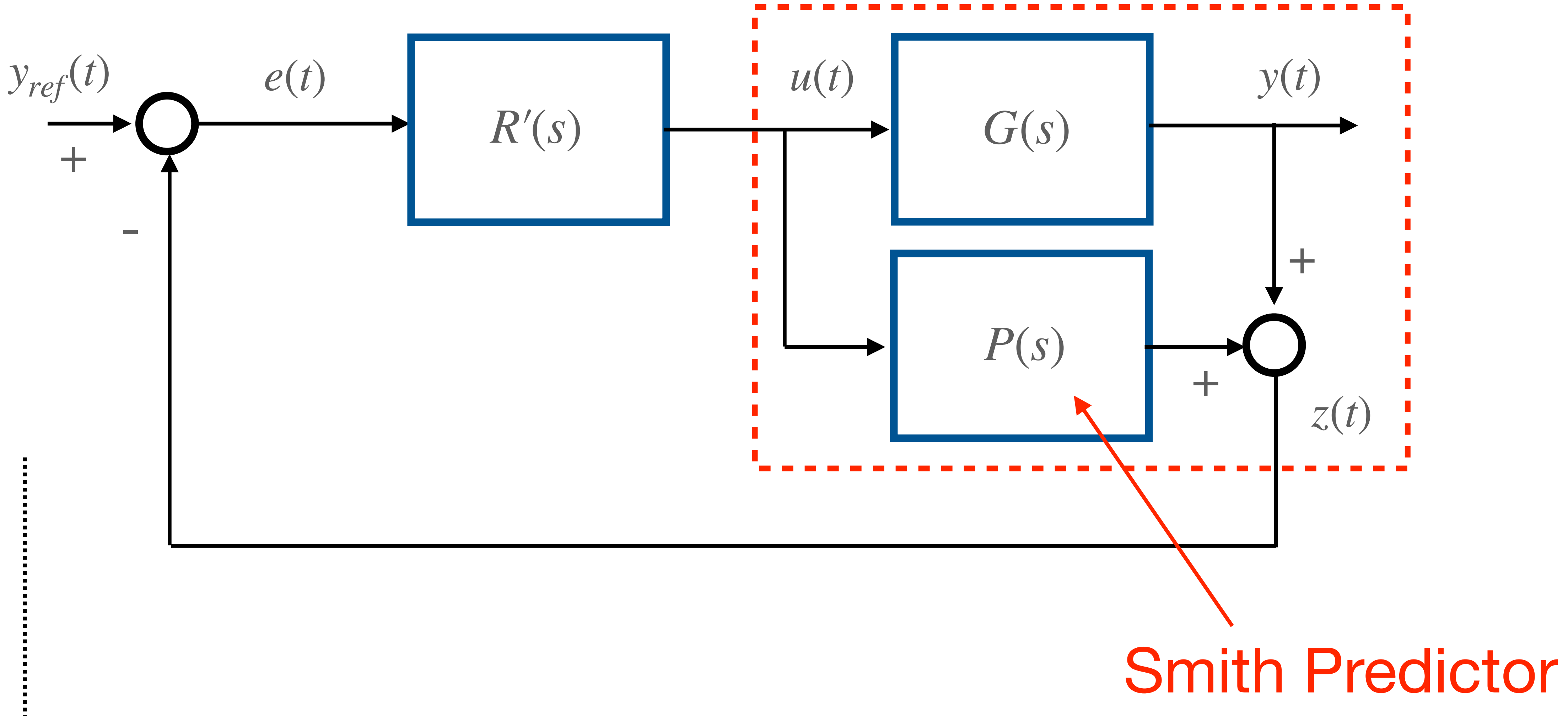
$$Z(s) = G'(s)\, U(s)$$

$$Y(s) = e^{-\tau s}\, G'(s)\, U(s)$$

$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

$\boxed{z(t) \neq y(t)}$ → $z(t)$?



In open loop:

$u \to z: \quad G'(s)$

$u \to y: \quad e^{-\tau s}\, G'(s)$

$\mathscr{L}$

$Z(s) = G'(s)\, U(s)$

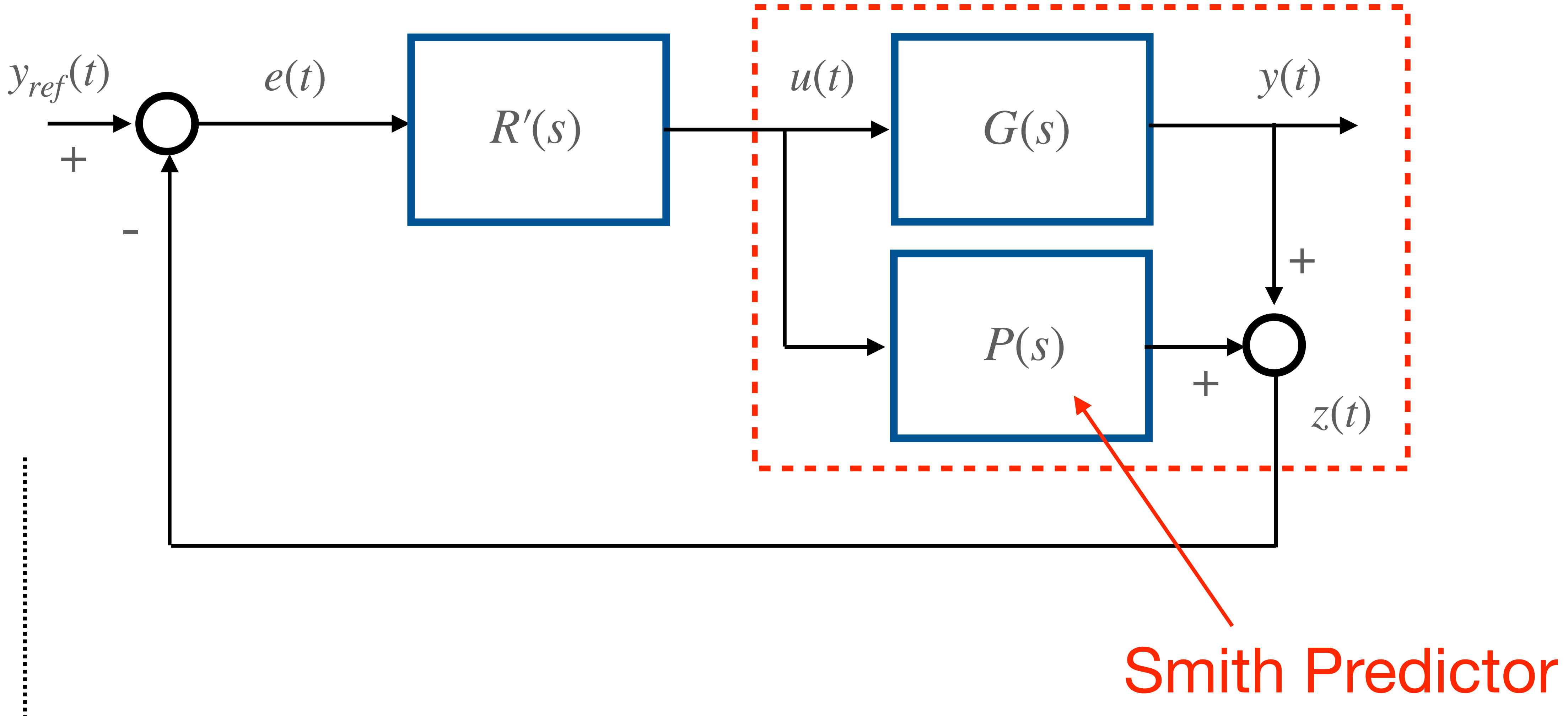
$Y(s) = e^{-\tau s}\, G'(s)\, U(s)$

$\boxed{Z(s) = e^{\tau s}\, Y(s)}$

$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

$\boxed{z(t) \neq y(t)}$ → $z(t)$?



In open loop:

$$u \rightarrow z: \quad G'(s)$$

$$u \rightarrow y: \quad e^{-\tau s}\, G'(s)$$

$\mathscr{L}$

$$Z(s) = G'(s)\, U(s)$$

$$Y(s) = e^{-\tau s}\, G'(s)\, U(s)$$

$z(t)$ is a prediction of $y(t)$

$$\boxed{Z(s) = e^{\tau s}\, Y(s)}$$

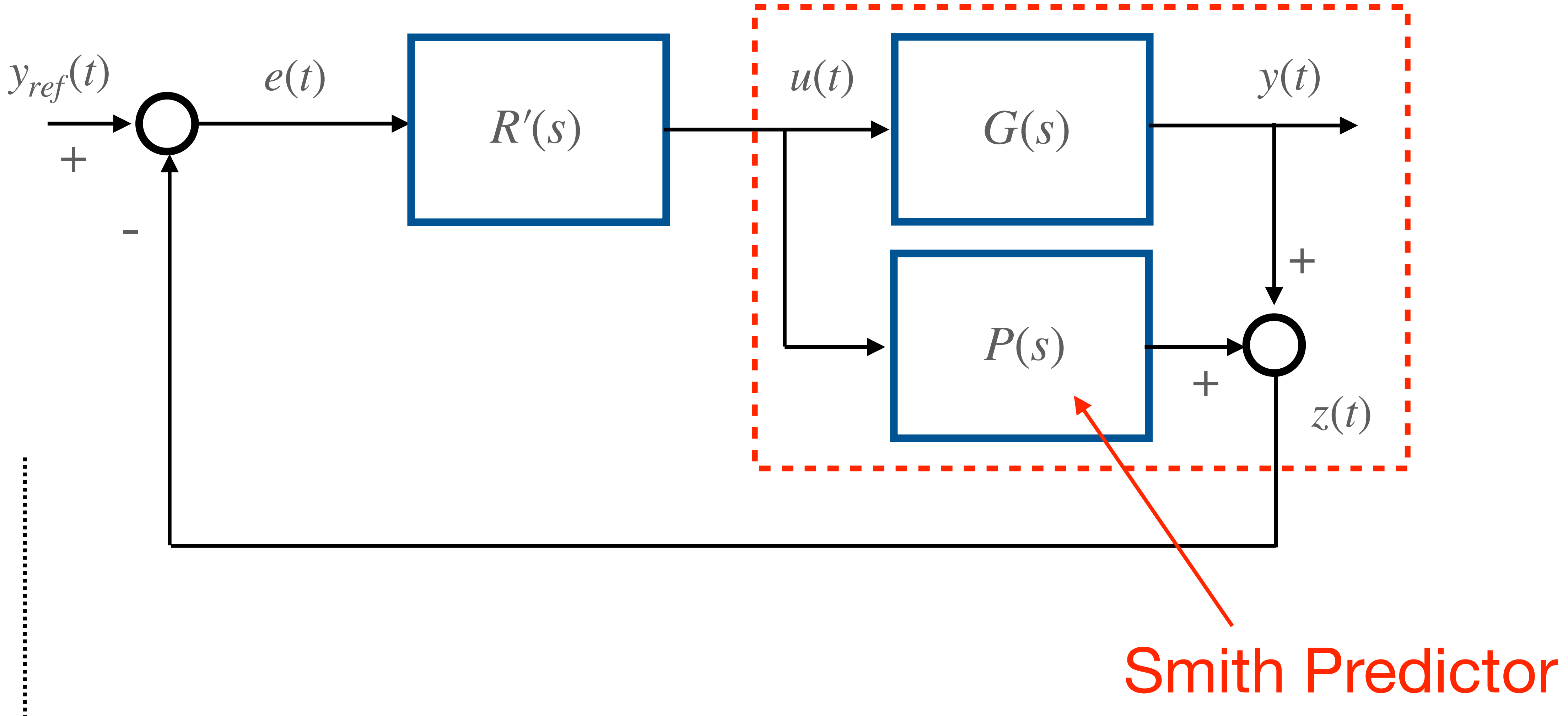
$$P(s) = (1 - e^{-\tau s})\,\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

Observation 1:

In the static case

$$P(0) = (1 - e^{-\tau 0})\, G'(0) = 0$$



$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

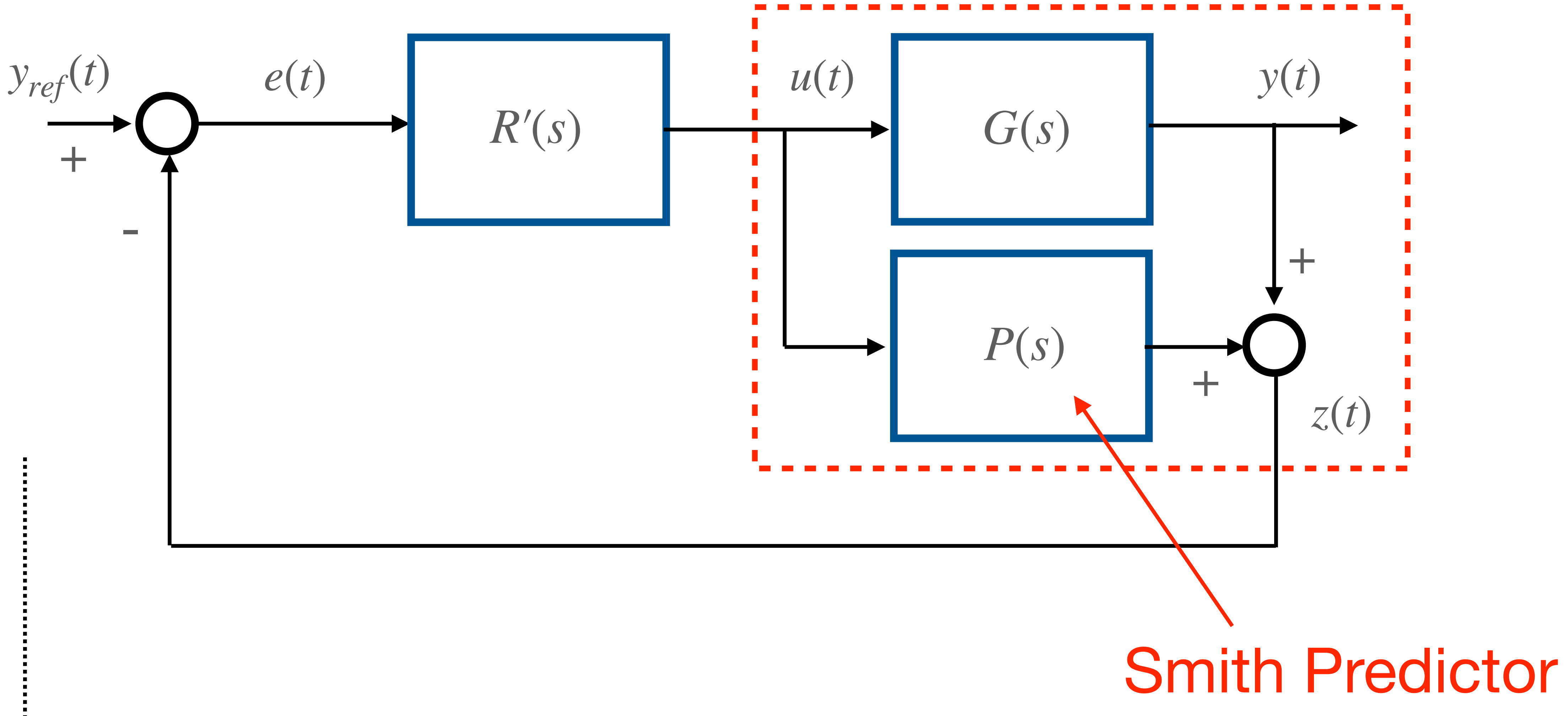
Observation 1:

In the static case

$$P(0) = (1 - e^{-\tau 0})\, G'(0) = 0$$

Then, if the closed loop system is As. Stable and $y_{ref}(t) = A\, step(t)$:

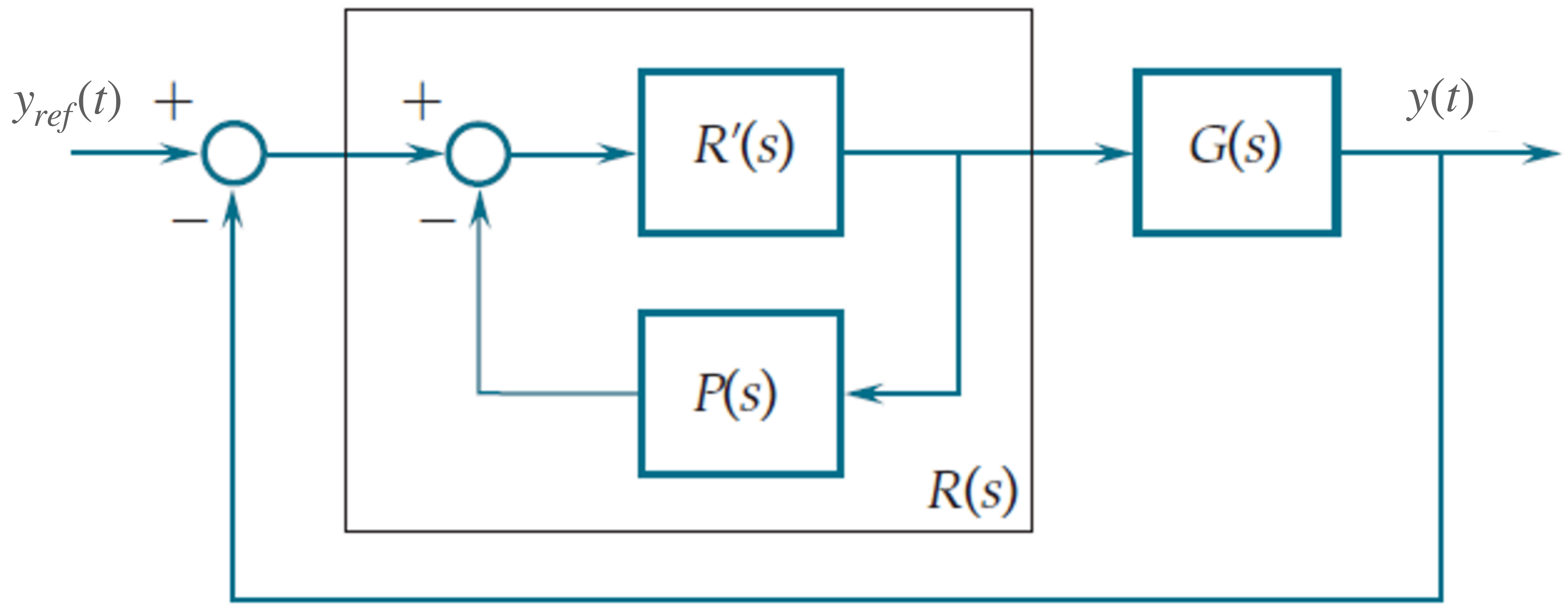
$$\boxed{z(t) \to y(t) \quad \text{for} \quad t \to \infty}$$

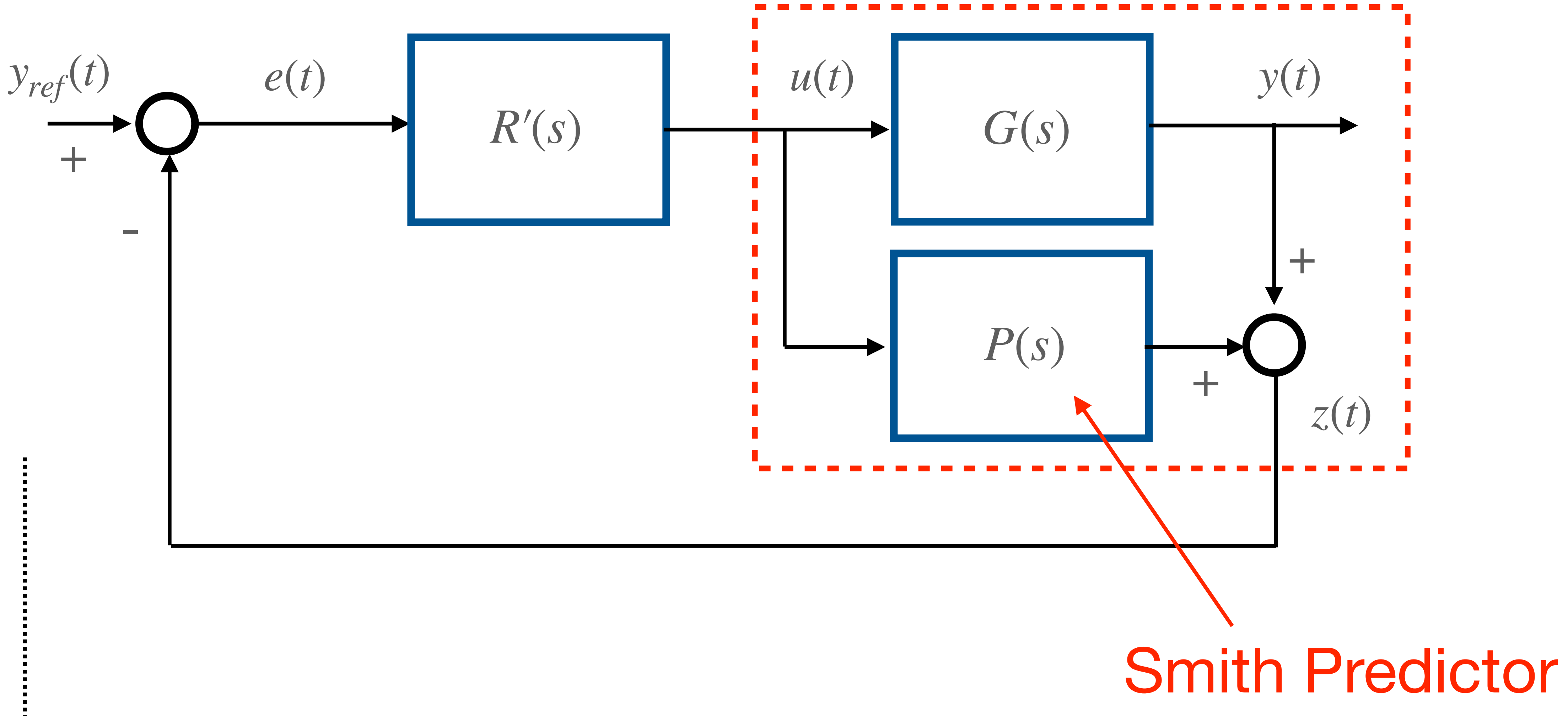


$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

Observation 2: alternative formulation

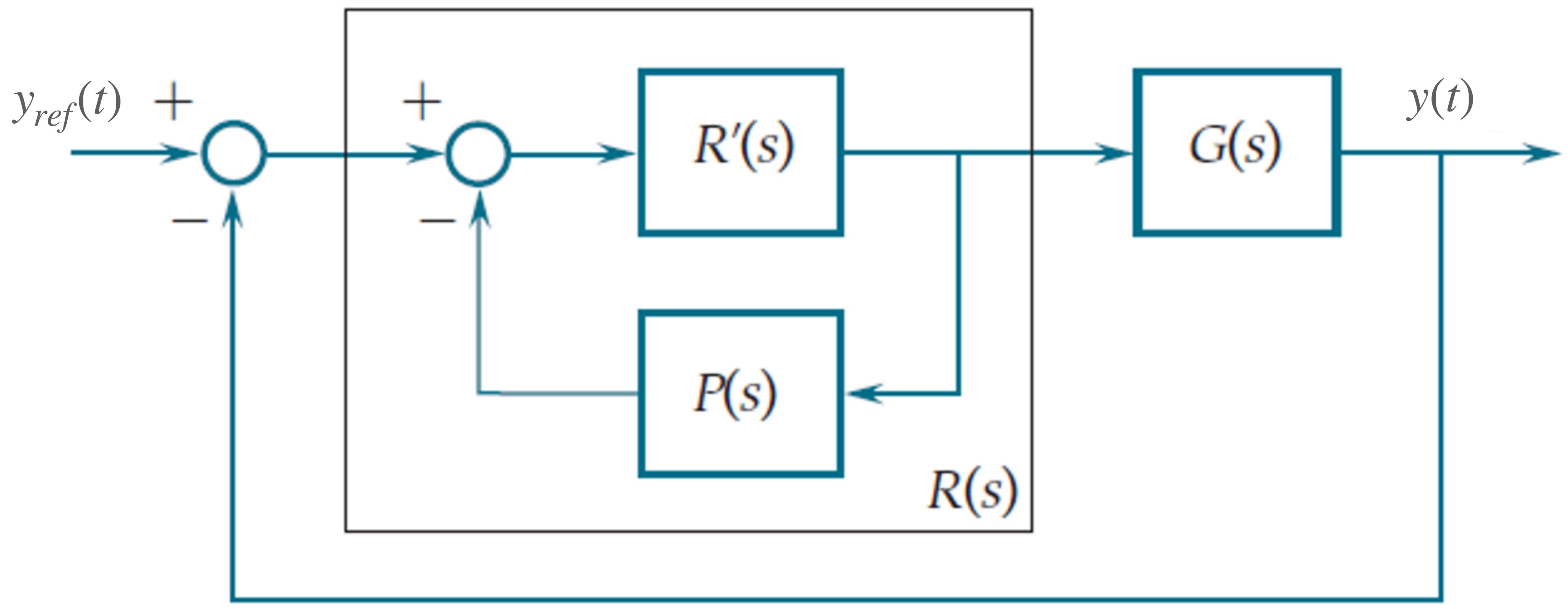




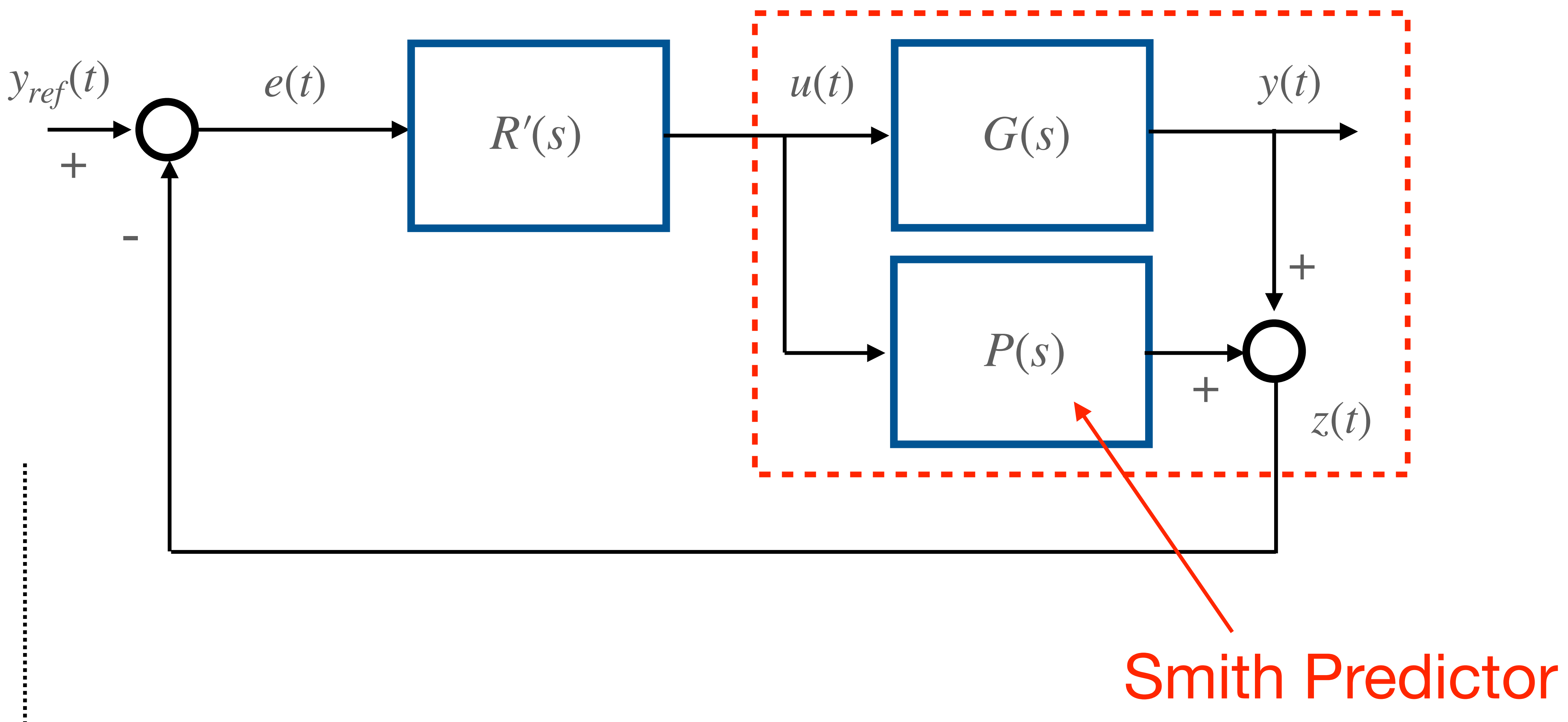
$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

Observation 2: alternative formulation



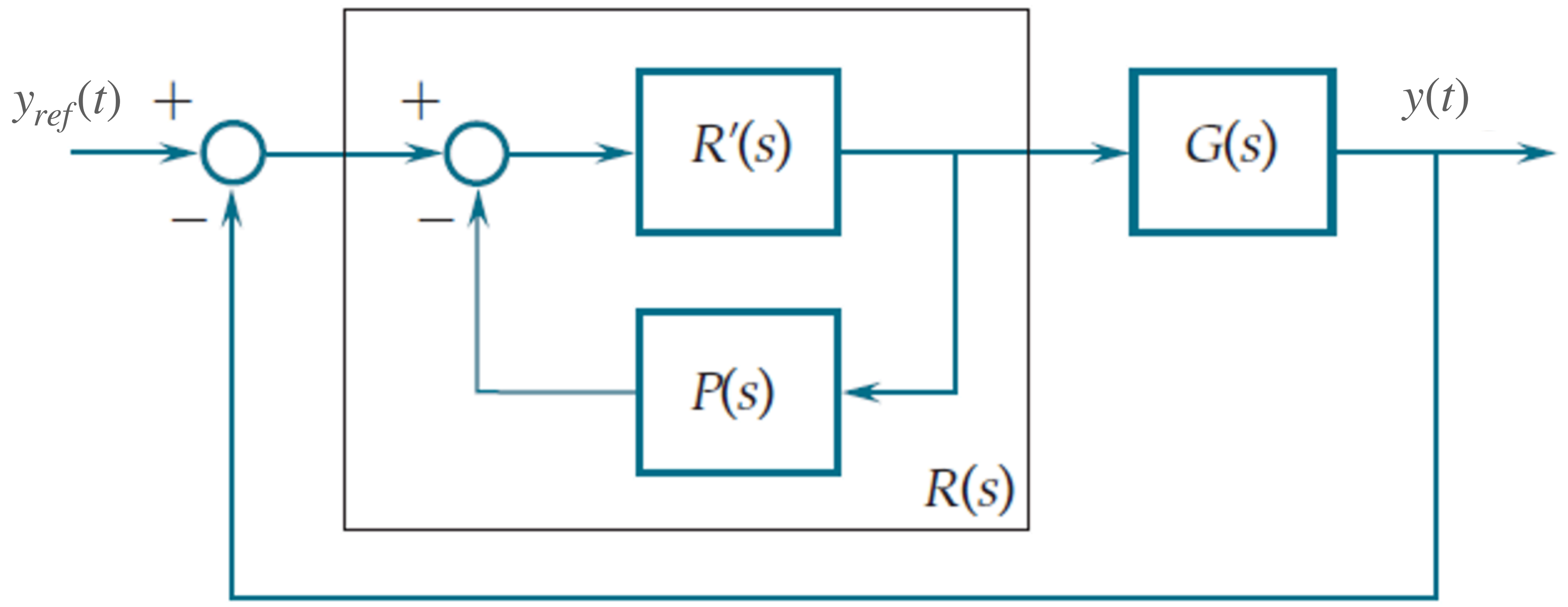
$$R(s) = \frac{R'(s)}{1 + R'(s)P(s)}$$



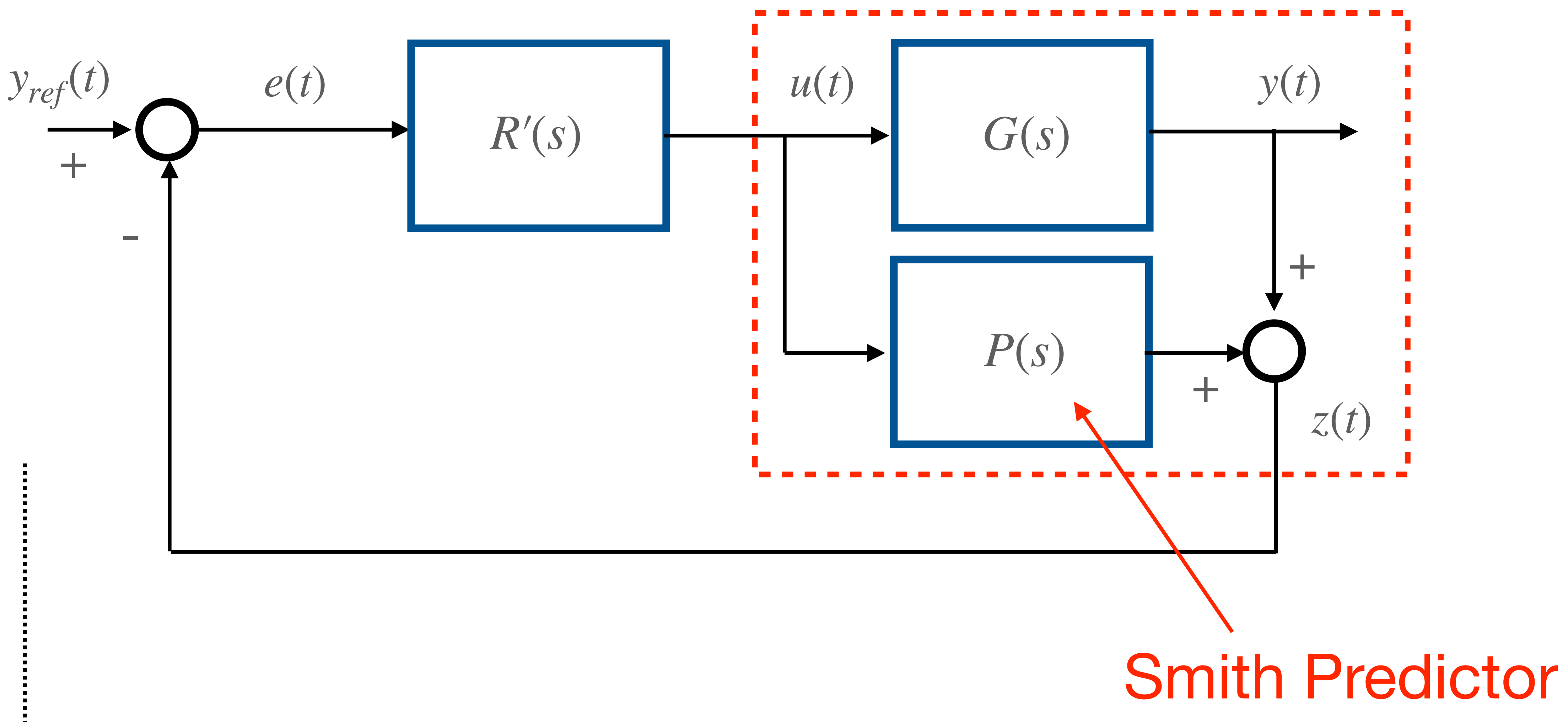
$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

Observation 2: alternative formulation



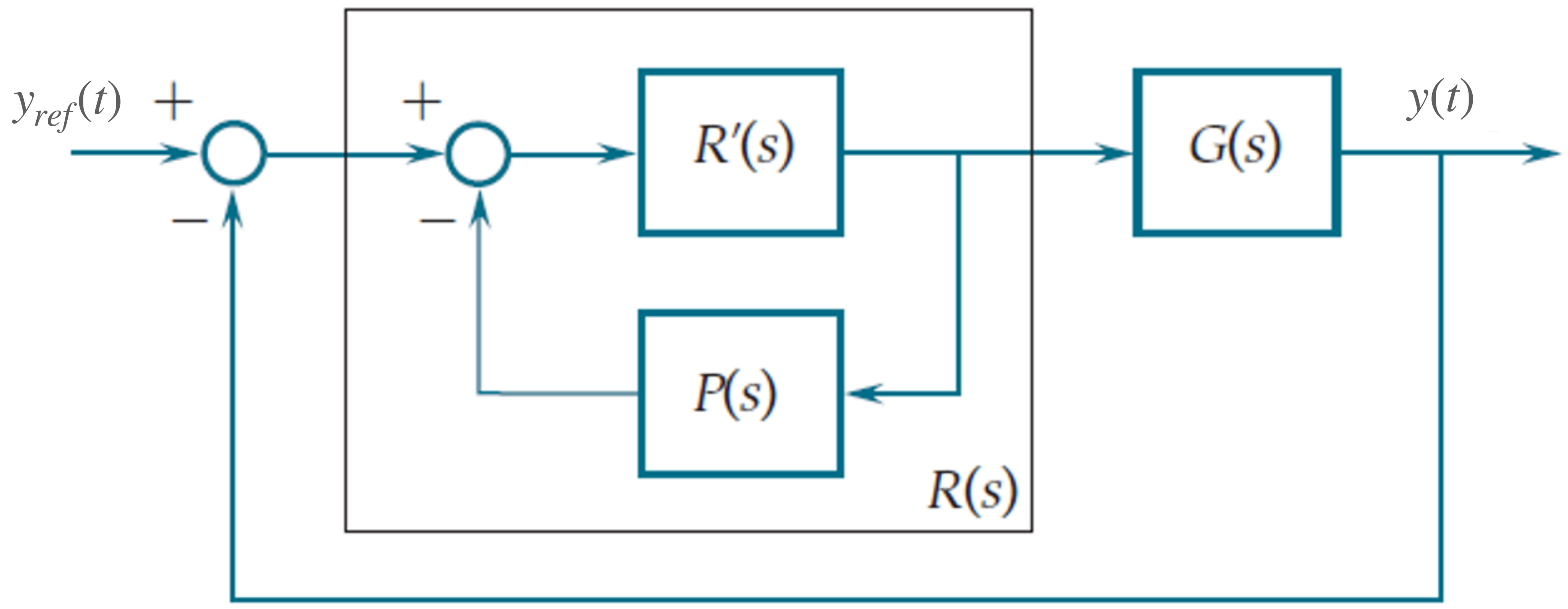
$$R(s) = \frac{R'(s)}{1 + R'(s)P(s)}$$
$$= \frac{N_{R'}(s)/D_{R'}(s)}{1 + (N_{R'}(s)/D_{R'}(s))(1 - e^{-\tau s})(N(s)/D(s))}$$



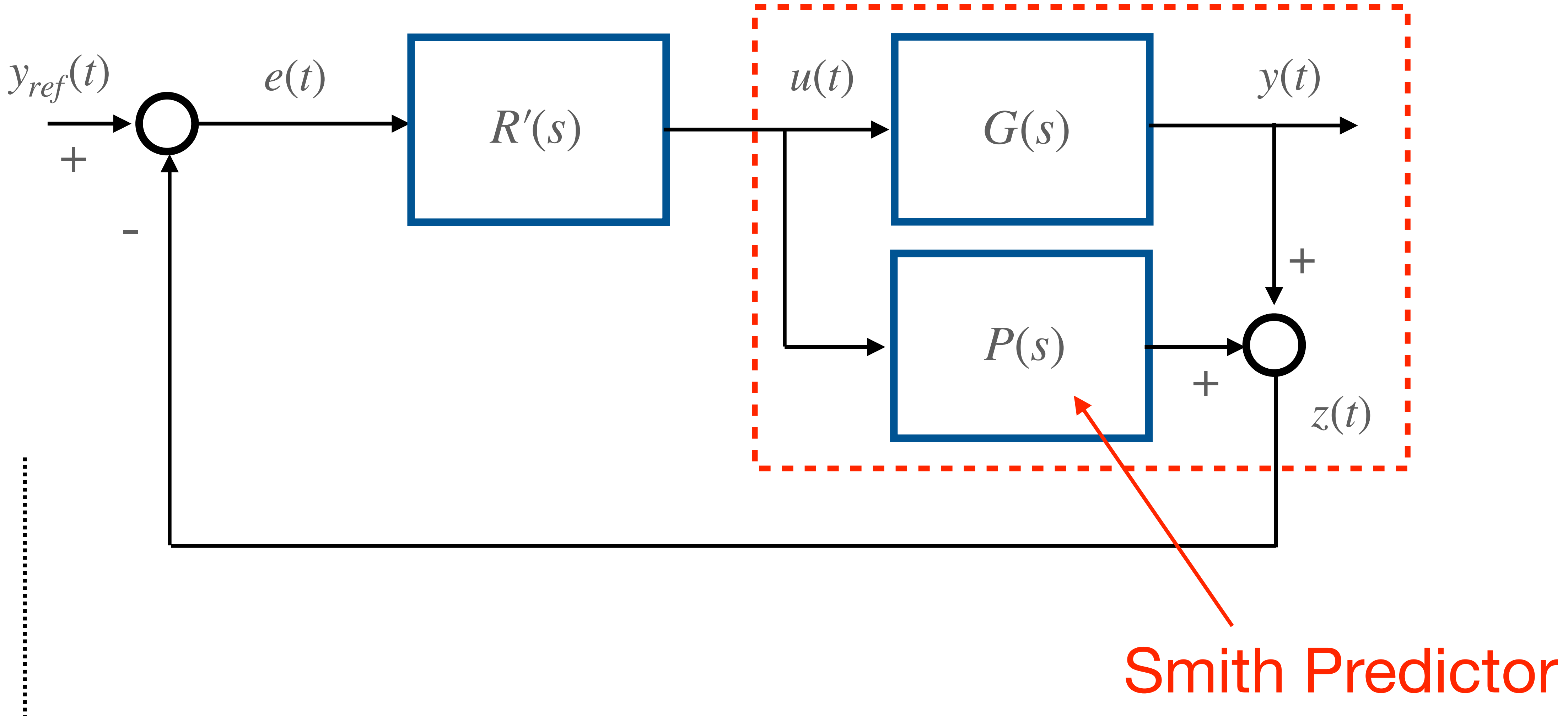
$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\,G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

Observation 2: alternative formulation



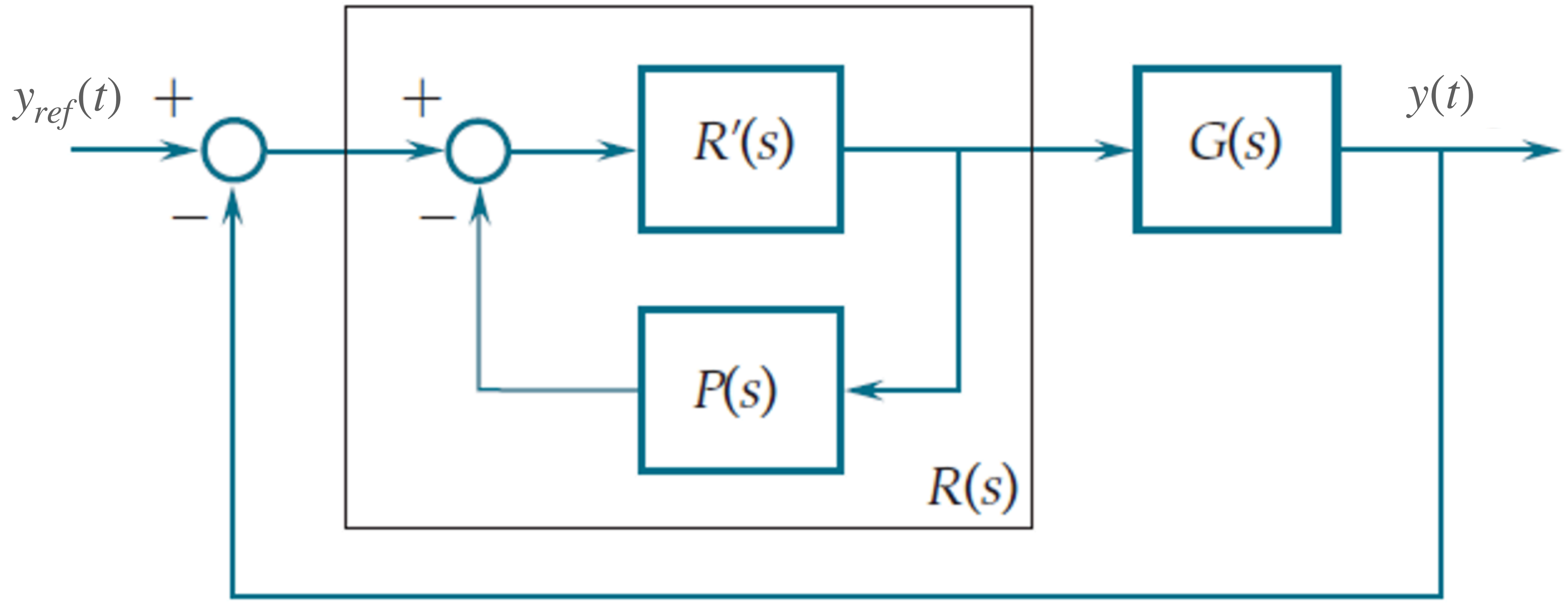
$$R(s) = \frac{R'(s)}{1 + R'(s)P(s)}$$
$$= \frac{N_{R'}(s)/D_{R'}(s)}{1 + (N_{R'}(s)/D_{R'}(s))(1 - e^{-\tau s})(N(s)/D(s))}$$
$$= \frac{N_{R'}(s)D(s)}{D_{R'}(s)D(s) + N_{R'}(s)N(s)(1 - e^{-\tau s})}$$

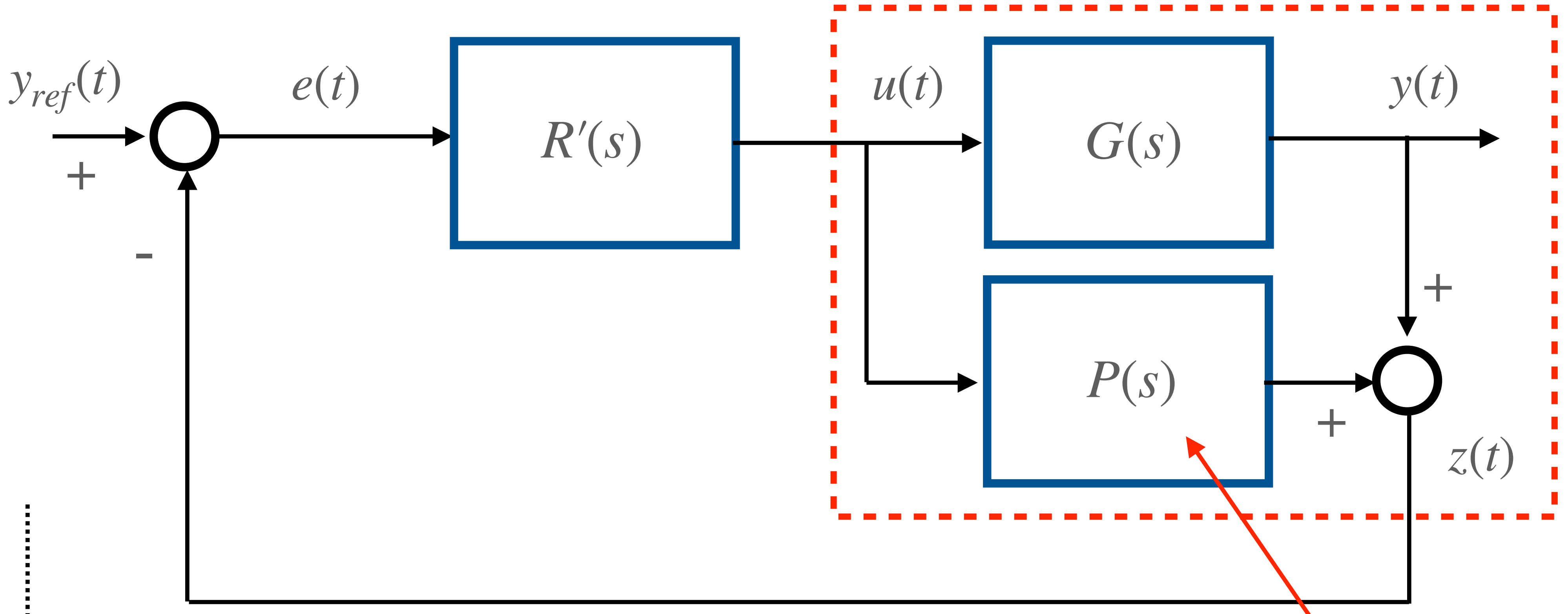


$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\,G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

Observation 2: alternative formulation





Smith Predictor

$$R(s) = \frac{R'(s)}{1 + R'(s)P(s)}$$

$$= \frac{N_{R'}(s)/D_{R'}(s)}{1 + (N_{R'}(s)/D_{R'}(s))(1 - e^{-\tau s})(N(s)/D(s))}$$

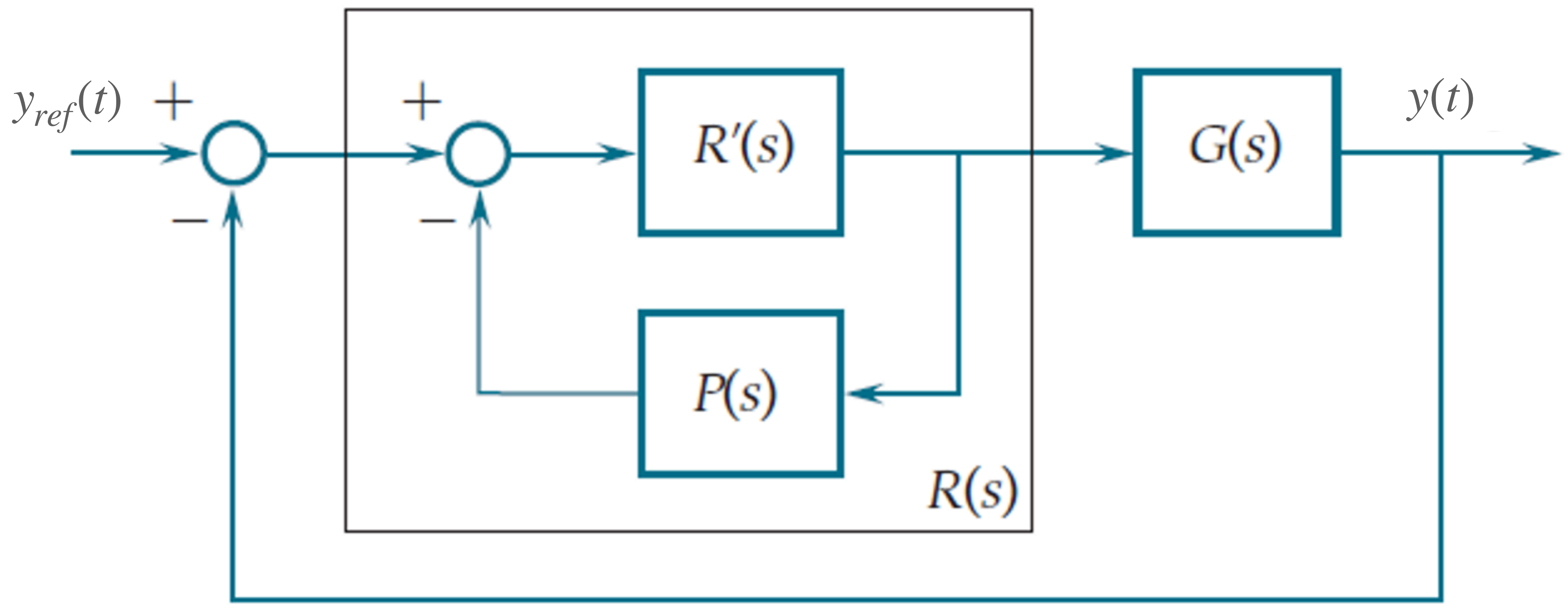
$$= \frac{N_{R'}(s)\boxed{D(s)}}{D_{R'}(s)D(s) + N_{R'}(s)N(s)(1 - e^{-\tau s})}$$

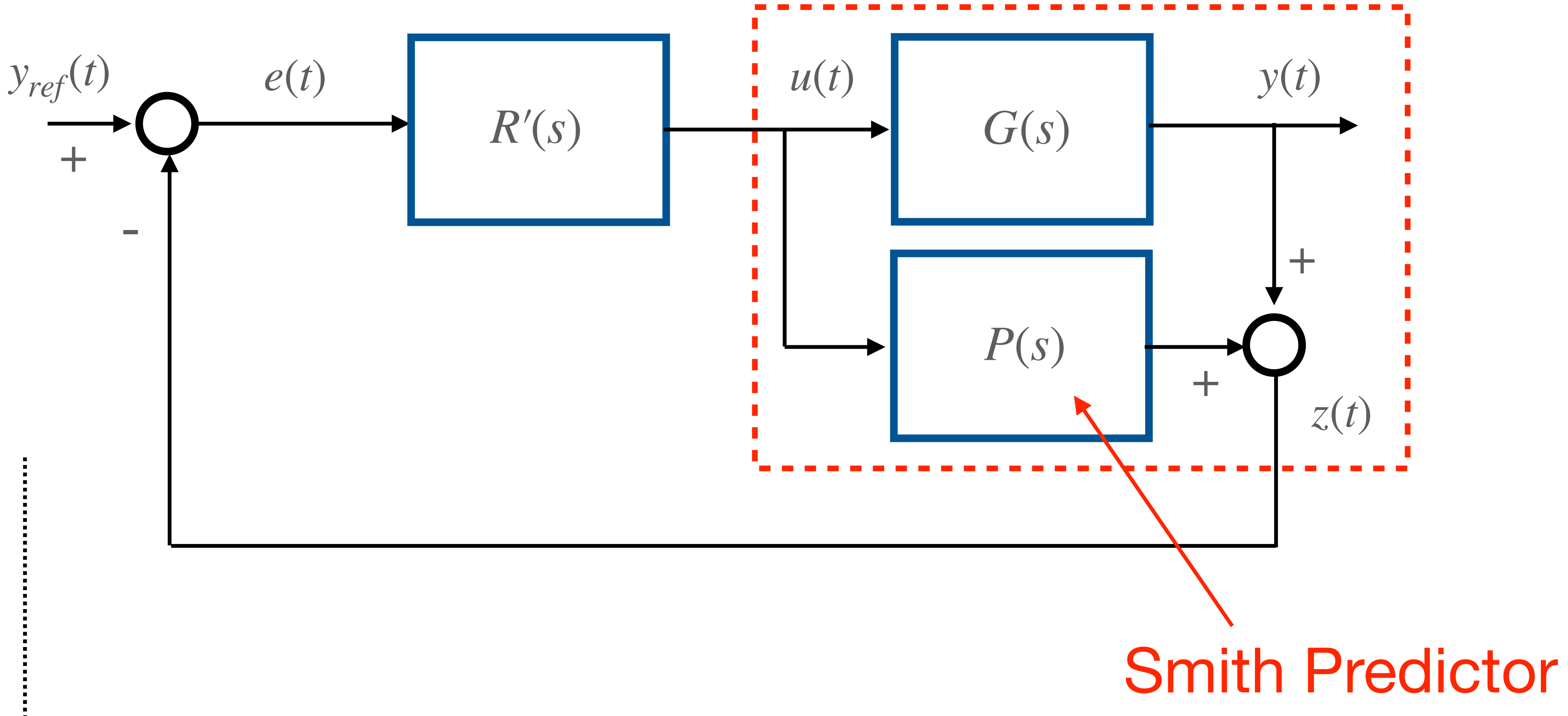
synthesis by cancellation

$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\,G'(s)$$

Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

Observation 3:





$$
\begin{aligned}
R(s) &= \frac{R'(s)}{1 + R'(s)P(s)} \\
&= \frac{N_{R'}(s)/D_{R'}(s)}{1 + (N_{R'}(s)/D_{R'}(s))(1 - e^{-\tau s})(N(s)/D(s))} \\
&= \frac{N_{R'}(s)D(s)}{D_{R'}(s)D(s) + N_{R'}(s)N(s)(1 - \boxed{e^{-\tau s}})}
\end{aligned}
$$

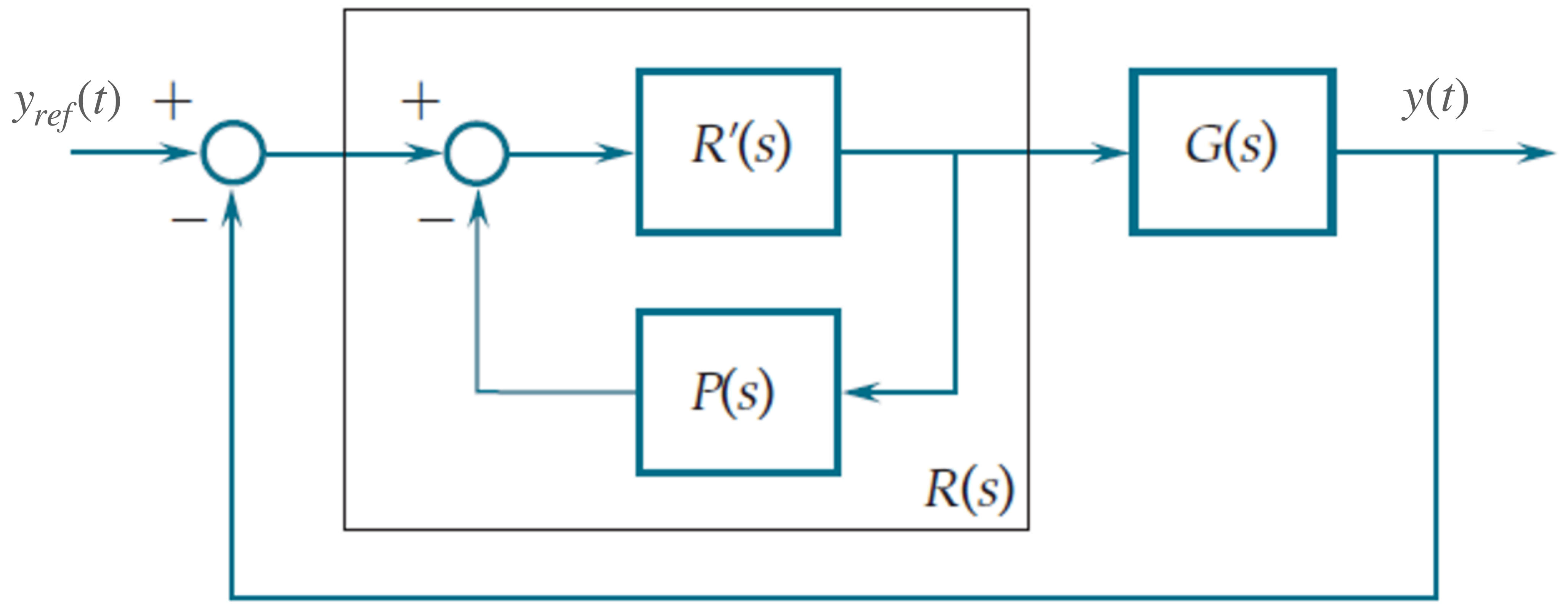
$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

$R(s)$ is a controller with a non-rational transfer function

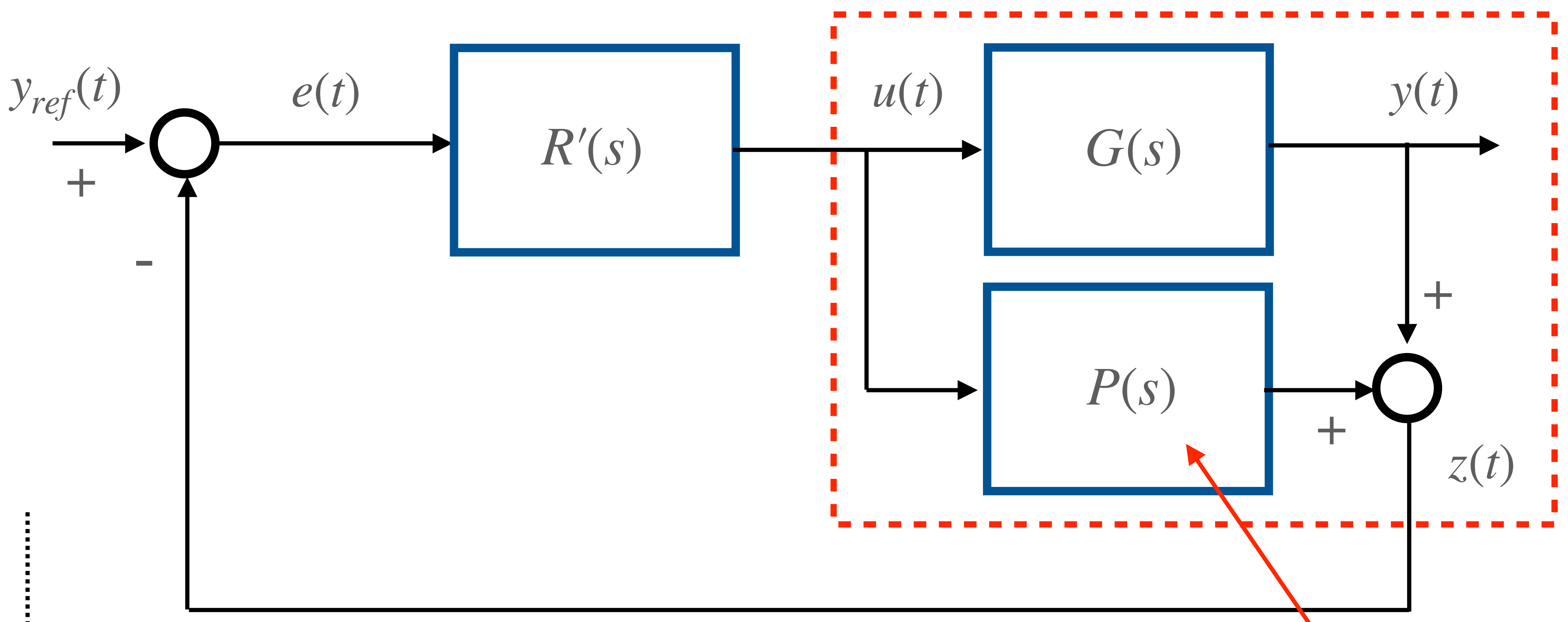
Control of LTI Systems with Delays: Smith Predictor Based Control Scheme

Summary:

Alternative scheme



Classical scheme



Smith Predictor

$z(t)$ is a prediction of $y(t)$

$$Z(s) = e^{\tau s}\, Y(s)$$

$y(t)$ is $z(t)$ delayed of τ seconds

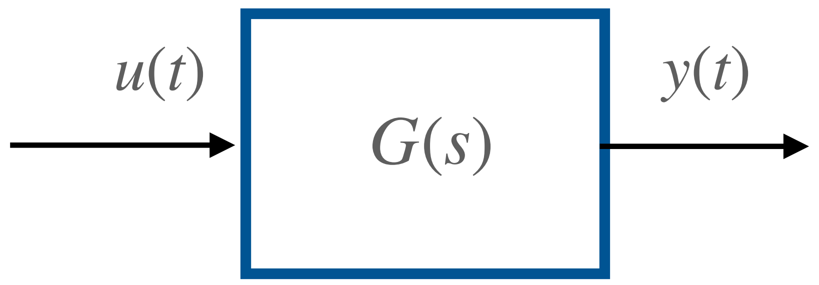
$$Y(s) = e^{-\tau s}\, Z(s)$$

$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\, G'(s)$$

the delay has not disappeared!

Control of LTI Systems with Delays: Padé Approximation

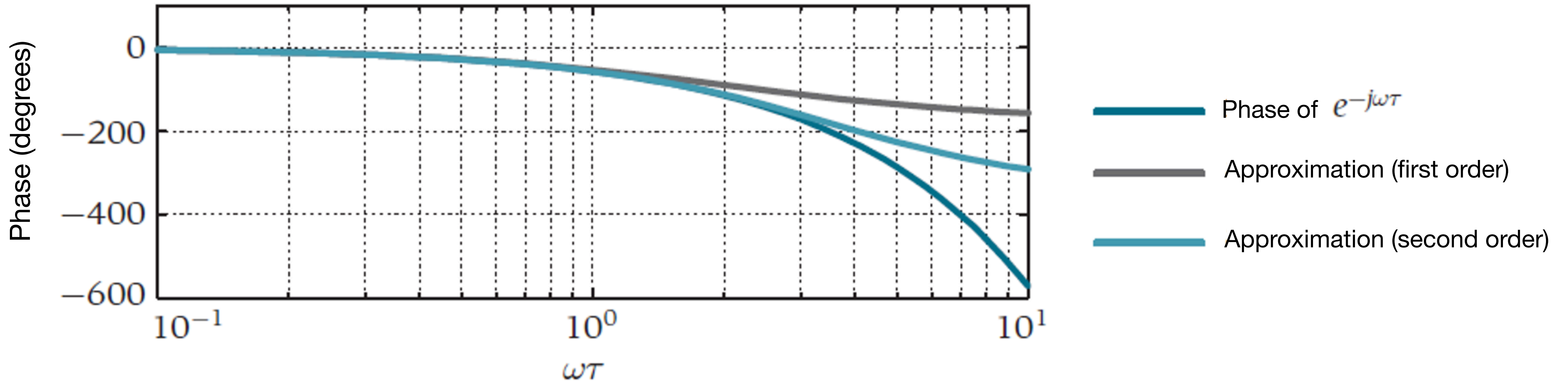
Process



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

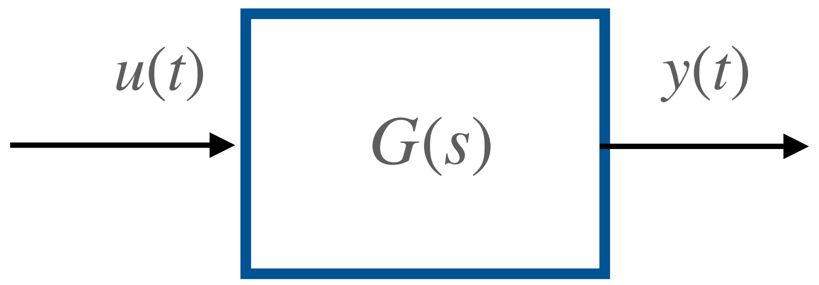
replace with an approximation
based on a rational function of s

Phase Lag due to the delay term

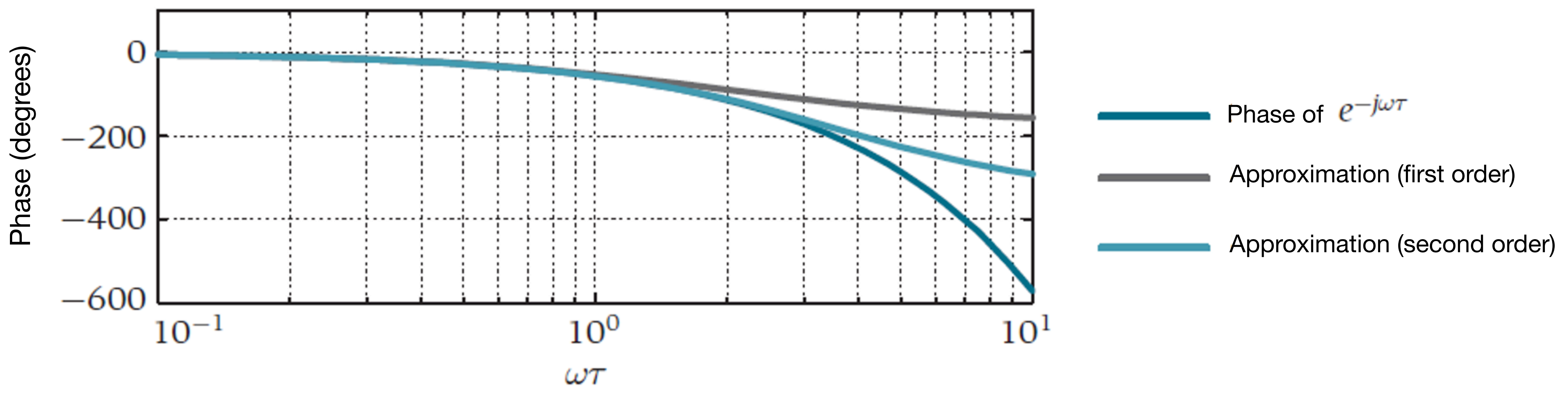


Control of LTI Systems with Delays: Padé Approximation

Process



Phase Lag due to the delay term



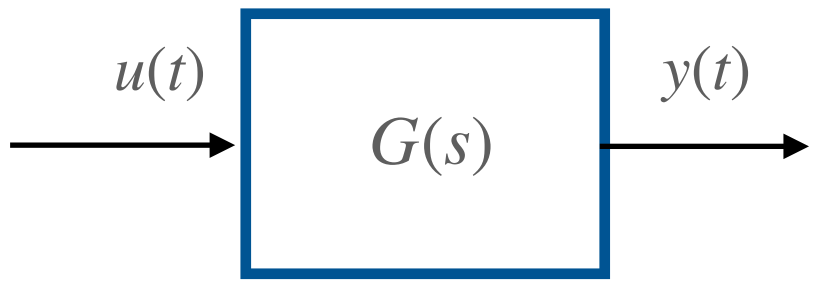
$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

Function to approximate: $e^{-\tau s}$

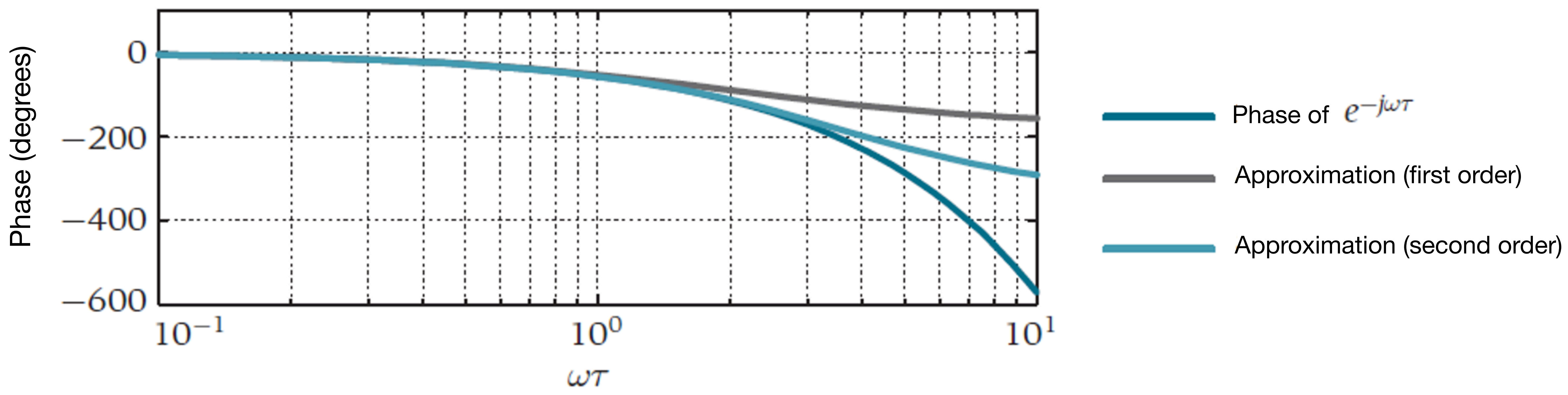
First order approximation: $G_I(s) = \mu\frac{1+as}{1+bs}$

Control of LTI Systems with Delays: Padé Approximation

Process



Phase Lag due to the delay term



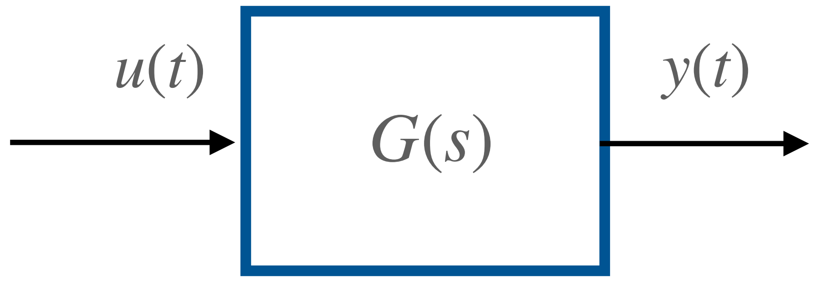
$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

Function to approximate: $e^{-\tau s}$

First order approximation: $G_I(s) = \mu\dfrac{1+as}{1+bs}$ $\longrightarrow$ How to select the parameters?

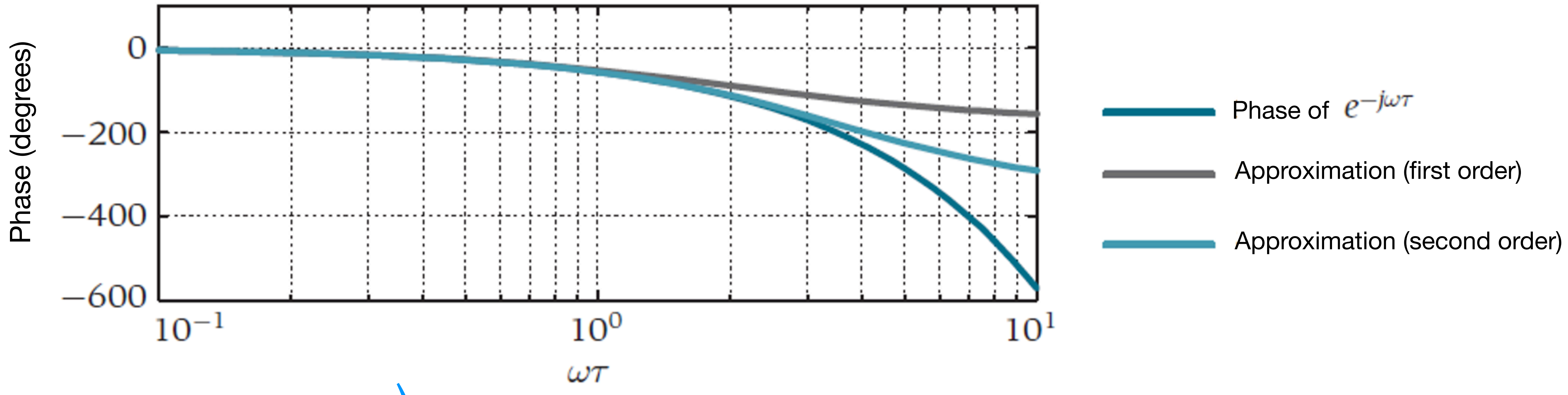
Control of LTI Systems with Delays: Padé Approximation

Process



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

Phase Lag due to the delay term



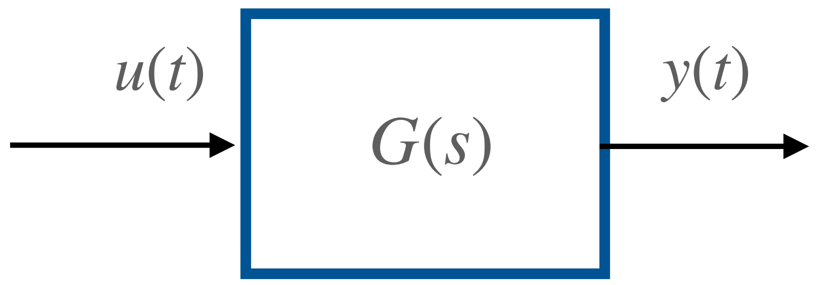
Function to approximate: $e^{-\tau s}$

First order approximation: $G_I(s) = \mu\frac{1+as}{1+bs}$

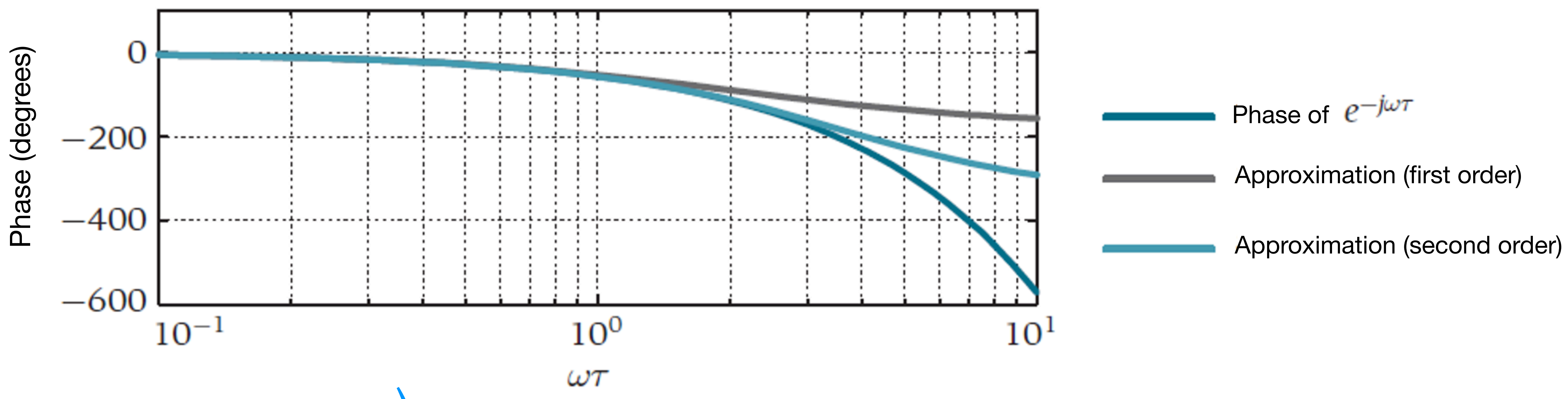
→ Use MacLaurin series expansion

Control of LTI Systems with Delays: Padé Approximation

Process



Phase Lag due to the delay term



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

Function to approximate: $e^{-\tau s}$

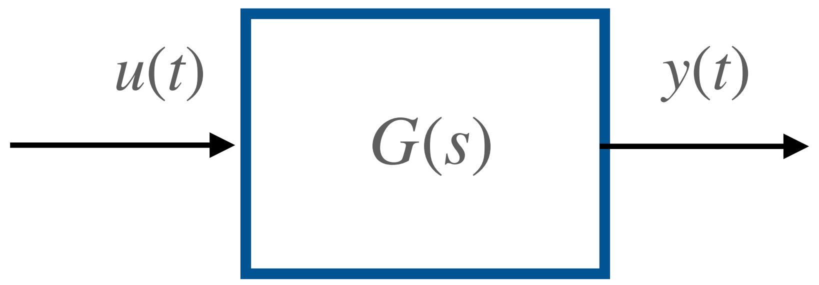
First order approximation: $G_I(s) = \mu\dfrac{1+as}{1+bs}$

Use MacLaurin series expansion

$$f(s) = f(0) + \frac{df}{ds}(0)\,s + \frac{1}{2!}\frac{d^2f}{ds^2}(0)\,s^2 + \frac{1}{3!}\frac{d^3f}{ds^3}(0)\,s^3 + \mathcal{O}(s^4)$$

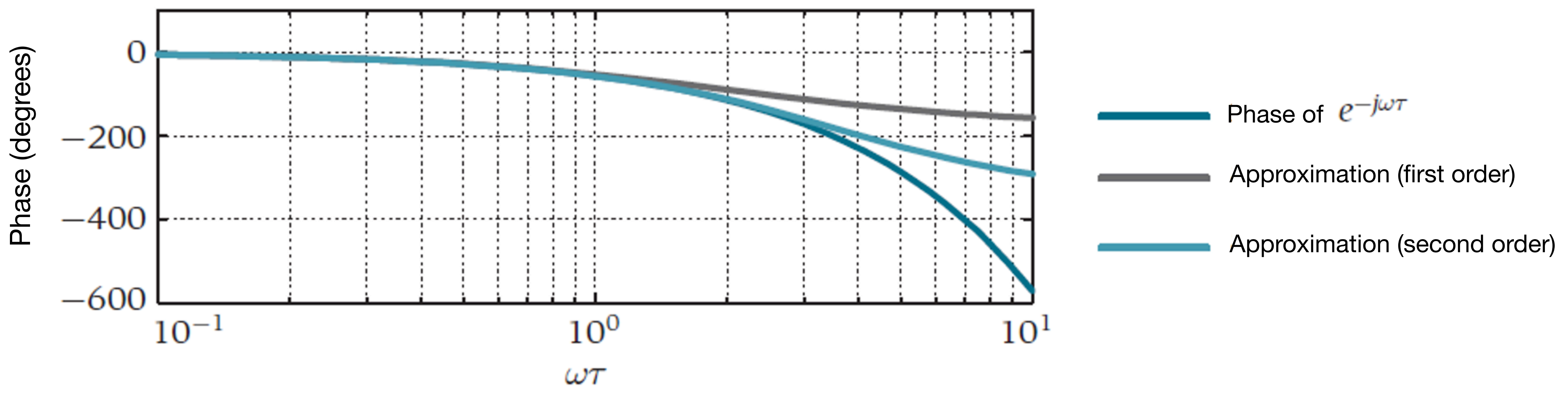
Control of LTI Systems with Delays: Padé Approximation

Process



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

Phase Lag due to the delay term



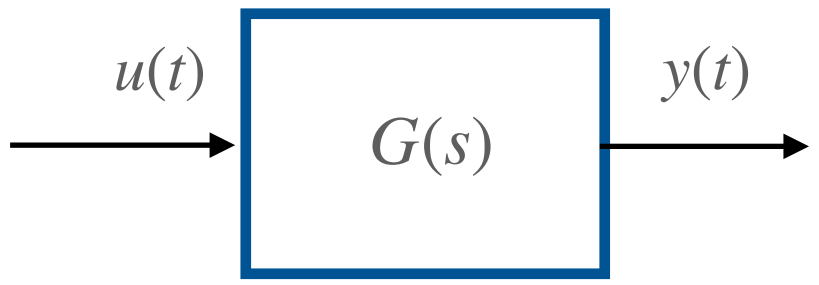
Function to approximate: $e^{-\tau s}$

McLaurin approximation: $e^{-\tau s} \approx 1 - \tau s + \dfrac{\tau^2 s^2}{2} + \ldots$

First order approximation: $G_I(s) = \mu \dfrac{1 + as}{1 + bs}$

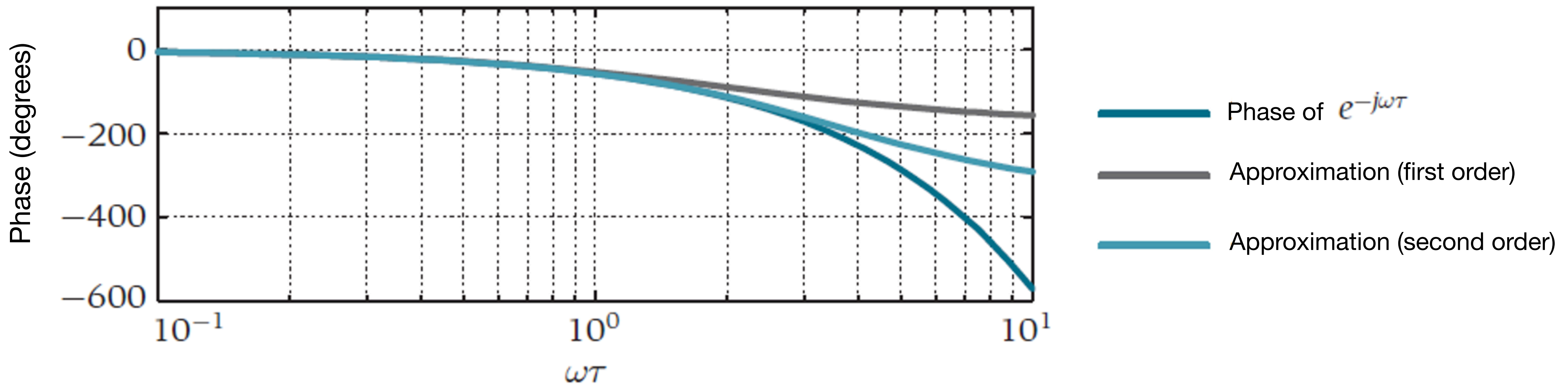
Control of LTI Systems with Delays: Padé Approximation

Process



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}} G'(s)$$

Phase Lag due to the delay term



Function to approximate: $e^{-\tau s}$

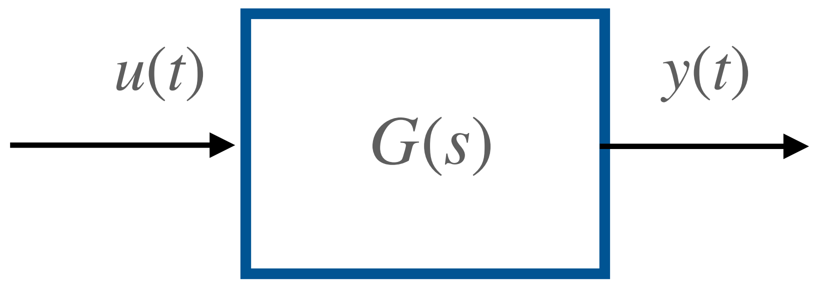
McLaurin approximation: $e^{-\tau s} \approx 1 - \tau s + \dfrac{\tau^2 s^2}{2} + \ldots$

First order approximation: $G_I(s) = \mu \dfrac{1 + as}{1 + bs}$

McLaurin approximation: $G_I(s) = \mu \dfrac{1 + as}{1 + bs} = \approx \mu + \mu(a - b)s - \mu b(a - b)s^2$

Control of LTI Systems with Delays: Padé Approximation

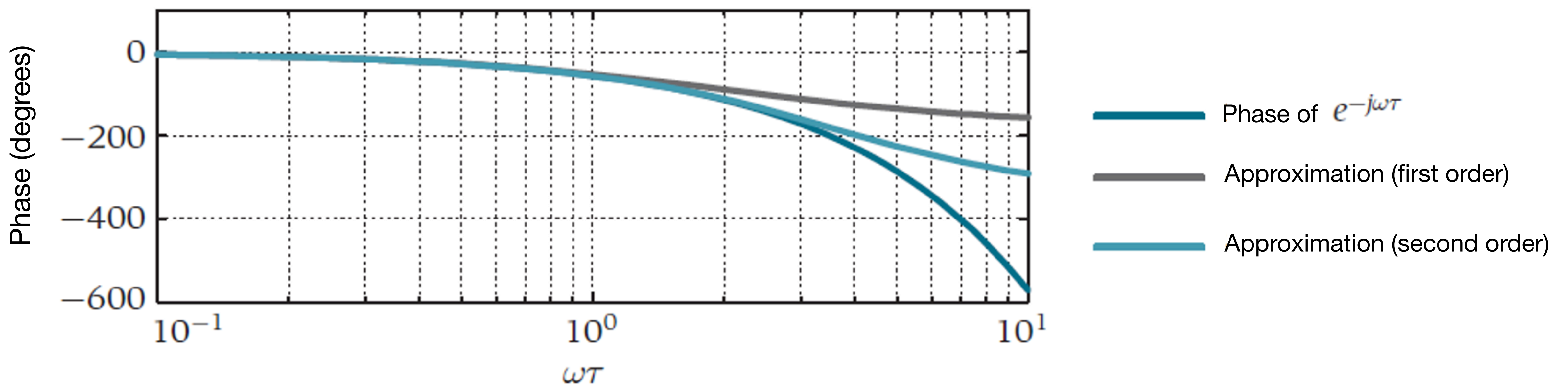
Process



$$f(s) = f(0) + \frac{df}{ds}(0)\,s + \frac{1}{2!}\frac{d^2f}{ds^2}(0)\,s^2 + \frac{1}{3!}\frac{d^3f}{ds^3}(0)\,s^3 + \mathcal{O}(s^4)$$

$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

Phase Lag due to the delay term



$$\mu = 1$$
$$\mu(a-b) = -\tau$$
$$-\mu b(a-b) = \frac{\tau^2}{2}$$

Function to approximate: $e^{-\tau s}$

McLaurin approximation: $e^{-\tau s} \approx 1 - \tau s + \frac{\tau^2 s^2}{2} + \dots$

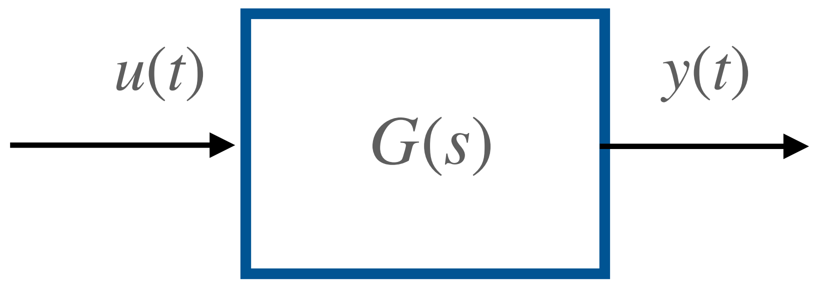
First order approximation: $G_I(s) = \mu\frac{1+as}{1+bs}$

McLaurin approximation: $G_I(s) = \mu\frac{1+as}{1+bs} = \approx \mu + \mu(a-b)s - \mu b(a-b)s^2$

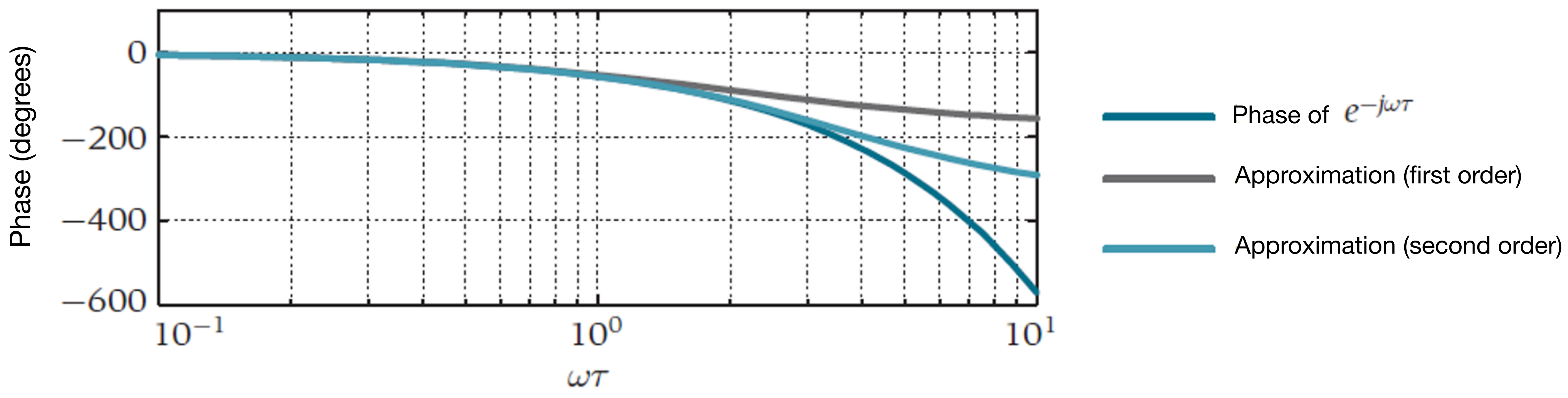
UNIVERSITÀ DI PAVIA

Control of LTI Systems with Delays: Padé Approximation

Process



Phase Lag due to the delay term



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

$$\boxed{\begin{aligned} a &= -\frac{\tau}{2} \\ b &= \frac{\tau}{2} \end{aligned}}$$

$$\mu = 1$$
$$\mu(a-b) = -\tau$$
$$-\mu b(a-b) = \frac{\tau^2}{2}$$

Function to approximate: $e^{-\tau s}$

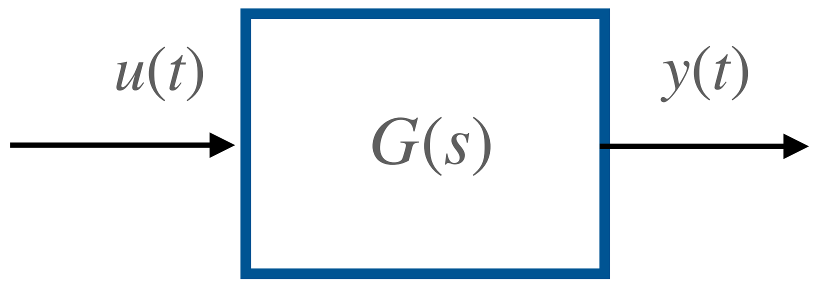
McLaurin approximation: $e^{-\tau s} \approx 1 - \tau s + \dfrac{\tau^2 s^2}{2} + \dots$

First order approximation: $G_I(s) = \mu\dfrac{1+as}{1+bs}$

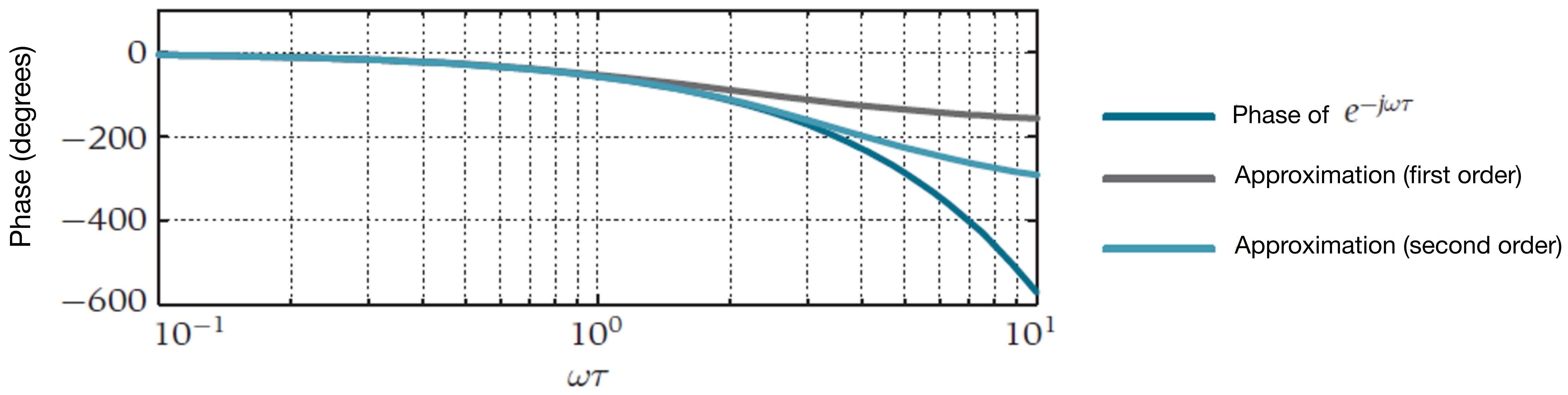
McLaurin approximation: $G_I(s) = \mu\dfrac{1+as}{1+bs} = \approx \mu + \mu(a-b)s - \mu b(a-b)s^2$

Control of LTI Systems with Delays: Padé Approximation

Process



Phase Lag due to the delay term



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

Function to approximate: $e^{-\tau s}$

McLaurin approximation: $e^{-\tau s} \approx 1 - \tau s + \dfrac{\tau^2 s^2}{2} + \dots$

First order approximation: $G_I(s) = \mu\dfrac{1+as}{1+bs}$ $\longrightarrow$ $\boxed{G_I(s) = \dfrac{1-\frac{\tau}{2}s}{1+\frac{\tau}{2}s}}$

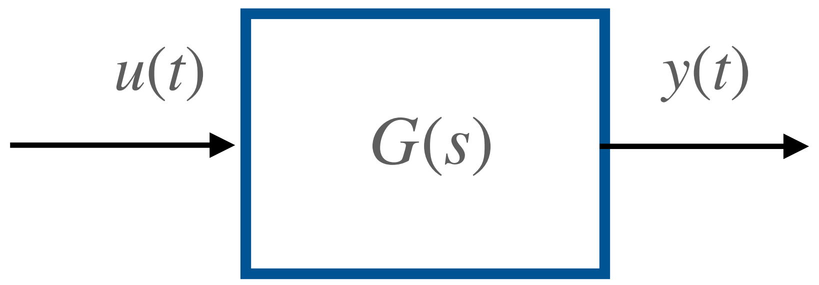
McLaurin approximation: $G_I(s) = \mu\dfrac{1+as}{1+bs} = \approx \mu + \mu(a-b)s - \mu b(a-b)s^2$

$$\mu = 1$$
$$\mu(a-b) = -\tau$$
$$-\mu b(a-b) = \frac{\tau^2}{2}$$

$$\boxed{\begin{aligned} a &= -\frac{\tau}{2} \\ b &= \frac{\tau}{2} \end{aligned}}$$

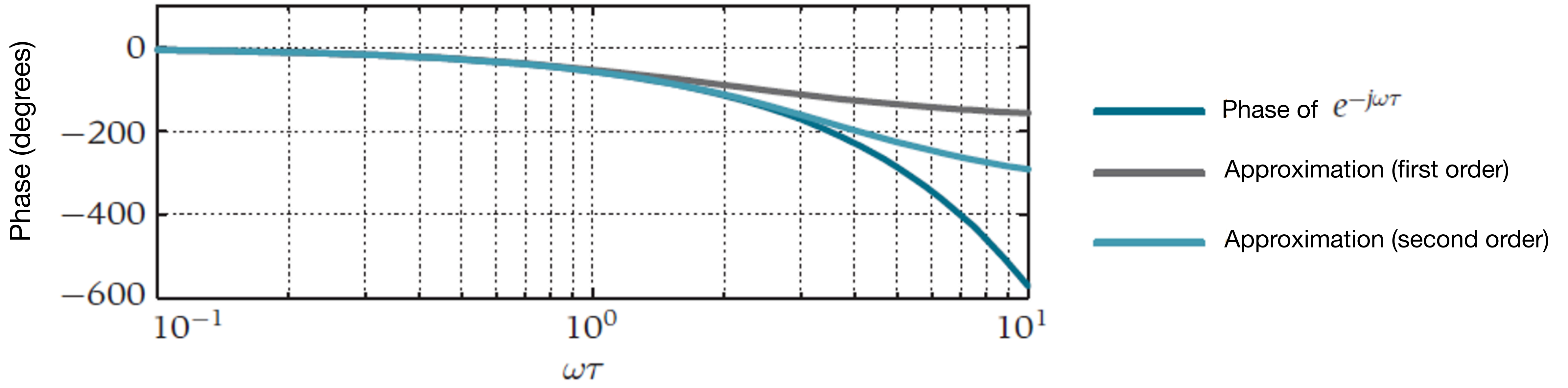
Control of LTI Systems with Delays: Padé Approximation

Process



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

Phase Lag due to the delay term



$$\boxed{\begin{aligned} a &= -\frac{\tau}{2} \\ b &= \frac{\tau}{2} \end{aligned}}$$

$$\mu = 1$$
$$\mu(a-b) = -\tau$$
$$-\mu b(a-b) = \frac{\tau^2}{2}$$

Function to approximate: $e^{-\tau s}$

McLaurin approximation: $e^{-\tau s} \approx 1 - \tau s + \dfrac{\tau^2 s^2}{2} + \dots$

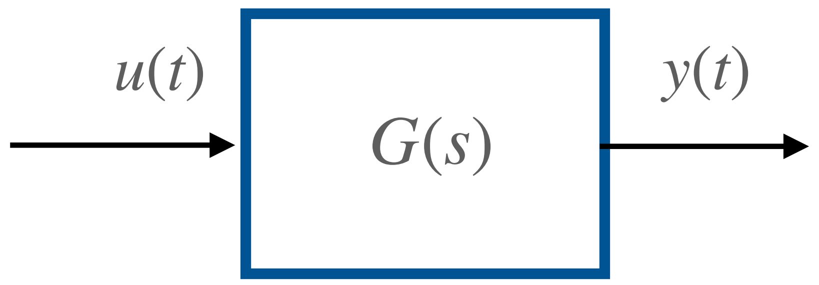
First order approximation: $G_I(s) = \mu\dfrac{1+as}{1+bs}$ $\longrightarrow$ $\boxed{G_I(s) = \dfrac{1-\frac{\tau}{2}s}{1+\frac{\tau}{2}s}}$

McLaurin approximation: $G_I(s) = \mu\dfrac{1+as}{1+bs} = \approx \mu + \mu(a-b)s - \mu b(a-b)s^2$

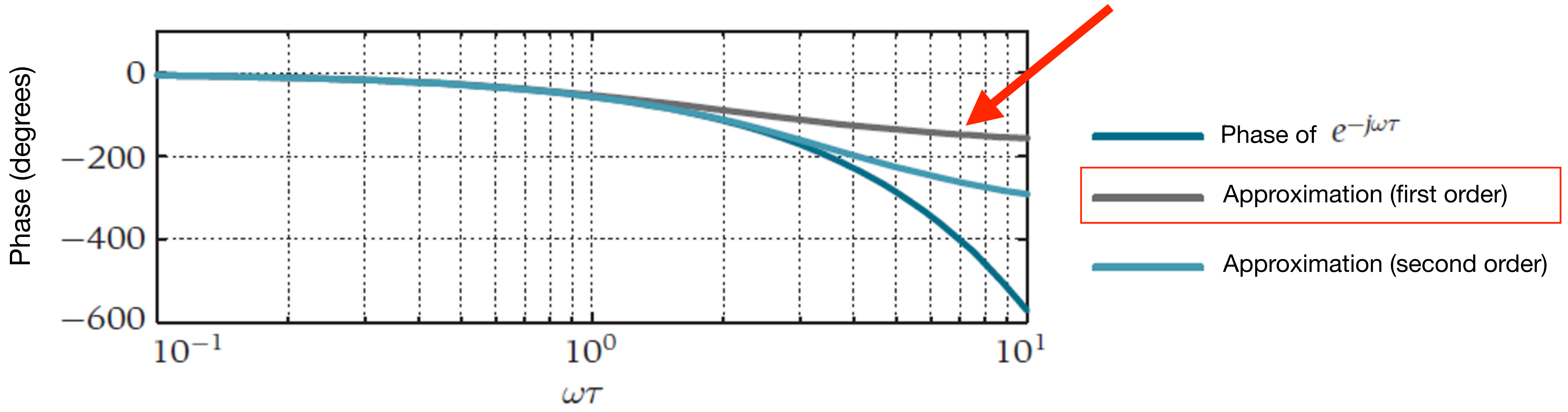
Magnitude and phase ?

Control of LTI Systems with Delays: Padé Approximation

Process



Phase Lag due to the delay term



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

$$\boxed{\begin{aligned} a &= -\frac{\tau}{2} \\ b &= \frac{\tau}{2} \end{aligned}}$$

$$\mu = 1$$

$$\mu(a-b) = -\tau$$

$$-\mu b(a-b) = \frac{\tau^2}{2}$$

Function to approximate: $e^{-\tau s}$

McLaurin approximation: $e^{-\tau s} \approx 1 - \tau s + \dfrac{\tau^2 s^2}{2} + \dots$

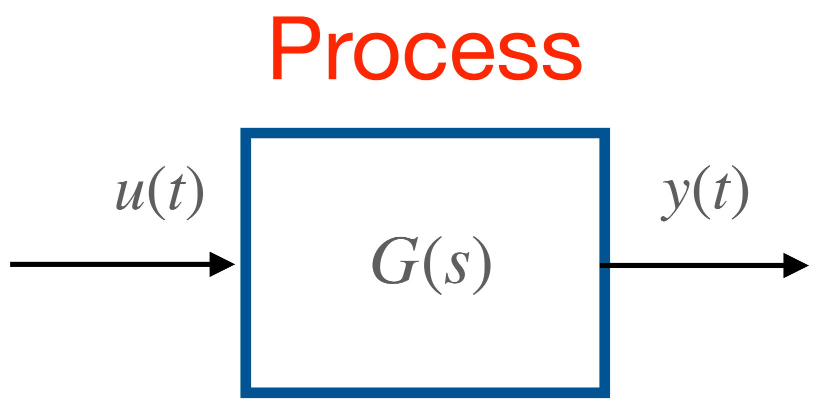
First order approximation: $G_I(s) = \mu\dfrac{1+as}{1+bs}$ $\longrightarrow$ $\boxed{G_I(s) = \dfrac{1-\frac{\tau}{2}s}{1+\frac{\tau}{2}s}}$

McLaurin approximation: $G_I(s) = \mu\dfrac{1+as}{1+bs} = \approx \mu + \mu(a-b)s - \mu b(a-b)s^2$

Magnitude and phase ?

Control of LTI Systems with Delays

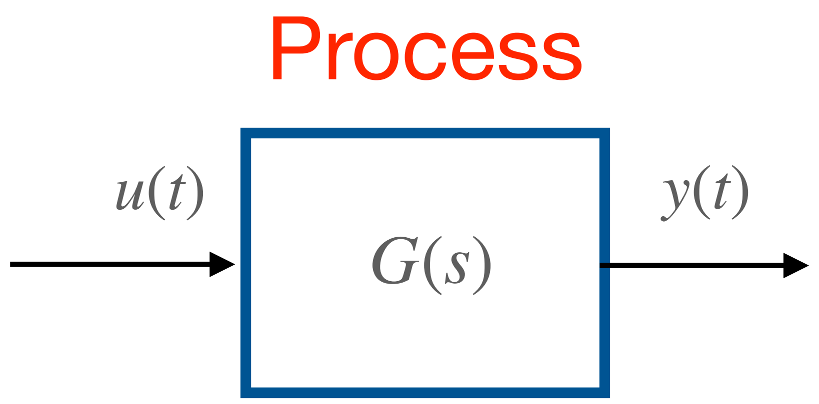
Summary:



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}G'(s)$$

Control of LTI Systems with Delays

Summary:



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

Modelling alternatives: $e^{-\tau s}$

$$G_I(s) = \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}$$

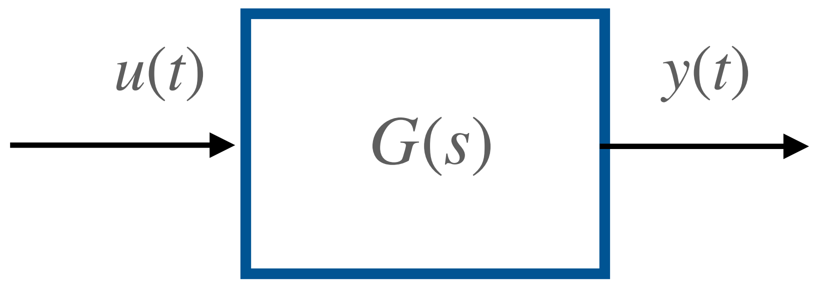
$$G_{II}(s) = \ldots$$

$$G_{III}(s) = \ldots$$

Control of LTI Systems with Delays

Summary:

Process



$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = \boxed{e^{-\tau s}}G'(s)$$

Modelling alternatives:

$$e^{-\tau s}$$

$$G_I(s) = \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}$$

$$G_{II}(s) = \dots$$

$$G_{III}(s) = \dots$$

Design alternatives for R(s):

$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}\,G'(s)$$

$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\,G'(s)$$

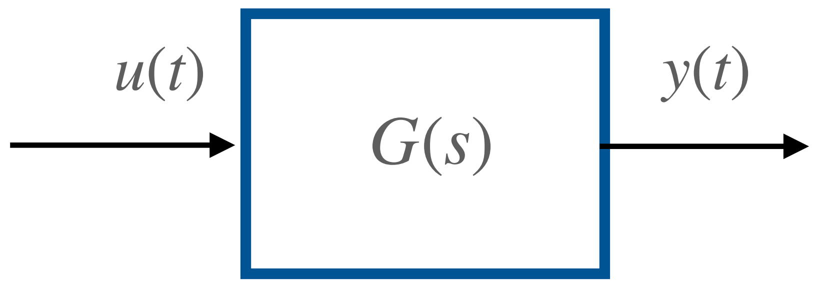
$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} \approx \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}\,G'(s)$$

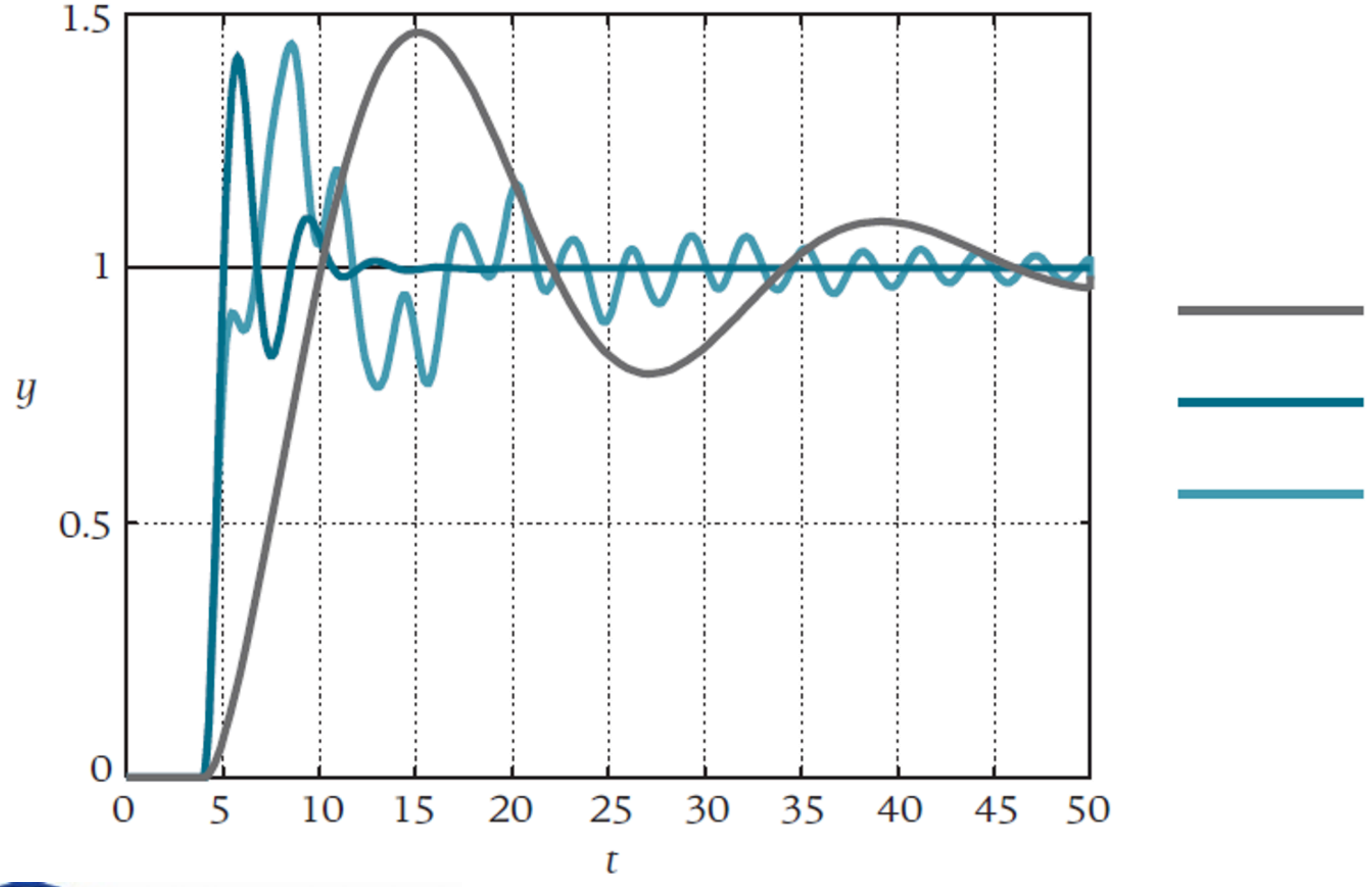
$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} \approx \left(1 - \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}\right)G'(s)$$

Control of LTI Systems with Delays

Summary:

Process





Design alternatives for R(s):

$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}\,G'(s)$$

$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\,G'(s)$$

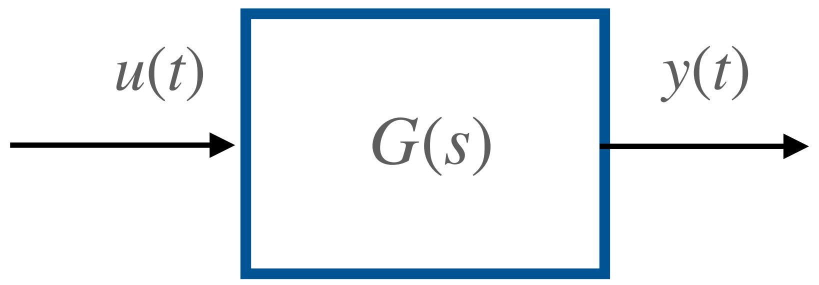
$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} \approx \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}\,G'(s)$$

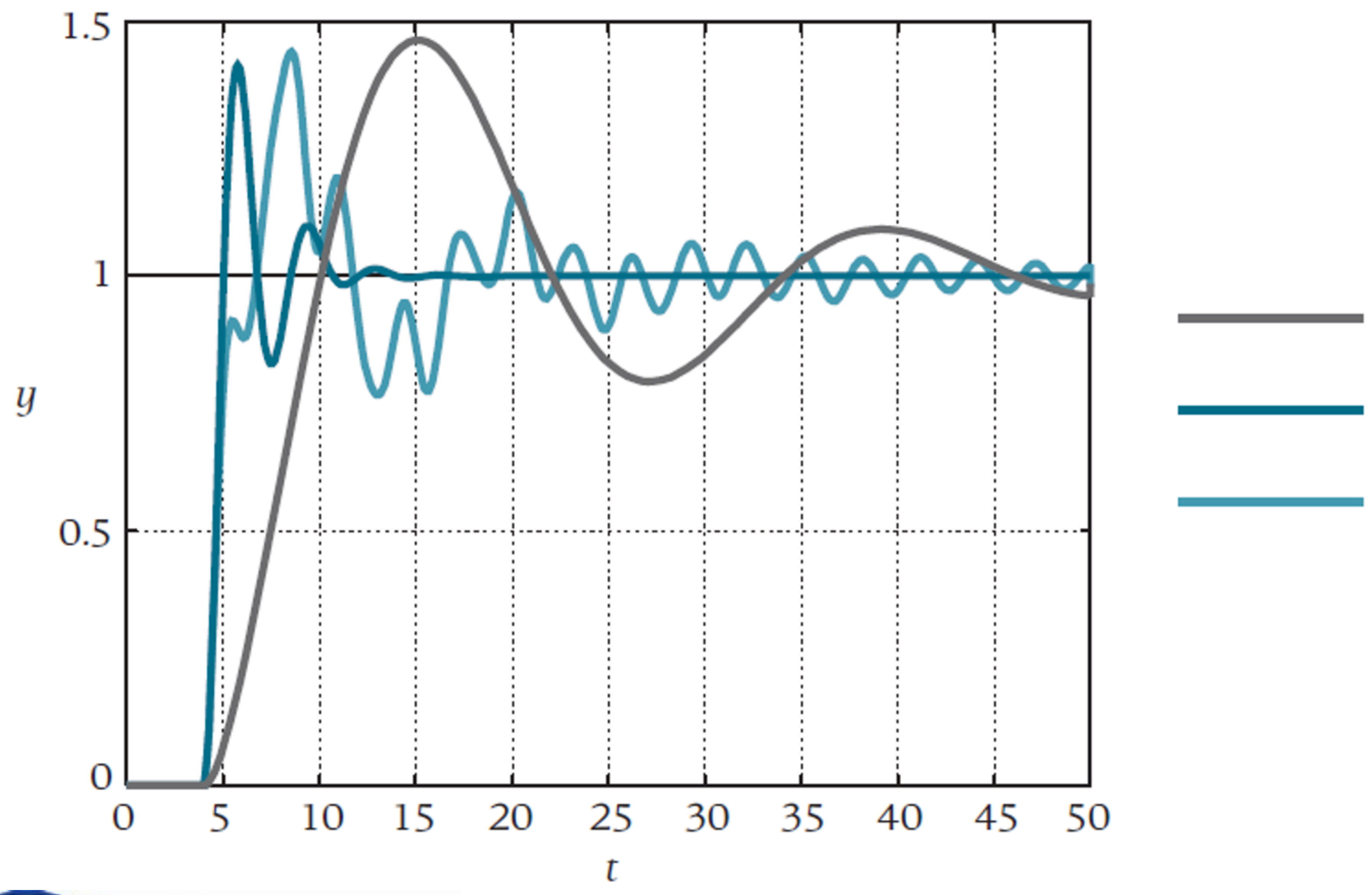
$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} \approx \left(1 - \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}\right)G'(s)$$

Control of LTI Systems with Delays

Summary:

Process





Design alternatives for R(s):

$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}\,G'(s)$$

$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\,G'(s)$$

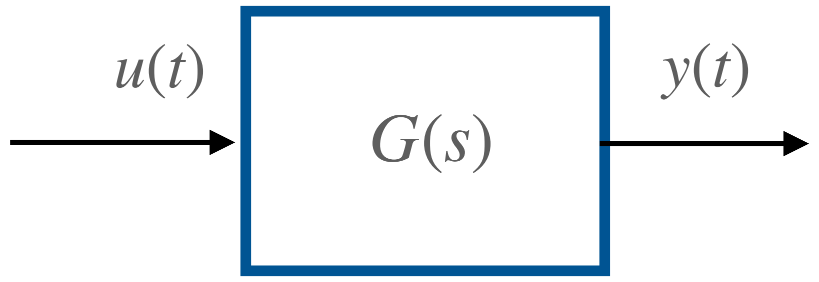
$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} \approx \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}\,G'(s)$$

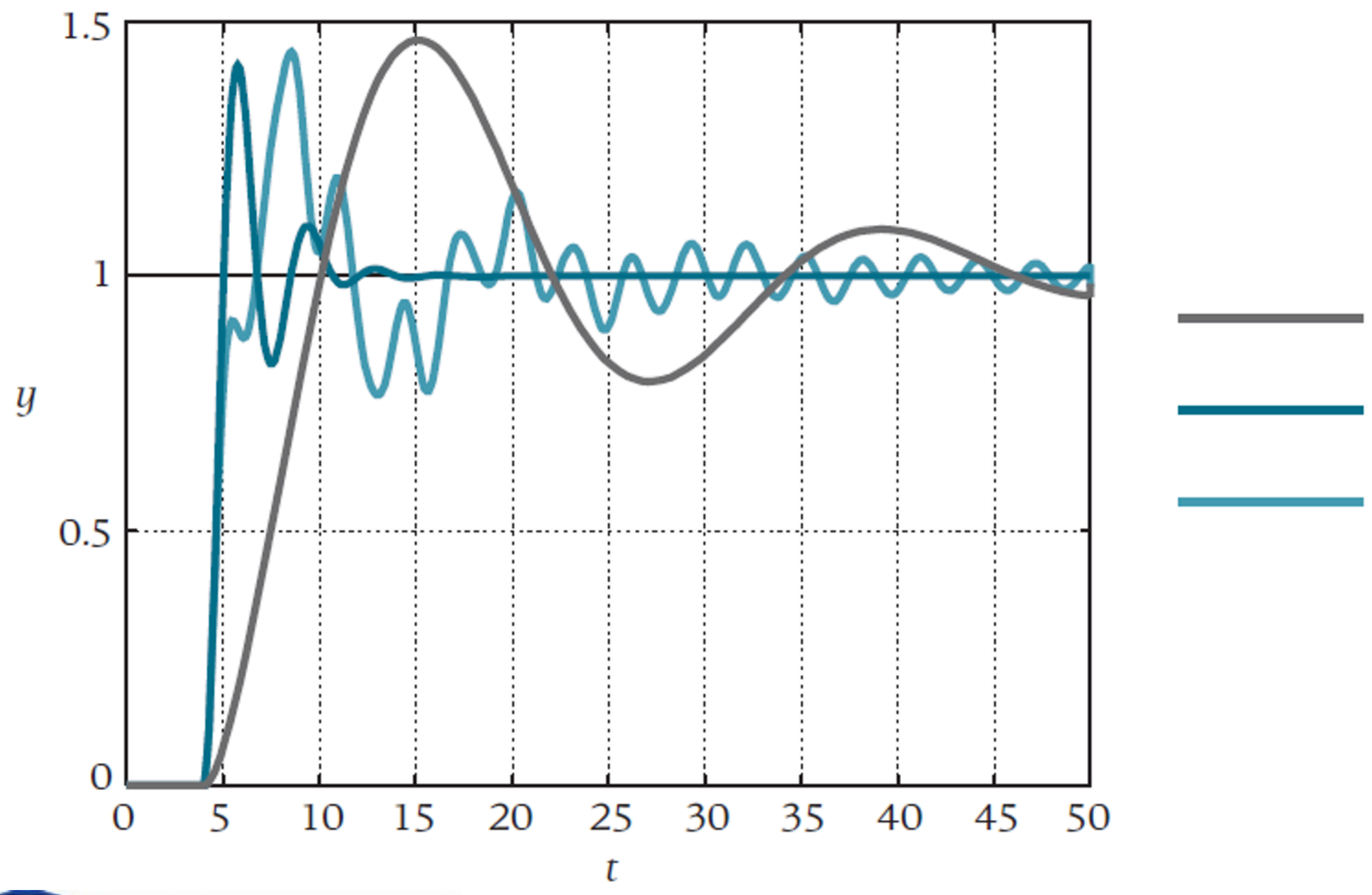
$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} \approx \left(1 - \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}\right)G'(s)$$

Control of LTI Systems with Delays

Summary:

Process





Design alternatives for R(s):

$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}\,G'(s)$$

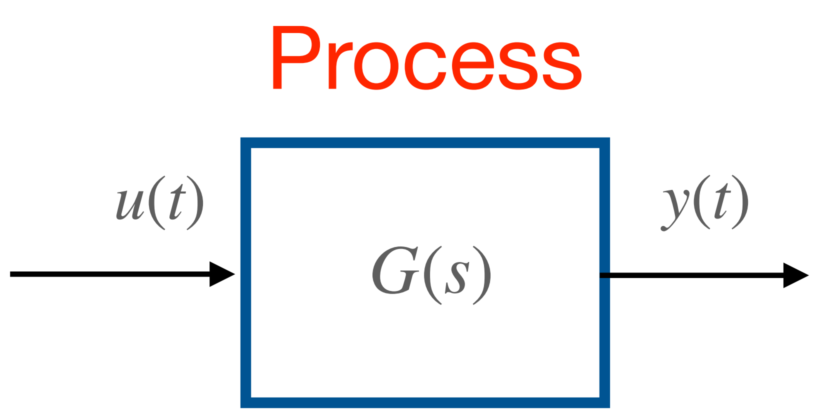
$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\,G'(s)$$

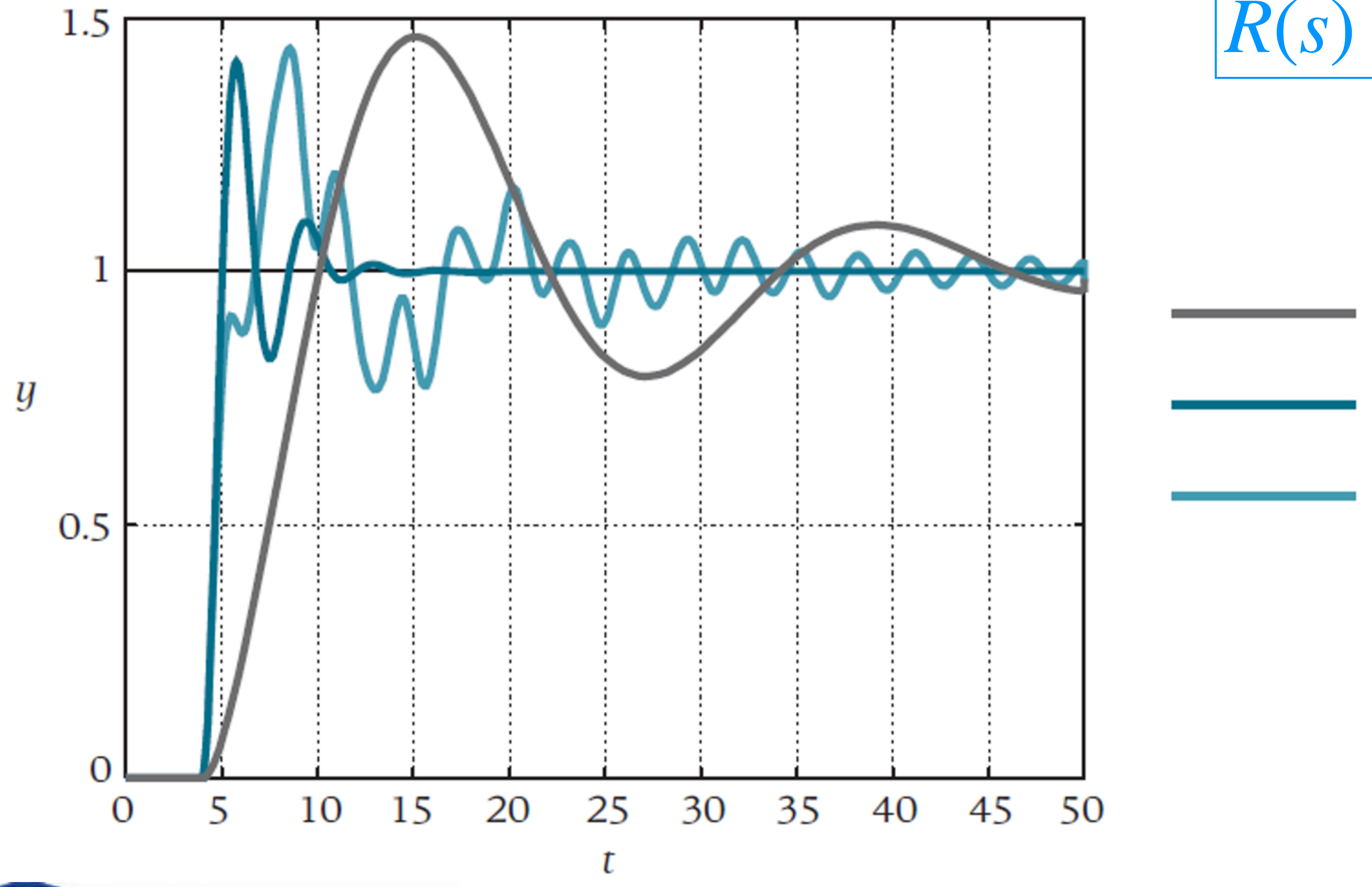
$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} \approx \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}\,G'(s)$$

$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} \approx \left(1 - \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}\right) G'(s)$$

Control of LTI Systems with Delays

Summary:





Exercise: Use the UFCS and design $R(s)$ relying on

Design alternatives for R(s):

$$G(s) = e^{-\tau s}\frac{N(s)}{D(s)} = e^{-\tau s}\,G'(s)$$

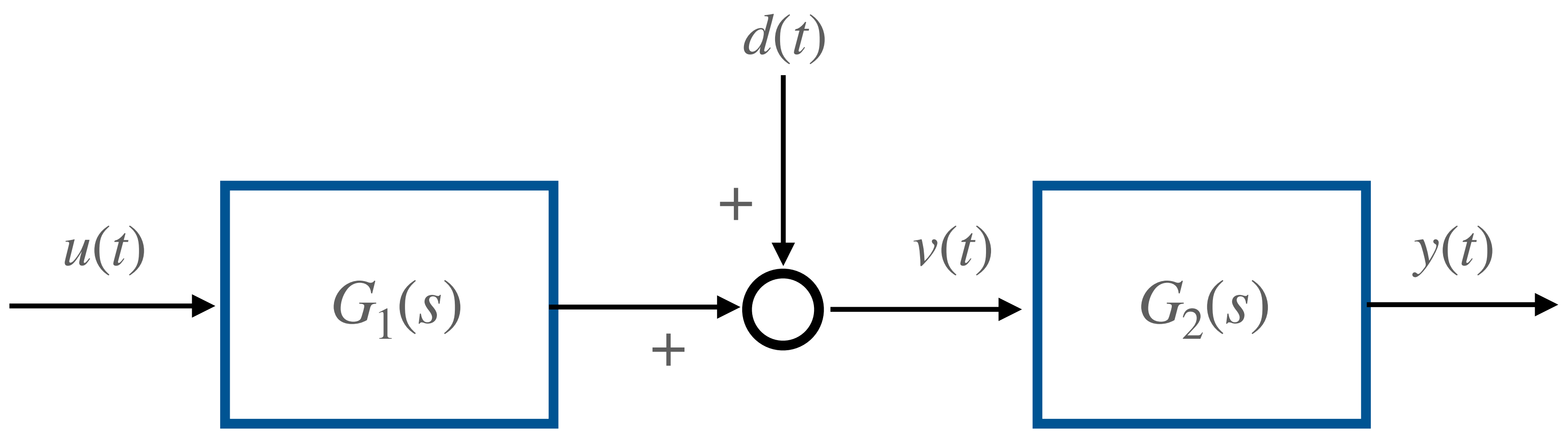
$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} = (1 - e^{-\tau s})\,G'(s)$$

$$\boxed{G(s) = e^{-\tau s}\frac{N(s)}{D(s)} \approx \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}\,G'(s)}$$

$$P(s) = (1 - e^{-\tau s})\frac{N(s)}{D(s)} \approx \left(1 - \frac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}\right)G'(s)$$

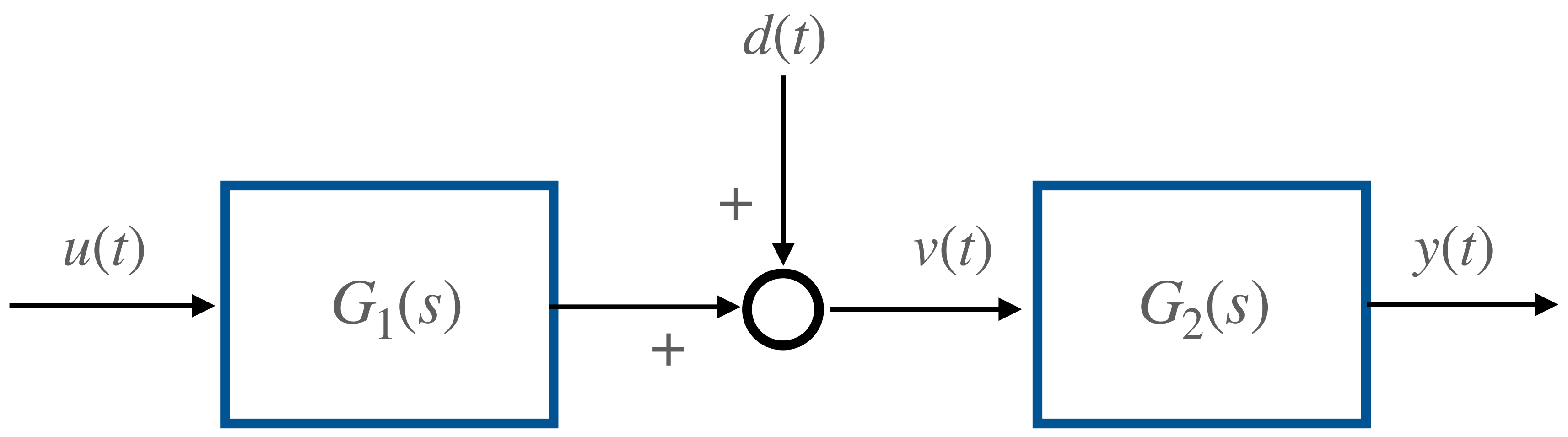
Control Scheme with Decoupling in the Frequency Domain

Process



Control Scheme with Decoupling in the Frequency Domain

Process

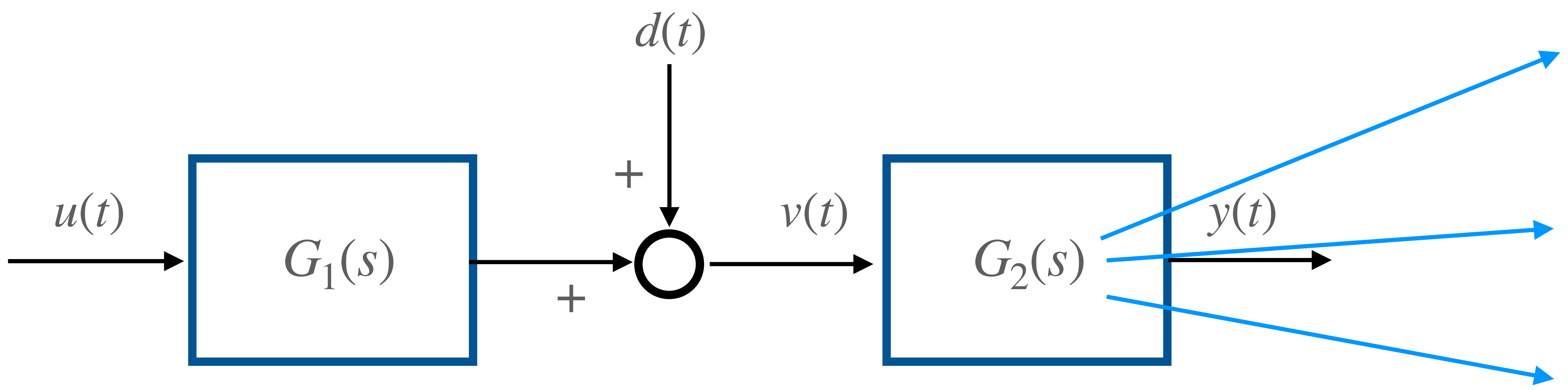


"good" dynamics

"bad" dynamics

Control Scheme with Decoupling in the Frequency Domain

Process



“good” dynamics

“bad” dynamics

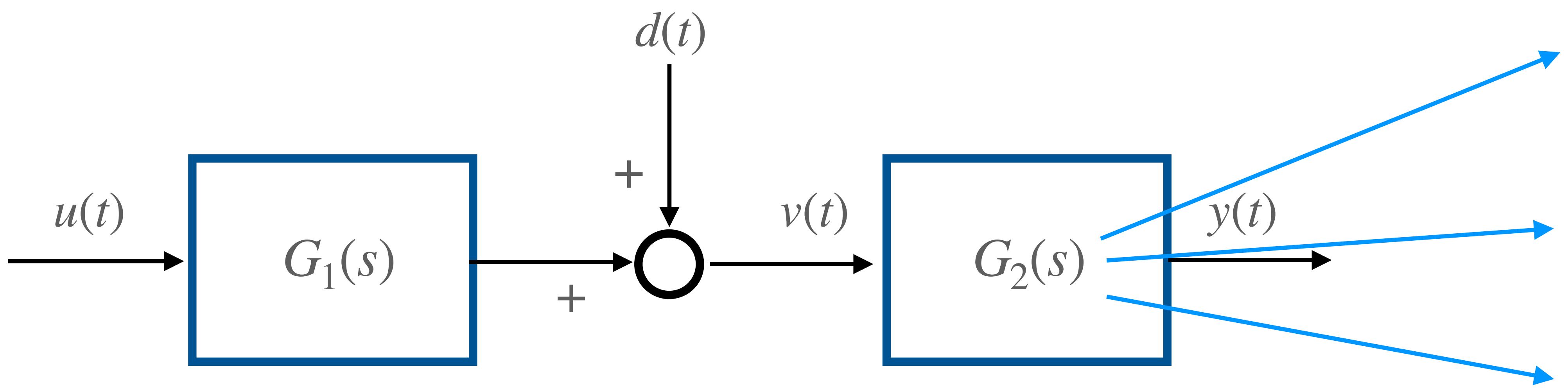
subsystem with con minimum phase zeros

subsystem affected by delay

subsystem affected by uncertainty at HF

Control Scheme with Decoupling in the Frequency Domain

Process



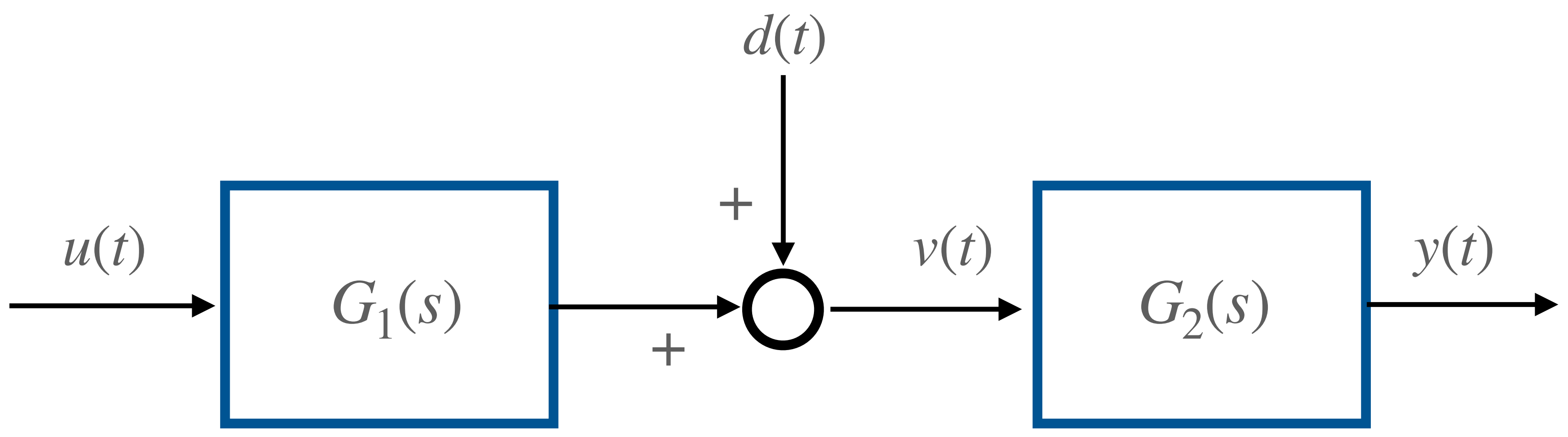
“good” dynamics

“bad” dynamics

- subsystem with con minimum phase zeros
- subsystem affected by delay
- subsystem affected by uncertainty at HF

Control Scheme with Decoupling in the Frequency Domain

Process



"good" dynamics

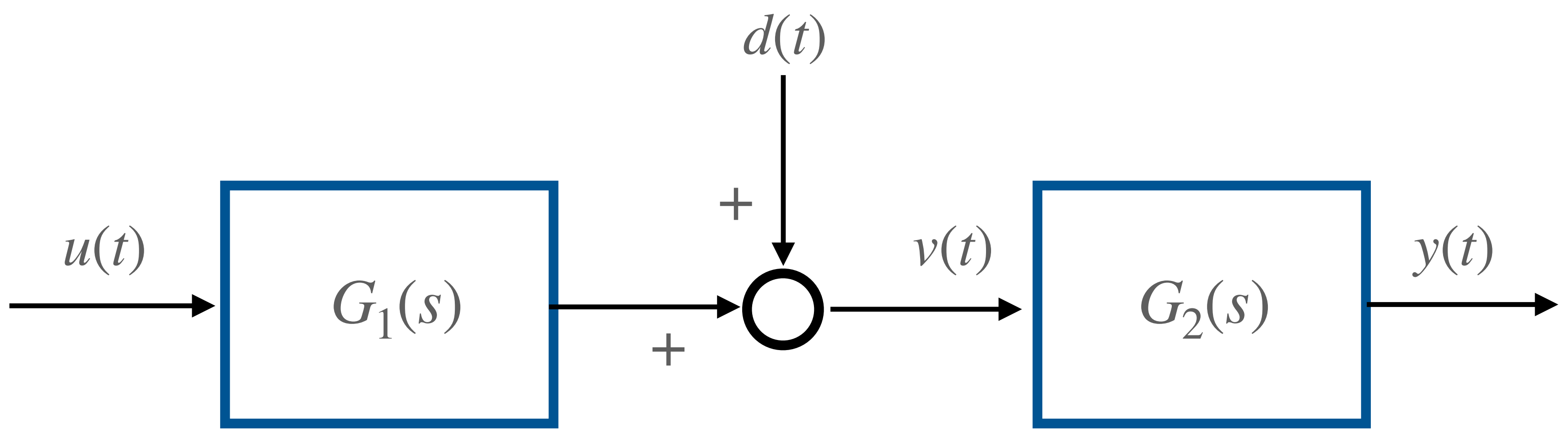
"bad" dynamics

Assumptions:

- The performances that can be obtained by designing a control system for $G_1(s)$ alone are better than those that can be obtained for the entire system $G_1(s)G_2(s)$
- v is accessible (measurable)

Control Scheme with Decoupling in the Frequency Domain

Process

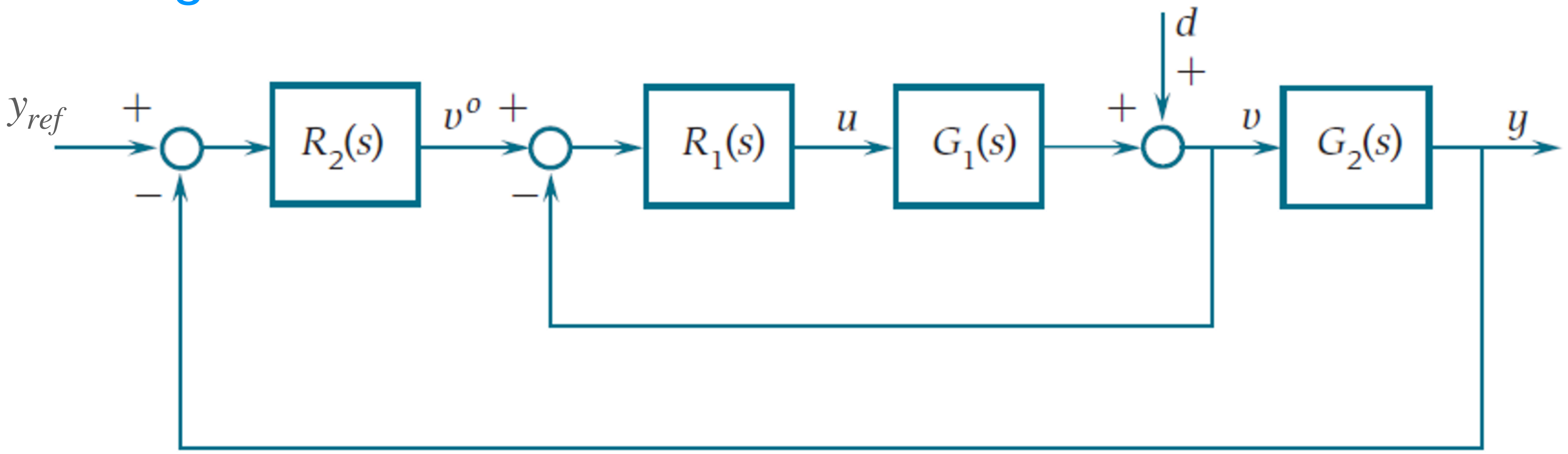


"good" dynamics   "bad" dynamics

Assumptions:

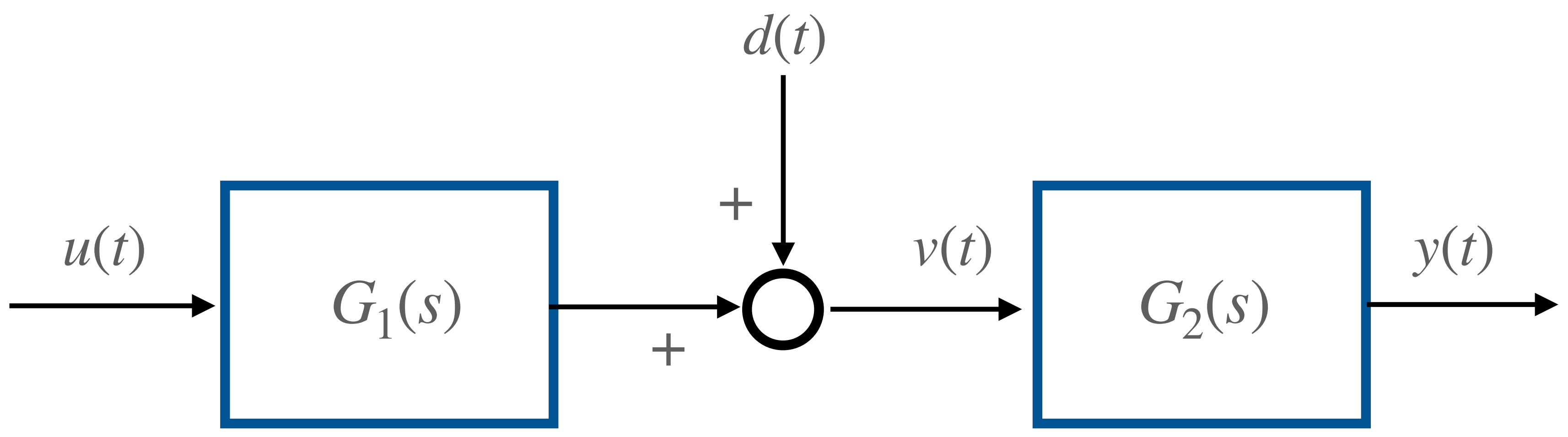
- The performances that can be obtained by designing a control system for $G_1(s)$ alone are better than those that can be obtained for the entire system $G_1(s)G_2(s)$
- v is accessible (measurable)

Control scheme design

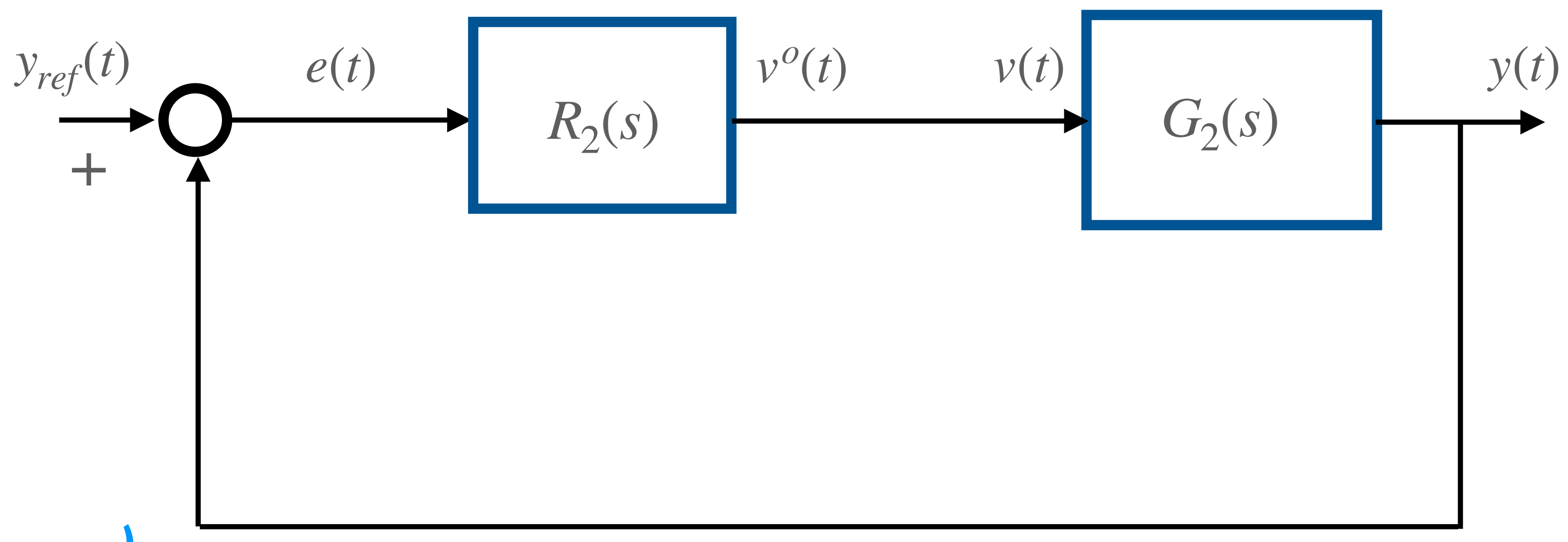


Control Scheme with Decoupling in the Frequency Domain

Process

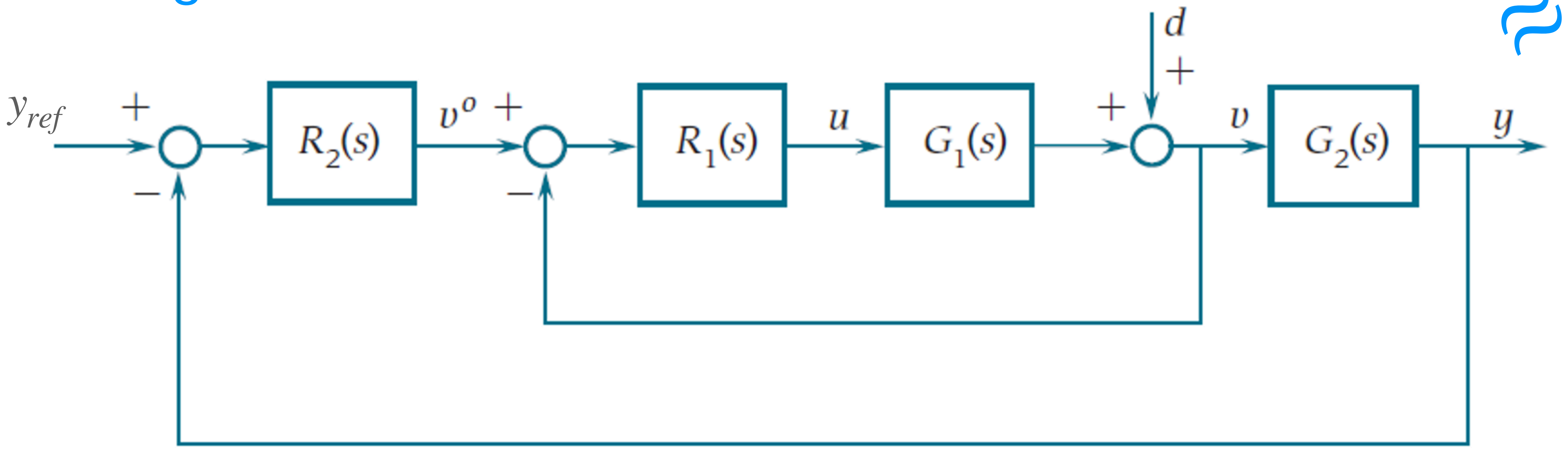


“good” dynamics — “bad” dynamics



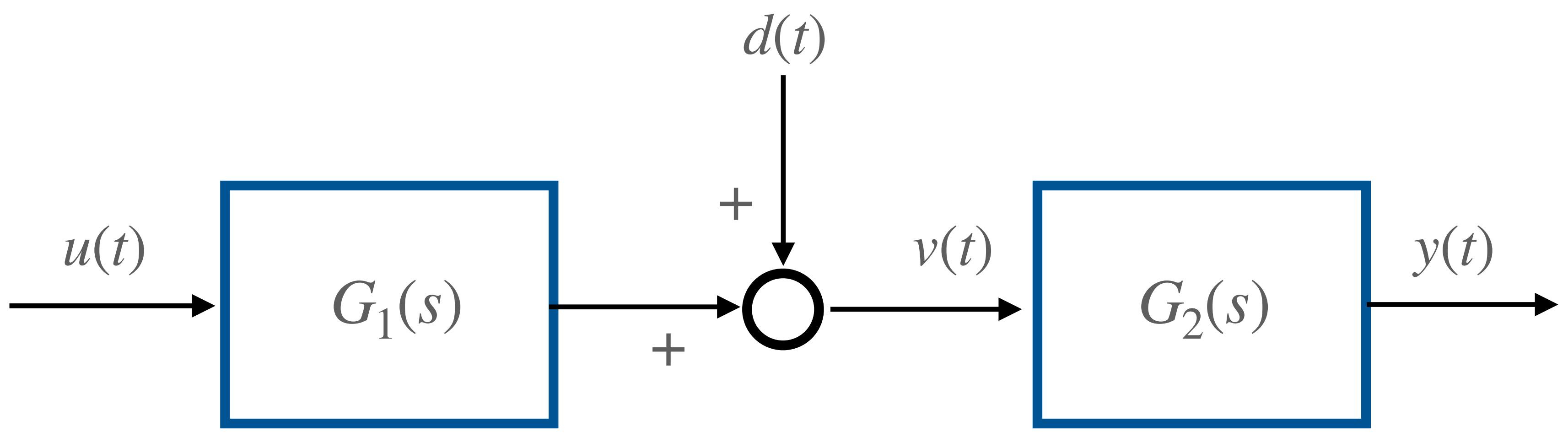
Control scheme design

≈



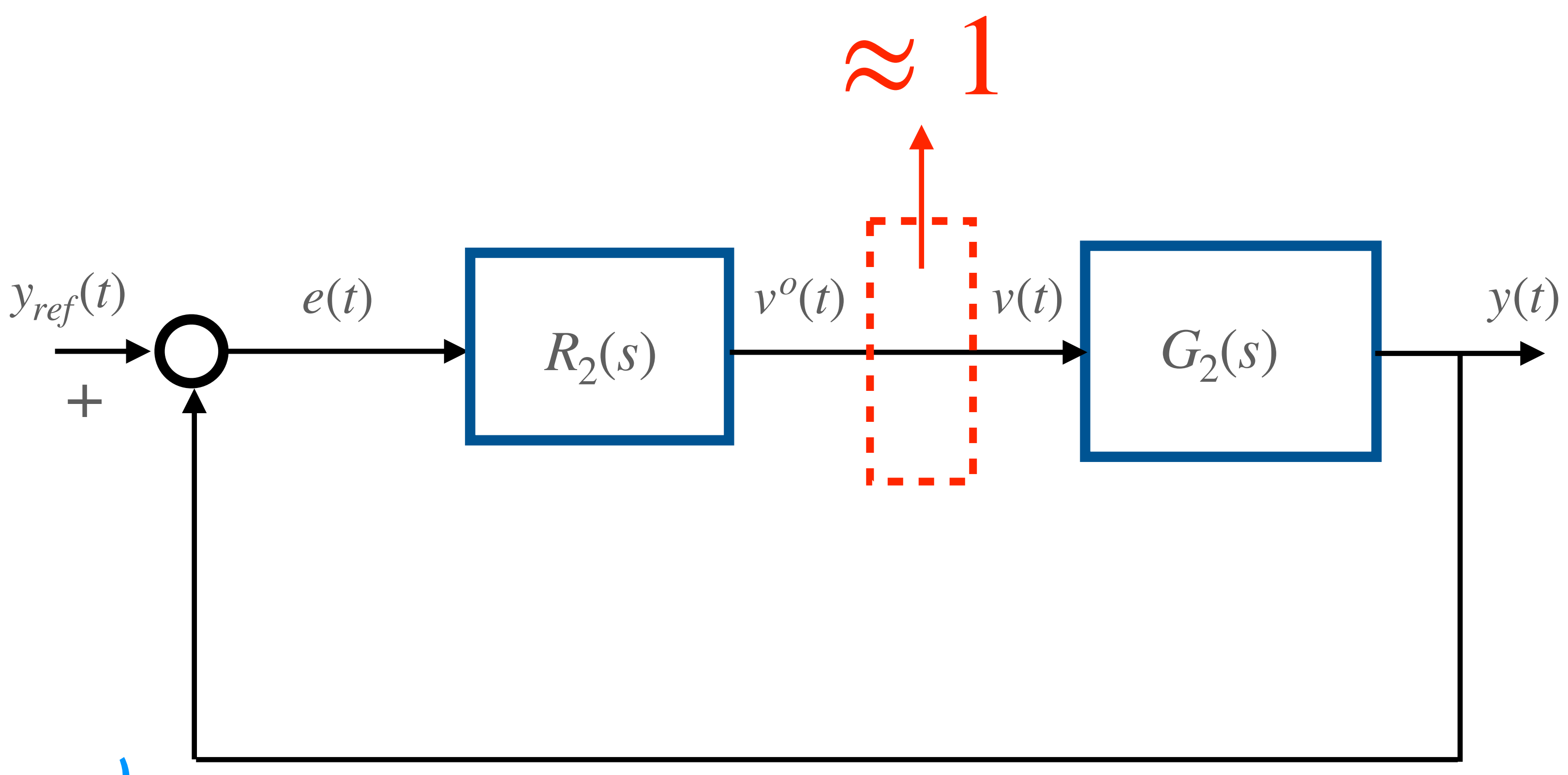
Control Scheme with Decoupling in the Frequency Domain

Process

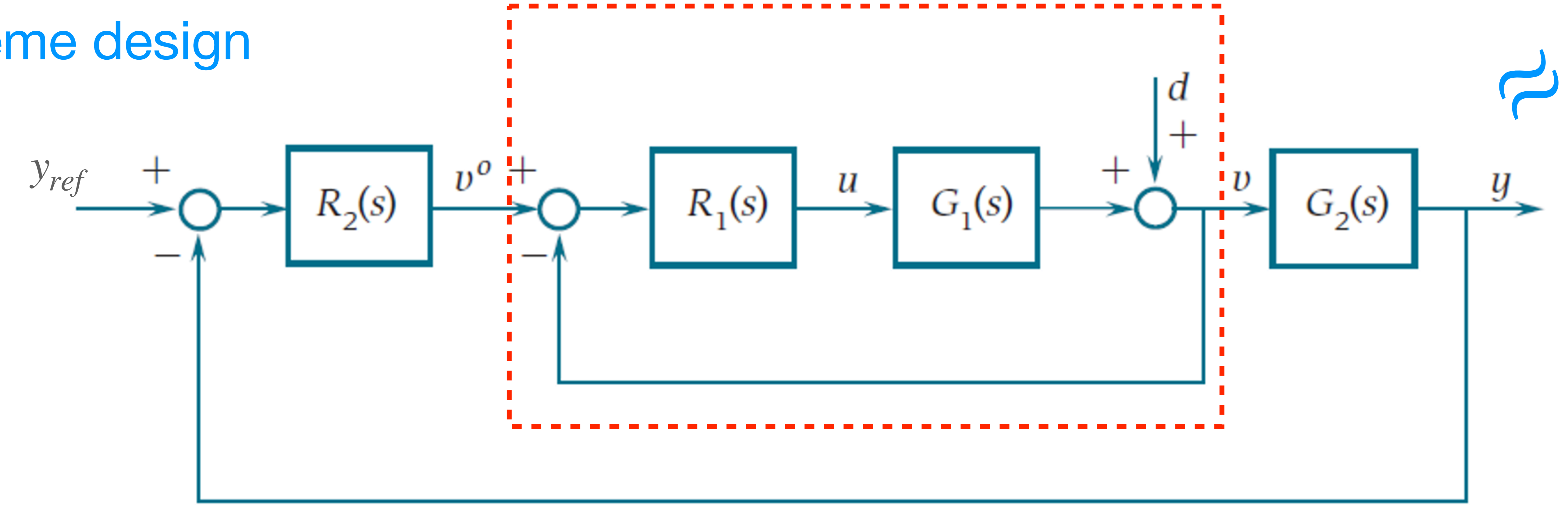


"good" dynamics

"bad" dynamics

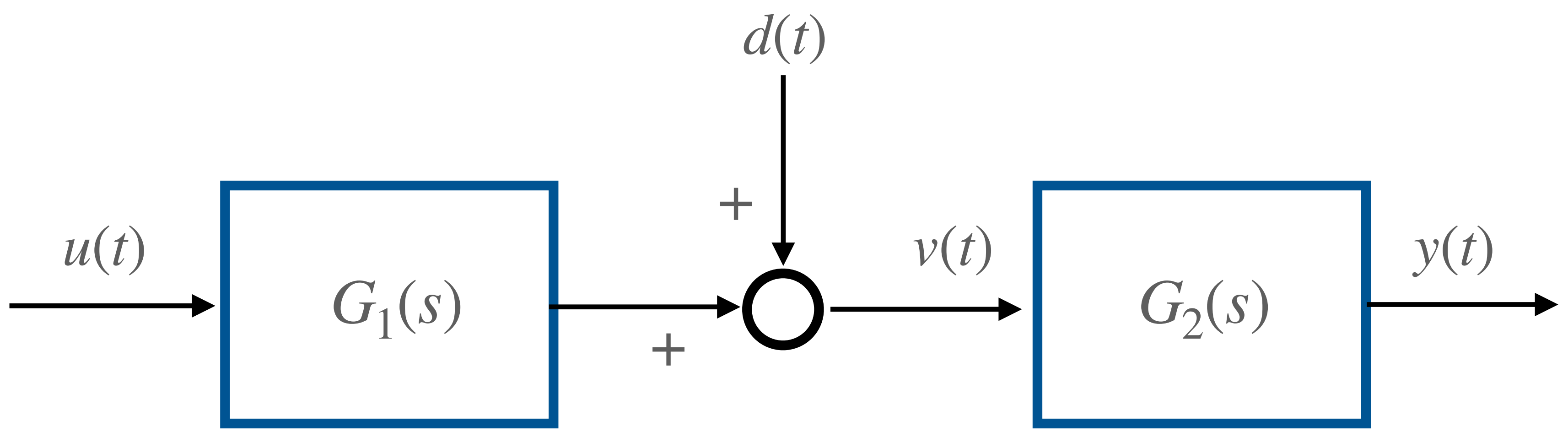


Control scheme design

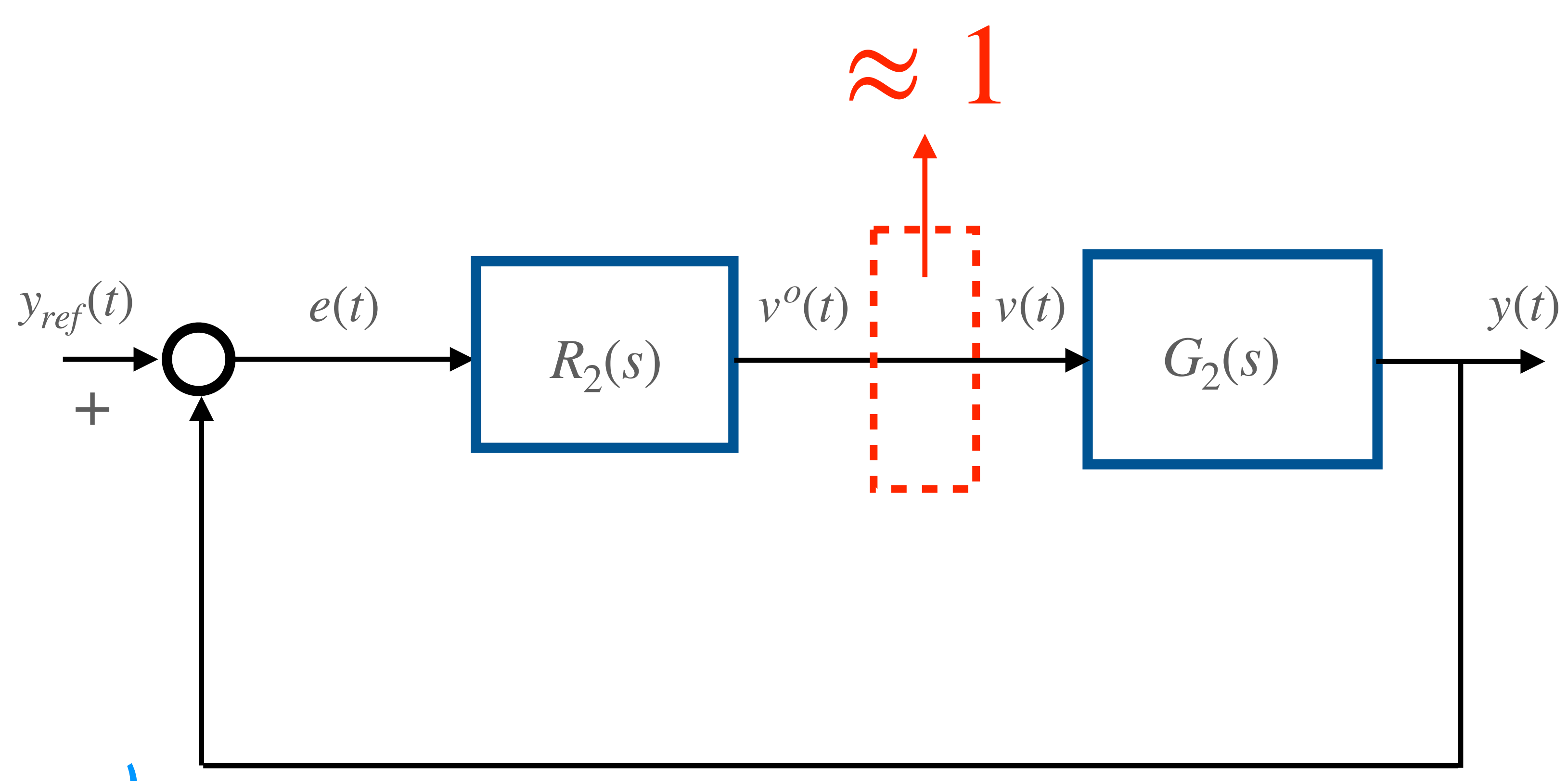


Control Scheme with Decoupling in the Frequency Domain

Process

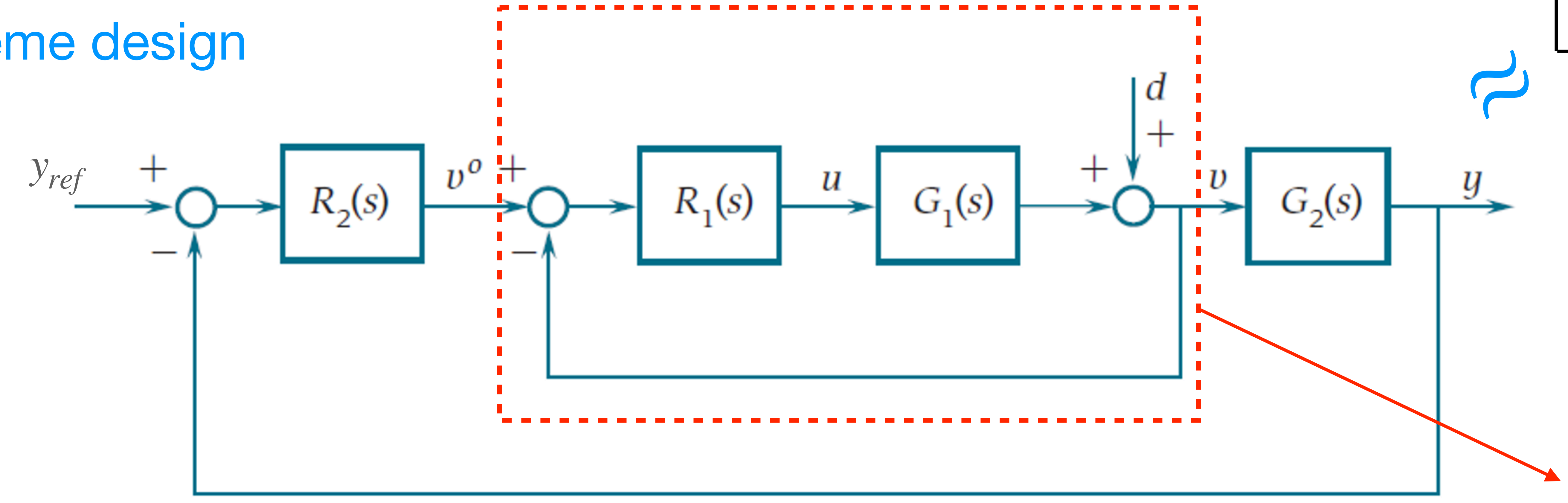


"good" dynamics   "bad" dynamics



Control scheme design

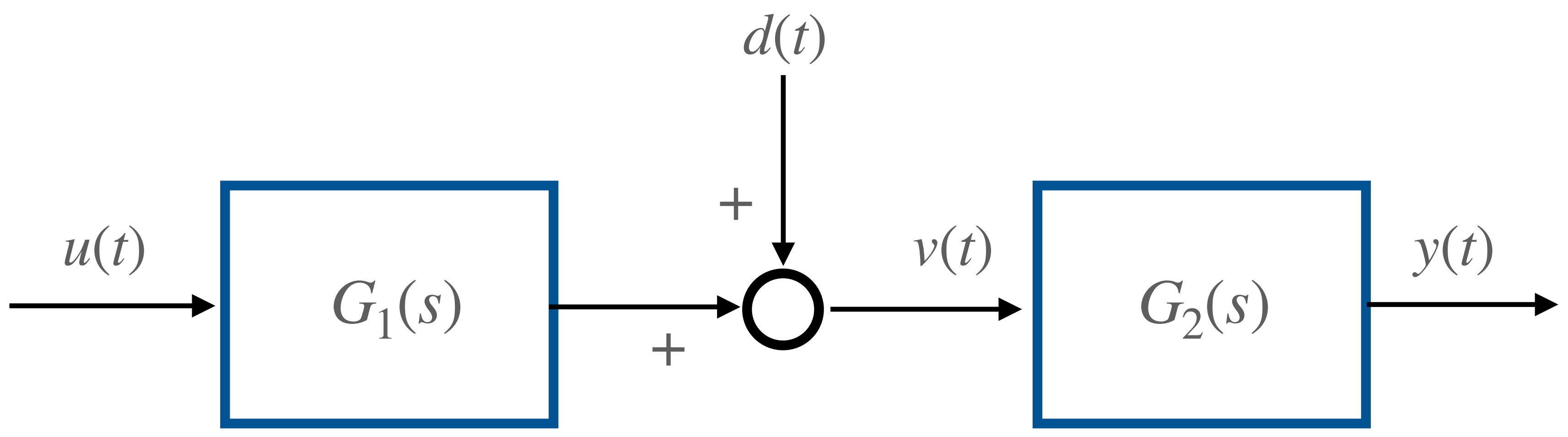
$\approx$



large $\omega_{BW_{CL_1}}$

Control Scheme with Decoupling in the Frequency Domain

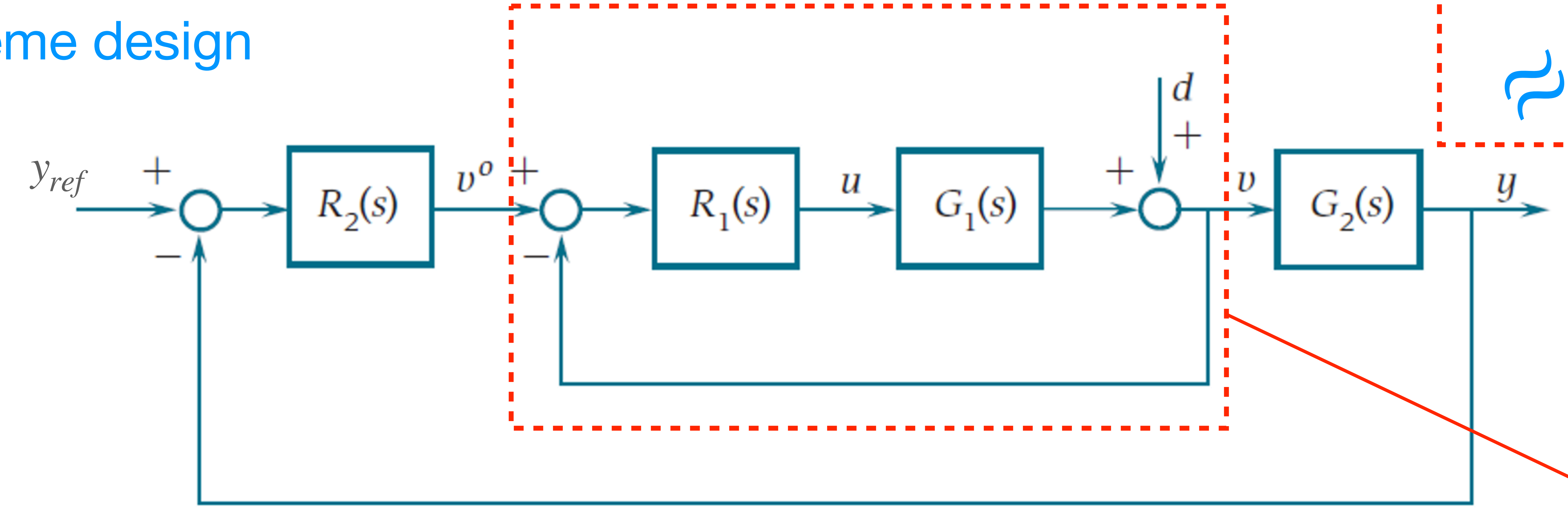
Process

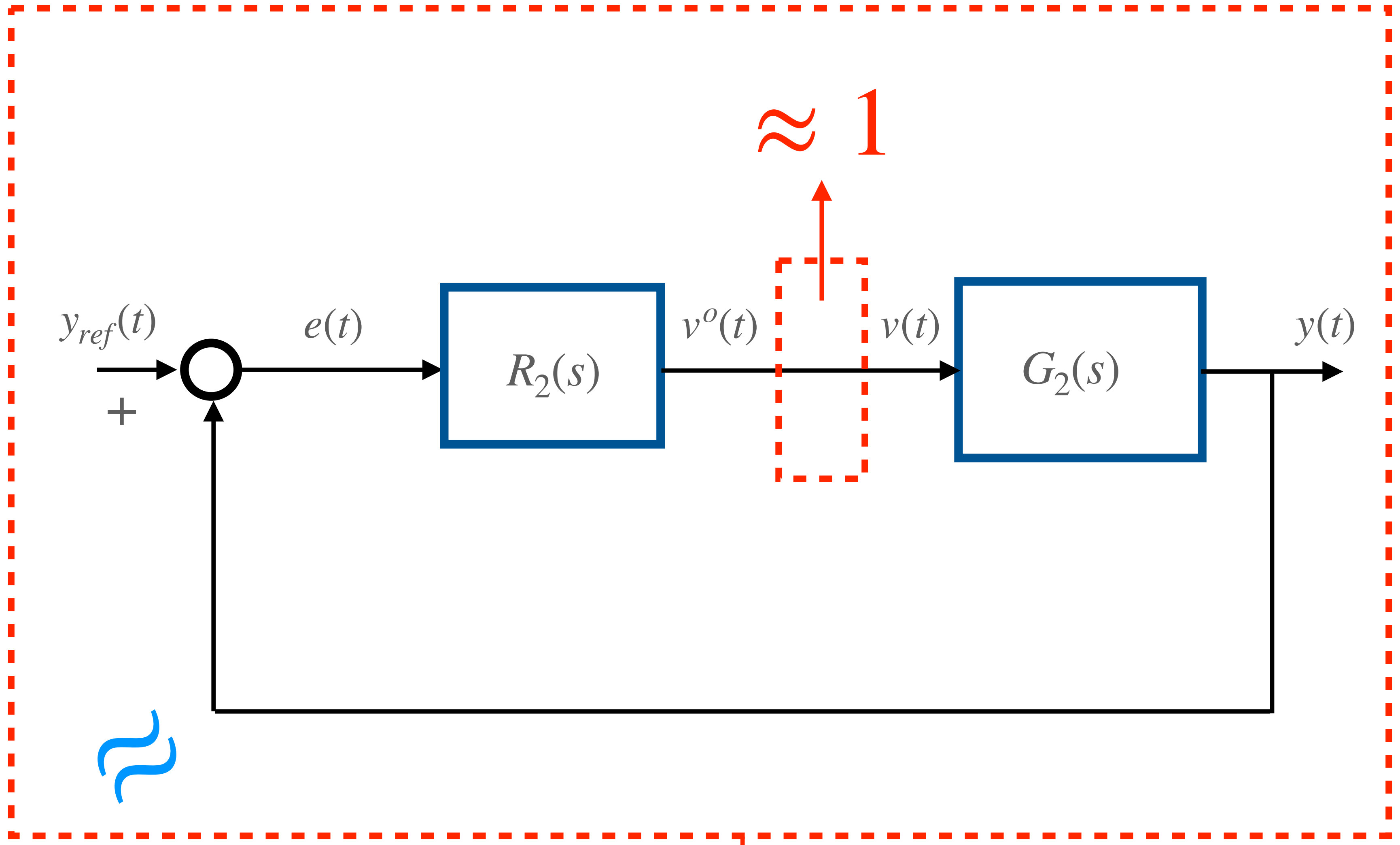


"good" dynamics

"bad" dynamics

Control scheme design





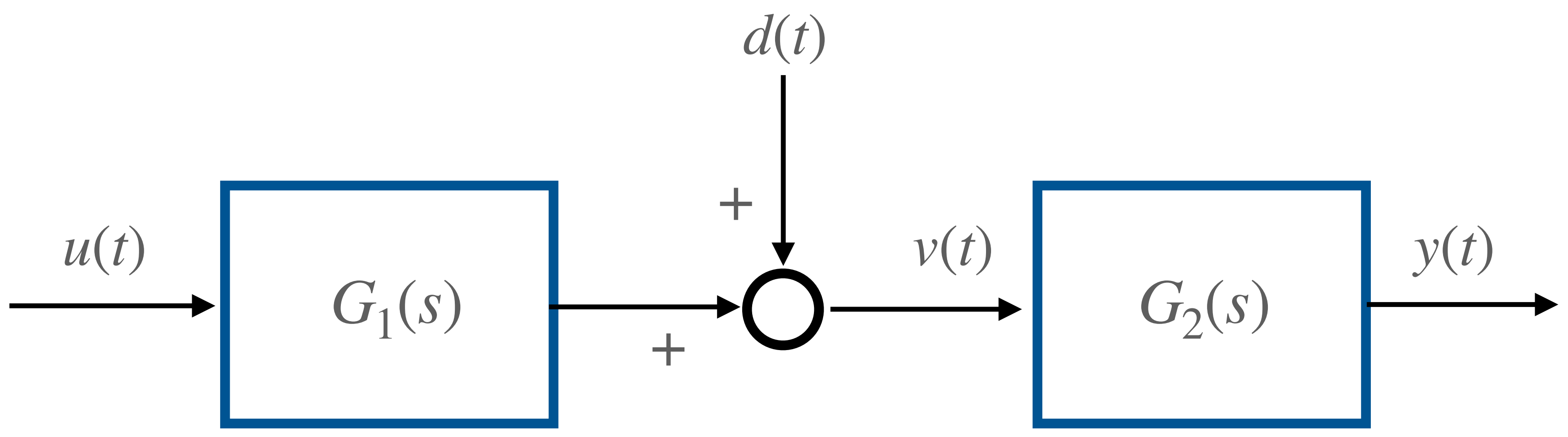
narrow $\omega_{BW_{CL_2}}$

large $\omega_{BW_{CL_1}}$

Control Scheme with Decoupling in the Frequency Domain

Example (from the textbook):

Process

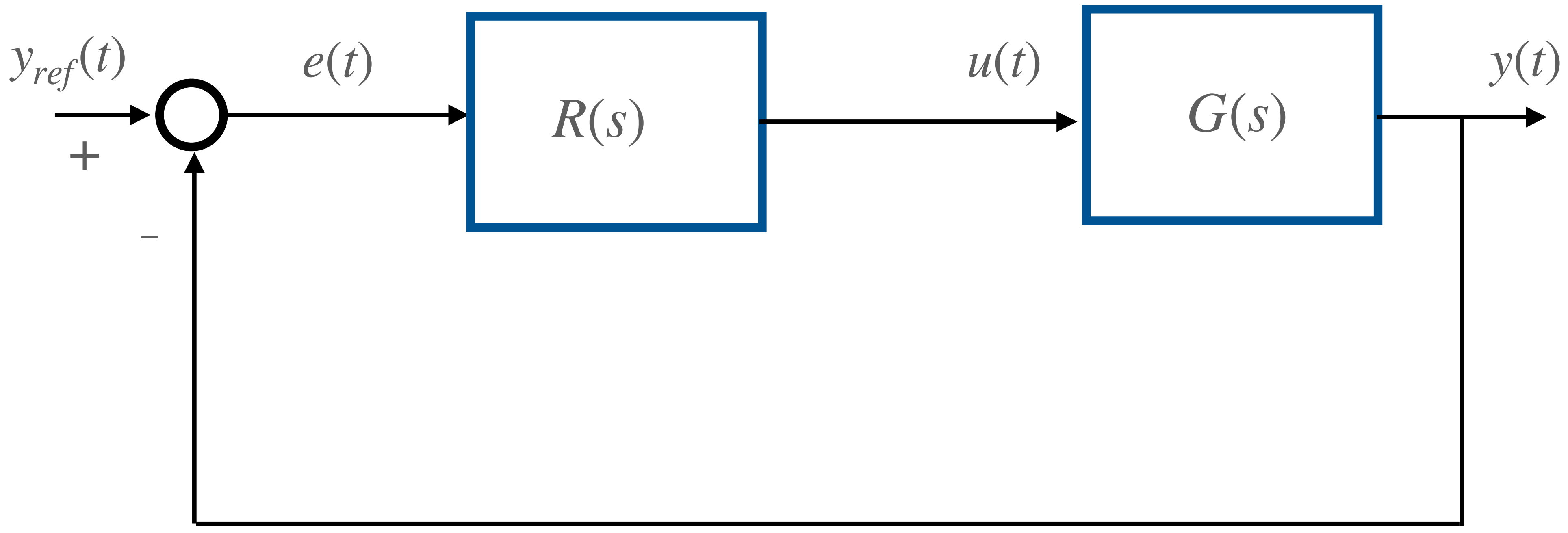


"good" dynamics "bad" dynamics

$$G_1(s) = \frac{1}{1 + 0.005s}$$

$$G_2(s) = \frac{e^{-4s}}{(1+s)^2}$$

Control scheme design: Solution 1

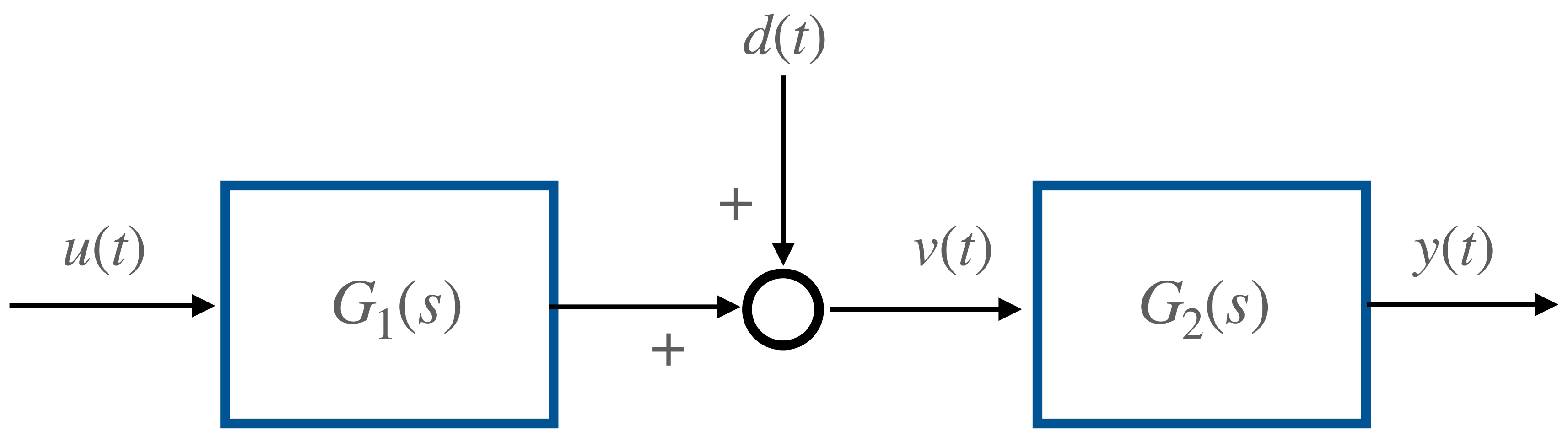


$$R(s) = \frac{0.2(s+1)}{s}$$

Control Scheme with Decoupling in the Frequency Domain

Example (from the textbook):

Process



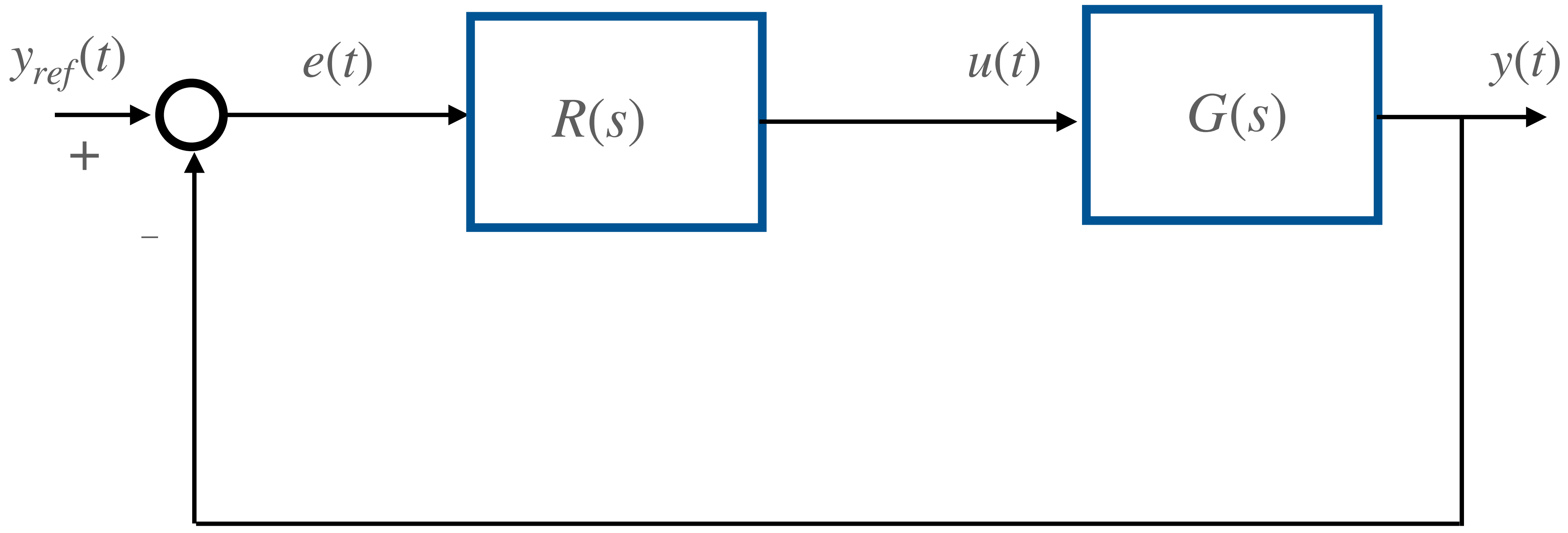
"good" dynamics

"bad" dynamics

$$G_1(s) = \frac{1}{1 + 0.005s}$$

$$G_2(s) = \frac{e^{-4s}}{(1+s)^2}$$

Control scheme design: Solution 1



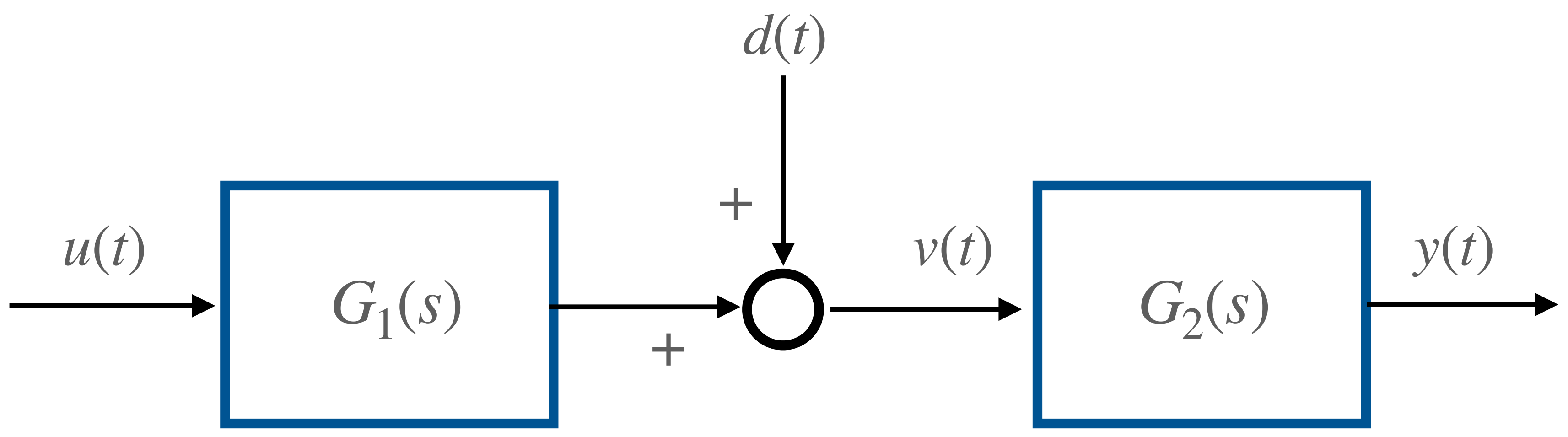
$$R(s) = \frac{0.2(s+1)}{s}$$

→ Compute the closed loop bandwidth

Control Scheme with Decoupling in the Frequency Domain

Example (from the textbook):

Process

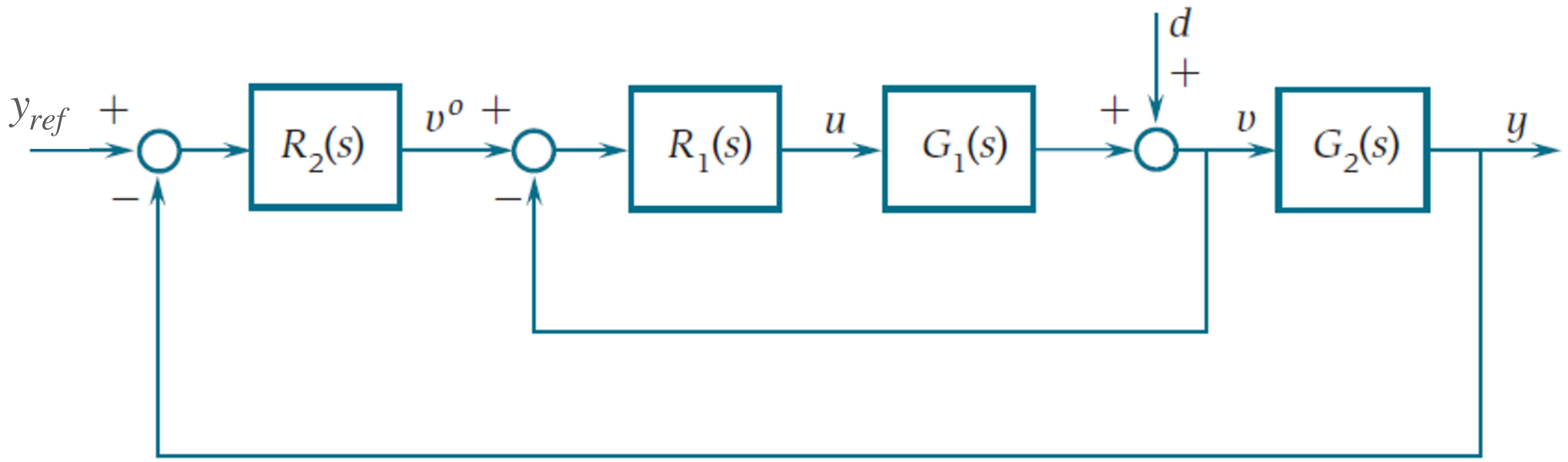


"good" dynamics "bad" dynamics

$$G_1(s) = \frac{1}{1+0.005s}$$

$$G_2(s) = \frac{e^{-4s}}{(1+s)^2}$$

Control scheme design: Solution 2



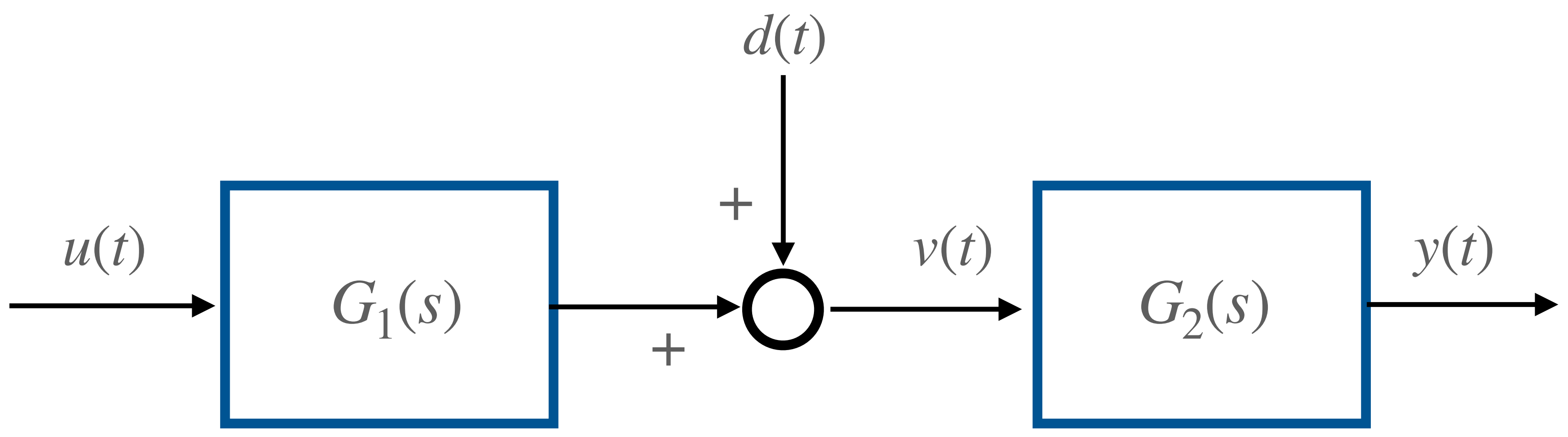
$$R_1(s) = 10$$

$$R_2(s) = \frac{0.2(s+1)}{s}$$

Control Scheme with Decoupling in the Frequency Domain

Example (from the textbook):

Process

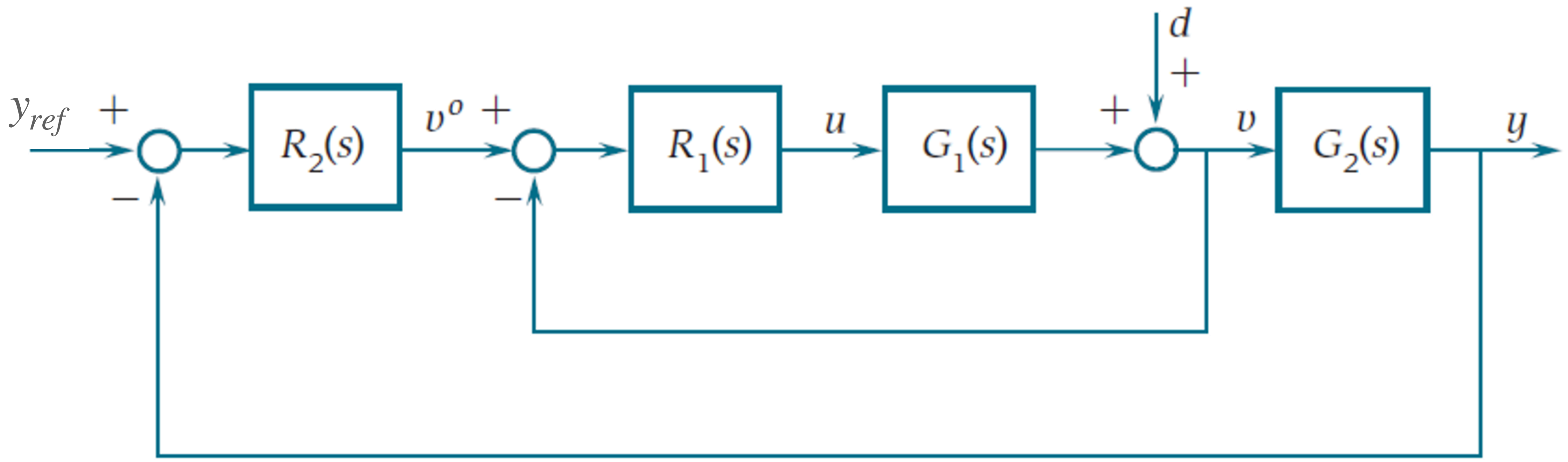


"good" dynamics "bad" dynamics

$$G_1(s) = \frac{1}{1 + 0.005s}$$

$$G_2(s) = \frac{e^{-4s}}{(1 + s)^2}$$

Control scheme design: Solution 2



$$R_1(s) = 10$$

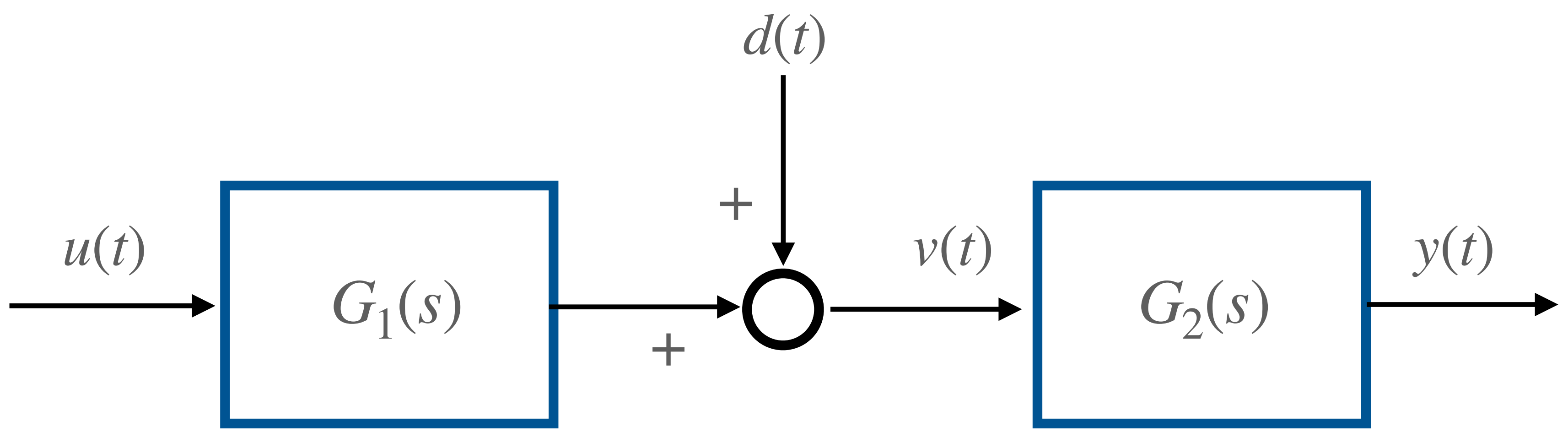
$$R_2(s) = \frac{0.2(s + 1)}{s}$$

Compute $\omega_{BW_{CL_1}}$ and $\omega_{BW_{CL_2}}$

Control Scheme with Decoupling in the Frequency Domain

Example (from the textbook):

Process

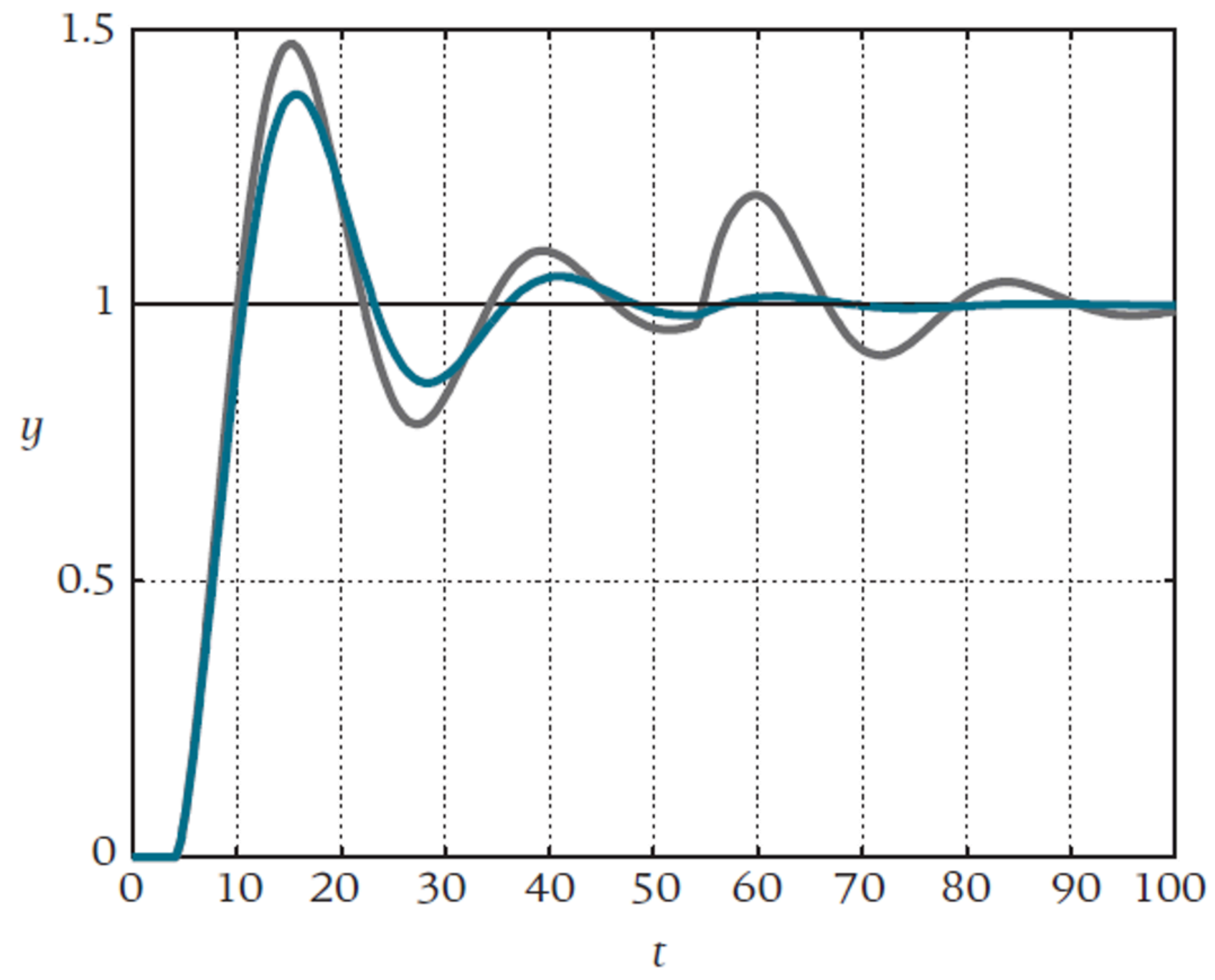


"good" dynamics

"bad" dynamics

$$G_1(s) = \frac{1}{1 + 0.005s}$$

$$G_2(s) = \frac{e^{-4s}}{(1 + s)^2}$$



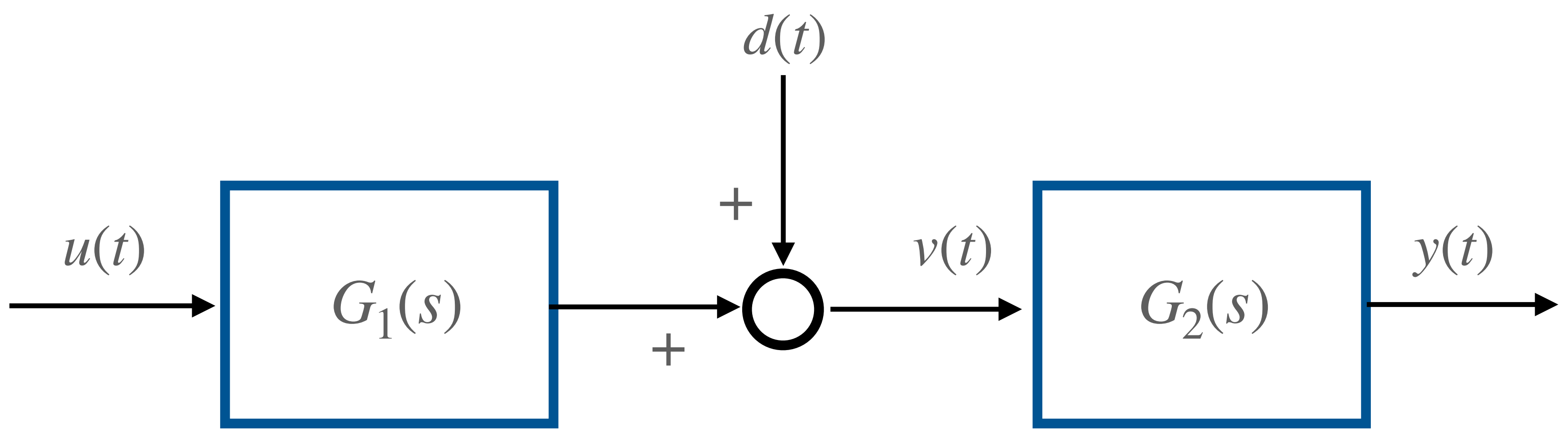
Solution 1

Solution 2

Control Scheme with Decoupling in the Frequency Domain

Example (from the textbook):

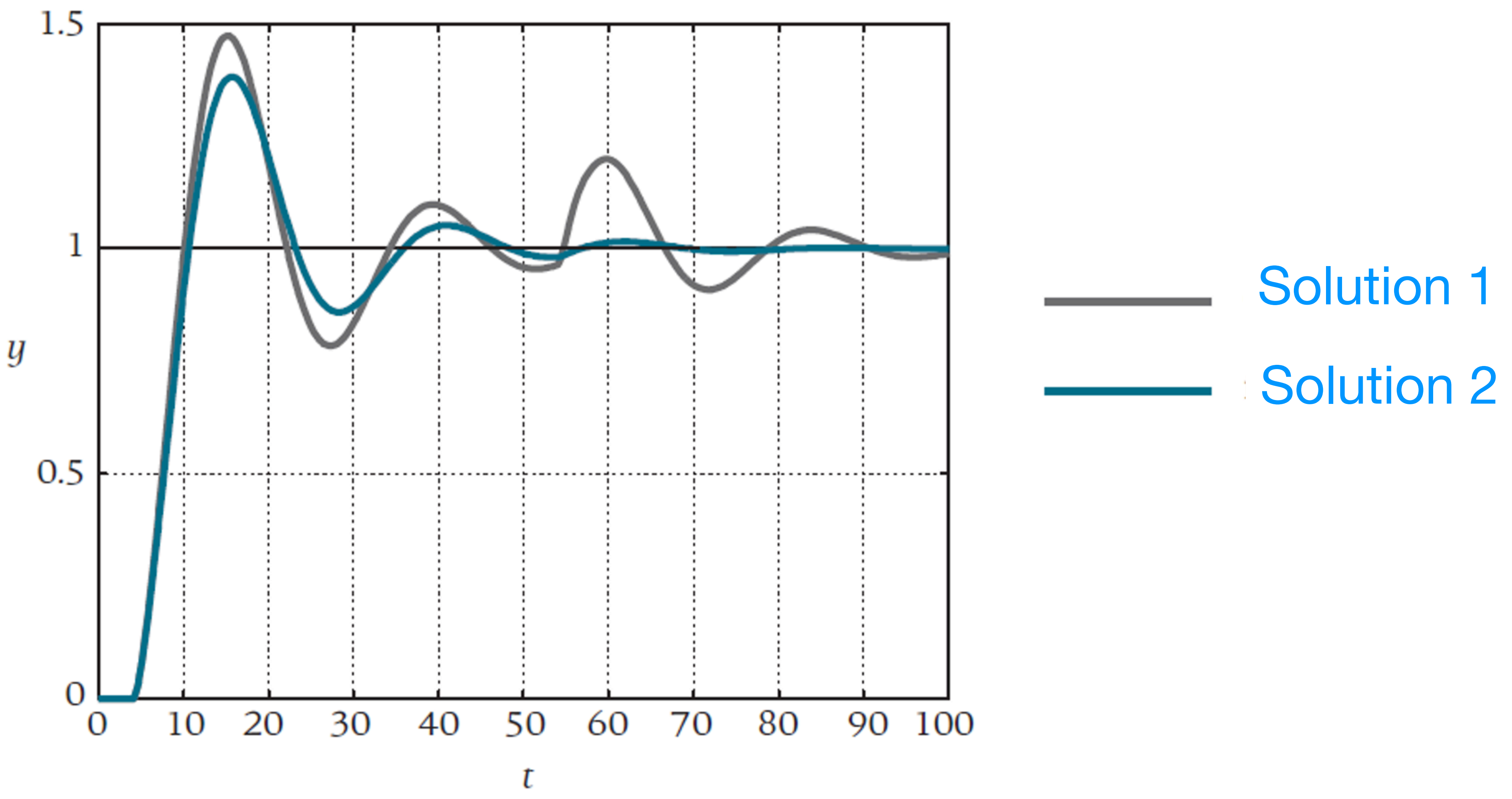
Process



"good" dynamics "bad" dynamics

$$G_1(s) = \frac{1}{1 + 0.005s}$$

$$G_2(s) = \frac{e^{-4s}}{(1+s)^2}$$

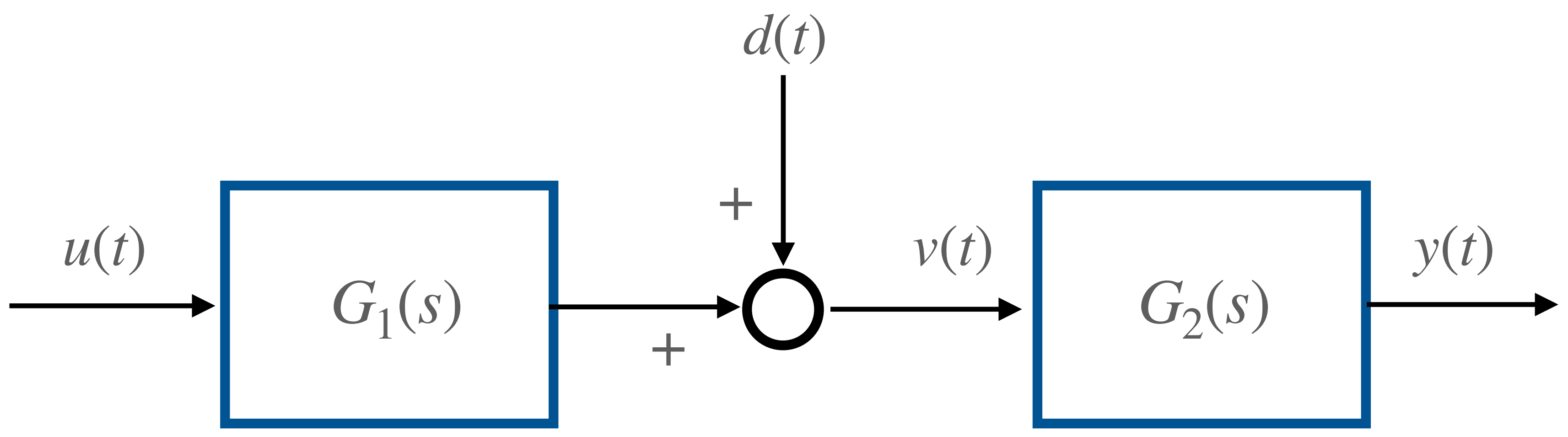


Exercise: formulate a similar example when $G_2(s)$ is rational with a real zero in $+1$

Control Scheme with Decoupling in the Frequency Domain

Example (from the textbook):

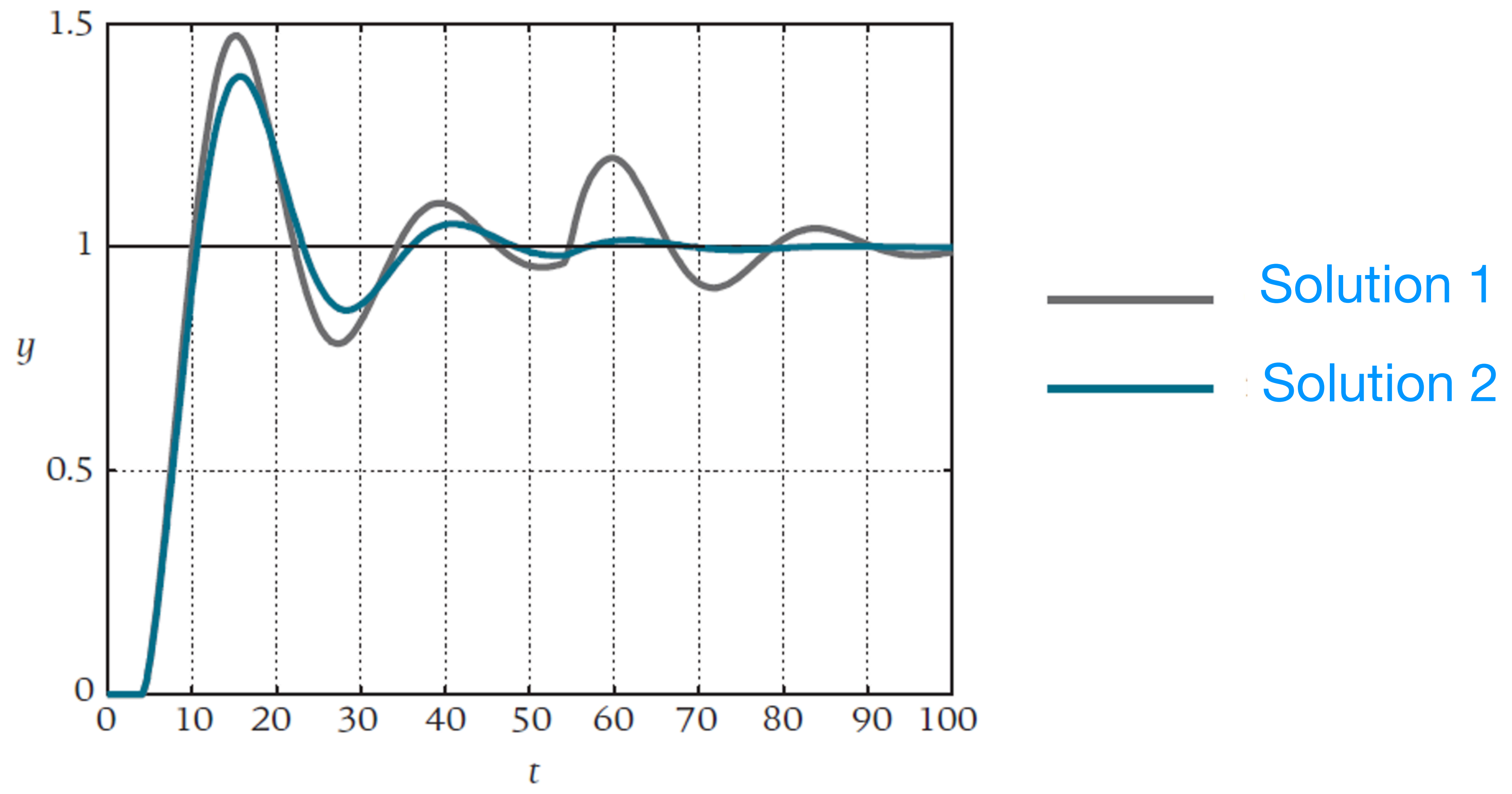
Process



"good" dynamics "bad" dynamics

$$G_1(s) = \frac{1}{1 + 0.005s}$$

$$G_2(s) = \frac{e^{-4s}}{(1+s)^2}$$



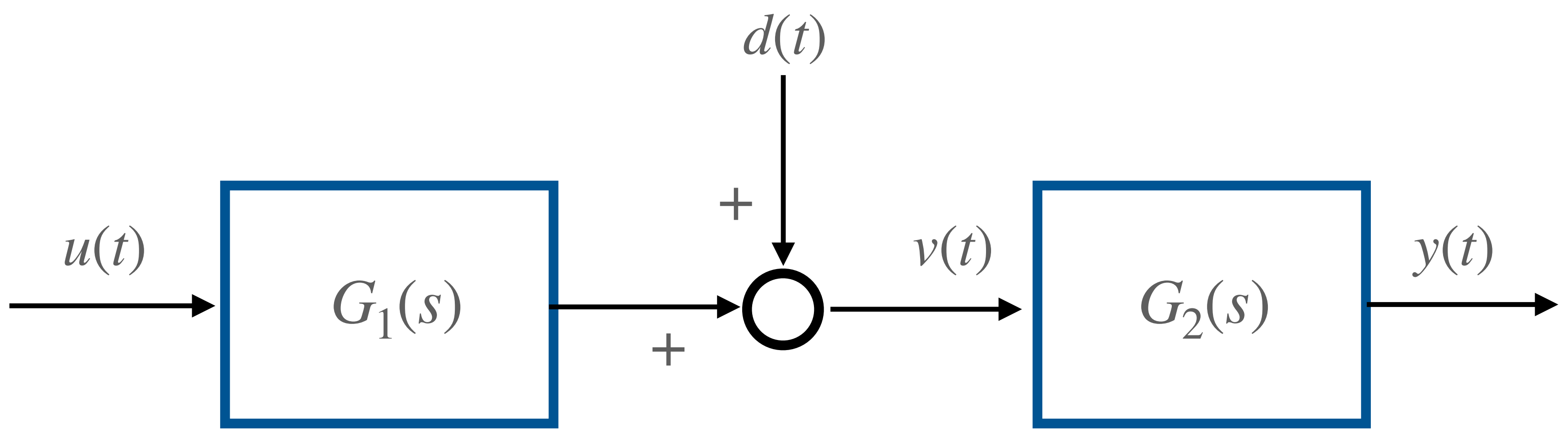
Exercise: formulate a similar example when $G_2(s)$ is rational with a real zero in $+1$

Can you use the unitary feedback control scheme and design a single controller $R(s)$? Try for different $d(t)$

Control Scheme with Decoupling in the Frequency Domain

Example (from the textbook):

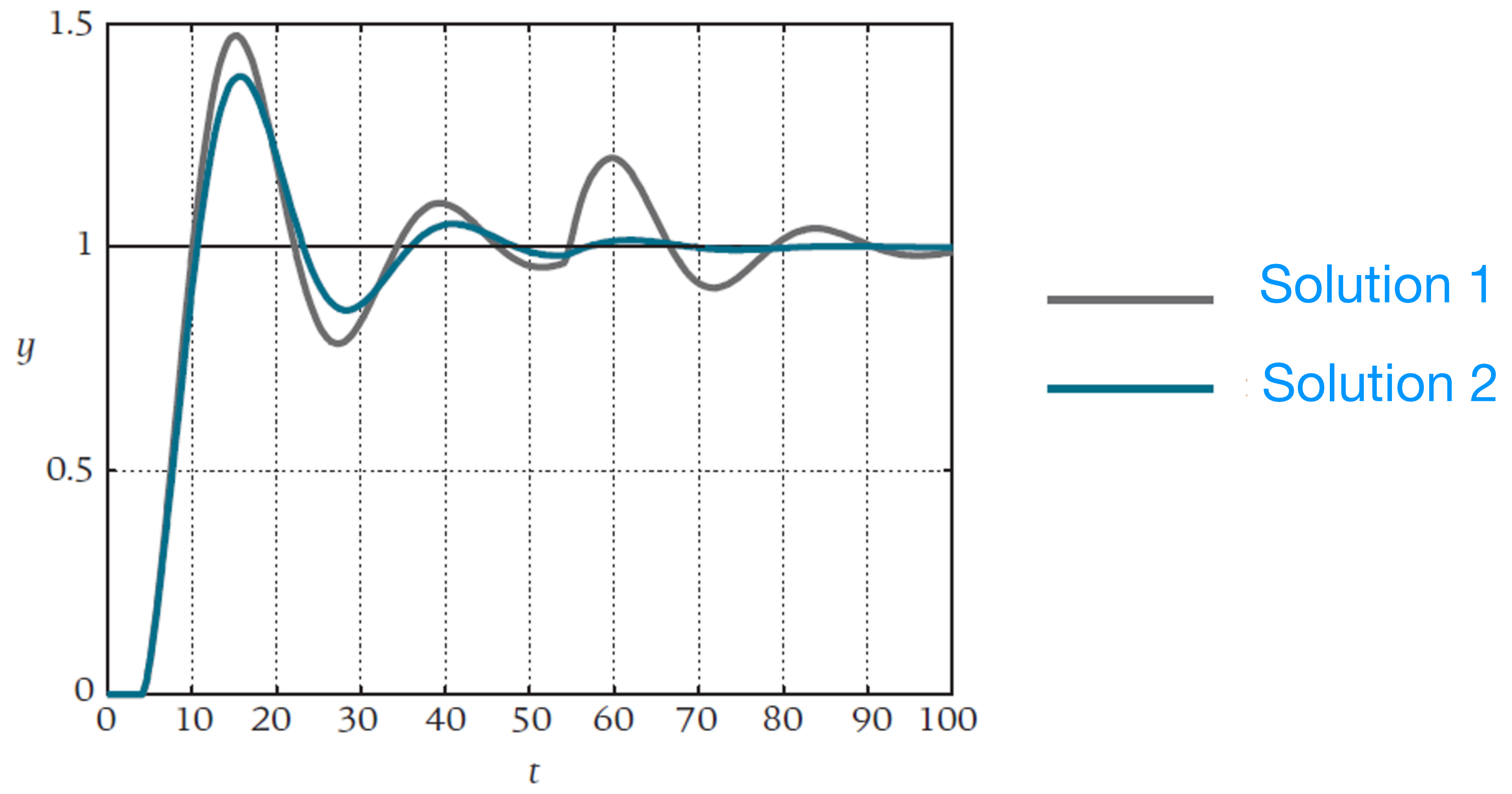
Process



"good" dynamics "bad" dynamics

$$G_1(s) = \frac{1}{1 + 0.005s}$$

$$G_2(s) = \frac{e^{-4s}}{(1 + s)^2}$$



Exercise: formulate a similar example when $G_2(s)$ is rational with a real zero in $+1$

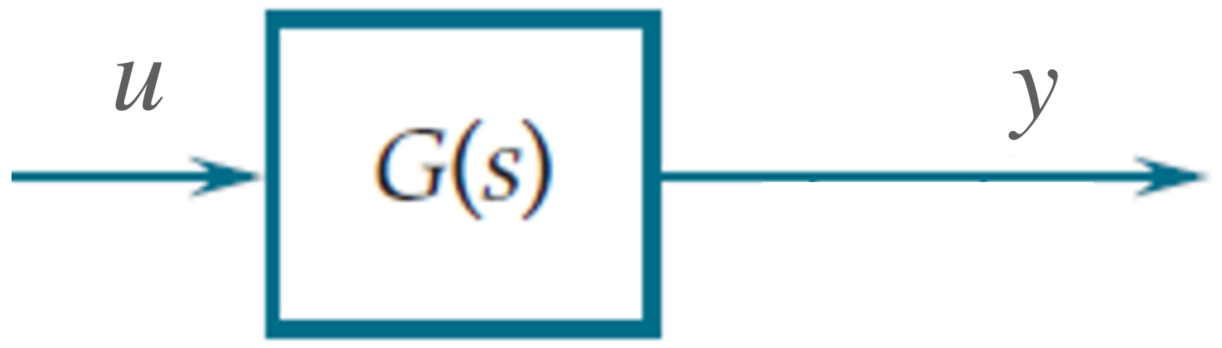
Use decoupling in the freq. dom. Find $R_1(s)$ and $R_2(s)$ and simulate the step response in MATLAB

Control of Open-loop Unstable Systems

Assumption:

- $G(s)$ open-loop unstable

Process

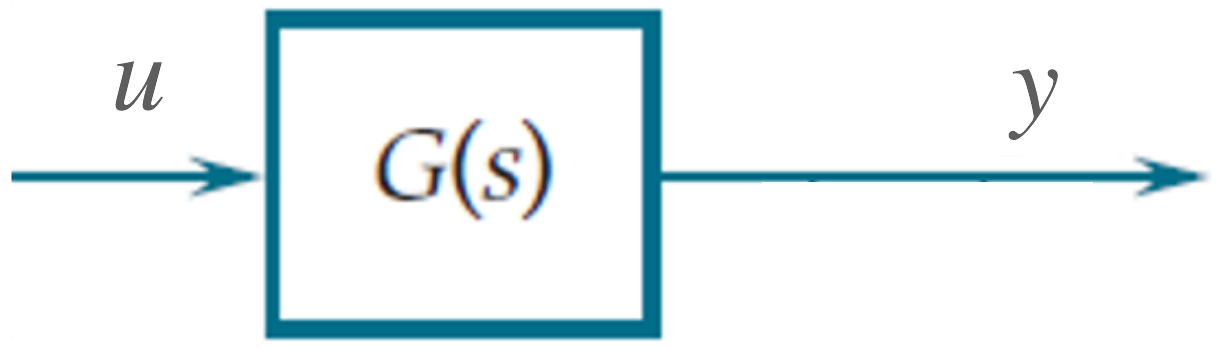


Control of Open-loop Unstable Systems

Assumption:

- $G(s)$ open-loop unstable

Process



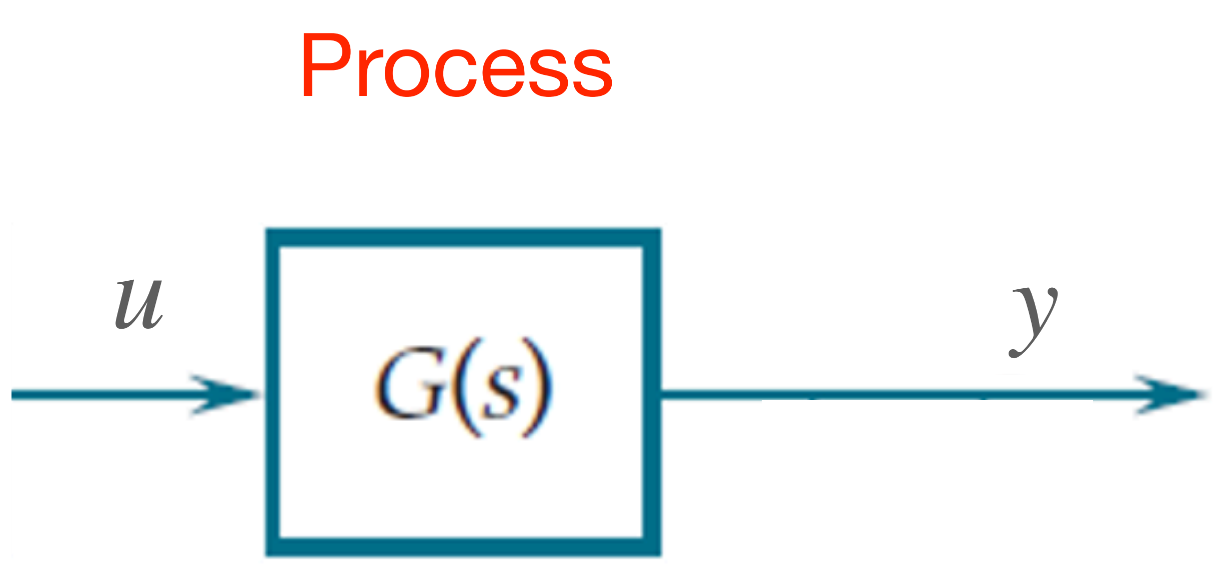
Bode Criterion: It requires that

- $L(s)$ has no poles with positive real part
- the Bode plot of the magnitude of $L(j\omega)$ crosses the 0 dB axis only once

Control of Open-loop Unstable Systems

Assumption:

- $G(s)$ open-loop unstable



Bode Criterion: It requires that

- $L(s)$ has no poles with positive real part
- the Bode plot of the magnitude of $L(j\omega)$ crosses the 0 dB axis only once

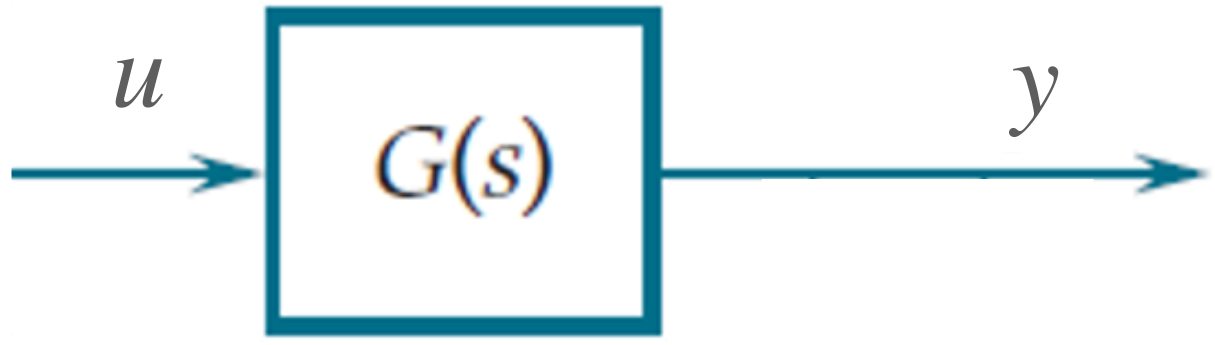
It cannot be applied in this case!

Control of Open-loop Unstable Systems

Assumption:

- $G(s)$ open-loop unstable

Process



Bode Criterion: It requires that

- $L(s)$ has no poles with positive real part
- the Bode plot of the magnitude of $L(j\omega)$ crosses the 0 dB axis only once

It cannot be applied in this case!

Alternatives:

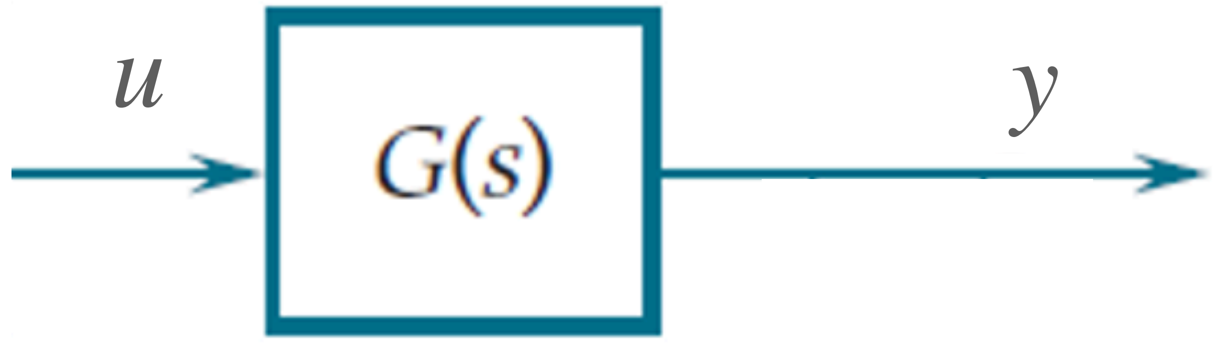
- Nyquist Criterion
- Poles Assignment
- Root Locus

Control of Open-loop Unstable Systems

Assumption:

- $G(s)$ open-loop unstable

Process



↓

Bode Criterion: It requires that

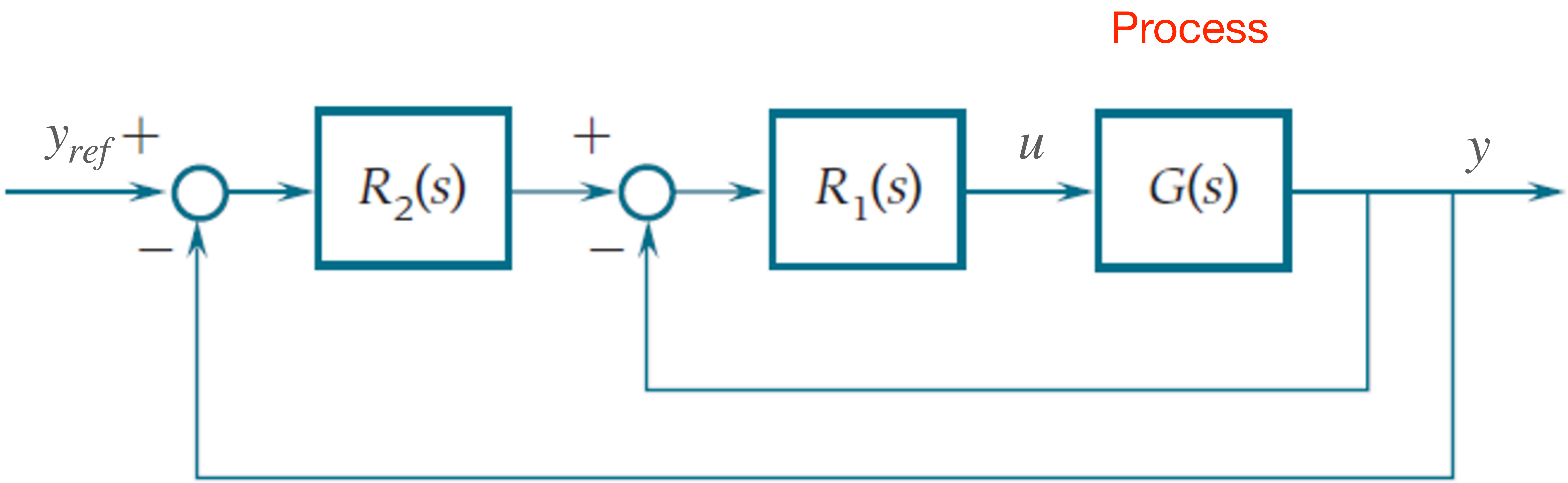
- $L(s)$ has no poles with positive real part
- the Bode plot of the magnitude of $L(j\omega)$ crosses the 0 dB axis only once

↘ It cannot be applied in this case! →

Alternatives:

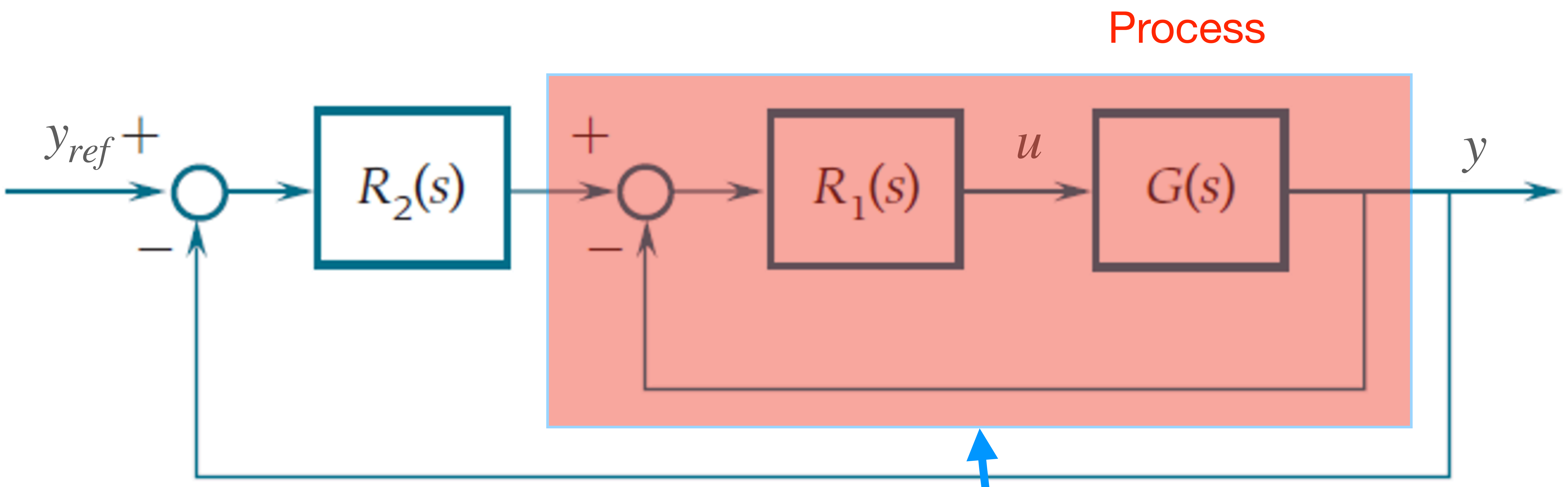
- Nyquist Criterion
- Poles Assignment
- Root Locus

Control of Open-loop Unstable Systems



Alternatives:
- Nyquist Criterion
- Poles Assignment
- Root Locus

Control of Open-loop Unstable Systems

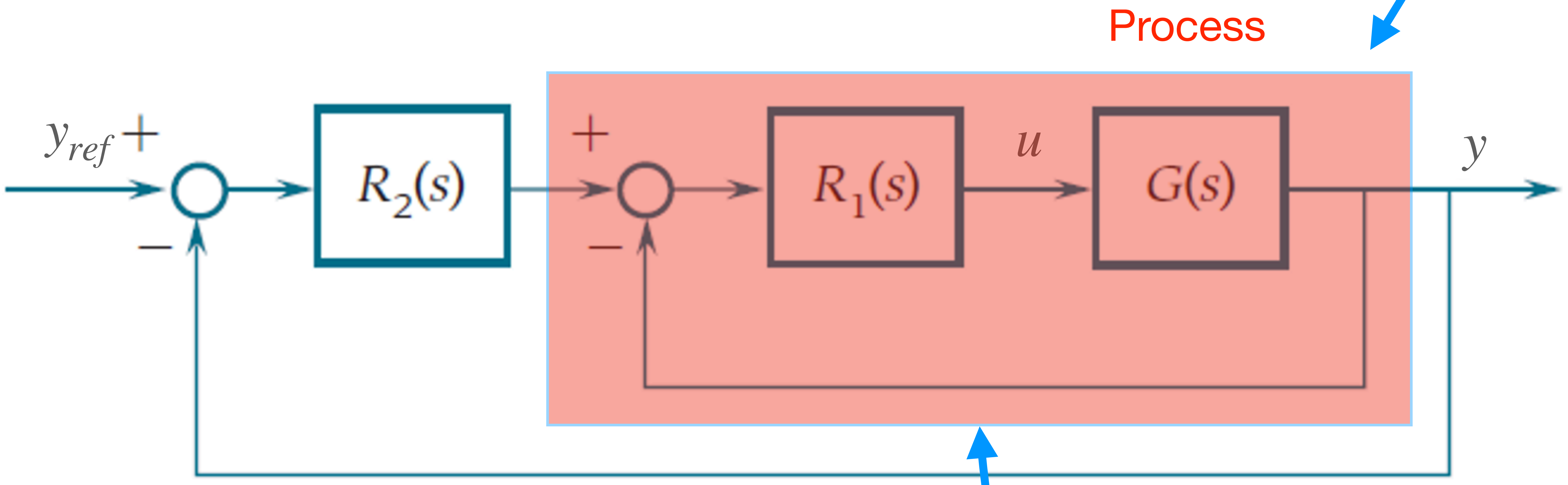


Alternatives:

- Nyquist Criterion
- Poles Assignment
- Root Locus

Control of Open-loop Unstable Systems

Objective: to stabilize the inner loop



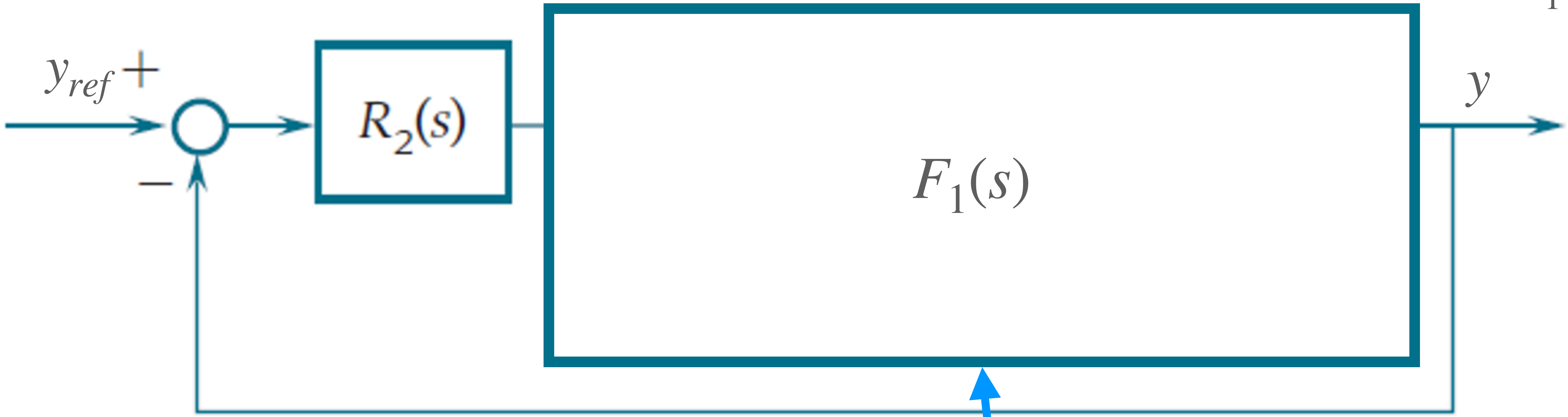
Alternatives:

- Nyquist Criterion
- Poles Assignment
- Root Locus

Control of Open-loop Unstable Systems

$$L_1(s) = R_1(s)\,G(s)$$

$$F_1(s) = \frac{L_1(s)}{1 + L_1(s)}$$



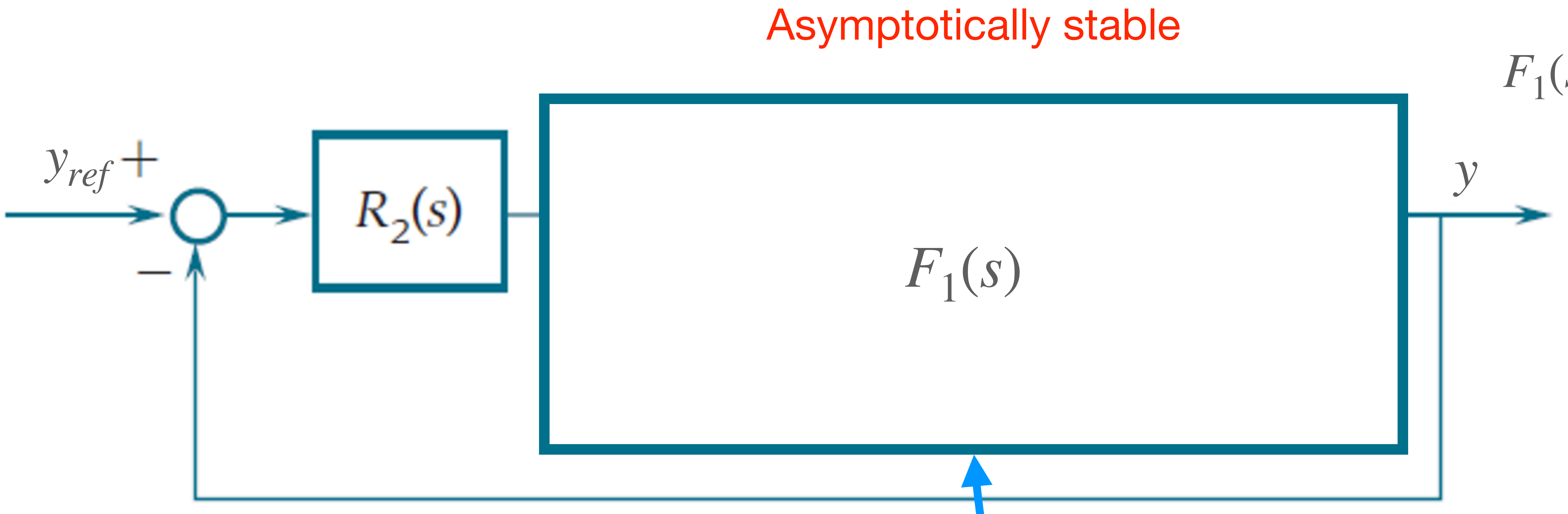
Alternatives:

- Nyquist Criterion
- Poles Assignment
- Root Locus

Control of Open-loop Unstable Systems

$$L_1(s) = R_1(s)\,G(s)$$

$$F_1(s) = \frac{L_1(s)}{1 + L_1(s)}$$



Alternatives:
- Nyquist Criterion
- Poles Assignment
- Root Locus

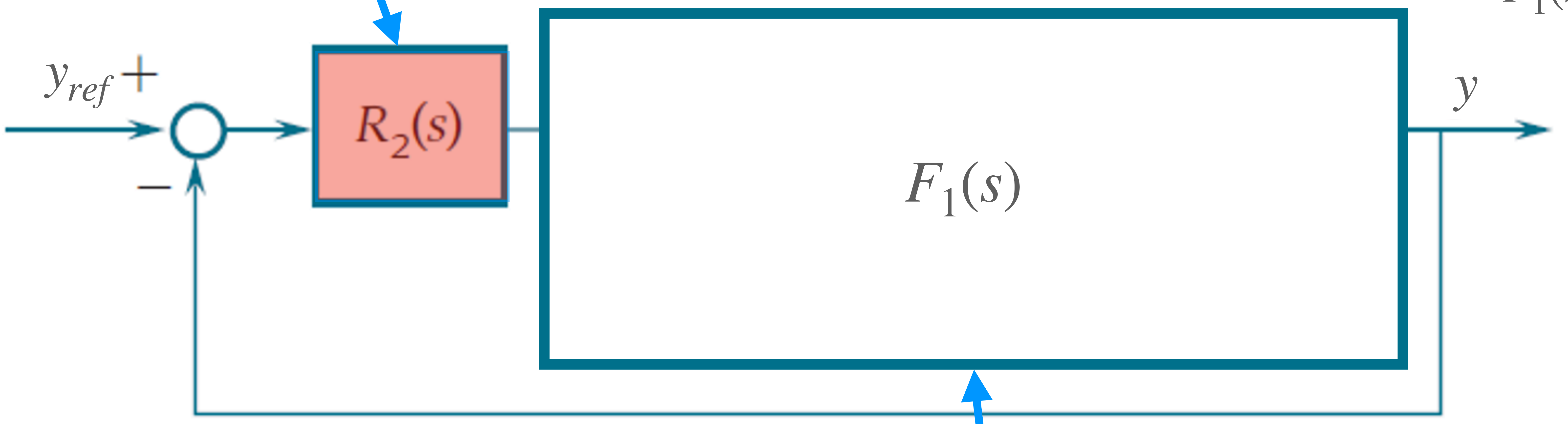
Control of Open-loop Unstable Systems

Objective: to stabilize the outer loop & satisfy the design requirements

Asymptotically stable

$$L_1(s) = R_1(s)\,G(s)$$

$$F_1(s) = \frac{L_1(s)}{1 + L_1(s)}$$

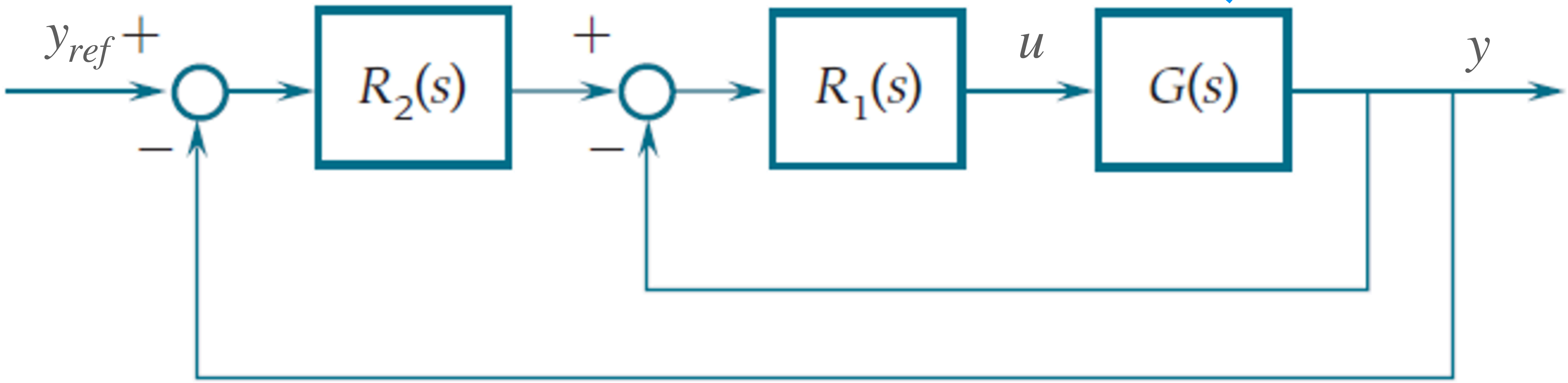


Alternatives:
- Nyquist Criterion
- Poles Assignment
- Root Locus

Control of Open-loop Unstable Systems

Example:

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

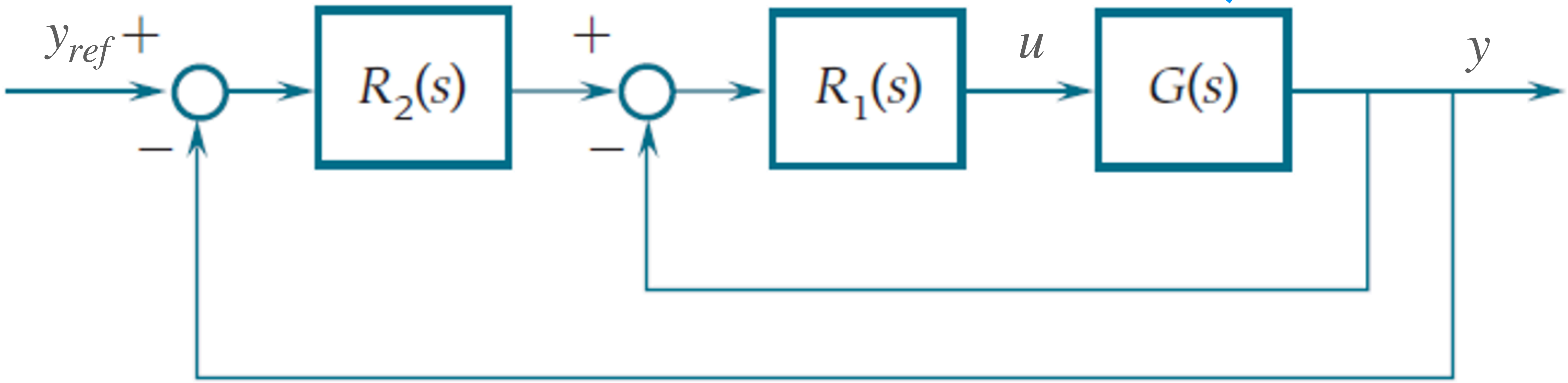


Control of Open-loop Unstable Systems

Example:

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



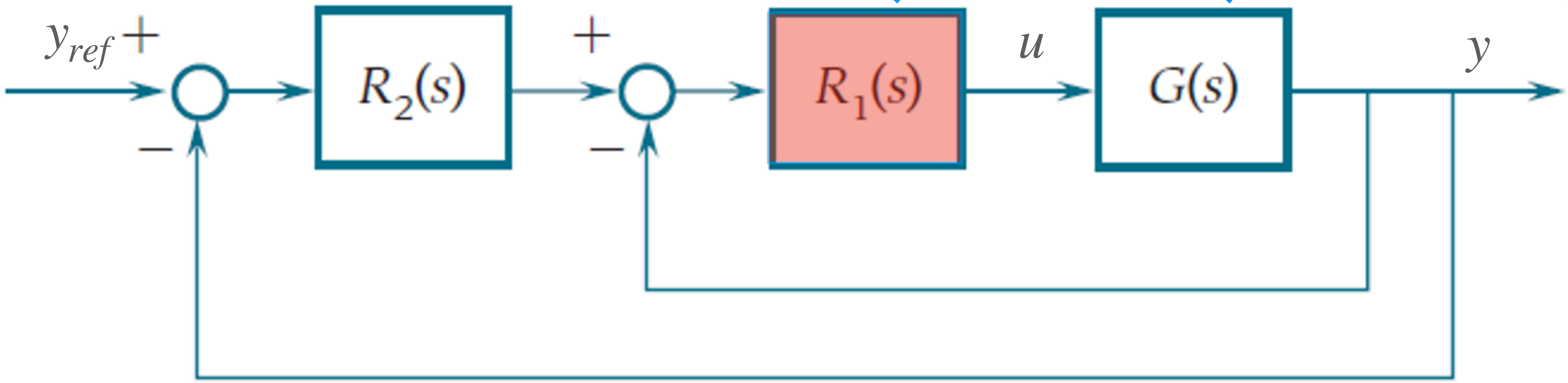
Control of Open-loop Unstable Systems

Example:

$$R_1(s) = k$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



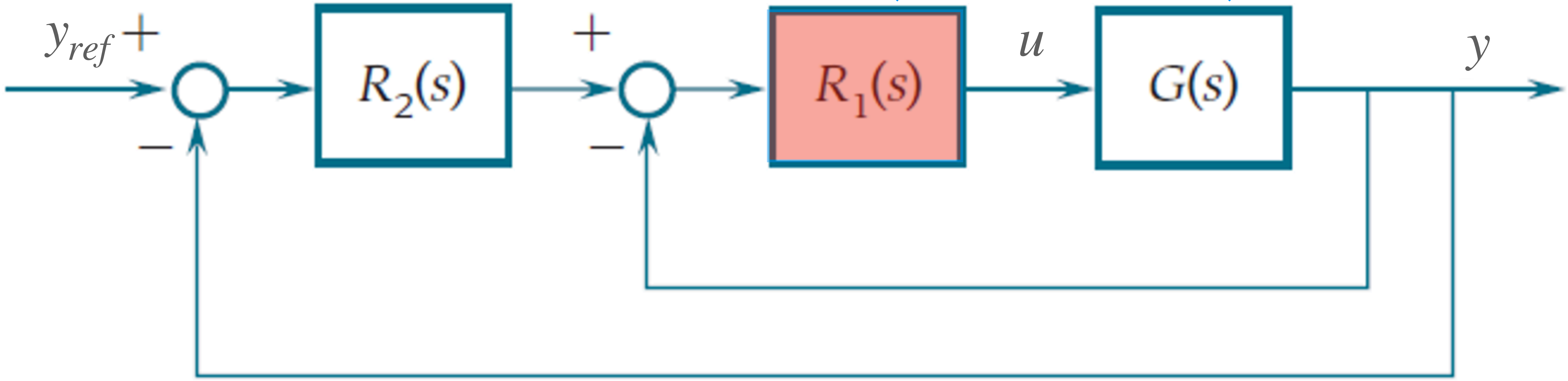
Control of Open-loop Unstable Systems

Example:

$$R_1(s) = k$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+3)(s-10)}$$

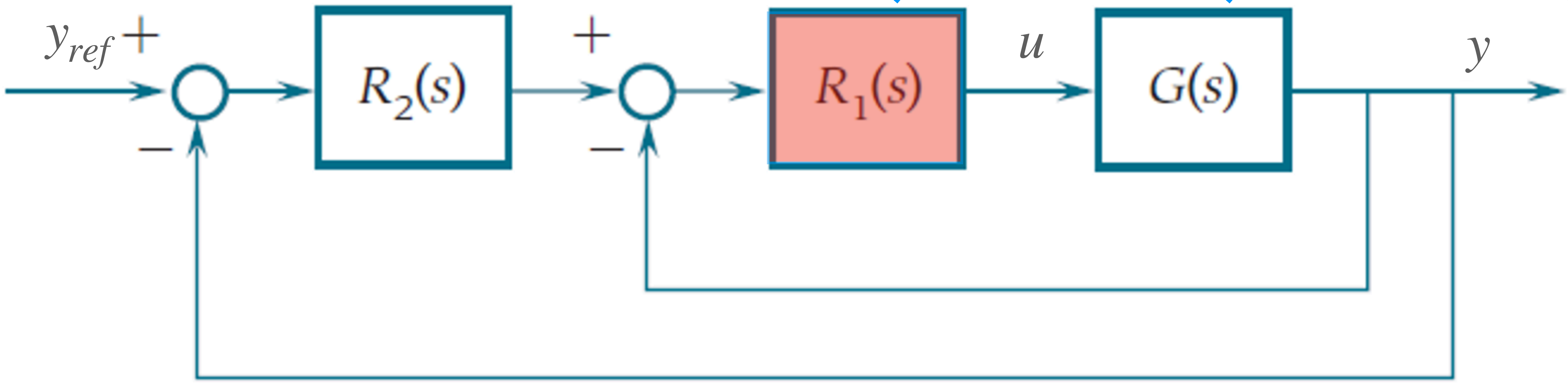
$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

Control of Open-loop Unstable Systems

Example:

$$R_1(s) = k \qquad G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



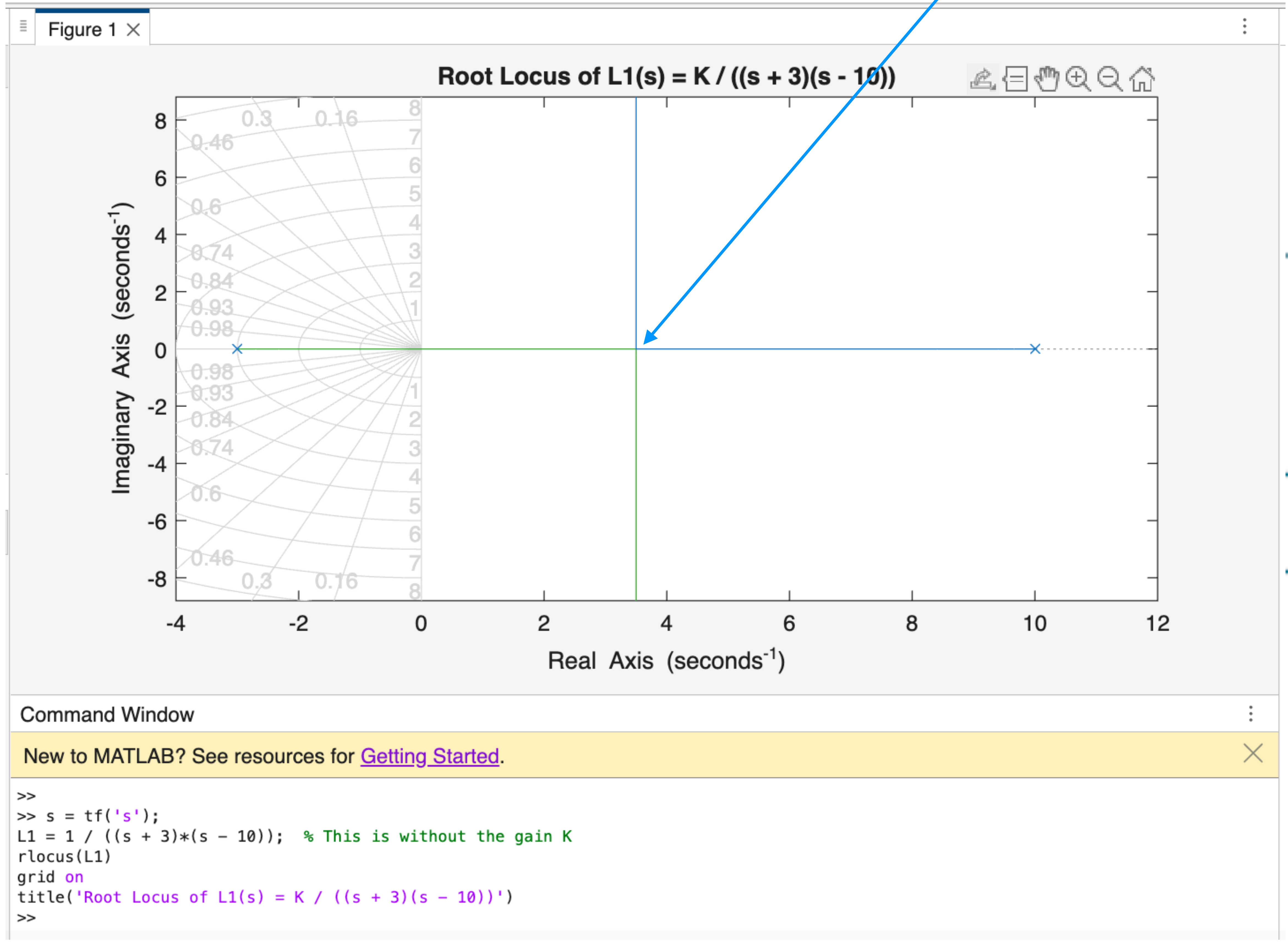
$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+3)(s-10)} = \frac{K}{(s+3)(s-10)}$$

$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

Control of Open-loop Unstable Systems

Example:

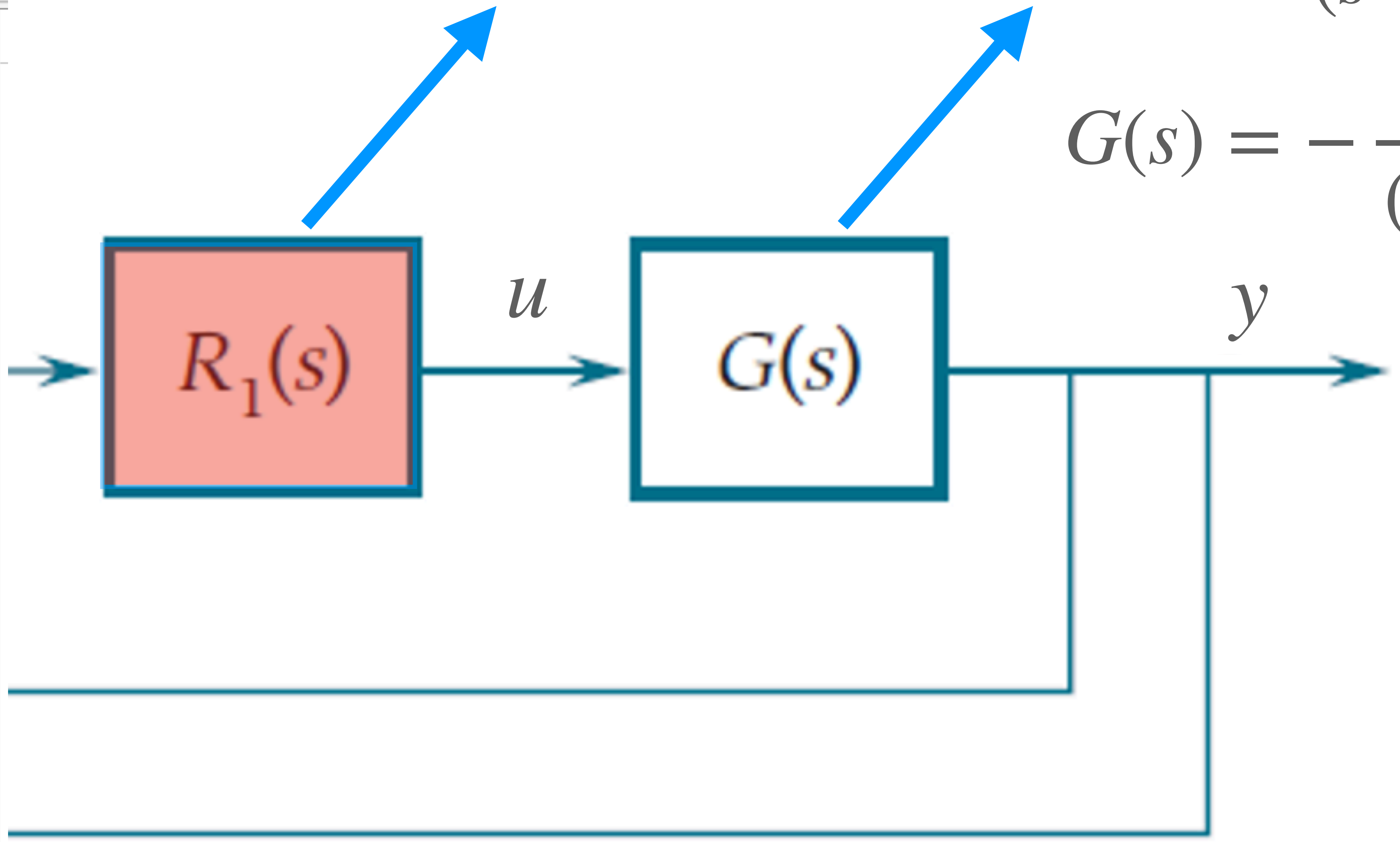
$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{-3+10}{2-0} = \frac{7}{2} = 3.5$$



$$R_1(s) = k$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



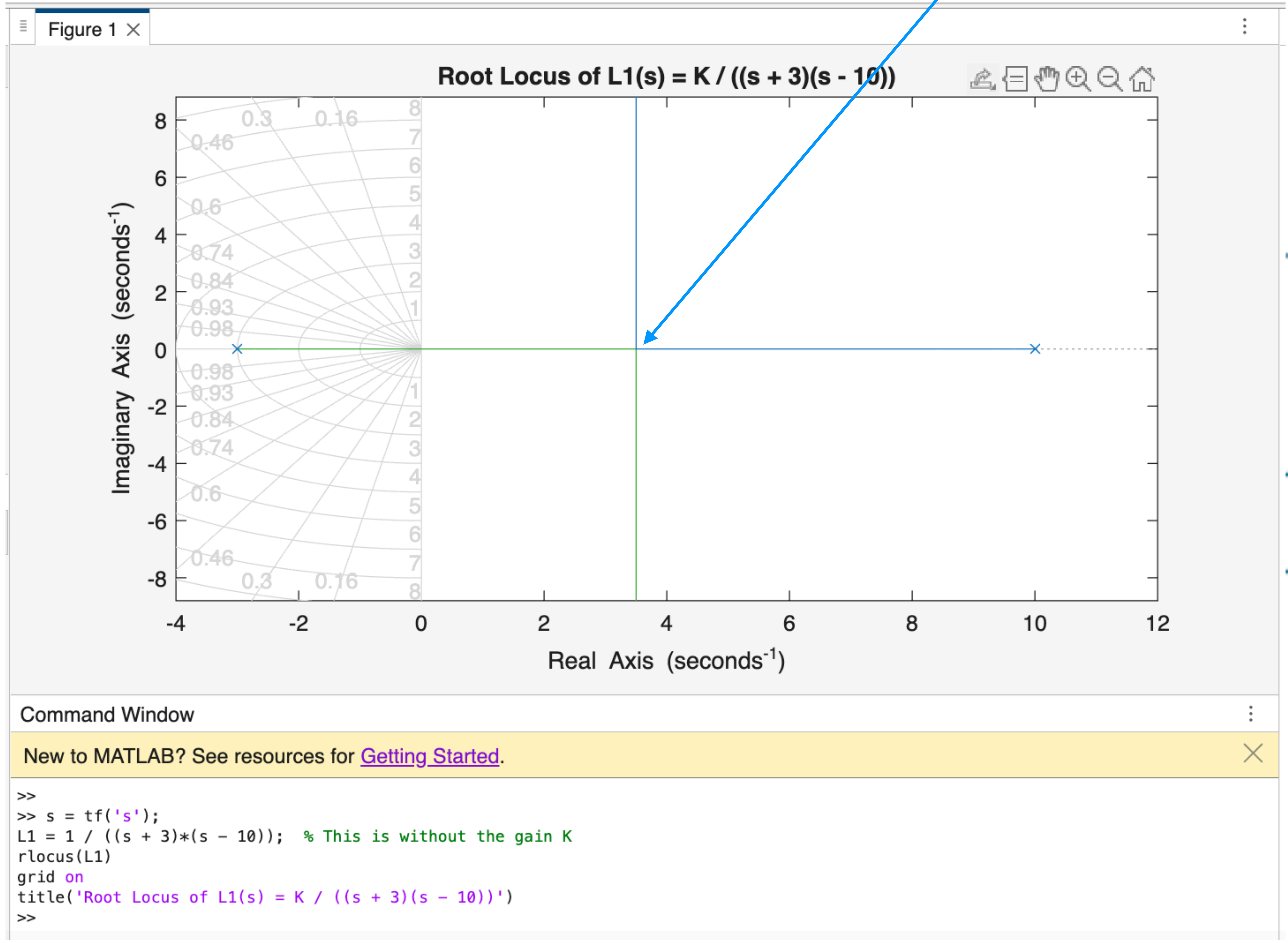
$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+3)(s-10)} = \frac{K}{(s+3)(s-10)}$$

$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

Control of Open-loop Unstable Systems

Example:

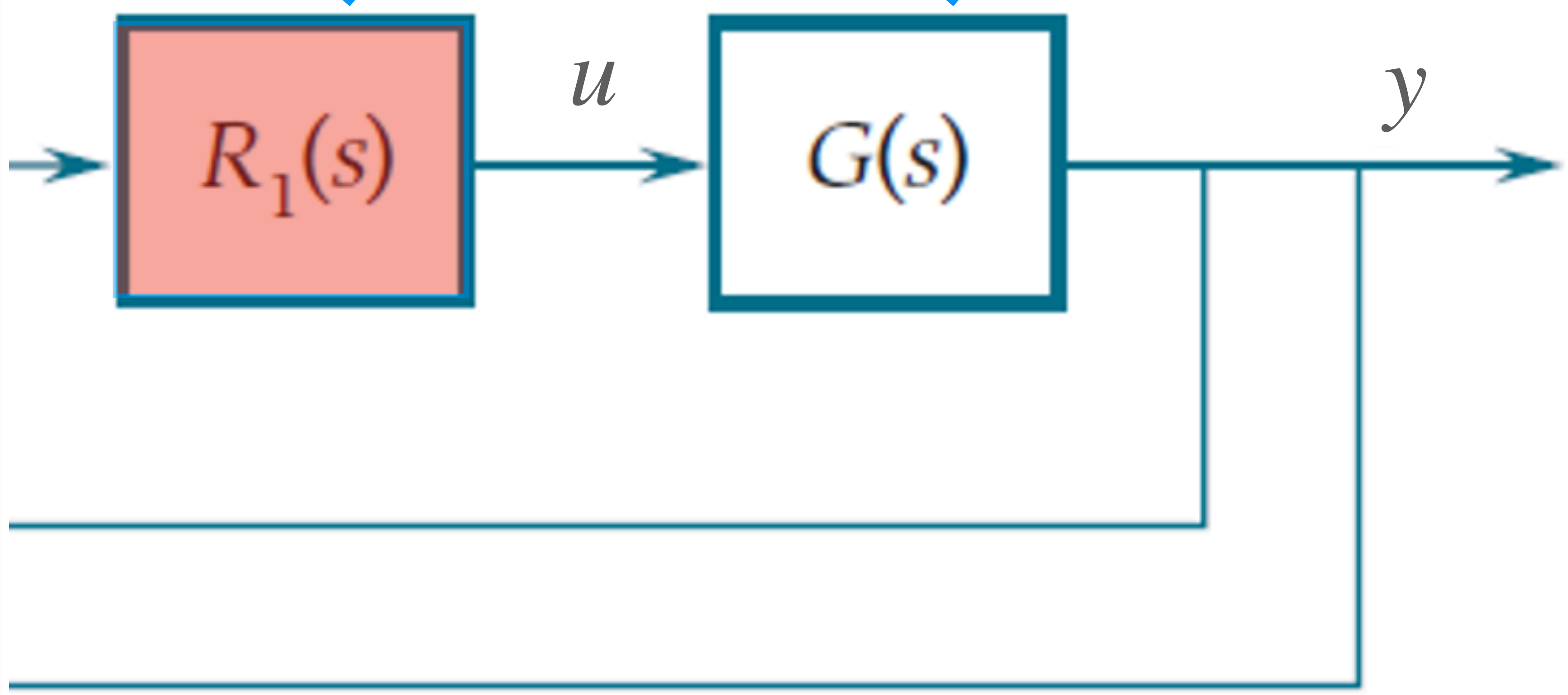
$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{-3+10}{2-0} = \frac{7}{2} = 3.5$$



$\cancel{R_1(s)} = k$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+3)(s-10)} = \frac{K}{(s+3)(s-10)}$$

$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

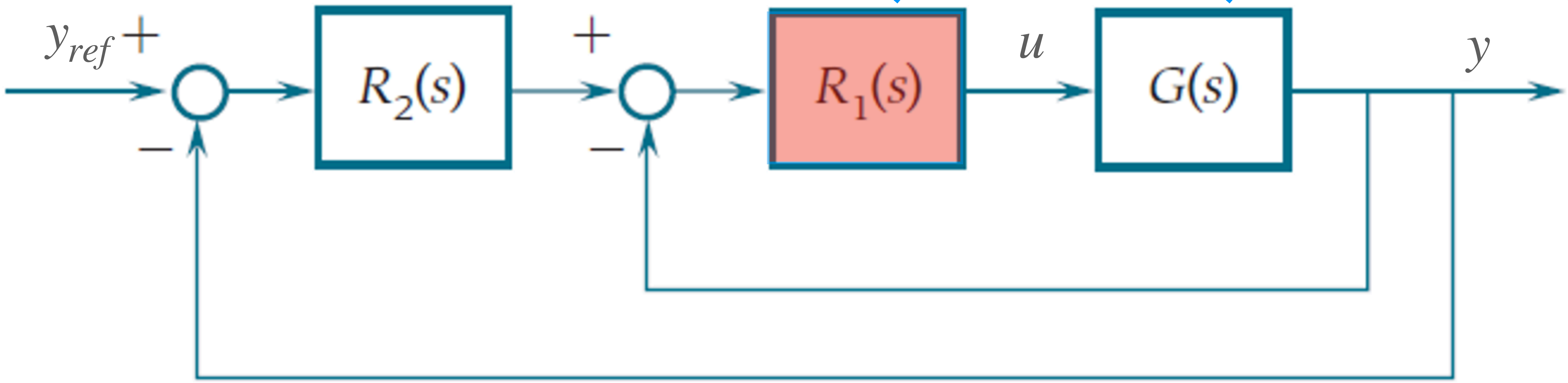
Control of Open-loop Unstable Systems

Example:

$$R_1(s) = \frac{k(s+3)}{s+p}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$

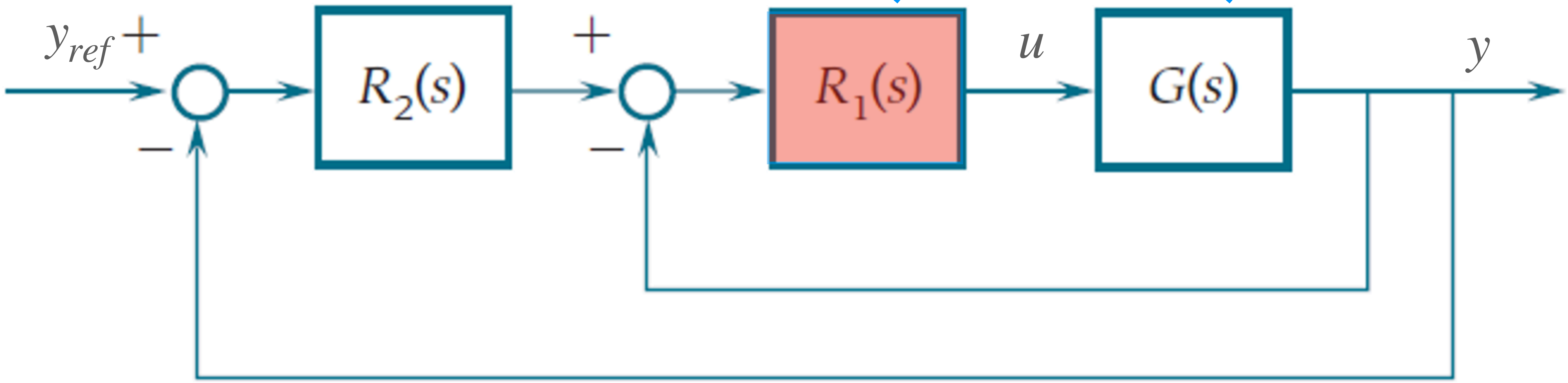


Control of Open-loop Unstable Systems

Example:

$$R_1(s) = \frac{k(s+3)}{s+p} \qquad G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+p)(s-10)} = \frac{K}{(s+p)(s-10)}$$

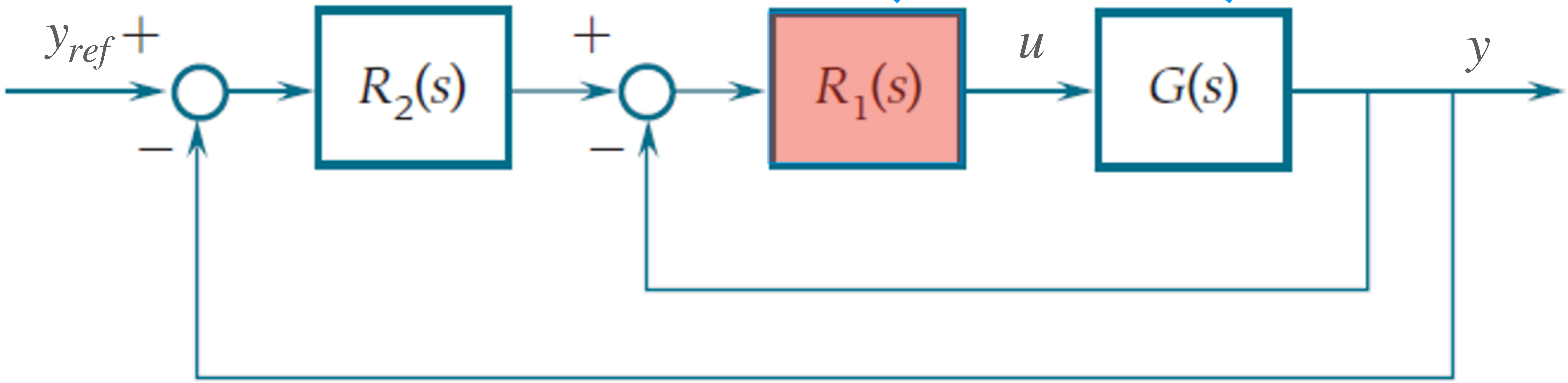
$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

Control of Open-loop Unstable Systems

Example:

$$R_1(s) = \frac{k(s+3)}{s+p} \qquad G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+p)(s-10)} = \frac{K}{(s+p)(s-10)}$$

?

$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

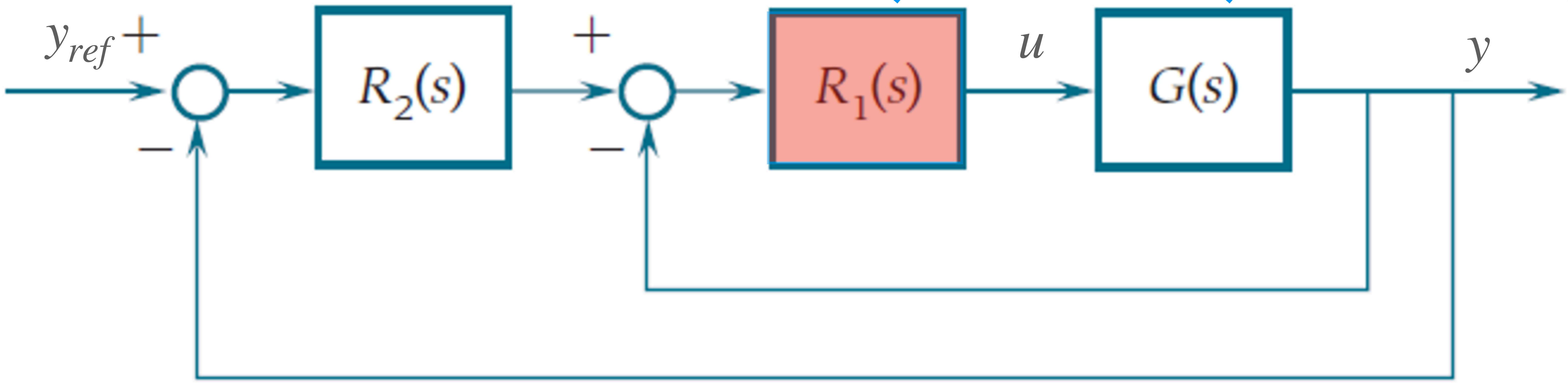
Control of Open-loop Unstable Systems

Example:

$$R_1(s) = \frac{k(s+3)}{s+p}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{(-p)+10}{2-0} = \frac{10-p}{2} =$$

$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+p)(s-10)} = \frac{K}{(s+p)(s-10)}$$

?

$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

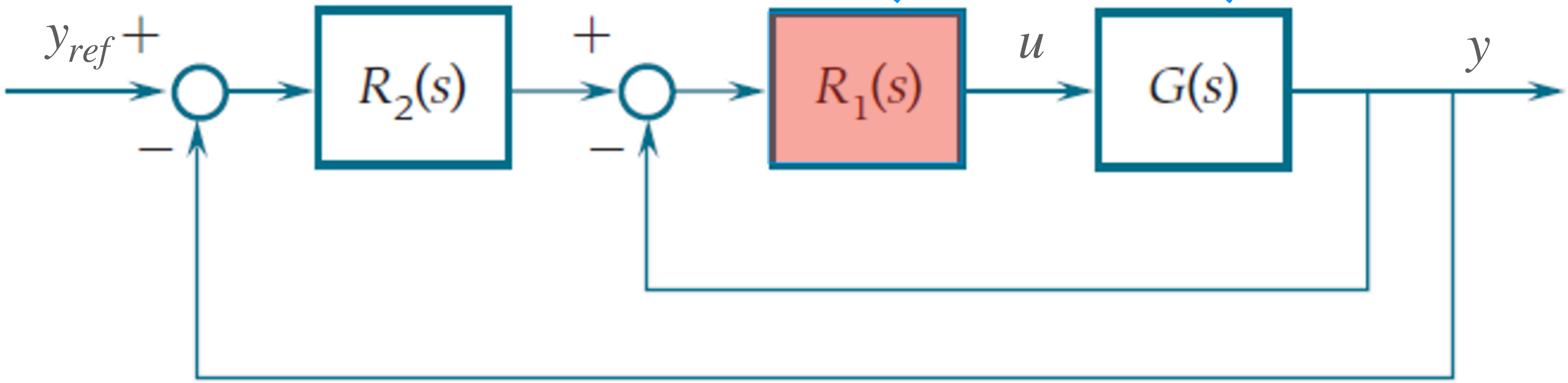
Control of Open-loop Unstable Systems

Example:

$$R_1(s) = \frac{k(s+3)}{s+p}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{(-p)+10}{2-0} = \frac{10-p}{2} = -10$$

$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+p)(s-10)} = \frac{K}{(s+p)(s-10)}$$

?

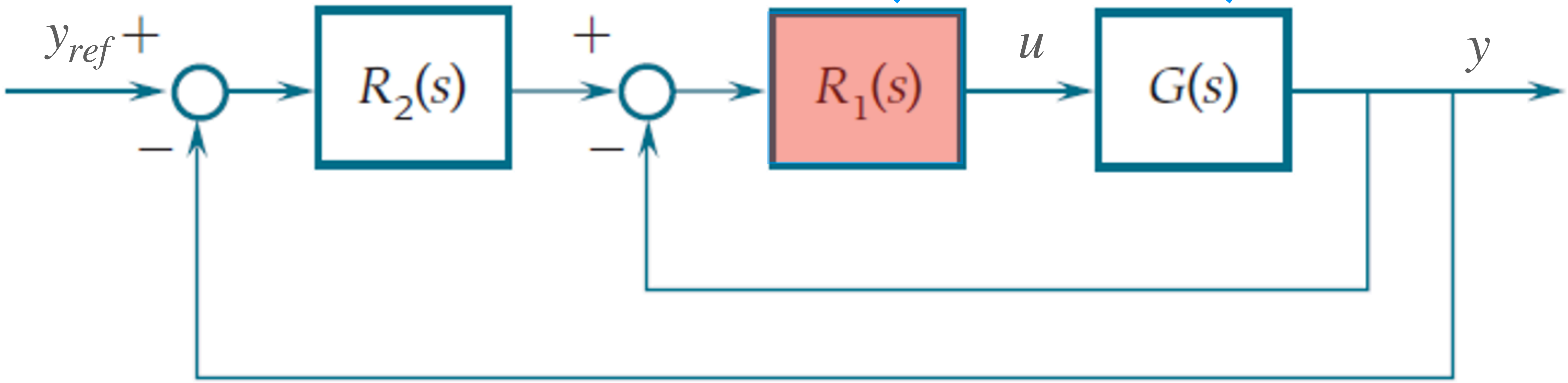
$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

Control of Open-loop Unstable Systems

Example:

$$R_1(s) = \frac{k(s+3)}{s+p} \qquad G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{(-p)+10}{2-0} = \frac{10-p}{2} = \boxed{-10}$$

$$p = 30$$

$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+p)(s-10)} = \frac{K}{(s+p)(s-10)}$$

?

$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

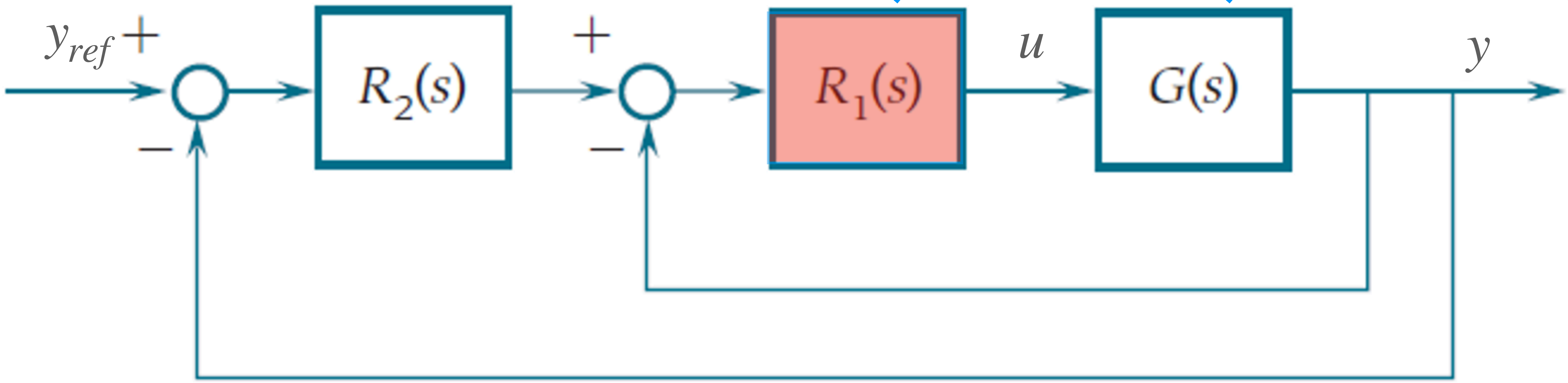
Control of Open-loop Unstable Systems

Example:

$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{(-p)+10}{2-0} = \frac{10-p}{2} = \boxed{-10}$$

$$p = 30$$

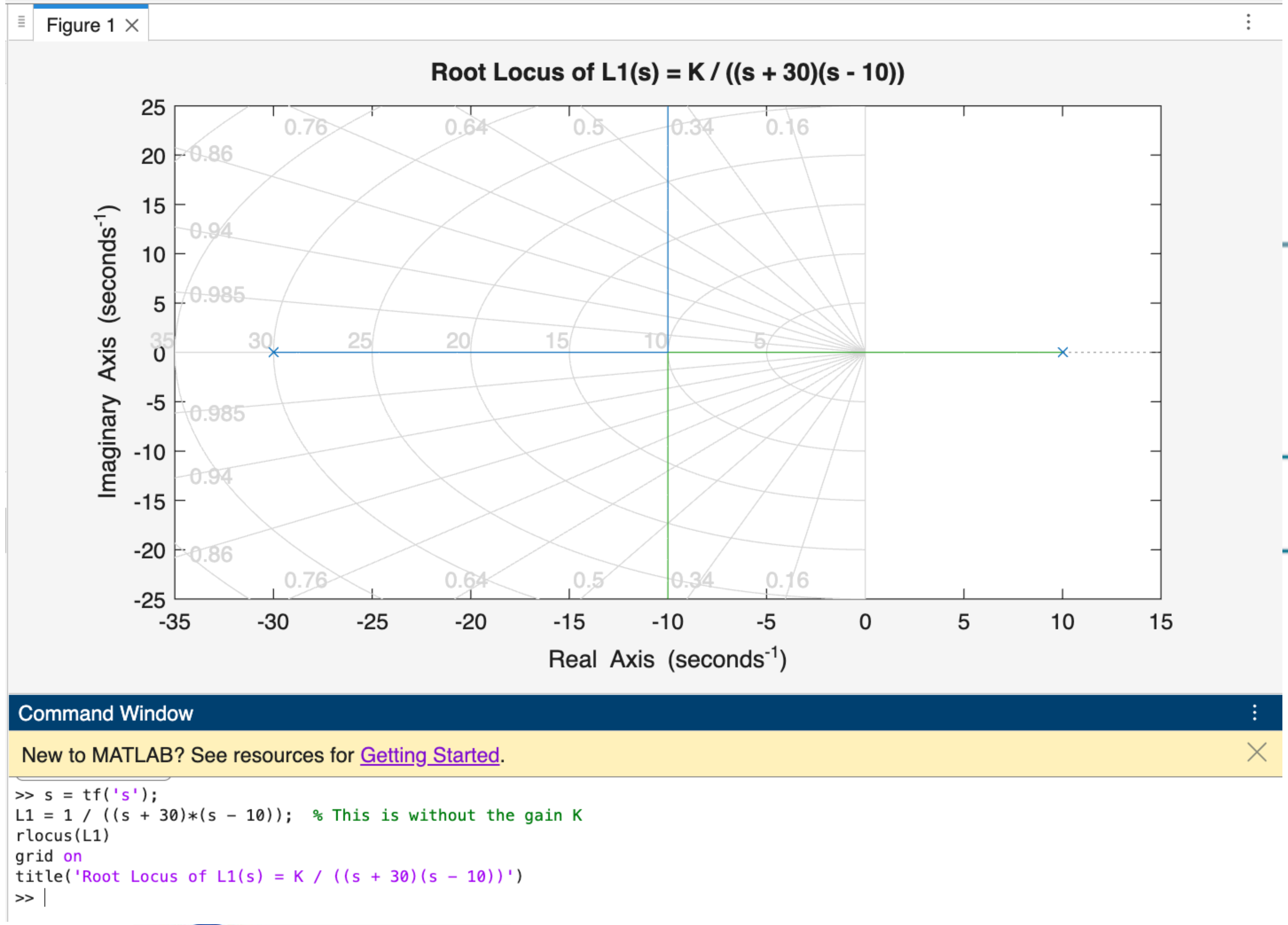
$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+p)(s-10)} = \frac{K}{(s+p)(s-10)}$$

?

$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

Control of Open-loop Unstable Systems

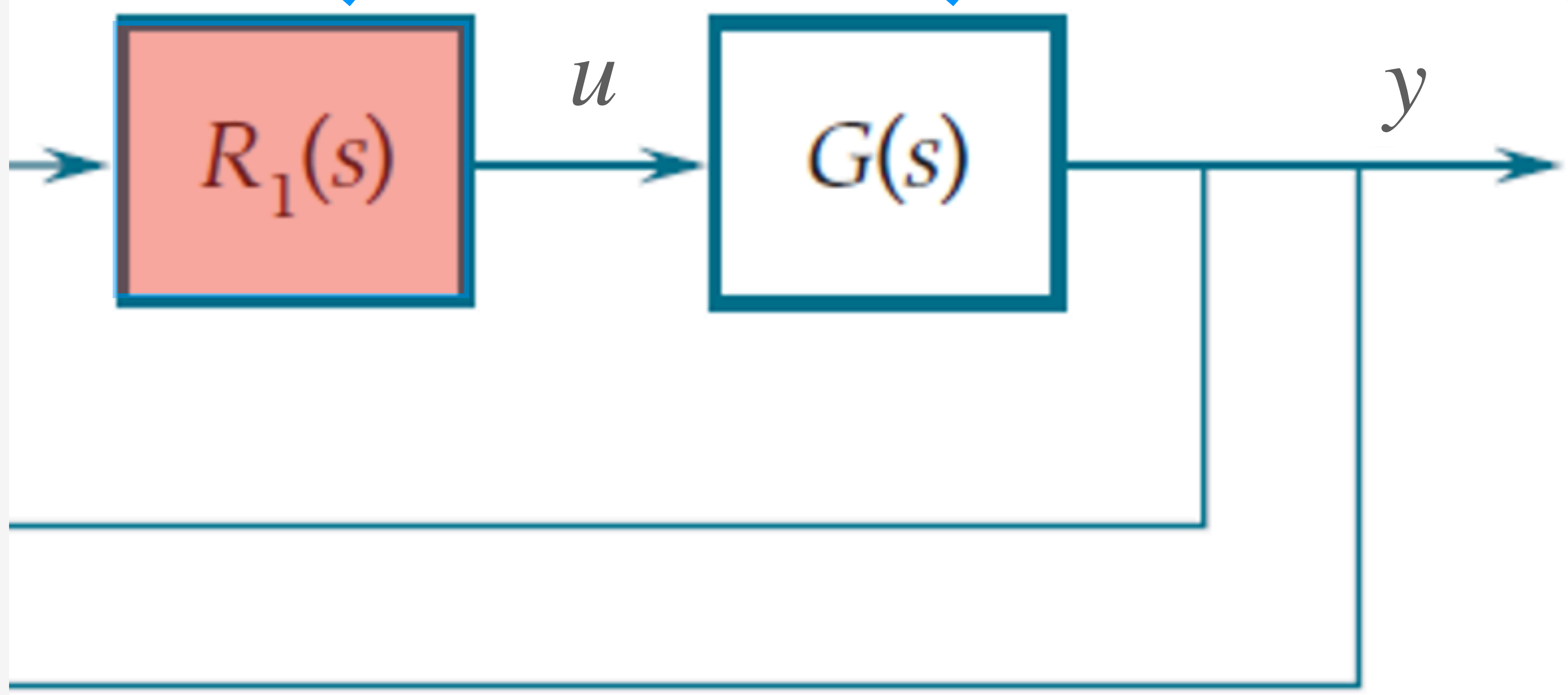
Example:



$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+p)(s-10)} = \frac{K}{(s+p)(s-10)}$$

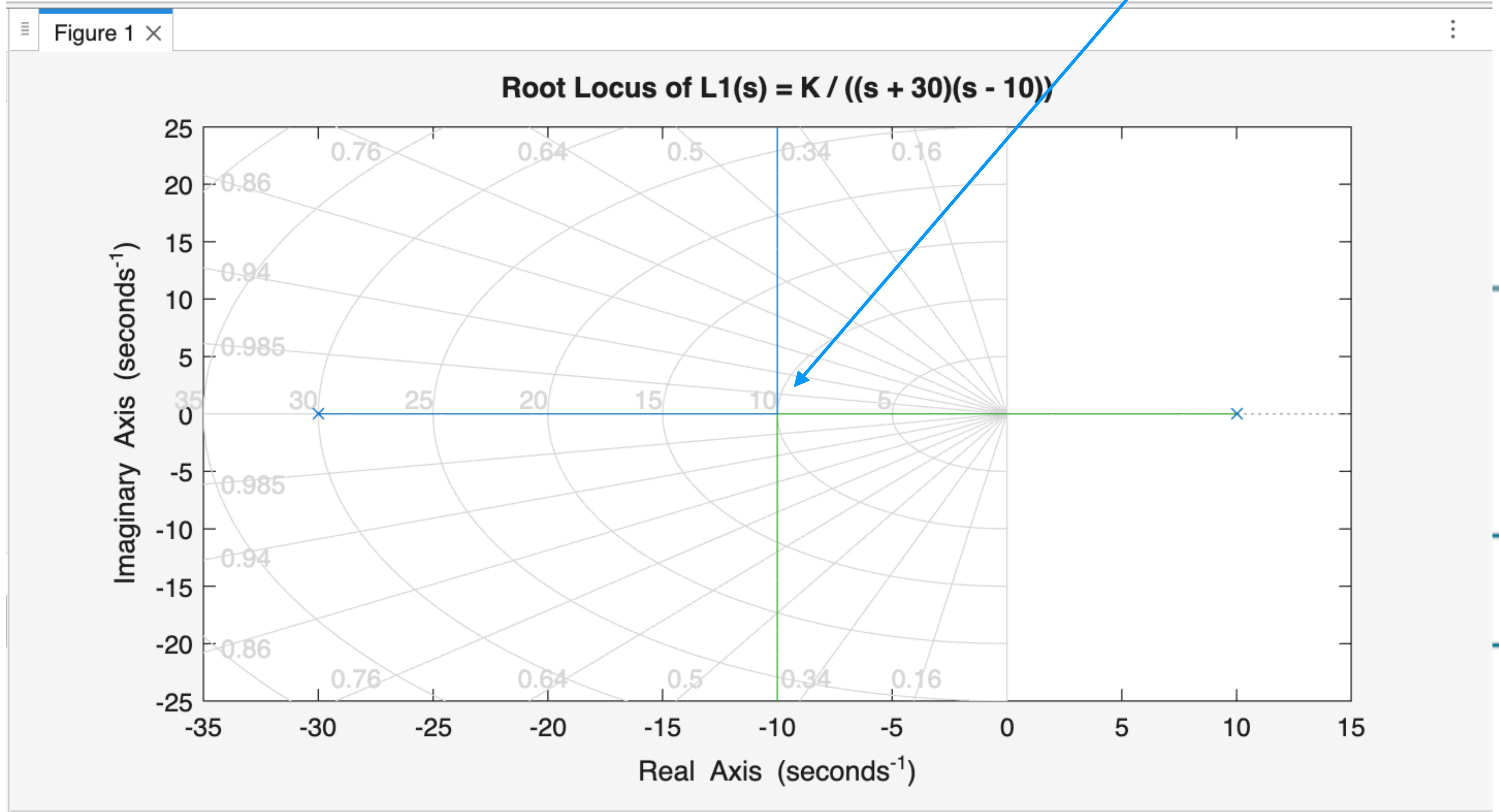
?

$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

Control of Open-loop Unstable Systems

Example:

$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{-30+10}{2-0} = \frac{-20}{2} = -10$$



Command Window

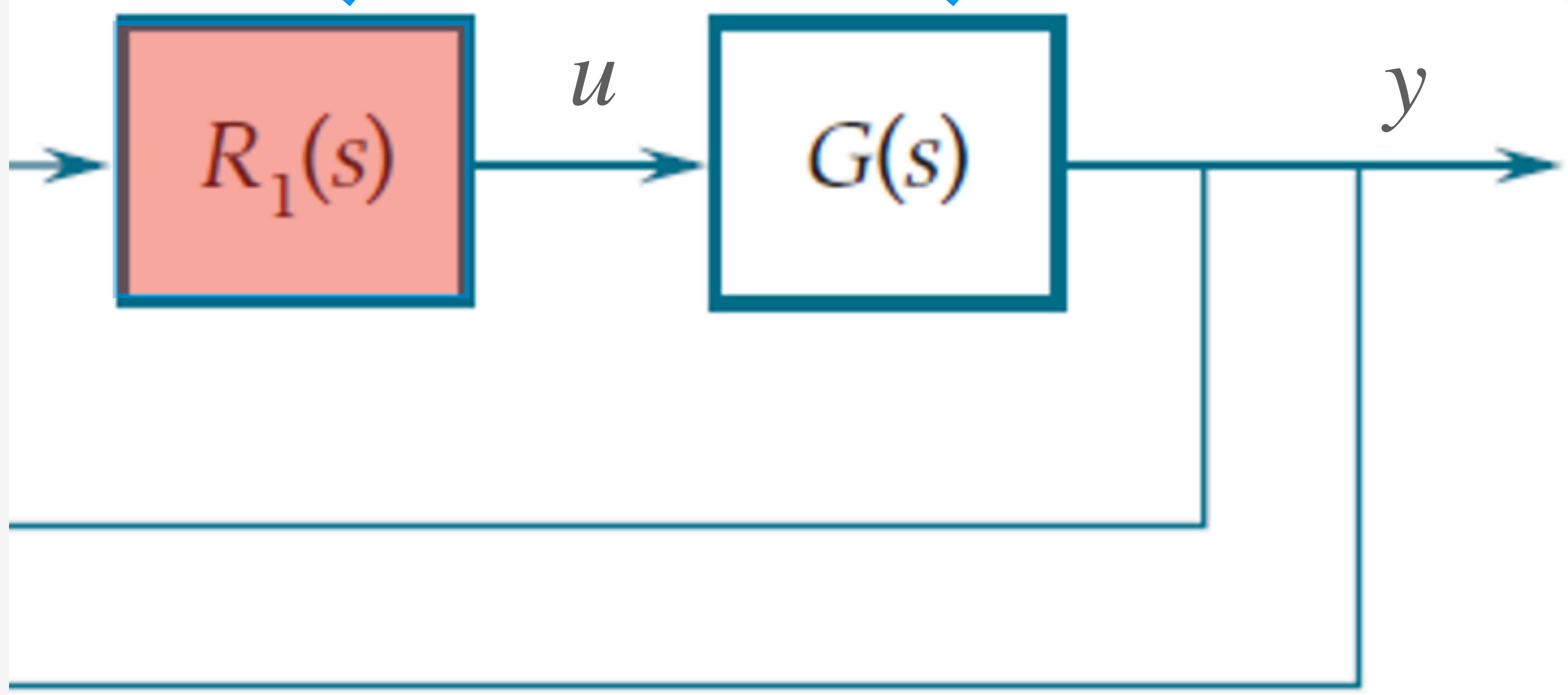
New to MATLAB? See resources for Getting Started.

```
>> s = tf('s');
L1 = 1 / ((s + 30)*(s - 10));  % This is without the gain K
rlocus(L1)
grid on
title('Root Locus of L1(s) = K / ((s + 30)(s - 10))')
>> |
```

$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+p)(s-10)} = \frac{K}{(s+p)(s-10)}$$

?

$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

Control of Open-loop Unstable Systems

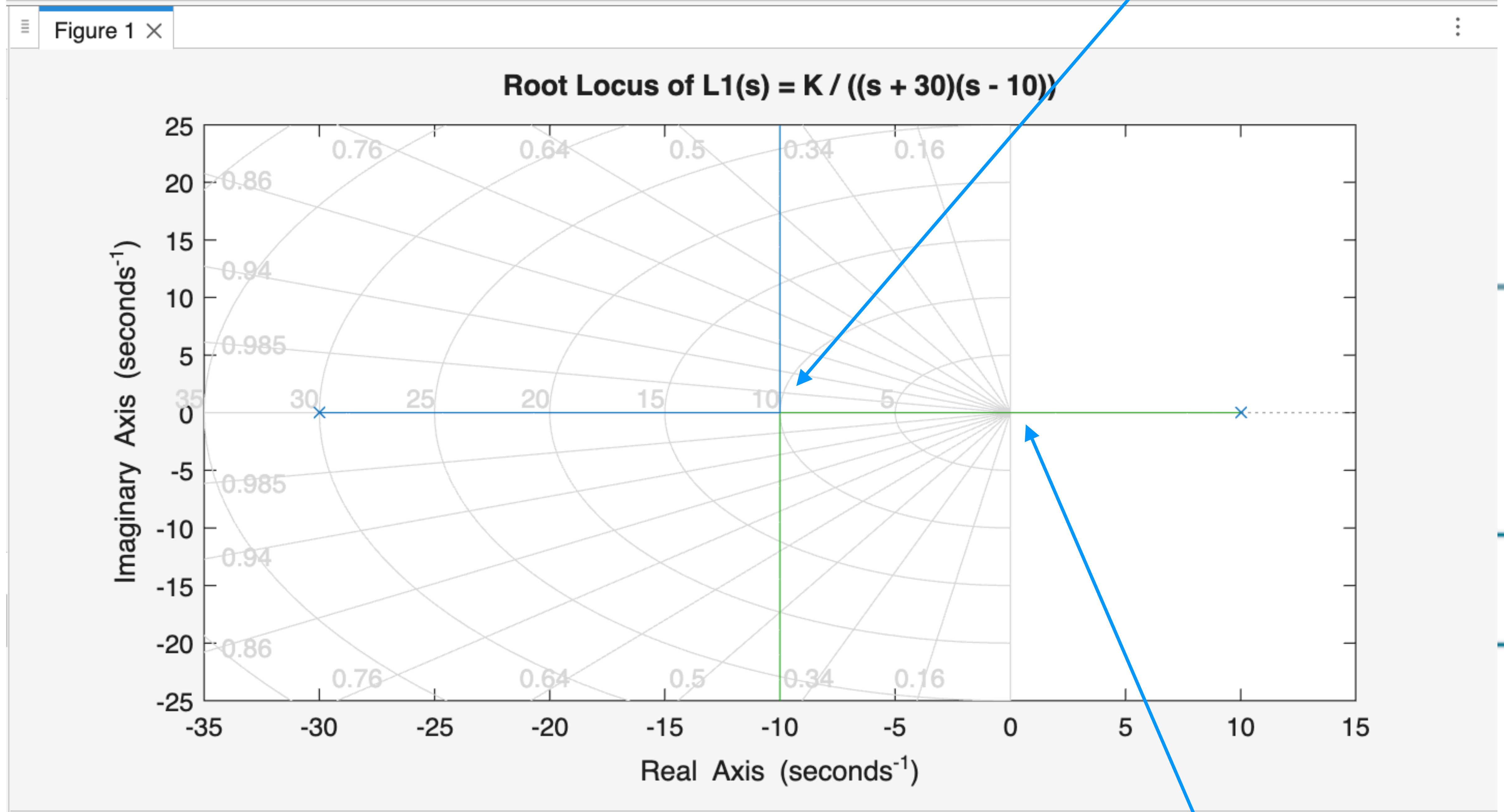
Example:

$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{-30+10}{2-0} = \frac{-20}{2} = -10$$

$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$

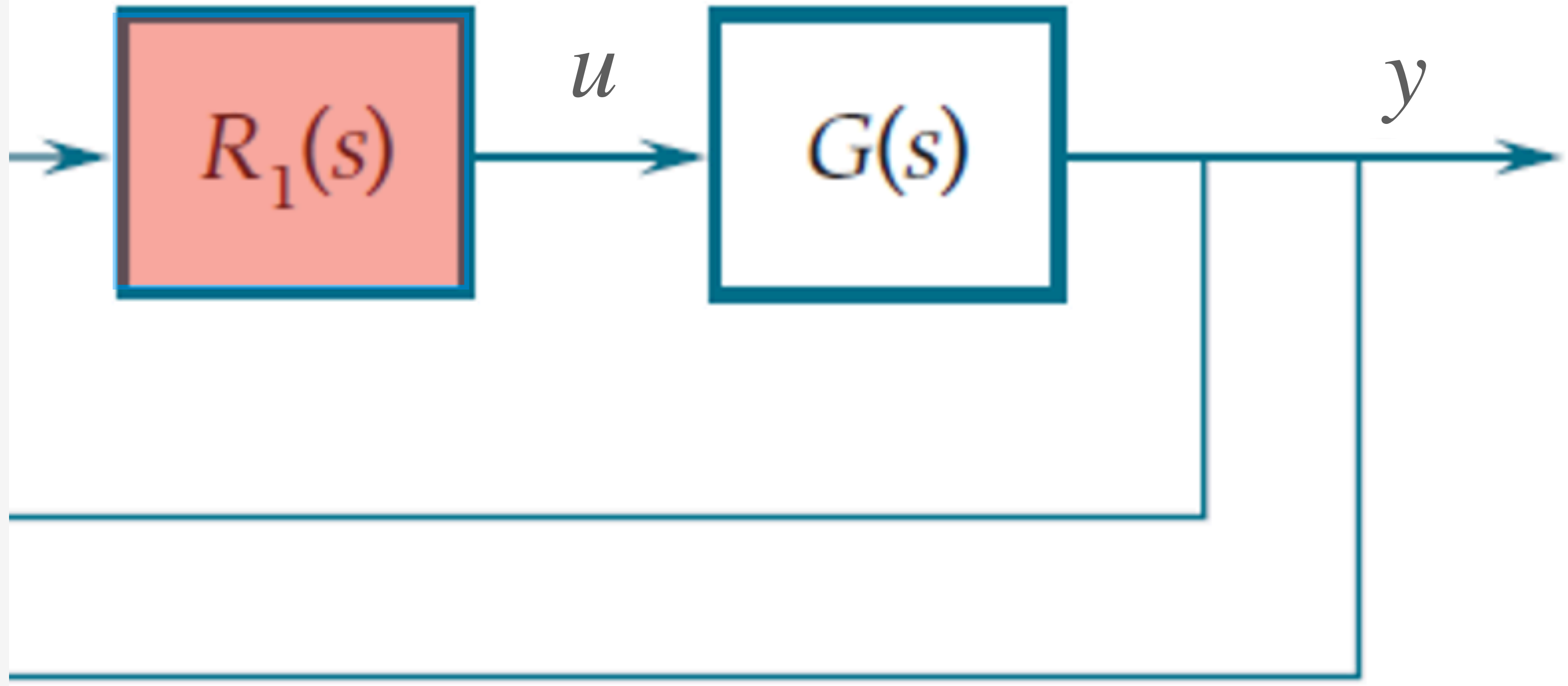


Command Window

New to MATLAB? See resources for Getting Started.

```
>> s = tf('s');
L1 = 1 / ((s + 30)*(s - 10));  % This is without the gain K
rlocus(L1)
grid on
title('Root Locus of L1(s) = K / ((s + 30)(s - 10))')
>> |
```

K^* ?



$$L_1(s) = R_1(s)\,G(s) = -\frac{100k}{(s+p)(s-10)} = \frac{K}{(s+p)(s-10)}$$

?

$$F_1(s) = \frac{L_1(s)}{1+L_1(s)}$$

Control of Open-loop Unstable Systems

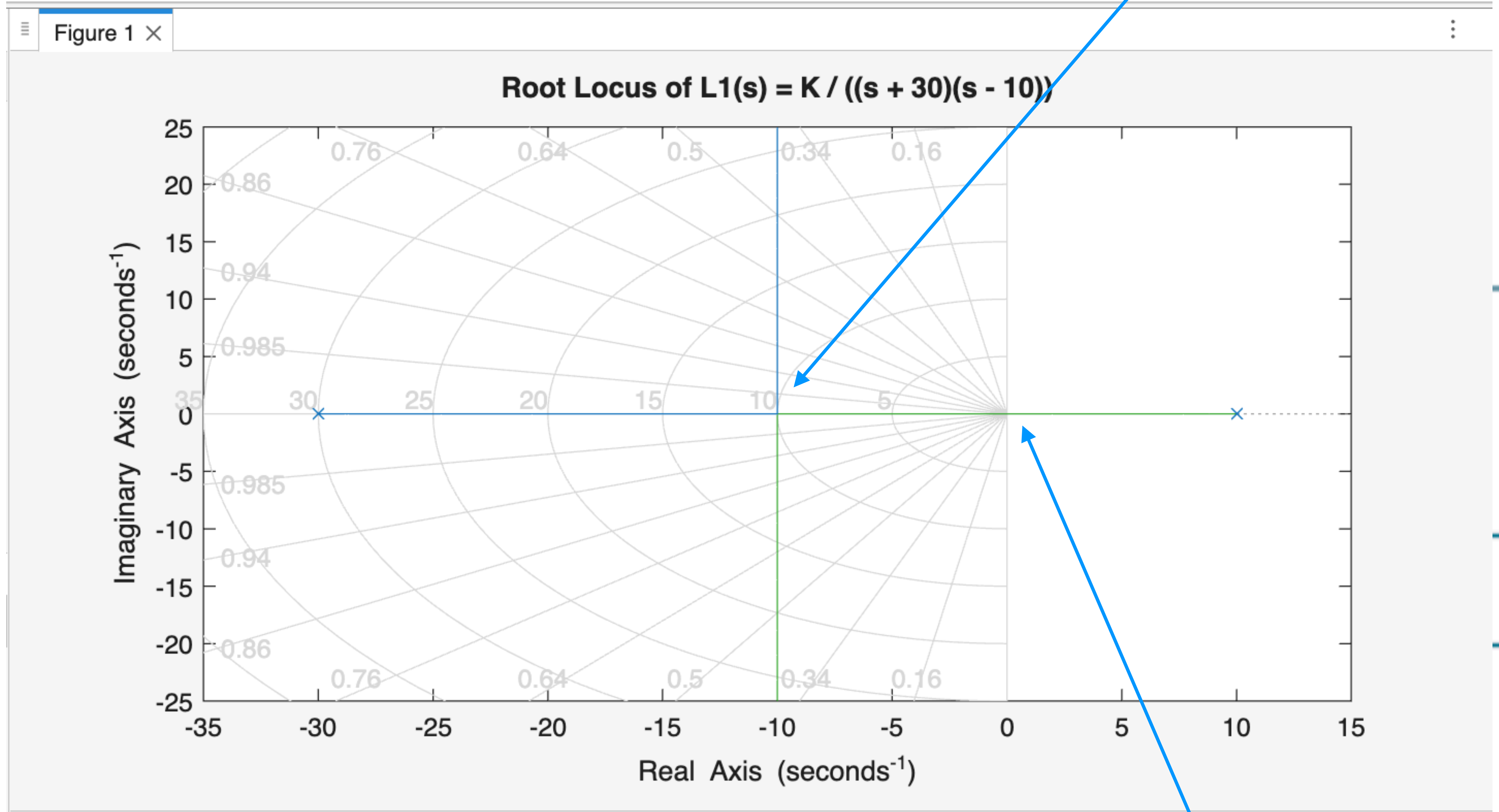
Example:

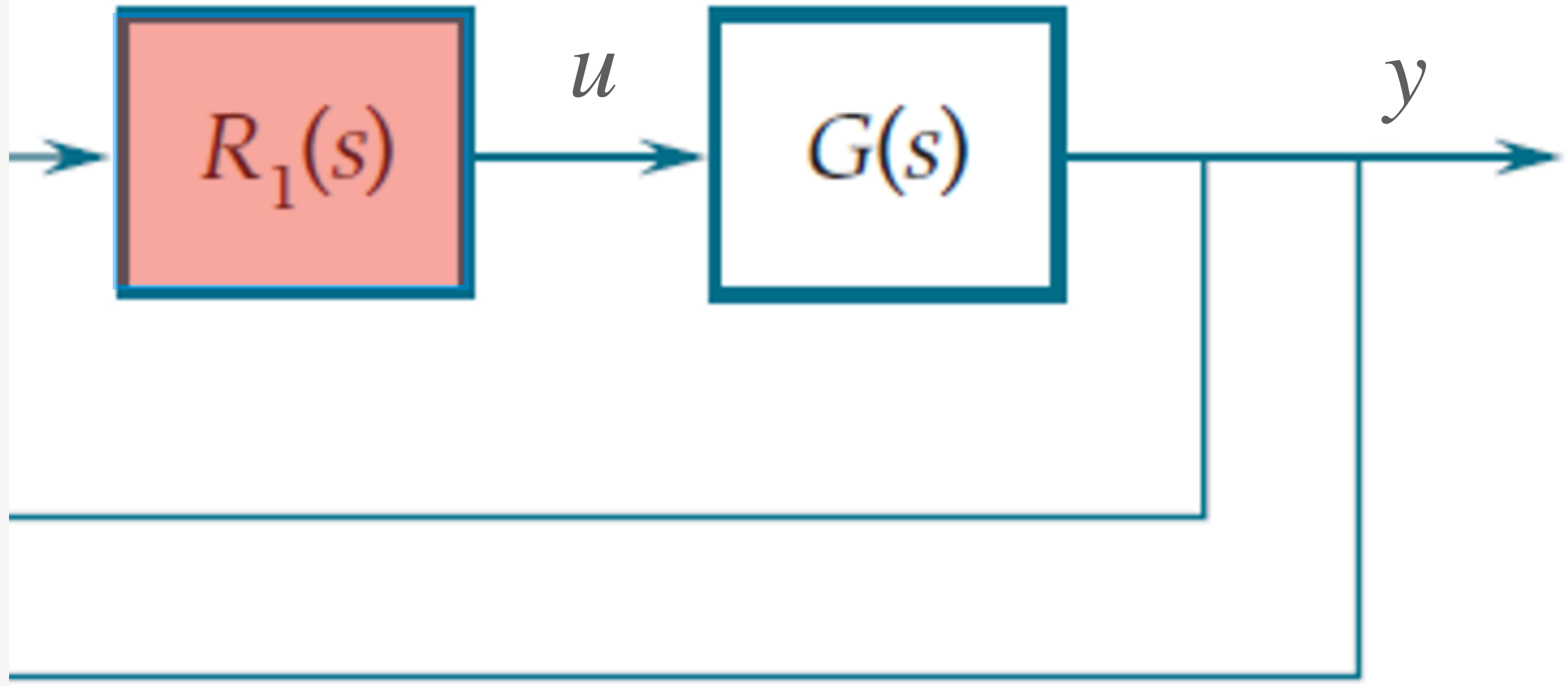
$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n - m} \qquad \sigma_a = \frac{-30+10}{2-0} = \frac{-20}{2} = -10$$

$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$





Command Window

New to MATLAB? See resources for Getting Started.

```
>> s = tf('s');
L1 = 1 / ((s + 30)*(s - 10));  % This is without the gain K
rlocus(L1)
grid on
title('Root Locus of L1(s) = K / ((s + 30)(s - 10))')
>> |
```

K^* ?

Command Window

New to MATLAB? See resources for Getting Started.

```
desired_pole = 0;
K_at_origin = rlocfind(L1, desired_pole);
K_at_origin

K_at_origin =

   300
```

Control of Open-loop Unstable Systems

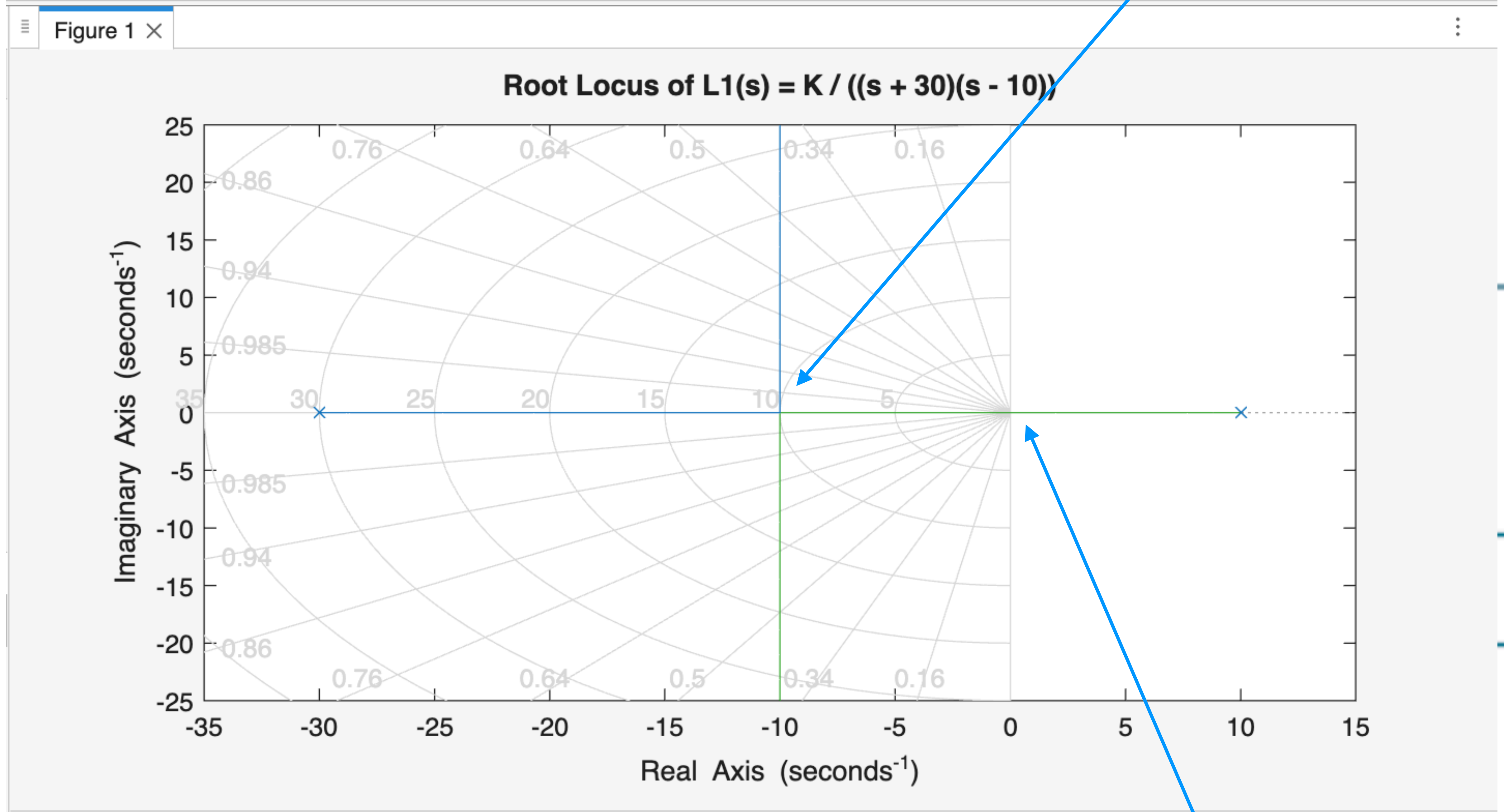
Example:

$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{-30+10}{2-0} = \frac{-20}{2} = -10$$

$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

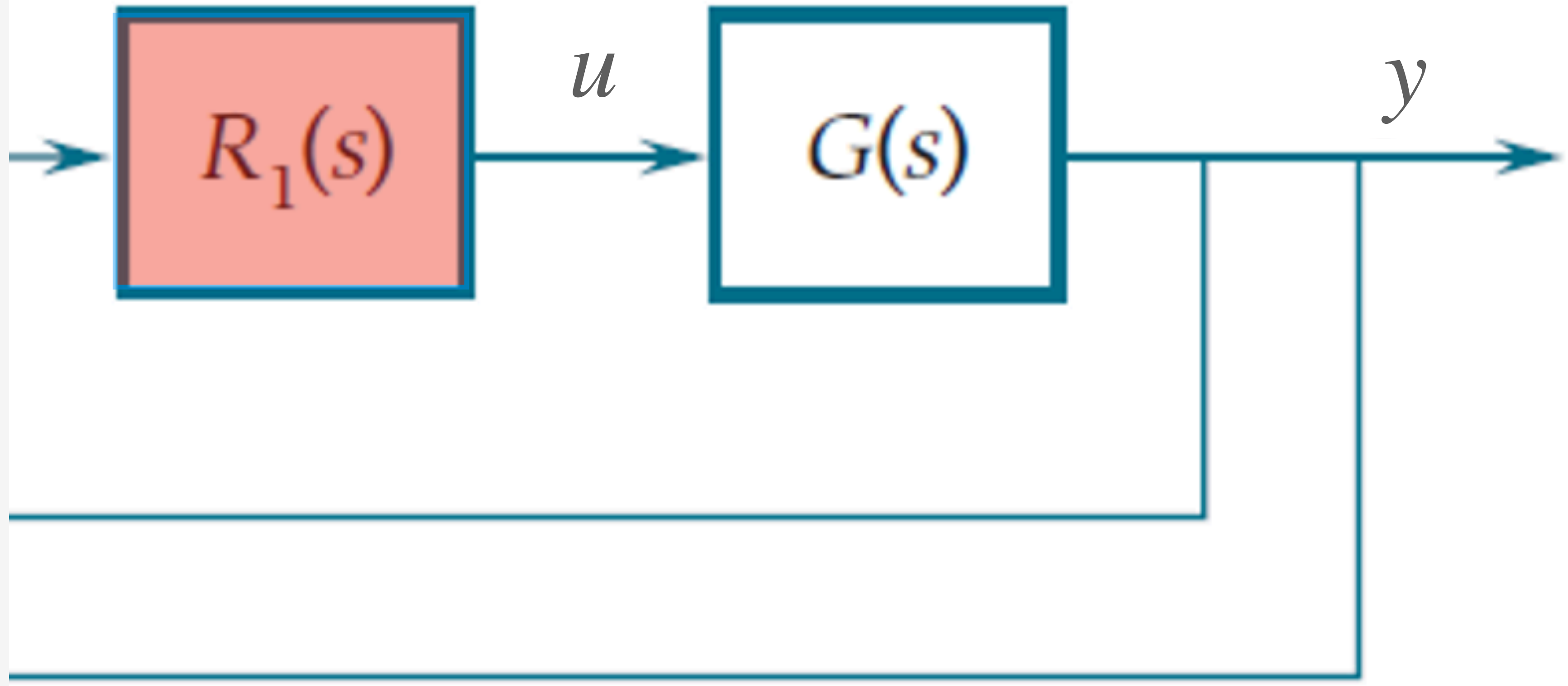
$$G(s) = -\frac{100}{(s+3)(s-10)}$$



Command Window

New to MATLAB? See resources for Getting Started.

```
>> s = tf('s');
L1 = 1 / ((s + 30)*(s - 10));  % This is without the gain K
rlocus(L1)
grid on
title('Root Locus of L1(s) = K / ((s + 30)(s - 10))')
>> |
```



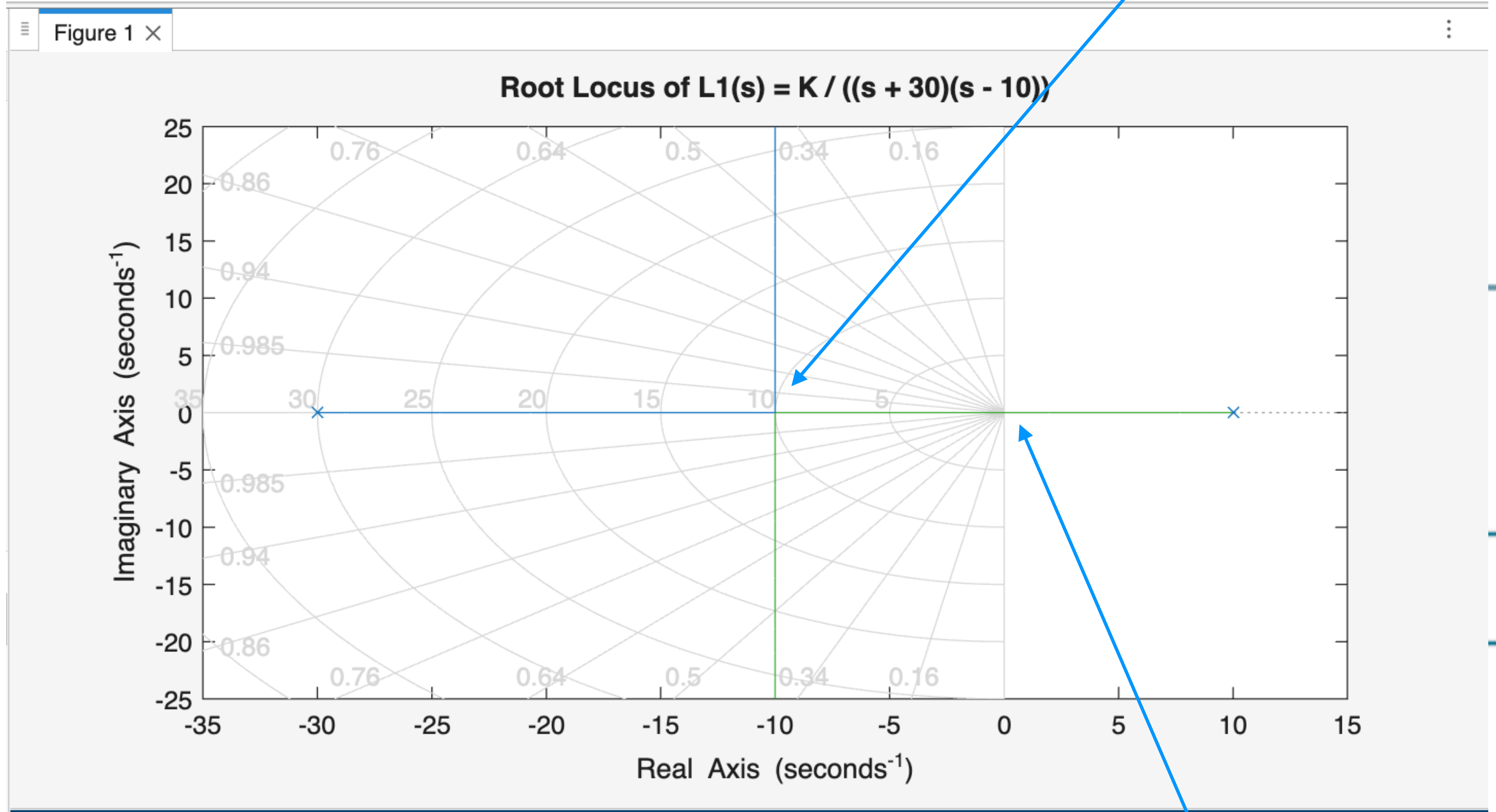
$$|K|_{s=s^*} = \left|\frac{D(s^*)}{N(s^*)}\right|$$

K^* ?

Control of Open-loop Unstable Systems

Example:

$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{-30+10}{2-0} = \frac{-20}{2} = -10$$

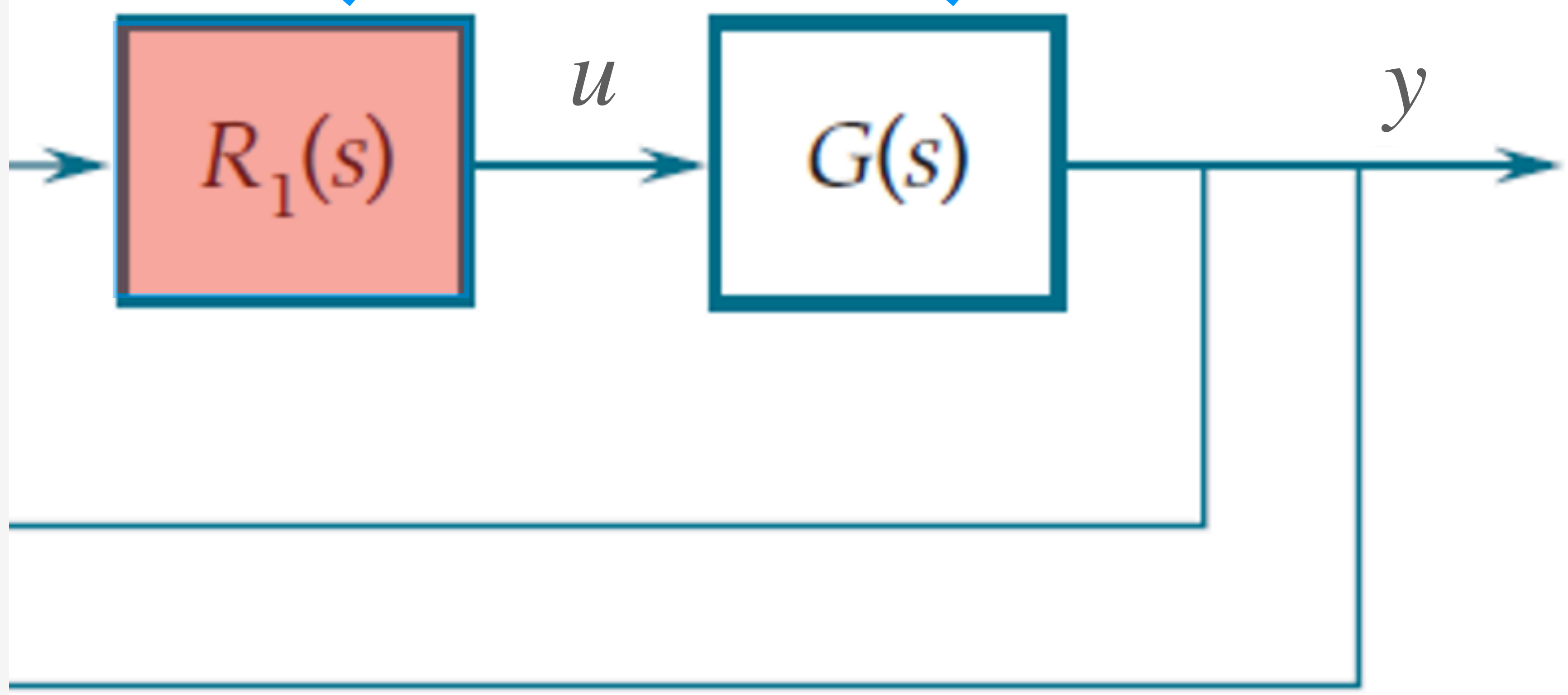



```
>> s = tf('s');
L1 = 1 / ((s + 30)*(s - 10));  % This is without the gain K
rlocus(L1)
grid on
title('Root Locus of L1(s) = K / ((s + 30)(s - 10))')
>> |
```

$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



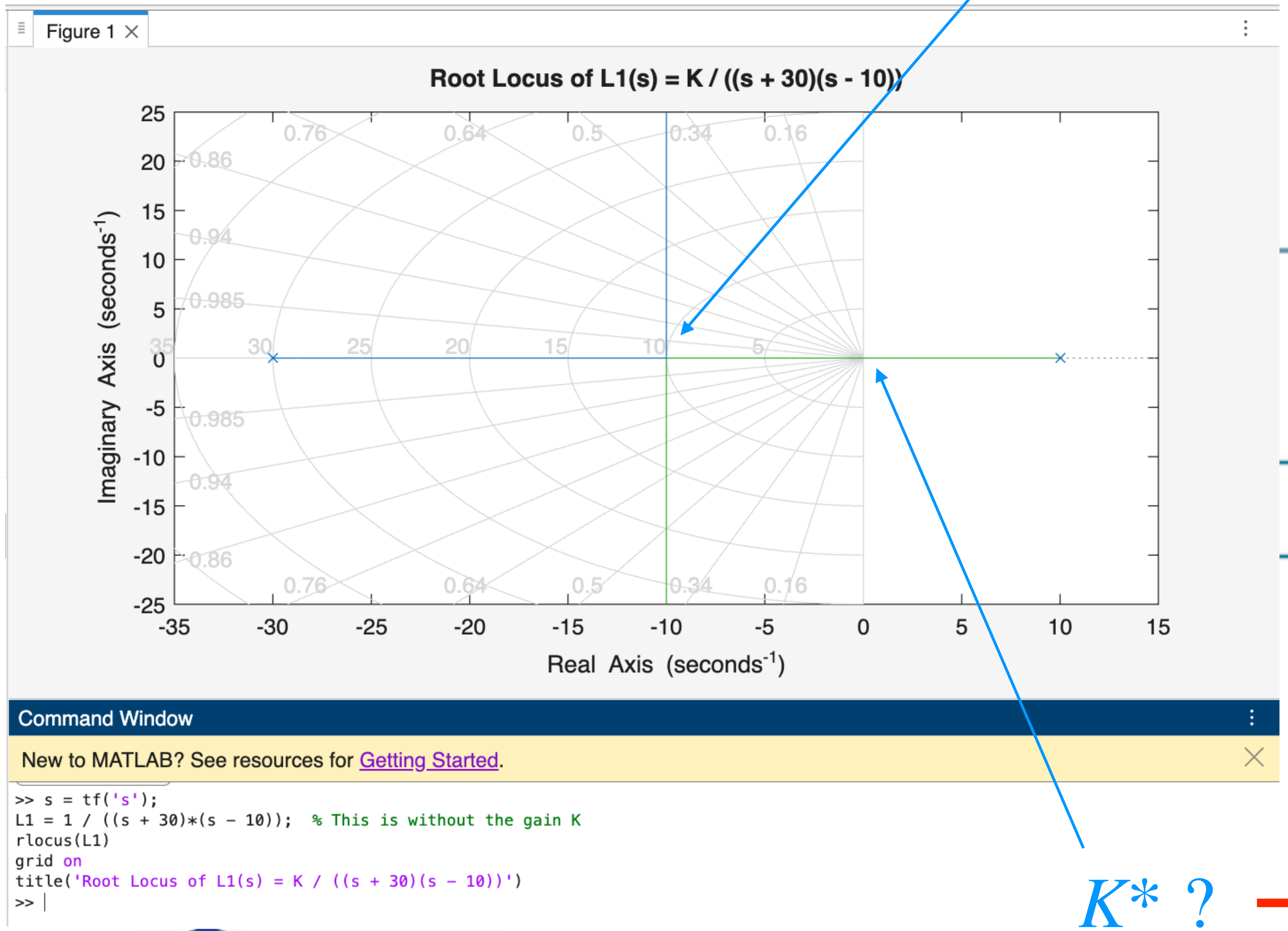
$$|K|_{s=s^*} = \left|\frac{D(s^*)}{N(s^*)}\right| = \left|\frac{(s^*+30)(s^*-10)}{1}\right|$$

K^* ?

Control of Open-loop Unstable Systems

Example:

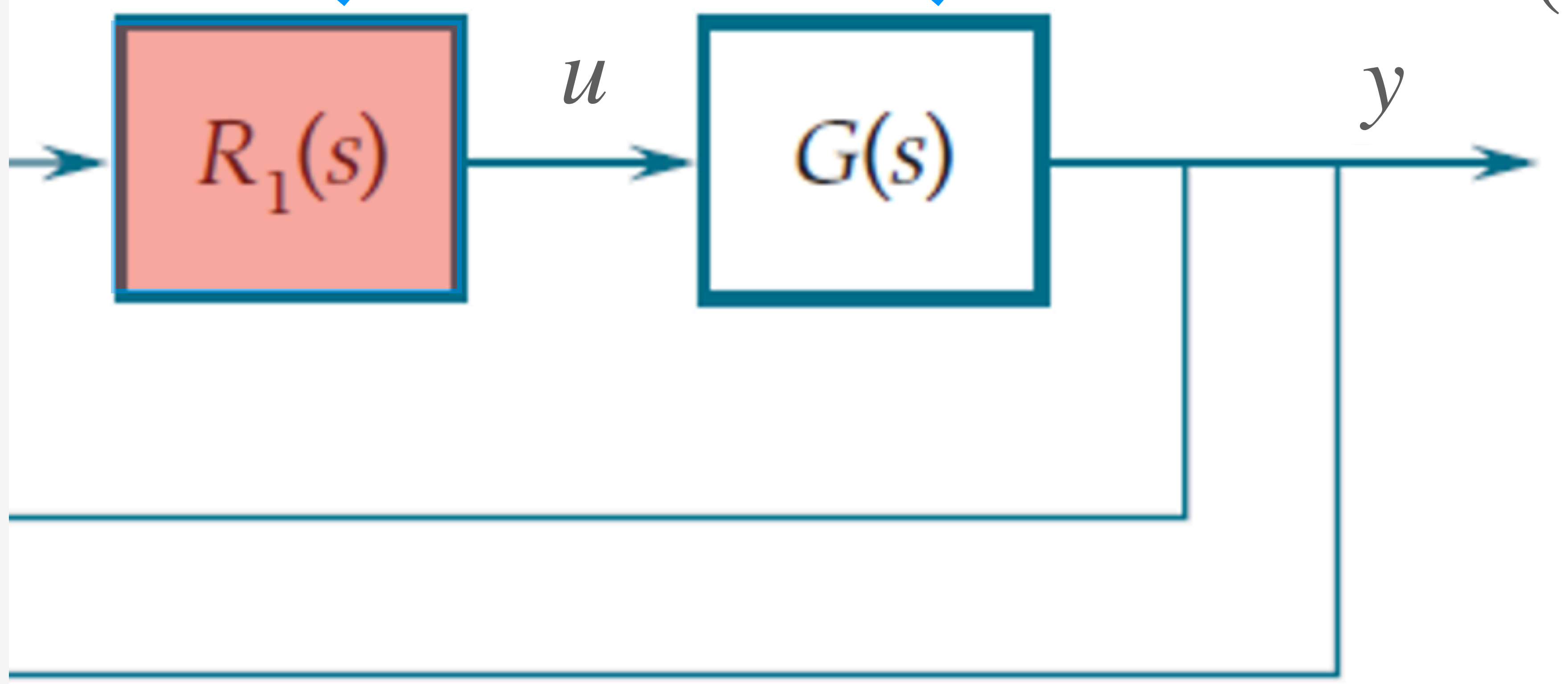
$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n - m} \qquad \sigma_a = \frac{-30+10}{2-0} = \frac{-20}{2} = -10$$



$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



$$|K|_{s=s^*} = \left|\frac{D(s^*)}{N(s^*)}\right| = \left|\frac{(s^*+30)(s^*-10)}{1}\right|$$

K^* ?

$$|K|_{s=0} = \left|\frac{D(0)}{N(0)}\right| = \left|\frac{(0+30)(0-10)}{1}\right|$$

Control of Open-loop Unstable Systems

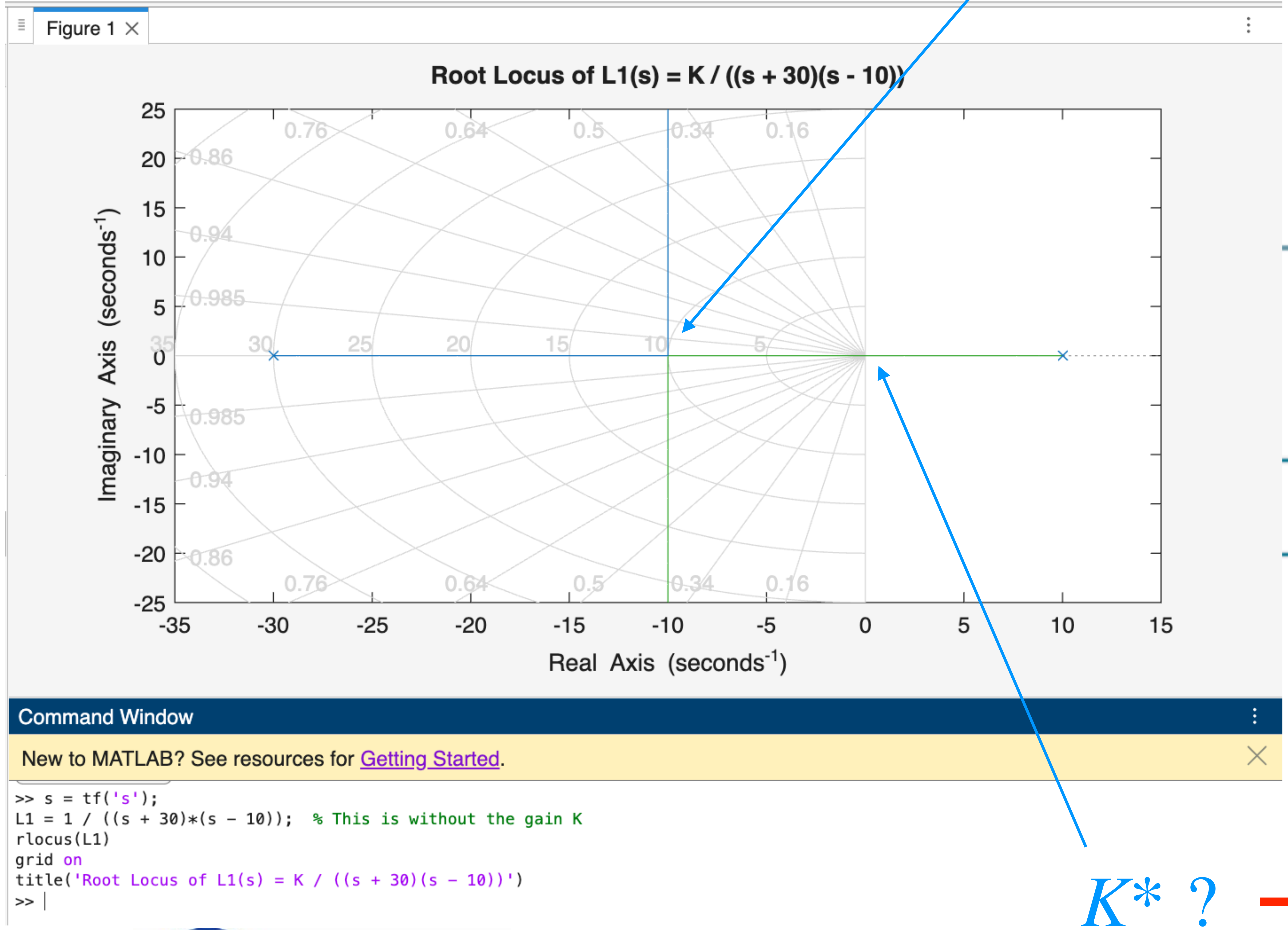
Example:

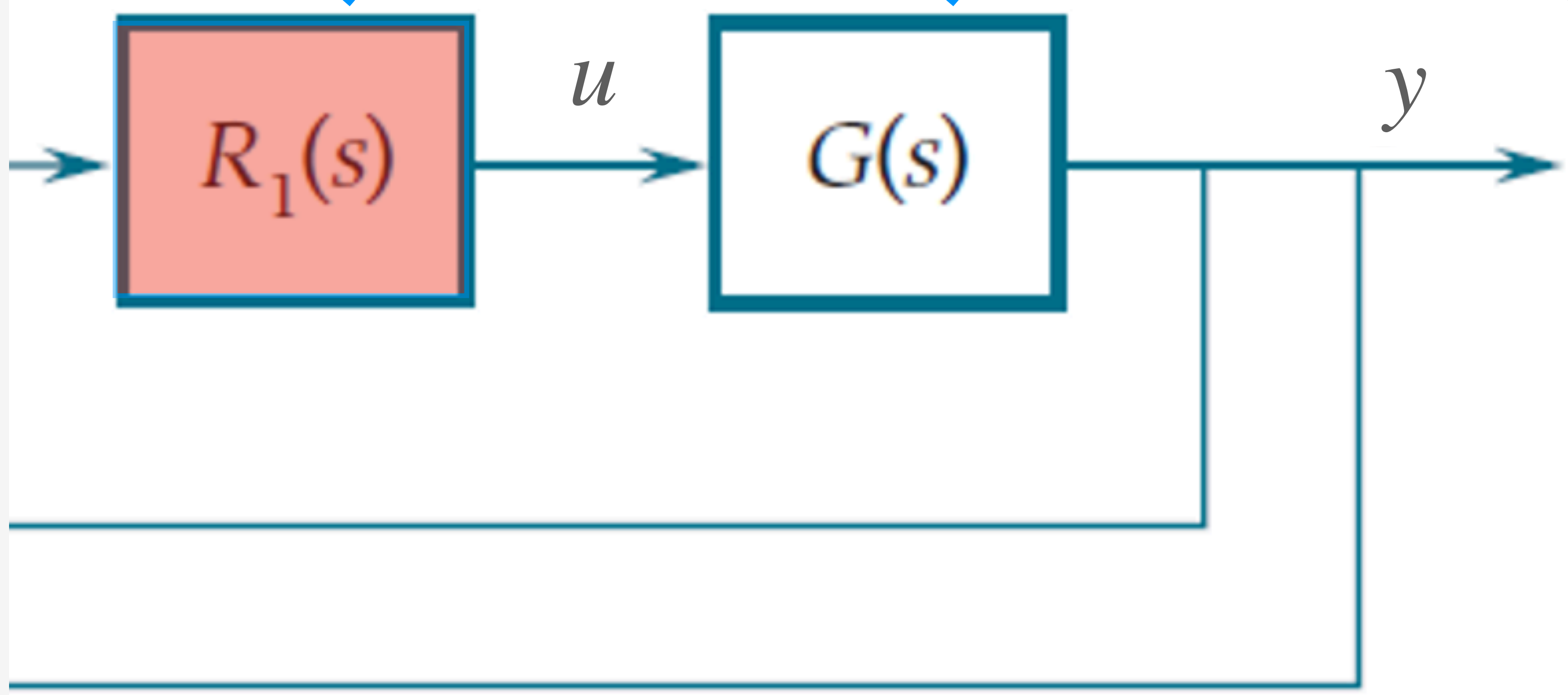
$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{-30+10}{2-0} = \frac{-20}{2} = -10$$

$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$





$$|K|_{s=s^*} = \left|\frac{D(s^*)}{N(s^*)}\right| = \left|\frac{(s^*+30)(s^*-10)}{1}\right|$$

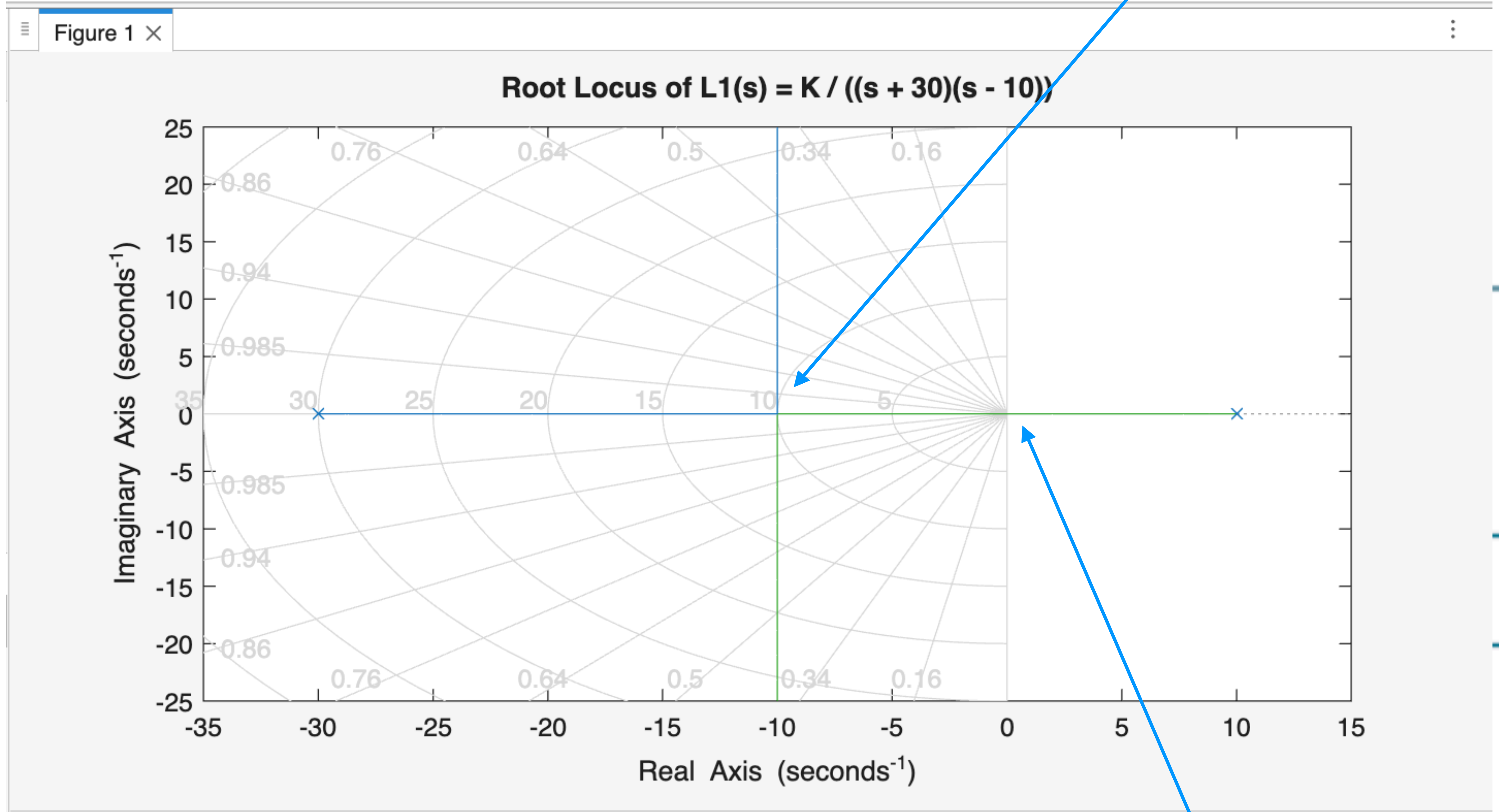
K^* ?

$$|K|_{s=0} = \left|\frac{D(0)}{N(0)}\right| = \left|\frac{(0+30)(0-10)}{1}\right| = 300$$

Control of Open-loop Unstable Systems

Example:

$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{-30+10}{2-0} = \frac{-20}{2} = -10$$



Command Window

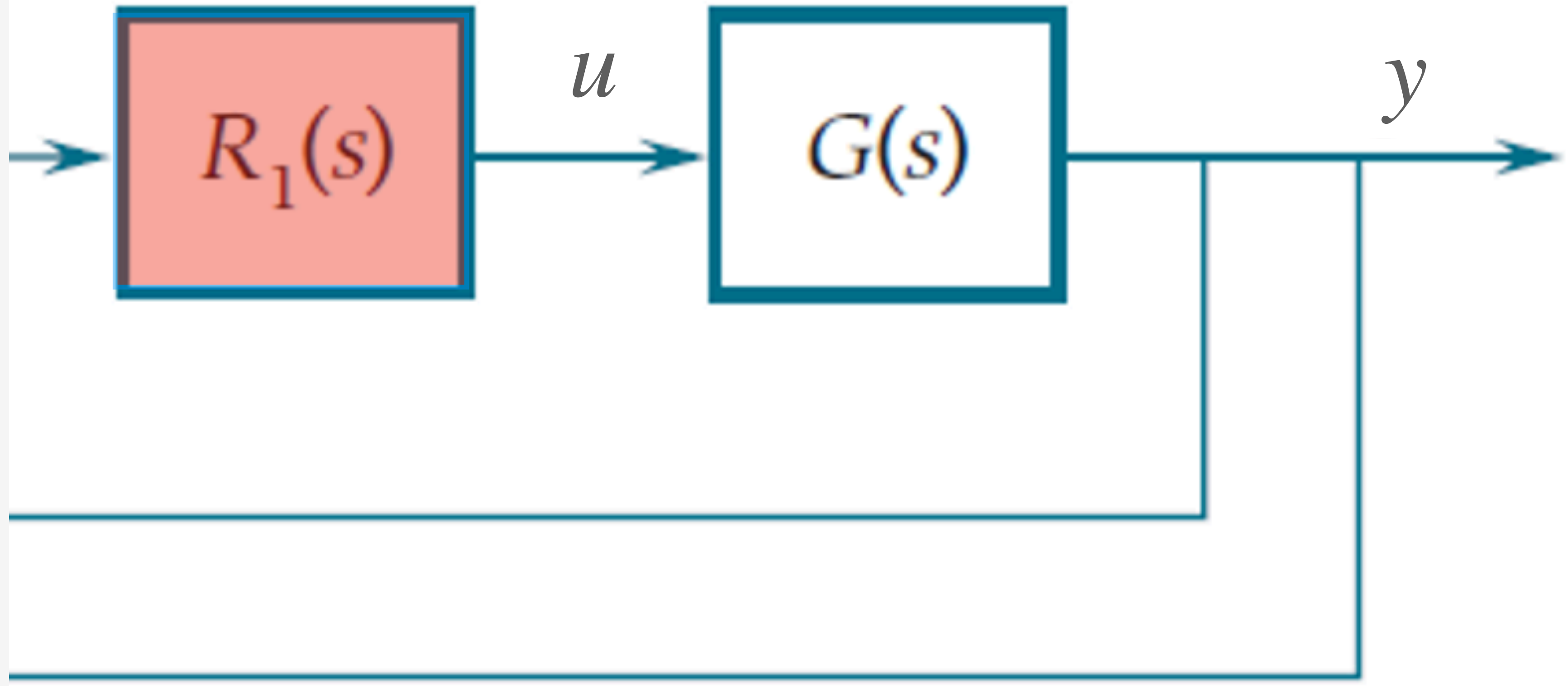
New to MATLAB? See resources for Getting Started.

```
>> s = tf('s');
L1 = 1 / ((s + 30)*(s - 10));  % This is without the gain K
rlocus(L1)
grid on
title('Root Locus of L1(s) = K / ((s + 30)(s - 10))')
>> |
```

$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



To conclude for $R_1(s)$:
- Choose K
- Find k considering that $-100k = K$

$$|K|_{s=s^*} = \left|\frac{D(s^*)}{N(s^*)}\right| = \left|\frac{(s^*+30)(s^*-10)}{1}\right|$$

K^* ?

$$|K|_{s=0} = \left|\frac{D(0)}{N(0)}\right| = \left|\frac{(0+30)(0-10)}{1}\right| = 300$$

Control of Open-loop Unstable Systems

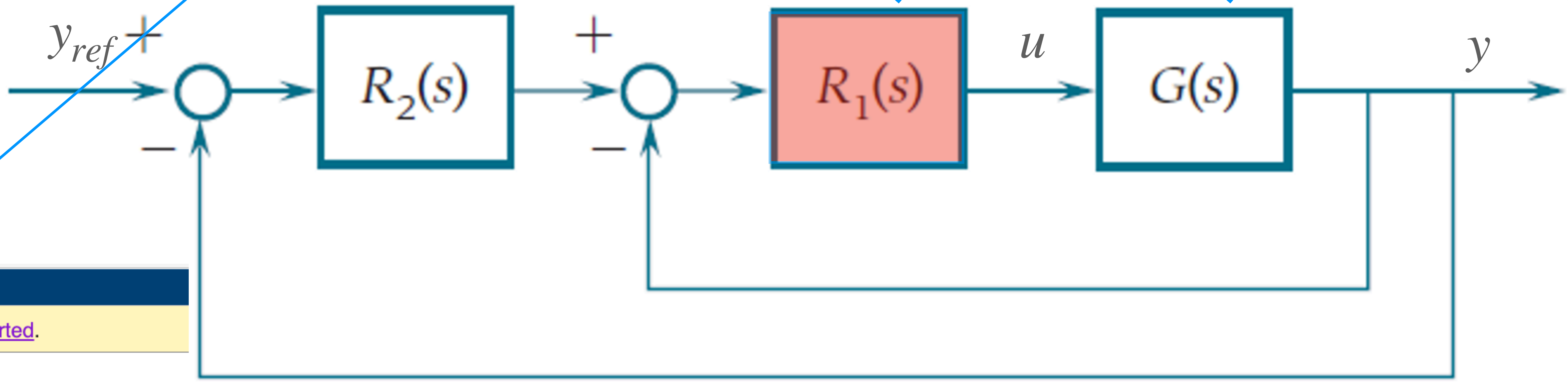
Example:

$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{-30+10}{2-0} = \frac{-20}{2} = -10$$

$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



To conclude for $R_1(s)$:

- Choose K
- Find k considering that $-100k = K$

My choice:

Command Window

New to MATLAB? See resources for Getting Started.

```
>> desired_pole = -10;
K_at_sigma_a = rlocfind(L1, desired_pole);
K_at_sigma_a

K_at_sigma_a =

   400
```

$$|K|_{s=s^*} = \left|\frac{D(s^*)}{N(s^*)}\right| = \left|\frac{(s^*+30)(s^*-10)}{1}\right|$$

K^* ?

$$|K|_{s=0} = \left|\frac{D(0)}{N(0)}\right| = \left|\frac{(0+30)(0-10)}{1}\right| = 300$$

Control of Open-loop Unstable Systems

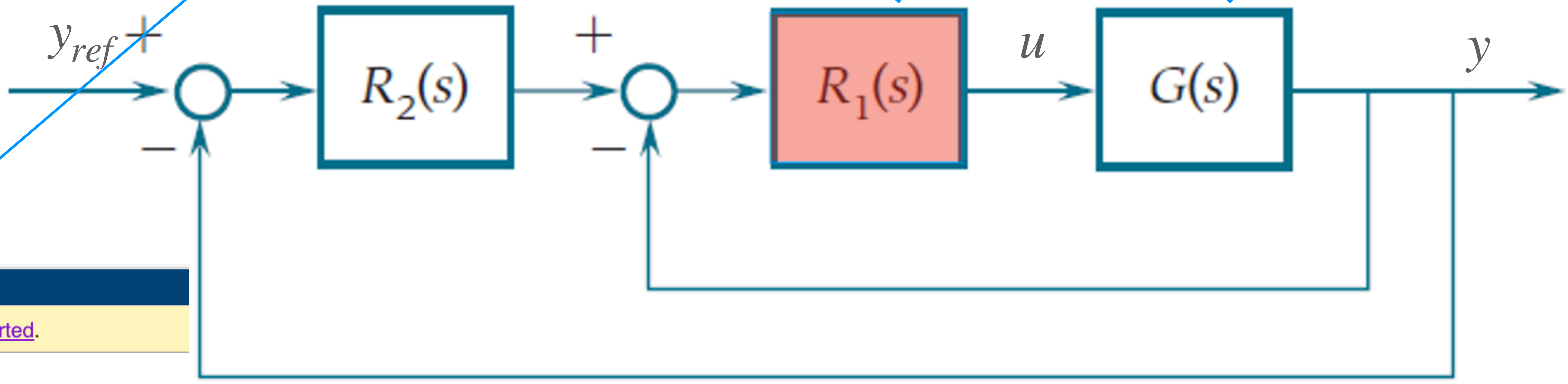
Example:

$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{-30+10}{2-0} = \frac{-20}{2} = -10$$

$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



To conclude for $R_1(s)$:
- Choose K
- Find k considering that $-100k = K$

My choice:

Command Window

New to MATLAB? See resources for Getting Started.

```
>> desired_pole = -10;
K_at_sigma_a = rlocfind(L1, desired_pole);
K_at_sigma_a

K_at_sigma_a =

   400
```

$$-100k = K$$

$$k = -\frac{K}{100}$$

$$|K|_{s=s^*} = \left|\frac{D(s^*)}{N(s^*)}\right| = \left|\frac{(s^*+30)(s^*-10)}{1}\right|$$

K^* ? $\rightarrow$ $$|K|_{s=0} = \left|\frac{D(0)}{N(0)}\right| = \left|\frac{(0+30)(0-10)}{1}\right| = 300$$

Control of Open-loop Unstable Systems

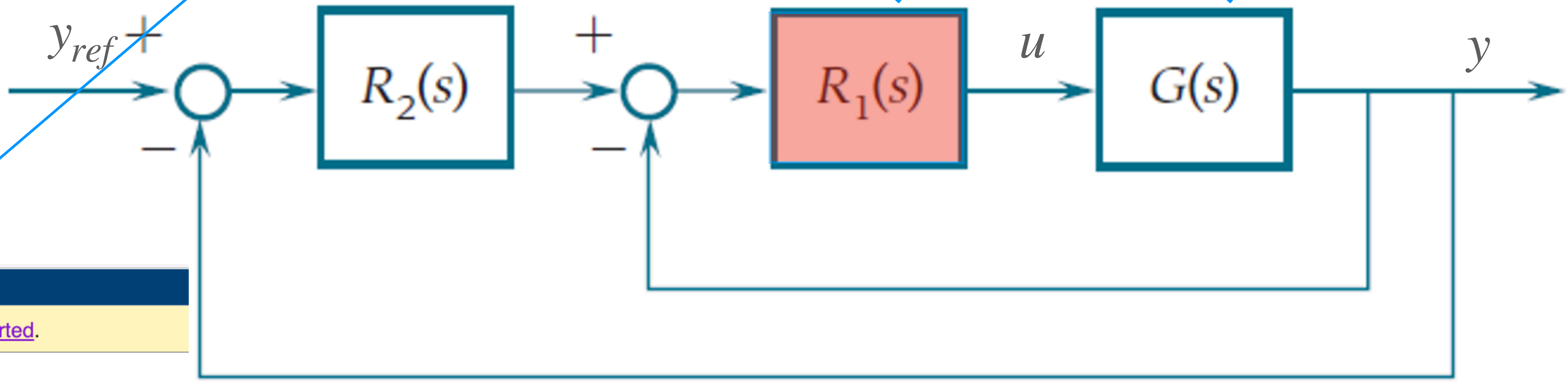
Example:

$$\sigma_a = \frac{\sum \text{poli} - \sum \text{zeri}}{n-m} \qquad \sigma_a = \frac{-30+10}{2-0} = \frac{-20}{2} = -10$$

$$R_1(s) = \frac{k(s+3)}{s+30}$$

$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



To conclude for $R_1(s)$:
- Choose K
- Find k considering that $-100k = K$

My choice:

Command Window

New to MATLAB? See resources for Getting Started.

```
>> desired_pole = -10;
K_at_sigma_a = rlocfind(L1, desired_pole);
K_at_sigma_a

K_at_sigma_a =

   400
```

$$-100k = K$$

$$k = -\frac{K}{100}$$

$$R_1(s) = -\frac{4(s+3)}{s+30}$$

$$|K|_{s=s^*} = \left|\frac{D(s^*)}{N(s^*)}\right| = \left|\frac{(s^*+30)(s^*-10)}{1}\right|$$

$$K^* \;? \longrightarrow |K|_{s=0} = \left|\frac{D(0)}{N(0)}\right| = \left|\frac{(0+30)(0-10)}{1}\right| = 300$$

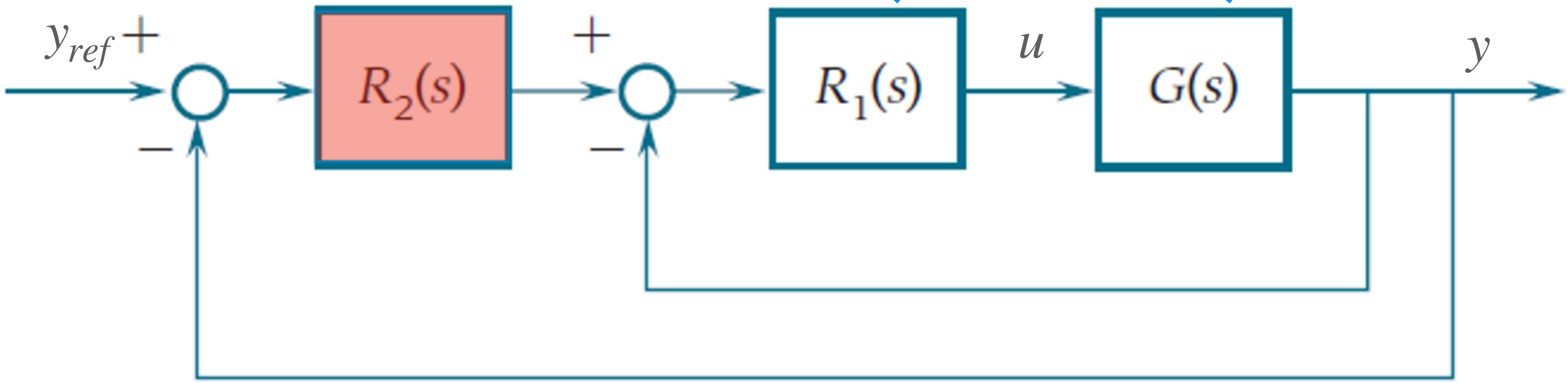
Control of Open-loop Unstable Systems

Example:

$$R_1(s) = -\frac{4(s+3)}{s+30}$$

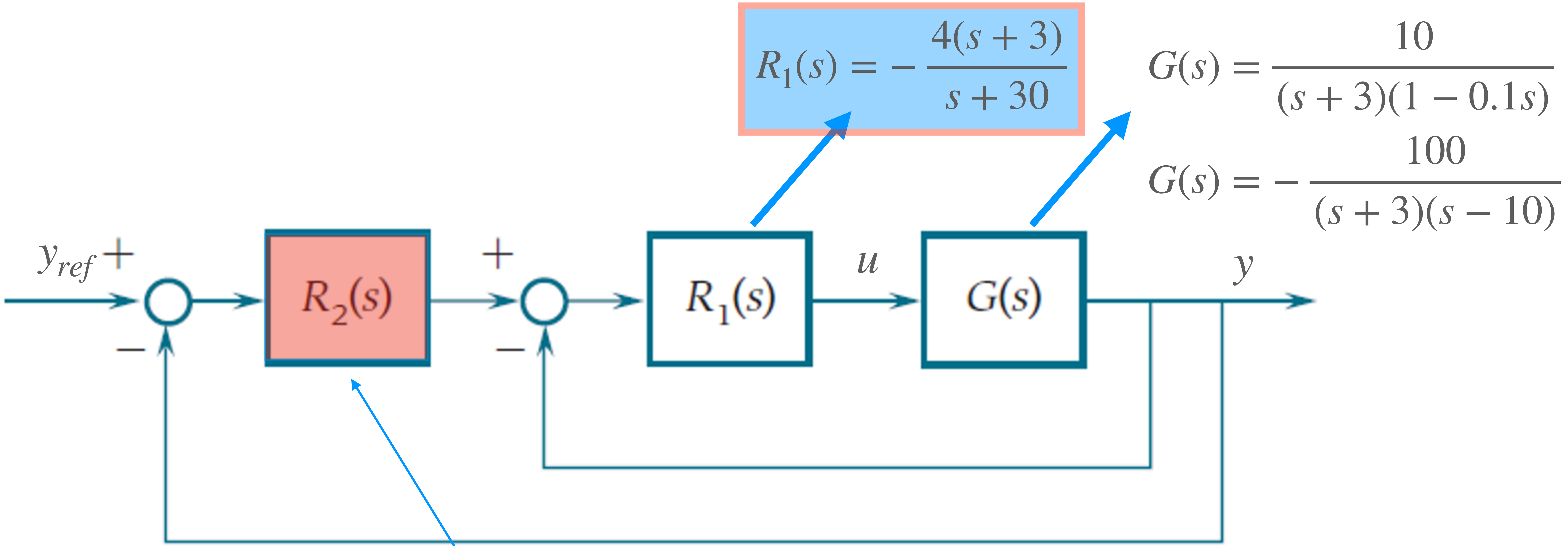
$$G(s) = \frac{10}{(s+3)(1-0.1s)}$$

$$G(s) = -\frac{100}{(s+3)(s-10)}$$



Control of Open-loop Unstable Systems

Example:



To conclude for $R_2(s)$:

- Compute $F_1(s)$
- Design $R_2(s)$ based on $F_1(s)$ using standard loop-shaping design guided by the Bode criterion