General Information

Prof. Antonella Ferrara https://scholar.google.com/citations?user=r5JuMskAAAAJ&hl=en

Course Teaching Material: KIRO UNIPV https://elearning.unipv.it/

- 504462 - PROCESS CONTROL 2025-26 - PROF.SSA FERRARA ANTONELLA

Lecture Time-table: http://www-3.unipv.it/ingserv/orario2526/1sem/insegnamenti/ProCont.html

Exams: https://studentionline.unipv.it/esse3/Home.do https://kirotesting.unipv.it/

Introduction

- Program of the course:

Advanced SISO control schemes:
Pre-filters and parallel compensators, two degrees of freedom control schemes, compensation of measurable disturbances, systems with delays and Smith Predictor, Padé approximation, decoupling in the frequency domain, control of open loop unstable systems.

Advanced MIMO control schemes:
Decoupling based control schemes, decentralized control, relative gain array.

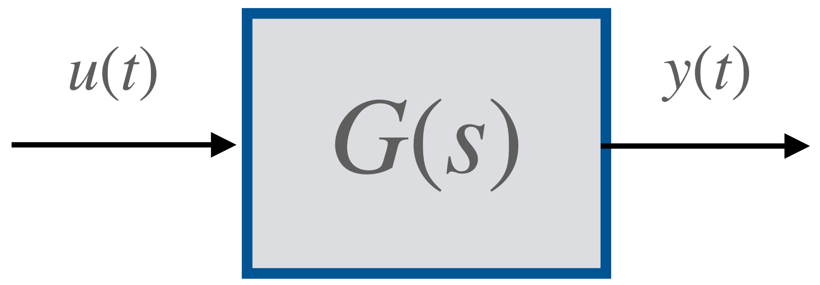
PID controllers:
Features and properties. Rules for the empirical calibration. Wind-up and anti wind-up schemes.

Digital control:
Discrete-time systems. The concept of equilibrium for discrete-time systems. Stability of linear time-invariant discrete-time systems. Jury test. Digital control schemes. Zeta transform and its properties. Transfer functions in the z domain. Sampling and aliasing. Choice of the sampling time. Zero-order-Hold. Discretization of continuous-time controllers. Bilinear transformation, Euler and Tustin methods.

Introduction

- Some of the figures in these slides, kindly provided by McGraw-Hill, are those of the Textbook:

Discrete-time Systems

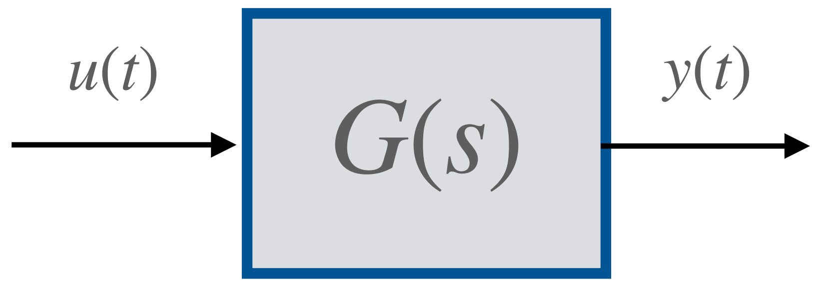


$$u(t) \in \mathbb{R}^m$$
$$y(t) \in \mathbb{R}^p$$

$$t \in \mathbb{R}^+_{\{0\}}$$

Continuous-time System

Discrete-time Systems



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$$y(t) \in \mathbb{R}^p$$

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Continuous-time System

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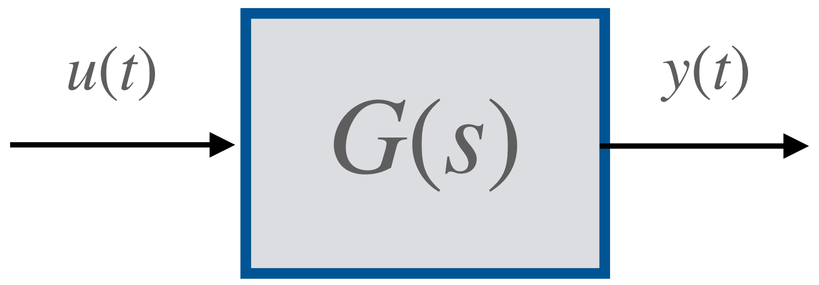
$$y(t) \in \mathbb{R}^p$$

$$t \in \mathbb{N}_{\{0\}}$$

Discrete-time System

Discrete-time Systems



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{N}_{\{0\}}$

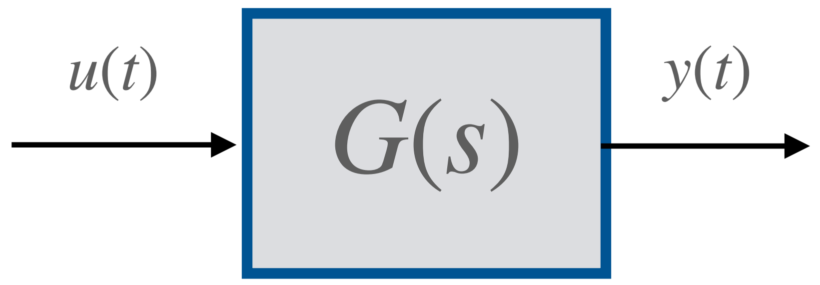
Discrete-time System

$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k), k) \\ y(k) &= g(x(k), u(k), k) \end{aligned}}$$

Discrete-time Systems

Given:

$$x(k_0) = x_0, \quad u(k) \geq k_0$$



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$$t \in \mathbb{R}^+_{\{0\}}$$

Continuous-time System

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$$t \in \mathbb{N}_{\{0\}}$$

Discrete-time System

$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k), k) \\ y(k) &= g(x(k), u(k), k) \end{aligned}}$$

Discrete-time Systems

$$u(t) \longrightarrow \boxed{G(s)} \longrightarrow y(t)$$

$$u(t) \in \mathbb{R}^m$$
$$y(t) \in \mathbb{R}^p$$

$$t \in \mathbb{R}^+_{\{0\}}$$

Continuous-time System

$$u(t) \longrightarrow \boxed{S} \longrightarrow y(t)$$

$$u(t) \in \mathbb{R}^m$$
$$y(t) \in \mathbb{R}^p$$

$$t \in \mathbb{N}_{\{0\}}$$

Discrete-time System

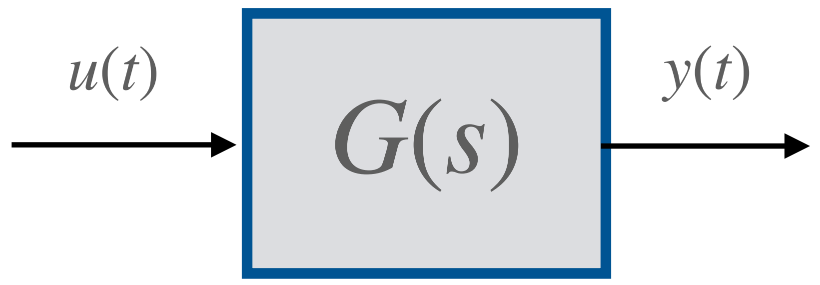
Given:

$$x(k_0) = x_0, \quad u(k) \geq k_0$$

$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k), k) \\ y(k) &= g(x(k), u(k), k) \end{aligned}}$$

$$x(1) = f\big(x_0, u(0)\big),$$
$$x(2) = f\big(x(1), u(1)\big),$$
$$\vdots$$
$$x(i) = f\big(x(i-1), u(i-1)\big)$$

Discrete-time Systems



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$$t \in \mathbb{R}^+_{\{0\}}$$

Continuous-time System

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$$t \in \mathbb{N}_{\{0\}}$$

Discrete-time System

Given:

$$x(k_0) = x_0, \;\; u(k) \geq k_0$$

$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k), k) \\ y(k) &= g(x(k), u(k), k) \end{aligned}}$$

$$x(1) = f\big(x_0, u(0)\big),$$

$$x(2) = f\big(x(1), u(1)\big),$$

$$\vdots$$

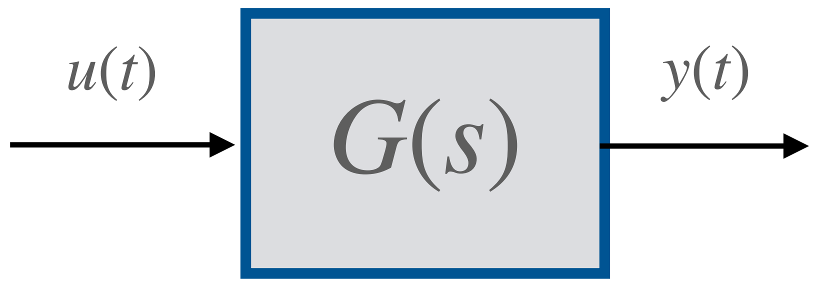
$$x(i) = f\big(x(i-1), u(i-1)\big)$$

$$y(k) = g\big(x(k), u(k)\big), \qquad k \geq 0$$

Discrete-time Systems: Classification

Given:

$$x(k_0) = x_0, \quad u(k) \geq k_0$$



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$ → MIMO

$t \in \mathbb{N}_{\{0\}}$

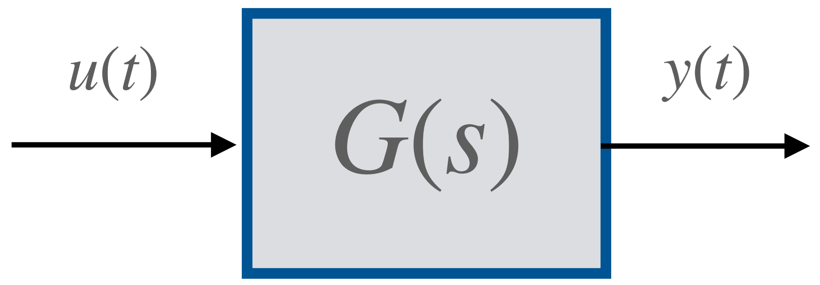
Discrete-time System

$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k), k) \\ y(k) &= g(x(k), u(k), k) \end{aligned}}$$

Discrete-time Systems: Classification

Given:

$$x(k_0) = x_0, \quad u(k) \geq k_0$$



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

$u(t) \in \mathbb{R}$

$y(t) \in \mathbb{R}$

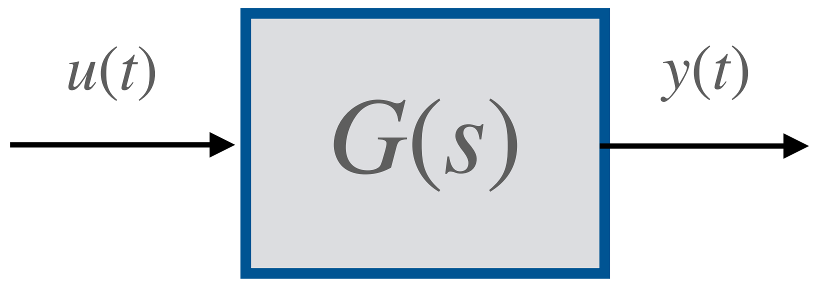
SISO

$t \in \mathbb{N}_{\{0\}}$

Discrete-time System

$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k), k) \\ y(k) &= g(x(k), u(k), k) \end{aligned}}$$

Discrete-time Systems: Classification



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{N}_{\{0\}}$

Discrete-time System

Given:

$$x(k_0) = x_0, \quad u(k) \geq k_0$$

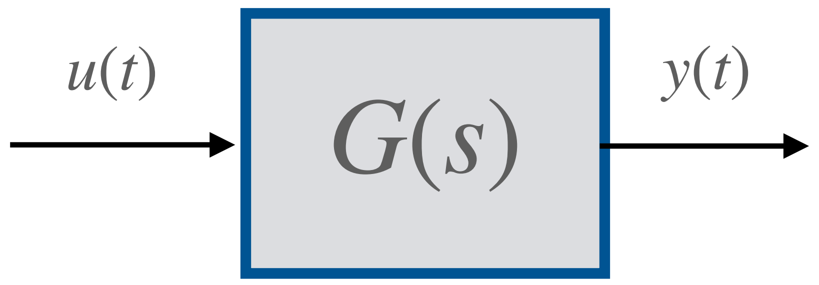
$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k), k) \\ y(k) &= g(x(k), u(k), k) \end{aligned}}$$

Proper

Discrete-time Systems: Classification

Given:

$$x(k_0) = x_0, \quad u(k) \geq k_0$$



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{N}_{\{0\}}$

Discrete-time System

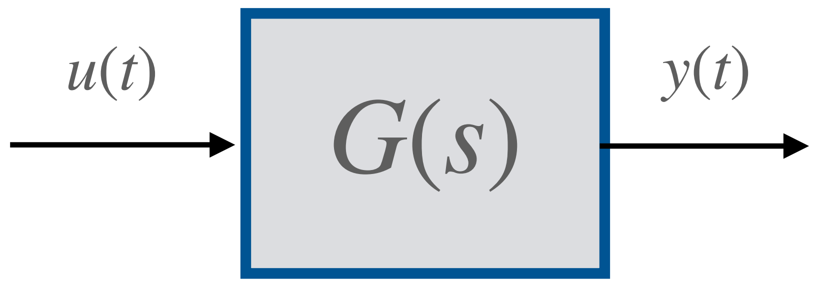
$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k), k) \\ y(k) &= g(x(k), k) \end{aligned}}$$

Strictly proper

Discrete-time Systems: Classification

Given:

$$x(k_0) = x_0, \quad u(k) \geq k_0$$



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{N}_{\{0\}}$

Discrete-time System

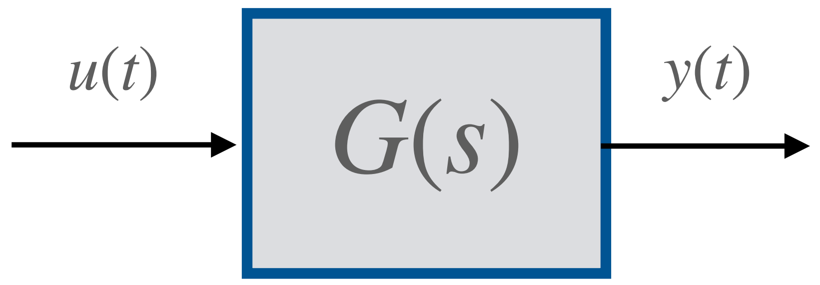
$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k), k) \\ y(k) &= g(x(k), u(k), k) \end{aligned}}$$

Time-varying

Discrete-time Systems: Classification

Given:

$$x(k_0) = x_0, \quad u(k) \geq k_0$$



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{N}_{\{0\}}$

Discrete-time System

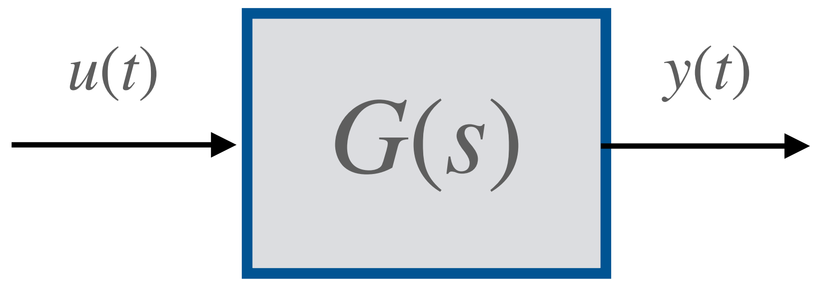
$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= g(x(k), u(k)) \end{aligned}}$$

Time-invariant

Discrete-time Systems: Classification

Given:

$$x(k_0) = x_0, \quad u(k) \geq k_0$$



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

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Continuous-time System

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{N}_{\{0\}}$

Discrete-time System

$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k), k) \\ y(k) &= g(x(k), u(k), k) \end{aligned}}$$

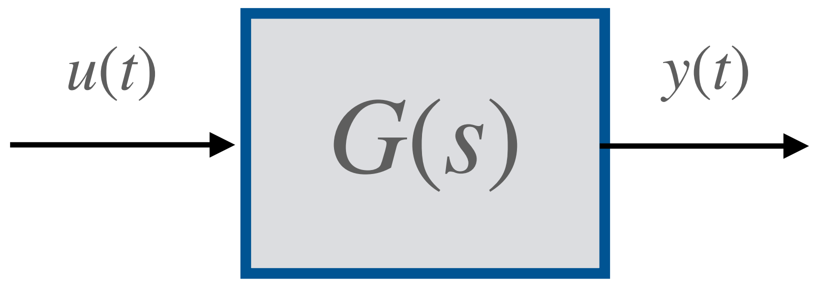
$\downarrow$

Nonlinear

Discrete-time Systems: Classification

Given:

$$x(k_0) = x_0, \quad u(k) \geq k_0$$



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{N}_{\{0\}}$

Discrete-time System

$$\boxed{\begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k) \\ y(k) &= C(k)x(k) + D(k)u(k) \end{aligned}}$$

Linear Time-varying

Discrete-time Systems: Classification

Given:

$$x(k_0) = x_0, \quad u(k) \geq k_0$$

$$u(t) \longrightarrow \boxed{G(s)} \longrightarrow y(t)$$

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

$$u(t) \longrightarrow \boxed{S} \longrightarrow y(t)$$

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{N}_{\{0\}}$

Discrete-time System

$$\boxed{\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}}$$

Linear Time-invariant

Discrete-time Systems: Equilibrium

Given:

$$u(k) = \bar{u}, \quad \forall\, k \geq 0$$

$$u(t) \longrightarrow \boxed{G(s)} \longrightarrow y(t)$$

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$$t \in \mathbb{R}^+_{\{0\}}$$

Continuous-time System

$$u(t) \longrightarrow \boxed{S} \longrightarrow y(t)$$

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$$t \in \mathbb{N}_{\{0\}}$$

Discrete-time System

$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= g(x(k), u(k)) \end{aligned}}$$

Time-invariant

Discrete-time Systems: Equilibrium

Given:

$u(k) = \bar{u}, \ \forall \ k \geq 0$

$$u(t) \longrightarrow \boxed{G(s)} \longrightarrow y(t)$$

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

$$\boxed{\dot{x}(t) = 0}$$

$$u(t) \longrightarrow \boxed{S} \longrightarrow y(t)$$

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{N}_{\{0\}}$

Discrete-time System

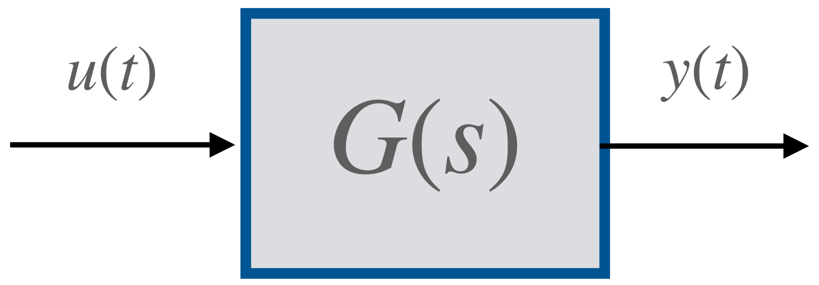
$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= g(x(k), u(k)) \end{aligned}}$$

Time-invariant

Discrete-time Systems: Equilibrium

Given:

$$u(k) = \bar{u}, \quad \forall\, k \geq 0$$



$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

$$\boxed{\dot{x}(t) = 0}$$

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{N}_{\{0\}}$

Discrete-time System

$$\boxed{x(k+1) = x(k)}$$

$$\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= g(x(k), u(k)) \end{aligned}$$

Time-invariant

Discrete-time Systems: Equilibrium

Given:

$$u(k) = \bar{u}, \quad \forall\, k \geq 0$$

$$u(t) \rightarrow \boxed{G(s)} \rightarrow y(t)$$

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

$$\boxed{\dot{x}(t) = 0}$$

$$u(t) \rightarrow \boxed{S} \rightarrow y(t)$$

$u(t) \in \mathbb{R}^m$

$y(t) \in \mathbb{R}^p$

$t \in \mathbb{N}_{\{0\}}$

Discrete-time System

$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= g(x(k), u(k)) \end{aligned}}$$

Time-invariant

$$\boxed{x(k+1) = x(k)}$$

If the solution exists:

$$x(k) = \bar{x}, \quad \forall\, k \geq 0$$

$$\bar{y}(k) = g(\bar{x}, \bar{u})$$

Discrete-time Systems: Stability of Equilibrium

Given:

$$x(k_0) = x_0, \quad u(k) = \bar{u}$$

$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= g(x(k), u(k)) \end{aligned}}$$

Time-invariant

An equilibrium state is STABLE if:

$$\forall\, \varepsilon > 0 \;\; \exists \;\; \delta > 0 \;\; \text{s.t.} \;\; \|x(0) - \bar{x}\| < \delta$$

$$\Longrightarrow \quad \|x(k) - \bar{x}\| < \varepsilon, \quad \forall k \geq 0$$

If the solution exists:

$$x(k) = \bar{x}, \;\; \forall\, k \geq 0$$
$$\bar{y}(k) = g(\bar{x}, \bar{u})$$

Discrete-time Systems: Stability of Equilibrium

Given:

$$x(k_0) = x_0, \quad u(k) = \bar{u}$$

$$x(k+1) = f(x(k), u(k))$$
$$y(k) = g(x(k), u(k))$$

Time-invariant

An equilibrium state is AS. STABLE if:

$$\forall\ \varepsilon > 0 \ \ \exists\ \ \delta > 0 \ \text{ s.t. } \ \|x(0) - \bar{x}\| < \delta$$

$$\Longrightarrow \quad \|x(k) - \bar{x}\| < \varepsilon, \quad \forall k \geq 0$$

and in addition:

$$\lim_{k \to \infty} \|x(k) - \bar{x}\| = 0$$

If the solution exists:

$$x(k) = \bar{x}, \ \ \forall\ k \geq 0$$
$$\bar{y}(k) = g(\bar{x}, \bar{u})$$

Discrete-time Systems: Stability of Equilibrium

Given:

$$x(k_0) = x_0, \quad u(k) = \bar{u}$$

An equilibrium state is UNSTABLE if:

it is not stable

$$x(k+1) = f(x(k), u(k))$$
$$y(k) = g(x(k), u(k))$$

Time-invariant

If the solution exists:

$$x(k) = \bar{x}, \quad \forall\, k \geq 0$$
$$\bar{y}(k) = g(\bar{x}, \bar{u})$$

Discrete-time Systems: Stability of a Generic Motion of the State

Given:

$$x(k_0) = x_0, \quad u(k) = \bar{u}$$

$$x(k+1) = f(x(k), u(k))$$
$$y(k) = g(x(k), u(k))$$

Time-invariant

An motion of the state $\tilde{x}(k)$ is STABLE if:

$$\forall\, \varepsilon > 0 \;\; \exists \;\; \delta > 0 \;\; \text{s.t.} \;\; \|x(0) - \tilde{x}(0)\| < \delta$$

$$\Longrightarrow \qquad \|x(k) - \tilde{x}(k)\| < \varepsilon, \quad \forall k \geq 0$$

An motion of the state $\tilde{x}(k)$ is AS. STABLE if:

$$\forall\, \varepsilon > 0 \;\; \exists \;\; \delta > 0 \;\; \text{s.t.} \;\; \|x(0) - \tilde{x}(0)\| < \delta$$

$$\Longrightarrow \qquad \|x(k) - \tilde{x}(k)\| < \varepsilon, \quad \forall k \geq 0$$

and in addition:

$$\lim_{k \to \infty} \|x(k) - \tilde{x}(k)\| = 0$$

If the solution exists:

$$x(k) = \bar{x}, \;\; \forall\, k \geq 0$$
$$\bar{y}(k) = g(\bar{x}, \bar{u})$$

Discrete-time Systems: Stability of a Generic Motion of the State

Given:

$$x(k_0) = x_0, \quad u(k) = \bar{u}$$

$$\boxed{\begin{aligned} x(k+1) &= f(x(k), u(k)) \\ y(k) &= g(x(k), u(k)) \end{aligned}}$$

Time-invariant

An motion of the state $\tilde{x}(k)$ is UNSTABLE if:

it is not stable

If the solution exists:

$$x(k) = \bar{x}, \quad \forall\, k \geq 0$$
$$\bar{y}(k) = g(\bar{x}, \bar{u})$$

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Given:

$$x(k_0) = x_0, \;\; u(k) \geq k_0$$

$$\boxed{\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}}$$

Linear Time-invariant

The stability property is referred to the system

Stability only depends on the eigenvalues of matrix A

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Given:

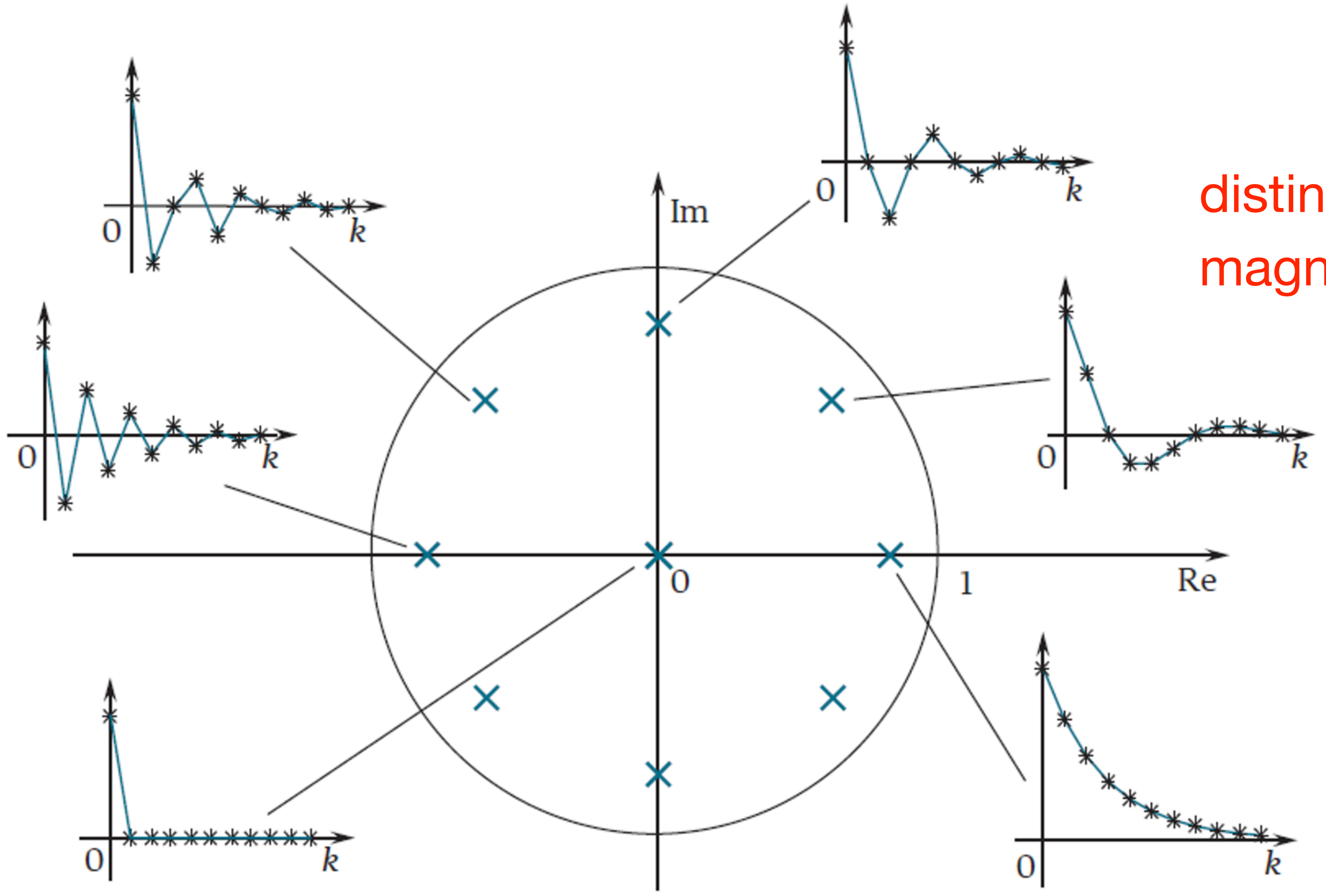
$$x(k_0) = x_0, \;\; u(k) \geq k_0$$

$$\boxed{\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}}$$

Linear Time-invariant

The stability property is referred to the system

Stability only depends on the eigenvalues of matrix A



distinct eigenvalues with magnitude < 1

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Given:

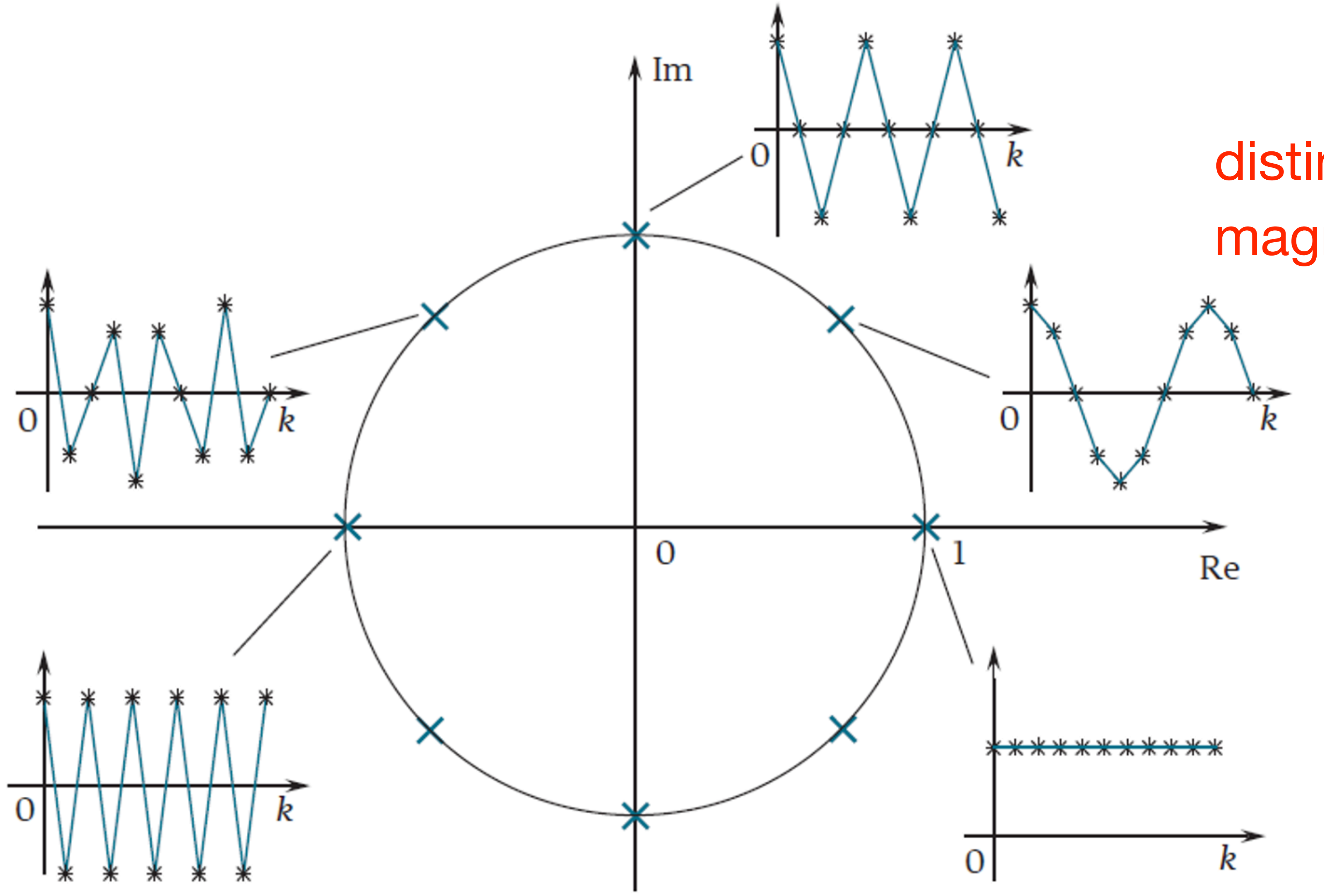
$$x(k_0) = x_0, \;\; u(k) \geq k_0$$

$$\boxed{\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}}$$

Linear Time-invariant

The stability property is referred to the system

Stability only depends on the eigenvalues of matrix A



distinct eigenvalues with magnitude $= 1$

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Given:

$$x(k_0) = x_0, \;\; u(k) \geq k_0$$

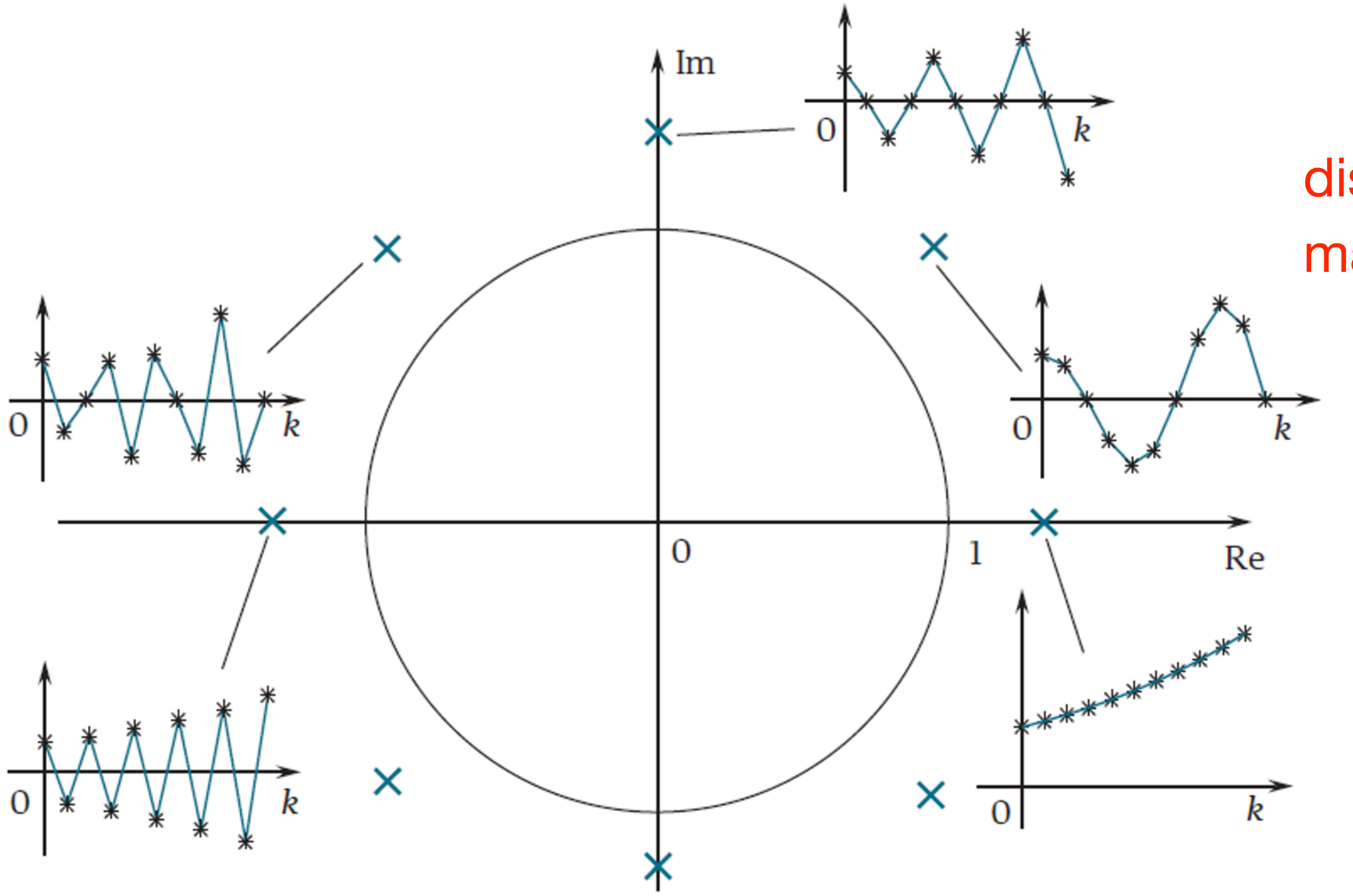
$$\boxed{\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}}$$

$\downarrow$

Linear Time-invariant

The stability property is referred to the system

Stability only depends on the eigenvalues of matrix A



distinct eigenvalues with magnitude > 1

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Given:

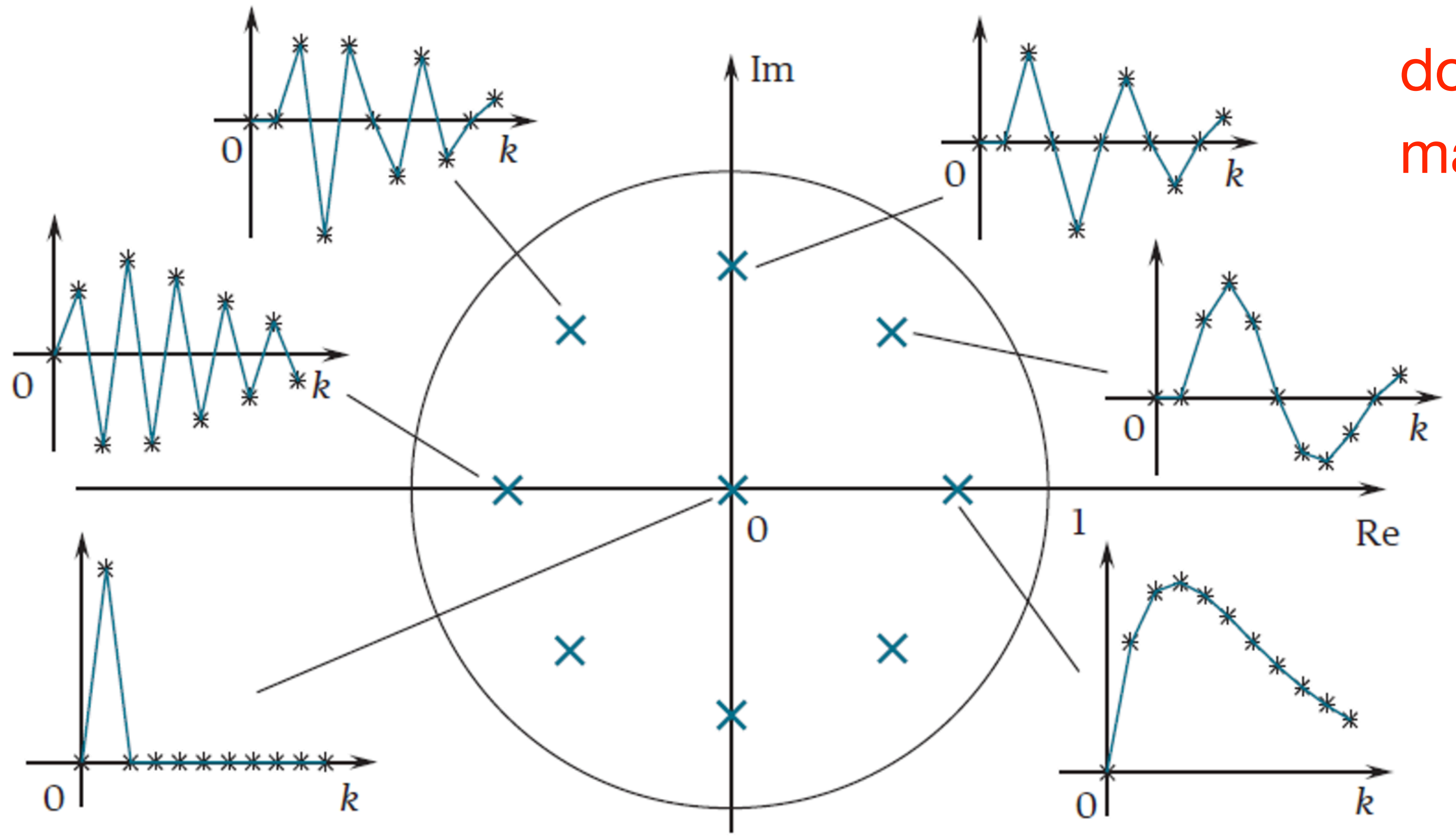
$$x(k_0) = x_0, \;\; u(k) \geq k_0$$

$$\boxed{\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}}$$

Linear Time-invariant

The stability property is referred to the system

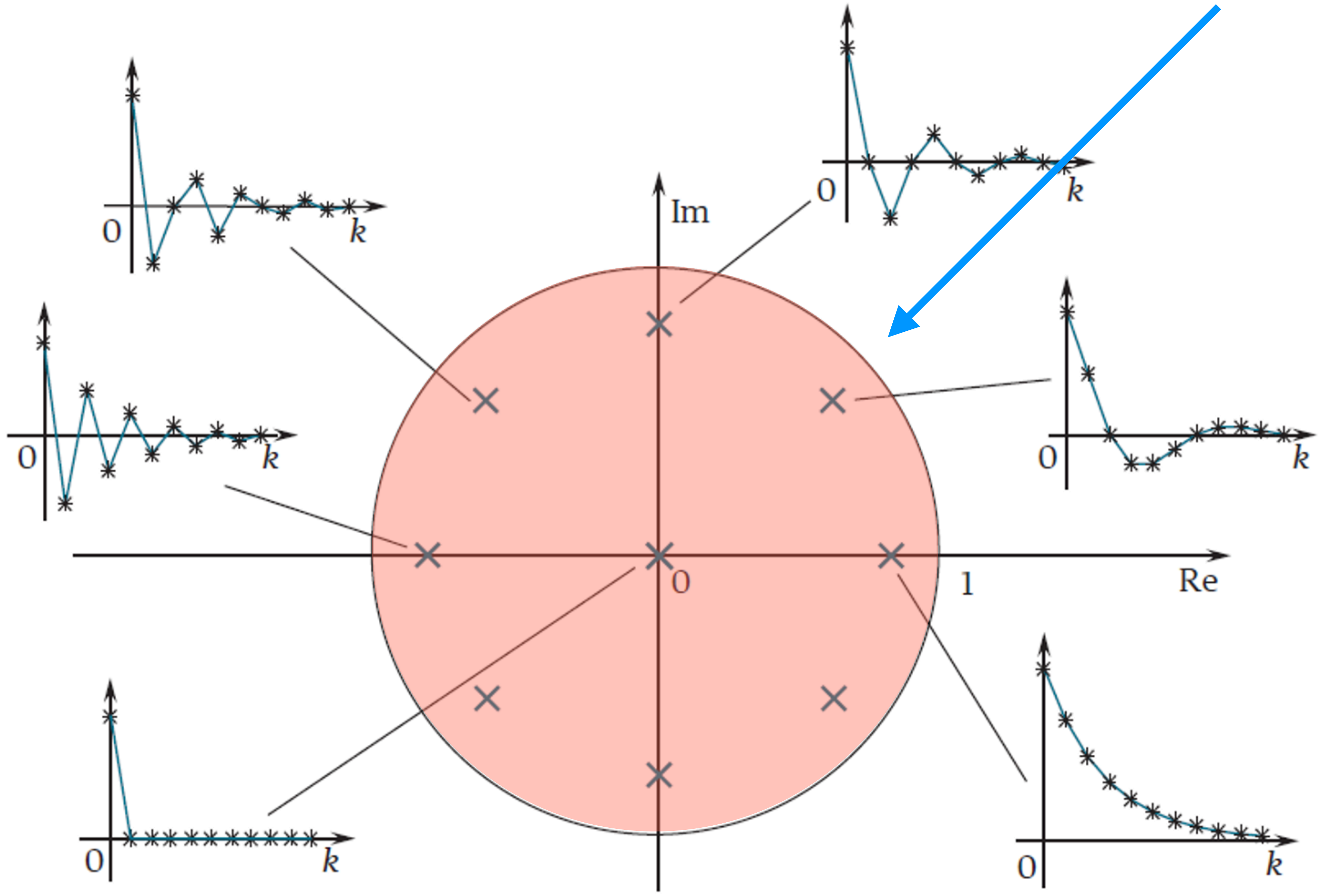
Stability only depends on the eigenvalues of matrix A



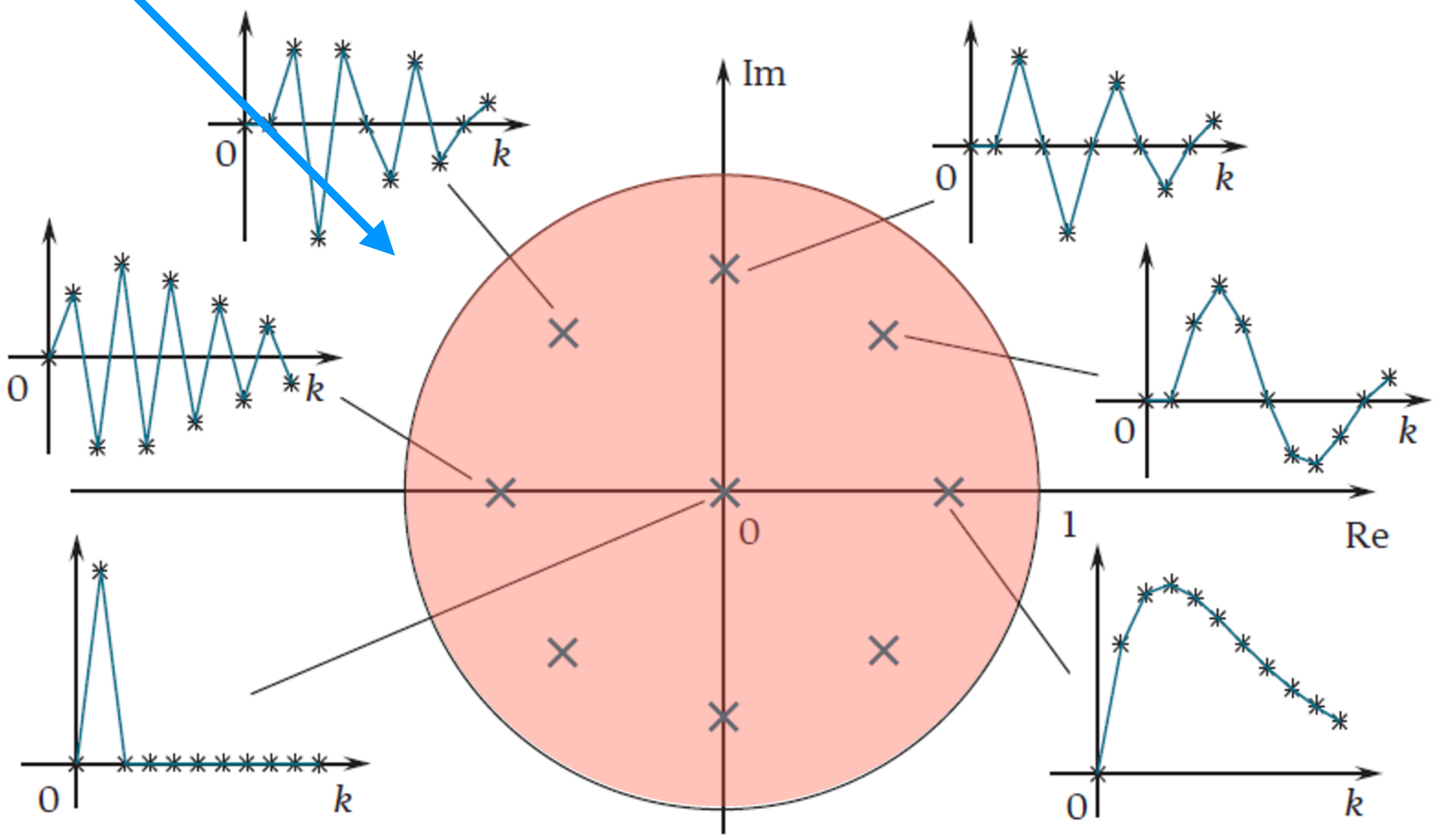
double eigenvalues with magnitude < 1

Discrete-time Systems: Stability of LTI Discrete-Time Systems

stability region



distinct eigenvalues with magnitude < 1



double eigenvalues with magnitude < 1

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Characteristic Polynomial

$$\varphi(z) = \det(zI - A) = \varphi_0 z^n + \varphi_1 z^{n-1} + \cdots + \varphi_{n-1} z + \varphi_n$$

To find the eigenvalues:

$$\varphi(z) = \det(zI - A) = 0$$

not straightforward if $n > 2$

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Characteristic Polynomial

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To find the eigenvalues:

$$\varphi(z) = \det(zI - A) = 0$$

not straightforward if $n > 2$

Alternatives:

Necessary or sufficient conditions based on $\varphi(z)$

Stability test (Jury's Test) based on $\varphi(z)$

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Characteristic Polynomial

$$\varphi(z) = \det(zI - A) = \varphi_0 z^n + \varphi_1 z^{n-1} + \cdots + \varphi_{n-1} z + \varphi_n$$

Necessary conditions based on $\varphi(z)$:

1. A necessary condition for a discrete-time dynamical system to be asymptotically stable is that
$$\left|\frac{\varphi_n}{\varphi_0}\right| < 1$$

2. A necessary condition for a discrete-time dynamical system to be asymptotically stable is that
$$\varphi_0\,\varphi(1) > 0$$

3. A necessary condition for a discrete-time dynamical system to be asymptotically stable is that
$$(-1)^n \varphi_0\,\varphi(-1) > 0$$

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Characteristic Polynomial

$$\varphi(z) = \det(zI - A) = \varphi_0 z^n + \varphi_1 z^{n-1} + \cdots + \varphi_{n-1} z + \varphi_n$$

Sufficient conditions based on $\varphi(z)$:

1. A sufficient condition for a discrete-time dynamical system to be asymptotically stable is that

$$\varphi_0 > \varphi_1 > \varphi_2 > \ \ldots \ > \varphi_{n-1} > \varphi_n > 0$$

2. A sufficient condition for a discrete-time dynamical system to be asymptotically stable is that

$$\sum_{i=1}^{n} |\varphi_i| < |\varphi_0|$$

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Characteristic Polynomial

$$\varphi(z) = \det(zI - A) = \varphi_0 z^n + \varphi_1 z^{n-1} + \cdots + \varphi_{n-1} z + \varphi_n$$

Jury's Test:

Eliahu Ibrahim Jury (May 23, 1923 – September 20, 2020)

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Characteristic Polynomial

$$\varphi(z) = \det(zI - A) = \varphi_0 z^n + \varphi_1 z^{n-1} + \cdots + \varphi_{n-1} z + \varphi_n$$

Jury's Test:

$$\begin{array}{cccccc} \varphi_0 & \varphi_1 & \varphi_2 & \cdots & \varphi_{n-1} & \varphi_n \\ \multicolumn{6}{c}{\cdots\cdots\cdots\cdots\cdots\cdots} \\ h_0 & h_1 & \ldots & h_{\nu-1} & h_\nu & \\ l_0 & l_1 & \ldots & l_{\nu-1} & & \\ \multicolumn{6}{l}{\cdots\cdots\cdots} \end{array}$$

Build the Jury's Table

Eliahu Ibrahim Jury (May 23, 1923 – September 20, 2020)

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Characteristic Polynomial

$$\varphi(z) = \det(zI - A) = \varphi_0 z^n + \varphi_1 z^{n-1} + \cdots + \varphi_{n-1} z + \varphi_n$$

Jury's Test:

$$\varphi_0 \quad \varphi_1 \quad \varphi_2 \quad \cdots \quad \varphi_{n-1} \quad \varphi_n$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

$$h_0 \quad h_1 \quad \dots \quad h_{\nu-1} \quad h_\nu$$

$$\boxed{l_0 \quad l_1 \quad \dots \quad l_{\nu-1}} \longleftarrow$$

$$\cdots\cdots\cdots$$

Fill the generic line

$$\boxed{l_i = \frac{1}{h_1} \det \begin{bmatrix} h_1 & h_{\nu-i+1} \\ h_\nu & h_i \end{bmatrix}}$$

Eliahu Ibrahim Jury (May 23, 1923 – September 20, 2020)

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Characteristic Polynomial

$$\varphi(z) = \det(zI - A) = \varphi_0 z^n + \varphi_1 z^{n-1} + \cdots + \varphi_{n-1} z + \varphi_n$$

Jury's Test:

Check the first column

$$\begin{array}{llllll}
\boxed{\varphi_0} & \varphi_1 & \varphi_2 & \cdots & \varphi_{n-1} & \varphi_n \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\boxed{h_0} & h_1 & \ldots & h_{\nu-1} & h_\nu & \\
\boxed{l_0} & l_1 & \ldots & l_{\nu-1} & & \\
\cdots & \cdots & \cdots & & &
\end{array}$$

Eliahu Ibrahim Jury (May 23, 1923 – September 20, 2020)

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Characteristic Polynomial

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$$\begin{array}{cccccc}
\boxed{\varphi_0} & \varphi_1 & \varphi_2 & \cdots & \varphi_{n-1} & \varphi_n \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
h_0 & h_1 & \dots & h_{\nu-1} & h_\nu & \\
l_0 & l_1 & \dots & l_{\nu-1} & & \\
\cdots & \cdots & \cdots & & &
\end{array}$$

Jury's Criterion:

A LTI system is asymptotically stable if and only if the Yuri Table associated with its characteristic polynomial is well defined and the elements appearing in the first column all have the same sign

Eliahu Ibrahim Jury (May 23, 1923 – September 20, 2020)

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Example

Characteristic Polynomial $\varphi(z) = \det(zI - A) = z^3 + 2z^2 + z + 3$

Jury's Test:

$$\begin{matrix} 1 & 2 & 1 & 3 \end{matrix}$$

$$\boxed{-8}$$

Fill the second line

$$\boxed{l_1 = \frac{1}{1}\det\begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix}}$$

$$\boxed{l_0 \quad l_1 \quad \dots \quad l_{\nu-1}}$$

Fill the generic line

$$\boxed{l_i = \frac{1}{h_1}\det\begin{bmatrix} h_1 & h_{\nu-i+1} \\ h_\nu & h_i \end{bmatrix}}$$

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Discrete-time Systems: Stability of LTI Discrete-Time Systems

Example

Characteristic Polynomial $\varphi(z) = \det(zI - A) = z^3 + 2z^2 + z + 3$

Jury's Test:

There is a change of sign

$$
\begin{array}{cccc}
1 & 2 & 1 & 3 \\
-8 & & & \\
 & & & \\
l_0 & l_1 & \dots & l_{\nu-1}
\end{array}
$$

Eliahu Ibrahim Jury (May 23, 1923 – September 20, 2020)

Discrete-time Systems: Stability of LTI Discrete-Time Systems

Example

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$$\begin{array}{cccc} 1 & 2 & 1 & 3 \\ \boxed{-8} & & & \\ & & & \\ \boxed{l_0} & l_1 & \dots & l_{\nu-1} \end{array}$$

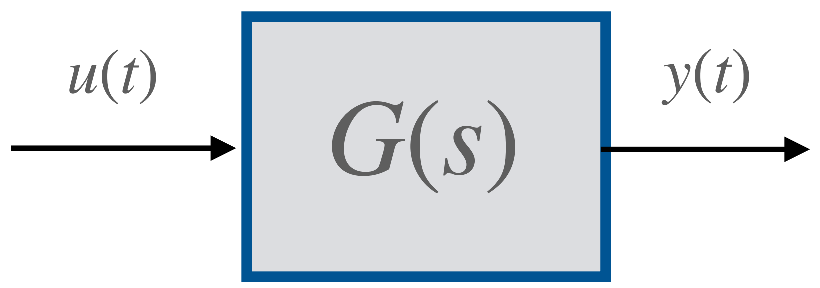
Jury's Criterion:

A LTI system is asymptotically stable if and only if the Yuri Table associated with its characteristic polynomial is well defined and the elements appearing in the first column all have the same sign

The system with characteristic polynomial $\varphi(z)$ is NOT As. Stable

Eliahu Ibrahim Jury (May 23, 1923 – September 20, 2020)

Digital Control Schemes



$u(t) \in \mathbb{R}^m$

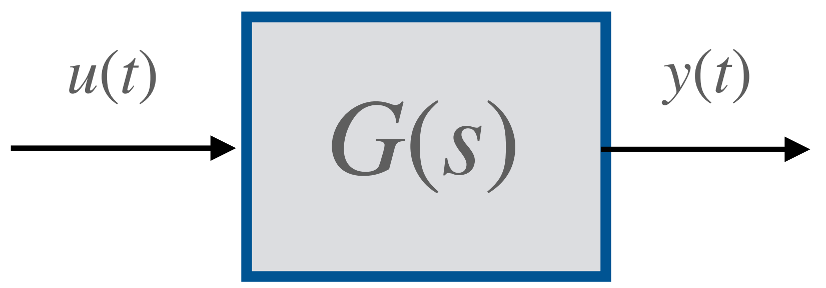
$y(t) \in \mathbb{R}^p$

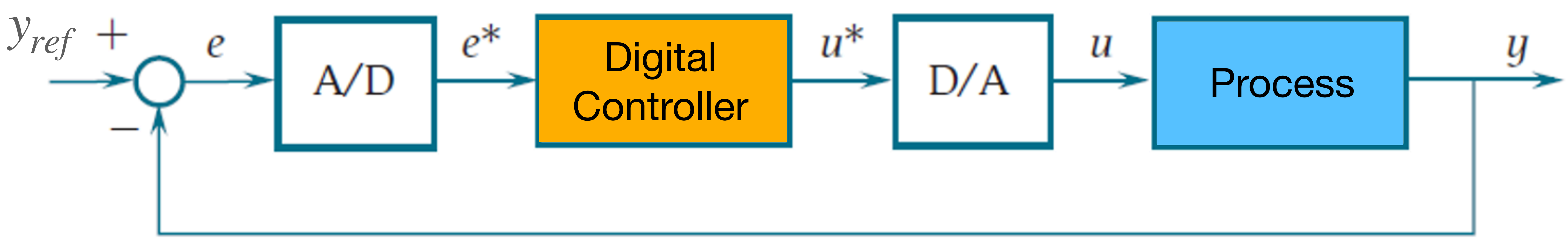
$t \in \mathbb{R}^+_{\{0\}}$

Continuous-time System

Digital Control Schemes

Digital control scheme with error sampling





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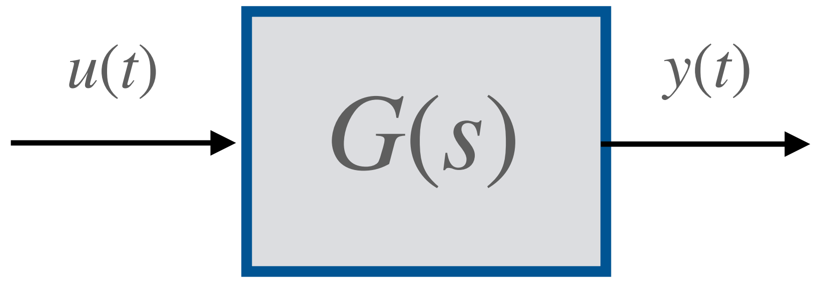
Continuous-time System

Digital Control Schemes



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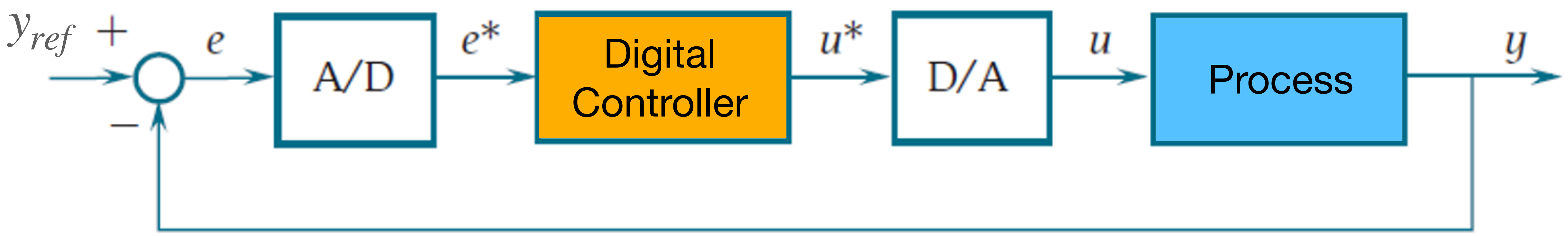
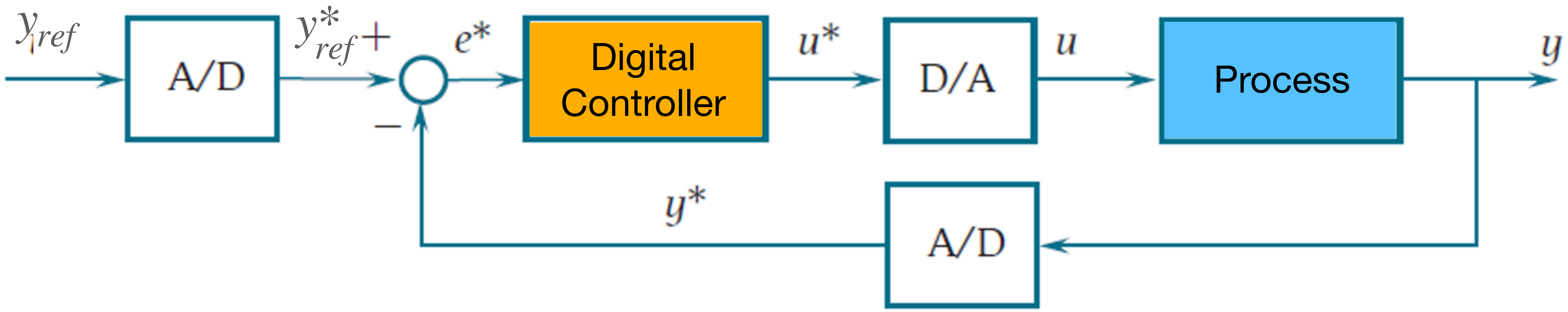
Continuous-time System

Digital control scheme with error sampling



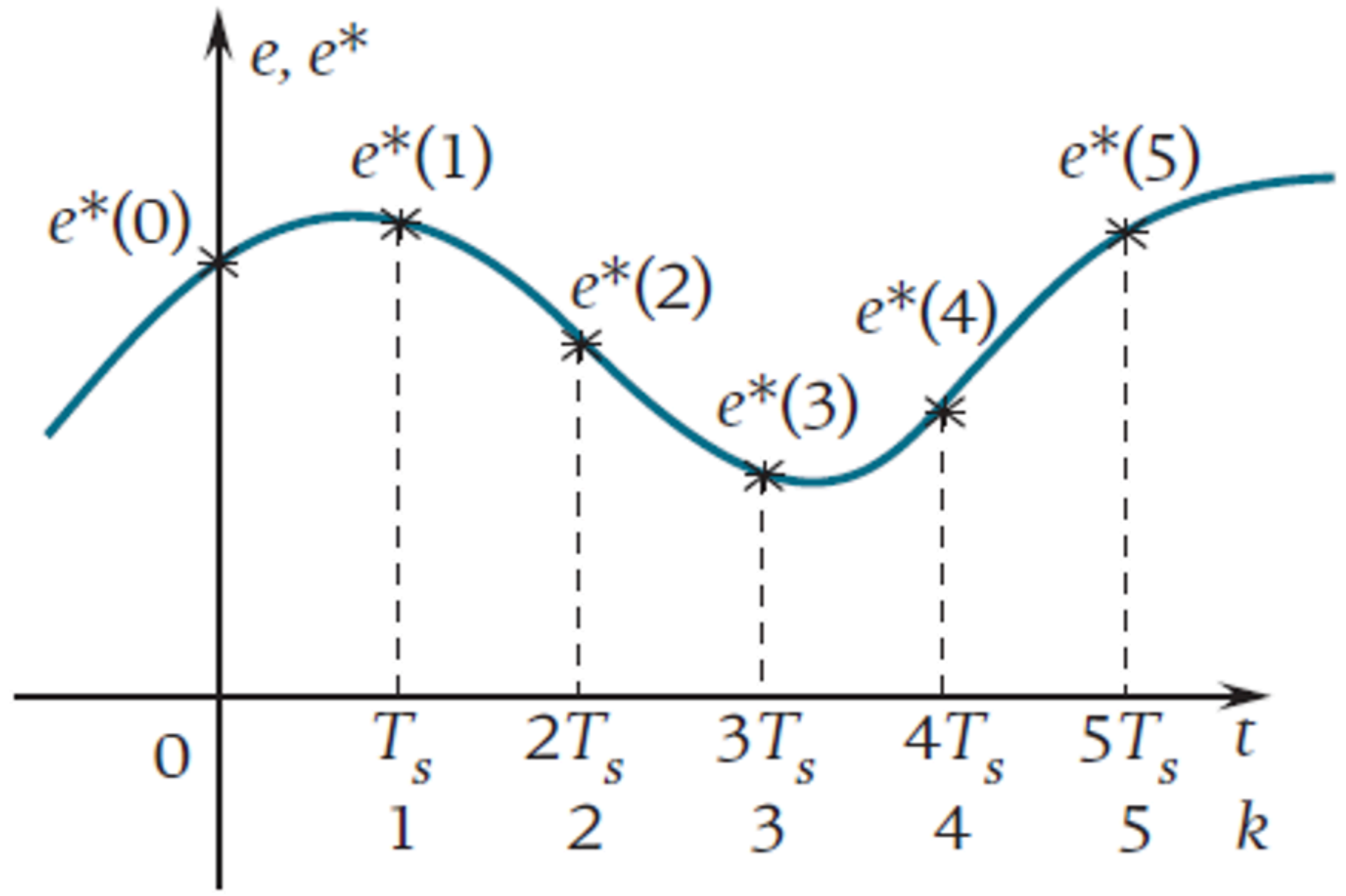
Digital control scheme with output and reference sampling



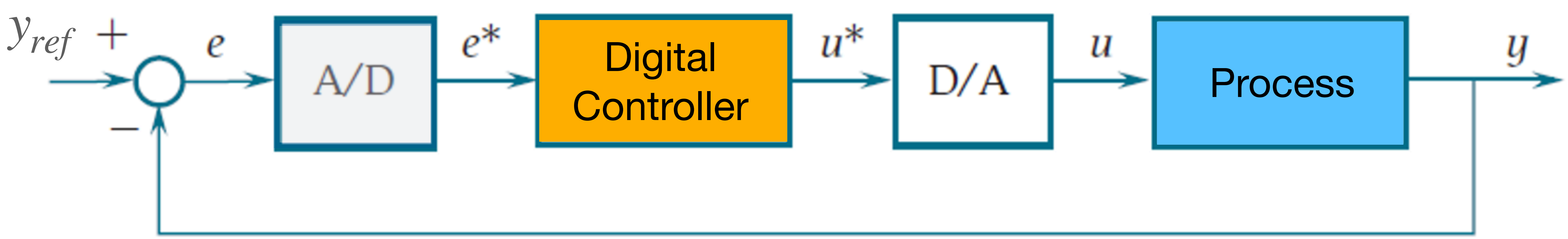
Digital Control Schemes

$\{e^*(k)\}, \ \{u^*(k)\}$ sequences

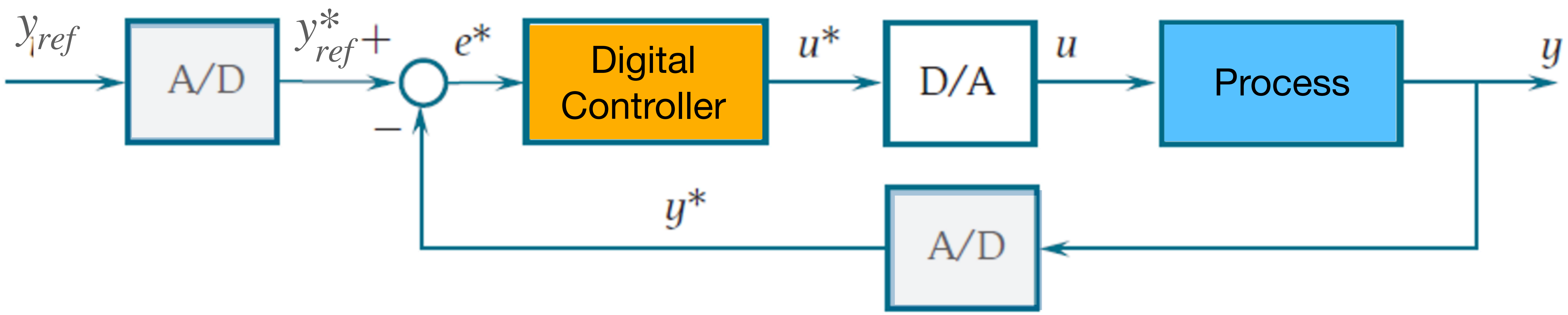
A/D converter: it is a sampler



Digital control scheme with error sampling



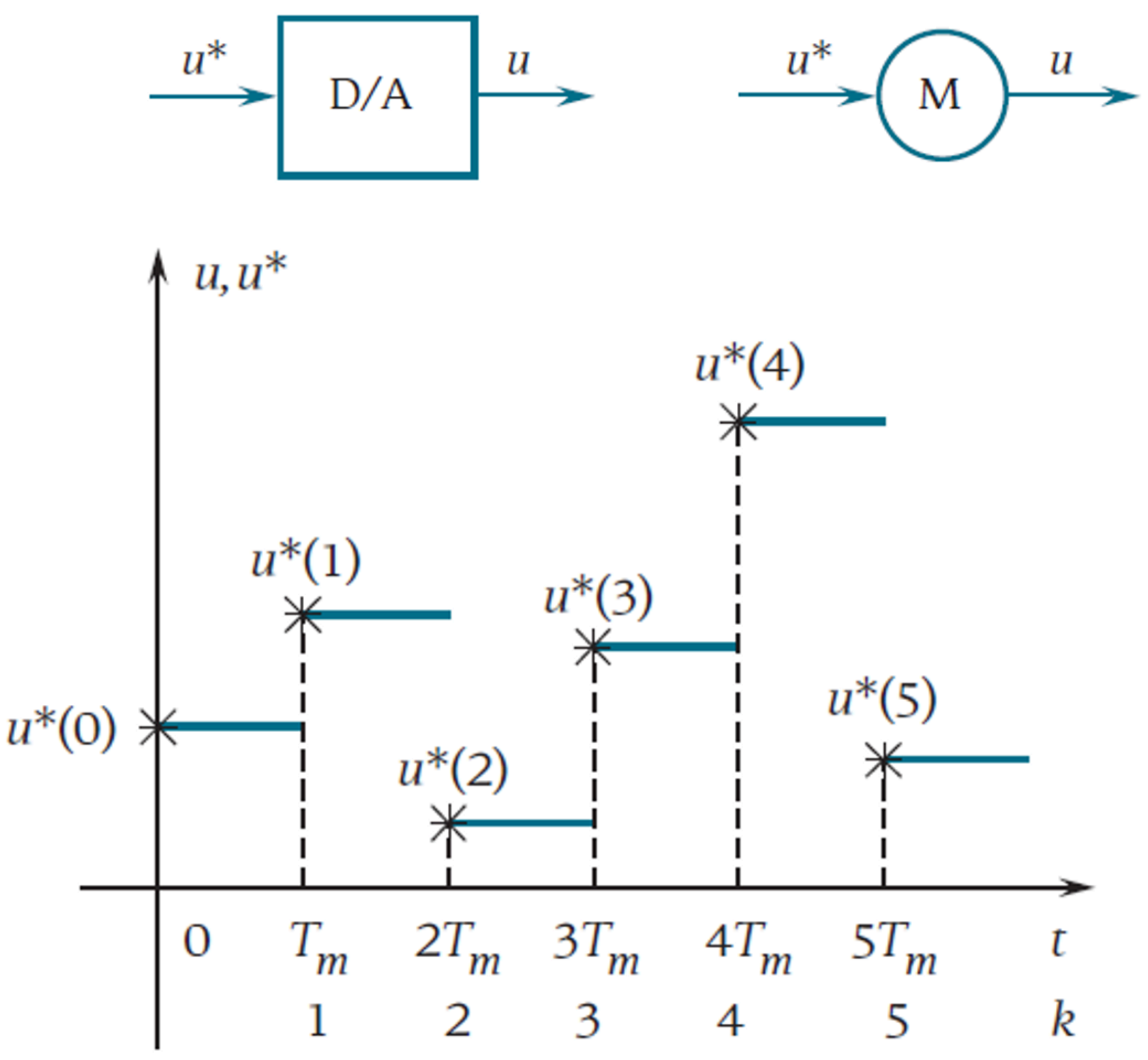
Digital control scheme with output and reference sampling



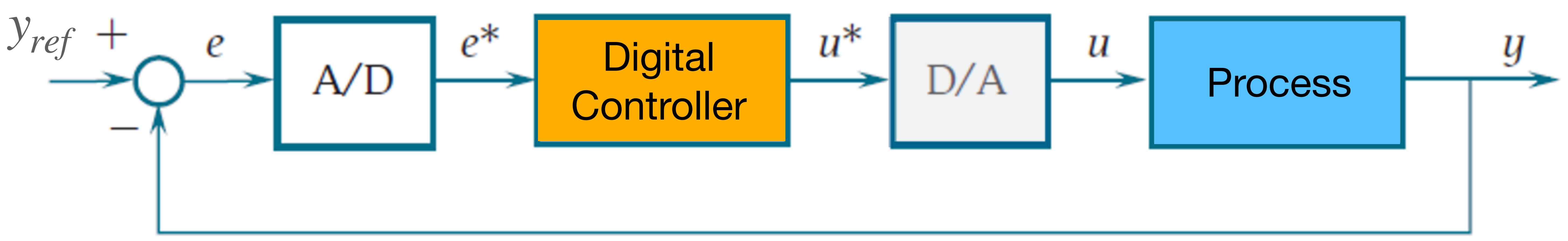
$\{y^*_{ref}(k)\}, \ \{y^*(k)\}$ sequences

Digital Control Schemes

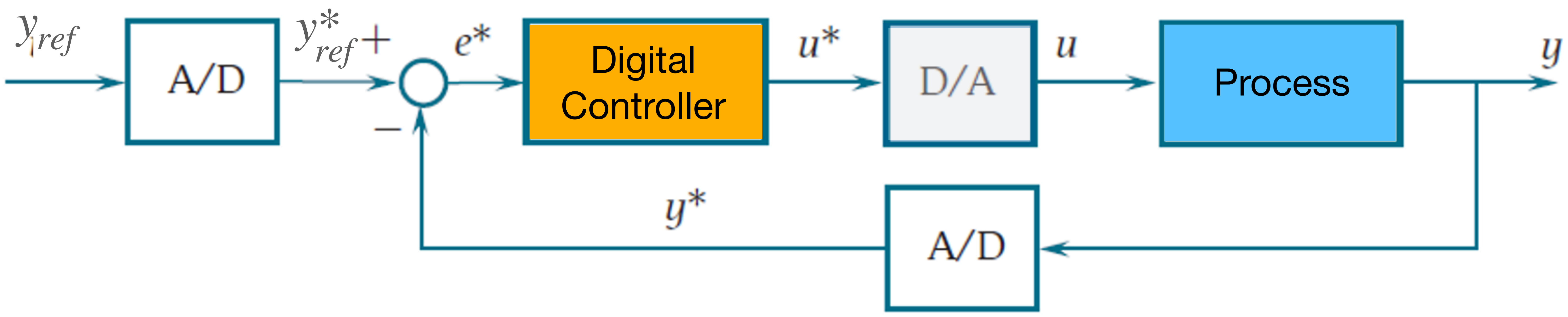
D/A converter: it is a hold



Digital control scheme with error sampling

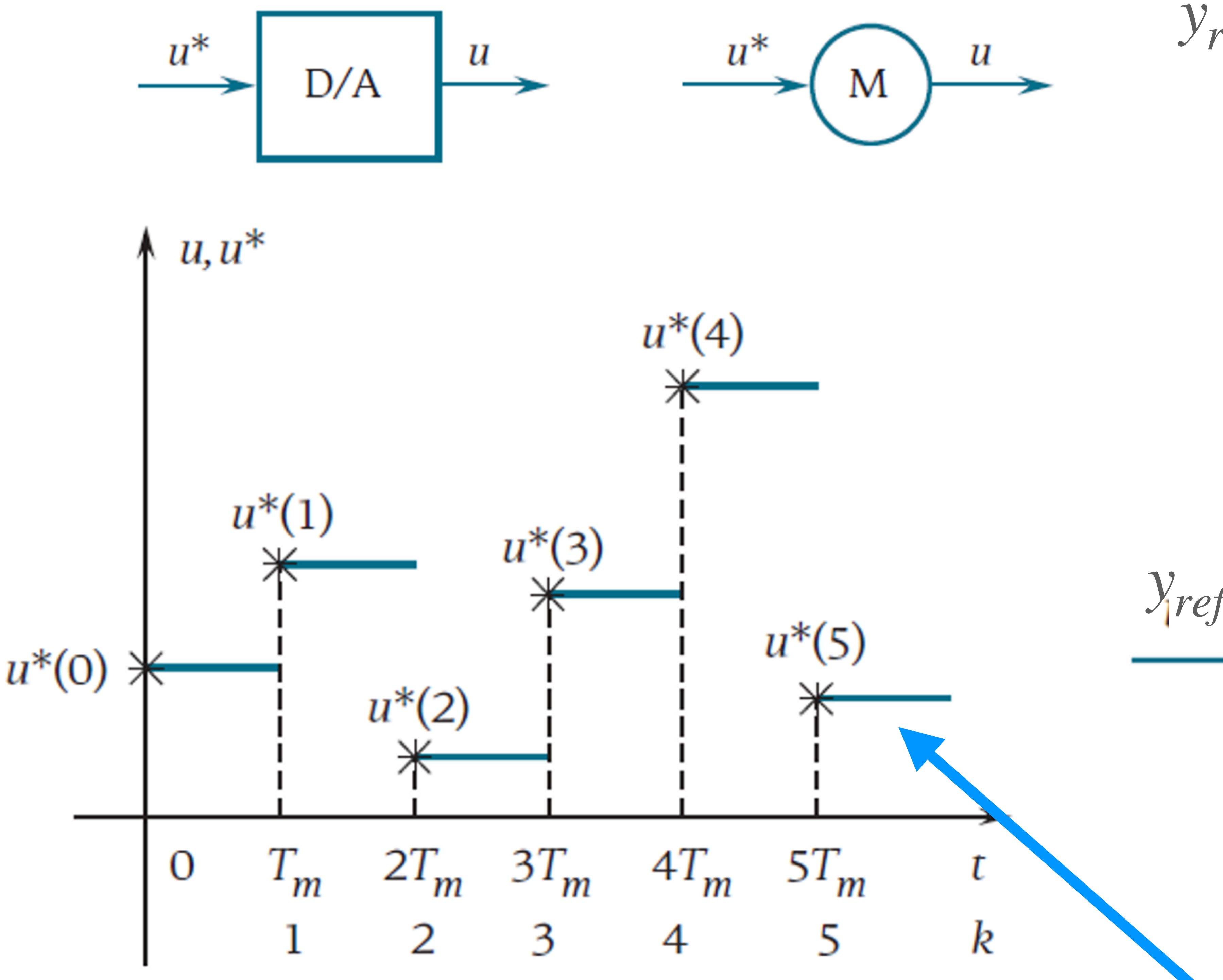


Digital control scheme with output and reference sampling

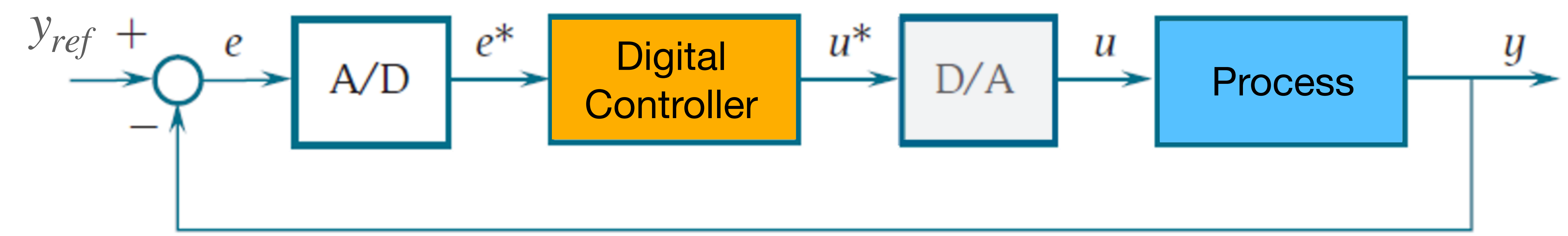


Digital Control Schemes

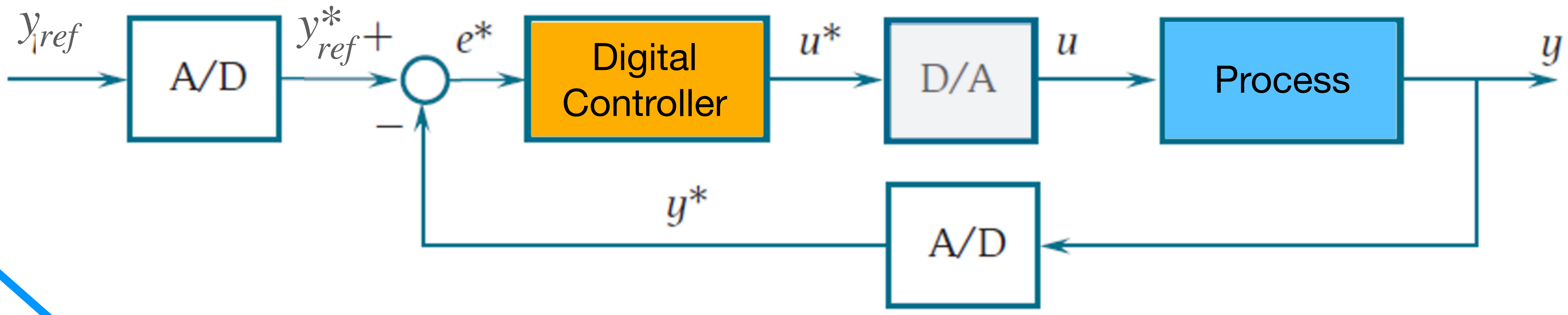
D/A converter: it is a hold



Digital control scheme with error sampling



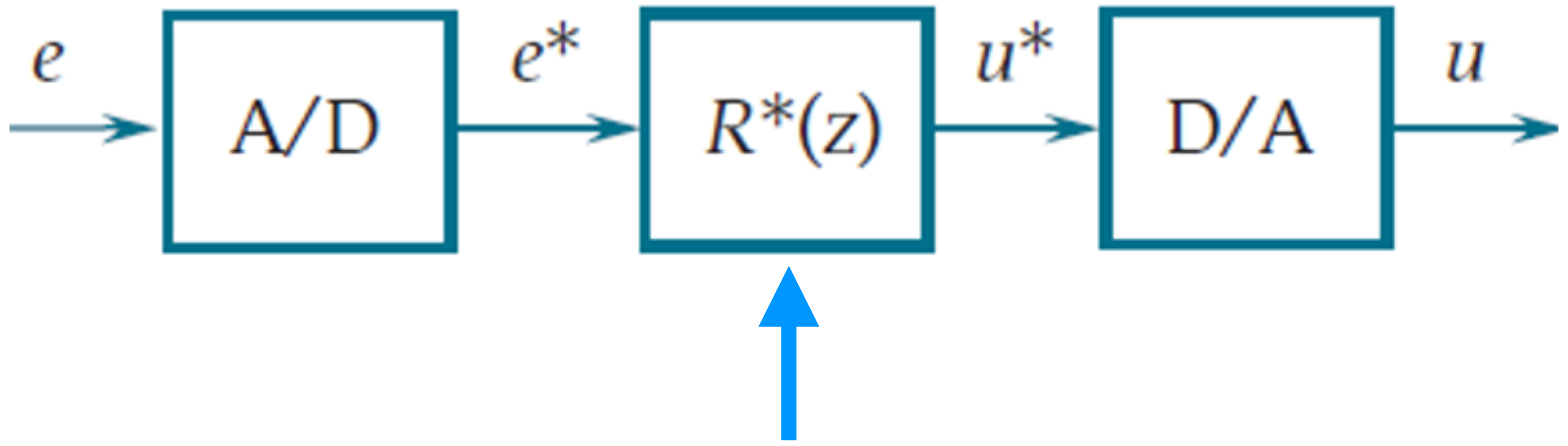
Digital control scheme with output and reference sampling



ZOH: zero-order hold

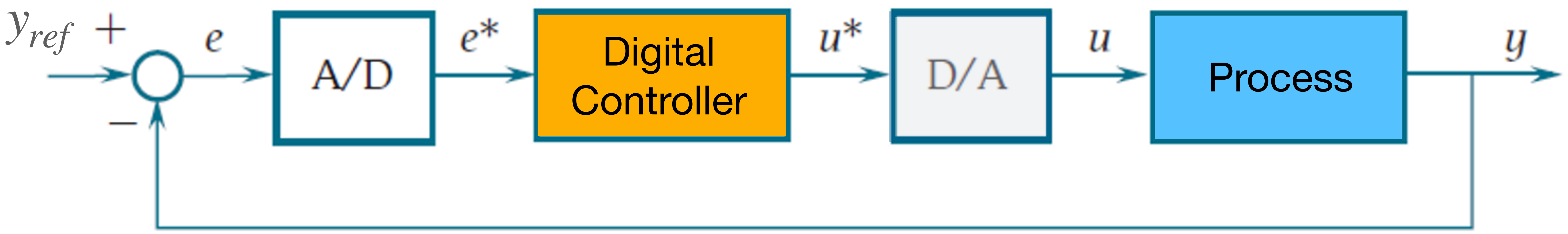
Digital Control Schemes

The control element:

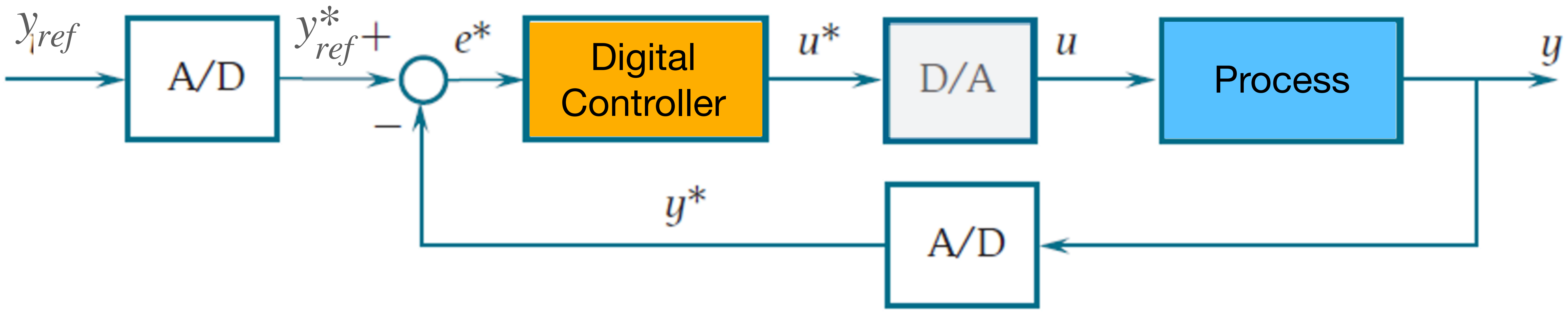


Digital Controller

Digital control scheme with error sampling

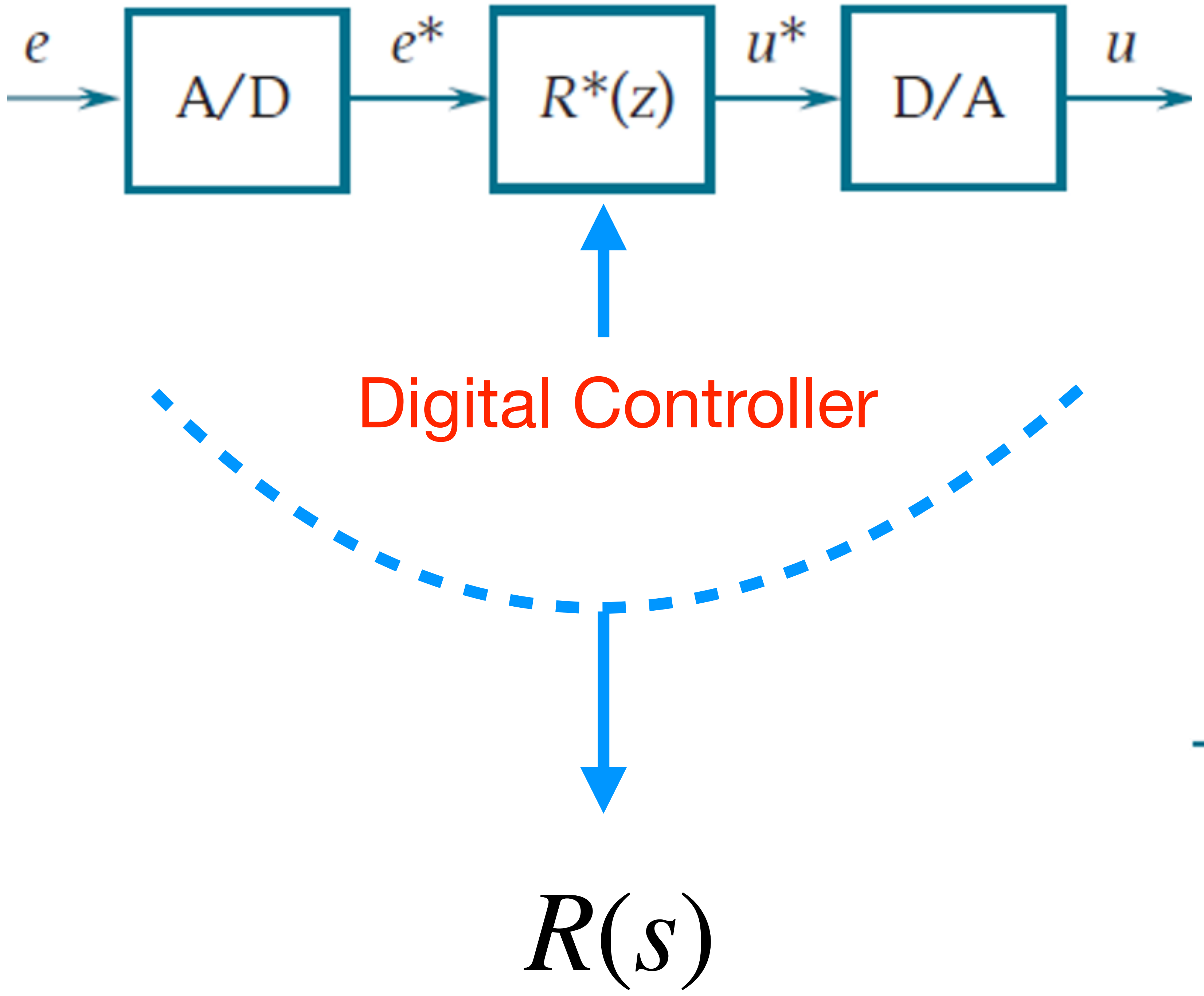


Digital control scheme with output and reference sampling

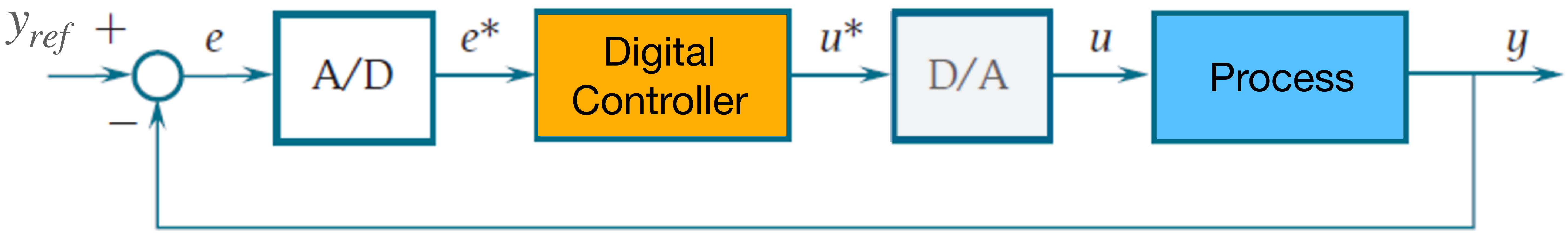


Digital Control Schemes

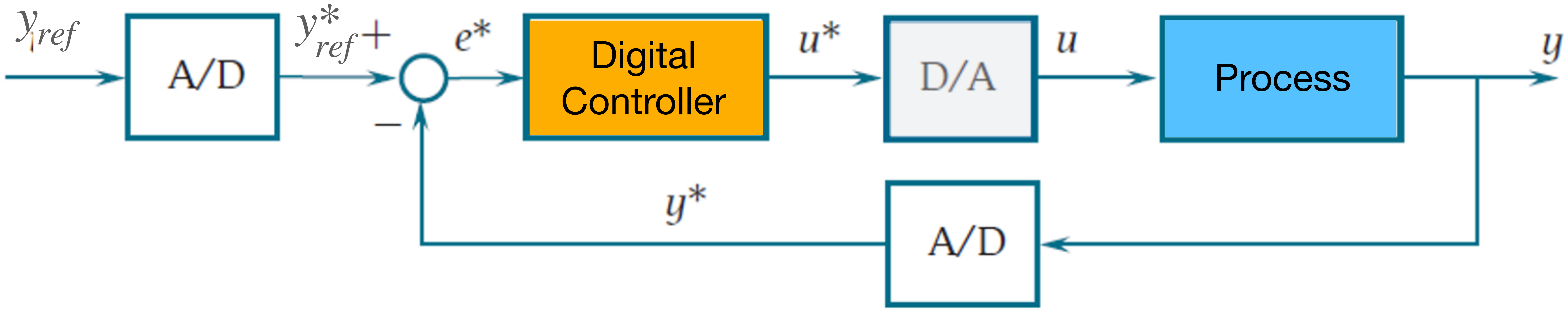
The control element:



Digital control scheme with error sampling

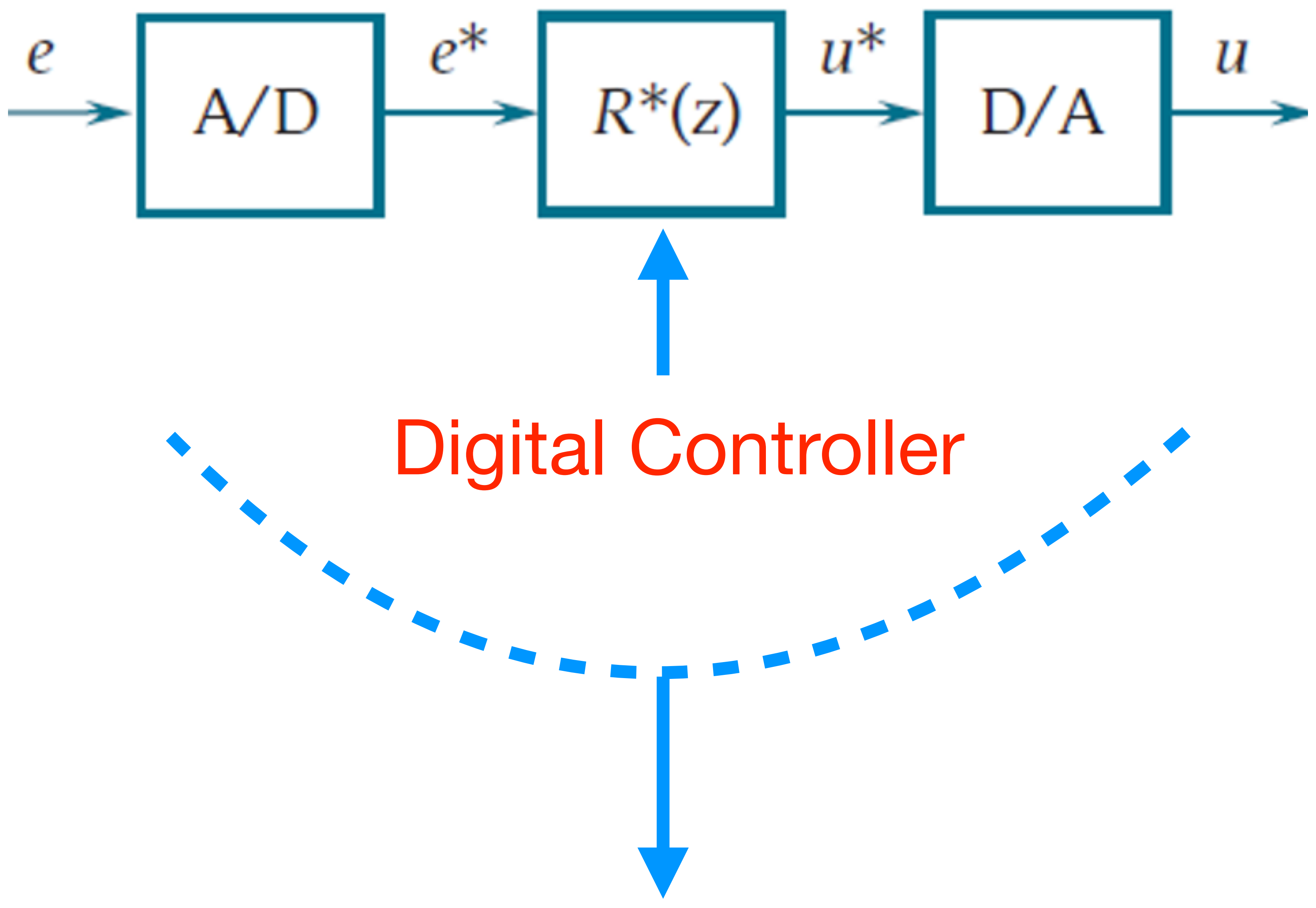


Digital control scheme with output and reference sampling



Digital Control Schemes: Problems to solve

The control element:



$R(s)$

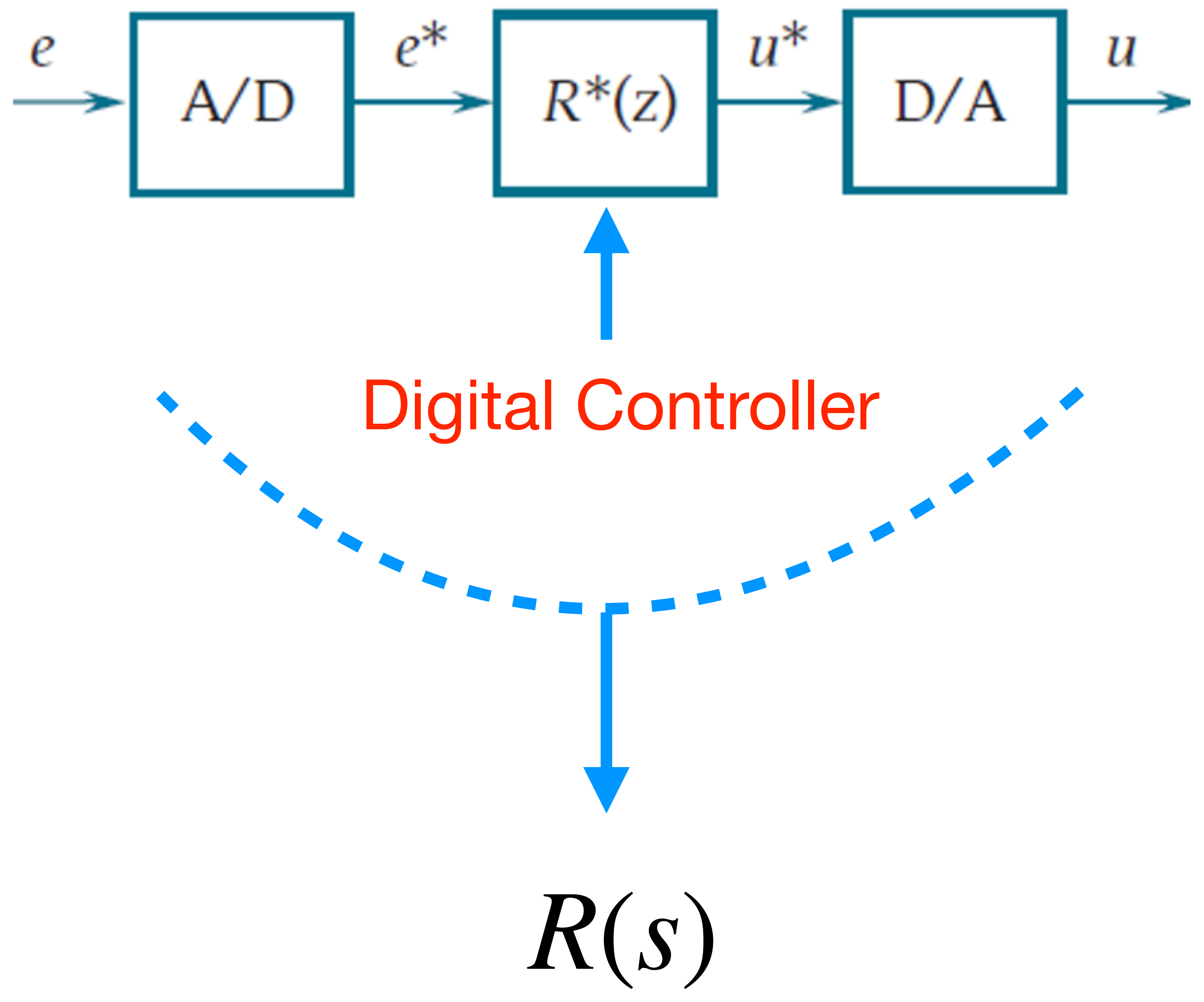
A/D converter: it is a sampler

D/A converter: it is a hold

Digital Control Schemes: Problems to solve

1. How to determine the transfer function of a discrete-time LTI system

The control element:



A/D converter: it is a sampler

D/A converter: it is a hold

Digital Control Schemes: Problems to solve

1. How to determine the transfer function of a discrete-time LTI system

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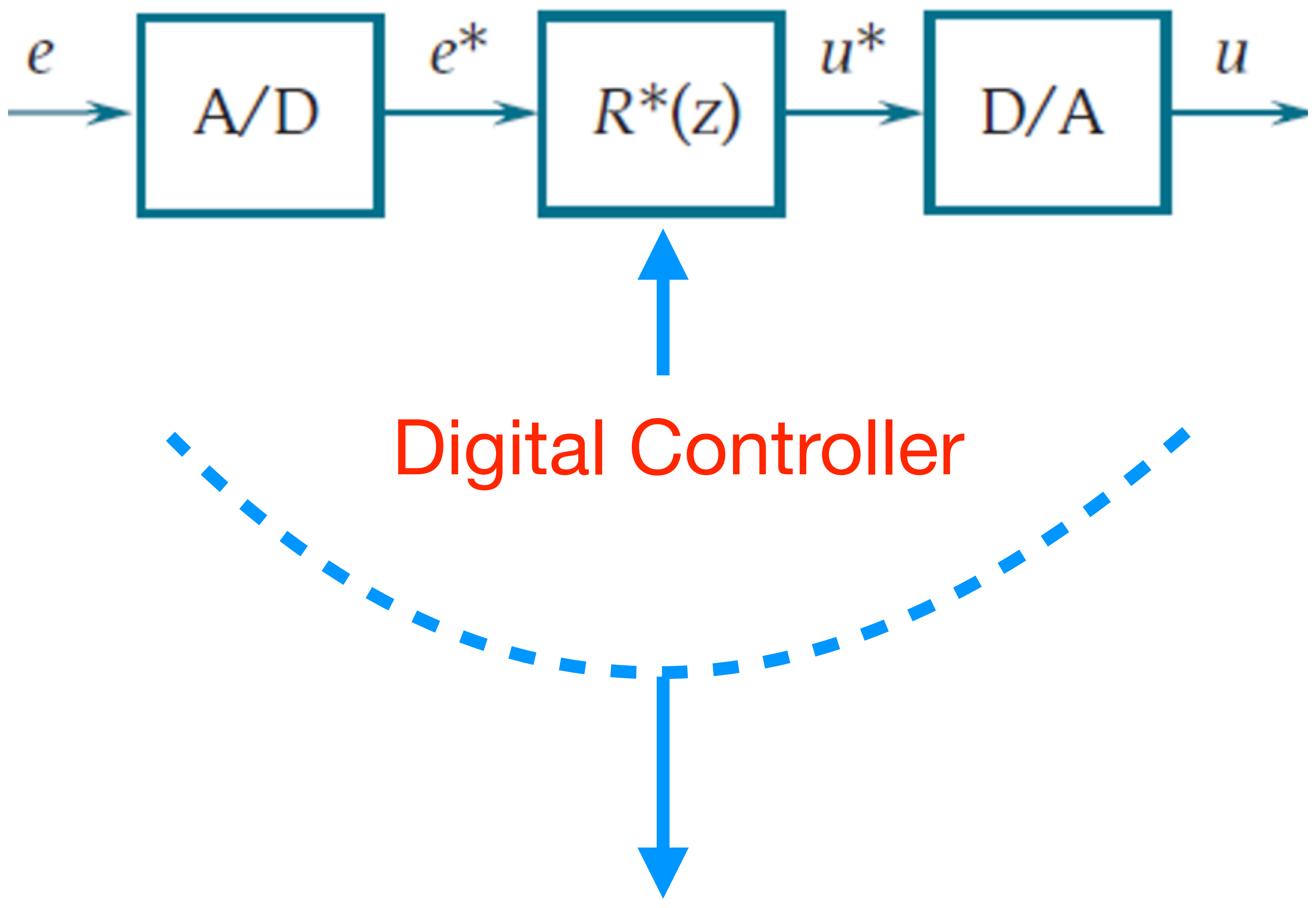


$R(s)$

A/D converter: it is a sampler

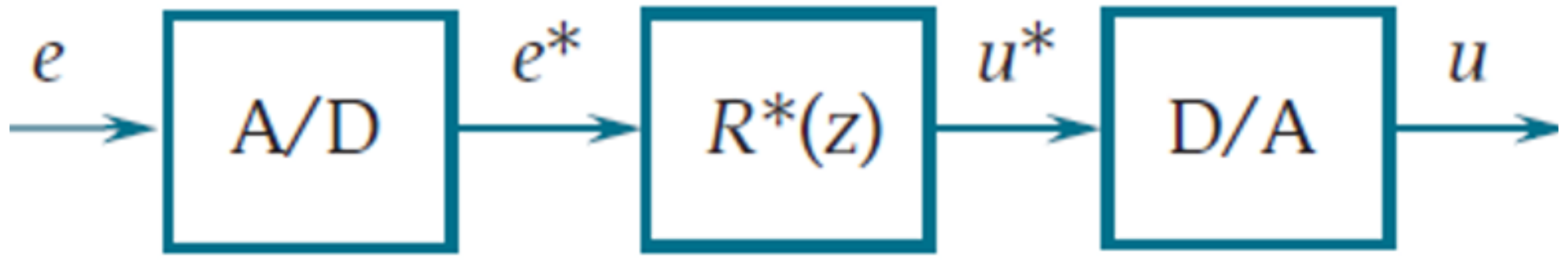
2. How to determine the sampling time T_s

D/A converter: it is a hold

Digital Control Schemes: Problems to solve

1. How to determine the transfer function of a discrete-time LTI system

The control element:



Digital Controller

$$R(s)$$

A/D converter: it is a sampler

2. How to determine the sampling time T_s

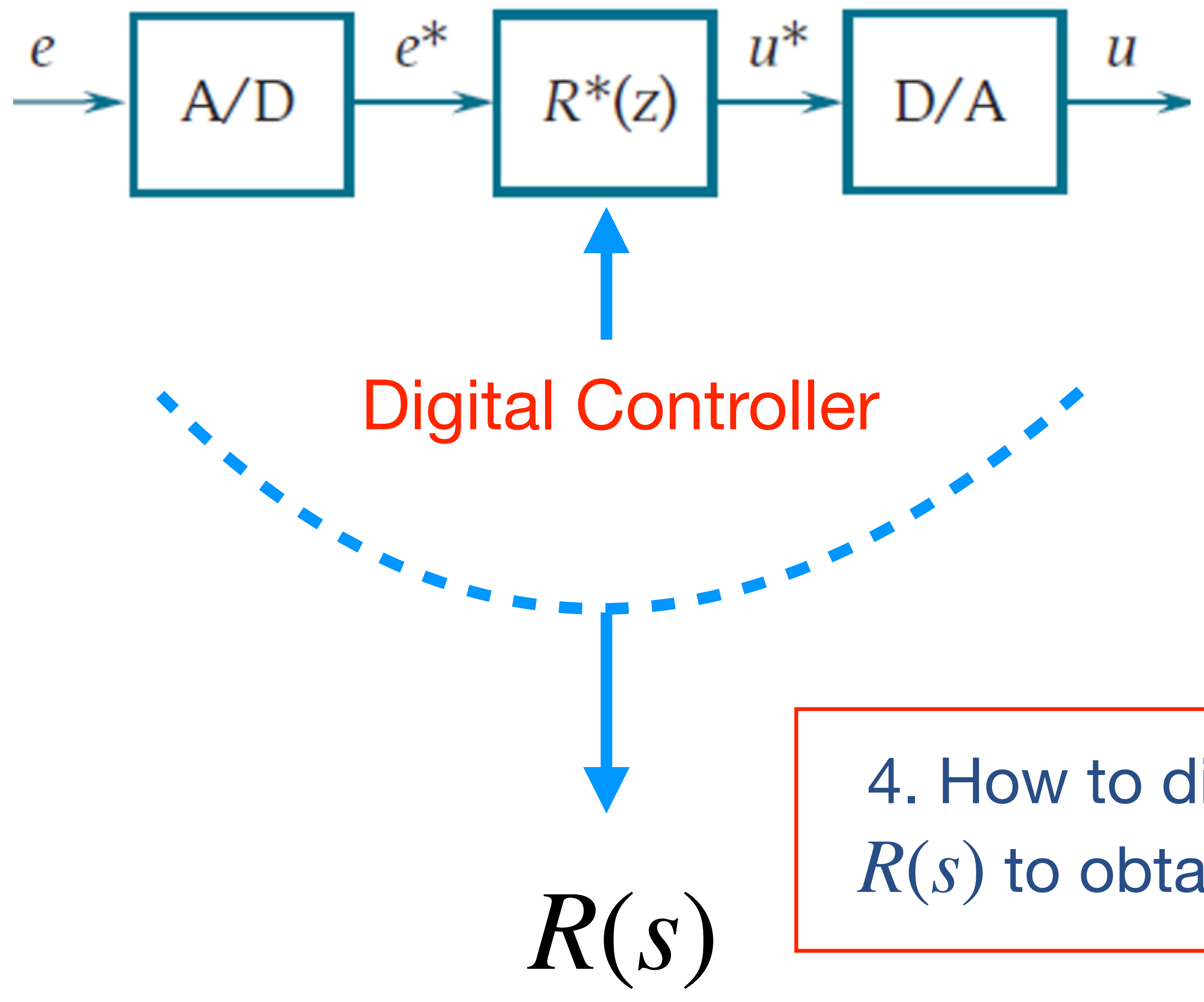
D/A converter: it is a hold

3. How to model the ZOH effect

Digital Control Schemes: Problems to solve

1. How to determine the transfer function of a discrete-time LTI system

The control element:



4. How to discretize $R(s)$ to obtain $R^*(z)$

A/D converter: it is a sampler

2. How to determine the sampling time T_s

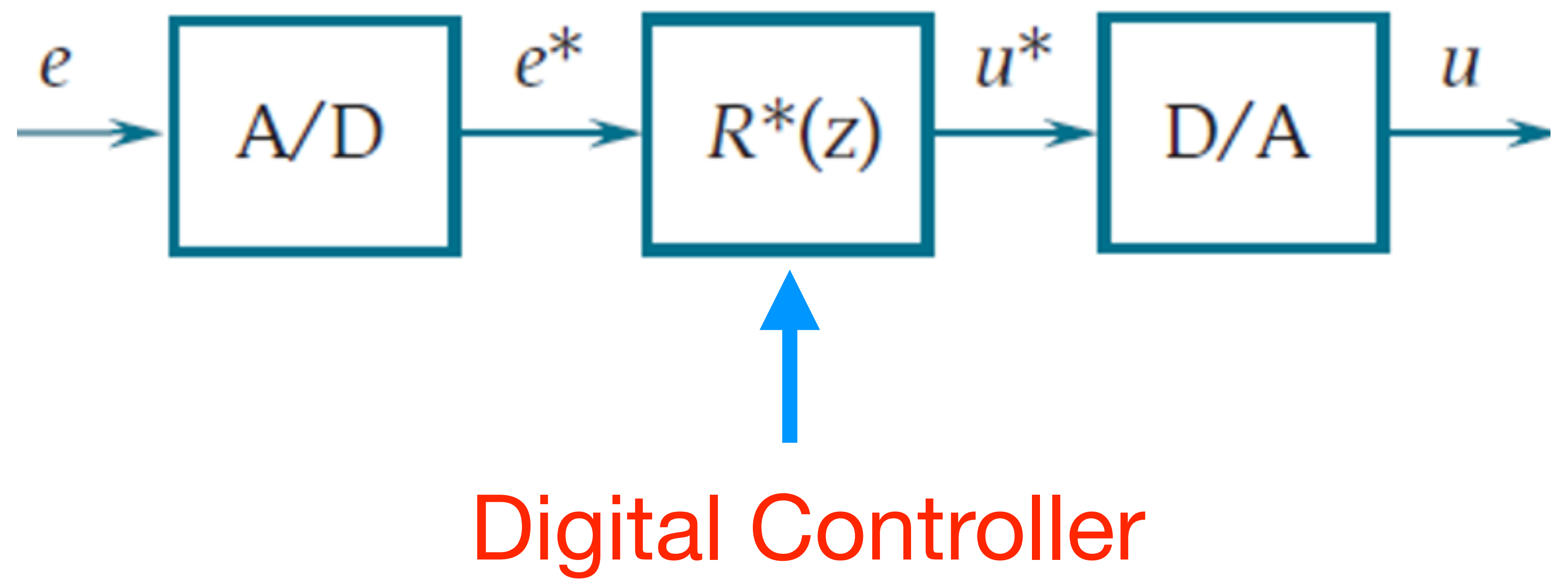
D/A converter: it is a hold

3. How to model the ZOH effect

Digital Control Schemes: Z-transform

1. How to determine the transfer function of a discrete-time LTI system

The control element:



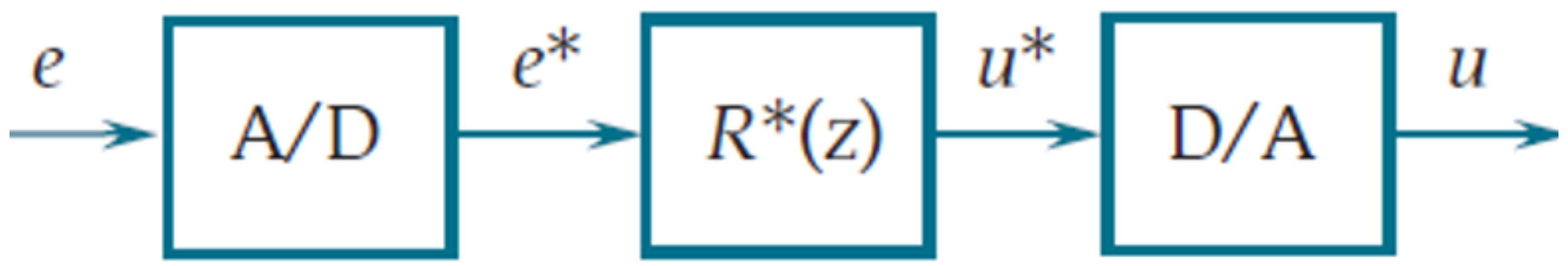
Given the discrete sequence $\{x(k)\}$, $k \geq 0$, its Z-transform is defined as:

$$X(z) = \mathscr{Z}\left[\{x(k)\}\right] = \sum_{k=0}^{\infty} x(k)\, z^{-k}, \quad \text{with} \quad z \in \mathbb{C}$$

Digital Control Schemes: Z-transform

1. How to determine the transfer function of a discrete-time LTI system

The control element:



Digital Controller

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$$X(z) = \mathscr{Z}\left[\{x(k)\}\right] = \sum_{k=0}^{\infty} x(k)\, z^{-k}, \quad \text{with} \quad z \in \mathbb{C}$$

Properties:

1) $\mathscr{Z}\left[a\,\{x(k)\} + b\,\{y(k)\}\right] = a\,X(z) + b\,Y(z)$

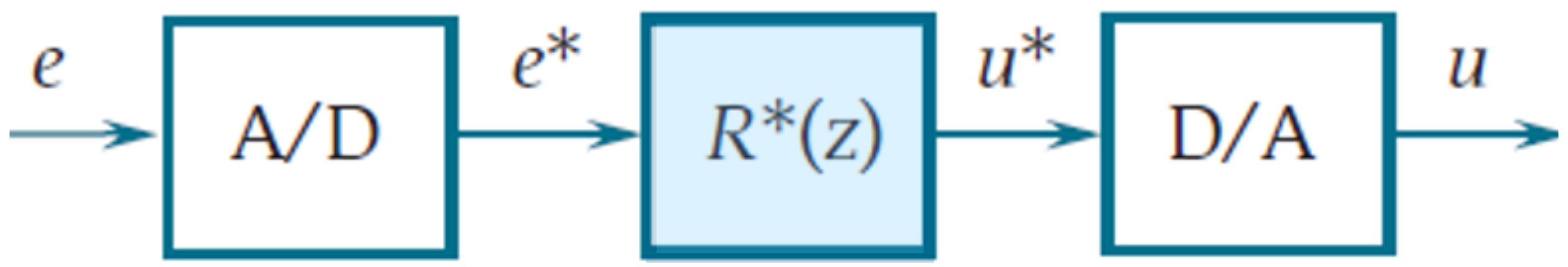
2) $\mathscr{Z}\left[\{x(k-n)\}\right] = z^{-n}X(z)$

3) $\mathscr{Z}\left[\{x(k+1)\}\right] = zX(z)$

Digital Control Schemes: Z-transform

1. How to determine the transfer function of a discrete-time LTI system

The control element:



Digital Controller

Given the discrete sequence $\{x(k)\},\ k \geq 0$,
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2) $\mathcal{Z}\left[\{x(k-n)\}\right] = z^{-n}X(z)$

3) $\mathcal{Z}\left[\{x(k+1)\}\right] = zX(z)$

Transfer Function:

$$R^*(z) = \frac{U^*(z)}{E^*(z)}$$