

- I want to take the BASIC EXAM  
 I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.
- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

**BASIC EXAM**

1. Compute the LU factorization (without pivoting) of the matrix

$$A = \begin{bmatrix} 4 & 2 & 0 \\ 3 & 1 & 2 \\ 1 & 1 & -1 \end{bmatrix}$$

showing the intermediate computations.

2. Write the pseudocode of the composite midpoint quadrature rule.

**ADVANCED EXAM**

3. Describe the Crank-Nicolson scheme for the solution of an ordinary differential equation and explain its relation with the trapezoidal quadrature rule. Then, compute one step of the Crank-Nicolson scheme for the problem

$$\begin{cases} y'(t) = e^t y(t) \\ y(0) = 1 \end{cases}$$

selecting  $\Delta t = 1$ .

4. State and prove the theorem on the existence and uniqueness of the Lagrange interpolant of a given function.