

- I want to take the BASIC EXAM
 I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.
- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

BASIC EXAM

1. Given the Cauchy problem:

$$\begin{cases} y'(t) = -2y(t) + t^2 & \text{for } t > 0 \\ y(0) = 2; \end{cases}$$

compute two steps by the implicit Euler method, with $\Delta t = 1/2$, in order to approximate $y(1)$. Report the intermediate computations.

2. Introduce the power method for the computation of the dominant eigenvalue and eigenvector, with pseudocode. State conditions that guarantee its convergence

ADVANCED EXAM

3. Describe the Crank-Nicolson scheme for the solution of an ODE and explain its relation with the trapezoidal quadrature rule. Then, compute one step of the Crank-Nicolson scheme for the problem

$$\begin{cases} y'(t) = (t+1)y(t) \\ y(0) = 1 \end{cases}$$

selecting $\Delta t = 2$.

4. State and prove the theorem on the existence and uniqueness of the Lagrange interpolant of a given function.