

- I want to take the BASIC EXAM  
 I want to take the ADVANCED EXAM

Exam rules:

- Basic exam: the maximum grade is 24/30.
- Advanced exam: the maximum grade is 30/30 cum laude.

Total time is 1 hour. Students who get a positive grade in the written part (i.e., at least 18/30) *might* choose to take an oral exam. For students who choose the basic written exam, the maximum grade obtainable can never exceed 24/30.

**BASIC EXAM**

1. Compute the linear regression  $r(x) = c_0 + c_1x$  for the set of points

$$(-3, 0), (-2, 0), (-1, 3), (1, 1), (2, 1), (3, 2).$$

2. Describe the Crank-Nicolson scheme for the solution of an ODE and explain its relation with the trapezoidal quadrature rule. Report its pseudocode. Finally, compute one step of the Crank-Nicolson scheme for the problem

$$\begin{cases} y'(t) = e^{-t} y(t) \\ y(0) = 1 \end{cases}$$

with step-size  $\Delta t = 1$ .

**ADVANCED EXAM**

3. Describe the LU-based algorithm that computes the inverse of a matrix  $A$ . Give its pseudocode that should include the LU factorization part (it could be with or without pivoting, your choice) and give an estimate of its computational cost
4. Write the pseudocode of the bisection method. Apply two bisection iterations to the equation

$$x^3 + 1 = 0 \quad \text{in } [-2, 2].$$