

MATH3322 Matrix Computation

Homework 3

Due date: 22 March, Monday

1. Let

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 1 & -3 \\ -1 & 1 \end{bmatrix}$$

Find the economic QR decomposition of \mathbf{A} .

2. Find the orthogonal projection of the vector

$$\mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$$

onto the subspace spanned by the two vectors

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad \mathbf{a}_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

3. Implement and compare the Gram-Schmidt procedure and the modified Gram-Schmidt for QR decomposition. A template file `gs_qr.m` is provided. Plot the curve of accuracy against test problems. Which algorithm is more accurate?
4. The QR decomposition is also of theoretical interest. Use the QR decomposition to prove that

$$|\det(\mathbf{A})| \leq \|\mathbf{a}_1\|_2 \cdot \|\mathbf{a}_2\|_2 \cdots \|\mathbf{a}_n\|_2,$$

where $\mathbf{A} \in \mathbb{R}^{n \times n}$ is a full rank square matrix with \mathbf{a}_i the i -th column, and $\det(\mathbf{A})$ denotes the determinant of \mathbf{A} . (*Hint: $|\det(\mathbf{H})| = 1$ if $\mathbf{H} \in \mathbb{R}^{n \times n}$ is orthogonal, and $\det(\mathbf{BC}) = \det(\mathbf{B})\det(\mathbf{C})$ for any $\mathbf{B}, \mathbf{C} \in \mathbb{R}^{n \times n}$.*)