

TAREA 1

1. Grafique las siguientes funciones

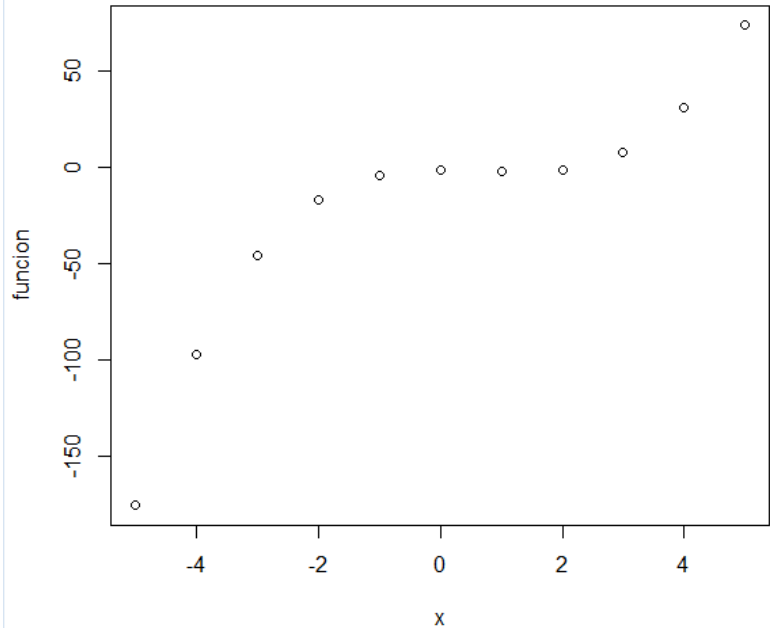
(a) $f(x) = x^3 - 2x^2 - 1$ en el intervalo $[-5, 5]$

Resolución

$$\begin{aligned} & (-5:5)^3 - 2(-5:5)^2 - 1 \\ & = -176 \ -97 \ -46 \ -17 \ -4 \ -1 \ -2 \ -1 \ 8 \\ & \quad 31 \ 74 \end{aligned}$$

Script

```
> x <- c(-5:5)
> funcion <- ((x^3) -
(2*(x^2)) -1)
> Plot(x, funcion)
```



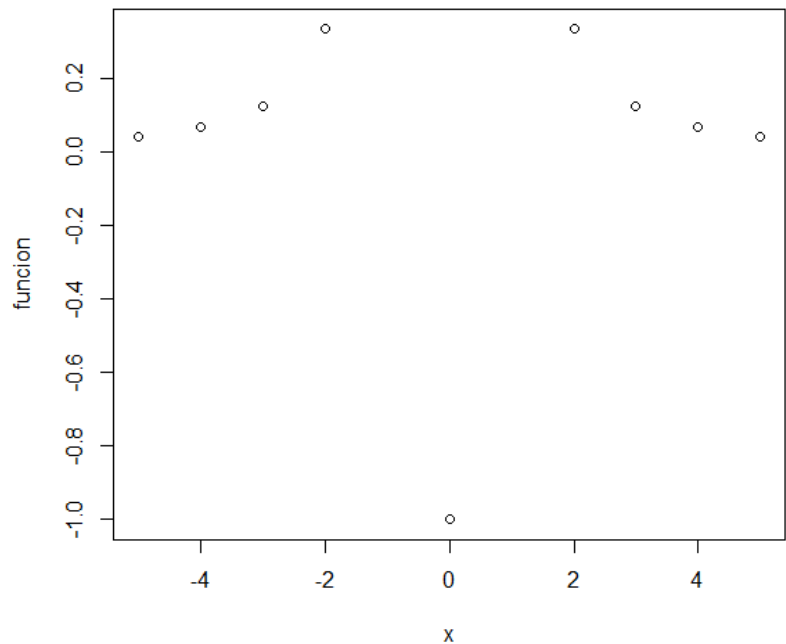
(b) $g(x) = \frac{1}{(x-1)(1+x)}$ en el intervalo $[-5, 5]$

Resolución

$$\begin{aligned} & \frac{1}{((-5:5)-1)(1+(-5:5))} = \\ & 0.04166667, 0.06666667, 0.12500000, \\ & 0.33333333, -\text{Inf}, -1.00000000, \text{Inf}, \\ & 0.33333333, 0.12500000, 0.06666667, \\ & 0.04166667 \end{aligned}$$

Script

```
> x <- c(-5:5)
> funcion <- (1/((x-1)*(1+x)))
> plot(x,funcion)
```



2. Calcule los siguientes límites

(a) $\lim_{x \rightarrow 0} \frac{(e^x - 1)}{e^x}$

Resolución

$$\frac{(e^{(0)} - 1)}{e^{(0)}} = 0$$

Script

```
> library(Ryacas)
> x<- Sym("x")
> Limit((((exp(x))-1)/(exp(x))),x,0)
expression(0)
```

(b) $\lim_{x \rightarrow \infty} \frac{x}{x+1}$

Resolución

$$\frac{(\infty)}{(\infty) + 1}$$

Script

```
> library(Ryacas)
> x<- Sym("x")
> Limit(((x)/(x+1)), x, Inf)
expression(Inf/(Inf + 1))
```

3. Encontrar las soluciones de las siguientes ecuaciones

(a) $x^2 - 2x + 1 = 0$

Resolución

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(1)}}{2(1)} \quad x = \frac{2 \pm \sqrt{(4) - 4}}{2} \quad x = \frac{2}{2} \quad x = 1$$

Script

```
> a <- -1
> b <- (-2)
> c <- 1
> x1 <- (((-b)+(sqrt((b^2)-(4*a*c))))/(2*1))
> x2 <- (((-b)-(sqrt((b^2)-(4*a*c))))/(2*1))
> x1
[1] 1
> x2
[1] 1
```

(b) $x^3 - x = 0$

Resolución

$$ax^3 + bx^2 + cx + d = 0$$

$$\begin{aligned} x^3 - x &= 0 \\ x &= 0 \end{aligned}$$

$$\begin{aligned} x(x^2 - 1) &= 0 \\ x &= 1 \end{aligned}$$

$$\begin{aligned} x(x+1)(x-1) &= 0 \\ x &= -1 \end{aligned}$$

Script

```
> a <- 1
> b <- 0
> c <- -1
> x1 <- (((-b)+(sqrt((b^2)-(4*a*c))))/(2*1))
> x2 <- (((-b)-(sqrt((b^2)-(4*a*c))))/(2*1))
> x1
[1] 1
> x2
[1] -1
X3 = 0
```

4. Encontrar las soluciones de los siguientes sistemas de ecuaciones

(a) $x - 2y = 0$

$-3x + 2y = -1$

Resolución

$$\begin{bmatrix} 1 & -2 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad \text{fila2} - (\text{fila1} * (-3)) \quad \begin{bmatrix} 1 & -2 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \quad \text{fila2} / -4 \quad \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{4} \end{bmatrix}$$

$$\text{fila1} - (\text{fila2} * (-2)) \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$$

$$x = \frac{1}{2}$$

$$y = \frac{1}{4}$$

Script

```
> Datos1 <- matrix(c(1,-2,-3,2), ncol=2, by=T)
> Datos2 <- c(0,-1)
> solve(Datos1,Datos2)
[1] 0.50 0.25
```

(a) $2x + 3y = 0$

$3x - 2y = 1$

Resolución

$$\begin{bmatrix} 2 & 3 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{fila1}/2 \quad \begin{bmatrix} 1 & \frac{3}{2} \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{fila2} - (\text{fila1} * 3) \quad \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & \frac{-13}{2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$\text{fila2}/(-13/2) \quad \begin{bmatrix} 1 & \frac{3}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{fila1} - (\text{fila2} * (3/2)) \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{13} \\ \frac{-2}{13} \end{bmatrix}$$
$$x = \frac{3}{13}$$
$$y = \frac{-2}{13}$$

Script

```
> Datos1 <- matrix(c(2,3,3,-2), ncol=2, by=T)
> Datos2 <- c(0,1)
> solve(Datos1, Datos2)
[1] 0.2307692 -0.1538462
```

5. Sea X la variable aleatoria que representa la suma del resultado al lanzar dos dados. Encontrar las siguientes probabilidades.

(a) $P(X = 3) =$

Script

```
> x <- c(1:6)
> xx <- c(x[1]+x[1:6], x[2]+x[1:6], x[3]+x[1:6], x[4]+x[1:6], x[5]+x[1:6], x[6]+x[1:6])
> xx
[1] 2 3 4 5 6 7 3 4 5 6 7 8 4 5 6 7 8 9 5 6 7 8 9 10 6
[26] 7 8 9 10 11 7 8 9 10 11 12
> sum(xx==3)/length(xx)
[1] 0.05555556
```

(b) $P(X = 15) =$

Script

```
> sum(xx==15)/length(xx)
[1] 0
```

(c) $P(X = 4 \text{ ó } 6) =$

Script

```
> sum(xx==4 | xx==6)/length(xx)
[1] 0.2222222
```

(d) $P(X \leq 4) =$

Script

```
> sum(xx<=4)/length(xx)  
[1] 0.1666667
```

(e) $P(X > 4) =$

Script

```
> sum(xx>4)/length(xx)  
[1] 0.8333333
```

TAREA 2

1. Escriba las instrucciones en R que calcule lo siguiente:

(a) $\sum_{i=1}^{500} (2i - 1) =$

Resolución

$$(2(1,2,3,\dots,500)) - 1 = 1 + 3 + 5 + \dots + 999 = 250000$$

Script

```
> i <- c(1:500)
> sum((2*i)-1)
[1] 250000
```

(b) $\sum_{i=1}^{500} (-1)^{(i+1)} (2i - 1) =$

Resolución

$$(-1)^{((1,2,3,\dots,500)+1)} (2(1,2,3,\dots,500)) - 1 = (1, -3, 5 \dots 999) = -500$$

Script

```
> i <- c(1:500)
> sum(((1)^{(i+1)})*((2*i)-1))
[1] -500
```

(c) $\sum_{i=1}^{500} (i)^2 =$

Resolución

$$(1,2,3 \dots 500)^2 = 1^2 + 2^2 + 3^2 + \dots + 1000^2 = 41791750$$

Script

```
> i <- c(1:500)
> sum(i^2)
[1] 41791750
```

2. Dados los vectores $x = c(3, 5, 6, 4, 2, 7, 8, 9)$ y $y = c(4, 3, 2, 5, 7, 4, 3, 8)$ escribir las instrucciones en R para que calcule lo siguiente:

(a) $\frac{\bar{x} = \sum_i^8 X_i}{n} = \frac{x1+x2+x3+x4+x5+x6+x7+x8}{n}$ $\frac{\bar{y} = \sum_i^8 y_i}{n} = \frac{y1+y2+y3+y4+y5+y6+y7+y8}{n}$

Resolución

$$\frac{\bar{x} = \sum_i^8 X_i}{n} = \frac{3 + 5 + 6 + 4 + 2 + 7 + 8 + 9}{8} = 5.5$$

$$\frac{\bar{y} = \sum_i^8 y_i}{n} = \frac{4 + 3 + 2 + 5 + 7 + 4 + 3 + 8}{8} = 4.5$$

Script

```
> x <- c(3, 5, 6, 4, 2, 7, 8, 9)
> sum(x)/(length(x))
[1] 5.5
```

```
> mean(x)
[1] 5.5
```

```
> y <- c(4,3,2,5,7,4,3,8)
> sum(y)/(length(y))
[1] 4.5
```

```
> mean(y)
[1] 4.5
```

(b) $va = \frac{\sum_{i=1}^8 (x_i - \bar{x})^2}{n-1}$

Script

```
> x <- c(3, 5, 6, 4, 2, 7, 8, 9)
> mx <- sum(x)/(length(x))
> sum(((x - mx)^2))/((length(x))-1)
[1] 6
```

```
> var(x)
[1] 6
```

(c)

$$ss = \frac{\sum_{i=1}^8 (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^8 (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^8 (y_i - \bar{y})^2}}$$

Script

```
> x <- c(3, 5, 6, 4, 2, 7, 8, 9)
> mx <- sum(x)/(length(x))
> my <- sum(y)/length(y)
> ss <- sum((x-mx)*(y-my))/((sqrt(sum((x-mx)^2)))*(sqrt(sum((y-my)^2))))
> ss
[1] -0.02817181
```