Ecuaciones de Lotka-Volterra para dos especies con retardo

Versión 1

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$$
\begin{aligned}
\dot{x}_1(t) &= x_1(t-20) - x_1(t-20)x_2(t-20) - 0.4x_1(t-20)u_1(t-20) \\
\dot{x}_2(t) &= -x_2(t-20) + x_1(t-20)x_2(t-20) - 0.2x_2(t-20)u_2(t-20), \quad (1)
\end{aligned}
$$

con las condiciones:

$$
\begin{aligned}
x_1(t) &= x_2(t) = 1, \quad -20 \leq t \leq 0 \qquad (2) \\
u_1(t) &= u_2(t) = 0, \quad -20 \leq t \leq 0 \qquad (3)
\end{aligned}
$$

Denotando:

$$
\begin{aligned}
y_1(t) &= x_1(t-20), \\
y_2(t) &= x_2(t-20), \\
v_1(t) &= u_1(t-20), \\
v_2(t) &= u_2(t-20),
\end{aligned}
$$

el Hamiltoniano aumentado tiene la forma:

$$
H(t,x,y,u,v) = x_1^2(t) + x_2^2(t) + u_1^2(t) + u_2^2(t) + p_1(t)y_1(t)v_1(t) + p_2 t)y_2(t)v_2(t) \quad (4)
$$

En cuanto al sistema adjunto, para $t \in [0, 40]$, se tiene

$$
\begin{aligned}
\dot{p}_1(t) &= -x_1(t) - \chi(t)_{[0,40]}\Big(p_1(t+20) - p_1(t+20)x_2(t-20) + p_2(t+20)x_2(t-20) - 0.4p_1(t+20)u_1(t-20)\Big) \\
\dot{p}_2(t) &= x_2(t) - \chi(t)_{[0,40]}\Big(- p_2(t+20) + p_2(t+20)x_1(t-20) - p_1(t+20)x_1(t-20) - 0.2p_2(t+20)u_2(t-20
\end{aligned}
$$

(5)

$$
\begin{aligned}
p_1(60) &= 0.5, \qquad (6) \\
p_1(60) &= 0.7, \qquad (7)
\end{aligned}
$$

$$0 \leq \quad u_1(t) \quad \leq 1, \tag{8}$$

$$0 \leq \quad u_2(t) \quad \leq 1, \tag{9}$$

$$\begin{aligned} u_1(t) &= -0.4p_1(t+20)x_1(t-20) \\ u_2(t) &= -0.2p_2(t+20)x_2(t-20) \end{aligned} \tag{10}$$