Ecuaciones de Lotka-Volterra para tres especies

Versión 3

January 25, 2023

1 Equations

$$
\begin{aligned}
dx_1(t) = (&\eta\, x_1(t) - \beta x_1(t)x_2(t)) - \delta x_1(t)x_3(t) \\
&-Ax_1(t)u_1(t))dt + \alpha_1\, dW_1(t) \\
&+x_1(t)u_1(t) \int_{\mathbb{R}^n} \xi(x_1(t))N(dt,dz) \\
dx_2(t) = (&\omega x_2(t) - \beta x_2(t)x_1(t) - \epsilon x_2(t)x_3(t) \\
&-Bx_2(t)u_2(t))dt + \alpha_2\, dW_2(t) \\
&+x_2(t)u_2(t) \int_{\mathbb{R}^n} \xi(x_1(t))N(dt,dz) \\
dx_3(t) = (&-\kappa x_3(t) + \delta x_3(t)x_1(t) + \epsilon x_3(t)x_2(t) \\
&-Cx_3(t)u_3(t))dt + \alpha_3\, dW_3(t) + \\
&x_3(t)u_3(t) \int_{\mathbb{R}^n} \xi(x_1(t))N(dt,dz)
\end{aligned} \tag{1}
$$

with the conditions:

$$x_1(0) = 0.7, \quad x_2(0) = 0.7, \quad x_3(0) = 0.5, \quad x_1(T) = 0.7, \quad x_2(T) = 0.7, x_3(0) = 0.5,$$

$$
\begin{aligned}
dp_1(t) &= \Big(\eta x_1(t) - p_1(t) + \beta p_1(t)x_2(t) + \delta p_1(t)x_3(t) \\
&+ Ap_1(t)u_1(t) + \beta p_2(t)x_2(t) - \delta p_3(t)x_3(t)\Big)dt \\
&+ p_1(t)dW_1(t) + u_1(t)x_1(t)\int_\gamma \xi(x_1(t))N(dt,d\gamma) \\
dp_2(t) &= \Big(\omega x_2(t) - p_2(t) + \beta p_2(t)x_1(t) + \epsilon p_2(t)x_3(t) \\
&+ Bp_2(t)u_2(t) + \beta p_1(t)x_1(t) - \epsilon p_3(t)x_3(t)\Big)dt \\
&+ p_2(t)dW_2(t) + u_2(t)x_2(t)\int_\gamma \xi(x_1(t))N(dt,d\gamma) \qquad (2) \\
dp_3(t) &= \Big(\kappa x_3(t) + p_3(t) - \delta p_3(t)x_1(t) - \epsilon p_3(t)x_2(t) \\
&+ Cp_3(t)u_3(t) + \delta p_1(t)x_1(t) + \epsilon p_2(t)x_2(t)\Big)dt \\
&+ p_3(t)dW_3(t) + u_3(t)x_3(t)\int_\gamma \xi(x_1(t))N(dt,d\gamma) \\
p_1(T) &= 0.5 \\
p_2(T) &= 0.5 \\
p_3(T) &= 0.7
\end{aligned}
$$

$$
\begin{aligned}
u_1(t) &= -Ap_1(t)x_1(t) + p_1(t)x_1(t)\int_\gamma \xi(x_1(t))N(dt,d\gamma) \\
u_2(t) &= -Bp_2(t)x_2(t) + p_2(t)x_2(t)\int_\gamma \xi(x_1(t))N(dt,d\gamma) \qquad (3) \\
u_3(t) &= -Cp_3(t)x_3 + p_3(t)x_3(t)\int_\gamma \xi(x_1(t))N(dt,d\gamma)(t)
\end{aligned}
$$