Dynamical behaviour of a Lotka–Volterra competitive-competitive–cooperative model with feedback controls and time delays - 2019

ABSTRACT

The aim of this paper is to investigate the dynamical behaviour of a class of three species Lotka–Volterra competitive-competitive-cooperative models with feedback controls and time delays. By developing a new analysis technique, we obtain some sufficient conditions that ensure these models have the dynamical property of permanence. We also give some sufficient conditions that guarantee the global attractivity of positive solutions for this system by constructing a new suitable Lyapunov function. Finally, we give some numerical simulations to illustrate our results in this paper.

El objetivo de este artículo es investigar el comportamiento dinámico de una clase de tres modelos Lotka-Volterra competitivo-competitivo-cooperativo con controles de retroalimentación y retardos temporales. Mediante el desarrollo de una nueva técnica de análisis, obtenemos condiciones suficientes que garantizan la propiedad dinámica de permanencia de estos modelos. También proporcionamos condiciones suficientes que garantizan la atracción global de soluciones positivas para este sistema mediante la construcción de una nueva función de Lyapunov adecuada. Finalmente, presentamos algunas simulaciones numéricas para ilustrar nuestros resultados.

1. Introduction

The modelling and analysis of the dynamics of biological populations by means of differential equations are of the primary concern in population growth problems. A well-known and extensively studied class of models in population dynamics is the Lotka–Volterra system which models certain types of interactions among various species. In the real world, the growth rate of a natural species will not often respond immediately to changes in its own species or that of an interacting species, but will rather do so after a time lag. Time delays are introduced to make the model respond better to impersonal law (see, [1–11]).

El modelado y análisis de la dinámica de poblaciones biológicas mediante ecuaciones diferenciales es fundamental en los problemas de crecimiento poblacional. Un modelo bien conocido y ampliamente estudiado en dinámica poblacional es el sistema Lotka-Volterra, que modela ciertos tipos de interacciones entre diversas especies. En el mundo real, la tasa de crecimiento de una especie natural no suele responder inmediatamente a los cambios en su propia especie o en la de una especie con la que interactúa, sino que lo hará con un cierto desfase temporal. Se introducen desfases temporales para que el modelo responda mejor a la ley impersonal (véase [1–11]).

Lu et al. in [2] proposed and studied the following Lotka–Volterra system with discrete delays

Lu et al. en [2] propusieron y estudiaron el siguiente sistema Lotka-Volterra con retrasos discretos

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1 - a_1x_1(t) - a_{11}x_1(t-\tau_{11}) + a_{12}x_2(t-\tau_{12}) \\ \dot{x}_2(t) = x_2(t)[r_2 - a_2x_2(t) - a_{21}x_1(t-\tau_{21}) + a_{22}x_2(t-\tau_{22}) \end{cases}$$

(1)

with initial conditions

x i (t) = i (t) 0, t [0 , 0]; i (0) ¿ 0, (i = 1, 2)

where ri, ai, aij and ij are constants with ai ¿ 0, aij 0(i, j = 1, 2) and 0 = max ij : i, j = 1, 2 , ij is continuous on [0 , 0]. They show that delays can change the permanence for Lotka–Volterra cooperative systems. For certain delays with the same length, the delayed system has a similar property to the corresponding system without delays in the sense of permanence, but for a general delay case, the delays may destroy the permanence for the system. In 2010, Nakata and Muroya considered the following nonautonomous Lotka–Volterra cooperative systems with time delays (see, [3])

donde ri, ai, aij y ij son constantes con ai ¿ 0, aij 0(i, j = 1, 2) y 0 = máx ij : i, j = 1, 2 , ij es continuo en [0 , 0]. Demuestran que los retrasos pueden cambiar la permanencia de los sistemas cooperativos Lotka-Volterra. Para ciertos retrasos con la misma duración, el sistema retrasado tiene una propiedad similar a la del sistema correspondiente sin retrasos en cuanto a la permanencia, pero para un caso general de retraso, los retrasos pueden destruir la permanencia del sistema. En 2010, Nakata y Muroya consideraron los siguientes sistemas cooperativos Lotka-Volterra no autónomos con retrasos temporales (véase [3]).

x ˙ 1 (t) = x 1 (t)[r 1 (t) a 11 1 (t)x 1 (t) a 11 2 (t)x 1 (t 2) + a 12 1 (t)x 2 (t)], x ˙ 2 (t) = x 2 (t)[r 2 (t) + a 21 0 (t)x 1 (t) + a 21 1 (t)x 1 (t) a 22 0 (t)x 2 (t) a 22 1 (t)x 2 (t)],

where xi(t)(i = 1, 2) denote the density of i-species at time t, is a positive constant and ri(t), aijl (t) (1 i, j 2; 0 l 2) are continuous, bounded and strictly positive functions as t [, +). They obtained some sufficient conditions for the permanence of the system (2). Xu and Zu [4] investigated the following two-species delayed competitive model with stage structure and harvesting

Donde xi(t)(i = 1, 2) denota la densidad de i-especies en el tiempo t, es una constante positiva y ri(t), aijl (t) (1 i, j 2; 0 l 2) son funciones continuas, acotadas y estrictamente positivas cuando t [, +]. Obtuvieron algunas condiciones suficientes para la permanencia del sistema (2). Xu y Zu [4] investigaron el siguiente modelo competitivo retardado de dos especies con estructura de etapas y cosecha.

dx 1 (t) = (t)x 2 (t) x 1 (t) (t)e x 2 (t), dt dx 2 (t) = (t)e x 2 (t) (t)x 2 2 (t) a 1 (t)x 2 (t)y(t) E(t)x 2 (t), dt dy(t) = y(t)(r 1 (t) a 2 (t)x 2 (t) b(t)y(t)).

By using the differential inequality theory, some new sufficient conditions which ensure the permanence of the system are established. In [5], the authors considered the following competitor-competitor–mutualist Lotka–Volterra systems with discrete time delays

Mediante la teoría de la desigualdad diferencial, se establecen nuevas condiciones suficientes que garantizan la permanencia del sistema. En [5], los autores consideraron los siguientes sistemas Lotka-Volterra competidor-competitivo-mutualista con retardos discretos.

x ˙ 1 (t) = x 1 (t)[r 1 (t) a 11 1 (t)x 1 (t) a 11 2 (t)x 1 (t 2) a 12 (t)x 2 (t 2) a 13 (t)x 3 (t)], + x ˙ 2 (t) = x 2 (t)[r 2 (t) a 21 (t)x 1 (t 2) a 22 1 (t)x 2 (t) a 22 2 (t)x 2 (t 2) + a 23 (t)x 3 (t)], x ˙ 3 (t) = x 3 (t)[r 3 (t) + a 31 (t)x 1 (t) + a 32 (t)x 2 (t) a 33 1 (t)x 3 (t) a 33 2 (t)x 3 (t)].

(4)

And some sufficient conditions which guarantee the boundedness, permanence and global attraction for system (4) were obtained. In 2011, Xu et al. [6] studied the dynamical behaviours for the following Lokta–Volterra predator–prey model with two delays

Se obtuvieron condiciones suficientes que garantizan la acotación, la permanencia y la atracción global del sistema (4). En 2011, Xu et al. [6] estudiaron los comportamientos dinámicos del siguiente modelo depredador-presa de Lokta-Volterra con dos retrasos.

x ˙ 1 (t) = x 1 (t)[r 1 a 11 x 1 (t 1) a 12 y(t 2)], x ˙ 2 (t) = x 2 (t)[r 2 a 21 x 1 (t 1) a 22 y(t 2)].

(5)

Its linear stability and Hopf bifurcation are investigated by analysing the associated characteristic transcendental equation. Some explicit formulate for determining the stability and the direction of the Hopf bifurcation periodic solutions are obtained by using normal form theory and centre manifold theory.

Se investiga su estabilidad lineal y la bifurcación de Hopf mediante el análisis de la ecuación trascendental característica asociada. Se obtienen fórmulas explícitas para determinar la estabilidad y la dirección de las soluciones periódicas de la bifurcación de Hopf mediante la teoría de la forma normal y la teoría de la variedad central.

One can find that an ecosystem in the real world is continuously distributed by some forces, which can result in changes in the biological parameters such as survival rates. The practical interest in ecology is the question of whether or not an ecosystem can withstand those disturbances which persist for a finite period of time. In the control systems, we regard the disturbance functions as control variables. These are of significance in the control of ecology balance. One of the methods to research it is to alter the system structurally by introducing feedback control variables. The feedback control mechanism might be implemented by means of some biological control schemes or harvesting procedure. In fact, during the last decade, the qualitative behaviour of the population dynamics with feedback control has been studied extensively. In 2009, Nie et al. [12] considered the following non-autonomous predator–prey Lotka–Volterra system with feedback controls

En la vida real, un ecosistema se distribuye continuamente por ciertas fuerzas, lo que puede provocar cambios en parámetros biológicos como las tasas de supervivencia. El interés práctico en ecología radica en determinar si un ecosistema puede soportar perturbaciones que persisten durante un período finito. En los sistemas de control, consideramos las funciones de perturbación como variables de control. Estas son importantes para controlar el equilibrio ecológico. Uno de los métodos para investigarlo es alterar la estructura del sistema mediante la introducción de variables de control de retroalimentación. El mecanismo de control de retroalimentación podría implementarse mediante esquemas de control biológico o procedimientos de cosecha. De hecho, durante la última década, se ha estudiado ampliamente el comportamiento cualitativo de la dinámica poblacional con control de retroalimentación. En 2009, Nie et al. [12] consideraron el siguiente sistema Lotka-Volterra depredador-presa no autónomo con controles de retroalimentación.

$\dot{x}_1(t) = x_1(t)[b_1(t)\ \ a_{11}(t)x_1(t)\ \ a_{12}(t)x_2(t) + c_1(t)u_1(t)]$, $\dot{x}_2(t) = x_2(t)[b_2(t) + a_{21}(t)x_1(t)\ \ a_{22}(t)x_2(t)\ \ c_2(t)u_2(t)]$, $\dot{u}_1(t) = f_1(t)\ \ e_1(t)u_1(t)\ \ d_1(t)x_1(t)$, $\dot{u}_2(t) = e_2(t)u_2(t) + d_2(t)x_2(t)$,

(6)

where x1(t) is the prey population density and x2(t) is the predator population density, b1(t) and a11(t) are the intrinsic growth rate and density-dependent coefficient of the prey, respectively; b2(t) and a 22 (t) are the intrinsic growth rate and densitydependent coefficient of the predator, respectively; a12(t) is the capturing rate of the predator and a21(t) is the rate of conversion of nutrient into the reproduction of the predator; ui(t)(i = 1, 2) are control variables. They studied whether or not the feedback controls have an influence on the permanence of a positive solution of the general nonautonomous predator–prey Lotka–Volterra-type systems and establish the general criteria on the permanence of system (6), which is independent of some feedback controls. In addition, by constructing an appropriate Lyapunov function, some sufficient conditions are obtained for the global stability of any positive solution to system (6). In [13], Yang, Wang and Chen proposed and studied the following cooperation system with feedback controls

donde x1(t) es la densidad de población de la presa y x2(t) es la densidad de población del depredador, b1(t) y a11(t) son la tasa de crecimiento intrínseca y el coeficiente dependiente de la densidad de la presa, respectivamente; b2(t) y a 22 (t) son la tasa de crecimiento intrínseca y el coeficiente dependiente de la densidad del depredador, respectivamente; a12(t) es la tasa de captura del depredador y a21(t) es la tasa de conversión de nutrientes en la reproducción del depredador; ui(t)(i =1, 2) son variables de control. Estudiaron si los controles de retroalimentación tienen o no influencia en la permanencia de una solución positiva de los sistemas generales no autónomos depredador-presa tipo Lotka-Volterra y establecen los criterios generales sobre la permanencia del sistema (6), que es independiente de algunos controles de retroalimentación. Además, al construir una función de Lyapunov apropiada, se obtienen algunas condiciones suficientes para la estabilidad global de cualquier solución positiva al sistema (6). En [13], Yang, Wang y Chen propusieron y estudiaron el siguiente sistema de cooperación con controles de retroalimentación

x ˙ 1 (t) = x 1 (b 1 a 11 x 1 (t) + a 12 x 2 (t) 1 u 1 (t)), x ˙ 2 (t) = x 2 (b 2 + a 21 x 1 (t) a 22 x 2 (t) 2 u 2 (t)), u ˙ 1 (t) = 1 u 1 (t) + a 1 x 1 (t), u ˙ 2 (t) = 2 u 2 (t) + a 2 x 2 (t),

(7)

where bi , aij , i , i , ai , i, j = 1, 2 are positive constants. xi(t), (i = 1, 2) are the densities of the species at time t, ui(t), (i = 1, 2) denote feedback controls. They showed that if system (7) has a positive equilibrium, then feedback controls can only influence the position of the positive equilibrium, and have no influence on the stability. In 2018, Wang et al. [14] considered the following three-species Lokta–Volterra predator–prey system with feedback

Donde bi, aij, i, i, ai, i, j = 1, 2 son constantes positivas. xi(t), (i = 1, 2) son las densidades de las especies en el tiempo t, ui(t), (i = 1, 2) representan controles de retroalimentación. Demostraron que si el sistema (7) tiene un equilibrio positivo, entonces los controles de retroalimentación solo pueden influir en la posición del equilibrio positivo y no en la estabilidad. En 2018, Wang et al. [14] consideraron el siguiente sistema depredador-presa de tres especies de Lokta-Volterra con retroalimentación.

a 12 (t)x 2 (t) x ˙ 1 (t) = x 1 (t)[r 1 (t) a 11 (t)x 1 (t) b 12 (t)x 2 (t) + x 1 (t) a 13 (t)x 3 (t) (t)u 1 (t)], b 13 (t)x 3 (t) + x 1 (t) d1 a 21 (t)x 1 (t) x ˙ 2 (t) = x 2 (t)[r 2 (t) + (t)x 3 (t) + d 2 (t)u 2 (t)], a23 b 12 (t)x 2 (t) + x 1 (t) a 31 (t)x 1 (t) x ˙ 3 (t) = x 3 (t)[r 3 (t) + (t)x 2 (t) + d 3 (t)u 3 (t)], b 13 (t)x 3 (t) x 1 (t) a32 + u ˙ 1 (t) = e 1 (t) f 1 (t)u 1 (t) + q 1 (t)x 1 (t), u ˙ 2 (t) = e 2 (t) f 2 (t)u 2 (t) q 2 (t)x 2 (t), u ˙ 3 (t) = e 3 (t) f 3 (t)u 3 (t) q 3 (t)x 3 (t),

(8)

By using a comparison theorem and constructing a suitable Lyapunov function as well as developing some new analysis techniques, the authors established a set of easily verifiable sufficient conditions which guarantee the permanence of the system and the global attractivity of positive solution for the predator–prey system (8). Furthermore, some conditions for the existence, uniqueness and stability of a positive periodic solution for the corresponding periodic system were obtained by using the fixed point theory and some new analysis method. More work on feedback controls can be found in (cf. [15–21] and the references cited therein). As is known to all, the Lotka–Volterra system with time delay and feedback control can respond better to impersonal law. In recent years, more and more attention has been paid to some ecosystem models with both feedback control and time delay (see, [22–27]). In 1993, Gopalsamy et al. [22] studied a class of autonomous single-species ecosystem with feedback control and time delay

Mediante el uso de un teorema de comparación y la construcción de una función de Lyapunov adecuada, así como el desarrollo de nuevas técnicas de análisis, los autores establecieron un conjunto de condiciones suficientes fácilmente verificables que garantizan la permanencia del sistema y la atracción global de una solución positiva para el sistema depredador-presa (8). Además, se obtuvieron algunas condiciones para la existencia, unicidad y estabilidad de una solución periódica positiva para el sistema periódico correspondiente mediante el uso de la teoría del punto fijo y un nuevo método de análisis. Se puede encontrar más información sobre controles de retroalimentación en (cf. [15–21] y las referencias citadas). Como es bien sabido, el sistema Lotka-Volterra con retardo temporal y control de retroalimentación puede responder mejor a la ley impersonal. En los últimos años, se ha prestado cada vez más atención a algunos modelos de ecosistemas con control de retroalimentación y retardo temporal (véase [22–27]). En 1993, Gopalsamy et al. [22] estudiaron una clase de ecosistema autónomo monoespecífico con control de retroalimentación y retardo temporal.

a1 dn(t) n(t) + a 2 n(t) = rn(t)[1 ( ) cu(t)], dt K du(t) = au(t) + bn(n), dt

(9)

where u(t) denotes an indirect control variable, , a2 , a, b, c, r (0,) and a 1 [0,). Some sufficient conditions were obtained for the global asymptotic stability of the positive equilibrium for the system (9). In order to show that whether the feedback control variables play an essential role on the persistent property of Lotka–Volterra cooperative systems or not, Xu and Chen [26] established and studied the following system with time delay and feedback control

x˙ 1 (t) = x 1 (t)[r 1 (t) a 1 (t)x 1 (t) a 11 (t)x 1 (t) + a 12 (t)x 2 (t) b 1 (t)u 1 (t 1)], x˙ 2 (t) = x 2 (t)[r 2 (t) a 2 (t)x 2 (t) + a 21 (t)x 1 (t) a 22 (t)x 2 (t) b 2 (t)u 2 (t 2)], u˙ 1 (t) = c 1 (t)u 1 (t) + d 1 (t)x 1 (t 1), u˙ 2 (t) = c 2 (t)u 2 (t) + d 2 (t)x 2 (t 2).

(10)

They obtained some new sufficient conditions which ensured the system to be permanent, and showed that feedback control variables had no influence on the permanence of the

system. In 2017, Xu and Li [27] considered the following competition and cooperation model of two enterprises with multiple delays and feedback controls

dx 1 (t) dt = x 1 (t)[r 1 (t) a 1 (t)x 1 (t) b 1 (t)(x 2 (t) c 2 (t)) 2 e 1 (t)u 1 (t 1 (t))], du 1 (t) = 1 (t)u 1 (t) + 1 (t)x 1 (t 1 (t)), dt dx 2 (t) = x 2 (t)[r 2 (t) a 2 (t)x 2 (t) + b 2 (t)(x 1 (t) c 1 (t)) 2 e 2 (t)u 2 (t 2 (t))], dt du 2 (t) = 2 (t)u 2 (t) + 2 (t)x 2 (t 2 (t)). dt

(11) Some sufficient conditions that guarantee the existence of a unique globally asymptotically stable nonnegative almost periodic solution for the system (11) were obtained by constructing a suitable Lyapunov functional

and using the comparison theorem of differential equations.

However, as far as we know, no work has been done until now for the three-species Lotka–Volterra system with feedback control and time delay. Motivated by the above work, we propose and investigate the following three species Lotka–Volterra competitive–cooperative model with feedback controls and time delays

x ˙ 1 (t) = x 1 (t)[r 1 (t) a 11 1 (t)x 1 (t) a 11 2 (t)x 1 (t 2) a 12 (t)x 2 (t 2) a 13 (t)x 3 (t) d 1 (t)u 1 (t)], + x ˙ 2 (t) = x 2 (t)[r 2 (t) a 21 (t)x 1 (t 2) a 22 1 (t)x 2 (t) a 22 2 (t)x 2 (t 2) + a 23 (t)x 3 (t) + d 2 (t)u 2 (t)], x ˙ 3 (t) = x 3 (t)[r 3 (t) + a 31 (t)x 1 (t) + a 32 (t)x 2 (t) a 33 1 (t)x 3 (t) a 33 2 (t)x 3 (t) + d 3 (t)u 3 (t)], u ˙ 1 (t) = e 1 (t) f 1 (t)u 1 (t) + q 1 (t)x 1 (t), u ˙ 2 (t) = e 2 (t) f 2 (t)u 2 (t) q 2 (t)x 2 (t), u ˙ 3 (t) = e 3 (t) f 3 (t)u 3 (t) q 3 (t)x 3 (t),

(12)

where x i (t), i = 1, 2, 3 stands for the densities of the species at time t, and u i (t), i = 1, 2, 3 are the indirect control variables. The given coefficients a 12 (t), a 13 (t), a 21 (t), a 23 (t), a 31 (t), a 32 (t), r i (t), d i (t),e i (t), f i (t), q i (t)(i = 1, 2, 3), a 11 l (t), a 22 l (t), a 33 l (t), (l = 1, 2) are continuous, bounded and strictly positive functions on [0,). r i (t), (i = 1, 2, 3) denote the intrinsic growth rate of the i-th species at time t. Especially, a ii l (t), (i = 1, 2, 3, l = 1, 2) denote the internal competitive coefficient of the three species at time t. a 12 (t), a 21 (t) are the competitive coefficient of species x 1 (t) and x 2 (t) at time t, respectively. a 13 (t), a 23 (t), a 31 (t), a 32 (t) are the cooperative coefficient of species x i (t), (i = 1, 2, 3) at time t, respectively. is a positive constant.

Due to the biological interpretation of the system (8), it is reasonable to consider only positive solution of the system (8), in other words, take admissible initial conditions

x i (t) = i (t), i = 1, 2, 3 for t [2, 0) and 1 (0) ¿0,

(13)

u i (t) = i (t), i = 1, 2, 3 for t ¿ 0 and 1 (0) ¿0.

Obviously, the solutions of system (12) with the initial values (13) are positive for all t 0.

Comparing the systems (4) and (12), one could see that we introduce the control variables u i (t) (i = 1, 2, 3) so as to implement a feedback control mechanism. Our main purpose in this paper is to establish some sufficient conditions which ensure the system to be permanence and global attractivity by constructing a new appropriate Lyapunov function and developing a new analysis technique. This paper is organized as follows: In Section 2, we provide the conditions for the permanence to system (12). In Section 3, by constructing a nonnegative Lyapunov function, we shall derive sufficient conditions for the global attractivity of positive solution for the Lotka–Volterra Competitive-Competitive–Cooperative model (12). Some numerical simulations to the system are given in Section 4.

2. Permanence

In order to establish a permanence result for the system (12), we need some preparations. Firstly, we introduce the following notations and definitions. Given a function g(t) defined on [t 0 +), we set

g m = sup g(t) — t 0 ¡ t ¡ + , g l = inf g(t) — t 0 ¡ t ¡ + .

Definition 2.1: System (12) is called permanent, if there exist positive constants M i , N i , m i , n i (i = 1, 2, 3), and T, such that m i x i (t) M i , n i u i (t) N i for any positive solution Z(t) = (x 1 (t), x 2 (t), x 3 (t), u 1 (t), u 2 (t), u 3 (t)) of (8) as t ¿ T.

As a direct corollary of Lemma 2.1 of Chen [1], we have.

Lemma 2.1: If a ¿ 0, b ¿ 0 and x ˙ b ax, when t 0 and x(0) ¿ 0, we have

lim inf x(t) b/a. t +

If a ¿ 0, b ¿ 0 and x ˙ b ax, when t 0 and x(0) ¿ 0, we have

lim sup x(t) b/a.

t +

Lemma 2.2 (see [3], Lemma 2.2): Assume that for y(t) ¿ 0, it holds that

m y ˙ (t) y(t)(l y(t l)) + D, l=0

with initial conditions y(t) = (t) 0 for t [m, 0) and (0) ¿ 0, where

m ¿ 0, l 0(l = 0, 1, 2, · · · , m), = l ¿ 0 and D 0, l=0

are constants. Then there exist a positive constant M y ¡ + such that

D D lim sup y(t) M y =) exp(m) ¡ + , (y + + t +

(14)

where y = y is the unique solution of equation y(y) + D = 0.

Lemma 2.3 (see [3], Lemma 2.3): Assume that for y(t) ¿ 0, it holds that

m y ˙ (t) y(t)[l y(t l)]. l=0

If the system (14) holds, then, there exists a positive constant m y ¿ 0 such that for =

m l ¿0 l=0

lim inf y(t) m y =)m ¿ 0. exp (My t +

For the system (12), let

m m e e N 2 = 2 ,N 3 = 3 , l l f 2 f 3

(r1 m + r 3 m + d3 m N3)2 P 1 = exp (r 1 m + r 3 m + d 3 m N 3) , a11 1l a33 2l

(r2 m + r 3 m + d2 m N 2 + d3 m N3)2 P 2 = exp (r 2 m + r 3 m + d 2 m N 2 + d 3 m N 3) , a22 1l a33 2l

a13 m P 1 P 1 + q m M1 a13 m e1 m 1 M 1 = + (+ x 1) exp(2r 1 m), N 1 = , l r 1 m r 1 m f 1

a P a P M 2 = + (+ x 2) exp(2(r 2 m + d 2 m N 2)), r 2 m + m 23 dm 2 2 N 2 r 2 m + m 23 dm 2 2 N2

M 3 = x 3 exp((r 3 m + (a 31 m + a 32 m + d 3 m) max M 1 , M 2 , N 3)),

r1 l a12 m M2 d1 m N1 m 1 = exp[(r 1 l a 12 m M 2 d 1 m N 1 (a 11 1m + a 11 2m)M 1)2], a 11 1m + a11 2m

r2 l a21 m M1 m 2 = exp[(r 2 l a 21 m M 1 (a 22 1m + a 22 2m)M 2)2], a 22 1m + a22 2m

r3 l + a31 l m 1 + a32 l m2 m 3 = exp[(r 3 l + a 31 l m 1 + a 32 l m 2 (a 33 1m + a 33 2m)M 3)], a 33 1m + a33 2m

+ q 1 l 2 q m m 1 q m M 2 3 M3 e3 l e1 l e2 l n 1 = ,n 2 = ,n 3 = , f 1 m f 2 m f 3 m

where x 1 is the unique positive solution of equation x 1 [r 1 m (a 11 1l + a 11 2l)x 1] + a 13 m P 1 = 0, x 2 is the unique positive solution of equation x 2 [r 2 m + d 2 m N 2 (a 22 1l + a 22 2l)x 2] + a 23 m P 2 = 0, and x 3 is the unique positive solution of equation x 3 (t)[r 3 m + (a 31 m + a 32 m + d 3 m) max M 1 , M 2 , N 3 (a 33 1l + a 33 2l)x 3] = 0.

Theorem 2.1: Assume the following conditions satisfy

(H 1) a 11 2l ¿ a 31 m , (H 2) a 12 l ¿ a 32 m , (H 3) a 33 1l ¿ a 13 m , (H 4) a 21 l ¿ a31 m

(H 5) a 22 2l ¿ a 32 m , (H 6) a 33 1l ¿ a 23 m , (H 7) r 1 l ¿ a 12 m M 2 + d 1 m N 1 ,

(H 8) r 2 l ¿ a 21 m M 1 , (H 9) e 2 l ¿ q 2 m M 2 , (H 10) e 3 l ¿ q 3 m M 3 .

Then the system (12) is permanent.

Proof. By the fifth equation of system (8), we have

u ˙ 2 (t) e 2 (t) f 2 (t)u 2 (t) e 2 m f 2 l u 2 (t).

e2 m lim sup u 2 (t) = N 2 . t + f2 l

(15)

Moreover, similar to the above discussion of the fifth equation of system (12), from the sixth of system (12), we have

e3 m lim sup u 3 (t) = N 3 . t + f3 l

(16)

Next, suppose that lim sup x 1 (t)x 3 (t) = + , then there exists a time sequence

t +

t k k=1

and

such that

lim sup x 1 (t k)x 3 (t k) = + ,

(17)

k +

d (x1 (t)x 3 (t)) — t=t k 0, k = 1, 2, · · · . dt

(18)

From system (12), one has

d (x1 (t)x 3 (t)) = x 1 (t)x 3 (t)[r 1 (t) + r 3 (t) a 11 1 (t)x 1 (t) dt

a 11 2 (t)x 1 (t 2) a 12 (t)x 2 (t 2) + a 13 (t)x 3 (t)

d 1 (t)u 1 (t) + a 31 (t)x 1 (t 2) + a 32 (t)x 2 (t 2)

a 33 1 (t)x 3 (t) a 33 2 (t)x 3 (t2) + d 3 (t)u 3 (t)]

x 1 (t)x 3 (t)[r 1 m + r 3 m + d 3 m N 3 (a 11 2l a 31 m)x 1 (t 2)

(a 12 l a 32 m)x 2 (t 2) (a 33 1l a 13 m)x 3 (t)

(19)

a 11 1l x 1 (t) a 33 2l x 3 (t 2) d 1 l u 1 (t)].

From (18), (19), we can obtain

(a 11 2l a 31 m)x 1 (t k 2) + (a 12 l a 32 m)x 2 (t k 2) + (a 33 1l a 13 m)x 3 (t k) + d 1 l u 1 (t k) + a 11 1l x 1 (t k) + a 33 2l x 3 (t k 2) r 1 m + r 3 m + d 3 m N 3 .

(20)

Thus, by the assumption of the Theorem 2.1 and (20), it holds that

r1 m + r 3 m + d3 m N 3 + r 3 m + d3 m N3 r1 m x 1 (t k) , x 3 (t k 2) . a 11 1l a33 2l

Moreover, by (19) and the assumption of the Theorem 2.1, it follows that

d (x1 (t)x 3 (t)) x 1 (t)x 3 (t)[r 1 m + r 3 m + d 3 m N 3]. dt

(21)

By integrating both sides of (21) from t k to t k further, we have

x 1 (t k)x 3 (t k) x 1 (t k)x 3 (t k 2) exp (r 1 m + r 3 m + d 3 m N 3) .

Therefore

(r1 m + r 3 m + d3 m N3)2 x 1 (t k)x 3 (t k) exp (r 1 m + r 3 m + d 3 m N 3) . a11 1l a33 2l

However, it leads to a contradiction with (17). Thus, we have

(r1 m + r 3 m + d3 m N3)2 lim sup(x 1 (t)x 3 (t)) P 1 = exp (r 1 m + r 3 m + d 3 m N 3) . (22) t + a 11 1l a33 2l

Moreover, similar to the above discussion, we can also obtain that

lim sup(x 2 (t)x 3 (t)) P2

t +

(r2 m + r 3 m + d2 m N 2 + d3 m N3)2 = exp (r 2 m + r 3 m + d 2 m N 2 + d 3 m N 3) . a22 1l a33 2l

(23)

According to the first equation of system (21) and (22), it follows that

x ˙ 1 (t) x 1 (t)[r 1 a 11 1 x 1 (t) a 11 2 x 1 (t 2) + a 13 x 3 (t)]

x 1 (t)[r 1 m a 11 1l x 1 (t) a 11 2l x 1 (t 2)] + a 13 m P 1 .

From Lemma 2.2, we have

a13 m P 1 P1 a13 m lim sup x 1 (t) + (+ x 1) exp(2r 1 m) = M 1 , t + r 1 m r1 m

(24)

where x 1 is the unique positive solution of equation x 1 [r 1 m (a11 1l +

a 11 2l)x 1] + a 13 m P 1 = 0.

Similar to the above discussion of the first equation of system (12), from (23) and the second equation of system (12), we obtain

x ˙ 2 (t) x 2 (t)[r 2 a 22 1 x 2 (t) a 22 2 x 2 (t 2) + a 23 x 3 (t) + d 2 u 2 (t)]

x 2 (t)[r 2 m a 22 1l x 2 (t) a 22 2l x 2 (t 2) + d 2 m N 2] + a 23 m P 2 .

So, we have

a P a P lim sup x 2 (t) + (+ x 2) exp((r 2 m + d 2 m N 2)2) = M 2 . (25) t + r 2 m + m 23 dm 2 2 N 2 r 2 m + m 23 dm 2 2 N2

where x 2 is the unique positive solution of the following equation

x 2 [r 2 m + d 2 m N 2 (a 22 1l + a 22 2l)x 2] + a 23 m P 2 = 0.

From the third equation of system (12), we obtain

x ˙ 3 (t) = x 3 (t)[r 3 + a 31 x 1 (t) + a 32 x 2 (t) a 33 1 x 3 (t) a 33 2 x 3 (t) + d 3 u 3 (t)]

x 3 (t)[r 3 m + a 31 m M 1 + a 32 m M 2 + d 3 m N 3 a 33 1l x 3 (t) a 33 2l x 3 (t)] x 3 (t)[r 3 m + (a 31 m + a 32 m + d 3 m) max M 1 , M 2 , N 3   a 33 1l x 3 (t) a 33 2l x 3 (t)].

By Lemma 2.2, it holds that

lim sup x 3 (t) x 3 exp((r 3 m + (a 31 m + a 32 m + d 3 m) max M 1 , M 2 , N 3)) = M 3 .

(26)

t +

where x 3 is the unique positive solution of the following equation

x 3 (t)[r 3 m + (a 31 m + a 32 m + d 3 m) max M 1 , M 2 , N 3   (a 33 1l + a 33 2l)x 3] = 0.

From the fourth equation of system (12), one has

u ˙ 1 (t) e 1 m f 1 l u 1 (t) + q 1 m M 1 .

Thus, according to Lemma 2.1, it follows that

e 1 m + q1 m M1 lim sup u 1 (t) = N 1 . t + f1 l

(27)

On the contrary, from the first equation of system (12), we have

x ˙ 1 (t) x 1 (t)[r 1 a 11 1 x 1 (t) a 11 2 x 1 (t 2) a 12 x 2 (t 2) d 1 (t)u 1 (t)]

x 1 (t)[r 1 l a 11 1m x 1 (t) a 11 2m x 1 (t 2) a 12 m M 2 d 1 m N 1].

By Lemma 2.3, one easily verifies that

lim inf x 1 (t) t +

r1 l a12 m M2 d1 m N1 exp[2(r 1 l a 12 m M 2 d 1 m N 1 (a 11 1m + a 11 2m)M 1)] = m 1 .

By the same way, from the second and third equations of system (12), we deduce

x ˙ 2 (t) x 2 (t)[r 2 a 21 x 1 (t 2) a 22 1 x 2 (t) a 22 2 x 2 (t 2)]

x 2 (t)[r 2 l a 21 m M 1 a 22 1m x 2 (t) a 22 2m x 2 (t 2)],

x ˙ 3 (t) x 3 (t)[r 3 + a 31 x 1 (t) + a 32 x 2 (t) a 33 1 x 3 (t) a 33 2 x 3 (t)]

x 3 (t)[r 3 l + a 31 l m 1 + a 32 l m 2 a 33 1m x 3 (t) a 33 2m x 3 (t)].

Thus, by Lemma 2.3, we have

r2 l a21 m M1 lim inf x 2 (t) exp[(r 2 l a 21 m M 1 (a 22 1m + a 22 2m)M 2)2] = m 2 , t + a 22 1m + a22 2m

(29)

and

lim inf x 3 (t) t +

r 3 l + a31 l m 1 + a32 l m2 exp[(r 3 l + a 31 l m 1 + a 32 l m 2 (a 33 1m + a 33 2m)M 3)] = m 3 . (30) a 33 1m + a33 2m

According to the fourth equation of system (12), we have

u ˙ 1 (t) e 1 l f 1 m u 1 (t) + q 1 l m 1 .

From Lemma 2.1, we can obtain

e 1 l + q1 l m1 lim inf u 1 (t) = n 1 . t + f1 m

(31)

Similarly, from the fifth and sixth equations of system (12), it follows that

u ˙ 2 (t) e 2 l f 2 m u 2 (t) q 2 m M 2 , u ˙ 3 (t) e 3 l f 3 m u 3 (t) q 3 m M 3 .

Moreover, by Lemma 2.1, it follows that

e2 l q2 m M2 lim inf u 2 (t) = n 2 , t + f2 m

(32)

and

e3 l q3 m M3 lim inf u 3 (t) = n 3 . t + f3 m

(33)

From (15), (16), and (24)–(33), this completes the proof of Theorem 2.1.

3. Globally attractive

In this section, we shall prove that the system (12) is globally attractive. To get the sufficient conditions for globally attractive of system (12), we give firstly the following definition and Lemma.

Definition 3.1: System (12) is said to be globally attractive, if there exists a positive solution X(t) = (x 1 (t), x 2 (t), x 3 (t), u 1 (t), u 2 (t), u 3 (t)) of the system (12) such that

lim — x i (t) y i (t) — = 0, lim — u i (t) v i (t) — = 0, t + t +

for any other positive solution Y(t) = (Y 1 (t), Y 2 (t), Y 3 (t), v 1 (t), v 2 (t), v 3 (t)) of the system (12).

Lemma 3.1 (See [28], Lemma 8.2): If the function f (t) : R + R is uniformly continuous, and the limit lim f (s)ds exists and is finite, thent lim f (t) = 0.

t +

t 0

+

Theorem 3.1: Let a 11 M = max a 11 1m , a 11 2m , a 22 M = max a 22 1m , a 22 2m . Assume that the system (12) satisfies (H 1) (H 10) and the following conditions satisfy

(H 11) lim inf A i (t) ¿ 0, B i ¿ 0, (i = 1, 2, 3) t +

where

2 t A 1 (t) = a 11 1l + a 11 2l a 11 k (s + k)ds[r 1 m + (a 11 1m + a 11 2m)M1 k=1 tk

2 t + k + a 12 m M 2 + a 13 m M 3 + d 1 m N 1] M 1 a 11 k (s + k)ds a 11 k (t + k) k=1 (t)

2 2M 1 a 11 M a 11 k (t + k) (1 + M 2 (a 22 1m + 2a 22 2m))a 21 (t + 2) k=1

(1 + M 3 a 33 2m)a 31 (t +) q 1 m ,

2 t A 2 (t) = a 22 1l + a 22 2l a 22 k (s + k)ds[r 2 m + a 21 m M 1 + (a 22 1m + a 22 2m)M2 k=1 tk

2 t + k + a 23 m M 3 + d 2 m N 2] M 2 a 22 k (s + k)ds a 22 k (t + k) k=1 (t)

2 2M 2 a 22 M a 22 k (t + k) k=1

(1 + M 1 (a 11 1m + 2a 11 2m))a 12 (t + 2) (1 + M 3 a 33 2m)a 32 (t +) q 2 m ,

t A 3 (t) = a 33 1l + a 33 2l (M 3 a 33 2m)(a 33 1m + a 33 2 (t +)) a 33 2 (s +)ds[r 3 m + a 31 m M1 t

+ a 32 m M 2 + (a 33 1m + a 33 2m)M 3 + d 3 m N 3] (1 + M 1 (a 11 1m + 2a 11 2m))a 13 (t +)

(1 + M 2 (a 22 1m + 2a 22 2m))a 23 (t +) q 3 m ,

B 1 = f 1 l (1 + M 1 (a 11 1m + 2a 11 2m))d 1 m ,

B 2 = f 2 l (1 + M 2 (a 22 1m + 2a 22 2m))d 2 m ,

B 3 = f 3 l (1 + M 3 a 33 2m)d 3 m .

Then system (12) is globally attractive.

Proof. Suppose that (x 1 (t), x 2 (t), x 3 (t), u 1 (t), u 2 (t), u 3 (t)) and (y 1 (t), y 2 (t), y 3 (t), v 1 (t), v 2 (t), v 3 (t)) are any two different positive solutions of the system (12). Then from Theorem 2.1, there exist positive constants M i , N i , m i , n i , i = 1, 2, 3 and T, such that

m i x i (t), y i (t) M i , n i u i (t), v i (t) N i , i = 1, 2, 3, for all t T.

Define

V 11 (t) = — ln x 1 (t) ln y 1 (t) — .

Calculating the upper right derivative of V 11 (t) along the solution of system (12), we obtain

D + V 11 (t) = sgn(x 1 (t) y 1 (t))[a 11 1 (t)(x 1 (t) y 1 (t))

a 11 2 (t)(x 1 (t 2) y 1 (t 2))

a 12 (t)(x 2 (t 2) y 2 (t 2)) + a 13 (t)(x 3 (t) y 3 (t))

d 1 (t)(u 1 (t) v 1 (t))]

= sgn(x 1 (t) y 1 (t))[(a 11 1 (t) + a 11 2 (t))(x 1 (t) y 1 (t)) d 1 (t)(u 1 (t) v 1 (t))

a 12 (t)(x 2 (t 2) y 2 (t 2)) + a 13 (t)(x 3 (t) y 3 (t))
t t ˙1 + a 11 1 (t) (˙x 1 () y ˙ 1 ()d + a 11 2 (t) (˙x 1 () y ()d] t t2
= sgn(x 1 (t) y 1 (t))[(a 11 1 (t) + a 11 2 (t))(x 1 (t) y 1 (t)) d 1 (t)(u 1 (t) v 1 (t))
a 12 (t)(x 2 (t 2) y 2 (t 2)) + a 13 (t)(x 3 (t) y 3 (t))
2 t + a 11 k (t) tk (x 1 ()[r 1 () a 11 1 ()x 1 ( ) k=1
a 11 2 ()x 1 (2) a 12 ()x 2 (2)
+ a 13 ()x 3 ( ) d 1 ()u 1 ()] y 1 ()[r 1 () a 11 1 ()y 1 ( )
a 11 2 ()y 1 (2)
a 12 ()y 2 (2) + a 13 ()y 3 ( ) d 1 ()v 1 ()])d]
= sgn(x 1 (t) y 1 (t))[(a 11 1 (t) + a 11 2 (t))(x 1 (t) y 1 (t)) d 1 (t)(u 1 (t) v 1 (t))
a 12 (t)(x 2 (t 2) y 2 (t 2)) + a 13 (t)(x 3 (t) y 3 (t))
2 t + a 11 k (t) tk ((x 1 () y 1 ())[r 1 () a 11 1 ()y 1 ( )
a 11 2 ()y 1 (2)
a 12 ()y 2 (2) + a 13 ()y 3 ( ) d 1 ()v 1 ()]
+ x 1 ()[a 11 1 ()(x 1 ( )
y 1 ( )) a 11 2 ()(x 1 (2) y 1 (2)) a 12 ()(x 2 (2)
y 2 (2))
+ a 13 ()(x 3 ( ) y 3 ( )) d 1 ()(u 1 () v 1 ())d]
(a 11 1 (t) + a 11 2 (t)) — x 1 (t) y 1 (t) — + d 1 (t) — u 1 (t)
v 1 (t) — + a 12 (t) — x 2 (t 2) y 2 (t 2) —
+ a 13 (t) — x 3 (t) y 3 (t) —
2 t + (a 11 k (t) tk ([r 1 () + a 11 1 ()y 1 ( ) + a 11 2 ()y 1 (2) k=1
+ a 12 ()y 2 (2) + a 13 ()y 3 ( ) + d 1 ()v 1 ()] — x 1 () y 1 () —
+ x 1 ()[a 11 1 ()
— x 1 ( ) y 1 ( ) — + a 11 2 () — x 1 (2) y 1 (2) —
+ a 12 () — x 2 (2) y 2 (2) —
(34)
+ a 13 () — x 3 ( ) y 3 ( ) — + d 1 () — u 1 () v 1 () —])d).

Next, we define that 2 t t V 12 (t) = a 11 k (s + k)([r 1 () + a 11 1 ()y 1 ( ) + a 11 2 ()y 1 (2) k=1 tk s
+ a 12 ()y 2 (2) + a 13 ()y 3 ( ) + d 1 ()v 1 ()] — x 1 () y 1 () —
+ x 1 ()[a 11 1 () — x 1 ( ) y 1 ( ) — + a 11 2 () — x 1 (2) y 1 (2) —
+ a 12 () — x 2 (2) y 2 (2) — + a 13 () — x 3 ( ) y 3 ( ) —
(35)
+ d 1 () — u 1 () v 1 () —)dds.

Then, from (34) and (35), we have

2 V 1i (t) (a 11 1 (t) + a 11 2 (t)) — x 1 (t) y 1 (t) — + d 1 (t) — u 1 (t) v 1 (t) — i=1

+ a 12 (t) — x 2 (t 2) y 2 (t 2) — + a 13 (t) — x 3 (t) y 3 (t) —

2 t + tk a 11 k (s + k)ds[r 1 (t) + (a 11 1 (t) + a 11 2 (t))M 1 + a 12 (t)M2 k=1

2 t + a 13 (t)M 3 + d 1 (t)N 1] — x 1 (t) y 1 (t) — + M 1 a 11 k (s + k)ds

[a 11 1 (t) — x 1 (t) y 1 (t) — + a 11 2 (t) — x 1 (t 2) y 1 (t 2) —

+ a 12 (t) — x 2 (t 2) y 2 (t 2) — + a 13 (t) — x 3 (t) y 3 (t) —

+ d 1 (t) — u 1 (t) v 1 (t) —]

(a 11 1 (t) + a 11 2 (t)) — x 1 (t) y 1 (t) — + (1 + M 1 (a 11 1m + 2a 11 2m))d 1 (t) — u 1 (t) v 1 (t) —

+ a 12 (t) — x 2 (t 2) y 2 (t 2) — + a 13 (t) — x 3 (t) y 3 (t) —

2 t + tk a 11 k (s + k)ds[r 1 (t) + (a 11 1 (t) + a 11 2 (t))M 1 + a 12 (t)M2 k=1

2 t + a 13 (t)M 3 + d 1 (t)N 1] — x 1 (t) y 1 (t) — + M 1 (a 11 k (s + k)ds k=1 tk 2 a 11 k (t) — x 1 (t k) y 1 (t k) —) + 2M 1 a 11 M (a 11 k (t) — x 1 (t k) k=1

y 1 (t k) —)

+ M 1 (a 11 1m + 2a 11 2m)[a 12 (t) — x 2 (t 2) y 2 (t 2) — + a 13 (t) — x 3 (t)

(36)

y 3 (t) —].

Define

2 t w + k V 13 (t) = M 1 a 11 k (s + k)a 11 k (w + k) — x 1 (w) y 1 (w) — dsdw k=1 tk w

2 t + 2M 1 a 11 M a 11 k (w + k) — (x 1 (w) y 1 (w) — dw k=1 tk

t + (1 + M 1 (a 11 1m + 2a 11 2m)) a 12 (w + 2) — x 2 (w) y 2 (w) — dw t2

t + (1 + M 1 (a 11 1m + 2a 11 2m)) a 13 (w +) — x 3 (w) y 3 (w) — dw. t

(37)

Let

V 1 (t) = V 11 (t) + V 12 (t) + V 13 (t).

(38)

According to (36) and (37), calculating the upper right derivative of V 1 (t), we have

2 t D + V 1 (t) a 11 1l + a 11 2l a 11 k (s + k)ds[r 1 m + (a 11 1m + a 11 2m)M1 k=1 tk

2 t + k + a 12 m M 2 + a 13 m M 3 + d 1 m N 1] M 1 (a 11 k (s + k)ds a 11 k (t + k)) k=1 t

2 2M 1 a 11 M a 11 k (t + k) — x 1 (t) y 1 (t) — + (1 + M 1 (a 11 1m + 2a 11 2m))a 12 (t + 2) k=1

— x 2 (t) y 2 (t) — + (1 + M 1 (a 11 1m + 2a 11 2m))a 13 (t +) — x 3 (t) y 3 (t) —

(39)

+ (1 + M 1 (a 11 1m + 2a 11 2m))d 1 m — u 1 (t) v 1 (t) — .

Similarly, we define V 21 (t) = — ln x 2 (t) ln y 2 (t) — , then one obtain

D + V 21 (t) = sgn(x 2 (t) y 2 (t))[a 21 (t)(x 1 (t 2) y 1 (t 2)) a 22 1 (t)(x 2 (t) y 2 (t))

a 22 2 (t)(x 2 (t 2) y 2 (t 2)) + a 23 (t)(x 3 (t) y 3 (t)) + d 2 (t)(u 2 (t) v 2 (t)]

= sgn(x 2 (t) y 2 (t))[(a 22 1 (t) + a 22 2 (t))(x 2 (t) y 2 (t)) a 21 (t)(x 1 (t 2) y 1 (t 2))

t ˙2 + a 23 (t)(x 3 (t) y 3 (t)) + d 2 (t)(u 2 (t) v 2 (t)) + a 22 1 (t) (˙x 2 () y ())d t

t ˙2 + a 22 2 (t) (˙x 2 () y ())d] t2

= sgn(x 2 (t) y 2 (t))[(a 22 1 (t) + a 22 2 (t))(x 2 (t) y 2 (t)) a 21 (t)(x 1 (t 2) y 1 (t 2))

2 t + a 23 (t)(x 3 (t) y 3 (t)) + d 2 (t)(u 2 (t) v 2 (t)) + a 22 k (t) (x 2 ()[r 2 () k=1 tk

a 21 ()x 1 (2) a 22 1 ()x 2 ( ) a 22 2 ()x 2 (2) + a 23 ()x 3 ( ) + d 2 ()u 2 ()]

y 2 ()[r 2 () a 21 ()y 1 (2) a 22 1 ()y 2 ( ) a 22 2 ()y 2 (2) + a 23 ()y 3 ( )

+

d 2 ()v 2 ()])d]

= sgn(x 2 (t) y 2 (t))[(a 22 1 (t) + a 22 2 (t))(x 2 (t) y 2 (t)) a 21 (t)(x 1 (t 2) y 1 (t 2))

2 t + a 23 (t)(x 3 (t) y 3 (t)) + d 2 (t)(u 2 (t) v 2 (t)) + a 22 k (t) ((x 2 () y 2 ()) k=1 tk

[r 2 () a 21 ()y 1 (2) a 22 1 ()y 2 ( ) a 22 2 ()y 2 (2) + a 23 ()y 3 ( )

+ d 2 ()v 2 ()] + x 2 ()[a 21 ()(x 1 (2) y 1 (2)) a 22 1 ()(x 2 ( ) y 2 ( ))

a 22 2 ()(x 2 (2) y 2 (2)) + a 23 ()(x 3 ( )y 3 ( )) + d 2 ()(u 2 ()v 2 ())])d]

(a 22 1 (t) + a 22 2 (t)) — x 2 (t) y 2 (t) — + d 2 (t) — u 2 (t) v 2 (t) — + a 21 (t) — x 1 (t 2) y 1 (t 2) —

2 t + a 23 (t) — x 3 (t) y 3 (t) — + (a 22 k (t) ([r 2 () + a 21 ()y 1 (2) + a 22 1 ()y 2 ( ) k=1 tk

+ a 22 2 ()y 2 (2) + a 23 ()y 3 ( ) + d 2 ()v 2 ()] — x 2 () y 2 () —

+ x 2 ()[a 21 () — x 1 (2) y 1 (2) — + a 22 1 () — x 2 ( ) y 2 ( ) —

+ a 22 2 () — x 2 (2) y 2 (2) — + a 23 () — x 3 ( ) y 3 ( ) — + d 2 () — u 2 ()v 2 () —])d).

(40)

On the other hand, define

2 t t V 22 (t) = a 22 k (s + k)([r 2 () + a 21 ()y 1 (2) + a 22 1 ()y 2 ( )   k=1 tk s

+ a 22 2 ()y 2 (2) + a 23 ()y 3 ( ) + d 2 ()v 2 ()] — x 2 () y 2 () —

+ x 2 ()[a 21 () — x 1 (2) y 1 (2) — + a 22 1 () — x 2 ( ) y 2 ( ) —

+ a 22 2 () — x 2 (2) y 2 (2) — + a 23 () — x 3 ( ) y 3 ( ) —

(41)

+ d 2 () — u 2 () v 2 () —])dds.

From (40), (41), we have

2 D + V 2i (a 22 1 (t) + a 22 2 (t)) — x 2 (t) y 2 (t) — + d 2 (t) — u 2 (t) v 2 (t) — i=1

+ a 21 (t) — x 1 (t 2) y 1 (t 2) —

2 t + a 23 (t) — x 3 (t) y 3 (t) — + a 22 k (s + k)ds[r 2 (t) + a 21 (t)M1 k=1 tk

+ (a 22 1 (t) + a 22 2 (t))M 2 + a 23 (t)M 3 + d 2 (t)N 2] — x 2 (t) y 2 (t) —

2 t + M 2 a 22 k (s + k)ds[a 21 (t) — x 1 (t 2) y 1 (t 2) — k=1 tk

+ a 22 1 (t) — x 2 (t) y 2 (t) —

+ a 22 2 (t) — x 2 (t 2) y 2 (t 2) — + a 23 (t) — x 3 (t) y 3 (t) —

+ d 2 (t) — u 2 (t) v 2 (t) —]

(a 22 1 (t) + a 22 2 (t)) — x 2 (t) y 2 (t) —

+ (1 + M 2 (a 22 1m + 2a 22 2m))d 2 (t) — u 2 (t) v 2 (t) —

+ a 21 (t) — x 1 (t 2) y 1 (t 2) — + a 23 (t) — x 3 (t) y 3 (t) —

2 t +   tk a 22 k (s + k)ds[r 2 (t) + a 21 (t)M 1 + (a 22 1 (t) + a 22 2 (t))M2 k=1

2 t + a 23 (t)M 3 + d 2 (t)N 2] — x 2 (t) y 2 (t) — + M 2 (a 22 k (s + k)ds k=1 tk 2 a 22 k (t) — x 2 (t k) y 2 (t k) —) + 2M 2 a 22 M (a 22 k (t) — x 2 (t k) k=1

y 2 (t k) —)

+ M 2 (a 22 1m + 2a 22 2m)[a 21 (t) — x 1 (t 2) y 1 (t 2) — + a 23 (t) — x 3 (t)

(42)

y 3 (t) —].

Furthermore, define

2 t w + k V 23 (t) = M 2 a 22 k (s + k)a 22 k (w + k) — x 2 (w) y 2 (w) — dsdw k=1 tk w

+ 2M 2 a 22 M a 22 k (w + k) — x 2 (w) y 2 (w) — dw k=1 tk

t + (1 + M 2 (a 22 1m + 2a 22 2m)) a 21 (w + 2) — x 1 (w) y 1 (w) — dw t2

t + (1 + M 2 (a 22 1m + 2a 22 2m)) a 23 (w +) — x 3 (w) y 3 (w) — dw. t

(43)

Let

V 2 (t) = V 21 (t) + V 22 (t) + V 23 (t).

(44)

From (42) and (43), we can get the upper right derivative of V 2 (t)

2 t D + V 2 (t) a 22 1l + a 22 2l a 22 k (s + k)ds[r 2 m + a 21 m M 1 + (a 22 1m + a 22 2m)M2 k=1 tk

2 t + k + a 23 m M 3 + d 2 m N 2] M 2 (a 22 k (s + k)ds a 22 k (t + k)) k=1 t

2 2M 2 a 22 M a 22 k (t + k) k=1

— x 2 (t) y 2 (t) — + (1 + M 2 (a 22 1m + 2a 22 2m)a 21 (t + 2) — x 1 (t) y 1 (t) —

+ (1 + M 2 (a 22 1m + 2a 22 2m)a 23 (t +) — x 3 (t) y 3 (t) —

(45)

+ (1 + M 2 (a 22 1m + 2a 22 2m))d 2 m — u 2 (t) v 2 (t) — .

Similarly, we define

V 31 (t) = — ln x 3 (t) ln y 3 (t) — .

Then, it follows that

D + V 31 = sgn(x 3 (t) y 3 (t))[a 31 (t)(x 1 (t) y 1 (t)) + a 32 (t)(x 2 (t)

y 2 (t)) a 33 1 (t)(x 3 (t) y 3 (t)) a 33 2 (t)(x 3 (t) y 3 (t))

+ d 3 (t)(u 3 (t) v 3 (t)]

= sgn(x 3 (t) y 3 (t))[a 31 (t)(x 1 (t) y 1 (t)) + a 32 (t)(x 2 (t)
y 2 (t)) (a 33 1 (t) + a 33 2 (t))(x 3 (t) y 3 (t)) + a 33 2 (t)
t (˙x 3 () y ˙ 3 ())d + d 3 (t)(u 3 (t) v 3 (t))] t
= sgn(x 3 (t) y 3 (t))[(a 33 1 (t) + a 33 2 (t))(x 3 (t) y 3 (t)) + d 3 (t)(u 3 (t) v 3 (t))
+ a 31 (t)(x 1 (t) y 1 (t)) + a 32 (t)(x 2 (t) y 2 (t))
t + a 33 2 (t) x 3 ()[r 3 () + a 31 ()x 1 ( ) + a 32 ()x 2 ( ) t
a 33 1 ()x 3 () a 33 2 ()x 3 ( ) + d 3 ()u 3 ()]
y 3 ()[r 3 () + a 31 ()y 1 ( ) + a 32 ()y 2 ( )
a 33 1 ()y 3 () a 33 2 ()y 3 ( ) + d 3 ()v 3 ()]d
= sgn(x 3 (t) y 3 (t))[(a 33 1 (t) + a 33 2 (t))(x 3 (t) y 3 (t)) + d 3 (t)(u 3 (t) v 3 (t))
+ a 31 (t)(x 1 (t) y 1 (t)) + a 32 (t)(x 2 (t) y 2 (t)) + a 33 2 (t)
t ((x 3 () y 3 ())[r 3 () + a 31 ()y 1 ( ) + a 32 ()y 2 ( ) t
a 33 1 ()y 3 () a 33 2 ()y 3 ( ) + d 3 ()v 3 ()] + x 3 ()[a 31 ()(x 1 ( )
y 1 ( )) + a 32 ()(x 2 ( ) y 2 ( )) a 33 1 ()(x 3 ()
y 3 ()) a 33 2 ()(x 3 ( ) y 3 ( )) + d 3 ()(u 3 () v 3 ())]])d]
(a 33 1 (t) + a 33 2 (t)) — x 3 (t) y 3 (t) — + d 3 (t) — u 3 (t) v 3 (t) — + a 31 (t) — x 1 (t)
y 1 (t) — + a 32 (t) — (x 2 (t) y 2 (t) — + a 33 2 (t)
t ([r 3 () + a 31 ()y 1 ( ) + a 32 ()y 2 ( ) + a 33 1 ()y 3 () t
+ a 33 2 ()y 3 ( ) + d 3 ()v 3 ()] — x 3 () y 3 () — + x 3 ()[a 31 () — x 1 ( )
y 1 ( ) — + a 32 () — x 2 ( ) y 2 ( ) — + a 33 1 () — x 3 () y 3 () —
(46)
+ a 33 2 () — x 3 ( ) y 3 ( ) — + d 3 () — u 3 () v 3 () —])d.

Let

t t V 32 (t) = a 33 2 (s +)([r 3 () + a 31 ()y 1 ( ) + a 32 ()y 2 ( ) t s
+ a 33 1 ()y 3 () + a 33 2 ()y 3 ( ) + d 3 ()v 3 ()] — x 3 () y 3 () —
+ x 3 ()[a 31 () — x 1 ( ) y 1 ( ) — + a 32 () — x 2 ( ) y 2 ( ) —
+ a 33 1 () — x 3 () y 3 () — + a 33 2 () — x 3 ( ) y 3 ( ) —
(47)
+ d 3 () — u 3 () v 3 () —])dds.

Then, we have

2 D + V 3i (a 33 1 (t) + a 33 2 (t)) — x 3 (t) y 3 (t) — + d 3 (t) — u 3 (t) v 3 (t) — i=1
+ a 31 (t) — x 1 (t) y 1 (t) — + a 32 (t) — (x 2 (t) y 2 (t) —
t + a 33 2 (s +)ds[r 3 (t) + a 31 (t)M 1 + a 32 (t)M2 t

+ (a 33 1 (t) + a 33 2 (t))M 3 + d 3 (t)N 3] — x 3 (t) y 3 (t) —
t + M 3 a 33 2 (s +)ds[a 31 (t) — x 1 (t) y 1 (t) — t
+ a 32 (t) — x 2 (t) y 2 (t) — + a 33 1 (t) — x 3 (t) y 3 (t) —
+ a 33 2 (t) — x 3 (t) y 3 (t) — + d 3 (t) — u 3 (t) v 3 (t) —]
(a 33 1 (t) + a 33 2 (t) M 3 a 33 2m a 33 1 (t)) — x 3 (t) y 3 (t) —
+ (1 + M 3 a 33 2m)d 3 (t) — u 3 (t) v 3 (t) — + a 31 (t) — x 1 (t) y 1 (t) —
t + a 32 (t) — (x 2 (t) y 2 (t) — + a 33 2 (s +)ds[r 3 (t) t
+ a 31 (t)M 1 + a 32 (t)M 2 + (a 33 1 (t) + a 33 2 (t))M 3 + d 3 (t)N 3] — x 3 (t) y 3 (t) —
+ M 3 a 33 2m [a 31 (t) — x 1 (t) y 1 (t) — + a 32 (t) — x 2 (t) y 2 (t) —
(48)
+ a 33 2 (t) — x 3 (t)y 3 (t) —].
Take
t V 33 (t) = (1 + M 3 a 33 2m) a 31 (w +) — x 1 (w) y 1 (w) — dw t
t + (1 + M 3 a 33 2m) a 32 (w +) — x 2 (w) y 2 (w) — dw t
t + M 3 a 33 2m a 33 2 (w +) — x 3 (w) y 3 (w) — dw. t
(49)
Moreover, we take
V 3 (t) = V 31 (t) + V 32 (t) + V 33 (t).
(50)
Then, we have
t D + V 3 (t)   a 33 1l + a 33 2l M 3 a 33 2m [a 33 1m + a 33 2 (t +)] a 33 2 (s +)ds t
[r 3 m + a 31 m M 1 + a 32 m M 2 + (a 33 1m + a 33 2m)M 3 + d 3 m N 3]
— x 3 (t) y 3 (t) — + (1 + M 3 a 33 2m)a 31 (t +) — x 1 (t) y 1 (t) —
+ (1 + M 3 a 33 2m)a 32 (t +) — (x 2 (t) y 2 (t) — + (1 + M 3 a 33 2m)d 3 m — u 3 (t)v 3 (t) — .
(51)
V4
(t)
=
—
ln
u1

(t)
ln
v1
(t)
—
,
V5
(t)
=
—
ln
u2
(t)
ln
v2
(t)
—
,
V6
(t)
=
—
ln
u3
(t)

Take ln v 3 (t) — , and calculate the upper right derivative of V 4 (t), V 5 (t), V 6 (t), we have

D + V 4 (t) sgn(u 1 (t) v 1 (t))[f 1 (t)(u 1 (t) v 1 (t)) + q 1 (t)(x 1 (t) y 1 (t))

(52)

f 1 l — u 1 (t) v 1 (t) — + q 1 m — x 1 (t) y 1 (t) — ,

D + V 5 (t) sgn(u 2 (t) v 2 (t))[f 2 (t)(u 2 (t) v 2 (t)) + q 2 (t)(x 2 (t) y 2 (t))

(53)

f 2 l — u 2 (t) v 2 (t) — + q 2 m — x 2 (t) y 2 (t) — ,

D + V 6 (t) sgn(u 3 (t) v 3 (t))[f 3 (t)(u 3 (t) v 3 (t)) + q 3 (t)(x 3 (t) y 3 (t))

(54)

f 3 l — u 3 (t) v 3 (t) — + q 3 m — x 3 (t) y 3 (t) — .

Moreover, we give a Lyapunov function as follows

6 V(t) = V i (t), i=1

from (39), (45), (51)–(54), we can obtain

3 D + V(t)   (A i (t) — x i (t) y i (t) — + B i — u i (t) v i (t) —) i=1

(55)

for all t t + .

In view of the conditions of Theorem 3.1, there exist a constant ¿ 0 and T such that for all t T , it holds that

T +

A i (t)   ¿ 0, B i   ¿ 0, (i = 1, 2, 3).

(56)

Integrating from T to t on both sides of (55) and by (56), we have

t 3 V(t) + ([— x i (s) y i (s) — + — u i (s) v i (s) —])ds V(T) ¡ + . T i=1

(57)

Therefore, V(t) is bounded on [T , +), and we have

3 V(T) ([— x i (t) y i (t) — + — u i (t) v i (t) —])ds ¡ + . T i=1

(58)

By (58), we have

3 (— x i (t) y i (t) — + — u i (t) v i (t) —) L 1 (T, +). i=1

(59)

3

From the uniformity permanence of the system (12),

i=1

[— x i (t) y i (t) — + — u i (t)

v i (t) —] is uniformity continuous on [T , +). By Lemma 3.1, we can obtain

lim — x i (t) y i (t) — = 0, lim — u i (t) v i (t) — = 0, (i = 1, 2, 3). t + t +

This completes the proof of Theorem 3.1.

4. Numerical simulation

In this section, we give some numerical simulations to support our theoretical analysis. As an example, we consider the following Lotka–Volterra competitivecompetitive–cooperative model with feedback controls and time delays and choose the

Figure 1. The numerical solution of systems (60) with the initial conditions (61) and = 0.075.

periodic function as the coefficients of the model

x˙ 1 (t) = x 1 (t)[2 + — sin(t) — 5 + — sin(t) — x 1 (t) 1 + — cos(t) — x 1 (t 2) 2 + sin(t) 550 x 2 (t 2)   2 3 5 + 2 + cos(t) 300 x 3 (t) (0.015 + 0.001 cos t)u 1 (t)],   x˙ 2 (t) = x 2 (t)[2 + — 2 cos(t) — 2 + cos(t) 550 x 1 (t 2) 6 + — 4 cos(t) — x 2 (t) 1 + — 5 sin(t) — x 2 (t 2)   + 2 + cos(t) 300 x 3 (t) + (0.0075 + 0.0005 sin t)u 2 (t)], x˙ 3 (t) = x 3 (t)[1 + — sin(t) — + 1.5 + cos(t) 1500 x 1 (t) + 1.5 + sin(t) 1500 x 2 (t) 5 + cos(t) x 3 (t)   2 2   6 5 + sin(t) — — x 3 (t) + (0.016 + 0.004 sin t)u 3 (t)],   u˙ 1 (t) = (0.35 + 0.05 sin t) (0.6 + 0.1 cos t)u 1 (t) + (0.0016 + 0.0003 sin t)x 1 (t),   u˙ 2 (t) = (0.3 + 0.05 cos t) (0.7 + 0.2 cos t)u 2 (t) (0.0015 + 0.0005 sin t)x 2 (t),   u˙ 3 (t) = (3 + 0.4 sin t) (5 + 0.5 sin t)u 3 (t) (0.0005 + 0.00015 sin t)x 3 (t).

(60)

By calculating, we have

P 1 5.50, P 2 6.14, x 1   1.22, x 2   1.09, x 3   0.37,

M 1 1.38, M 2 1.54, M 3 0.4,

N 1 0.81, N 2 0.7, N 3 0.76, m 1 0.43, m 2 0.51, m 3 0.21,

n 1 0.52, n 2 0.36,

n 3 0.47, r 1 l a 12 m M 2 d 1 m N 1 0.98, r 2 l a21 m

M 1 0.99, e 2 l q 2 m M 2 0.25, e 3 l q 3 m M 3 2.60,

lim inf A 1 0.35,(t) lim inf A 2 (t) 0.40, lim inf A 3 (t) 0.48, t + t + t +

B 1 0.48, B 2 0.49, B 3 4.48.

It is easy to show that the system (60) satisfies the conditions of Theorem 3.1. It follows from Theorem 3.1 that the Lotka–Volterra competitive-competitive–cooperative model (60) is permanent and globally attractive. By employing the software package MATLAB 7.1, we can solve the numerical solutions of the system (60) which are shown in Figures 1–3. Figure 1 shows that the permanence of the systems (60) with time delay = 0.075 and the

initial conditions

x 1 (t 0) = 0.7, x 2 (t 0) = 0.7, x 3 (t 0) = 0.7, u 1 (t 0) = 0.5, u 2 (t 0) = 0.5, u 3 (t 0) = 0.5. (61)

From Figure 2, it is not difficult to find that the system (60) is globally attractive. Figure 3 shows the dynamical behaviour of the systems (60).

5. Conclusion

This paper presents the use of Lyapunov stability theorem for system of nonlinear differential equations. This method is a powerful tool for solving nonlinear differential equations in mathematical physics, chemistry and engineering etc. The technique constructing an appropriate Lyapunov function provides a new efficient method to handle the nonlinear structure with time delay and feedback control.

We have dealt with the problem of positive solution for a class of three-species Lotka–Volterra competitive- competitive–cooperative with feedback controls and time delays. By developing some new analysis techniques and constructing a new suitable Lyapunov function, we obtain some sufficient conditions which ensure the system to be permanent and globally attractive. Our results show that feedback control variables and time delay terms have influence both the persistent property and global attractive of system (12). Moreover, some numerical simulations to the system (60) are given to illustrate our results obtained in this paper. In particular, the sufficient conditions that we obtained are very simple and practical, which provide flexibility for the application and analysis of the Lotka–Volterra models with feedback controls and time delays.

Remark: The main contribution and innovation of this paper are as follows: (1) The control variables are introduced to the known model (4) to implement a feedback control mechanism, and the new model can better describe the interactions among multi-species. Obviously, system (4) is the special case of the new system (12). To the best of the author's

knowledge, this is the first time such a system is proposed. (2) To study the new model, we obtain some new methods and skills (such as the new structure method of the Lyapunov function, the applications of delay differential inequalities) that can also be used to research other related models with multi-delays and feedback controls. Because of the complexity of the new system, the structure of the Lyapunov function is completed step by step to overcome the difficulties brought about by the multiple time delays and feedback controls, please see pages 10–17. (3) In this paper, the research contents are richer than the related references. We study not only the permanence and global attractivity of the new system but also some numerical simulations to the new system (12) are given to illustrate our results obtained in this paper. (4) The sufficient conditions obtained herein are new, general, and easily verifiable, which provide flexibility for the application and analysis of three-species multi-delays Lotka–Volterra predator–prey model with feedback controls.

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No potential conflict of interest was reported by the authors.

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