

stat-V

1) In each of the following situations, state whether it is a correctly stated hypothesis testing problem and why.

1) $H_0: \mu = 25$ $H_1: \mu \neq 25$

Yes. - We will always state the null hypothesis as an equality so that the probability of type I error.

② $H_0: \sigma > 10$ $H_1: \sigma = 10$

No.

③ $H_0: \bar{x} = 50$ $H_1: \bar{x} \neq 50$

No.

④ $H_0: P = 0.1$ $H_1: P = 0.5$

No.

⑤ $H_0: S$ $H_1: S \geq 30$

No.

state hypothesis

②. Given values

$$\mu = 52 \quad \sigma = 4.50 \quad n = 100$$

$$\alpha = 0.05 \quad Z_{\alpha/2} = 1.65 \quad \bar{x} = 52.80$$

step: 1

$$H_0: \mu = 52$$

$$H_a: \mu > 52$$

LL hypothesis of type I

③ Compute the test statistic, z :

$$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$
$$= \frac{52.80 - 52}{4.50 / \sqrt{100}}$$

$$= 0.8$$

$$Z = 1.78$$

④ We find the rejection Region. Here we use significance level of $\alpha = 0.05$. Therefore rejection Region is when $Z > 1.65$.

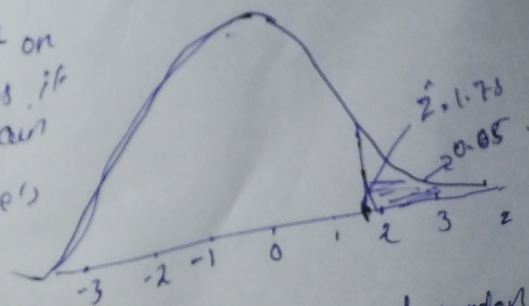
Conclusion: Since $Z = 1.78 > 1.65$ we reject H_0 .

We compute the p-value of the test:

$$P(\bar{X} > 52) = P(Z > 1.78) = 0.0358$$

* In sample size 100 with one tailed test on the right and $\alpha = 0.05$, it seems as though we are not believe the bookstore's claim that the mean of cats of total box is 52.

The true value of μ is 52.80 probably not due to random chance.



3) $\mu = 34$ $\sigma = 8$ $\alpha = 0.01$ $\bar{x} = 32.5$ $n = 50$

$H_0: \mu = 34$

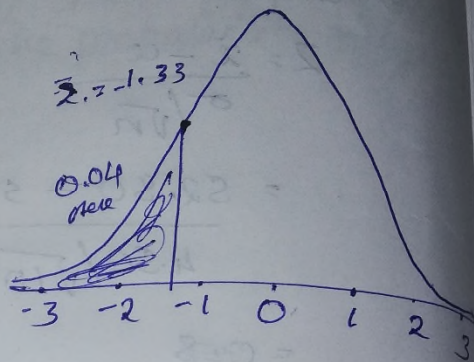
$H_a: \mu < 34$

compute the test statistic

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{32.5 - 34}{8 / \sqrt{50}}$$

$z = -1.33$



The test statistic lies on the Acceptance Region for H_0 .

Do not Reject H_0 . Based on this, on sample of size 50 with a one-tailed test on the left and $\alpha = 0.04$, it seems as though we can not believe the factory's claim that the mean amount of pollutant is less than 33 ppm. The lower value of 32.5 ppm is probably due to random chance.

4) $\mu = 1135$ $\alpha = 0.05$

Hypothesis:

$H_0: \mu = 1135$

$H_1: \mu \neq 1135$

The significance level is 5%.

$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}}$$

$\bar{x} = 1031.32$

$$Z = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$$= \frac{1031.32 - 1135}{240.37 / \sqrt{22}}$$

$$= \frac{-103.68}{51.25}$$

$$= -2.02$$

The critical value of z is -1.96 and $+1.96$
 The critical value is $z = \pm 1.96$ for two tailed test at 5% level of significance.

The compute value falls in rejection region.
 we reject the null hypothesis.

$$5) \mu = 48432 \quad s = 2000 \quad n = 400$$

$$\bar{x} = 48574$$

Hypothesis:

$$H_0: \mu = 48432$$

$$H_1: \mu \neq 48432$$

* Significance level: 10%

$$Z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

$$= \frac{48574 - 48432}{2000/\sqrt{400}}$$

$$= \frac{142}{100}$$

$$= \underline{\underline{1.42}}$$

The critical value of Z is -1.645 and $+1.645$ for two tailed test at 5% level of significance. Since the computed value of $Z = 1.42$ falls in acceptance region.

We accept the null hypothesis.

$$6) \mu = 32.28 \quad s = 1.29 \quad n = 19 \quad \bar{x} = 31.67$$

Hypothesis:

$$H_0: \mu = 32.28$$

$$H_1: \mu \neq 32.28$$

Significance level: 5%.

$$Z = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

$$= \frac{31.67 - 32.28}{1.29 / \sqrt{19}}$$

$$= \frac{-0.61}{0.29}$$

$$= -2.1$$

The critical value of z is -1.96 and $+1.96$.

The critical value is $z = \pm 1.96$ for two tailed test at 5% level of significance. Since the

computed value of $z = -2.1$ fall in

Reject region: we reject the null hypothesis.

The average price per square foot for warehouse has changed now.

1.645
significance

7).

Acceptance Region	sample size	α	$\mu = 5.2$	$\mu = 50.5$
$48.5 < \bar{x} < 51.5$	10	0.0576	0.2643	0.8923
$48 < \bar{x} < 52$	10	0.0114	0.5000	0.9701
$48.5 < \bar{x} < 51.5$	16	0.0164	0.2119	0.9445
$48 < \bar{x} < 52$	16	0.0014	0.5000	0.9914

$$z_1 = \frac{48.5 - 5.2}{0.79} \quad \text{and} \quad z_2 = \frac{51.5 - 5.2}{0.79} = -0.63$$

$$B = P(-4.43 \leq z \leq -0.63) = P(z \leq -0.63) - P(z \leq -4.43) \\ = 0.2643 - 0.0000 = 0.2643.$$

when $\mu = 50.5$

$$B = P(48.5 \leq \bar{x} \leq 51.5 \text{ when } 50.5)$$

$$z_1 = \frac{48.5 - 50.5}{0.79} = -2.53 \quad z_2 = \frac{51.5 - 50.5}{0.79} = 1.27$$

$$B = P(-2.53 \leq z \leq 1.27) = P(z \leq 1.27) - P(z \leq -2.53) \\ = 0.8980 - 0.0057 = 0.8923$$

* increasing the sample size results in a decrease in the probability of type II error.

$$s) \quad n=16 \quad u=10 \quad \bar{x}=12 \quad s=1.5 \quad n=16$$

$$df = n - 1 = 16 - 1 = \underline{15}$$

$$t = \frac{\bar{x} - u_{hyp}}{s_{\bar{x}}}$$

$$= \frac{12 - 10}{1.5}$$

$$= \frac{2}{1.5}$$

$$t = 1.34$$

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{1.5}{4} = 0.375$$

$$t = \frac{12 - 10}{0.375}$$

$$t = \underline{\underline{5.333}}$$

$$9) n = 16 \quad \alpha = 0.01$$

$$df = n - 1 = 16 - 1 = 15$$

$$1 - \alpha = 0.99$$

$$t_{0.99} = -t_{0.01}$$

$$t_{0.99} = -\underline{\underline{2.602}}$$

$$10) n = 25 \quad \bar{x} = 60 \quad \sigma = 4 \quad \alpha = 0.05$$

$$df = 25 - 1 = 24 \quad \bar{x} = 60 \quad s = 4$$

$$s_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$$= \frac{4}{\sqrt{25}}$$

$$= 0.8$$

$$= 0.8$$

\bar{x} = sample mean

μ = population mean

s = standard dev.

n = sample

$$t = \frac{\bar{x} - \mu_{hyp}}{s_{\bar{x}}}$$

$$= 60 -$$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$n = 25 \quad \mu = 60 \quad s = 4 \quad \alpha = 0.05$$

$$df = n - 1 = 25 - 1 = 24$$

$$t_{0.95} = \sigma = s \sqrt{n}$$

$$\sigma = 4 \sqrt{25}$$