

stat VI

$$1) n_1 = 1200 \quad \bar{x}_1 = 452 \quad \sigma_1 = 212$$

$$n_2 = 800 \quad \bar{x}_2 = 523 \quad \sigma_2 = 185$$

The null and alternative hypothesis are,

$$H_0: \mu_1 - \mu_2 = 0$$

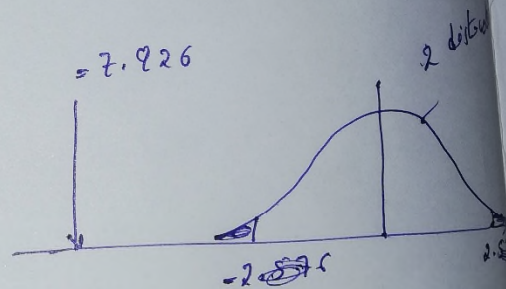
$$H_1: \mu_1 - \mu_2 \neq 0.$$

The value of test statistics is.

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)_0}{\sqrt{S_1^2/n_1 + S_2^2/n_2}}$$

$$Z = \frac{452 - 523}{\sqrt{212^2/1200 + 185^2/800}}$$

$$Z = -7.926$$



The computed value of the  $Z$  falls in the left-hand region for any commonly used  $\alpha$  and the p-value is very small. We conclude that there is a statistically significant difference in average monthly charges between Bangalore to Chennai and Bangalore to Hosur.

$$n_1 = 100 \quad x_1 = 308 \quad s_1 = 84$$

$$n_2 = 100 \quad x_2 = 254 \quad s_2 = 67$$

Our null and alternative hypothesis are

$$H_0: \mu_1 - \mu_2 \leq 0 \quad \mu_1 - \mu_2 \leq 0$$

$$H_1: \mu_1 - \mu_2 > 0 \quad \mu_1 - \mu_2 > 0$$

$$H_0: \mu_1 - \mu_2 \leq 0$$

$$H_1: \mu_1 - \mu_2 > 0$$

The value of test statistic is

$$z = \frac{\bar{x}_1 - \bar{x}_2 - (\mu_1 - \mu_2)}{\sqrt{s_1^2/n_1 + s_2^2/n_2}}$$

$$= \frac{308 - 254 - 0}{\sqrt{84^2/100 + 67^2/100}} = \frac{54}{\sqrt{115.48}}$$

=

$$= \underline{\underline{5.025}}$$

The value fall in the non rejection region of our right-tailed test at any conventional level of significance  $\alpha$ .



$$\begin{array}{lll} a) & n_1 = 15 & \bar{x}_1 = 6598 \quad s_1 = 844 \\ & n_2 = 12 & \bar{x}_2 = 6870 \quad s_2 = 669 \end{array}$$

This is one tailed test.

$$H_0: \mu_1 - \mu_2 \geq 0$$

$$H_1: \mu_1 - \mu_2 < 0$$

$$\begin{aligned} \text{degrees of freedom} &= n_1 + n_2 - 2 \\ &= 15 + 12 - 2 \\ &= 25 \end{aligned}$$

$$t = \frac{(\bar{x}_2 - \bar{x}_1)}{\frac{(s_1^2 + s_2^2)}{25} \left( \frac{1}{15} + \frac{1}{12} \right)}$$

$$= 0.91$$

\* This value of the statistic falls inside the non rejection region for any usual level of significance.

$$\begin{array}{lll} 5) & n_1 = 1000 & x_1 = 53 & p_1^* = 0.53 \\ & n_2 = 100 & x_2 = 43 & p_2^* = 0.83 \end{array}$$

$$H_0: p_1 - p_2 = 0$$

$$H_a: p_1 - p_2 \neq 0.$$

$$\hat{p}^* = \frac{x_1 + x_2}{n_1 + n_2} = \frac{53 + 43}{1000 + 100} = 0.48$$

$$1 - \hat{p}^* = 0.52$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}^*(1-\hat{p}^*) \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}$$

$$= \frac{0.53 - 0.43}{\sqrt{(0.48)(0.52) \left( \frac{1}{1000} + \frac{1}{100} \right)}}$$

$$= 1.415$$

\* This value of the test statistic falls in the nonrejection region even if we use  $\alpha = 0.10$ .



$$6) n_1 = 300 \quad x_1 = 120 \quad \hat{p}_1 = 0.40$$

$$n_2 = 700 \quad x_2 = 140 \quad \hat{p}_2 = 0.20$$

$$H_0: p_1 - p_2 \leq 0$$

$$H_1: p_1 - p_2 > 0.10$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2 - D}{\sqrt{\hat{p}_1(1-\hat{p}_1)/n_1 + \hat{p}_2(1-\hat{p}_2)/n_2}}$$

$$= \frac{120/300 - 140/700 - 0.10}{\sqrt{[(120/300)(180/300)]/300 + [(140/700)(560/700)]/700}}$$

$$= \frac{(0.4 - 0.2) - 0.1}{\sqrt{(0.4)(0.6)/300 + (0.2)(0.8)/700}}$$

$$= \underline{\underline{3.118}}$$

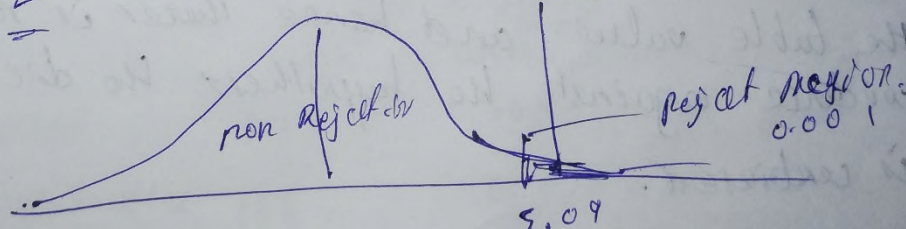
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ues.

this value of the test falls in the rejection region for  $\alpha = 0.001$ . The p-value is therefore less than 0.001 and the null hypothesis is rejected.

$$0.4 - 0.2 \pm 1.96 \sqrt{\frac{(0.4)(0.6)}{300} + \frac{(0.2)(0.8)}{700}} = 0.2 \pm 1.96$$

$$= [0.137, 0.263]$$

Test value  
= 3.118



2) Null hypothesis: Set up the null hypothesis that the die is unbiased.  
On the basis of hypothesis that the die is unbiased, we expect each number to turn up  $132/6 = 22$  times.

Apply  $\chi^2$ -test

O	E	$(O-E)^2$	$\frac{(O-E)^2}{E}$
16	22	36	1.64
20	22	4	0.18
25	22	9	0.41
14	22	64	2.91
29	22	49	2.23
23	22	36	1.64

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= 9.01$$

No. of degrees of freedom =  $n-1 = 6-1 = 5$

For 5 degrees of freedom at 5% level of significance, the table value of  $\chi^2 = 11.07$ .

The calculated value of  $\chi^2$  is less than the table value and hence there is no evidence against the hypothesis the die is unbiased.



Porter

8) We have

H<sub>0</sub>: Gender is independent of voting  
 H<sub>1</sub>: Gender and Voting are dependent.

	Men	Women	
voted	2792	3591	6386
Didn't vote	1486	2131	3617
	4278	5722	10000

$$\begin{aligned} \text{Expected value} \\ \text{men voted} &= \frac{6386 \times 4278}{10000} \\ &= 2730.6474 \\ &= \underline{2731} \end{aligned}$$

$$\begin{aligned} \text{Expected value} \\ \text{women voted} &= \frac{6386 \times 5722}{10000} = 3652.352 \\ &= \underline{3652} \end{aligned}$$

$$\begin{aligned} \text{Expected value} \\ \text{men not voted} &= \frac{3617 \times 4278}{10000} \\ &= \underline{1547} \end{aligned}$$

$$\begin{aligned} \text{Expected value} \\ \text{women not voted} &= \frac{3617 \times 5722}{10000} \\ &= \underline{2070} \end{aligned}$$

	men	women
voted	2731	3652
Didn't vote	1547	2070

$$\chi^2 = \frac{(2782-2731)^2}{2731} + \frac{(3591-3652)^2}{3652} + \frac{(1486-1547)^2}{1547} + \frac{(2131-2070)^2}{2070}$$

$$= 6.6$$

Degree of freedom = 1

$$t_{\alpha} = 3.84$$

Since the chi-square goodness of fit value 6.6 exceeds the critical  $\chi^2(3.84)$  we will reject the null-hypothesis.

9) The A is appropriate is that the candidates roughly support our exp

0
E
(O-E)
(O-E)
(O-E)
E

$$\chi^2 =$$

deg

The significance obtain the 1.



9) The Chi-Squared Goodness-of-fit test is appropriate here. The null hypothesis is that there is no preference for any of the candidates. If this is so, we would expect roughly equal number of votes to support each candidate.

Our expected frequencies  $100/4 = 25$  per candidate.

O	41	19	24	16
E	25	25	25	25
(O-E)	16	-6	-1	-9
(O-E) <sup>2</sup>	256	36	1	81
(O-E) <sup>2</sup> /E	10.24	1.44	0.04	3.24
$\Sigma$				

$$\chi^2 = \frac{\sum (O-E)^2}{E} = 10.24 + 1.44 + 0.04 + 3.24$$

$$= 14.96$$

$$\text{degrees of freedom} = k - 1 = \underline{3}$$

The critical value of chi-squared for 0.05 significance level and 3 d.f is 7.82. Our obtained chi-square value is greater than this, and so we conclude that our obtained value is unlikely to have occurred merely by chance.

10)

age of child	A	B	C	row totals
5-6 years	18	22	20	60
6-8 years	2	28	40	70
9-10 years	20	10	40	70
Column totals	40	60	100	200

For each above table, gives us:

O	18	22	20	2	28	40	20	10	40
E	12	18	30	14	21	35	14	21	35
(O-E)	6	4	-10	-12	7	5	6	-11	5
(O-E) <sup>2</sup>	36	16	100	144	49	25	36	121	25
$\frac{(O-E)^2}{E}$	3	0.89	3.33	10.29	2.33	0.71	2.57	5.76	0.71

Chi-square is the sum of these.

$$\chi^2 = 29.60$$

$$d.f. = (row-1) \times (column-1) = 2 \times 2 = 4$$

The critical value of chi-square in the table for a 0.001 significance level and 4 d.f. is 18.46. Our obtained value is bigger than this. Therefore we have a chi-square value which is so large that it would occur by chance only, about

11)

Conform
not Conform
Column totals

E

$$(10-E) - 0.5$$

$$(10-E) - 0.5$$

$$\frac{10-E-0.5}{E}$$

$$\chi^2 = 8$$

$$d.f. = 1$$

Our critical value is 1.64. Our obtained value is 8.0. Therefore we have a chi-square value which is so large that it would occur by chance only, about



(1)

	Support	no support	row totals
Conform:	18	40	58
not conform	32	10	42
Column totals:	50	50	100

E	18	40	32	10
	29	29	21	21

$$(O-E) = 0.8 \quad 10.5 \quad 10.5 \quad 10.5 \quad 10.5$$

$$(O-E)^2 = 110.25 \quad 110.25 \quad 110.25 \quad 110.25$$

$$\frac{(O-E)^2}{E} \quad 3.80 \quad 3.80 \quad 5.25 \quad 5.25$$

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

$$= 18.10$$

$$d.f. = (row - 1) \times (column - 1) = 1 \times 1 = 1$$

Our obtained value is bigger than the critical value of chi-squared for 0.001 significant level!

13)

	Married	Widowed, divorced or separated	Never married
Employed	679	103	114
Unemployed	63	10	20
Not in labor force	42	18	28

O: 679 103 114 63 10 20  
E 654 109 133 68 11 14

O: 42 18 28  
E 62 10 13

$$\chi^2 = \frac{(679 - 654)^2}{654} + \frac{(103 - 109)^2}{109} + \frac{(114 - 133)^2}{133}$$

$$+ \frac{(63 - 68)^2}{68} + \frac{(10 - 11)^2}{11} + \frac{(20 - 14)^2}{14}$$

$$+ \frac{(42 - 62)^2}{62} + \frac{(18 - 10)^2}{10} + \frac{(28 - 13)^2}{13}$$

$$= 30.96 \text{ with 4 df.}$$

Looking at table of chi-square distribution  
with  $(3-1)(3-1) = 2 \times 2 = 4$ .

Since  $30.96 >> 13.28$ , we conclude from the  
table that:

so we reject  $H_0$  with all confidence. Marital  
status seems to be related to job status in this way.



25 The null hypothesis  $H_0$ :  $X \sim \text{Poisson}$   
 $H_1$ :  $X$  does not follow a Poisson distribution

$$\hat{\mu} = \frac{(32 \times 0) + (15 \times 1) + (9 \times 2) + (4 \times 3)}{60}$$

$$\hat{\mu} = 0.75$$

$$P_0 = P(X=0) = \frac{e^{-0.75} (0.75)^0}{0!}$$

$$= 0.472$$

$$E_0 = 0.472 \times 60 = 28.32$$

$$P_1 = P(X=1) = \frac{e^{-0.75} (0.75)^1}{1!} = 0.354$$

$$E_1 = 0.354 \times 60 = 21.24$$

$$P_2 = P(X=2) = \frac{e^{-0.75} (0.75)^2}{2!} = 0.133$$

$$E_2 = 0.133 \times 60 = 7.98$$

$$P_3 = P(X \geq 3) = 1 - (P_0 + P_1 + P_2) = 0.041$$

$$E_3 = 0.041 \times 60 = 2.46$$

No. of Pet.	Observed Fr.	Expected
0	32	28.32
1	15	21.24
2 or more	4	10.44

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

$$= \frac{(32 - 28.32)^2}{28.32} + \dots$$

$$= 2.94$$

The degrees of  
 $k=3$   $p=1$   
 so,  $3-1-1=$

if you look  
 distribution w  
 p-value of 0.0

We conclude  
 evidence to su  
 follow Poiso

$$\begin{aligned}
 \chi^2 &= \sum \frac{(O - E)^2}{E} \\
 &= \frac{(32 - 28.32)^2}{28.32} + \frac{(15 - 21.24)^2}{21.24} + \frac{(13 - 10.44)^2}{10.44} \\
 &= \underline{2.94}
 \end{aligned}$$

The degrees of freedom is  $k - p - 1$

$$k = 3 \quad p = 1$$

$$\text{so, } 3 - 1 - 1 = 1 \text{ d.f.}$$

if you look 2.94 table chi-square distribution with  $df = 1$ , we obtain a p-value of  $0.05 < p < 0.1$ .

We conclude that there is no real evidence to suggest the data do not follow poisson distribution.



