

stat-11

1) A test is conducted which consisting of 20 MCQs, (multiple choice question) with every MCQ having its four options out of which only one is correct. Determine the probability that a person undertaking that test has answered exactly 5 question wrong.

=> Here, $n=20$, $n-x=5$, $x=20-5=15$

Probability of success $= p = \frac{1}{4}$

Probability of failure $= 1-p$

$$= 1 - \frac{1}{4} =$$

$$= \frac{3}{4}$$

To calculate the probability associated with each value of x

$$P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

for $x=0, 1, 2, \dots, n$.

$$\begin{aligned} P(5 \text{ out of } 20) &= \frac{20!}{5! 15!} \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^{15} \\ &= \frac{(20 \times 19 \times 18 \times 17 \times 16) 15!}{5! 15!} \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^{15} \end{aligned}$$

$$= 0.0000034 \text{ (approx.)}$$

Thus the required probability is 0.0000034 approximately.

② A die marked A to E rolled 50 times. Find the probability of getting a "D" exactly 5 times.

\Rightarrow Here, $n = 50$, $x = 5$, $n - x = 45$

The probability of success = The probability of getting a "D" $= 5 = \frac{1}{5}$

Hence the probability of failure = the probability of not getting "D" $= 1 - 5 = \frac{4}{5}$

The p.m.f given by $P(X=x) = \binom{n}{x} p^x q^{n-x}$

$$P(D \text{ 5 times}) = P(X=5) = \binom{50}{5} \left(\frac{1}{5}\right)^5 \left(\frac{4}{5}\right)^{45}$$

$$= 0.02953 \text{ (Approx)}^{45}$$

$$= {}^{50}C_5 \times \left(\frac{1}{5}\right)^5 \left(\frac{4}{5}\right)^{45}$$

$$= 0.02953$$

③ Customers arrive at a Rate of 72 per hour to my shop. What is the probability of 4 customers arriving in 4 minutes?

④ 5 customers

⑤ no more than 3 customers.

Give a pictorial representation of the same to validate your answer.

$$\lambda = 72 \text{ (hour)} \div 60 \text{ min} = 1.2 \text{ per min.}$$

$$\text{for 4 minutes} = (4 \times 1.2) = 4.8$$

a) 5 Customers

$$P(5) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$= \frac{(4.8)^5 e^{-4.8}}{5!}$$

$$= 0.1747$$

b) not more than 3 customers

$$P(k \leq 3) = \sum_{i=0}^3 \frac{\lambda^i e^{-\lambda}}{i!}$$

$$= \frac{(4.8)^0 e^{-4.8}}{0!} + \frac{(4.8)^1 e^{-4.8}}{1!} + \frac{(4.8)^2 e^{-4.8}}{2!} + \frac{(4.8)^3 e^{-4.8}}{3!}$$

$$= 0.00823 + 0.039503 + 0.094807 + 0.1516$$

$$= 0.29422$$

c) more than 3 customers

$$P(k > 3) = 1 - P(k \leq 3)$$

$$= 1 - 0.294229916$$

$$= 0.705770084$$