

Problem Statement 1: [100 marks]

A company manufactures LED bulbs with a faulty rate of 30%. If I randomly select 6 chosen LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also evaluate the standard deviation associated with it.

Stat-II
1) A company manufacture LED bulb, with a faulty rate of 30%. If randomly select 6 LEDs, what is the probability of having 2 faulty LEDs in my sample? Calculate the average value of this process. Also standard deviation associated with it.

$$\Rightarrow \text{Faulty Rate} = 30\% \\ \text{Probability of faulty Rate} = 0.3 \\ n = 6 \\ \text{Probability of success rate} = 1 - p = 1 - 0.3 = 0.7$$

$$\text{Probability of having 2 faulty in sample} \\ = {}^6C_2 \times (0.3)^2 \times (0.7)^4$$

$$= \frac{6!}{2!4!} \times (0.3)^2 \times (0.7)^4$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 4 \times 1} \times (0.3)^2 \times (0.7)^4$$

$$= 15 \times 0.09 \times (0.7)^4 \\ = 0.324135$$

$$\text{Average value} : u = np = 6 \times 0.3 = 1.8$$

$$\text{Standard Deviation} : \sigma = \sqrt{np(1-p)} \\ = \sqrt{6 \times 0.3 \times 0.7} \\ = 1.1225$$

Problem Statement 2: [100 marks]

Gaurav and Barakha are both preparing for entrance exams. Gaurav attempts to solve 8 questions per day

with a correction rate of 75%, while Barakha averages around 12 questions per day with a correction rate

of 45%. What is the probability that each of them will solve 5 questions correctly? What happens in cases

of 4 and 6 correct solutions? What do you infer from it? What are the two main governing factors

affecting their ability to solve questions correctly? Give a pictorial representation of the same to validate your answer.

Q. Solution: Answer

No of question per day $(n) = 8$

Correction Rate = $75\% = 0.75$

Probability of incorrect answer = $1 - 0.75 = 0.25$

Probability to solve 5 question correct.

$$P(x) = {}^nC_x \times p^x \times (1-p)^{n-x}$$

$$P(5) = {}^8C_5 \times (0.75)^5 \times (0.25)^{8-5}$$

$$= \frac{8!}{5! \times (8-5)!} \times (0.75)^5 \times (0.25)^3$$

$$= \frac{8 \times 7 \times 6 \times 5!}{5! \times 3!} \times (0.75)^5 \times (0.25)^3$$

$$= 56 \times 0.237305 \times 0.015625$$

$$= 0.207641602$$

Barika:

No of question per day $(n) = 12$

Correction Rate = $45\% = 0.45$

Probability of incorrect = $1 - p = 1 - 0.45 = 0.55$

Probability to solve 5 question correctly,

$$P(5) = {}^{12}C_5 \times (0.45)^5 \times (0.55)^{12-5}$$

$$= 792 \times (0.45)^5 \times (0.55)^7$$

$$= 0.2225$$

5. Probability of answering 4 questions correctly by Chauran -

$$\begin{aligned} 1) \quad P(4) &= {}^8C_4 \times (0.75)^4 \times (0.25)^4 \\ &= 70 \times (0.75)^4 \times (0.25)^4 \\ &= 0.0865 \end{aligned}$$

Probability of answering 4 questions correctly by Barak

$$\begin{aligned} P(4) &= {}^{12}C_4 \times (0.45)^4 \times (0.55)^{12-4} \\ &= 495 \times (0.45)^4 \times (0.55)^8 \\ &= 0.16996 \\ &\approx \underline{\underline{0.17}} \end{aligned}$$

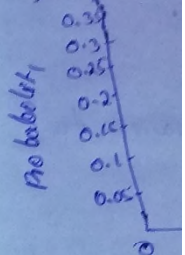
Probability of answering 6 questions correctly by Chauran

$$\begin{aligned} P(6) &= {}^8C_6 \times (0.75)^6 \times (0.25)^{8-6} \\ &= 28 \times (0.75)^6 \times (0.25)^2 \\ &= 0.3115 \end{aligned}$$

Probability of answering 6 questions correctly by Barak

$$\begin{aligned} P(6) &= {}^{12}C_6 \times (0.45)^6 \times (0.55)^{12-6} \\ &= 924 \times (0.45)^6 \times (0.55)^6 \\ &= 0.2124 \end{aligned}$$

Probability by Chauran



Probability by Barakha

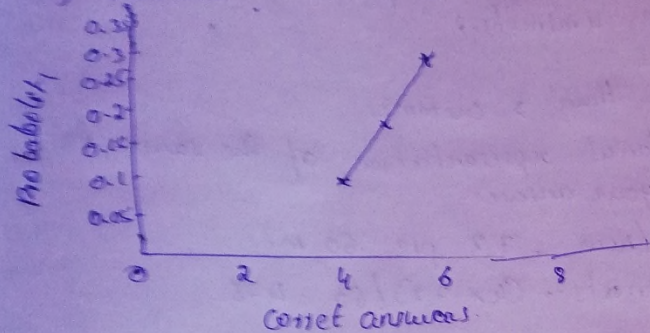
0.2
0.2
0.1
0.1
0.0

* Number two

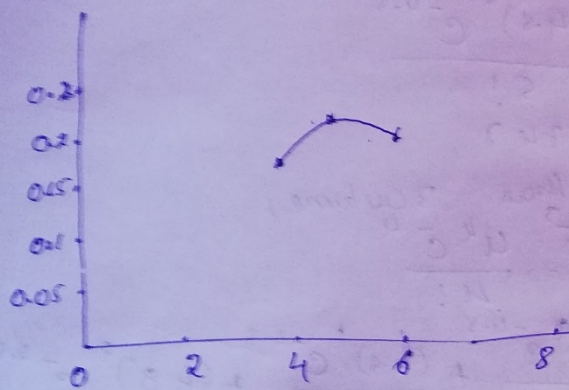
* Cost que ab cor

T

Probability of answering correct answers by Barun



Probability of answering correct answer by Barakha



* Number of questions attempted and correction rate
two main factors affecting these abilities.

* Correction rate is important rather than number of questions attempted. In case of Barakha, he has ability to answer 12 questions per day. But his correction rate is less.

There is no point in answering all the questions with wrong answers, so if correction rate is good, the probability of answering a question will be good.

Problem Statement 3: [100 marks]

Customers arrive at a rate of 72 per hour to my shop. What is the probability of k customers arriving in 4 minutes? a) 5 customers, b) not more than 3 customers, c) more than 3 customers. Give a pictorial representation of the same to validate your answer.

③ Customers arrive at a rate of 72 per hour to my shop. What is the probability of k customers arriving in 4 minutes?

④ 5 customers

⑤ no more than 3 customers

Give a pictorial representation of the same to validate your answer.

$\Rightarrow \lambda = 72 \text{ (hour)} \div 72 \text{ per } 60 \text{ min.}$
for 4 minutes $= (4 \times 72) / 60 = 4.8$

a) 5 customers

$$P(5) = \frac{\lambda^k e^{-\lambda}}{k!}$$
$$= \frac{(4.8)^5 e^{-4.8}}{5!}$$
$$= 0.17467$$

b) not more than 3 customers

$$P(k \leq 3) = \sum_{i=0}^3 \frac{\lambda^k e^{-\lambda}}{k!}$$
$$= \frac{(4.8)^0 e^{-4.8}}{0!} + \frac{(4.8)^1 e^{-4.8}}{1!} + \frac{(4.8)^2 e^{-4.8}}{2!} + \frac{(4.8)^3 e^{-4.8}}{3!}$$
$$= 0.00823 + 0.039503 + 0.094807 + 0.1510$$
$$= 0.29422$$

c) more than 3 customers

$$P(k > 3) = 1 - P(k \leq 3)$$
$$= 1 - 0.294229916$$
$$= 0.705770084$$