(ii) Let the complex f^n be given by: $H(x,y) = f(x,y) + i g(x,y)$

Cauchy-Riemann Criteria: $\frac{\partial f}{\partial x} = \frac{\partial g}{\partial y}$ & $\frac{\partial f}{\partial y} = -\frac{\partial g}{\partial x}$

In Newton's method:

$$x_{n+1} = x_n - \frac{H(x_n, y)}{\partial H/\partial x|_{x_n}} \quad \& \quad y_{n+1} = y_n - \frac{H(x, y_n)}{\partial H/\partial y|_{y_n}}$$

Let us define: $z_n = x_n + i y_n$

$\therefore$ From the above eqn we get:

$$(x_{n+1} + i y_{n+1}) = (x_n + i y_n) - \left[\frac{H(x_n, y)}{\partial H/\partial x} + i \frac{H(x, y_n)}{\partial H/\partial y} \right]$$

$$\Rightarrow \frac{\partial H}{\partial x} = \frac{\partial f}{\partial x} + i \frac{\partial g}{\partial x}$$

$$= \frac{\partial f}{\partial x} - i \frac{\partial f}{\partial y} \quad \text{(Cauchy's criteria)}$$

$$\Rightarrow \frac{\partial H}{\partial y} = \frac{\partial f}{\partial y} + i \frac{\partial g}{\partial y}$$

$$= \frac{i \partial H}{\partial x}$$

$$\therefore z_{n+1} = z_n - \left[\frac{H(x_n, y)}{\partial H/\partial x} + i \frac{H(x, y_n)}{i \partial H/\partial x} \right] = z_n - \left(\frac{H(x_n, y) + H(x, y_n)}{\partial H/\partial x} \right)$$

$$\text{Also } z_{n+1} = z_n - \left(\frac{i H(x_n, y) + i H(x, y_n)}{\partial H/\partial y} \right)$$

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Comparing the above two eqn:

$$\frac{i\{[H(x_n,y)+H(x,y_n)]\}}{\frac{\partial H}{\partial y}} = \frac{H(x_n,y)+H(x,y_n)}{\partial H/\partial x}$$

$$\Rightarrow \boxed{\frac{\partial H}{\partial x} = \frac{-i\,\partial H(x,y)}{\partial y}}$$

$$Z = x+iy \Rightarrow \frac{\partial}{\partial x} = \frac{\partial}{\partial z} \quad \& \quad \frac{\partial}{\partial y} = \frac{i\,\partial}{\partial z}$$

$$\cancel{\frac{\partial H}{\partial x}} = -i\cancel{\frac{\partial H}{\partial y}} \Rightarrow \cancel{\frac{\partial H}{\partial x}}.$$

$$Z_{n+1} = Z_n - \left\{\frac{H(Z_n)}{\partial H/\partial z} + \frac{i\,H(Z_n)}{i\,\partial H/\partial z}\right\}$$

$$Z_{n+1} = Z_n - \frac{H(Z_n)}{\partial H/\partial z|_{Z_n}}$$

Thus we ~~Ate~~ see that Newton's method reduces to one complex variable.