

10 a)

Range : $\phi \rightarrow 0 \text{ to } 2\pi$
 $\theta \rightarrow 0 \text{ to } \pi$

$$\int p(\phi) d\phi = \int_0^{2\pi} \frac{d\phi}{2\pi} = 1 \quad \text{Proved.}$$

$$\int_0^{\pi} p(\theta) d\theta = \int_0^{\pi} \frac{\sin\theta d\theta}{2} = \left[\frac{\cos\theta}{2} \right]_{\pi}^0 = \frac{2}{2} = 1 \quad \text{Proved.}$$

(b) Distribution :

$$p(\phi) d\phi = q(z) dz$$

$$\Rightarrow \int_0^{\phi(z)} \frac{d\phi}{2\pi} = z.$$

$$\int_0^{\theta(z)} \frac{\sin\theta d\theta}{2} = z.$$

$$\therefore \boxed{\phi(z) = 2\pi z}$$

$$1 - \cos(\theta(z)) = 2z$$

$$\therefore \boxed{\theta(z) = \cos^{-1}\{1 - 2z\}}$$