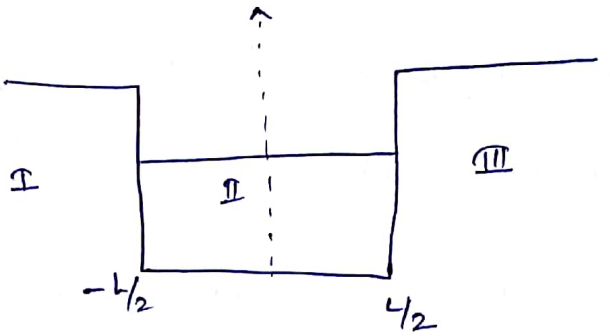
(7)

Finite potential well :



Region – 1 :

$x \leq -\frac{L}{2} \qquad V(x) = V_0$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V_0\psi = E$$

$$\Rightarrow \quad \frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(V_0 - E)\psi$$

Applying the BC : $\psi \to 0$ as $x \to -\infty$

$$\psi = Be^{kx} \qquad \text{where} \quad k = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}$$

Region – 3 : Same calculation as above :

$x > L/2 \qquad V(x) = V_0$

$$\psi = Ae^{-kx} \qquad \{\psi \to 0 \quad \text{as} \quad x \to +\infty\}$$

Region – 2 :

$$-\frac{L}{2} < x < \frac{L}{2}, \qquad V(x) = 0$$

$$\Rightarrow \quad \frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}E\psi$$

$$\therefore \quad \psi = C\cos(kx) \quad \text{(even parity)} \qquad \text{with} \quad k = \sqrt{\frac{2mE}{\hbar^2}}$$

$$\& \quad \psi = D\sin(kx) \quad \text{(odd ")}$$

Matching the soln at boundary:

Even parity:

$$C\cos\left(\frac{kL}{2}\right) = Ae^{-\kappa L/2}$$

$$-C\frac{k}{\kappa}\sin\left(\frac{kL}{2}\right) = -Ae^{-\kappa L/2}$$

$$\therefore \frac{\kappa}{k} = \tan\left(\frac{kL}{2}\right)$$

$$\frac{\kappa}{k} = \tan\left(\frac{kL}{2}\right) \quad \text{even parity}$$

$$\frac{\kappa}{k} = \tan\left(\frac{kL}{2} + \frac{\pi}{2}\right) \quad \text{odd parity}$$

This gives the energy level in finite potential well.

To solve it numerically:

Define $\eta = \frac{kL}{2} = \frac{L}{2}\sqrt{\frac{2mE}{\hbar^2}}$

$$\eta_0 = \frac{L}{2}\sqrt{\frac{2mV_0}{\hbar^2}}$$

This gives

$$\tan(\eta) = \sqrt{\left(\frac{\eta_0}{\eta}\right)^2 - 1} \quad \text{even parity}$$

$$-\cot(\eta) = \sqrt{\left(\frac{\eta_0}{\eta}\right)^2 - 1} \quad \text{odd parity}$$