CTEVT, DIPLOMA, QUESTION & SOLUTION

⇐ SECOND EDITION ⇒

Engineering Mathematics-I

(for Diploma I Yrs. I Part)

First Semester

(Engineering All)

By

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S.No	Exam Year, Month
1.	2076 Falgun Regular/Back
2.	2078 Bharda Regular/Back
3.	2079 Ashad Regular (2021 New)
4.	2079 Ashad Back (Old)
5.	2080 Baishakh Regular (2021 New)
6.	2080 Baishakh Back (Old)
7.	2080/81 Chaitra Regular (2021 New)
8.	2080/81 Chaitra Back (Old)
9.	2080 Poush Regular (scholarship 2021)

Engineering Mathematics I_(Engg. All) 1^{st} Sem

(2076) Question Paper Solution.

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1. a) In any triangle ABC, If $a^4+b^4+c^4-2c^2(a^2+b^2)=0$; prove that $\angle c = 45°$ or $135°$.

➢ **Solution:-**

$$a^4+b^4+c^4-2c^2(a^2+b^2)=0$$

$$\text{or, } a^4+b^4+c^4-2c^2a^2-2c^2b^2=0$$

$$\text{or, } a^4+b^4+c^4+2a^2b^2-2b^2c^2-2c^2a^2=2a^2b^2$$

$$\text{or, } (a^2+b^2-c^2)^2=(2a^2b^2).2\times\frac{1}{2}$$

$$[\because (a+b-c)^2=a^2+b^2+2ab-2bc-2ac]$$

$$\text{or, } \left(\frac{a^2+b^2-c^2}{2ab}\right)^2=\frac{1}{2} \quad \text{or, } \cos^2 C=\frac{1}{2}$$

$$\text{or, } \cos C=\pm\frac{1}{\sqrt{2}}$$

$$\text{or, } \cos C=\cos 45° \text{ or } \cos 135°$$

$$\boxed{\therefore C=45° \text{ or } 135°}$$

b) Prove that: $2\text{Tan}^{-1}\frac{1}{3}+\text{Tan}^{-1}\frac{1}{7}=\frac{1}{4}\pi$.

➢ **Solution:-**

$$\text{L.H.S}: \ 2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{3}\cdot\frac{1}{7}}$$

$$= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{10}{20}$$

$$= \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$$

$$= \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\cdot\frac{1}{2}}\right)$$

$$= \tan^{-1}\left(\frac{5}{5}\right)$$

$$\boxed{= \tan^{-1}1 = \frac{\pi}{4} = \text{R.H.S}}$$

OR) Find the general solution of the equation

$$\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0.$$

➢ **Solution:-**

$$\sin^2\theta - 2\cos\theta + \frac{1}{4} = 0$$

$$1-\cos^2\theta - 2\cos\theta + \frac{1}{4} = 0$$

$$\cos^2\theta + 2\cos\theta + \left(\frac{-5}{4}\right) = 0$$

$$\cos^2\theta + 2\cos\theta + \frac{-5}{4} = 0$$

$$4\cos^2\theta + 8\cos\theta - 5 = 0$$

$$4\cos^2\theta + 8\cos\theta - 5 = 0$$

$$4\cos^2\theta + 10\cos\theta - 2\cos\theta - 5 = 0$$

$$2\cos\theta\,(2\cos\theta + 5) - 1(2\cos\theta + 5) = 0$$

$$(2\cos\theta - 1)\;(2\cos\theta + 5) = 0$$

$$\cos\theta = \frac{1}{2} \text{ or, } \frac{-5}{2}$$

$$\cos\theta \neq \frac{-5}{2} = -2.5 \text{ as } [-1 \leq \cos\theta \leq +1]$$

$$\boxed{\therefore \cos\theta = \frac{1}{2} = \cos\frac{\pi}{3} \quad i.e,\; \theta = 2n\pi \pm \frac{\pi}{3}}$$

2. a) Evaluate: $\lim\limits_{x \to y} \dfrac{x\,Tan\,y - y\,Tan\,x}{x-y}$.

➢ **Solution:-**

The given function takes the indeterminate form $\left(\frac{0}{0}\right)$,when $x = y$.

Given,

$$\lim_{x \to y} \frac{xTany - yTanx}{x - y}$$

$$\lim_{x \to y} \frac{x\,Tany - y\,Tany + y\,Tany - y\,Tanx}{x - y}$$

$$\lim_{x \to y} \frac{(x - y)\,Tany}{(x - y)} + \frac{y(Tany - Tanx)}{(x - y)}$$

$$\lim_{x \to y} \left\{\text{Tan}\, y + \frac{y}{(x - y)} \times \left(\frac{\sin y}{\cos y} - \frac{\sin x}{\cos x}\right)\right\}$$

$$\lim_{x \to y} Tan\,y + \lim_{x \to y} \left\{y \frac{\sin y \cos x - \cos y \sin x}{(x - y)\cos y \cos x}\right\}$$

$$Tan\ y + \lim_{x \to y} \frac{y}{\cos y} \frac{\sin(y-x)}{(x-y)\cos x}$$

$$Tan\ y + \frac{y}{\cos y}\left\{\lim_{x \to y} \frac{\sin(y-x)}{-(y-x)\cos x}\right\}$$

$$Tan\ y + \frac{y}{\cos y} \times (-1) \times \frac{1}{\cos y} \qquad \left\{\because \lim_{x \to y} \frac{\sin\theta}{\theta} = 1\right\}$$

$$\boxed{\therefore \mathbf{Tan\, y - y.sec^2 y}.}$$

b) Test the continuity of $f(x) = \frac{x^2-64}{8-x}$ at $x = 8$.

➢ **Solution:-**

$$f(x) = \frac{x^2-64}{8-x} \ at\ x = 8$$

Left hand limit at x = 8 $= \lim_{x \to 8^-} f(x)$

$$= \lim_{x \to 8^-} \frac{(x^2-64)}{8-x} \left(\frac{0}{0}\right)$$

$$= \lim_{x \to 8^-} \frac{(x^2-8^2)}{8-x}$$

$$= \lim_{x \to 8^-} \frac{(x+8)(x-8)}{-(x-8)}$$

$$= \lim_{x \to 8^-} -(x+8)$$

$$= -(8+8)$$

$$= \mathbf{-16}$$

Right hand limit at x = 8 = $\lim_{x \to 8^+} f(x)$

$$= \lim_{x \to 8^+} \frac{x^2 - 64}{8 - x} \left(\frac{0}{0}\right)$$

$$= \lim_{x \to 8^+} \frac{(x+8)(x-8)}{-(x-8)}$$

$$= \lim_{x \to 8^+} -(x+8)$$

$$= -(8+8)$$

$$= \mathbf{-16}$$

Function value at x = 8 , $f(8) = \frac{8^2 - 64}{8 - x} = \frac{0}{0}$

Hence, $\lim_{x \to 8^-} f(x) = \lim_{x \to 8^+} f(x) \neq f(8)$

The function Discontinues *at x* = 8.

3. a) Show that lines joining the origin to the point of intersection of the line fx−gy = λ and x² + hxy − y² + gx + fy = 0 are at right angles for all values of λ ≠ 0.

➢ **Solution:-**

$x^2 + hxy - y^2 + 9x + fy = 0$ ……… (1)

$f(x) = gy = \lambda$ ……… (2)

$x^2 + hxy - y^2 + 9x = -fy$ ……… (3)

$fx - gy = \lambda$ ……… (4)

From equation (4)

$$\frac{fx - gy}{\lambda} = 1$$

$$x^2 + hxy - y^2 + 9x\left(\frac{fx - gy}{\lambda}\right) + fy\left(\frac{fx - gy}{\lambda}\right) = 0$$

$$\lambda x^2 + 1hxy - \lambda y^2 + fgx^2 - g^2xy + f^2xy - fgy^2 = 0$$

$$x^2(\lambda + fg) + xy(\lambda h + f^2 - g^2) + y^2(-\lambda - fg) = 0 \quad \cdots\cdots(5)$$

This is the equation of line joining the origin and the point common to line (1) and line (2).

Comparing (5) with

$$ax^2 + 2hxy + by^2 = 0$$

We get,

$$a = \lambda + fg$$

$$h = \frac{\lambda h + f^2 - g^2}{2}$$

$$b = (-\lambda - fg)$$

$$\boxed{For\ Right\ angle,\ a + b = 0}$$

$$\lambda + fg + (-\lambda - fg) = 0 \quad \Rightarrow 0 = 0$$

∴**Hence, lines joining the origin to the point of intersection of the line are at right angles**

b) Find the egⁿ of a straight line passing through $(-2, -3)$ and making angle $45°$ with the line $2x - 3y + 5 = 0$

➢ **Solution:-**

Given, $\theta = 45°$

$$2x - 3y + 5 = 0$$

$$y = \frac{2x+5}{3}$$

$$y = \frac{2}{3}x + \frac{5}{3}$$

Hence,

$$y = mx + c$$

So, $m_2 = \frac{2}{3}$

Now,

Let $\theta = 45°$ be Angle between them

$$\tan 45 = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$1 = \left|\frac{m_1 - \frac{2}{3}}{1 + m_1 \frac{2}{3}}\right|$$

$$1 = \pm\frac{(3m_1 - 2)}{3 + 2m_1}$$

$$3 + 2m_1 = \pm(3m_1 - 2)$$

Taking +ve , Sign

$$3 + 2m_1 = (3m_1 - 2)$$

$$5 = m_1$$

Equation of line passing through $(-2, -3)$

$$y - (-3) = m_1(x - (-2))$$

$$y + 3 = m_1(x + 2) \quad \cdots\cdots\cdots (i)$$

$$y + 3 = 5(x + 2)$$

$$y + 3 = 5x + 10$$

$$y = 5x - 7$$

$$\boxed{\boldsymbol{y - 5x + 7 = 0}}$$

and , Taking – ve sign

$$1 = -\left(\frac{3m_1 - 2}{3 + 2m_1}\right)$$

$$3 + 2m_1 = -3m_1 + 2$$

$$5m_1 = -1$$

$$m_1 = \frac{-1}{5}$$

So,

Equation of lines,

$$y - (-3) = m_1(x - (-2))$$

$$y + 3 = -\frac{1}{5}(x + 2)$$

$$5y + 15 = -x - 2$$

$$\boxed{\boldsymbol{x + 5y + 17 = 0}}$$

Hence,

Equation of lines are

$$\boldsymbol{y - 5x + 7 = 0}$$

$$\mathbf{x + 5y + 17 = 0}$$

4) Find the sum of n terms of the series

$1 + 11 + 111 + 1111 + \ldots\ldots\ldots\ldots$

➢ **Solution:-**

Let S_n be the required sum then

$$S_n = 1 + 11 + 111 + 1111 + \cdots \text{to n terms}$$
$$= \frac{1}{9}[9 + 99 + 999 + \cdots \text{to n terms}]$$
$$= \frac{1}{9}[(10-1) + (100-1) + (1000-1) + \cdots \text{to n terms}]$$
$$= \frac{1}{9}[10 + 10^2 + 10^3 + \cdots \textit{to n terms}] - (1 + 1 + 1 + \cdots \textit{to n terms})$$
$$= \frac{1}{9}\left[\frac{10(10^n-1)}{10-1} - n\right] \qquad \left[\because s = \frac{a(r^n-1)}{(r-1)}\right]$$
$$= \frac{10}{81}(10^n - 1) - \frac{n}{9}$$
$$= \frac{1}{9}\left\{\frac{10^{n+1}-10}{9} - n\right\}$$

$$\boxed{\therefore\ \mathbf{S_n = \frac{1}{9}\left\{\frac{10^{n+1}-10}{9} - n\right\}}}$$

5) Show that the quadratic equation $ax^2 + bx + c = 0$ can not have more than two roots.

➢ **Solution:-**

The given quadratic equation is **$ax^2 + bx + c = 0$.**

For, if possible, let α, β, γ be three different roots of quadratics Equation:

$$ax^2 + bx + c = 0 \qquad \cdots\cdots\cdots (a \neq 0)$$

Then, since each of these values must satisfy the equation;

We have,

$$a\alpha^2 + b\alpha + c = 0 \quad \cdots\cdots\cdots\cdots \ (i)$$
$$a\beta^2 + b\beta + c = 0 \quad \cdots\cdots\cdots\cdots \ (ii)$$
$$a\gamma^2 + b\gamma + c = 0 \quad \cdots\cdots\cdots\cdots \ (iii)$$

From equation (i) and (ii) by subtraction; we get,

$$a(\alpha^2 - \beta^2) + b(\alpha - \beta) = 0$$

Since $\alpha \neq \beta$, divide out by $\alpha - \beta$; Then,

$$a(\alpha + \beta) + b = 0$$

Similarly, from equation (ii) and (iii); we get

$$a(\beta + \gamma) + b = 0$$

Hence by subtraction; we get

$a(\alpha - \gamma) = 0$ which is impossible , since, by hypothes is $a \neq 0$, and α is not equal to γ.

Hence, there cannot be more than two different roots.

6) Find the middle term in the expansion of $\left(x + \frac{2}{2x^2}\right)^{12}$.

➢ **Solution:-**

Given, $\left(x + \dfrac{2}{2x^2}\right)^{12}$

Here, the number of terms in the expansion of $\left(x + \frac{2}{2x^2}\right)^{12}$ is $(12 + 1)$

$i.e.$, 13 so, there is only one middle term.

$i.e., t_{\left(\frac{12}{2}\right)+1}$

$= t_{6+1}$

$= c(12,6)\ (x)^{12-6} \left(\frac{2}{2x^2}\right)^6$

$= \frac{(12)!}{(6!)(6!)} \times n^6 . \frac{2^6}{2^6 . x^{12}}$

$= \frac{(12)!}{(6!)^2} \times \frac{1}{x^6}$

$= \frac{12!}{(6!)^2} \times \frac{1}{x^6}$

Hence,

Middle term $= \frac{12!}{(6!)^2} \times \frac{1}{x^6}$

7) Prove that: $1 + \frac{1}{3.2^2} + \frac{1}{5.2^4} + \frac{1}{7.2^6} + \ldots\ldots\ldots\ldots = \log_e 3.$

➢ **Solution:**

We have;

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} - \cdots\cdots (i)$$

$$\log_e(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \frac{x^5}{5} - \frac{x^6}{6} - \frac{x^7}{7} - \cdots\cdots (ii)$$

Subtracting equation (ii) from (i) we get,

$\log_e(1+x) - \log_e(1-x)$

$$= x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-\frac{x^6}{6}+\frac{x^7}{7}-\cdots\cdots+\left\{-\left(-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}-\frac{x^5}{5}\right.\right.$$

$$\left.\left.-\frac{x^6}{6}-\frac{x^7}{7}-\cdots\cdots\right)\right\}$$

Using properties of log, $\log\left(\frac{\text{b}}{\text{a}}\right)= \log(\text{b})-\log(\text{a})$

$$\text{so, } \log_e\left(\frac{1+x}{1-x}\right)= x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-\frac{x^6}{6}+\frac{x^7}{7}-\cdots\cdots+x+\frac{x^2}{2}$$

$$+\frac{x^3}{3}+\frac{x^4}{4}+\frac{x^5}{5}+\frac{x^6}{6}+\frac{x^7}{7}+\cdots$$

On Simplifying ,we get

$$\log_e\left(\frac{1+x}{1-x}\right)=2\left(x+\frac{x^3}{3}+\frac{x^5}{5}+\cdots\cdots\right)$$

Putting $x=\frac{1}{2}$,

$$\log_e 3 = 1+\frac{1}{3}.\frac{1}{2^2}+\frac{1}{5}.\frac{1}{2^4}+\cdots\cdots$$

$$\therefore \mathbf{1+\frac{1}{3.2^2}+\frac{1}{5.2^4}+\frac{1}{7.2^6}+\cdots\cdots=\log_e 3.}$$

8) Find, from first principle, the derivative of $f(x)=\frac{1}{\sqrt{x}}$

➢ **Solution:**

$$Let\ f(x) = y = \frac{1}{\sqrt{x}} \quad \cdots\cdots\cdots(i)$$

Let Δx *be* small increment in x and Δy be the corresponding small increment in y Then,

$$y + \Delta y = \frac{1}{\sqrt{x + \Delta x}} \quad \cdots\cdots\cdots (ii)$$

Subtracting eqn (i) from (ii) we get,

$$y + \Delta y - y = \frac{1}{\sqrt{x + \Delta x}} - \frac{1}{\sqrt{x}}$$

$$\Delta y = \frac{1}{\sqrt{x + \Delta x}} - \frac{1}{\sqrt{x}}$$

$$\Delta y = \frac{\sqrt{x} - \sqrt{x + \Delta x}}{\sqrt{x + \Delta x}\ \sqrt{x}}$$

$$\Delta y = \frac{\sqrt{x} - \sqrt{x + \Delta x}}{\sqrt{x + \Delta x}\ \sqrt{x}} \times \frac{\sqrt{x} + \sqrt{x + \Delta x}}{\sqrt{x} + \sqrt{x + \Delta x}}$$

$$\Delta y = \frac{x - (x + \Delta x)}{\sqrt{x + \Delta x}\ \sqrt{x}} \times \frac{1}{\sqrt{x} + \sqrt{x + \Delta x}}$$

$$\Delta y = \frac{-\Delta x}{\sqrt{x + \Delta x}\ \sqrt{x}} \times \frac{1}{\sqrt{x} + \sqrt{x + \Delta x}}$$

$$\frac{\Delta y}{\Delta x} = \frac{\Delta x}{\Delta x\sqrt{x + \Delta x}\ \sqrt{x}} \times \frac{-1}{\sqrt{x} + \sqrt{x + \Delta x}}$$

$$\frac{\Delta y}{\Delta x} = \frac{1}{\sqrt{x + \Delta x}\ \sqrt{x}} \times \frac{1}{\sqrt{x} + \sqrt{x + \Delta x}}$$

By first Principle,

$$\frac{dy}{dx} = \Delta x \overset{\lim}{\longrightarrow} 0 \frac{\Delta y}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{1}{\sqrt{x+\Delta x}\ \sqrt{x}} \times \frac{-1}{\sqrt{x}+\sqrt{x+\Delta x}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x+0}\ \sqrt{x}} \times \frac{-1}{\sqrt{x}+\sqrt{x+0}}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x}\ \sqrt{x}} \times \frac{-1}{\sqrt{x}+\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{1}{x} \times \frac{-1}{2\sqrt{x}}$$

$$\therefore \boldsymbol{\frac{dy}{dx} = \frac{-1}{2x^{\frac{3}{2}}}}$$

or) $f(x) = Tan\, 4x.$

➢ **Solution:**

$f(x) = Tan\, 4x$

$Let\ y = f(x) = Tan\, 4x \quad \cdots\cdots\cdots (i)$

Let Δx be a small increment in x and Δy be corresponding increment in y.

Then, $y + \Delta y = \text{Tan}\, 4(x + \Delta x) \quad \cdots\cdots\cdots (ii)$

Subtracting (i) from (ii),

$y + \Delta y - y = Tan\, 4(x + \Delta x) - tan\, 4x$

$\Delta y = Tan\, 4(x + \Delta x) - tan\, 4x$

Dividing both side by Δx,

$$\frac{\Delta y}{\Delta x} = \frac{\tan 4(x + \Delta x) - \tan 4x}{\Delta x}$$

By first Principle,

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\tan 4(x+\Delta x) - \tan 4x}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\frac{\sin 4(x+\Delta x)}{\cos 4(x+\Delta x)} - \frac{\sin 4x}{\cos 4x}}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\frac{\sin 4(x+\Delta x).\cos 4x - \sin 4(x).\cos 4(x+\Delta x)}{\cos 4(x+\Delta x).\cos 4x}}{\Delta x}$$

$$\because [\sin(a-b) = \sin a.\cos b - \cos a.\sin b]$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\frac{\sin[4(x+\Delta x) - 4x]}{\cos 4(x+\Delta x).\cos 4x}}{\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\sin 4\Delta x}{\cos 4(x+\Delta x).\cos 4x.\Delta x}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{1}{\cos 4(x+\Delta x).\cos 4x} \times \frac{\sin 4\Delta x}{4\Delta x} \times 4$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{1}{\cos 4(x+\Delta x).\cos 4x} \times \lim_{\Delta x \to 0} \frac{\sin 4\Delta x}{4\Delta x} \times 4$$

$$\frac{dy}{dx} = \frac{1}{\cos 4(x+0).\cos 4x} \times 1 \times 4 \qquad \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$\frac{dy}{dx} = \frac{1}{\cos 4x.\cos 4x} \times 1 \times 4$$

$$\therefore \frac{dy}{dx} = 4\sec^2 4x$$

9) Find $\frac{dy}{dx}$ (Any one):

i) $x^3 + y^3 = 3xy^2$

➢ **Solution:**

Given, $x^3 + y^3 = 3xy^2$

Differentiating both sides with respect to x, we have

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(3xy^2)$$

$$or,\ \frac{dx^3}{dx} + \frac{dy^3}{dy}.\frac{dy}{dx} = 3x\ \frac{dy^2}{dy}.\frac{dy}{dx} + 3y^2\frac{dx}{dx}$$

$$or,\ 3x^2 + 3y^2\frac{dy}{dx} = 3x.2y\ \frac{dy}{dx} + 3y^2$$

$$or,\ (3x^2 - 6xy)\frac{dy}{dx} = 3y^2 - 3x^2$$

$$or,\ \frac{dy}{dx} = \frac{3y^2 - 3x^2}{3y^2 - 6xy} = \frac{y^2 - x^2}{y^2 - 2xy}$$

$$\therefore\ \boldsymbol{\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 - 2xy}}$$

ii) $x^2 + y^2 = Tan\ xy$

➢ **Solution:**

Given, $x^2 + y^2 = Tan\ xy$

Differenting both side w.r.t 'x'

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}Tan(xy)$$

$$\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = \frac{d}{dx}tan(xy)$$

$$2x + \frac{dy^2}{dy} \times \frac{dy}{dx} = \frac{dtan(xy)}{d(xy)} \times \frac{d(xy)}{dx}$$

$$2x + 2y\,\frac{dy}{dx} = sec^2(xy) \times \left[x\frac{dy}{dx} + y.\frac{dx}{dx}\right]$$

$$2x + 2y\,\frac{dy}{dx} = sec^2(xy)\left[x\frac{dy}{dx} + y\right]$$

$$2x + 2y\,\frac{dy}{dx} = x\,sec^2(xy)\frac{dy}{dx} + y\,sec^2(xy)$$

$$2x - y\,sec^2(xy) = (x\,sec^2(xy) - 2y)\frac{dy}{dx}$$

$$\frac{2x - y\,sec^2(xy)}{x\,sec^2(xy) - 2y} = \frac{dy}{dx}$$

$$\therefore\ \boldsymbol{\frac{dy}{dx} = \frac{2x - y\,sec^2(xy)}{x\,sec^2(xy) - 2y}}$$

10) Evaluate: $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-4}}$

➢ **Solution:** $Let, I = \int \frac{dx}{x^2\sqrt{x^2-4}}$

$$Let\ x = 2\,sec\theta, cos\theta = \frac{2}{x}, sin\theta = \sqrt{1 - \frac{4}{x^2}} = \frac{\sqrt{x^2-4}}{x}$$

$$or, dx = 2\,sec\theta\tan\theta\,d\theta$$

Integral becomes,

$$I = \int \frac{2\sec\theta \tan\theta\, d\theta}{(2\sec\theta)^2\sqrt{(2\sec\theta)^2 - 4}}$$

$$I = \int \frac{2\sec\theta \tan\theta\, d\theta}{(2\sec\theta)^2\sqrt{4[(\sec\theta)^2 - 1]}}$$

$$= \int \frac{2\sec\theta \tan\theta\, d\theta}{4\sec^2\theta \times \sqrt{4\tan^2\theta}}$$

$$[\because \sec^2\theta - 1 = \tan^2\theta]$$

$$= \int \frac{2\sec\theta \tan\theta\, d\theta}{4\sec^2\theta \times 2.\tan\theta}$$

$$= \int \frac{1}{4}\frac{1}{\sec\theta}\, d\theta$$

$$I = \frac{1}{4}\int \cos\theta\, d\theta$$

$$\therefore \mathbf{I} = \frac{\mathbf{1}}{\mathbf{4}}\frac{\sqrt{x^2 - 4}}{x} + c$$

11) Evaluate: $\int_0^{\pi/4} Tan^3 x dx.$

➢ **Solution:**

First, we evaluate $\int tan^3 x\, dx$

Let $I = \int tan^3 x\, dx$

$$= \int \tan x \ tan^2 x \, dx$$

$$= \int \tan x \,(sec^2 x - 1.) dx$$

$$= \int (\tan x \ \ sec^2 x - \tan x \,) \ dx$$

$$= \int \tan x \ \ sec^2 x \ dx - \int \tan x \ dx = \mathrm{I}_2 - \mathrm{I}_1$$

***Let,* $\boldsymbol{y = \ tan\, x}$*, then* $\boldsymbol{dy = sec^2\, x \ \ dx}$**

$$\therefore \int \tan x \, sec^2 x \, dx = \int y \, dy = \frac{y^2}{2} + c_1 = \frac{1}{2} tan^2 x + c_2$$

$$and, \ \ \mathrm{I}_2 = \int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$$

Let $\boldsymbol{y \ = \ \cos x, \mathrm{d}y = -\sin x \, \mathrm{d}x}$

$$\therefore \ \mathrm{I}_2 = \int \frac{\sin x}{\cos x} \ dx$$

$$= \int \frac{dy}{y}$$

$$= -\log y + c_2$$

$$= -\log \cos x + c_2$$

$$\therefore \mathbf{I} = \mathbf{I_1} - \mathbf{I_2}$$

$$= \frac{1}{2} tan^2 x + c_1 - \{-\log \cos x + c_2\}$$

$$= \frac{1}{2} \tan^2 x + \log \cos x + c_1 - c_2$$

$$= \frac{1}{2}\tan^2 x + \log\cos x + c \qquad [\because c = c_1 - c_2]$$

$$\therefore I = \int_0^{\pi/4} Tan^3 x dx$$

$$= \left[\frac{1}{2} tan^2 x + log\ cos\ x + c\right]_0^{\pi/4}$$

$$= \frac{1}{2} tan^2\left(\frac{\pi}{4}\right) + log\ cos\left(\frac{\pi}{4}\right) - tan^2 0 - log\ cos\ 0$$

$$= \frac{1}{2} + log\left(\frac{1}{\sqrt{2}}\right) - 0 - \log 1$$

$$I = \frac{1}{2} + log\left(\frac{1}{\sqrt{2}}\right)$$

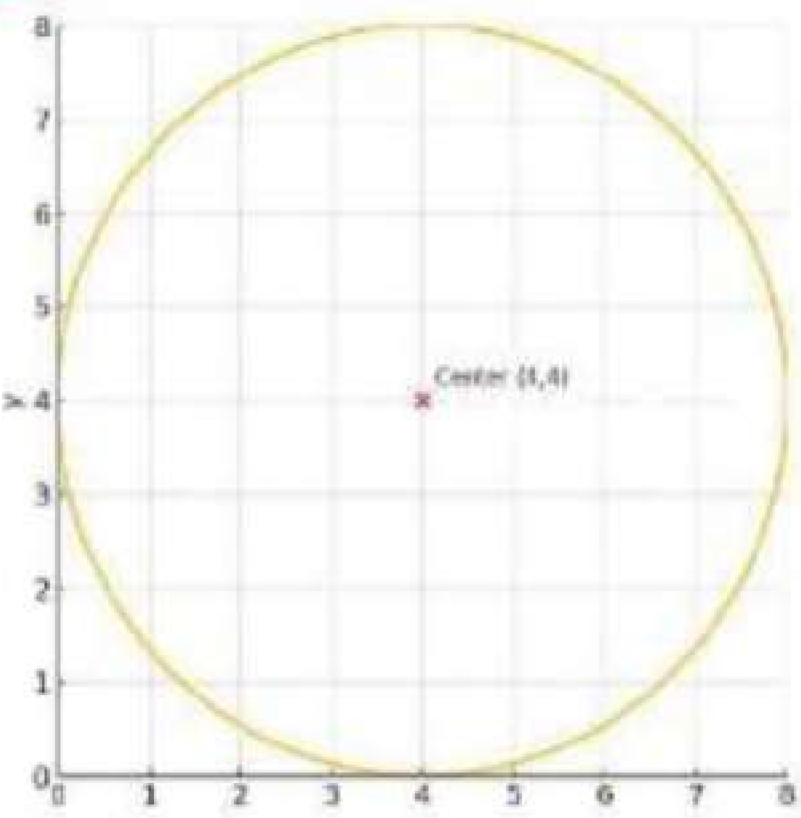
12) Find the equation of a circle which touches both axes and radius is 4.

➢ **Solution:**

Given, Radius is $r = 4$.

Circle touches both axes *i.e*,

$$h = k = r = 4$$



Equation of circle is

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-4)^2 + (y-4)^2 = 4^2$$

$$x^2 - 8x + 16 + y^2 - 8y + 16 = 16$$

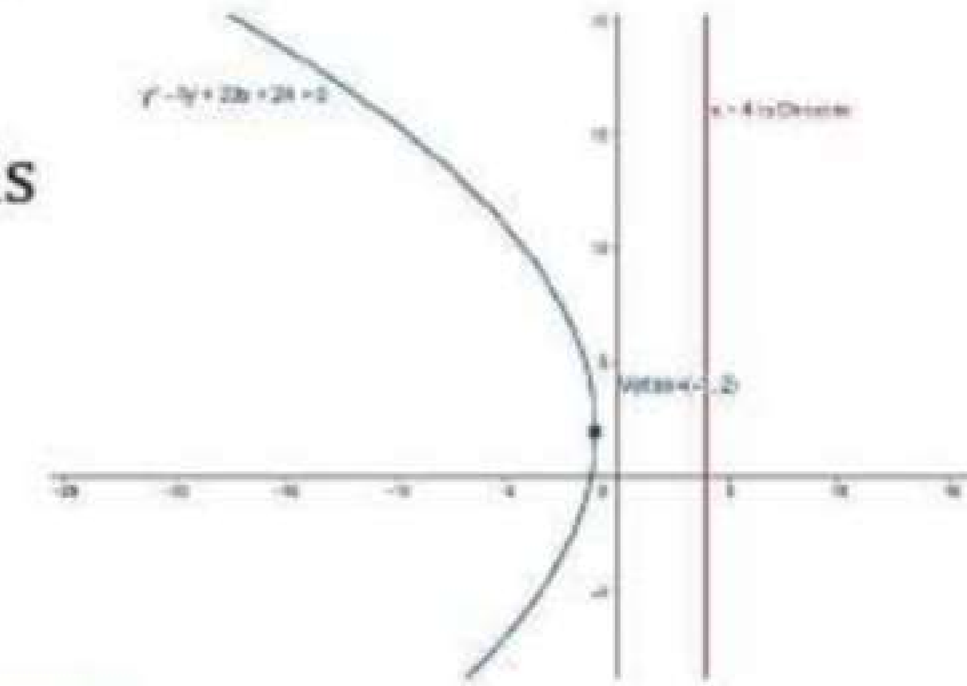
$$x^2 - 8x + 16 + y^2 - 8y + 16 - 16 = 0$$

$\therefore \boldsymbol{x^2 + y^2 - 8x - 8y + 16 = 0}$ **is required equation.**

13) Find the equation of the parabola whose vertex is at $(-1, 2)$ and directrix $x = 4$.

➢ **Solution:**

Here vertex of parabola $(h, k) = (-1, 2)$ and equation of directrix is $x = 4$



We know that the equation of directrix is

$$x - h + a = 0$$

$$or,\ x = h - a$$

$$\therefore 4 = -1 - a$$

$$or,\ a = -5$$

Hence the required equation of parabola is

$$(y - k)^2 = 4a(x - h)$$

$$(y - 2)^2 = 4(-5)(x + 1)$$

$$or,\ (y - 2)^2 = -20(x + 1)$$

$$or,\ y^2 - 4y + 4 + 20x + 20 = 0$$

$$or,\ y^2 - 4y + 20x + 24 = 0$$

Hence, The Required equation of the parabola is $y^2 - 4y + 20x + 24 = 0$

14) From 6 gentlemen and 4 laties a committee of 5 is to be formed. In how many ways can this be done so as to include at least one lady.

➢ **Solution:**

The selection of member in the committee can made as follows:

Gentlemen(6)	Ladies(4)	Selection
4	1	C(6,4) × C(4,1)
3	2	C(6,3) × C(4,2)
2	3	C(6,2) × C(4,3)
1	4	C(6,1) × C(4,4)
Total = 5 member to formed		

The required no. of committee such that to include at least one lady.

$$= C(6,4) \times C(4,1) + C(6,3) \times C(4,2) + C(6,2) \times C(4,3) + C(6,1) \times C(4,4)$$

$$= \frac{6!}{2!\,4!} \times \frac{4!}{3!\,1!} + \frac{6!}{3!\,3!} \times \frac{4!}{2!\,2!} + \frac{6!}{4!\,2!} \times \frac{4!}{1!\,3!} + \frac{6!}{5!\,1!} \times \frac{4!}{0!\,4!}$$

$$= 15 \times 4 + 20 \times 6 + 15 \times 4 + 6 \times 1$$

$$= 60 + 120 + 60 + 6$$

$$= \mathbf{246}\ \boldsymbol{ways}.$$

15) Define inverse of a function. In which condition does the inverse of function exist? If $f(x) = x^2 - 3$ find $f^{-1}(x)$.

- Let f: A → B be a function from set A to set B. **If f is one-one and onto both, then there exists a function from set B to set A such that each elements of set B is associated with unique elements of set A is known as the *inverse function of f*.** If f: A → B is one-one and onto, then f^1 : B → A is the inverse function of f. The required condition for the function to have **its inverse** is that the **function one to one and onto both.**

$Again, f(x) = x^3 - 3$

Let, $\boldsymbol{x_1, x_2 \in R}$

$Then,\ f(x_1) = (x_1)^3 - 3$

$and\ \ f(x_2) = (x_2)^3 - 3$

Now, $f(x_1) = f(x_2)$

$\Rightarrow (x_1)^3 - 3 = (x_2)^3 - 3$

$\Rightarrow (x_1)^3 = (x_2)^3$

$\Rightarrow x_1 = x_2$

$\therefore$ F is one – one function.

Let $\boldsymbol{y = f(x)}$ ***with*** $\boldsymbol{y \in R}$***, then***

$y = x^3 - 3$

$or,\ x = (y + 3)^{1/3}$

For each $y \in R$, $x \in R$, so f is onto. Since, f is one-one and onto both, f^{-1} exists.

Let x be the image of y under $\boldsymbol{f^{-1}}$***, ie.*** $\boldsymbol{x = f^{-1}(y)}$.

Solving for x in terms of y, we have

$x = (y + 3)^{1/3}$

$i.e., \boldsymbol{f^{-1}}(y) = (y + 3)^{1/3}$

$\therefore \boldsymbol{f^{-1}(x) = (x + 3)^{1/3}}$

–The End –

Engineering Mathematics I_(Engg. All) 1^{st} Sem

(2078) Question Paper Solution.

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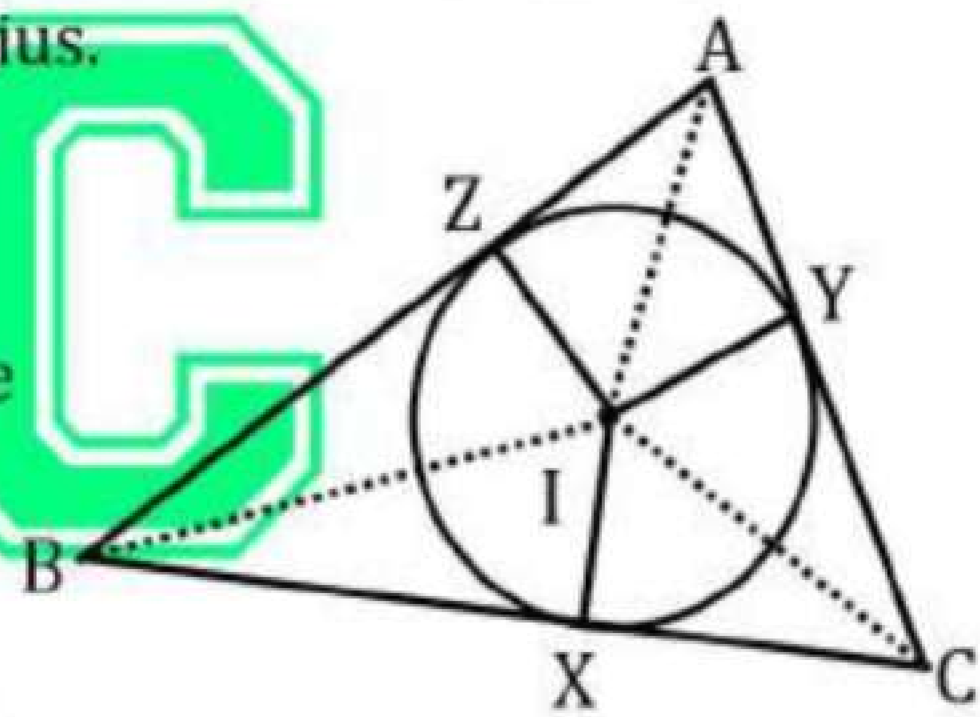
1. a) Define in circle. In any triangle ABC, establish the relation $r = \frac{\Delta}{s}$ where symbols have their usual meanings.

➢ A circle drawn within the triangle such that each side is tangent to the circle is known as the in-circle. The centre and the radius in the circle are known as the in-centre and in-radius.



A circle drawn outside the triangle such that a side and two remaining side after producing are tangent to the circle, is known as ex-circle.

The centre and the radius in this circle are known as the ex-centre and the ex-radius respectively.
Three ex-circles can be drawn in a triangle

Let I be the in-centre of the triangle ABC. Let X,Y and Z be the points of contacts of the in-circle with the sides BC,CA and AB respectively.

$IX = IY = IZ = r =$ radius of the circle.

Also IX, IY and IZ are perpendicular to BCCA and AB respectively.

Join IA, IB and IC.

$$\Delta\, ABC = \Delta\, BIC + \Delta CIA + \Delta AIB$$

$$= \frac{1}{2}\, BC.\, 1X + \frac{1}{2}\, CA.\, IY + \frac{1}{2}\, AB.\, IZ$$

$$\Delta = \frac{1}{2}\, a.r + \frac{1}{2}\left(b.r + \frac{1}{2}\, c.r\right)$$

$$= \frac{1}{2}\, r(a + b + c) \qquad \because \frac{(a+b+c)}{2} = s$$

$$= \frac{1}{2}\, r.2s$$

$$\therefore \quad \boldsymbol{r = \frac{\Delta}{s}}$$

b) Find the general solution of $\mathbf{Cos\theta + Cos3\theta + Cos5\theta = 0}$

➢ **Solution:-**

$$Cos\theta + Cos3\theta + Cos5\theta = 0$$

$$(Cos5\theta + Cos\theta) + Cos3\theta = 0$$

$$2\cos\frac{5\theta + \theta}{2}\cos\frac{5\theta - \theta}{2} + \cos 3\theta = 0$$

$$2\cos 3\theta \cos 4\theta + \cos 3\theta = 0$$

$$\cos 3\theta\,(2\cos 4\theta + 1) = 0$$

Either,

$$\cos 3\,\theta = 0$$

$$3\theta = (2n+1)\frac{\pi}{2}$$

$$\boldsymbol{\theta = (2n+1)\frac{\pi}{6}}$$

OR

$$2\cos 4\,\theta + 1 = 0$$

$2\cos 4\theta = -1$

$\cos 4\theta = -\frac{1}{2}$

$\cos 4\theta = cos\frac{2\pi}{3}$

$4\theta = 2n\pi \pm \frac{2\pi}{3}$

$\theta = \frac{n\pi}{2} \pm \frac{\pi}{6}$

$\boldsymbol{\theta = \frac{\pi}{6}(3n \pm 1)}$

Hence,

$\boldsymbol{\theta = (2n+1)\frac{\pi}{6}, \ \ \theta = \frac{\pi}{6}(3n \pm 1)}$

or) Prove that: $\boldsymbol{tan^{-1}x - tan^{-1}y = tan^{-1}\frac{x-y}{1+xy}}$

➢**Solution:-**

Let $tan^{-1}x = A$ *and* $tan^{-1}y = B$ *then,*

$tan\,A = x$ *and* $tan\,B = y$

Now,

$$tan(A+B) = \frac{tan\,A + tan\,B}{1 - tan\,A\,tan\,B}$$

$$= \frac{x+y}{1-xy}$$

Hence,

$$\boldsymbol{\therefore tan^{-1}x - tan^{-1}y = tan^{-1}(\frac{x+y}{1-xy})}$$

2. a) Define continuity of a function at a given point. Test the continuity of the function at a given point where

$$f(x) = \begin{bmatrix} 3 + 2x & for & \frac{-3}{2} \le x \le 0 \\ 3 - x^2 & for & 0 < x \le {}^3/_2 \end{bmatrix} \{\text{at } x = 0$$

➢ *A function f is said to be continuous at $x = a$ if,*

- $f(a)$ *is defined (i.e, it exists at* $x = a$*)*
- $\lim_{x \to a} f(x)$ *exists.*
- $\lim_{x \to a} f(x) = f(a)$

➢ **Solution :-**

$$f(x) = \begin{bmatrix} 3 + 2x & for & \frac{-3}{2} \le x \le 0 \\ 3 - x^2 & for & 0 < x \le {}^3/_2 \end{bmatrix} \{at\ x = 0$$

LHS limit at $x = 0$

$$\lim_{x \to 0^-} f(x) = 3 + 2x$$
$$= 3 + 0$$
$$= 3$$

RHS Limit at $x = 0$

$$\lim_{x \to 0^+} f(x) = 3 + x^2$$
$$= 3 + 0$$
$$= 3$$

$$\therefore \quad \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x) = 3$$

f(x)is continuous at $x = 0$

Or) Evaluate the limit of lim of $\lim_{x \to \theta} \frac{x \sin\theta - \theta \sin x}{x - \theta}$

➢ **Solution:-**

$$\lim_{x \to \theta} \frac{x \sin\theta - \theta \sin x}{x - \theta}$$

$$= \lim_{x \to \theta} \frac{x \sin\theta - \theta \sin\theta + \theta \sin\theta - \theta \sin x}{x - \theta}$$

$$= \lim_{x \to \theta} \frac{\sin(x - \theta) + \theta(\sin\theta - \theta \sin x)}{x - \theta}$$

$$= \lim_{x \to \theta} \frac{\sin\theta.(x - \theta)}{x - \theta} + \lim_{x \to \theta} \frac{\theta(\sin\theta - \sin x)}{x - \theta}$$

$$= \sin\theta + \lim_{x \to \theta} \frac{-2\theta \sin\left(\frac{x - \theta}{2}\right) \cos\left(\frac{\theta + x}{2}\right)}{x - \theta}$$

$$= \sin\theta - \theta \lim_{x \to \theta} \frac{\sin\left(\frac{x - \theta}{2}\right)}{\frac{x - \theta}{2}} \lim_{x \to \theta} \cos\left(\frac{\theta + x}{2}\right)$$

$\therefore \sin\theta - \theta\cos\theta.$

b) Find from first principle the derivatives of $y = \tan x$ or $y = e^{ax}$

➢ **Solution:-**

Let $y = \tan x$

Let δx be the increment of x and δy, the corresponding increment of y.

$$y + \delta y = \tan(x + \delta x)$$

On Subtracting

$$\delta y = \tan(x + \delta x) - \tan x$$

$$\delta y = \left[\frac{\sin(x + \delta x)}{\cos(x + \delta x)}\right] - \frac{\sin x}{\cos x}$$

$$= \frac{\sin(x + \delta x)\cos x - \cos(x + \delta x)\, sinx}{\cos x \cos(x + \delta x)}$$

$$= \frac{\sin(x + \delta - x)}{\cos x \cos(x + \delta x)}$$

$$= \frac{sin\delta x}{\cos x \cos(x + \delta x)}$$

$$[\because \sin A \cos B - \cos A \sin B = \sin(A - B)]$$

$$\frac{\delta y}{\delta x} = \frac{\boldsymbol{sin}\delta x}{\delta x} \frac{\mathbf{1}}{\boldsymbol{cos\, x\, cos}(x + \delta x)}$$

Taking limit $\delta x \to 0$ on both sides,

$$\lim_{\delta x \to \theta} \frac{\delta y}{\delta x} = \lim_{\delta x \to \theta} \frac{\sin \delta x}{\delta x} \frac{1}{\cos x \cos(x + \delta x)}$$

$$\frac{dy}{dx} = \frac{1}{\cos x \cos(x + 0)}$$

$$\frac{dy}{dx} = \frac{1}{\cos x \cos x}$$

$$\frac{dy}{dx} = \frac{1}{cos^2 x} = sec^2 x$$

$$\therefore \frac{\boldsymbol{d}(\tan x)}{\boldsymbol{dx}} = \boldsymbol{sec^2 x}$$

or) $y = e^{ax}$

➢ **Solution:-**

Let $\boldsymbol{y = e^{ax}}$

Let δx *be the increment of* x *and* δy, *the corresponding increment of* y.

$$\boldsymbol{y + \delta y = \tan(x + \delta x)}$$

On Subtracting

$$\delta y = e^{a(x+\delta x)} - e^{ax}$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} e^{ax} \frac{[e^{a\delta x} - 1]}{a\,\delta x} \times a$$

$$\frac{dy}{dx} = ae^{ax} \lim_{\delta x \to 0} \frac{[e^{a\delta x} - 1]}{a\delta x}$$

$$\therefore \boldsymbol{\frac{dy}{dx} = ae^{ax}}$$

3. a) What is homogenous equation of second degree? Prove that the homogenous equation of second degree represent a pair or straight line through the origin.

➢ A polynomial equation of two independent variable x and y is said to be homogeneous equation of degree n if sum indices of x and y in each term is n.

$ax^2 + 2hxy + by^2$ is a homogeneous equation of second degree in x and y.

Let us take the homogeneous equation of second degree in x and y as

$$ax^2 + 2hxy + by^2 \quad \cdots\cdots\cdots (i)$$

Dividing both sides by x^2

$$b\left(\frac{y}{x}\right)^2 + 2h\left(\frac{y}{x}\right) + a = 0 \quad \cdots\cdots\cdots (ii)$$

Substituting $\frac{y}{x} =$ m, Eqn(ii) becomes

$$bm^2 + 2hm + a = 0 \quad \cdots\cdots\cdots (iii)$$

Which is quadratic eqn. in m.

Let m_1 and m_2 be roots of this quadratic equation,

[If α and β are roots of $ax^2 + bx + c$, then $ax^2 + bx + c = a(x - \alpha)(x - \beta)$]

***For passes through orgin* $a = 0, h = 0$, Eqn(iii) Becomes.**

$$b(m - m_1)(m - m_2) = 0$$

$$b\left(\frac{y}{x} - m_1\right)\left(\frac{y}{x} - m_2\right) = 0$$

$$\frac{y}{x} - m_1 = 0 \;\&\; \frac{y}{x} - m_2 = 0$$

$$\boldsymbol{y = m_1 x \;\; and \;\; y = m_2 x}$$

Which pass through the origin.

b) Find the equation of the straight line through the point $(2, 3)$ and perpendicular to the line $5x - 2y = 8$.

➢ **Solution:-**

The equation of line through (2 ,3) is

$$y - 3 = m(x - 2) \quad \cdots\cdots\cdots\cdots (i)$$

The given line is

$$5x - 2y = 8 \quad \cdots\cdots\cdots\cdots (ii)$$

Slope of line (i) $m_1 = m$

Slope of line (ii)

$$m_2 = -\frac{coefficient\ of\ x}{coefficient\ of\ y}$$

$$= -\frac{5}{-2} = \frac{5}{2}$$

If the linear (i) and (ii) are perpendicular,

$$m_1 m_2 = -1$$

$$m \times \frac{5}{2} = -1$$

$$m = -\frac{2}{5}$$

Substituting value of $m = \frac{-2}{5}$ in equation, (i), we get

$$y - 3 = -\frac{2}{5}(x - 2)$$

$$5y - 15 = -2x + 4$$

$$2x + 5y - 19 = 0$$

$\therefore$ **$2x + 5y - 19$ is the required Equation.**

4) Sum to n terms of 7+77+777+……………..

➢ **Solution:-**

Let s_n be the required sum then

$$s_n = 7 + 77 + 777 + \cdots \ to\ n\ terms$$

$$= \frac{7}{9}\,[9 + 99 + 999 + \cdots .\ to\ n\ terms]$$

$$= \frac{7}{9}\,[(10-1) + (100-1) + (1000-1) + \cdots .\ to\ n\ terms]$$

$$= \frac{7}{9}\,[10 + 10^2 + 10^3 + \cdots to\ n\ terms\,] - (1 + 1 + 1 + \cdots to\ n\ terms)$$

$$= \frac{7}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right]$$

$$\therefore\ \ S_n = \frac{70(10^n - 1)}{81} - \frac{7n}{9}$$

5) From a group of 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done so as to include at most 2 lady?

➢ **Solution:-**

For including at least most two girls, the following case arise

i) (1 lady out of 4) and (4 men out of 6)
ii) Or (2 ladies out of 4) and (3 men out of 6)

The number of ways of these selection are

i) 1 ladies + 4 gentlemen $in\ {}^4C_1 \times {}^6C_4 = 15 \times 4 = 60$

ii) 2 ladies +3 gentlemen $= {}^4C_2 \times {}^4C_3 = 20 \times 6 = 120$

Total number of ways committee can be formed is $60 + 120 = \mathbf{180}$.

6) Find the middle term (s) in the expansion of $\left(1+\frac{x}{2}\right)^{15}$.

➢ **Solution:-**

Here n = 15. Since it is odd. It has two middle terms which are

$$\left(\frac{15+1}{2}\right)^{th} \text{ and } \left(\frac{15+1}{2}+1\right)^{th} \text{ or } T_8 \text{ and } T_9$$

$$T_8 = T_{7+1} = C(15,7)(1)^{9-7}\left(\frac{x}{2}\right)^7 \quad [\because T_{r+1} = C(n,r)a^{n-r}x^r]$$

$$= C(15,7)\frac{x^7}{128}$$

$$= \frac{C(15,7)x^7}{128}$$

$$= \frac{6435x^7}{128}$$

Similarly,

$$T_9 = T_{8+1} = C(15,8)(1)^{9-8}\left(\frac{x}{2}\right)^8$$

$$= \frac{\mathbf{6435x^8}}{\mathbf{256}}$$

7) Prove that every quadratic equation cannot have more than two roots.

➢ **Refer to the solution 2076 of Q. No 5 on page 11.**

8) Find the equation of the circle through the intersection of the circles $x^2 + y^2 - 8x - 2y + 7 = 0$ and $x^2 + y^2 - 4x + 10y + 8 = 0$ and passes through the point $(-1, -2)$.

➢ **Solution:-** Equation of circle through the intersection of the given circles is

$(x^2 + y^2 - 8x - 2y + 7) + \boldsymbol{k}(x^2 + y^2 - 4x + 10y + 8) = 0 \cdots\cdots (i)$

Since, $(\boldsymbol{i})$ passes through the point $(-1, -2)$,

$$(1 + 4 + 8 + 4 + 7) + k(1 + 4 + 4 - 20 + 8) = 0$$

$$24 + k \times -3 = 0$$

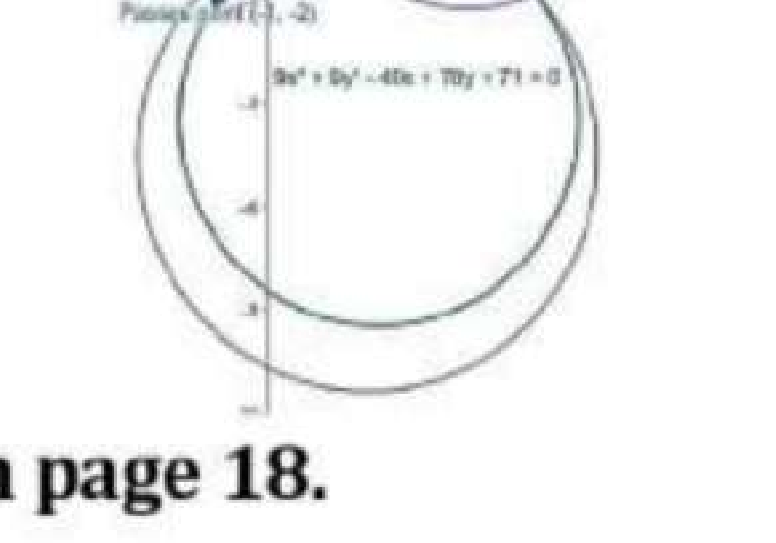
$$or, \;\; k = 8$$

Putting the value of k in $(\boldsymbol{i})$ we get

$$(x^2 + y^2 - 8x - 2y + 7) + 8(x^2 + y^2 - 4x + 10y + 8) = 0$$

$$or, \;\; 9x^2 + 9y^2 - 40x + 78y + 71 = 0$$

Hence, which is the required equation.



9) Find $\frac{dy}{dx}$: $(Any\ One)$

a) $x^3 + y^3 = 3xy^2$

➢ **Refer to the solution 2076 of Q. No 9 $(\boldsymbol{i})$ on page 18.**

b) $x = Tan\, t, y = sin\, t\, cos\, t$

➢**Solution:** $x = Tant \cdots\cdots\cdots (i)$

$y = sint\, cos\, t \cdots\cdots\cdots (ii)$

$Differenting\ (i)\ and\ (ii)\ with\ to\ t, we\ get$

$$\frac{dx}{dt} = sec^2 t$$

$$\& \frac{dy}{dt} = sint.\frac{dcost}{dt} + cost.\frac{dsint}{dt}$$

$$= -sin^2 t + cos^2 t$$

$$= cos^2 t - sin^2 t$$

$$= \mathbf{cos2\, t}$$

We have, By Chain Rule:

$$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{\mathbf{cos2t}}{\mathbf{sec^2 t}}$$

10) Integrate : (Any One)

i) $\int x^2\, sinx\, dx$

➢**Solution:**

$$\Rightarrow \int x^2 sindx = x^2 \int sinxdx - \int \left(\frac{dx^2}{dx} \int sinxdx\right) dx$$

$$= -x^2 cosx - \int 2x(-cosx)dx$$

$$= -x^2 cosx + 2\int x\ cosxdx$$

$$= -x^2 cosx + 2\left[x\int\ cosxdx - \int\left(\frac{dx}{dx}\int cosxdx\right)dx\right]$$

$$= -x^2 cosx + 2\left[xsinx - \int\ sinxdx\right]$$

$$= -x^2 cosx + 2[xsinx - (-\cos x)] + c$$

$$\therefore \int \boldsymbol{x^2 sindx = -x^2 cosx + 2(xsinx + cosx) + c}$$

ii) $\int e^{ax} cosbx\ dx$

➢Solution:

$$Let, I = \int e^{ax} cosbx\ dx$$

$$= \cos bx \int e^{ax} dx - \int\left(\frac{dcosbx}{dx}\int e^{ax} dx\right)$$

$$= \cos bx \frac{e^{ax}}{a} - \int -b.\sin bx\,.\frac{e^{ax}}{a}dx$$

$$= \frac{1}{a}e^{ax} cos\ bx + \frac{b}{a}\int e^{ax}\ sin\ bxdx$$

$$= \frac{1}{a}e^{ax} cos\ bx + \frac{b}{a}\left[\sin bx\int e^{ax} dx - \int\left(\frac{dsindx}{dx}\int e^{ax} dx\right)dx\right]$$

$$= \frac{1}{a}e^{ax} cos\ bx + \frac{b}{a}\left[\sin bx\,.\frac{e^{ax}}{a} - \int b\cos bx\,.\frac{e^{ax}}{a}dx\right]$$

$$= \frac{1}{a} e^{ax} \cos bx + \frac{b}{a^{a}} e^{ax} sinbx - \frac{b^2}{a^2} \int e^{ax} \cos bx \, dx$$

$$\therefore \boldsymbol{I = e^{ax} \left[\frac{1}{a} \cos bx + \frac{b}{a^2} \sin bx\right] - \frac{b^2}{a^2} I}$$

$$I \times \left(1 + \frac{b^2}{a^2}\right) = e^{ax}. \left[\frac{1}{a} cosbx + \frac{b}{a^2} sinbx\right]$$

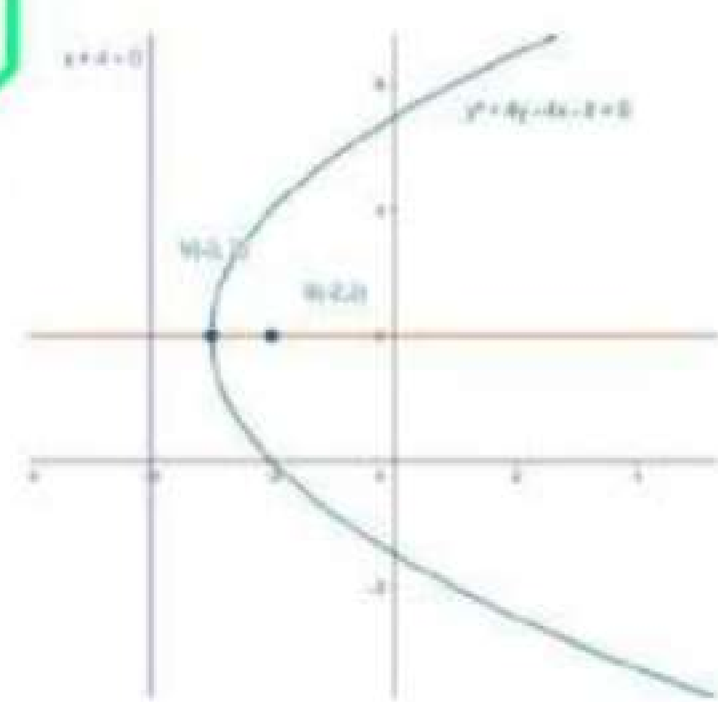
$$I \times \left(1 + \frac{b^2}{a^2}\right) = e^{ax} . \frac{a \cos bx + b \sin bx}{a^2}$$

$$\therefore \boldsymbol{I = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + bsin\, bx) + c}$$

11) Find the vertex, focus, equation of directrix and length of latus rectum of the parabola : $y^2 - 4y - 4x - 8 = 0$.

➢ **Solution:**

The given equation is



$y^2 - 4y - 4x - 8 = 0.$

$y^2 - 4y - 4x - 8$

$y^2 - 2 \times y \times 2 + 2^2 = 4x + 12$

$(y^2 - 2)^2 = 4(x + 3)$

Comparing with $\boldsymbol{(y - k)^2 = 4a(x - h)}$***, we have***

$h = -3, k = 2, 4a = 4 \;\; or \;\; a = 1$

The vertex of parabola is $V(h, k) = V(-3, 2)$

The focus is $S(h + a, k) = (-3 + 1, 2) = S(-2, 2)$

The equation of directrix is $x - h + a = 0$

$$x + 3 + 1 = 0$$

$$x + 4 = 0$$

The length of latus rectum $= 4a = 4 \times 1 = 4$

12) Let $f: R \rightarrow R$ and $R \rightarrow R$ be defined by $f(x) = x^3 + 1$ and $g(x) = x + 5$, Find $(i)\ fog(x)$ $(ii)\ gof\ (x)$

➢ **Solution:**

Given, $(x) = x^3 + 1$ *and* $g(x) = x + 5$

We have,

(i) $fog(x) = f(x+5)$

$$= (x+5)^3 + 1$$

(ii) $gof\ (x) = g(x^3 + 1)$

$$= (x^3 + 1) + 5$$

$$= x^3 + 6$$

or) $If\ \dfrac{\log x}{y-z} = \dfrac{\log y}{z-x} = \dfrac{\log z}{z-y}, prove\ that\ x^x . y^y . z^z = 1$

➢ **Solution:**

$$Let, \quad \frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{x-y} = k$$

$$\log x = k(y-z) \Rightarrow x.\log x = kx(y-z)$$

Similarly, $y.\log y = ky(z-x)$,

$$z.\log z = kz(x-y)$$

Adding All Above Equations,

$$x\log x + y\log y + z\log z = \boldsymbol{kx(y-z) + ky(z-x) + kz(x-y)}$$

$$\log x^x + \log y^y + \log z^z = \boldsymbol{k[xy - xz + yz - xy + xz - yz]}$$

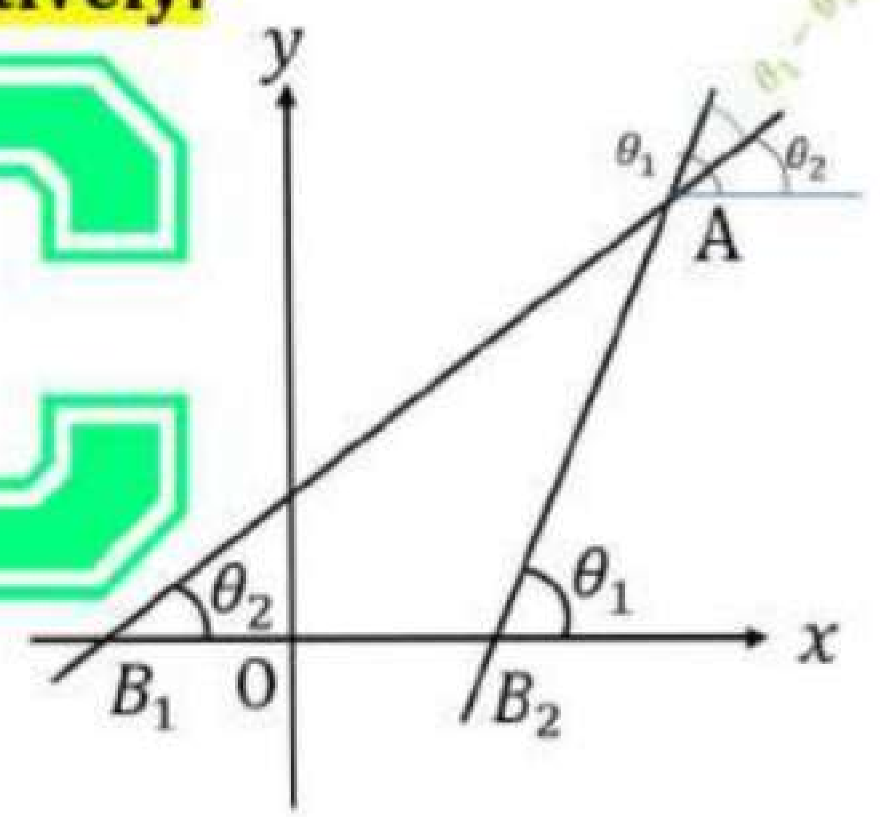
$$\log(x^x y^y z^z) = 0 \qquad \text{By properties}, x\log x = \log x^x$$

$$\log(x^x y^y z^z) = \log 1 \qquad \because \text{Log}1 = 0$$

$$\therefore \boldsymbol{x^x y^y z^z = 1}, \textbf{\textit{proved}}.$$

13) Prove that the angle between two straight *lines* $y = m_1 + c_1$ and $y = m_2x + c_2$ is $\tan\theta = \pm\left(\frac{m_1-m_2}{1+m_1m_2}\right)$. Also, prove that the two lines are parallel and perpendicular to each other if $m_1 = m_2$ and $m_1 \times m_2 = -1$ respectively.

➢ **Solution:**



Let the two straight lines be AB_1 & AB_2 meeting x –axis in B_1 & B_2.

Let the equations of be

$y = m_1x + c_1 \cdots\cdots\cdots (1)$

and $y = m_2x + c_2 \cdots\cdots\cdots (2)$

If θ_1 and θ_2 be the angles that the lines make with OX, then

$$\tan\theta_1 = m_1 \text{ and } \tan\theta_2 = m_2$$

Now, $\angle B_2 AB_1 = \theta_1 - \theta_2 = \theta$

$$\therefore \tan\angle B_2 AB_1 = \tan(\theta_1 - \theta_2)$$

$$= \frac{\tan\theta_1 - \tan\theta_2}{1 + \tan\theta_1 . \tan\theta_2} = \frac{m_1 - m_2}{1 + m_1m_2}$$

Since, $\angle B_2AB_1$ *is* θ, $\quad \tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|$

Condition of Perpendicularity,

$\theta = 90°$ $\tan 90 = \frac{1}{0} = \frac{m_1 - m_2}{1 + m_1 m_2}$

$$1 + m_1 m_2 = 0$$

$m_1 \times m_2 = -1$, **Which is condition of perpendicularity.**

Condition of Parallel, , $\theta = 0°$ $\tan 0 = 0 = \frac{m_1 - m_2}{1 + m_1 m_2}$

$$m_1 - m_2 = 0 \Rightarrow \boldsymbol{m_1 = m_2}$$

Which is condition of parallelism.

–The End –

1. a) If A = {2, 3, 4, 5, 6, 7}, B = {4,5,6,7,8) and c = {1,2,3,4,5), find i) (A ∪ B) ∩ C   ii) (A ∩ B) ∪ C

➢ **Solution:**

Given, A={2,3,4,5,6,7} , B={4,5,6,7,8} and C={1,2,3,4,5}

Find : i) (A U B) ∩ C   ii) (A ∩ B) U C

Now,

A U B = {2,3,4,5,6,7} U {4,5,6,7,8}

={2,3,4,5,6,7,8}

A ∩ B ={2,3,**4,5,6,7**} U {**4,5,6,7**,8}

={**4,5,6,7**}

i) (A U B) ∩ C

= {2,3,4,5,6,7,8} ∩ {1,2,3,4,5}

= {2,3,4,5}

ii) (A ∩ B) U C

= {4,5,6,7} U {1,2,3,4,5}

= {1,2,3,4,5,6,7}

b) Rewrite $|2x-1| \le 5$ without using absolute value sign.

➢ **Solution:**

Given, $|\mathbf{2x-1}| \le \mathbf{5}$

We have,

$$|x| \le a \quad \Rightarrow \quad -a \le x \le a$$

$or,\ -5 \le 2x-1 \le 5$

$or,\ -5+1 \le 2x \le 5+1$

$or,\ -4 \le 2x \le 6$

$or,\ \frac{-4}{2} \le x \le \frac{6}{2}$

$or,\ -2 \le x \le 3$

Hence, $-2 \le x \le 3$

2. a)Prove : $\sin(2sin^{-1}x) = 2x\sqrt{1-x^2}$

➢ **Solution:**

Let $sin^{-1}x = \theta \quad \cdots\cdots\cdots (i)$

$or,\ x = sin\theta \quad \cdots\cdots\cdots (ii)$

$$cos\theta = \sqrt{1-sin\theta^2} = \sqrt{1-x^2} \quad \cdots\cdots\cdots (iii)$$

L.H.S, $\sin(2sin^{-1}x)$ [$\because$ from (i)]

$= \mathrm{Sin}(2\theta)$

$= 2sin\,\theta.cos\,\theta$

$= \mathbf{2x\sqrt{1-x^2}}$ $R.H.S$ *proved.* [$\because$ *from* (ii) *and* (iii)]

b) In any ΔABC, show that. $c(a\ cosB - b\ cosA) = a^2 - b^2$.

➢ **Solution:-**

L.H.S,

$$c(a\ cosB - b\ cosA)$$

We Know that, $\because \cos B = \frac{a^2+c^2-b^2}{2ac}$, $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$$= c\left\{a.\frac{(a^2+c^2-b^2)}{2ac} - b.\frac{(b^2+c^2-a^2)}{2bc}\right\}$$

$$= c\left\{\frac{a^2+c^2-b^2}{2c} - \frac{b^2+c^2-a^2}{2c}\right\}$$

$$= C\frac{(a^2+c^2-b^2-b^2+c^2-a^2)}{2c}$$

$$= \frac{2a^2-2b^2}{2}$$

$$= \boldsymbol{a^2 - b^2}$$

R.H.S, proved.

3. a) If $\frac{CosA}{a} = \frac{CosB}{b}$, prove that the triangle is an isosceles.

➢ **Solution:-**

Given,

$\frac{CosA}{a} = \frac{CosB}{b}$, To Prove: Δ **is isosceles**

We have,

$$\frac{CosA}{a} = \frac{CosB}{b}$$

$$\frac{\frac{b^2+c^2-a^2}{2bc}}{a}=\frac{\frac{c^2+a^2-b^2}{2ac}}{b}$$

$$or,\ \frac{b^2+c^2-a^2}{2abc}=\frac{c^2+a^2-b^2}{2abc}$$

$$or,\ b^2+c^2-a^2=c^2+a^2-b^2$$

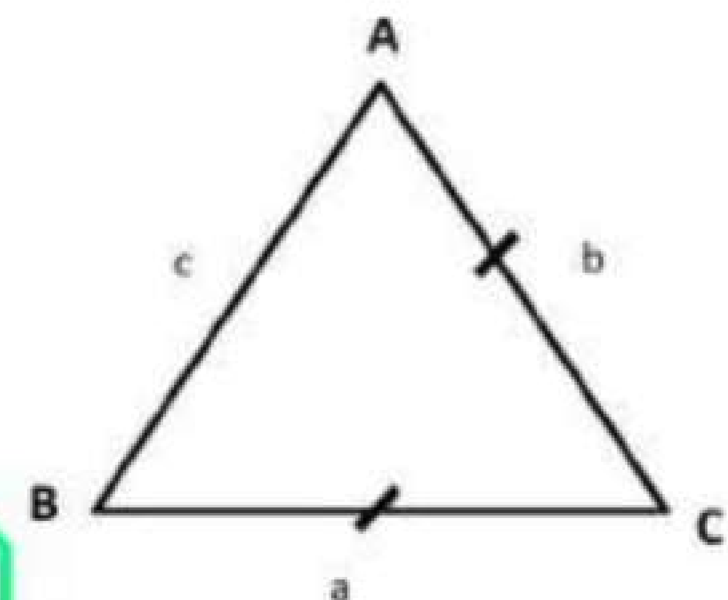
$$or,\ b^2-a^2=a^2-b^2$$

$$or,\ b^2+b^2=a^2+a^2$$

$$or,\ 2b^2=2a^2$$

$$or,\ b^2=a^2$$

$$\therefore \boldsymbol{b=\pm a}$$



Hence, Sides (a) and Sides (b) are equal So,Triangle is an isosceles.

b) Evaluate: $\frac{Lt}{x\to 2}\frac{x^2-5x+6}{x^2-x-2}$.

➢ **Solution:-**

$$\frac{Lim}{x\to 2}\ \frac{x^2-5x+6}{x^2-x-2}$$

When $x=2$ *Then, limit takes the form* $\left(\frac{0}{0}\right)$ *So,*

$$=\frac{Lim}{x\to 2}\ \frac{x^2-5x+6}{x^2-x-2}$$

$$=\frac{Lim}{x\to 2}\ \frac{x^2-3x-2x+6}{x^2-2x+x-2}$$

$$= \lim_{x \to 2} \frac{x(x-3) - 2(x-3)}{x(x-2) + 1(x-2)}$$

$$= \lim_{x \to 2} \frac{(x-2)(x-3)}{(x-2)(x+1)}$$

$$= \lim_{x \to 2} \frac{x-3}{x+1}$$

$$= \frac{2-3}{2+1}$$

$\therefore$ ***Value of Limit*** $= \dfrac{\mathbf{-1}}{\mathbf{3}}$

4. a) Find $\dfrac{dy}{dx}$; *when* $y = \dfrac{1}{\sqrt{ax^2 + bx + c}}$.

➢ **Solution:-**

Given,

$$y = \frac{1}{\sqrt{ax^2 + bx + c}} = \frac{1}{(ax^2 + bx + c)^{\frac{1}{2}}} = (ax^2 + bx + c)^{-\frac{1}{2}}$$

Differentiating both side *w.r.t x* we get,

$$\frac{dy}{dx} = \frac{d}{dx}\ (ax^2 + bx + c)^{-\frac{1}{2}}$$

Using chain rule,

$$\frac{dy}{dx} = \frac{d}{d\,(ax^2 + bx + c)}\ (ax^2 + bx + c)^{-\frac{1}{2}} \times \frac{d(ax^2 + bx + c)}{dx}$$

$$\frac{dy}{dx} = -\frac{1}{2}\ (ax^2 + bx + c)^{-\frac{1}{2}-1} \times (2ax + b)$$

$$\therefore \frac{dy}{dx} = -\frac{1}{2}\ (ax^2 + bx + c)^{-\frac{3}{2}} \times (2ax + b)$$

b) Find $\frac{dy}{dx}$ *when* $= cos\ (sin\sqrt{3x+5})$.

➢ **Solution:-**

Given,

$$y = cos(sin(\sqrt{3x+5})$$

Differentiating both side $w.r.t\ x$ we get,

$$\frac{dy}{dx} = \frac{d}{dx}\ cos(sin(\sqrt{3x+5})$$

Using chain rule,

$$\frac{dy}{dx} = \frac{d\ cos(sin(\sqrt{3x+5})}{dsin(\sqrt{3x+5})} \times \frac{dsin(\sqrt{3x+5})}{d\sqrt{3x+5}} \times \frac{(3x+5)^{\frac{1}{2}}}{d\ (3x+5)} \times \frac{d\ 3x+5}{dx}$$

$$\frac{dy}{dx} = -sin(sin(\sqrt{3x+5})) \times cos(\sqrt{3x+5}) \times \frac{(3x+5)^{\frac{1}{2}-1}}{2} \times 3$$

$$\therefore \frac{dy}{dx} = -\frac{3}{2} sin\left(sin(\sqrt{3x+5})\right).cos(\sqrt{3x+5})\ \times (3x+5)^{-\frac{1}{2}}$$

5. a) Integrate: $\int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$.

➢ **Solution:-**

Given,

$$I = \int \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right) dx$$

$$= \int x^{\frac{1}{2}} dx - \int x^{\frac{-1}{2}} dx \qquad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} + C\right]$$

$$= \frac{x^{\frac{1}{2}+1}}{\left(\frac{1}{2}+1\right)} - \frac{x^{\frac{-1}{2}+1}}{\left(\frac{-1}{2}+1\right)} + C$$

$$= \frac{x^{3/2}}{\frac{3}{2}} - \frac{x^{1/2}}{\frac{1}{2}} + C$$

$$\therefore I = \frac{2}{3} x^{3/2} - 2x^{1/2} + C.$$

b) The sum of an infinite G.S. is 15 and the first term is 3. Find the common ratio.

➢ **Solution:-**

Given, First term $(a) = 3$

Sum of an infinite G.S is 15

$i.e.,\ a + ar + ar^2 + ar^3 + \cdots\cdots\cdots\infty = S_\infty = 15$

We have,

$$S_\infty = \frac{a}{1-r}$$

$$So,\ 15 = \frac{3}{1-r}$$

$$1 - r = \frac{3}{15}$$

$$1 - r = \frac{1}{5}$$

$$or, \left(1 - \frac{1}{5}\right) = r$$

$$\left(\frac{5-1}{5}\right) = r$$

$$\therefore r = \frac{4}{5}$$

6. a) In how many ways can the letters of the word "MATHEMATICS" be arranged?

➢ **Solution:-**

In word, MATHEMATICS
Number of word (n) = 11
P = Number of "M" = 2
q = Number of "A" = 2
r = Number of "T" =2

Hence,

Number of ways the word can be arranged is with 2,2,2 like word.

$$\frac{n!}{p!\ q!\ r!}$$

$$\therefore \textbf{Number of ways} = \frac{11!}{2!\ 2!\ 2!} =$$

b) Find the seventh term in the expansion of $\left(3x^2 - \frac{1}{2x}\right)^{12}$.

➢ **Solution:**

Given,

To find T_7 in $\left(3x^2 - \frac{1}{2x}\right)^{12}$

We have,

$$T_{n+1} = C(n\,,r)\, a^{n-r}\, x^r \text{ in expansion } (a+x)_n$$

$$T_7 = T_{n+1} = C\,(12, 6)\,(3x^2)^{12-6}.\left(\frac{-1}{2x}\right)^{-6}$$

$$= \frac{12!}{6!\ \ 6!} \times\ (3x^2)^6\,.\,(-1)^6$$

$$= \frac{12!}{(6!)^2} \times 3^6 . x^{12} . \frac{1}{2^6 x^6}$$

$$= \frac{12!}{(6!)^2} \times \frac{3^6}{2^6} . x^6$$

$$= \frac{12!}{(6!)^2} \times \frac{729}{64} . x^6$$

$$\therefore \frac{\mathbf{729}}{\mathbf{64}} . \frac{\mathbf{12!}}{\mathbf{(6!)^2}} \, \boldsymbol{x^6}$$

7. a) Find the distance between the parallel lines $3x + 4y - 5 = 0$ and $6x + 8y + 17 = 0$.

➢ **Solution:**

Given lines,

$$3x + 4y - 5 = 0 \quad \cdots\cdots\cdots (i)$$

$$6x + 8y + 17 = 0 \quad \cdots\cdots\cdots (ii)$$

When $x = 0$, in (i) Then,

$$3 \times 0 + 4y - 5 = 0$$

$$y = \frac{5}{4}$$

$\therefore$ Point $\left(0, \frac{5}{4}\right)$ lies on the line (i)

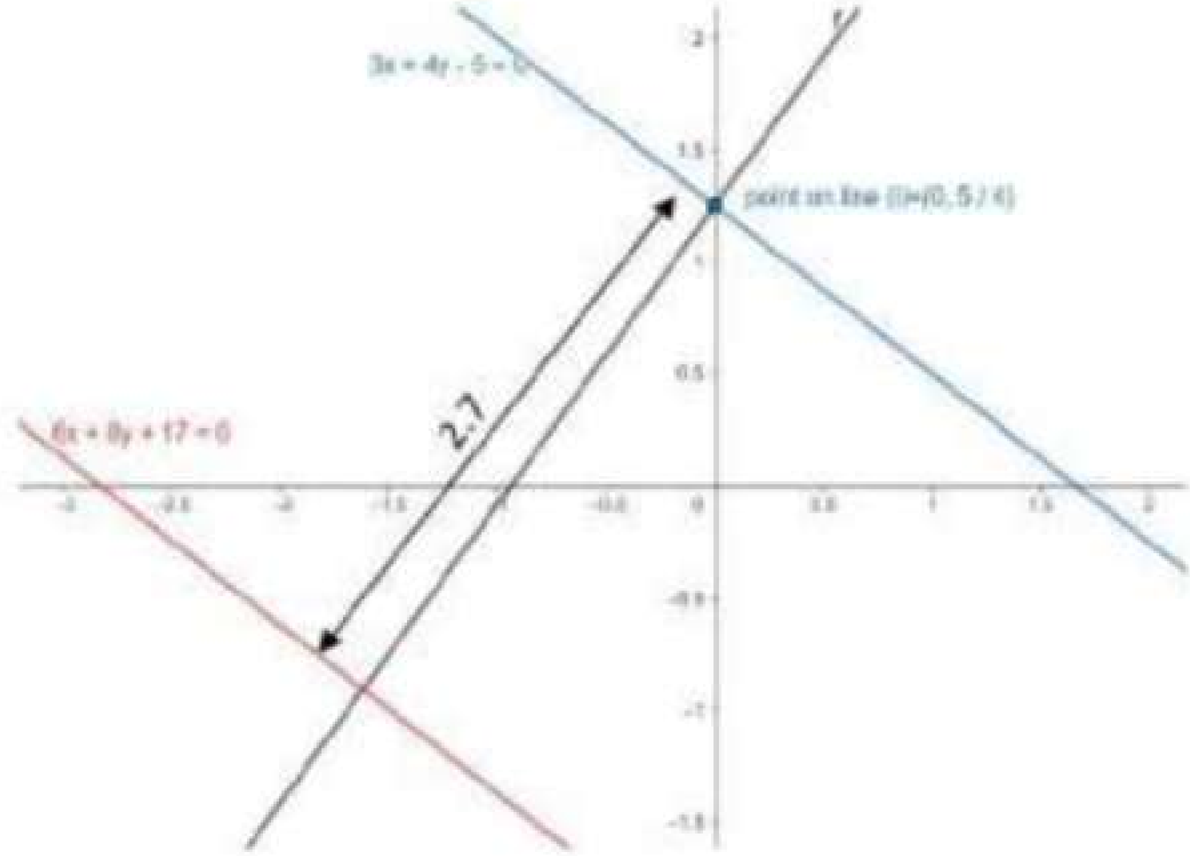
Let 'P' be length of perpendicular drawn from point

$\left(0, \frac{5}{4}\right)$ lies on the line $6x + 8y + 17 = 0$ *is*

$$P = \left|\frac{6 \times 0 + 8 \times \frac{5}{4} + 17}{\sqrt{6^2 + 8^2}}\right|$$

$$P = \left|\frac{2 \times 5 + 17}{\sqrt{100}}\right|$$

$$P = \left|\frac{10 + 17}{\sqrt{100}}\right|$$



$$\therefore\ \boldsymbol{P = \frac{27}{10} = 2.7\ unit.}$$

b) Find the angle between two lines represented by $x^2 - 2xy\ cot\theta - y^2\ =\ 0.$

➢ **Solution:**

Given,

$$x^2 - 2xy\ cot\theta - y^2 = 0.$$

Comparing with $ax^2 - 2hxy - by^2 = 0.$

$$a = 1, h = -\cot\theta\ , b = -1$$

Now, Let 'α' be angle between two lines so,

$$\tan\alpha = \pm\frac{2\sqrt{h^2 - ab}}{a + b}$$

$$\tan\alpha = \frac{2\sqrt{(-\cot\theta)^2 - 1 \times -1}}{1 - 1}$$

$$\tan\alpha = \frac{2\sqrt{(-\cot\theta)^2 + 1}}{0}$$

$$\tan\alpha = \infty$$

$$\alpha = tan^{-1}(\infty)$$

$$\therefore\ \boldsymbol{\alpha = \frac{\pi}{2}}$$

8) If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{z-y}$, prove that $x^2 . y^y . z^z = 1$

➢ **Refer to the solution 2078 of Q. No 12 (or) on page 41.**

OR) Let $f: R \longrightarrow R, g: R \longrightarrow R$ which are defined by $f(x) = x^3 + 1$ and $g(x) = x^5$ respectively then find

a) $fog(x)$ b) $gof(x)$ c) $f^{-1}(x)$

➢ **Solution:**

Given, $f(x) = x^3 + 1$ *and* $g(x) = x^5$

We have,

$(\boldsymbol{a})\ \boldsymbol{fog}(\boldsymbol{x}) = f\{g(x)\} = f\{x^5\} = (x^3)^5 + 1 = x^{15} + 1$

$(\boldsymbol{b})\ \boldsymbol{gof}\ (\boldsymbol{x}) = g\{f(x)\} = g\{x^3 + 1\} = (x^3 + 1)^5$

$(\boldsymbol{c})\ \boldsymbol{f^{-1}}(\boldsymbol{x})$

$Let\ y = f(x) = x^3 + 1$

Interchanging $\boldsymbol{x}$ and $\boldsymbol{y}$ we get,

$or,\ x = y^3 + 1$

$or,\ y^3 = x - 1$

$or,\ y = (x-1)^{\frac{1}{3}}$

$\boldsymbol{or,\ f^{-1}(x) = (x-1)^{\frac{1}{3}}\ x \in R}$

9) Solve: $tan^2x = secx + 1$.

➢ **Solution:**

Given, $tan^2x = secx + 1$

$$\left(\frac{sinx}{cosx}\right)^2 = \frac{1}{cosx} + 1$$

$$sin^2x = cos^2x \times \left(\frac{1}{cosx} + 1\right)$$

$$sin^2x = cosx + cos^2x$$

$$1 - cos^2x = cosx + cos^2x$$

$$1 = cosx + 2cos^2x$$

$$2cos^2x + cosx - 1 = 0$$

$$2cos^2x + 2cosx - cosx - 1 = 0$$

$$2cosx(cosx + 1) - 1.(cosx + 1) = 0$$

$$(2cosx - 1)(cosx + 1) = 0$$

Either, $(2cosx - 1) = 0$ *or,* $(cosx + 1) = 0$

$cosx = \frac{1}{2}$,	*or,* $cosx = -1$
$cosx = \left(\frac{1}{2}\right)$,	$cosx = cos(\pi)$
$cosx = cos\left(\frac{\pi}{3}\right)$	$x = 2n\pi + \pi$
$\boldsymbol{i.e\ x = 2n\pi \pm \left(\frac{\pi}{3}\right)}$	$\boldsymbol{x = (2n + 1)\pi}$

or) Solve: $Sin^{-1}\frac{2a}{1+a^2} - cos^{-1}\frac{1-b^2}{1+b^2} = 2tan^{-1}x$

➢ **Solution:**

Given, $sin^{-1}\frac{2a}{1+a^2} - cos^{-1}\frac{1-b^2}{1+b^2} = 2tan^{-1}x$

Let $a = tan\alpha$, $b = tan\beta$

$tan^{-1}a = \alpha \text{ and } tan^{-1}b = \beta$

$sin^{-1}\frac{2tan\alpha}{1+tan\alpha^2} - cos^{-1}\frac{1-tan\beta^2}{1+tan\beta^2} = 2tan^{-1}x$

$\left[\because sin2\theta = \frac{2tan\theta}{1+tan\theta^2}, cos\beta = \frac{1-tan\beta^2}{1+tan\beta^2}\right]$

$sin^{-1}(sin2\alpha) - cos^{-1}(cos2\beta) = 2tan^{-1}x$

$2\alpha - 2\beta = 2tan^{-1}x$

$2tan^{-1}a - 2tan^{-1}b = 2tan^{-1}x$

$tan^{-1}a - tan^{-1}b = tan^{-1}x$

$tan^{-1}\left(\frac{a-b}{1+ab}\right) = tan^{-1}x$

$\left(\frac{a-b}{1+ab}\right) = \tan\left(tan^{-1}x\right)$

$\therefore \left(\frac{\boldsymbol{a}-\boldsymbol{b}}{\boldsymbol{1}+\boldsymbol{ab}}\right) = \boldsymbol{x}$

10) If $a^4 + b^4 + c^4 = 2a^2(b^2 + c^2)$ prove that A = 45°or 135°.

➢ **Solution:**

Given, $a^4 + b^4 + c^4 = 2a^2(b^2 + c^2)$

$$a^4 + b^4 + c^4 - 2a^2b^2 - 2a^2c^2 = 0$$

$$a^4 + b^4 + c^4 - 2a^2b^2 + \underline{2b^2c^2} - 2a^2c^2 = \underline{2b^2c^2}$$

$$(b^2 + c^2 - a^2) = 2b^2c^2$$

$$[\because (a + b + c)^2 = a^2 + b^2 + 2ab + 2bc + 2ac]$$

$$(b^2 + c^2 - a^2) = 4 \times \frac{1}{2} b^2c^2$$

$$\left[\frac{b^2 + c^2 - a^2}{2bc}\right]^2 = \frac{1}{2}$$

$$Cos^2A = \frac{1}{2}$$

$$A = \frac{1}{\sqrt{2}}$$

$\therefore$ **$A = 45°$ *or* $135°$**

OR) Solve the ΔABC, if $b = \sqrt{3}, c = 1$ and $A = 30°$.

➢ **Solution:**

Given, $b = \sqrt{3}, c = 1, A = 30°$.

We have,

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos(30°) = \frac{3 + 1 - a^2}{2 \times \sqrt{3} \times 1}$$

$$\frac{\sqrt{3}}{2} = \frac{4 - a^2}{2\sqrt{3}}$$

$$3 = 4 - a^2$$

$$a^2 = 1$$

$$\therefore \boldsymbol{a = 1}$$

Now,

Using Sin law,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{1}{\sin A} = \frac{\sqrt{3}}{\sin B}$$

$$\sin B = \sqrt{3}.\sin 30°$$

$$Sin\ B = \frac{\sqrt{3}}{2}$$

$$B = sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\therefore\ \boldsymbol{B = 60°}$$

So, $A + B + C = 180$

$$C = 180 - (30 + 60)$$

$$\therefore \boldsymbol{C = 90°}$$

Hence,

$$\boldsymbol{A = 1,\ \ B = 60°,\ \ C = 90°}$$

11) Evaluate: $\underset{x \to \theta}{Lim} \dfrac{x \tan \theta - \theta \tan x}{x - \theta}$.

➢ **Solution:**

The given function takes the indeterminate form $\left(\frac{0}{0}\right)$ when $x = \theta$.

$$\underset{x \to \theta}{lim} \frac{x\ tan\theta - \theta tanx}{x - \theta}$$

$$\underset{x \to \theta}{lim} \frac{x\ - \theta\ tan\theta + \theta\ tan\theta - \theta\ tanx}{x - \theta}$$

$$\underset{x \to \theta}{lim} \frac{(x\ - y)\ tan\theta}{(x - \theta)} + \frac{\theta(tan\theta - tan\theta)}{(x - \theta)}$$

$$\underset{x \to \theta}{lim} \left\{tan\,\theta + \frac{\theta}{(x - \theta)} \times \left(\frac{sin\,\theta}{cos\,\theta} - \frac{sin\,x}{cos\,x}\right)\right\}$$

$$\underset{x \to \theta}{lim}\ tan\,\theta + \underset{x \to \theta}{lime} \left\{\theta \frac{sin\,\theta\,cos\,x - cos\,\theta\,sin\,x}{(x - \theta)\,cos\,\theta\,cos\,x}\right\}$$

$$tan\,\theta + \underset{x \to \theta}{lim} \frac{\theta}{cos\,\theta}\ \frac{sin\,(\theta - x)}{(x - \theta)\,cos\,x}$$

$$tan\,\theta + \frac{\theta}{cos\,\theta}\left\{\underset{x \to \theta}{lim}\ \frac{sin\,(\theta - x)}{-(\theta - x)\,cos\,x}\right\}$$

$$tan\,\theta + \frac{\theta}{cos\,\theta} \times (-1) \times \frac{1}{cos\,\theta} \qquad \left\{\because \underset{x \to \theta}{lim} \frac{sin\,\theta}{\theta} = 1\right\}$$

$\therefore$ $\boldsymbol{tan\,\theta - \theta sec^2\theta}$.

OR) A function $f(x)$ is defined as follows.

$$\int(x) = \begin{cases} 2x+1 & for\ x<1 \\ 2 & for\ x=1 \\ 3x & for\ x>1 \end{cases}$$

Is the function continuous at $x = 1$? If not, how can it be made continuous at $x = 1$?

➢ **Solution:**

$$Left\ hand\ limit\ at\ x = 1 = \lim_{x \to 1^-} f(x)$$

$$= \lim_{x \to 1^-} (2x+1)$$

$$= 2 \times 1 + 1$$

$$= 3$$

$$Right\ hand\ limit\ at\ x = 1 = \lim_{x \to 1^+} f(x)$$

$$= \lim_{x \to 1^+} 3x$$

$$= \lim_{x \to 8^+}$$

$$= 3 \times 1$$

$$= 3$$

Function value at $f(1) = 2$

Since,

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \neq . f(1)$$

Hence,

Given, The function Discontinues $at\ x = 1$. To make continuous, the value 2 is replaced by 3

$$\int(x) = \begin{cases} 2x+1 & for\ x<1 \\ 2 & for\ x=1 \\ 3x & for\ x>1 \end{cases}$$

12) Find from first principle, the derivatives of $\sqrt{tanx}$

➢ **Solution:**

$$Let\ y = \sqrt{tanx}$$

By definition of derivative,

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sqrt{tan(x+h)}-\sqrt{tanx}}{h}$$

$$= \lim_{h\to 0}\frac{tan(x+h)-tanx}{\sqrt{tan(x+h)}-\sqrt{tanx}}$$

$$= \lim_{h\to 0}\frac{\frac{sin(x+h)}{cos(x+h)}-\frac{sin\,x}{cos\,x}}{\sqrt{tan(x+h)}-\sqrt{tanx}}$$

$$= \lim_{h\to 0}\frac{sin(x+h-x)}{cosx\ cos\ (x+h)(\sqrt{tan(x+h)}+\sqrt{tanx})}$$

$$= \lim_{h\to 0}\frac{sinh}{h}.\lim_{h\to 0}\frac{1}{cosx\ cos\ (x+h)(\sqrt{tan(x+h)}+\sqrt{tanx})}$$

$$= 1.\frac{1}{cosx\ cos\ x.(\sqrt{tanx}+\sqrt{tanx})} = \frac{\sec x\ .\sec x}{(\sqrt{tanx}+\sqrt{tanx})}$$

$$\therefore \frac{dy}{dx} = \frac{sec^2x}{2\sqrt{tan\,x}}.$$

or) $f(x) = \dfrac{1}{\sqrt{4-5x}}$

➢ **Solution:**

$$\text{Let } f(x) = y = \frac{1}{\sqrt{4-5x}} \cdots\cdots\cdots (i)$$

Let Δx *be* small increment in x and Δy be the corresponding small increment in y Then,

$$y + \Delta y = \frac{1}{\sqrt{4-5(x+\Delta x)}} \cdots\cdots\cdots (ii)$$

Subtracting eqn (i) from (ii) we get,

$$y + \Delta y - y = \frac{1}{\sqrt{4-5(x+\Delta x)}} - \frac{1}{\sqrt{4-5x}}$$

$$\Delta y = \frac{1}{\sqrt{4-5(x+\Delta x)}} - \frac{1}{\sqrt{4-5x}}$$

$$\Delta y = \frac{\sqrt{4-5x} - \sqrt{4-5(x+\Delta x)}}{\sqrt{4-5(x+\Delta x)}\ \sqrt{4-5x}}$$

$$\Delta y = \frac{\sqrt{4-5x} - \sqrt{4-5(x+\Delta x)}}{\sqrt{4-5(x+\Delta x)}\ \sqrt{4-5x}} \times \frac{\sqrt{4-5x} + \sqrt{4-5(x+\Delta x)}}{\sqrt{4-5x} + \sqrt{4-5(x+\Delta x)}}$$

$$\Delta y = \frac{(4-5x) - (4-5(x+\Delta x))}{\sqrt{4-5(x+\Delta x)}\ \sqrt{4-5x}} \times \frac{1}{\sqrt{4-5x} + \sqrt{4-5(x+\Delta x)}}$$

$$\Delta y = \frac{4-5x-4+5x-5\Delta x}{\sqrt{4-5(x+\Delta x)}\ \sqrt{4-5x}} \times \frac{1}{\sqrt{4-5x} + \sqrt{4-5(x+\Delta x)}}$$

$$\Delta y = \frac{-5\Delta x}{\sqrt{4-5(x+\Delta x)}\ \sqrt{4-5x}} \times \frac{1}{\sqrt{4-5x}+\sqrt{4-5(x+\Delta x)}}$$

$$\frac{\Delta y}{\Delta x} = \frac{-5\Delta x}{\Delta x.\sqrt{4-5(x+\Delta x)}\ \sqrt{4-5x}} \times \frac{1}{\sqrt{4-5x}+\sqrt{4-5(x+\Delta x)}}$$

By first Principle,

$$\frac{dy}{dx} = \Delta x \xrightarrow{\lim} 0\, \frac{\Delta y}{\Delta x}$$

$$\frac{dy}{dx} = \Delta x \xrightarrow{\lim} 0\, \frac{-5}{\sqrt{4-5(x+\Delta x)}\ \sqrt{4-5x}} \times \frac{1}{\sqrt{4-5x}+\sqrt{4-5(x+\Delta x)}}$$

$$\frac{dy}{dx} = \frac{-5}{\sqrt{4-5(x+0)}\ \sqrt{4-5x}} \times \frac{1}{\sqrt{4-5x}+\sqrt{4-5(x+0)}}$$

$$\frac{dy}{dx} = \frac{-5}{\sqrt{4-5x}\ \sqrt{4-5x}} \times \frac{1}{\sqrt{4-5x}+\sqrt{4-5x}}$$

$$\frac{dy}{dx} = \frac{-5}{(4-5x)^1} \times \frac{1}{2\sqrt{4-5x}}$$

$$\therefore \boldsymbol{\frac{dy}{dx} = \frac{-5}{2(4-5x)^{\frac{3}{2}}}}$$

13) Integrate (<u>any one</u>)

a) $\displaystyle\int \frac{dx}{x^2\sqrt{9-x^2}}$

➢ **Solution:**

Let, $I = \int \frac{dx}{x^2\sqrt{9-x^2}}$

Let $x = 3\sin\theta$, $\sin\theta = \frac{x}{3}$, $\cos\theta = \sqrt{1-\frac{x^2}{9}} = \frac{\sqrt{9-x^2}}{3}$

or, $dx = 3\cos\theta d\theta$

Integral becomes,

$$I = \int \frac{3\cos\theta d\theta}{(3\sin\theta)^2\sqrt{9-(3\sin\theta)^2}}$$

$$I = \int \frac{3\cos\theta\, d\theta}{(3\sin\theta)^2\sqrt{9[1-(\sin\theta)^2]}}$$

$$= \int \frac{3\cos\theta\, d\theta}{9(\sin\theta)^2\ .3Cos\theta}$$

$[\because 1-\sin^2\theta = \cos^2\theta]$

$$= \int \frac{d\theta}{9(\sin\theta)^2}$$

$$= \int \frac{1}{9}\ cosec^2\theta d\theta$$

$$I = \frac{1}{9} - Cot\theta + c$$

$$\therefore I = -\frac{1}{9}\frac{\sqrt{9-x^2}}{x} + c$$

$$\left[\because cot\theta = \frac{cos\theta}{sin\theta} = \frac{\frac{\sqrt{9-x^2}}{3}}{\frac{x}{3}} = \frac{\sqrt{9-x^2}}{x}\right]$$

$$\therefore I = \frac{Secx\,tanx}{2} + \frac{1}{2}log|secx + tanx| + C$$

14) Prove that the AM, GM and HM between any two unequal positive numbers satisfy the relation.

i) $(G.M.)^2 = A.M. \times H.M.$ ii) $A.M. > G.M. > H.M.$

➢ **Proof:**

Let a and b be two unequal positive number. Then,

$$A.M. = \frac{a+b}{2} \quad , \quad G.M. = \sqrt{ab}$$

and, $H.M. = \dfrac{2ab}{a+b}$

To prove: the first part, we have

$$A.M. \times H.M. = \frac{a+b}{2} \times \frac{2ab}{a+b}$$

$$= ab = \left(\sqrt{ab}\right)^2$$

$$= (G.M.)^2$$

This result shows that G.M. is again the geometric mean between A.M. and H.M.

To prove the second Part, consider

$$A.M. - G.M. = \frac{a+b}{2} - \sqrt{ab}$$

$$= \frac{a+b-2\sqrt{ab}}{2}$$

$= \frac{1}{2}\left(\sqrt{a} - \sqrt{b}\right)^2$ **,which is square ,always greater**

15) From 6 gentleman and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done as to include at most two ladies?

➢ **Solution:**

For including at least most two ladies, the following case arise

i) $(1\ lady\ out\ of\ 4)\ and\ (4\ men\ out\ of\ 6)$

ii) $(2\ ladies\ out\ of\ 4)\ and\ (3\ men\ out\ of\ 6)$

The number of ways of these selection are :

i) 1 ladies + 4 gentlemen $in\ {}^4C_1 \times {}^6C_4 = 15 \times 4 = 60$

ii) 2 ladies + 3 gentlemen = ${}^4C_2 \times {}^4C_3 = 20 \times 6 = 120$

∴ Total number of ways committee can be formed **=60+ 120=180.**

16) Prove that: $\frac{1.2}{1!} + \frac{2.3}{2!} + \frac{3.4}{3!} + = 3e.$

➢ **Solution:**

Let t_n be n^{th} term.

$$t_n = \frac{n(n+1)}{n!}$$

$$t_n = \frac{n^2+1}{n!}$$

$$= \frac{n^2+1}{n!} + \frac{n(n+1)}{n!}$$

$$= \frac{n(n-1+1)}{n((n-1)!} + \frac{1}{(n-1)!}$$

$$= \frac{n(n-1)+n}{n((n-1)!} + \frac{1}{(n-1)!}$$

$$= \frac{n(n-1)}{n(n-1)!} + \frac{n}{n(n-1)!} + \frac{1}{(n-1)!}$$

$$= \frac{n(n-1)}{n(n-1)(n-2)!} + \frac{n}{n(n-1)!} + \frac{1}{(n-1)!}$$

$$\boldsymbol{t_n = \frac{1}{(n-2)!} + \frac{2}{(n-1)!}}$$

Sum of series $= \sum t_n$

$$= \sum \left\{\frac{1}{(n-2)!} + \frac{2}{(n-1)!}\right\}$$

$$= \sum_{n=}^{\infty} \frac{1}{(n-2)!} + 2\sum_{n=1}^{\infty} \frac{1}{(n-1)!}$$

$$= \left\{\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\right\} + 2\left\{\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} \cdots\right\}$$

$$= \left\{1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\right\} + 2\left\{1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} \cdots\right\}$$

$$\left\{\because \text{since. } e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\right\}$$

$$= e + 2e = 3e$$

$\therefore$ ***sum of series = 3e*** **Proved.**

Taking − ve sign,

$$52x - 39y + 13 = -(60x - 25y + 35)$$

Solving, $\mathbf{112x - 64y + 48 = 0}$ is one of bisector

$$\textbf{\textit{Let } } \boldsymbol{m_2} \textbf{\textit{ slope of line then, }} \boldsymbol{m_2} = \frac{-coeff.of\ x}{coeff\ of\ y}$$

$$= \frac{-112}{-64} = \frac{\mathbf{7}}{\mathbf{4}}$$

To prove: The bisectors at right angle

For this, checking condition of perpendicularity:

$$\boldsymbol{m_1 . m_2 = -1}$$

$$m_1 . m_2 = \frac{-8}{14} \times \frac{7}{4} = -1$$

Hence,

The bisectors of the angle between the lines are at Right angled.

18) Find the separate equations represented by
$2x^2 + xy - 3y^2 + 9x + 26y - 35 = 0.$
Also find the angle between them.

➢ **Solution:**

$$2x^2 + xy - 3y^2 + 9x + 26y - 35 = 0$$

$$2x^2 + x(y + 9) - 3y^2 + 26y - 35 = 0$$

Which is quadratic in x so,

$$x = \frac{-(y+9) \pm \sqrt{(y+9)^2 - 4.2(-3y2 + 26y - 35)}}{2.2}$$

$$x = \frac{-(y+9) \pm \sqrt{y^2 + 18y + 81 - 8(-3y2 + 26y - 35)}}{2.2}$$

$$x = \frac{-(y+9) \pm \sqrt{y^2 + 18y + 81 + 24y2 - 208y + 280}}{2.2}$$

$$x = \frac{-(y+9) \pm \sqrt{25y^2 - 190y + 361}}{4}$$

$$x = \frac{-(y+9) \pm \sqrt{(5y-19)^2}}{4}$$

$$x = \frac{-(y+9) \pm (5y-19)}{4}$$

$\boldsymbol{Taking\ \ +ve\ sign,}\ x = \frac{-(y+9) + (5y-19)}{4}$

$$4x = -(y+9) + (5y-19)$$

$$\mathbf{4x - 4y + 28 = 0}$$

$\boldsymbol{Let\ m_1\ slope\ of\ line\ then, m_1} = \frac{-coeff.of\ x}{coeff\ of\ y}$

$$\boldsymbol{m_1 = \frac{-4}{-4} = 1}$$

$\boldsymbol{Taking\ \ -ve\ sign,}\ \ x = \frac{-(y+9) - (5y-19)}{4}$

$$4x = -(y+9) - (5y-19)$$

$$\mathbf{4x - 6y - 10 = 0}$$

Making Homogenous equation of degree (2) with equation (3)

we get, $x^2+y^2=c^2\left(\frac{bx+ay}{ab}\right)^2$

$$a^2b^2(x^2+y^2)=c^2(bx+ay)^2$$

$$a^2b^2(x^2+y^2)=c^2(b^2x^2+2abxy+a^2y^2)$$

$$(a^2b^2-c^2b^2)x^2+(a^2b^2-a^2c^2)y^2-2abc^2xy=0$$

Which represent pair of lines through origin.

The line pair will be right angles if

Coeff. Of x^2 + Coeff. Of y^2=0

$$a^2b^2-c^2b^2+a^2b^2-a^2c^2=0$$

$$c^2b^2+a^2c^2=2a^2b^2$$

Dividing both side by $a^2b^2c^2$,

$$\frac{c^2b^2+a^2c^2}{a^2b^2c^2}=\frac{2a^2b^2}{a^2b^2c^2}$$

$$\therefore \frac{1}{a^2}+\frac{1}{b^2}=\frac{2}{c^2}\ \textit{proved.}$$

19) Find the equation of circle passing through the points (3,-2) and (-2, 0) whose centre lies on the line $2x$ - y = 3.

➢ **Solution:**

$$Let, x^2+y^2+2gx+2fy+c=0 \quad \cdots\cdots\cdots (i)$$

Be the required equation of circle. Let it pass through (3,-2) and (-2,0). Then ,

$(3,-2)$: $\quad 9 + 4 + 6g - 4f + c = 0$

$$i.e.\ \ 6g - 4f + 13 + c = 0 \ \cdots\cdots\cdots (ii)$$

$(-2,0)$: $\quad 4 - 4g + c = 0$

$$i.e.\ \ -4g + 4 + c = 0 \ \cdots\cdots\cdots (iii)$$

Subtracting Eqn(ii) and Eqn (iii) weget, $\mathbf{10g - 4f + 9 = 0 \cdots (iv)}$

Also centre$(-g, -f)$ of the circle lies on the line $2x - y = 3$ Then,

$$\mathbf{-2g + f = 3 \ \cdots\cdots\cdots (v)}$$

Solving the equations (iv) and (v) we get,

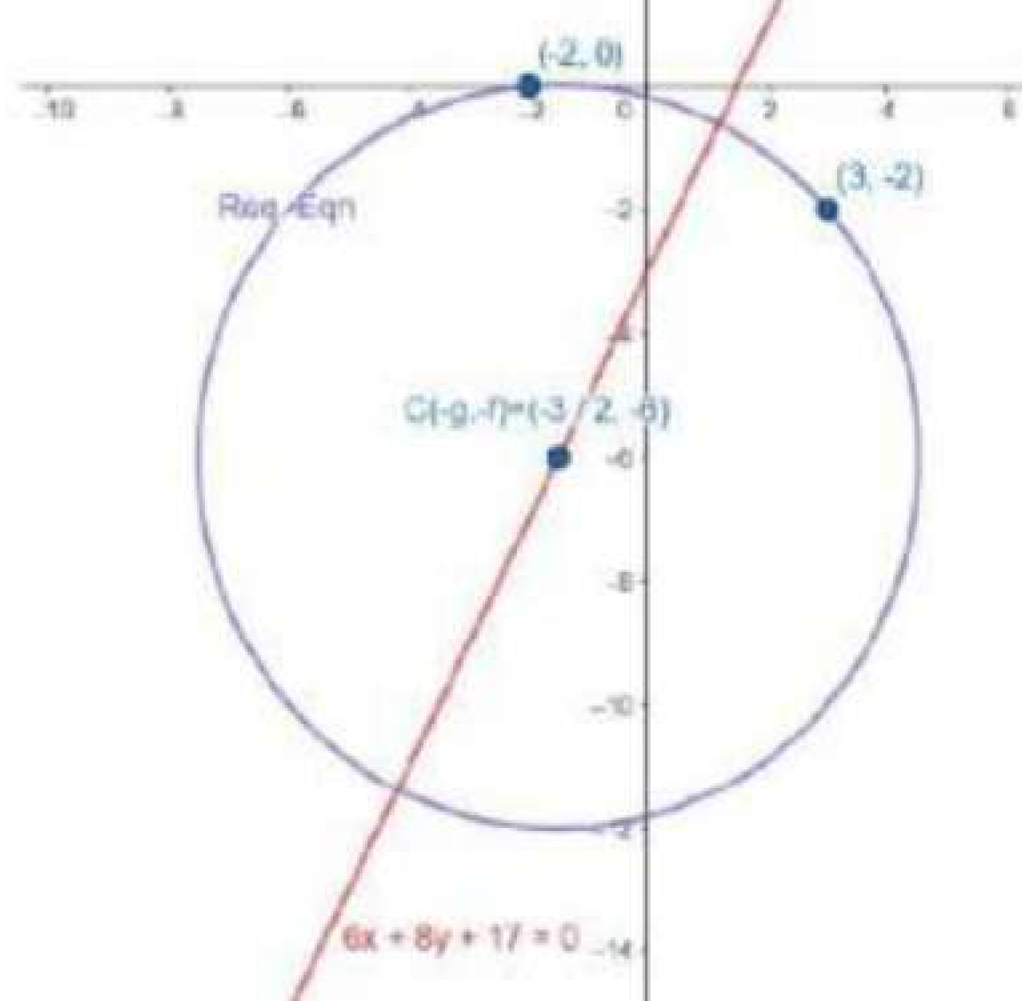
$$g = \frac{3}{2}, f = 6$$

$$From\ (iii),\ -4.\frac{3}{2} + 6 + c = 0 \ \Rightarrow C = 2$$

Puting these values in (i) we get,

$$x^2 + y^2 + 2.\frac{3}{2}x + 2.6y + 2 = 0$$

$$\therefore\ \mathbf{x^2 + y^2 + 3x + 12y + 2 = 0}, \textit{which is req.equation}$$



20) *Find* $\frac{dy}{dx}$ *(any one)*

i) $x^2y^2 = tan\,(xy)$

➢ **Solution:**

$$x^2y^2 = tan\,(xy)$$

Differenting both side w.r.t $'x'$

$$\frac{d}{dx}(x^2y^2) = \frac{d}{dx}tan(xy)$$

$$x^2\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(x^2) = \frac{d}{dx}tan(xy)$$

$$x^2\ \frac{dy^2}{dy} \times \frac{dy}{dx} + y^2\ 2x = \frac{dtan(xy)}{d(xy)} \times \frac{d(xy)}{dx}$$

$$x^2\ 2y \times \frac{dy}{dx} + y^2\ 2x = sec^2(xy) \times \left[x\frac{dy}{dx} + y.\frac{dx}{dx}\right]$$

$$2x^2y \times \frac{dy}{dx} + 2xy^2 = sec^2(xy)\left[x\frac{dy}{dx} + y\right]$$

$$2x^2y \times \frac{dy}{dx} + 2xy^2 = x\,sec^2(xy)\frac{dy}{dx} + y\,sec^2(xy)$$

$$\{2x^2y - x\,sec^2(xy)\} \times \frac{dy}{dx} = y\,sec^2(xy) - 2xy^2$$

$$\therefore \frac{dy}{dx} = \frac{y\,sec^2(xy) - 2xy^2}{2x^2y - x\,sec^2(xy)}$$

ii) $x^y . y^x = a$

➢ **Solution:**

$$x^y . y^x = a$$

Taking both side log we get,

$$log(x^y . y^x) = log\ a$$

Using properties $log(x^y . y^x) = log(x^y) + log(y^x)$

$$\boldsymbol{log(x^y) + log(y^x) = log\ a}$$

Using properties $log(x^y) = ylogx$

$$y.\log x + x.\log y = \log(a)$$

Differenting both side w.r.t $'x'$

$$\frac{d}{dx}(y.\log x + x.\log y) = \frac{d}{dx}\log(a)$$

$$\frac{d}{dx}(y.\log x) + \frac{d}{dx}(x.\log y) = 0$$

$$y\frac{d}{dx}(\log x) + \log x\frac{d}{dx}(y) + x\frac{d}{dx}(\log y) + \log y\frac{d}{dx}(x) = 0$$

$$y\frac{1}{x} + \log x\,\frac{dy}{dx} + x\left(\frac{d\,logy}{dy}\times\frac{dy}{dx}\right) + \log y.1 = 0$$

$$y\frac{1}{x} + \log x\,\frac{dy}{dx} + x\left(\frac{1}{y}\times\frac{dy}{dx}\right) + \log y = 0$$

$$\frac{y}{x} + \log x\,\frac{dy}{dx} + \frac{x}{y}\times\frac{dy}{dx} + \log y = 0$$

$$\frac{y}{x} + \left(\log x + \frac{x}{y}\right)\frac{dy}{dx} + \log y = 0$$

$$\left(\log x + \frac{x}{y}\right)\frac{dy}{dx} = -\left(\log y + \frac{y}{x}\right)$$

$$\frac{dy}{dx} = -\frac{\left(\log y + \frac{y}{x}\right)}{\left(\log x + \frac{x}{y}\right)}$$

$$\therefore \frac{dy}{dx} = -\frac{y(x\log y + y)}{x(y\log x \ + \ x)}$$

–The End –

1. a) State and prove cosine law.

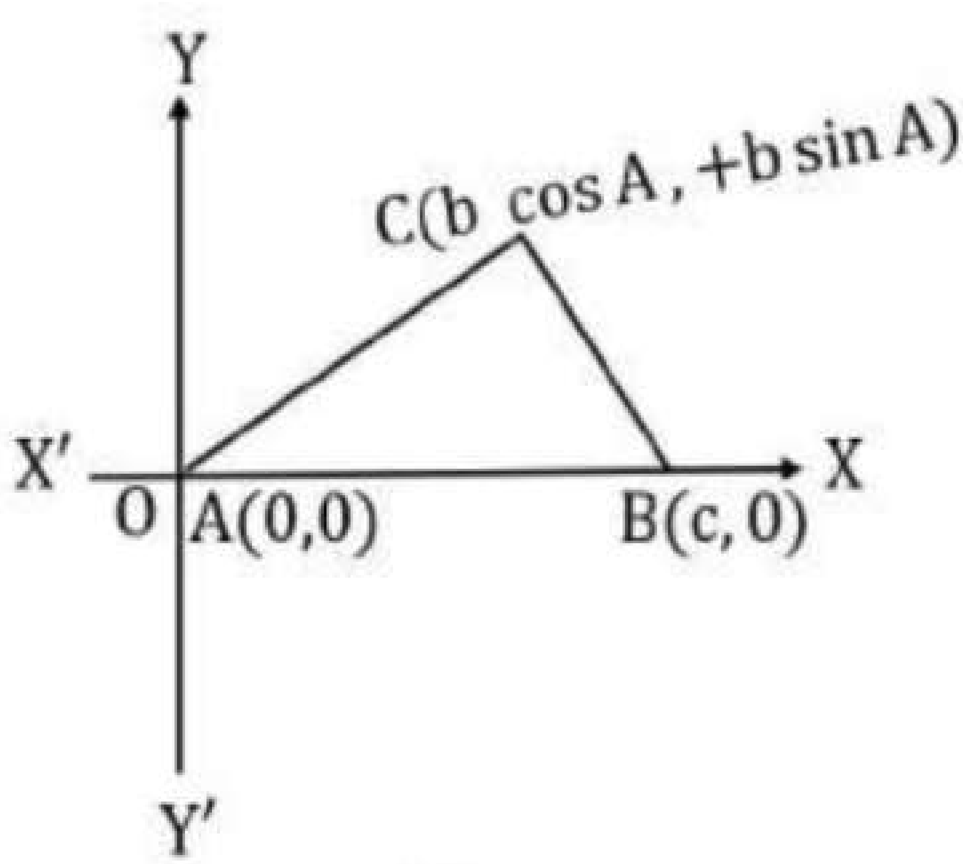
➢ **Solution:**

Statement: In any triangle ABC,

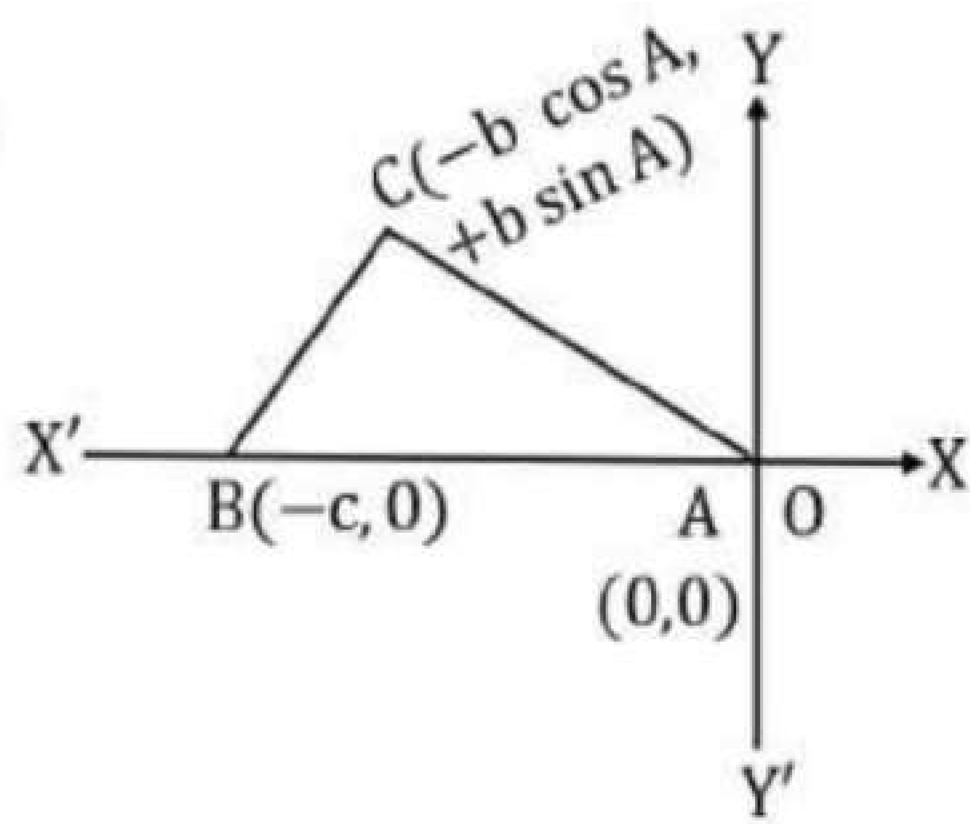
$$Cos\ A = \frac{b^2 + c^2 - a^2}{2\,bc} \quad or, \quad a^2 = b^2 + c^2 - 2bc\cos A,$$

$$Cos\ B = \frac{c^2 + a^2 - b^2}{2\,ca} \quad or, \quad b^2 = c^2 + a^2 - 2ca\cos B,$$

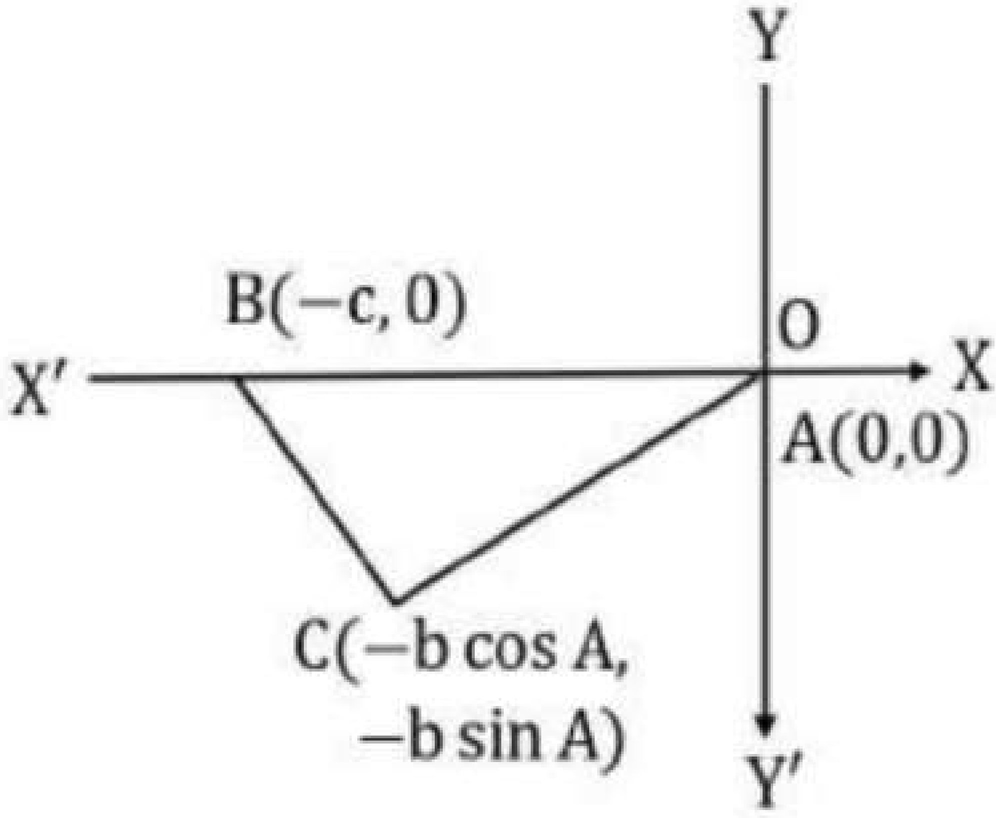
$$Cos\ C = \frac{a^2 + b^2 - c^2}{2\,ab} \quad or, \quad c^2 = a^2 + b^2 - 2ab\cos C.$$



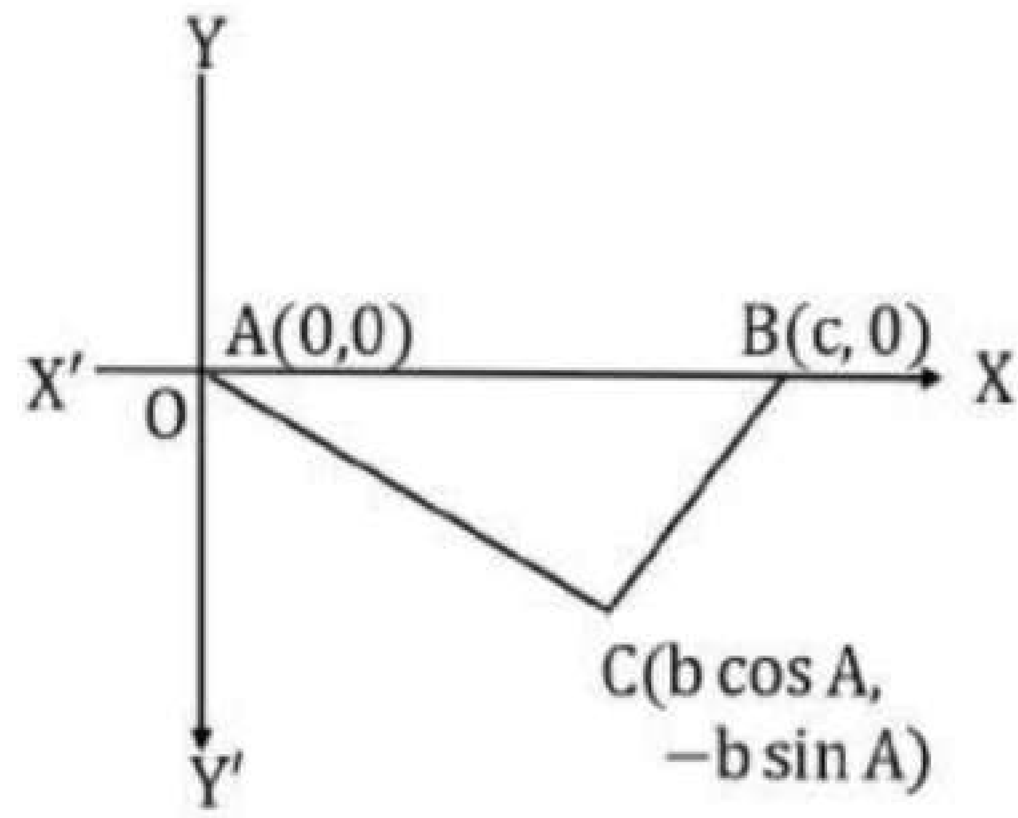
(a)



(b)



(c)



(d)

- we place the triangle ABC in the standard position with the vertex A at the **origin** and the side AB along the **positive x-axis.** Then, the coordinates of three vertices A(0, 0), B(c, 0) and $C(b\cos A, \pm b\sin A)$ respectively. { ∵ The +ve sign if the vertex C is above the x-axis and the -ve sign if it is below the x-axis.}
- Now, using the distance formula, **we have**

$$BC^2 = (b\cos A - c)^2 + (\pm b\sin A + 0)^2$$

$$or, \quad a^2 = b^2(\cos^2 A + \sin^2 A) + c^2 - 2bc\cos A$$

$$Hence, \ a^2 = b^2 + c^2 - 2bc\cos A$$

$$\therefore \boldsymbol{\cos A} = \frac{(\boldsymbol{b^2 + c^2 - a^2})}{\boldsymbol{2\,bc}}$$

Similarly,*it can Prove For Cos B and Cos C*

$$Cos\ B = \frac{c^2 + a^2 - b^2}{2\,ca}, \qquad Cos\ C = \frac{a^2 + b^2 - c^2}{2\,ab}$$

b) If $tan^{-1}x + tan^{-1}y + tan^{-1}z = \pi$, prove that $x + y + z = xyz$.

➢ **Solution:**

Given,

$$tan^{-1}x + tan^{-1}y + tan^{-1}z = \pi$$

$$tan^{-1}x + tan^{-1}y = \pi - tan^{-1}z$$

$$tan^{-1}\left(\frac{x+y}{1-xy}\right) = \pi - tan^{-1}z$$

$$\frac{x+y}{1-xy} = tan(\pi - tan^{-1}z)$$

$$\frac{x+y}{1-xy} = -tan\,(tan^{-1}z)$$

$$\frac{x+y}{1-xy} = -z$$

$$x + y = -z + xyz$$

$$\therefore \boldsymbol{x + y + z = xyz}$$

2. a) Prove that $\lim_{\theta \to 0} \frac{sin\theta}{\theta} = 1$.

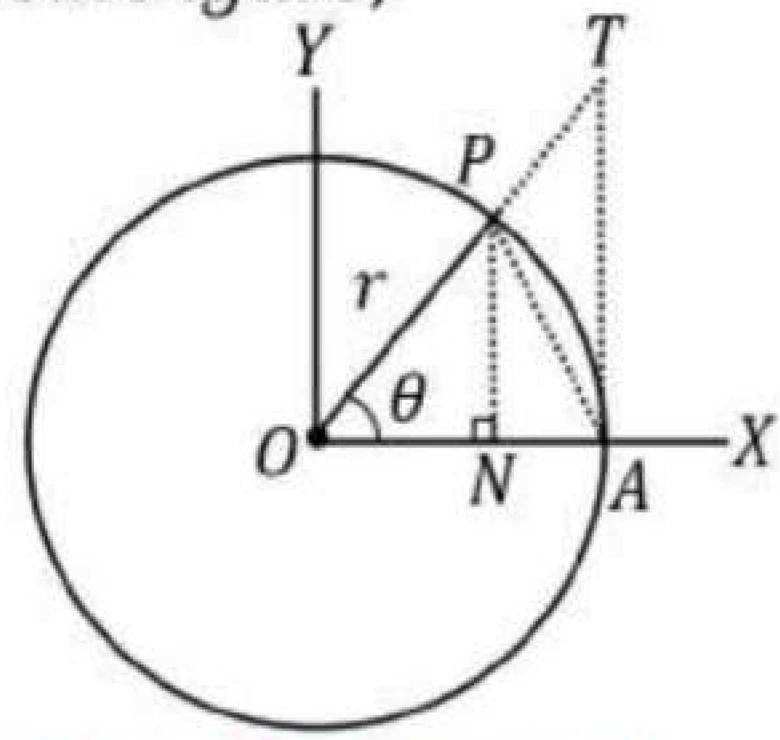
➢ **Proof:**

Consider a circle having centre O and radius $OP = r$.

Let $\angle XOP = \theta$ (in radian). $\left(0 \le \theta \le \frac{\pi}{2}\right)$.

➢ *Let the circle cut OX at A. Draw PN ⊥ OA. Draw a tangent at A to meet OP produced at T. Join PA. From Figure,*

$$\Delta \boldsymbol{OAP} < \Delta \boldsymbol{Sector\ OAP} < \Delta \boldsymbol{OAT}$$



$$\Rightarrow \frac{1}{2}.OA.PN < \frac{1}{2}r^2\theta < \frac{1}{2}.OA.AT$$

$$\Rightarrow \frac{1}{2}r.r\sin\theta < \frac{1}{2}r^2\theta < \frac{1}{2}r.r\tan\theta$$

$[\because In\ rt.\Delta PON, PN = r\sin\theta, in\ r.t\ \Delta TOA, AT = r\tan\theta]$

$$\Rightarrow \sin\theta < \theta < \tan\theta$$

$$\Rightarrow 1 < \frac{\theta}{\sin\theta} < \frac{1}{\cos\theta}$$ $[\because Dividing\ each\ term\ by \sin\theta]$

$$\Rightarrow 1 > \frac{\sin\theta}{\theta} > \cos\theta$$

$$\Rightarrow \lim_{\theta\to 0} 1 \geq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \geq \lim_{\theta\to 0}\cos\theta$$

$$\Rightarrow 1 \geq \lim_{\theta\to 0}\frac{\sin\theta}{\theta} \geq 1$$

$$\Rightarrow \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

$$\therefore \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

b) Test the continuity of :

$$f(x) = \begin{cases} 2x & for \quad x \le 3 \\ 3x - 3 & for \quad x > 3 \end{cases} \text{ at } x = 3$$

➢ **Solution:**

$$\textit{Left hand limit at } x = 3 = \lim_{x \to 3^-} f(x)$$

$$= \lim_{x \to 3^-} 2x$$

$$= 2 \times 3$$

$$= 6$$

$$\textit{Right hand limit at } x = 3 = \lim_{x \to 3^+} f(x)$$

$$= \lim_{x \to 3^+} 3x - 3$$

$$= 3 \times 3 - 3$$

$$= 6$$

Functional value at $x = 3$, $f(3) = 2 \times 3 = 6$

$$\textit{Hence, } \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$$

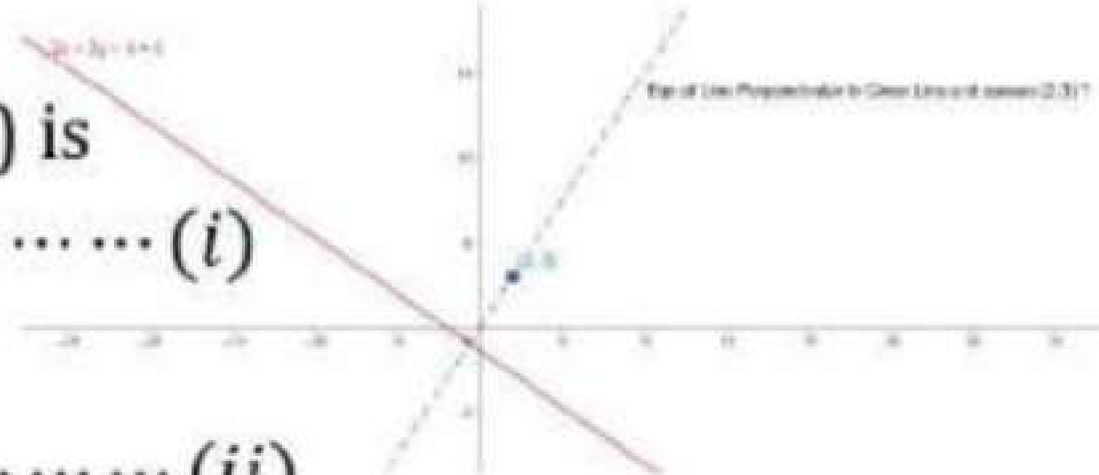
The function continuous $at\ x = 3$.

3. a) Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are at right angles if $\frac{1}{a^2} + \frac{1}{b^2} + \frac{2}{c^2}$.

➢ **Refer to the solution 2079 New of Q. No 18 (or) on Page 73.**

b) Find the equation of straight line at point $(2, 3)$ and equation of perpendicular $2x + 3y + 4 = 0$

➢ **Solution:-**



The equation of line through $(2 , 3)$ is

$$y - 3 = m(x - 2) \qquad \cdots\cdots\cdots\cdots (i)$$

The given line is

$$\mathbf{2x + 3y + 4 = 0} \qquad \cdots\cdots\cdots\cdots (ii)$$

Slope of line (i) $m_1 = m$

Slope of line (ii)

$$m_2 = -\frac{coefficient\ of\ x}{coefficient\ of\ y} = -\frac{2}{3}$$

If the linear (i) and (ii) are perpendicular,

$$m_1 m_2 = -1$$

$$m \times \frac{-2}{3} = -1$$

$$m = \frac{3}{2}$$

Substituting value of $\mathbf{m = \frac{3}{2}}$ in equation (i), we get

$$y - 3 = \frac{3}{2}(x - 2)$$

$$2y - 6 = 3x - 0$$

$$3x - 2y = 0$$

$\therefore$ **$3x - 2y = 0$ is the required Equation.**

4) Find the sum of squares of first n natural numbers.

➢ **Solution:-**

Sum of square of first n natural numbers:

Let $S_n = 1^2 + 2^2 + 3^2 + \ldots\ldots + n^2$

We have,

$$(a-1)^3 = a^3 - 3a^2 + 3a - 1$$

or, $a^3 - (a-1)^3 = 3a^2 + 3a - 1$

Putting $\boldsymbol{a = 1, 2, 3, \ldots\ldots\ldots, n}$ *we get,*

$$1^3 - 0^3 = 3.1^2 - 3.1 + 1$$

$$2^3 - 1^3 = 3.2^2 - 3.2 + 1$$

$$3^3 - 2^3 = 3.3^2 - 3.3 + 1$$

....

$$n^3 - (n-1)^3 = 3.n^2 - 3.n + 1$$

Adding all, we get,

$$n^3 = 3(1^2 + 2^2 + 3^2 + \cdots + n^2) - 3(1 + 2 + 3 + \cdots + n)$$
$$+ (1 + 1 + \cdots \text{ to n terms.})$$

$$or, n^3 = 3\,S_n - 3\frac{n(n+1)}{2} + n$$

$$or, 3\,S_n = n^3 - n\frac{3n(n+1)}{2}$$

$$or, 6\,S_n = 2n(n^2 - 1) + 3n(n+1) = n(n+1)(2n+1)$$

$$\therefore\ S_n = \frac{\boldsymbol{n(n+1)(2n+1)}}{\mathbf{6}}$$

5) Find the term free form x in the expansion of $\left(2x-\frac{1}{x}\right)^{10}$.

➢ **Solution:-**

Given, $\left(2x+\frac{1}{x}\right)^{10}$

General terms in the above expansion is given by

$$
\begin{aligned}
t_{r+1} &= c(10,r).(2x)^{10-r}\left(\frac{-1}{x}\right)^{r} \\
&= c(10,r).(2x)^{10-r}.(-1)^{r}\,x^{-r} \\
&= c(10,r).(2)^{10-r}.(-1)^{r}\,x^{10-r-r} \\
&= c(10,r)2^{10-r}.(-1)^{r}\,x^{\mathbf{10-2r}}
\end{aligned}
$$

If this term is independent to x then,

$$10-2r=0 \quad i.e., \qquad r=5$$

Hence,

$$Term = t_{5+1} = t_6 \text{ is independent of } x$$

$$
\begin{aligned}
and.\ t_{5+1} &= c\,(10,5)2^{10-5}\,(-1)^{5}\,x^{10-2\times 5} \\
&= \frac{10!}{5!\ 5!}\times 2^{5}(-1).x^{0} \\
&= -\frac{10!}{5!\ 5!}\times 2^{5}
\end{aligned}
$$

$\therefore$ ***Term Free from x = −8064.***

6)Prove that : $1+\frac{1+3}{2!}+\frac{1+3+5}{3!}+\frac{1+3+5+7}{4!}+\cdots\cdots=2e.$

➢ **Solution:-**

Let t_n be the n^{th} term of the given series.

$$\text{Then, } t_n = \frac{n^2}{n!} = \frac{n}{(n-1)!} \quad \text{or,} \quad t_n = \frac{n-1+1}{(n-1)!}$$

$$\therefore\ t_n = \frac{n}{(n-1)!} + \frac{n}{(n-1)!}$$

$$t_1 = 0 + \frac{1}{0!}$$

$$t_2 = \frac{1}{0!} + \frac{1}{1!}$$

$$t_3 = \frac{1}{1!} + \frac{1}{2!}$$

$$t_4 = \frac{1}{2!} + \frac{1}{3!}$$

....

Now, by addition, the sum of given series

$$s_\infty = \left(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \cdots\right) + \left(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\right)$$

$$= e + e$$

$$\therefore s_\infty = 2e$$

7) Prove that every quadratic equation cannot have more than two roots.

➢ **Refer to the solution 2076 of Q. No 5 on page 11.**

12) Find form first principle, the derivative $\frac{1}{\sqrt{x}}$.

➢ **Solution:**

$$Let\ y = \frac{1}{\sqrt{x}} = \frac{1}{x^{\frac{1}{2}}} \quad \cdots\cdots\cdots (i)$$

Let Δx *be* small increment in x and Δy be the corresponding small increment in y Then,

$$y + \Delta y = \frac{1}{(x+\Delta x)^{\frac{1}{2}}} \quad \cdots\cdots\cdots (ii)$$

Subtracting eqn (i) from (ii) we get,

$$\Delta y = \frac{1}{(x+\Delta x)^{\frac{1}{2}}} - \frac{1}{x^{\frac{1}{2}}}$$

$$= \frac{x^{\frac{1}{2}} - (x+\Delta x)^{\frac{1}{2}}}{(x+\Delta x)^{\frac{1}{2}}.x^{\frac{1}{2}}}$$

$$\frac{\Delta y}{\Delta x} = \frac{\left[(x+\Delta x)^{\frac{1}{2}} - x^{\frac{1}{2}}\right]}{\Delta x} \times \frac{1}{(x+\Delta x)^{\frac{1}{2}}.x^{\frac{1}{2}}}$$

By first Principle,

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\left[(x+\Delta x)^{\frac{1}{2}} - x^{\frac{1}{2}}\right]}{\Delta x} \times \frac{1}{(x+\Delta x)^{\frac{1}{2}}.x^{\frac{1}{2}}}$$

$$= \lim_{x+\Delta x \to x} \frac{-\left[(x+\Delta x)^{\frac{1}{2}} - x^{\frac{1}{2}}\right]}{x+\Delta x - x} \times \lim_{\Delta x \to 0} \frac{1}{(x+\Delta x)^{\frac{1}{2}}.x^{\frac{1}{2}}}$$

$$= \frac{1}{2}x^{\frac{1}{2}-1} \times \frac{1}{x^{\frac{1}{2}}.x^{\frac{1}{2}}}$$

$$= \frac{1}{2x^{\frac{1}{2}}} \times \frac{1}{x^{\frac{1}{2}}} \times \frac{1}{x^{\frac{1}{2}}}$$

$$= -\frac{1}{2x^{\frac{3}{2}}}$$

$$\therefore \boldsymbol{\frac{dy}{dx} = -\frac{1}{2x^{\frac{3}{2}}}}$$

13) Find $\left(\frac{dy}{dx}\right)$: (Any One)

i) $x^2 + 2hxy + by^2 = 0$

➢ **Solution:**

$x^2 + 2hxy + by^2 = 0$

Differentiating both sides with respect to x, we have

$$\frac{d}{dx}(x^2 + 2\text{hxy} + y^2) = \frac{d}{dx}(0)$$

$$or,\quad \frac{dx^2}{dx} + 2\text{h}\frac{d(xy)}{dx} + \text{b}\frac{dy^2}{dy}.\frac{dy}{dx} = 0$$

$$or,\quad 2x + 2h\left[x\frac{dy}{dx} + y.\frac{dx}{dx}\right] + 2by\frac{dy}{dx} = 0$$

$$or,\ 2x + 2h\left[x\frac{dy}{dx} + y.1\right] + 2by\frac{dy}{dx} = 0$$

$$or,\ 2x + 2hx\frac{dy}{dx} + 2hy + 2by\frac{dy}{dx} = 0$$

$$or,\ 2\left[(hx + by)\frac{dy}{dx} + (hy + x)\right] = 0$$

$$\therefore \frac{\boldsymbol{dy}}{\boldsymbol{dx}} = \frac{-(\boldsymbol{hy} + \boldsymbol{x})}{(\boldsymbol{h}x + \boldsymbol{by})}$$

ii) $x^2 + y^2 = sec\ xy^2$

➢ **Solution:**

$$x^2 + y^2 = sec\ xy^2$$

Differentiating both sides with respect to x, we have

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}sec(xy^2)$$

$$\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = \frac{d}{dx}sec(xy^2)$$

$$2x + \frac{dy^2}{dy} \times \frac{dy}{dx} = \frac{dsec(xy^2)}{d(xy^2)} \times \frac{d(xy^2)}{dx}$$

$$2x + 2y\frac{dy}{dx} = sec(xy^2).\tan(xy^2) \times \left[x\frac{dy^2}{dx} + y^2.\frac{dx}{dx}\right]$$

$$2x + 2y\frac{dy}{dx} = sec(xy^2).\tan(xy^2) \times \left[x\frac{dy^2}{dy} \times \frac{dy}{dx} + y^2\right]$$

$$2x + 2y\frac{dy}{dx} = sec(xy^2).\tan(xy^2) \times \left[2xy \times \frac{dy}{dx} + y^2\right]$$

$$2x - y^2 sec(xy^2).\tan(xy^2)$$

$$= \{2xysec(xy^2).\tan(xy^2) - 2y\}\frac{dy}{dx}$$

$$\frac{2x - y^2 sec(xy^2).\tan(xy^2)}{2xysec(xy^2).\tan(xy^2) - 2y} = \frac{dy}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{2x - y^2\ sec(xy).\tan\ (xy^2)}{2xy\ sec(xy).\tan\ (xy^2) - 2y}.$$

14) Evaluate: i) $\int \frac{x^2\,dx}{(1+x^2)^2}$.

➢ **Solution:**

$$Let, I = \int \frac{x^2\ dx}{(1 + x^2)^2}$$

$Let\ x = tan\theta$, $tan\theta = \frac{x}{1} = \frac{p}{b}$, $h = \sqrt{p^2 + b^2} = \sqrt{1 + x^2}$

$tan^{-1}x = \theta$, $sin\theta = \frac{p}{h} = \frac{x}{\sqrt{1+x^2}}$, $cos\theta = \frac{b}{h} = \frac{1}{\sqrt{1+x^2}}$

$or,\ x = tan\theta \Rightarrow dx = sec^2\theta.d\theta$

Integral becomes,

$$I = \int \frac{(tan\theta)^2\ sec^2\theta d\theta}{(1 + (tan\theta)^2)^2}$$

$$I = \int \frac{(tan\theta)^2\ sec^2\theta d\theta}{(sec^2\theta)^2}$$

$[\because \sec^2\theta = 1 + tan^2\theta\,]$

$$= \int \frac{(tan\theta)^2 d\theta}{sec^2\theta}$$

$$= \int sin^2\theta \, d\theta$$

$$I = \frac{1}{2}\int 1 - \cos 2\theta \;\, d\theta$$

$$I = \frac{1}{2}(\theta - 2sin2\theta) + c \qquad \because sin2\theta = sin\theta.cos\theta = \frac{1}{\sqrt{1+x^2}}.\frac{x}{\sqrt{1+x^2}} = \frac{x}{1+x^2}$$

$$\therefore I = \frac{1}{2}\left(tan^{-1}\theta - 2\frac{x}{1+x^2}\right) + c$$

ii) $\int e^{ax} cosbx \, dx$

➢ **Solution:**

$$Let\ I = \int e^{ax} cosbx\, dx$$

$$= \cos bx \int e^{ax} dx - \int \left(\frac{dcosbx}{dx} \int e^{ax} dx\right)$$

$$= \cos bx \frac{e^{ax}}{a} - \int -bsinbx.\frac{e^{ax}}{a} dx$$

$$= \frac{1}{a} e^{ax} cos\ bx + \frac{b}{a}\int e^{ax} \sin bx dx$$

$$= \frac{1}{a} e^{ax} cos\ bx + \frac{b}{a}\left[\sin bx \int e^{ax} dx - \int \left(\frac{dsindx}{dx}\int e^{ax} dx\right) dx\right]$$

$$= \frac{1}{a} e^{ax} \cos bx + \frac{b}{a}\left[\sin bx \frac{e^{ax}}{a} - \int b.\cos bx.\frac{e^{ax}}{a} dx\right]$$

$$= \frac{1}{a} e^{ax} \cos bx + \frac{b}{a^2}.e^{ax} \sin bx - \frac{b^2}{a^2}\int e^{ax} \cos bx\, dx$$

$$\therefore I = e^{ax}\left[\frac{1}{a}\cos bx + \frac{b}{a^2}\sin bx\right] - \frac{b^2}{a^2} I$$

$$I \times \left(1 + \frac{b^2}{a^2}\right) = e^{ax}\left[\frac{1}{a}\cos bx + \frac{b}{a^2}\sin bx\right]$$

$$I \times \left(\frac{a^2+b^2}{a^2}\right) = e^{ax}.\frac{a\cos bx + b\sin bx}{a^2}$$

$$\therefore \boldsymbol{I = \frac{e^{ax}}{a^2+b^2}\,(a\cos bx + b\sin bx) + c}$$

15) Evaluate : $\int_0^{\pi/2} (1 + Cosx)^2 \sin x\, dx$

➢ **Solution:**

$Let, y = 1 + \cos x$, then $dy = -\sin x\, dx$

When $x = 0, y = 2$ and when $x = \frac{\pi}{2}, y = 1$

$$\therefore I = \int_0^{\pi/2} (1 + Cosx)^2 \sin x\, dx = -\int_2^1 y^2\, dy$$

$$= -\left[\frac{y^3}{3}\right]_2^1$$

$$= -\left[\frac{1^3}{3} - \frac{2^3}{3}\right]$$

$$= -\left[\frac{1}{3} - \frac{8}{3}\right]$$

$$I = \frac{7}{3}$$

–The End –

Engineering Mathematics I_(Engg. All) 1^{st} Sem

(2080 New) Question Paper Solution.

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1 a) If $U = \{x: x\ is\ a\ vowel\ of\ English\ Alphabet\}$, $A = \{a, e, i\}, B = \{e, i, o\}$. Verify that $\overline{A \cap B} = \overline{A} \cup \overline{B}$.

➤ **Solution:**

$U = \{x: x\ is\ a\ vowel\ of\ English\ Alphabet\}$

$U = \{a, e, i, o, u\}$

Now,

$$A \cap B = \{a, e, i\} \cap \{e, i, o\}$$

$$A \cap B = \{e, i\}$$

$$\overline{A \cap B} = U - A \cap B$$

$$= \{a, e, i, o, u\} - \{e, i\}$$

$$\boxed{\therefore\ \overline{A \cap B} = \{a, o, u\}}$$

$$\overline{A} = U - A = \{a, e, i, o, u\} - \{a, e, i\} = \{o, u\}$$

$$\overline{B} = U - B = \{a, e, i, o, u\} - \{e, i, o\} = \{a, u\}$$

$$\overline{A} \cup \overline{B} = \{o, u\} \cup \{a, u\}$$

$$\boxed{\overline{A} \cup \overline{B} = \{a, o, u\}}$$

Hence, $\overline{A \cap B} = \overline{A} \cup \overline{B}$ *Verified.*

b) Let $f: R \to R$ and $R \to R$ be defined by $f(x) = x^3 + 1$ and $g(x) = x + 5$, Find $(i)\ fog(x)$ $(ii)\ gof\,(x)$

➢ **Refer to the solution 2078 of Q. No 12 on page 41.**

2. a) Solve : $Cos\ 2x - sin\ x = 0$

➢ **Solution:**

$$Cos\ 2x - sin\ x = 0$$

$$1 - 2sin^2x - sin\ x = 0$$

$$2sin^2x + \sin x - 1 = 0$$

$$2sin^2x + 2\sin x - sin\ x - 1 = 0$$

$$2sin\ x(sin\ x + 1) - (\sin x + 1) = 0$$

$$(2sin\ x - 1)(sin\ x + 1) = 0$$

Either, $2sin\ x - 1 = 0$ *or,* **$sin\ x + 1 = 0$**

$$2sin\ x - 1 = 0$$

$$or,\ sin\ x = \frac{1}{2} = sin\ \frac{\pi}{6}$$

$$x = n\pi + (-1)^n\frac{\pi}{6}$$

$$sin\ x + 1 = 0$$

$$or,\ sin\ x = -1 = sin\frac{3\pi}{2}$$

$$x = n\pi + (-1)^n\frac{3\pi}{2}$$

Hence,

$$\therefore x = n\pi + (-1)^n\frac{\pi}{6},\quad n\pi + (-1)^n\frac{3\pi}{2}$$

b) Evaluate : $tan^{-1}3 + tan^{-1}\frac{1}{3}$.

➢ **Solution:**

$$tan^{-1}3 + tan^{-1}\frac{1}{3}$$

$$= tan^{-1}\left(\frac{3+\frac{1}{3}}{1-3\times\frac{1}{3}}\right)$$

$$= tan^{-1}\left(\frac{3+\frac{1}{3}}{0}\right)$$

$$= tan^{-1}(\infty)$$

$$= \frac{\pi}{2}$$

3. a) If $2cosA = sinB: sinC$, Prove that the triangle is isosceles.

➢ **Solution:** $2cosA = sinB: sinC$

$$2cosA = \frac{sinB}{sinC}$$

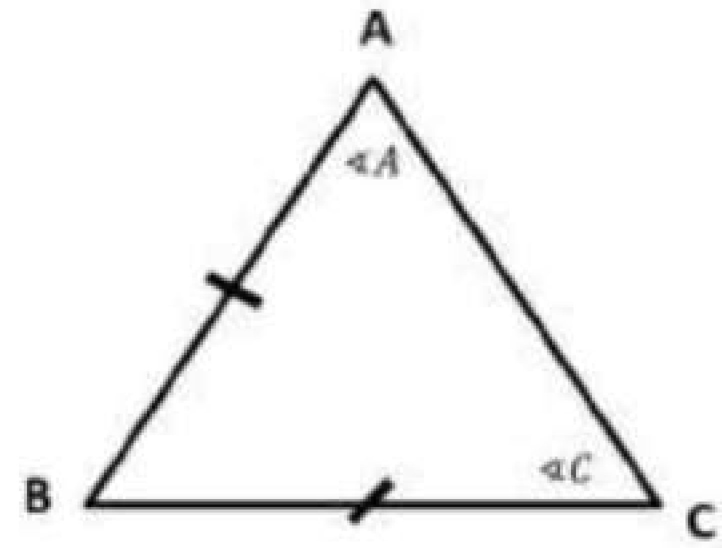
$$2cosA\,SinC = Sin\,B$$

$$Sin\,B = 2cosA.sinC$$

$$Sin\,B = Sin\,(A+C) - Sin\,(A-C)$$

$$Sin\,B = sin(\pi - B) - Sin\,(A-C) \quad [\because A+B+C=\pi]$$

$$Sin\ B = Sin\ B - Sin\ (A - C)$$



$$Sin\ (A - C) = 0$$

$$\Rightarrow A - C = 0$$

$$\measuredangle A = \measuredangle C$$

Hence, the triangle is isosceles.

b) Evaluate : $\displaystyle \lim_{x \to a} \frac{\sqrt{2x} - \sqrt{x+a}}{x - a}$

➢**Solution:**

$$\lim_{x \to a} \frac{\sqrt{2x} - \sqrt{x+a}}{x - a}$$

$$\lim_{x \to a} \frac{\sqrt{2x} - \sqrt{x+a}}{x - a} \times \frac{\sqrt{2x} + \sqrt{x+a}}{\sqrt{2x} + \sqrt{x+a}}$$

$$= \lim_{x \to a} \frac{2x - (x+a)}{x - a} \times \frac{1}{\sqrt{2x} + \sqrt{x+a}}$$

$$= \lim_{x \to a} \frac{x - a}{x - a} \times \frac{1}{\sqrt{2x} + \sqrt{x+a}}$$

$$= \lim_{x \to a} \frac{1}{\sqrt{2x} + \sqrt{x+a}}$$

$$= \frac{1}{\sqrt{2a} + \sqrt{a+a}}$$

$$= \frac{1}{\sqrt{2a} + \sqrt{2a}}$$

$$\therefore The\ Value\ of\ Limit = \frac{1}{2\sqrt{2a}}$$

4. a) Find $\frac{dy}{dx}$ when $x^2 + y^2 = a^2$.

➢ **Solution:**

$$x^2 + y^2 = a^2$$

Differentiating it w.r.t. x on both sides,

$$\frac{d}{dx}(x^2 + y^2) = \frac{da^2}{dx}$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx} = -\frac{x}{y}$$

b) Integrate : $\int cos^2x\ dx$

➢ **Solution:**

$$Let, I = \int cos^2x\ dx$$

$$= \int \frac{cos2x + 1}{2} dx \quad \left[\because cos^2 x = \frac{cos\ 2x + 1}{2}\right]$$

$$= \frac{1}{2}\int \cos 2x \, dx + \frac{1}{2}\int dx$$

$$= \frac{1}{2} \times \frac{\sin 2x}{2} + \frac{x}{2} + C$$

$$\therefore I = \frac{1}{4}\sin 2x + \frac{x}{2} + C$$

5. a) Evaluate : $\displaystyle\int_0^1 \frac{dx}{x+2}$.

➢ **Solution:**

$$I = \int \frac{dx}{x+2} = \log(x+2) \qquad \because \int \frac{dx}{x} = \log x$$

$$Now, \int_0^1 \frac{dx}{x+2} = \Big|\log(x+2)\Big|_0^1$$

$$= \log 3 - \log 2$$

$$= \log\frac{3}{2}$$

Hence $\mathbf{I = \log\frac{3}{2}}$

b) Find the sum of infinite geometric series:

$$\mathbf{1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots}$$

➢ **Solution:**

Given series is $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots\cdots$

$$a=1, r=\frac{1}{3}$$

Now,

$$\boldsymbol{S_\infty=\frac{a}{1-r}}$$

$$S_\infty=\frac{1}{1-\frac{1}{3}}=\frac{1}{\frac{2}{3}}$$

$$\therefore \boldsymbol{S_\infty=\frac{3}{2}}$$

6. a) In how many ways can the letters of the word "MISSISSIPPI" be Arranged?

➢ **Solution:**

The word **MISSISSIPPI** has one M, four I's, fours S's, two P's and a total of 11 Letters.

The number of all type of arrangements possible with the given Alphabets

$$=\frac{11!}{4!\,4!\,2!}=\frac{11\times10\times9\times8\times7\times6\times5\times4!}{4!\times4\times3\times2\times1\times2}=\mathbf{34650}.$$

b) Prove that : $\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+\cdots=\frac{1}{e}$

➤ **L. H. S** $=\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+\cdots$

$$Let, t_n=\frac{2n}{(2n+1)!}=\frac{2n+1-1}{(2n+1)!}=\frac{1}{2n!}-\frac{1}{(2n+1)!}$$

$$\therefore \text{L. H. S}=\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+\cdots$$

$$=\sum_{n=1}^{\infty} t_n=\sum_{n=1}^{\infty}\frac{1}{(2n)!}-\sum_{n=1}^{\infty}\frac{1}{(2n+1)!}$$

$$=\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\cdots\cdots\infty$$

$$=1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{5!}+\cdots\cdots\infty$$

$$=e^{-1}=\frac{1}{e}$$ **R. H. S = L. H. S Proved.**

7. a) Find the angle between the lines : $2x-y+3=0$ and $x+y-2=0$

➤ **Solution:**

Given: The equations of the lines are

$$2x-y+3=0 \quad \cdots\cdots\cdots(i)$$
$$x+y-2=0 \quad \cdots\cdots\cdots(ii)$$

Let m_1 and m_2 be the slopes of these lines.

$m_1 = 2, \ \ m_2 = -1$

Let θ be the angle between the lines. Then,

By using the formula,

$$\tan\theta = \frac{(m_1 - m_2)}{(1 + m_1 m_2)}$$

$$= \frac{(2 + 1)}{1 + (2)(-1)}$$

$$\tan\theta = \frac{3}{-1}$$

$$So, \theta = tan^{-1}(-3)$$

∴The angle between the lines is $\tan^{-1}(-3)$

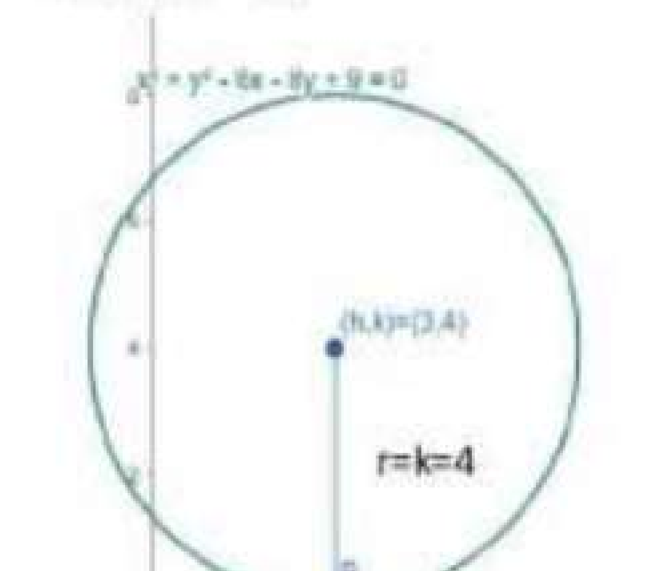
b) Find the equation of circle with centre $(3, 4)$ and touching the $x - axis$.

➢ **Solution:**

Given, Centre $(h, k) = (3, 4)$

Equation of the circle with touching $x - axis$ is

$$(x-h)^2 + (y-k)^2 = k^2$$
$$(x-3)^2 + (y-4)^2 = 4^2$$
$$x^2 - 6x + 9 + y^2 - 8y + 16 = 16$$
$$\boldsymbol{x^2 + y^2 - 6x - 8y + 9 = 0}$$



Hence,

Required Equation of circle is $x^2 + y^2 - 6x - 8y + 9 = 0$.

8) If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{z-y}$, prove that $x^x . y^y . z^z = 1$

➢ Refer to the solution 2078 of Q. No 12 (or) on page 41.

9) Solve: $tanx + tan2x + \sqrt{3}tanx\ tan2x = \sqrt{3}$.

➢ **Solution:**

$$tanx + tan2x + \sqrt{3}tanx\ tan2x = \sqrt{3}$$

$$or,\ tanx + tan2x = \sqrt{3} - \sqrt{3}tanx\ .\ tan2x$$

$$or,\ tanx + tan2x = \sqrt{3}\ (1 - tanx\ .\ tan2x)$$

$$or,\ \frac{\tan x + \tan 2x}{1 - \tan x\ .\ \tan 2x} = \sqrt{3}$$

$$or,\ tan(x + 2x) = \sqrt{3}$$

$$or,\ tan\ (x + 2x) = tan\ {}^{\pi}/_{3}$$

$$or,\ tan\ 3x = tan\ {}^{\pi}/_{3}$$

$$or,\ 3x = n\pi + {}^{\pi}/_{3}$$

$$or,\ x = \frac{n\pi}{3} + \frac{\pi}{9}$$

$$\therefore\ x = \frac{n\pi}{3} + \frac{\pi}{9}$$

10) If $(a + b + c)(b + c - a) = 3bc$, show that $A = 60°$.

➢ **Solution:**

Given, $(a + b + c)(b + c - a) = 3bc$ in ΔABC

To prove, A = 60°.

Now,

$$(a+b+c)(b+c-a) = 3bc$$

$$or,\ \{(b+c)+a\}\{(b+c)-a\} = 3bc$$

$$or,\ (b+c)^2 - a^2 = 3bc \qquad [\because a^2 - b^2 = (a+b)(a-b)]$$

$$or,\ b^2 + c^2 + 2bc - a^2 = 3bc$$

$$or,\ b^2 + c^2 - a^2 = bc$$

$$or,\ \frac{b^2+c^2-a^2}{2bc} = \frac{1}{2} \cdots\cdots (i) \qquad [dividing\ both\ side\ by\ 2]$$

By cosine rule,

$$\text{Cos A} = \frac{b^2+c^2-a^2}{2bc} = \frac{1}{2}$$

Equation (*i*)

$$\text{Cos A} = \frac{1}{2}$$

$$\text{Cos A} = \cos 60°$$

$$\therefore \mathbf{A = 60°}\ \textbf{Proved}$$

11) ***Let f(x) be defined by*** $f(x) = \begin{cases} 2x+1 & for\ x<1 \\ 2x & for\ x=1 \\ 3x & for\ x>1 \end{cases}$

i) ***Does*** $\lim_{x \to 1} f(x)$ ***exist?*** ii) ***Is*** $f(x)$ ***continuous at*** $x = 1$?

➢**Solution:**

Left hand limit at x = 1 $= \lim_{x \to 1^-} f(x)$

$= \lim_{x \to 1^-}(2x + 1)$

$= 2 \times 1 + 1$

$= 3$

Right hand limit at x = 1 $= \lim_{x \to 1^+} f(x)$

$= \lim_{x \to 1^+} 3$

$= 3 \times 1$

$= 3$

Function value at $f(1) = 2 \times 1 = 2$

i) *Since,* $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$ *so,*

Limit exist and value $\lim_{x \to 1} f(x) = 3$

ii) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \neq. f(1)$

Hence,

Given, The function Discontinuous $at\ x = 1$.

12) Find derivatives of *cosx* by definition.

➢**Solution:**

$$f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f(x) = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h\to 0} \left[\frac{\cos x\ \cos h - \sin x\ \sin h - \cos x}{h}\right]$$

$$= \lim_{h\to 0} \left[\frac{-\cos x\ (1 - \cos h) - \sin x\ \sin h}{h}\right]$$

$$= \lim_{h\to 0} \left[\frac{-\cos x\ (1 - \cos h) - \sin x\ \sin h}{h}\right]$$

$$= \lim_{h\to 0} \left[\frac{-\cos x\ (1 - \cos h)}{h} - \frac{\sin x\ \sin h}{h}\right]$$

$$= -\cos x \left(\lim_{h\to 0} \frac{1 - \cos h}{h}\right) - \sin x \lim_{h\to 0} \left(\frac{\sin h}{h}\right)$$

$$= -\cos x\ (0) - \sin x\ (1)$$

$$\therefore -\sin x$$

13) Find $\frac{dy}{dx}$ when $x = \frac{1-t^2}{1+t^2}, y = \frac{2t}{1+t^2}$.

➢ **Solution:**

Given: $x = \frac{1-t^2}{1+t^2}$, *and* $y = \frac{2t}{1+t^2}$

Let t = $\tan\theta$ in both the equation, we get

$$x = \frac{1 - tan^2\theta}{1 + tan^2\theta} = cos2\theta \quad \cdots\cdots\cdots (i)$$

$$y = \frac{2tan\theta}{1 + tan^2\theta} = sin\, 2\theta \quad \cdots\cdots\cdots (ii)$$

On differentiating both the equation (i) and (ii) $w.r.t.$ $'\theta'$ *we get,*

$$\frac{dx}{d\theta} = -2\, sin\, 2\theta \quad and \quad \frac{dy}{d\theta} = 2\, cos\, 2\theta$$

Now,

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2\cos 2\theta}{-2\sin 2\theta} = -\cot 2\theta$$

$$\therefore \frac{dy}{dx} = -\cot\{\cos^{-1} x\} \qquad [from(i), 2\theta = \cos^{-1} x]$$

14) *Compute the integral* : $\int e^{sin2x} 2x\, dx$.

➢ **Solution:**

$$Let\ I = \int e^{sin2x}.cos2x\, dx.$$

$$Let\ \ sin2x = u$$

Differentiating both side w.r.t 'x' $\frac{du}{dx} = -\frac{cos2x}{2}$

$$or,\ 2.du = -cos2x.dx$$

$$I = -\times \int e^{sin2x}.(-cos2x\, dx.)$$

$$I = -\int e^{u}\ 2.du = -2\int e^{u}\ du$$

$$= -2\,.e^{u} + c$$

$$I\ = \ \mathbf{-2.e^{sin2x} + c.}$$

15) If $a^x = b^y = c^z$ and a, b, c, are in GP prove that x, y, z are in H.P.

➢ **Solution:**

Since, a,b,c are in GP,

$$b = \sqrt{ac}\ \ i.e,\ \ \boldsymbol{b^2 = ac} \quad \cdots\cdots\cdots (i)$$

Also, $a^x = b^y = c^z = k(say)$

$$\Rightarrow a = k^{\frac{1}{x}}, b = k^{\frac{1}{y}}, c = k^{\frac{1}{z}}$$

Putting the value of a, b, c in (i) we get,

$$\Rightarrow k^{\frac{2}{y}}\ \ =\ \ k^{\frac{1}{x}}.k^{\frac{1}{z}}$$

$$\Rightarrow k^{\frac{2}{y}}\ \ =\ \ k^{\frac{1}{x}+\frac{1}{z}}$$

$$\Rightarrow \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\Rightarrow y = \frac{2xz}{x+z}$$

$\Rightarrow$ **x, y, z are in HP**

16) $\boldsymbol{Solve\ for\ n : C(n+2, 4) = 6. C(n, 2).}$

➢**Solution:**

$$C(n+2,4) = 6.C(n,2).$$

$$or,\ \frac{(n+2)!}{(n+2-4)!\,4!} = 6\frac{n!}{(n-2)!\,2!}$$

$$or,\ \frac{(n+2)(n+1)}{(n-2)!\,4!} = \frac{6}{(n-2)!\,2!}$$

$$or,\ \frac{(n+2)(n+1)}{4\times3\times2\times1} = \frac{6}{2\times1}$$

$$or,\ x^2 + n + 2n + 2 = 72$$

$$or,\ x^2 + 3n - 70 = 0$$

$$or,\ (n+10)(n-7) = 0$$

$$\therefore \boldsymbol{n = 7\ \ (As\ \ n \neq -10\ \ is - ve\ value\,)}$$

17) Find the term independent of x in the expansion of $\left(\mathbf{x}^2 - \frac{\mathbf{1}}{\mathbf{x}^2}\right)^{\mathbf{10}}$.

➢**Solution:**

Let t_{r+1} *be the independent of* x

$$t_{r+1} = c(10\,,r).(x^2)^{10-r}.\left(\frac{-1}{x^2}\right)^r$$

$$= c(10\,,r).x^{20-2r}.\frac{-1^{r}}{x^{2r}}$$

$$= c(10\,,r).x^{20-2r-2r}.(-1)^{r}$$

x is independent so,

$$x^{20-4r} = x^{0}$$

$$20 - 4r = 0$$

$$4r = 20$$

$$\therefore \boldsymbol{r = 5}$$

So,

$$t_{5+1} = t_{6} \text{ is independent}$$

$$t_{5+1} = c(10,5).(-1)^{5}$$

$$= -C(10,5)$$

$$= -\frac{10!}{5!\,5!} = -252$$

18) Find the equations of the bisectors of the angles between the lines $4x - 3y + 1 = 0$ *and* $12x - 5y + 7 = 0$. Also show that bisectors are at right angle.

➢ **Refer to the solution 2079 New of Q. No 17 on Page 70.**

19) Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a}+\frac{y}{b} = 1$ and the curve $x^2+y^2=c^2$ are at right angles if $\frac{1}{a^2}+\frac{1}{b^2}+\frac{2}{c^2}$.

➢ **Refer to the solution 2079 New of Q. No 18 (or) on Page 73.**

20) Find the equation of tangent to the circle $x^2+y^2 - 2x-4y-4=0$, Perpendicular to the line $3x-4x=1$.

➢ **Solution:**

Given, Equation of circle $x^2+y^2-2x-4y-4=0$

$g=-1, g=-2, f=-1$

$Radius = \sqrt{g^2+f^2-c}$,

$center(-g,-f) = (1,2)$

Equation of lines is $3x-4x=1$.

Now,

Let the equation of tangent perpendicular to

$3x-4y=1$ *be* $4x+3y+K=0$

Now,

Radius = Distance of center (1,2) form line

$$\sqrt{1^2+2^2-(-4)} = \left|\frac{4(1)+3(2)+K}{\sqrt{16+9}}\right|$$

$$or,\ 3 \times 5 = 4 + 6 + K$$

$$or,\ K = 5$$

Hence,

Equation of tangent to the circle and perpendicular to $\mathbf{3x - 4y = 1}$ is $\mathbf{4x + 3y + 5 = 0}$

–The End –

Engineering Mathematics I_(Engg. All) 1^{st} Sem

(2080 Old) Question Paper Solution.

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1. a) State and prove cosine law.

➢ Refer to the solution 2079 Old of Q. No 1 (a) on Page 79.

b) If $\frac{R}{r} = \frac{4}{3}$, *Prove that* $\cos A + \cos B + \cos C = \frac{7}{4}$.

➢ **Solution:**

$$\text{LHS} = \cos A + \cos B + \cos C$$

$$= 1 + 4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} \quad \text{[Using conditional identity]}$$

$$= 1 + \frac{1}{R}.4.R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$= 1 + \frac{1}{R}.r \quad \left[\because r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right]$$

$$= 1 + \frac{r}{R} \quad \left[\because \frac{r}{R} = \frac{3}{4}\ as, \frac{R}{r} = \frac{4}{3.}\right]$$

$$= 1 + \frac{3}{4}$$

$$= \frac{7}{4}, \quad \text{RHS Proved.}$$

2. a) Evaluate : $\lim\limits_{x \to \theta} \dfrac{x\sin\theta - \theta\sin x}{x - \theta}$.

➢ **Refer to the solution 2078 of Q. No 2.a (or) on Page 30.**

b) Test the continuity of : $f(x) = \left.\begin{matrix} 3x+2 & for & x<1 \\ 7 & for & x=1 \\ 6x-1 & for & x>1 \end{matrix}\right\}$ $at\ x = 1.$

If the function is not continuous at $x = 1$. How can you make it Continuous ?

➢ **Solution:**

$$Left\ hand\ limit\ at\ x = 1 = \lim_{x \to 1^-} f(x)$$
$$= \lim_{x \to 1^-} 3x + 2$$
$$= 3(1) + 2$$
$$= 5$$

$$Right\ hand\ limit\ at\ x = 1 = \lim_{x \to 1^+} f(x)$$
$$= \lim_{x \to 1^+} 6x - 1$$
$$= 6(1) - 1$$
$$= 5$$

Functional value at $x = 1$, $f(1) = 7$,

$$Hence,\ \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \neq f(1)$$

The function discontinuous $at\ x = 1$.

It can be made continuous re-stating the function as below :

$$f(x)=\begin{cases}3x+2 & for \quad x<1\\ 5 & for \quad x=1 \\ 6x-1 & for \quad x>1\end{cases} \quad at\; x=1$$

3. a) Find the condition that the general equation of second degree $ax^2+2hxy+by^2+2gx+2fy+c=0$ always the pair of straight line.

➢ **Solution:**

A second degree equation in x and y is,

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

Making it quadratic in x, we get,

$$ax^2+2(hy+g)x+(by^2+2fy+c)=0$$

$$or, x=\frac{-2(hy+g)\pm\sqrt{4(hy+g)^2-4a(by^2+2fy+c)}}{2a}$$

$$or, ax=-(hy+g)\pm\sqrt{(\boldsymbol{hy}+\boldsymbol{g})^2-\boldsymbol{a}(\boldsymbol{by}^2+\mathbf{2}\,\boldsymbol{fy}+\boldsymbol{c})}$$

This will give two linear equations if the **discriminant=0.** So the given equation represents a
line pair if,

$$(hy+g)^2-a(by^2+2fy+c)=0$$

$$or,\; h^2y^2+2ghy+g^2-aby^2-2afy-\mathrm{ca}=0$$

$$or, (h^2-ab)y^2+2(gh-af)y+g^2-\mathrm{ca}=0$$

$$or,\ 4(gh-af)^2 - 4.(h^2 - ab)(g^2 - ca) = 0$$

$$or, (gh-af)^2 - (h^2-ab)(g^2 - ca) = 0$$

$$or, g^2h^2 - 2afgh + a^2f^2 - g^2h^2 + h^2ca + abg^2 - a^2bc = 0$$

$$or,\ a(abc + 2fgh - af^2 - bg - ch^2) = 0$$

As, a$\neq 0 \Longrightarrow abc + 2fgh - af^2 - bg - ch^2 = 0$

$\therefore \boldsymbol{ax^2 + 2hxy + by + 2gx + 2fy + c = 0}$ represents a line pair if $abc + 2fgh - af^2 - bg - ch^2 = 0.$

b) The sum of three number in AP is 36. When the number are increased by 1, 4, 43 respectively, the resulting numbers are in GP. Find the numbers.

➢ **Solution:**

Let the numbers in AP be $a-d, a, a+d$. Then by question

$$a - d + a + a + d = 36$$

$$i.e.\ a = 12$$

Therefore the numbers are $12 - d, 12, 12 + d$

Again by question when 1,4,43 respectively added, the resulting numbers are in GP $i.e.$

$13 - d, 16, 55 + d$ are in GP. It follows that

$$(13 - d)(55 + d) = 16^2$$

$$or,\ 715 + 13d - 55d - d^2 = 256$$

$$or,\ d^2 + 42d - 459 = 0$$

$$or,\ d^2 + 51d - 9d - 459 = 0$$

$or,\ d(d+51)-9(d+51)=0$

$or,\ (d+51)(d-9)=0$

$or,\ \boldsymbol{d=-51,\ 9}$

When d = −51, the number are 63, 12 , −39

When d = 9, the number are 3, 12 , 21

4) Find derivative of first principle method of : (any ONE)

a) Sin x

- **Solution:**

$Let\ y=\sin x$

Let Δx *be* small increment in x and Δy be the corresponding small increment in y. Then,

$$y+\Delta y=\sin(x+\Delta x)$$

$$\Delta y=\sin(x+\Delta x)-y$$

$$\Delta y=\sin(x+\Delta x)-\sin x$$

$$\frac{\Delta y}{\Delta x}=\frac{\sin(x+\Delta x)-\sin x}{\Delta x}$$

By first Principle,

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}$$

$$\frac{dy}{dx}=\lim_{\Delta x\to 0}\frac{\sin(x+\Delta x)-\sin x}{\Delta x}$$

$$=\lim_{\Delta x \to 0} \frac{2\cos\left(\frac{x+\Delta x+x}{2}\right)\sin\left(\frac{x+\Delta x+x}{2}\right)}{\Delta x}$$

$$=\lim_{\Delta x \to 0} \frac{2\cos\left(x+\frac{\Delta x}{2}\right)\sin\left(\frac{\Delta x}{2}\right)}{\Delta x}$$

$$=\lim_{\Delta x \to 0} \left[\cos\left(x+\frac{\Delta x}{2}\right).\frac{\sin\left(\frac{\Delta x}{2}\right)}{\frac{\Delta x}{2}}\right]$$

$$= \cos x.\ 1$$

$$= \mathbf{\cos x}$$

b) $x + \sqrt{x}$

➢Solution:

Let $f(x) = y = x + \sqrt{x} \quad \cdots\cdots\cdots (i)$

Let Δx be small increment in x and Δy be the corresponding small increment in y Then,

$$y + \Delta y = x + \Delta x + \sqrt{x + \Delta x} \quad \cdots\cdots\cdots (ii)$$

Subtracting eqn (i) from (ii) *we get*,

$$y + \Delta y - y = x + \Delta x + \sqrt{x + \Delta x} - (x + \sqrt{x})$$

$$\Delta y = \Delta x + \sqrt{x + \Delta x} - \sqrt{x}$$

$$\Delta y = \Delta x + \sqrt{x + \Delta x} - \sqrt{x} \times \frac{\sqrt{x + \Delta x} + \sqrt{x}}{\sqrt{x + \Delta x} + \sqrt{x}}$$

$$\Delta y = \Delta x + \frac{x + \Delta x - x}{\sqrt{x} + \sqrt{x + \Delta x}}$$

$$\Delta y = \Delta x + \frac{\Delta x}{\sqrt{x} + \sqrt{x + \Delta x}}$$

$$\Delta y = \Delta x \left(1 + \frac{1}{\sqrt{x} + \sqrt{x + \Delta x}}\right)$$

$$\frac{\Delta y}{\Delta x} = 1 + \frac{1}{\sqrt{x} + \sqrt{x + \Delta x}}$$

By first Principle,

$$\boldsymbol{\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}}$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \left(1 + \frac{1}{\sqrt{x} + \sqrt{x + \Delta x}}\right)$$

$$\frac{dy}{dx} = \left(1 + \frac{1}{\sqrt{x} + \sqrt{x + 0}}\right)$$

$$\frac{dy}{dx} = \left(1 + \frac{1}{2\sqrt{x}}\right)$$

$$\boldsymbol{\therefore \frac{dy}{dx} = 1 + \frac{1}{2\sqrt{x}}}$$

5) Find $\dfrac{dy}{dx}$: (any ONE)

i) $x^2 + y^2 = 2x^2 + 3xy$

➢ **Solution:**

$$x^2 + y^2 = 2x^2 + 3xy$$

$$or, -x^2 + y^2 = 3xy$$

Differentiating both sides with respect to x, we have

$$\frac{d}{dx}(-x^2 + y^2) = \frac{d}{dx}(3xy)$$

$$or,\quad -2x + 2y\frac{dy}{dx} = 3y + 3x\frac{dy}{dx}$$

$$or,\quad 2y\frac{dy}{dx} - 3x\frac{dy}{dx} = 2x + 3y$$

$$or,\quad (2y - 3x)\frac{dy}{dx} = 2x + 3y$$

$$\therefore\quad \frac{dy}{dx} = \frac{2x + 3y}{2y - 3x}$$

ii) $x = a^2 \tan t\,,\ \ y = 2a \sec t$

➢ **Solution:**

$$x = a^2 \tan t\,,\ \ y = 2a\, sect$$

$$\text{Now,} \frac{dx}{dt} = \frac{d}{dt}(a^2 \tan t) = a^2\frac{d}{dt}(\tan t) = \boldsymbol{a^2 \sec^2 t}$$

and, $\frac{dy}{dt} = \frac{d}{dt}(2a\,sect) = a^2\frac{d}{dt}(\sec t) = \mathbf{2a\,sec\,t.\ tan\,t}$

We have, By chain Rule,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx} = \frac{2a\,sec\,t.\ tan\,t}{a^2\,sec^2\,t}$$

$$= \frac{2\ tan\,t}{sec\,t}$$

$$= \frac{2\ tan\,t}{cos\,t}$$

$$= \frac{cost\ t}{1}$$

$$= \mathbf{2\,sin\,t}$$

6) Integrate : (any ONE)

a) $\int e^{ax}\,sin\,bx\,dx$

➢**Solution:**

$Let, I = \int e^{ax} sin\,bx\,dx$

$$= e^{ax}\int sin\,bx\,dx - \int\left\{\frac{de^{ax}}{dx}\int sin\,bx\,dx\right\}dx$$

$$= e^{ax}\frac{(-\cos bx)}{b} - \int a\, e^{ax}\frac{(-\cos bx)}{b}\ dx$$

$$= -\frac{e^{ax}}{b}\cos bx + \frac{a}{b}\int e^{ax}\ \cos bx\, dx$$

$$= -\frac{e^{ax}}{b}\cos bx + \frac{a}{b}\left[e^{ax}\int \cos bx\ bx - \int\left\{\frac{de^{ax}}{dx}\int cosbx\ bx\right\}dx\right]$$

$$= -\frac{e^{ax}}{b}\cos bx + \frac{a}{b}\left[e^{ax}\frac{\sin bx}{b} - \int a.e^{ax}.\frac{\sin bx}{b}dx\right]$$

$$= -\frac{e^{ax}}{b}\cos bx + \frac{ae^{ax}}{b^2}\sin bx - \frac{a^2}{b^2}\int \boldsymbol{e^{ax}.\sin bx\, dx}$$

$$= -\frac{e^{ax}}{b}\cos bx + \frac{ae^{ax}}{b^2}\sin bx - \frac{a^2}{b^2}I$$

$$or, \left(1 + \frac{a^2}{b^2}\right)I = -\frac{e^{ax}}{b}\cos bx + \frac{ae^{ax}}{b^2}\sin bx$$

$$or, \left(\frac{b^2 + a^2}{b^2}\right)I = \frac{e^{ax}(-b\ \cos bx + a\ \sin bx)}{b^2}$$

$$or, I = \frac{e^{ax}}{a^2 + b^2}(-b\ \cos bx + a\ \sin bx) + C$$

$$\boldsymbol{\therefore I = \frac{e^{ax}}{a^2 + b^2}(a\ \sin bx - b\ \sin bx) + C,\ \ C\ is\ constant}$$

b) $\displaystyle\int \frac{e^x - 1}{e^x + 1}dx$

➢ **Solution:**

$$Let, I = \int \frac{e^x - 1}{e^x + 1} dx = \int \frac{e^{-x/2}(e^x - 1)}{e^{-x/2}(e^x + 1)} dx$$

$$= \int \frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2}} dx$$

$$Put\ y = e^{x/2} + e^{-x/2}, then\ dy = \frac{1}{2}\left(e^{x/2} - e^{-x/2}\right)dx$$

$$or,\ 2dy = \left(e^{x/2} - e^{-x/2}\right)dx$$

$$I = \int \frac{e^{x/2} - e^{-x/2}}{e^{x/2} + e^{-x/2}} dx = \int \frac{2dy}{y}$$

$$= 2\int \frac{dy}{y}$$

$$= 2\log y + c$$

$$\therefore I = 2\log\left(e^{x/2} + e^{-x/2}\right) + c$$

7) Show that the number of ways in which the letters of the word 'ARRANGE' can be arranged so that no two R's comes together is 900.

- **Solution:** In word, ARRANGE

Number of word (n) = 7
P = Number of "A" = 2
q = Number of "R" = 2

Hence,

Number of ways the word can be arranged is with 2R, 2A like word.

$$\frac{n!}{p!\ q!}$$

Total Number of ways the word can be arranged(M) $= \dfrac{\mathbf{7!}}{\mathbf{2!\ 2!}}$

For No two R's come together, first we find Two R's come together then it is subtracted from total number of ways

Hence, The word A,(RR),A,N,G,E Can be arranged in

= (the word arranged such that two R's taken as single word) × (Two R's arranged in ways)

$$(N) = \frac{6!}{2!} \times \left(\frac{2!}{2!}\right) = \frac{6!}{2!}$$

The number of ways in which the letters of the word 'ARRANGE' can be arranged so that no two R's comes together is

$$= M - N = \frac{7!}{2!\ 2!} - \frac{6!}{2!}$$

$$= 1260 - 360$$

$$= \mathbf{900.}$$

8) Find the middle term in the expansion of $\left(1+x^2\right)^{10}$.

➢ **Solution:**

Given,

$$(1+x^2)^{10}$$

Here, the number of terms in the expansion of

$$(1+x^2)^{10} \; is \; (10+1)$$

i.e., 11 which is odd, so, there is only one middle term.

$$i.e., t_{\left(\frac{10}{2}\right)+1}$$

$$= t_{5+1}$$

$$= c(10,5)\;(1)^{10-5}\,(x^2)^5$$

$$= \frac{(10)!}{(5!)(5!)} \times 1^5 . x^{10}$$

$$= \frac{(10)!}{(5!)^2} \times x^{10}$$

Hence, Middle term $= \frac{(10)!}{(5!)^2} \times x^{10}$

9) Show that : $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \cdots = \frac{1}{e}$

➢ **Refer to the solution 2080 New of Q. No 6 (b) on Page 106.**

OR) $\log_{e^2} = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \cdots$

➢ **Solution:**

$$log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \frac{x^7}{7} - \cdots\cdots$$

when, $x = 1$

$$log_e(2) = 1 - \frac{1^2}{2} + \frac{1^3}{3} - \frac{1^4}{4} + \frac{1^5}{5} - \frac{1^6}{6} + \cdots\cdots$$

$$log_e 2 = \left(1 - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + \cdots\cdots$$

$$log_e 2 = \left(\frac{2-1}{1.2}\right) + \left(\frac{4-3}{3.4}\right) + \left(\frac{6-5}{5.6}\right) + \cdots\cdots$$

$$\boldsymbol{log_e 2 = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \cdots\cdots}$$

10) Find the equation of straight line through the point $(2, 3)$ and perpendicular to $4x + 5y + 3 = 0$

➢ **Solution:-**

The equation of line through $(2, 3)$ is

$$y - 3 = m(x - 2) \quad \cdots\cdots\cdots\cdots (i)$$

The given line is

$$4x + 5y + 3 = 0 \quad \cdots\cdots\cdots\cdots (ii)$$

Slope of line (i) $m_1 = m$

Slope of line (ii)

$$m_2 = -\frac{coefficient\ of\ x}{coefficient\ of\ y} = -\frac{4}{5}$$

If the linear (i) and (ii) are perpendicular,

$$m_1 m_2 = -1$$

$$m \times \frac{-4}{5} = -1$$

$$m = \frac{5}{4}$$

Substituting value of $\mathbf{m = \frac{5}{4}}$ **in equation** (i), **we get**

$$y - 3 = \frac{5}{4}(x - 2)$$

$$4y - 12 = 5x - 10$$

$$5x - 4y + 2 = 0$$

$\therefore$ $\mathbf{5x - 4y + 2 = 0}$ **is the required answer.**

11) Find the equation of tangent and normal to the circle $\mathbf{x^2 + y^2 - 2x + 4y + 3 = 0}$ **at** $(\mathbf{2, 3})$.

➢ **Solution:-**

Comparing with, $x^2 + y^2 + 2gx + 2fy + c = 0$

$g = 1, f = -2, c = 3$

The equation of tangent to the $\boldsymbol{x^2 + y^2 + 2gx + 2fy + c = 0}$ **circle at point** $(\boldsymbol{x_1, y_1})$ **is**

$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + C = 0,\ At\ (2, 3),$

$$x.2 + y.3 + 1(x + 2) + (-2)(y + 3) + 3 = 0$$

$$3x + y + 2 - 6 + 3 = 0$$

$3x + y - 1 = 0$ **is Eqn. of Tangent.**

$$Slope\ of\ Tangent\ \ m_T = -\frac{coefficient\ of\ x}{coefficient\ of\ y} = -\frac{3}{1} = -3.$$

If the Tangent and Normal are perpendicular,

$$m_T.m_N = -1$$

$$3m_N = -1$$

$$m_N = \frac{1}{3}$$

The equation of Normal To Circle at $(2, 3)$ is

$$y - 3 = m_N(x - 2) \quad \cdots\cdots\cdots\cdots (i)$$

The equation of Normal To Circle at $(2, 3)$ is

$$y - 3 = \frac{1}{3}(x - 2)$$

$$3y - 9 = (x - 2)$$

$$x - 3y + 7 = 0$$

$\therefore$ $\boldsymbol{x - 3y + 7 = 0}$ **is Eqn. of Normal to the Circle.**

12) If the roots of the equation $\boldsymbol{lx^2 + nx + x + n = 0}$ be in the ratio p: q, Prove that : $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

➢ **Solution:-**

Let α and β be to root of $lx^2 + nx + n = 0$, *then* $\frac{\alpha}{\beta} = \frac{p}{q}$

$\alpha + \beta = -\frac{n}{l}$ *and* $\alpha\beta = \frac{n}{l}$

Now,

$$\frac{\boldsymbol{\alpha}+\boldsymbol{\beta}}{\sqrt{\boldsymbol{\alpha\beta}}}=\frac{-\frac{n}{l}}{\sqrt{\frac{n}{l}}}$$

$$or, \frac{\alpha}{\sqrt{\alpha\beta}}+\frac{\beta}{\sqrt{\alpha\beta}}=-\sqrt{\frac{n}{l}}$$

$$or, \sqrt{\frac{\alpha}{\beta}}+\sqrt{\frac{\beta}{\alpha}}+\sqrt{\frac{n}{l}}=0$$

$$\therefore \sqrt{\frac{\boldsymbol{p}}{\boldsymbol{q}}}+\sqrt{\frac{\boldsymbol{q}}{\boldsymbol{p}}}+\sqrt{\frac{\boldsymbol{n}}{\boldsymbol{l}}}=\mathbf{0} \qquad \left[\because \frac{\alpha}{\beta}=\frac{p}{q}\right]$$

13) Find the equation of parabola vertex (2, 3) and focus (6, 3).

➢ **Solution:-**

Here, Vertex $(h,k)=(2,3)$ *and focus* $(h+a,k)=(6,3)$

$\therefore \boldsymbol{a}=\mathbf{4}, \boldsymbol{h}=\mathbf{2}$ ***and*** $\boldsymbol{k}=\mathbf{3}$

Required equation of parabola is

$$(\mathrm{y}-\mathrm{k})^2=4\mathrm{a}(\mathrm{x}-\mathrm{h})$$

$$or, (\mathrm{y}-3)^2=4\times 4(\mathrm{x}-2)$$

$$or, \mathrm{y}^2-6\mathrm{y}+9=16(\mathrm{x}-2)$$

$$or,\ \mathrm{y}^2-6\mathrm{y}+9=16\mathrm{x}-32$$

$\therefore y^2 - 16x - 6y + 41 = 0$ is the required answer.

14) If $\dfrac{\log x}{y-z} = \dfrac{\log y}{z-x} = \dfrac{\log z}{z-y}$, prove that $x^x . y^y . z^z = 1$

➢ Refer to the solution 2078 of Q. No 12 (or) on page 41.

–The End –

Engineering Mathematics I_(Engg. All) 1^{st} Sem

(2080/81 New) Question Paper Solution.

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1. a) Define power set of a set. If $s = \{1, 2\}$, find its power set.

➢ The set of all the subsets of the given set S is called ***power set of S.*** It is denoted by 2^n

➢ ***Solution:***

$S = \{1,2\}$

Possible power set $= 2^n = 2^2 = \mathbf{4}$

$\therefore$ **Power set =** $\{\{\emptyset\}, \{1\}, \{2\}, \{1,2\}\}$

b) Prove that: $log\,(1 + 2 + 3) = log1 + log2 + log3.$

➢ ***Solution:***

$log\,(1 + 2 + 3) = log1 + log2 + log3$

$L.H.S = log\,(1 + 2 + 3)$

$= log\,6$

$= log\,(1.2.3)$ $\quad [\because logxy = logx + logy]$ *using log properties*

$= \mathbf{log\,1 + log\,2 + log\,3}$ $\quad$ ***RHS proved.***

2. a) Find the general solution of $Cos4\theta = Cos2\theta$

➢ **Solution:**

$$Cos4\theta = Cos2\theta$$

$$Cos4\theta - Cos2\theta = 0$$

$$-2\,Sin\,\frac{4\theta + 2\theta}{2}\,Sin\,\frac{4\theta - 2\theta}{2} = 0$$

$$-Sin\,3\theta\,Sin\,\theta = 0$$

$$Sin\,3\theta\,,Sin\,\theta = 0$$

Either,

$$\text{Sin}\,3\theta = 0 \qquad or \qquad \text{Sin}\,\theta = 0$$

$$3\theta = n\pi \qquad\qquad \theta = n\pi$$

$$\theta = \frac{n\pi}{3}$$

Hence, $\boldsymbol{\theta = \frac{n\pi}{3}, n\pi}$

b) Prove that : $sin\,(2sin^{-1}x) = 2x\sqrt{1 - x^2}$

➢ **Solution:**

Let $sin^{-1}x = \theta \quad \cdots\cdots\cdots\cdots (i)$

$or,\ x = sin\theta \quad \cdots\cdots\cdots\cdots (ii)$

$cos\theta = \sqrt{1 - sin\theta^2} = \sqrt{1 - x^2} \cdots\cdots\cdots\cdots (iii)$

$\boldsymbol{L.H.S,} = sin\,(2sin^{-1}x) \qquad [\,from\,(i)]$

$= \sin\,(2\theta)$

$$= 2\sin\theta.\cos\theta$$

$$= 2x\sqrt{1-x^2} \qquad [\ from\,(ii)\ and\ (ii)\,]$$

R.H.S Proved

3. a) Prove that: $\tan^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$

➢ **Solution:**

$Let\ \tan^{-1}x = \theta$

$or, x = \tan\theta \qquad \cdots\cdots\cdots\cdots (i)$

Now,

$$\tan 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$or,\ \tan 2\theta = \frac{2x}{1+x^2} \qquad [\because\ form\ Equation\ (i)\,]$$

$$or,\ \tan^{-1}\left(\frac{2x}{1+x^2}\right) = 2\boldsymbol{\theta}$$

$$\therefore \tan^{-1}\left(\frac{2x}{1+x^2}\right) = 2\,\mathbf{tan^{-1}}\,\boldsymbol{x}$$

b) Define indetermined form of limit with example.

➢ If a function $f(x)$ takes the form $\frac{0}{0}, \frac{\infty}{\infty}, 1^{\infty}$ for some value of x, then these forms are called ***indeterminate forms***.

Example: $\lim_{n\to 0}\left(\frac{x^2}{x}\right) = \frac{0^2}{0} = \frac{\mathbf{0}}{\mathbf{0}} \quad , (\mathbf{0^+}, \mathbf{0^-})$

4 a) Evaluate: $\dfrac{Lt}{x \to 2} \dfrac{x^2 - 3x + 2}{x - 2}$.

➢ **Solution:**

$$\frac{Lim}{x \to 2} \frac{x^2 - 3x + 2}{x - 2}$$

When $x = 2$ Then, limit takes the form $\left(\frac{0}{0}\right)$ So,

$$\frac{Lim}{x \to 2} \frac{x^2 - 3x + 2}{x - 2}$$

$$\frac{Lim}{x \to 2} \frac{x^2 - 2x - x + 2}{x - 2}$$

$$\frac{Lim}{x \to 2} \frac{x(x - 2) - 1(x - 2)}{(x - 2)}$$

$$\frac{Lim}{x \to 2} \frac{(x - 2)(x - 1)}{(x - 2)}$$

$$\frac{Lim}{x \to 2} (x - 1)$$

$$= (2 - 1)$$

$$= \mathbf{1}$$

b) Evaluate: $\dfrac{Lt}{p \to 0} \dfrac{\sin p\theta}{\theta}$.

➢ **Solution:**

$$\frac{Lim}{\theta \to 0} \frac{\sin p\theta}{\theta}$$

When $\theta = 0$ Then, limit takes the form $\left(\frac{0}{0}\right)$ So,

$$= \frac{Lim}{\theta \to 0} \frac{\sin p\theta}{p.\theta} \times p$$

$$= \left(\frac{Lim}{\theta \to 0} \frac{\sin p\theta}{p\theta}\right) \times p \qquad \left[\because \frac{Lim}{\theta \to 0} \frac{\sin \theta}{\theta} = 1\right]$$

$$= 1 \times p$$

$$= \boldsymbol{p}$$

5. a) Find $\frac{dy}{dx}$ $of\ y = ax^4 + 6x^3 + c\,x^2$.

➢ **Solution:**

Given,

$$y = ax^4 + 6x^3 + cx^2$$

Differentiating both side w.r.t 'x' we get,

$$\frac{dy}{dx} = \frac{d}{dx}(ax^4 + 6x^3 + cx^2)$$

$$or,\ \frac{dy}{dx} = a\frac{d\,x^4}{dx} + 6\frac{d\,x^3}{dx} + c\frac{d\,x^2}{dx}$$

$$or,\ \frac{dy}{dx} = a\ 4.x^3 + 6.3\,x^2 + c.2.x$$

$$\therefore\ \frac{dy}{dx} = 4ax^3 + 18x^2 + 2cx$$

b) Establish the relation:

i) $A.M > G.M > H.M$

ii) $(G.M)^2 = A.M \times H.M$ $for\ the\ number$ 9 and 36

➢ **Solution:**

Let a and b be two unequal positive number.

$a = 9 \text{ and } b = 36$

Then,

$$\text{A. M.} = \frac{a+b}{2} = \frac{9+36}{2} = \mathbf{22.5}$$

$$\text{G. M.} = \sqrt{ab} = \sqrt{9 \times 36} = 3 \times 6 = \mathbf{18}$$

$$and, \text{H. M.} = \frac{2ab}{a+b} = \frac{2 \times 9 \times 36}{9+36} = \frac{648}{45} = \mathbf{14.4}$$

Hence, from above values,

i) $\boldsymbol{A.M(=22.5) > G.M(=18) > H.M(=14.4)}$ *verified*

ii) $\boldsymbol{(G.M)^2 = A.M \times H.M}$

$18^2 = 22.5 \times 14.4$

$324 = 324$ *verified.*

6. a) How many number between 4000 and 5000 can be formed with the digits 2, 3 ,4 ,5 ,6,7 ?

➢ **Solution:**

Numbers Between 4000 and 5000 can be formed with the digits 2,3,4,5,6,7?

Here, 4 is filled is first place, remaining place can filled (2,3,5,6,7)

4			
Fixed	**(remaining place filled by 2,3,5,6,7)**		

So, 5 digit can filled in 3 place in can be arranged in $5P3$ *ways*
5 digit can filled in 3 place in can be arranged in

$$\frac{5!}{(5-3)!} = \frac{5.4.3.2!}{2!} ways$$
$$= 60 \ ways$$

b) Prove that: $\frac{1.}{1!} + \frac{2.}{2!} + \frac{3.}{3!} + \cdots \infty = e.$

➢ **Solution:**

Let t_n be n^{th} term.

$$t_n = \frac{n}{n!} = \frac{n}{n(n-1)!}$$
$$= \frac{1}{(n-1)!}$$
$$t_n = \frac{1}{(n-1)!}$$

$$Sum\ of\ series = \sum t_n$$
$$= \sum\left\{\frac{1}{(n-1)!}\right\}$$
$$= \sum_{n=1}^{\infty} \frac{1}{(n-1)!}$$
$$= \left\{1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} \ldots\right\} \quad \left\{\because\ e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots\right\}$$
$$= e \quad RHS\ Proved.$$

7. a) Find the equation of straight line passing through $(2, 1)$ and perpendicular to the line $2x + y = 0$

➢ **Solution:-**

The equation of line through $(2 , 1)$ is

$$y - 1 = m(x - 2) \quad \cdots\cdots\cdots\cdots (i)$$

The given line is

$$2x + y = 0 \quad \cdots\cdots\cdots\cdots (ii)$$

Slope of line (i) $m_1 = m$

Slope of line (ii)

$$m_2 = -\frac{coefficient\ of\ x}{coefficient\ of\ y} = -\frac{2}{1} = -2$$

If the linear (i) and (ii) are perpendicular,

$$m_1 m_2 = -1$$

$$m \times -2 = -1$$

$$m = \frac{1}{2}$$

Substituting value of $m = \frac{1}{2}$ in equation (i), we get

$$y - 1 = \frac{1}{2}(x - 2)$$

$$2y - 2 = x - 2$$

$$x - 2y = 0$$

$\therefore$ **$x - 2y = 0$ is the required answer.**

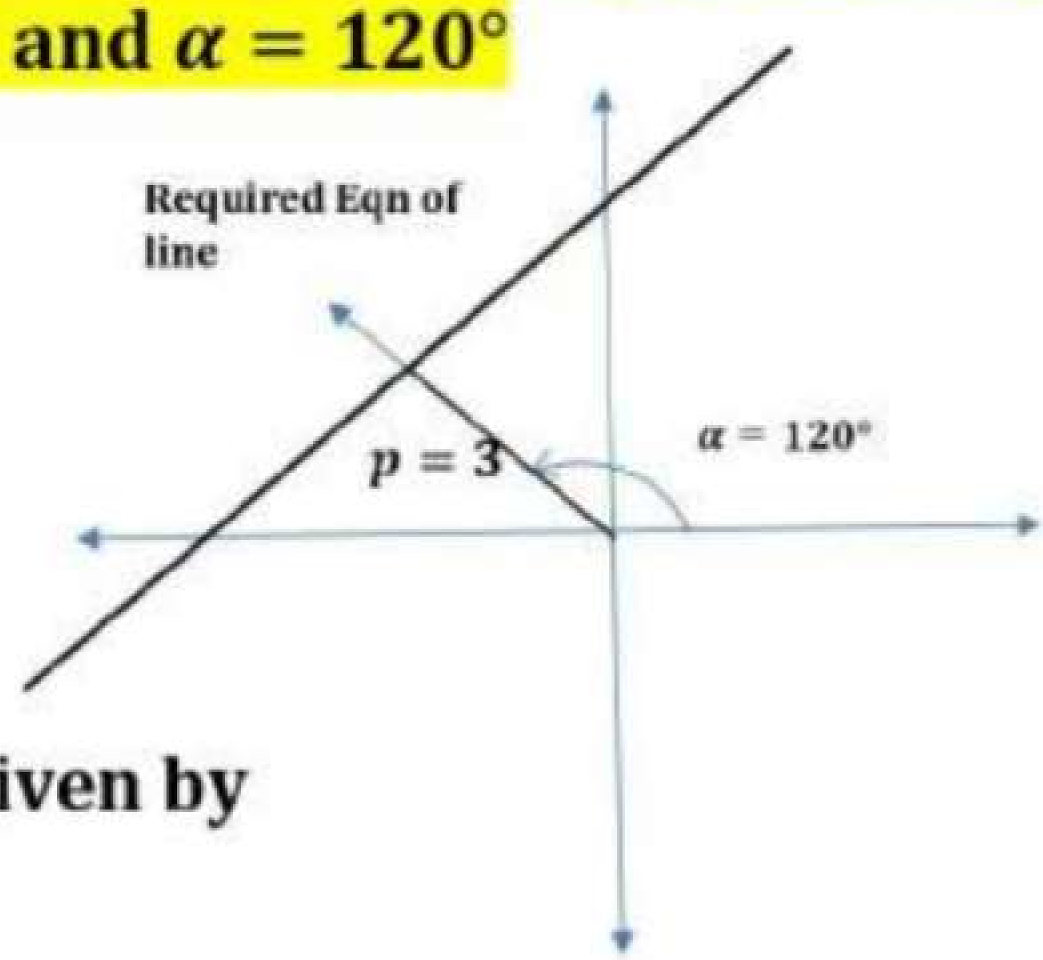
b) Find the equation of the straight line whose perpendicular distance from the origin $p = 3$ and $\alpha = 120°$

➢ **Solution:-**

Given,

$$p = 3$$

$$\alpha = 120°$$



Eqn of line in normal form is given by

$$x\cos\alpha + y\sin\alpha = p$$

$$x\cos 120° + y\sin 120° = 3$$

$$x \times -\frac{1}{2} + y \times \frac{\sqrt{3}}{2} = 3$$

$\therefore -x + \sqrt{3}\,y = 6$ **is the required eqn of line**

8) $f(x) = 3x + 4$ and $g(x) = 2x + 2$ respectively then prove $fog(x) = gof(x)$ and $f^{-1}og(x)$

➢ **Solution:-**

Given, $f(x) = x^3 + 1, \quad g(x) = x^5$

$$fog\,(x) = f\,\{g(x)\} = f\,\{2x + 2\} = 2(3x + 4) + 2 = 6x + 10$$

$$gof\,(x) = g\,\{f(x)\} = g\,\{3x + 4\} = 3(2x + 2) + 4 = 6x + 10$$

Hence, $fog(x) = gof(x)$ ***Proved.***

i) For next part first $f^{-1}(x)$ to find $f^{-1}og(x)$

$f^{-1}(x)$

$Let\ y = f(x) = 3x + 4$

Interchanging x and y we get,

$$or, x = 3y + 4$$

$$or, 3y = x - 4$$

$$or, y = \frac{x-4}{3}$$

$$\therefore f^{-1}(x) = \frac{x-4}{3}\quad x \in R$$

$$Now:, f^{-1}og(x) = f^{-1}(g(x))$$

$$= f^{-1}(2x+2)$$

$$= 2\left(\frac{x-4}{3}\right) + 2$$

$$= \frac{2x-8+6}{3}$$

$$\therefore f^{-1}og(x) = \frac{2x-2}{3}$$

$or)$ *If* $x = \log_{2a}(a), y = \log_{3a}(2a), z = \log_{4a}(3a)$ Prove that, $1 + xyz = 2yz$

➢ **Solution:-**

LHS $= 1 + xyz$

$$= 1 + \log_{2a} a \times \log_{3a} 2a \times \log_{4a} 3a$$

$$= 1 + \boldsymbol{\log_{3a} a} \times \log_{4a} 3a$$

Base changing log properties

$\because \boldsymbol{\log_a b \times \log_c a = \log_c b}$

$\boldsymbol{\log_a b = \frac{\log_c b}{\log_c a}}$

$$= 1 + \boldsymbol{\log_{4a} a}$$

$$= 1 + \frac{\log_e a}{\log_e 4a}$$

$$= \frac{\log_e a + \log_e 4a}{\log_e 4a} = \frac{\log_e 4a^2}{\log_e 4a}$$

$$= \log_{4a} 4a^2$$

$$= \log_{4a} (2a)^2$$

By power rule,

$\because \boldsymbol{\log a^n = n \log a}$

$$= \boldsymbol{2.\log_{4a} 2a}$$

RHS = $\boldsymbol{2y.z}$

$$= 2.\log_{3a} 2a \,.\log_{4a} 3a$$

$$= \boldsymbol{2 \log_{4a} 2a}$$

Base changing log properties

$\because \boldsymbol{\log_a b \times \log_c a = \log_c b}$

Hence , LHS = RHS proved.

9) Show that : $\boldsymbol{x^2 + y^2 + z^2 + 2xyz = 1}$ if

$$\boldsymbol{cos^{-1}x + cos^{-1}y + cos^{-1}z = \pi,}$$

➢ **Solution:-**

Given, $cos^{-1}x + cos^{-1}y + cos^{-1}z = \pi,$

$$\cos^{-1}x + \cos^{-1}y = \pi - \cos^{-1}z$$

$$\text{or, } \cos\left(\cos^{-1}x + \cos^{-1}y\right) = \cos\left(\pi - \cos^{-1}z\right)$$

$$\text{or, } \cos\left(\cos^{-1}x + \cos^{-1}y\right) = \cos\left(\pi - \cos^{-1}z\right)$$

$$\cos\left[\cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right) = -z\right.$$

$\because$ $\cos(a+b) = \cos a \cos b - \sin a . \sin b$, $[\cos \cos^{-1}x = x\ ]$

$$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$$

$$xy + z = \sqrt{1-x^2}\sqrt{1-y^2}$$

Squaring we get, $(xy+z)^2 = (1-x^2)(1-y^2)$

$$x^2y^2 + z^2 + 2xyz = 1 - y^2 - x^2 + x^2y^2$$

$$x^2 + y^2 + z^2 + 2xyz = 1$$

$$\therefore x^2 + y^2 + z^2 + 2xyz = 1$$

10) Prove that: $\dfrac{a+b}{c}\sin\dfrac{C}{2} = \cos\dfrac{A-B}{2}$

➢ **Solution:-**

We Know that, $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = \dfrac{1}{2R}$

LHS, $\dfrac{a+b}{c}\sin\dfrac{C}{2}$

$$= \frac{2R(\sin A + \sin B)}{2R \sin C} . \sin\frac{C}{2}$$

$$= \frac{(\text{Sin A} + \text{Sin B})}{2.\,\text{Sin}\frac{C}{2}.\,\text{Cos}\frac{C}{2}} \sin\frac{C}{2}$$

$$= \frac{\text{Sin}\frac{A+B}{2}.\,\text{Cos}\frac{A-B}{2}}{\text{Cos}\frac{C}{2}}$$

$$= \frac{\text{Sin}\frac{\pi-C}{2}.\,\text{Cos}\frac{A-B}{2}}{\text{Cos}\frac{C}{2}} \qquad [\because In\ \Delta,\ A+B+C=\pi]$$

$$= \frac{\text{Cos}\frac{C}{2}.\,\text{Cos}\frac{A-B}{2}}{\text{Cos}\frac{C}{2}}$$

$$= \text{Cos}\frac{A-B}{2}$$ **R.H.S Proved**

11) Find the sum to infinity

$1 - 3a + 5a^2 - 7a^3 + \cdots\cdots (|a| < 1)$

➢ **Solution:-**

$$s_\infty = 1 - 3a + 5a^2 - 7a^3 + \cdots\cdots\cdots$$

$$or,\ as_\infty = a - 3a^2 + 5a^3 + \cdots\cdots\cdots\cdots$$

Adding both equations we get,

$$(s_\infty + as_\infty) = 1 - 2a + 2a^2 - 2a^3 + \cdots\cdots\cdots$$

$$(1+a).S_\infty = 1 - 2[a - a^2 + a^3 + \cdots\cdots\cdots]$$

$$= 1 - 2\left[\frac{a}{1-(-a)}\right] \qquad \left[\because s_\infty = \frac{a}{1-r}\right]$$

$$= 1 - \frac{2a}{1+a}$$

$$= \frac{1+a-2a}{1+a}$$

$$(1+a).S_\infty = \frac{1-a}{1+a}$$

$$\therefore \boldsymbol{S = \frac{1-a}{(1+a)^2}}$$

12) From 5boys and 4 girls, a committee of 5 is to be formed. In how many ways can this be done as to include at least one girls?

➢ **Solution:-**

For including at least one girl, the following case arise

Boys (5)	Girls (4)	Selection of 5 committee so that include at least 1 girl
$C(5,4)$	$C(4,1)$	$C(5,4) \times C(4,1)$
$C(5,3)$	$C(4,2)$	$C(5,3) \times C(4,2)$
$C(5,2)$	$C(4,3)$	$C(5,2) \times C(4,3)$
$C(5,1)$	$C(4,4)$	$C(5,1) \times C(4,4)$

Total number of ways committee of 5 can be formed

$= C(5,4) \times C(4,1) + C(5,3) \times C(4,2) + C(5,2) \times C(4,3) + C(5,1) \times C(4,4)$

$= 5 \times 4 + 10 \times 6 + 10 \times 4 + 5 \times 1$

$= \boldsymbol{125\ ways}$

Hence,

The number of ways to form a committee of 5 with at least one girl is 125 ways.

13) If the coefficient of x in the expansion of $\left(x^2+\frac{k}{x}\right)^5$ is 270, Find k.

➢ **Solution:-**

To find k =?

Coefficient of x in the expansion of $\left(x^2+\frac{k}{x}\right)^5$ is 270

We have,

$T_{n+1} = C(n, r)\, a^{n-r}\, x^r$ in expansion $(a+x)^n$

$T_{n+1} = C(5, r)\cdot(x^2)^{5-r}\cdot\left(\frac{k}{x}\right)^r$

$= C(5, r) \times (x^{10-2r})\cdot(k)^r\cdot(x^{-r})$

$\mathbf{T_{n+1} = C(5, r) \times (x^{10-3r})\,(k)^r}$

For coefficient of x, $10-3r = \mathbf{1}$

or, $9 = 3r$

or, $r = 3$

$Coefficient\ of\ x = C(5,3) \times (x)\,(k)^3$

$Coefficient\ of\ x = C(5,3)\,(k)^3$

$or, C(5,3)\,(k)^3 = 270$

$or, 10\,(k)^3 = 270$

$or, (k)^3 = 27$

$or, k = \sqrt[3]{27} = 3$

Hence, The value of K = 3

14) Prove that the equation of the straight line through the point $(aCos^3\theta, aSin^3\theta)$ and Perependicular to line $x.sec\theta + y.cosec\theta = a$ is $x.cos\theta - y.sin\theta = acos2\theta$.

➢ **Solution:-**

Given line, $x.sec\theta + y.cosec\theta = a$

Let equation of line that perpendicular to the line ,

$x.sec\theta + y.cosec\theta = a$ is $x.cosec\theta - y.sec\theta = k$ ⋯⋯ (i)

where , k is scalar As it line (i) passes through point $(aCos^3\theta, aSin^3\theta)$ so,

$$aCos^3\theta.cosec\theta - aSin^3\theta\ sec\theta = k$$

$$aCos^3\theta.\frac{1}{\sin\theta} - aSin^3\theta\ \frac{1}{\cos\theta} = k$$

$$\frac{a[Cos^4\theta.-Sin^4\theta\,]}{\sin\theta.\cos\theta} = k$$

$$\frac{a[(Cos^2\theta - Sin^2\theta)(Cos^2\theta + Sin^2\theta)\,]}{\sin\theta.\cos\theta} = k$$

$$\frac{a\ Cos2\theta}{\sin\theta.\cos\theta} = k \qquad [\because Cos^2\theta + Sin^2\theta = 1]$$

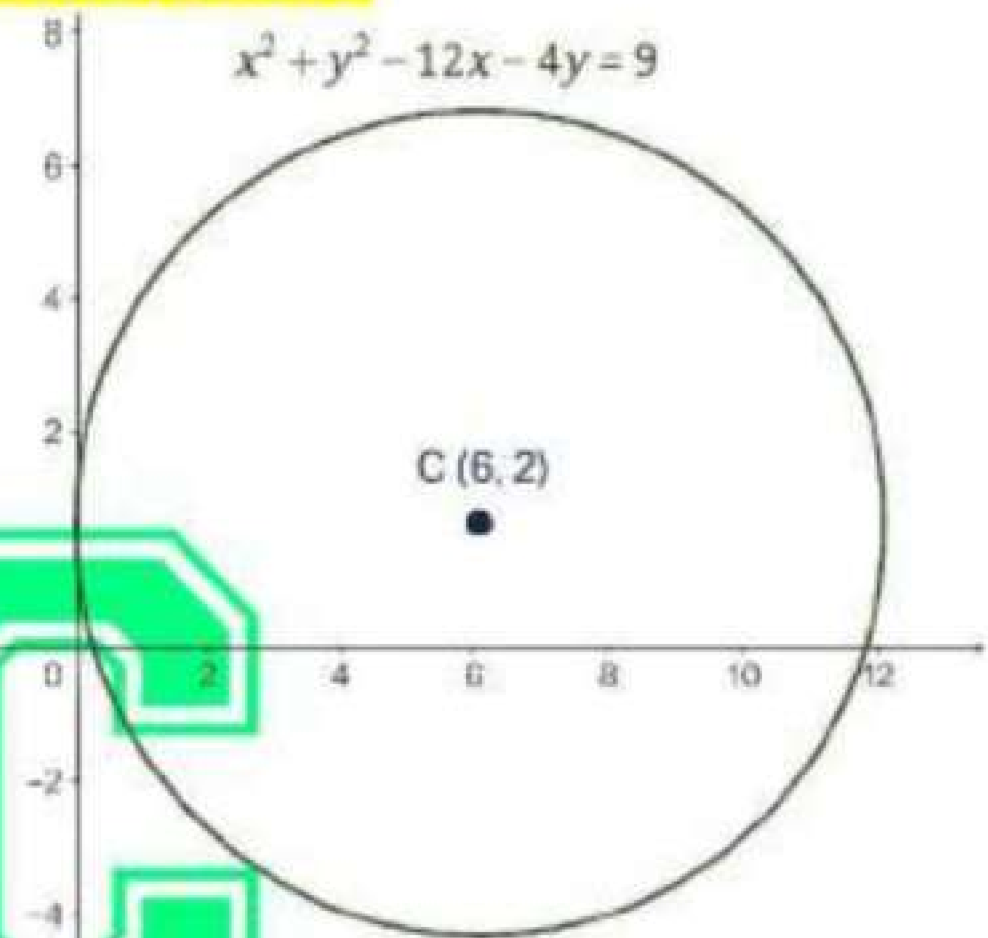
Substituting the value of k in (i),

$$x.cosec\theta - y.sec\theta = \frac{a\ Cos2\theta}{\sin\theta.\cos\theta}$$

$$x.cosec\theta.\sin\theta.\cos\theta - y.sec\theta.\sin\theta.\cos\theta = a\ Cos2\theta$$

$$\therefore\ \boldsymbol{x.\cos\theta - y.\sin\theta = a\ Cos2\theta} \quad \textbf{proved}.$$

15) Find the center and radius of the circle:
$x^2 + y^2 - 12x - 4y = 9$



➢ **Solution:-**

Given, $\boldsymbol{x^2 + y^2 - 12x - 4y = 9}$

$$x^2 + y^2 - 12x - 4y - 9 = 0$$

Comparing with,

$$\boldsymbol{x^2 + y^2 + 2gx + 2fy + c = 0}$$

$$f = -2,\ g = -6,\ c = -9.$$

Center of circle(h, k) $= (-g, -f) = \boldsymbol{(6, 2)}$

$$Radius\ of\ circle(R) = \sqrt{g^2 + f^2 - c}$$

$$= \sqrt{36 + 4 + 9}$$

$$= \sqrt{49}$$

$$= \mathbf{7}$$

16) P and Q are two points on the line $x - y + 1 = 0$ *and* are at distance 5 from the origin. Find the area of the triangle OPQ.

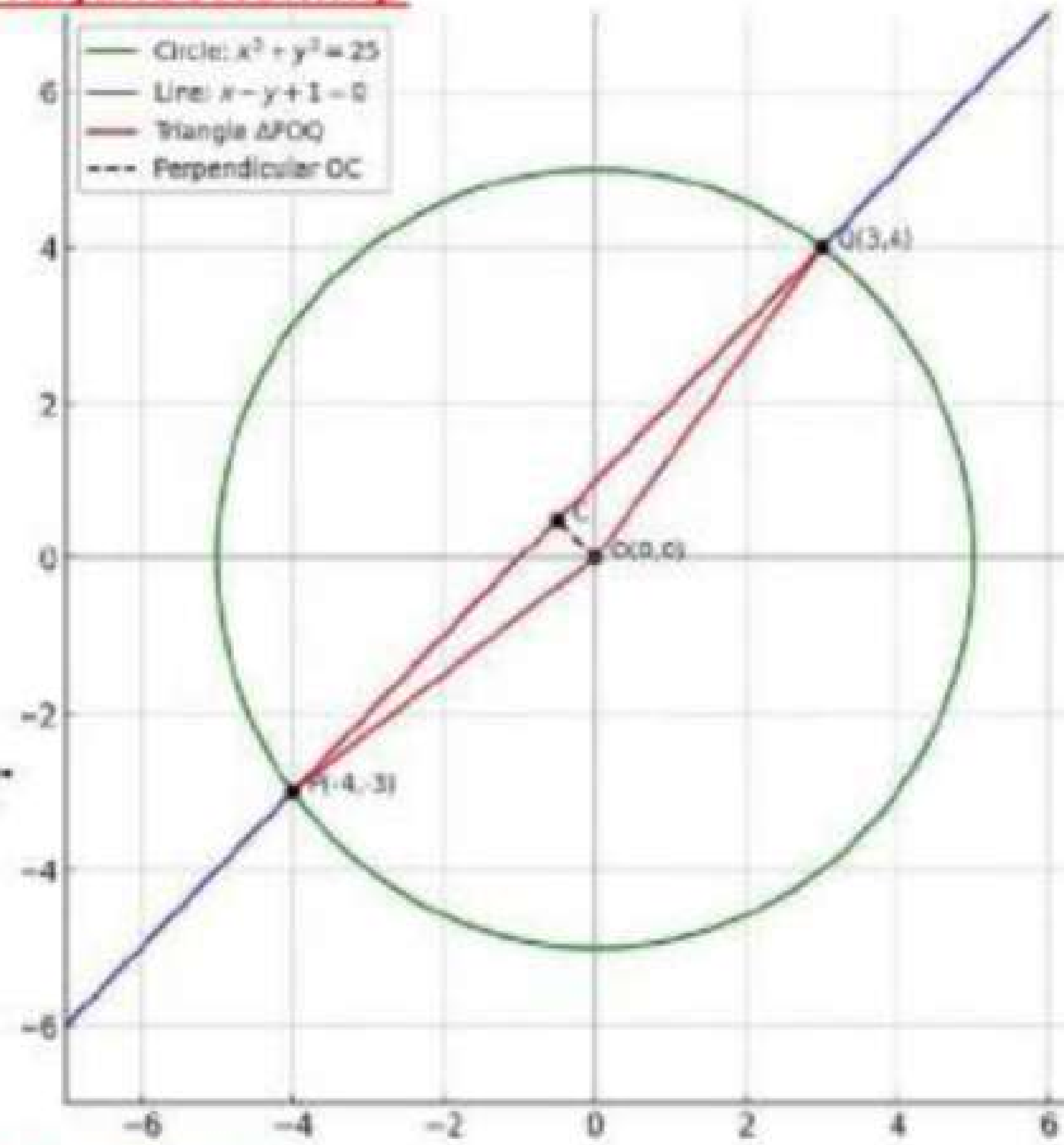
➢ **Solution:-**

P and Q are two points on the line $x - y + 1 = 0$ *and*

P and Q are at distance 5 from the origin . $i.e\ PO = QO = 5$

(a circle of constant radius R=5 is to be drawn)

Drawn perpendicular to line and join OC



Let 'CO' be length of perpendicular drawn from point O(0,0) on the line $x - y + 1 = 0$ is

$$CO = \left|\frac{0 - 0 + 1}{\sqrt{1^2 + -1^2}}\right|$$

$$CO = \left|\frac{1}{\sqrt{2}}\right|$$

$$\therefore CO = \frac{1}{\sqrt{2}}\ unit$$

From triangle POC, $PC = \sqrt{(PO)^2 - (CO)^2}$

$$= \sqrt{(5)^2 - (\frac{1}{\sqrt{2}})^2} = \sqrt{25 - \frac{1}{2}} = \sqrt{\frac{49}{2}} = \frac{7}{\sqrt{2}}$$

$$So, Area\ of\ triangle\ \Delta POC = \frac{1}{2} \times PC \times CO$$

$$= \frac{1}{2} \times \frac{7}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{7}{4}\ sq.unit$$

Hence, $\mathbf{Area\ of\ \Delta POQ = 2 \times Area\ of\ POC = 2 \times \frac{7}{4} = \frac{7}{2} = 3.5\ sq.\ unit}$

17) Show that: $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

➢ **Solution:-**

$$|x| = \begin{bmatrix} x & for & x > 0 \\ -x & for & x \leq 0 \end{bmatrix}$$

Left hand limit at $x = 0 = \lim_{x \to 0^-} \frac{|x|}{x}$

$$= \lim_{x \to 0^-} \frac{-x}{x} \qquad \left(\frac{0}{0}\right) form$$

$$= \lim_{x \to 0^-} -1$$

$$= -1$$

Right hand limit at $x = 0 = \lim_{x \to 0^+} \frac{|x|}{x}$

$$= \lim_{x \to 0^+} \frac{x}{x} \qquad \left(\frac{0}{0}\right) form$$

$$= \lim_{x \to 0^+} 1$$

$$= 1$$

Hence, $\lim_{x \to 0^-} \frac{|x|}{x} \neq \lim_{x \to 0^+} \frac{|x|}{x}$

Since, Left hand limit is not equal to **Right hand limit** so, **Limit does not exist.**

or) Evaluate the limit of $\lim\limits_{x \to \theta} \dfrac{x\cos\theta - \theta\cos x}{x - \theta}$

➢ **Solution:-**

$$\lim_{x \to \theta} \frac{x\cos\theta - \theta\cos x}{x - \theta}$$

When $x \to \theta$ the given function takes the form $\left(\frac{0}{0}\right)$ so,

$$= \lim_{x \to \theta} \frac{x\cos\theta - \theta\cos\theta + \theta\cos\theta - \theta\cos x}{x - \theta}$$

$$= \lim_{x \to \theta} \frac{\cos\theta.(x - \theta) + \theta(\cos\theta - \cos x)}{x - \theta}$$

$$= \lim_{x \to \theta} \frac{\cos\theta.(x - \theta)}{x - \theta} + \lim_{x \to \theta} \frac{\theta(\cos\theta - \cos x)}{x - \theta}$$

$$= \cos\theta + \lim_{x \to \theta} \frac{\theta \times -2\sin\left(\frac{\theta + x}{2}\right)\sin\left(\frac{\theta - x}{2}\right)}{x - \theta}$$

$$= \cos\theta - \theta\left[\lim_{x \to \theta} \frac{\sin\left(\frac{x - \theta}{2}\right)}{\frac{x - \theta}{2}}\right] \times \lim_{x \to \theta} - \sin\left(\frac{\theta + x}{2}\right)$$

$$\left[\because \lim_{x \to 0} \frac{\sin(x)}{x} = 1\right]$$

$$= \cos\theta + \theta.1.\sin\theta$$

$$= \boldsymbol{\cos\theta + \theta.\sin\theta}$$

18) Find from first principle the derivatives of $\cos x$.

➢ **Solution:-**

Let $y = \cos x$

Let δx *be the increment of* x *and* δy, *the corresponding increment of* y.

$$y + \delta y = \cos(x + \delta x)$$

On Subtracting,

$$\delta y = \cos(x + \delta x) - \cos x$$

$$= -2\sin\left(\frac{x + \delta x + x}{2}\right).\sin\left(\frac{x + \delta x - x}{2}\right)$$

$$= -2\sin\left(x + \frac{\delta x}{2}\right).\sin\left(\frac{\delta x}{2}\right)$$

$$\frac{\delta y}{\delta x} = \frac{-2\sin\left(x + \frac{\delta x}{2}\right).\sin\left(\frac{\delta x}{2}\right)}{\delta x}$$

Taking limit $\delta x \to 0$, by first principle

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$$

$$= \lim_{\delta x \to 0} \frac{-2.\sin\left(\frac{\delta x}{2}\right)}{\delta x} \times \lim_{\delta x \to 0} \sin\left(x + \frac{\delta x}{2}\right)$$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{-.\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}} \times \lim_{\delta x \to 0} \sin\left(x + \frac{\delta x}{2}\right)$$

$$\frac{dy}{dx} = -1 \times \sin\left(x + \frac{0}{2}\right) \qquad \left[\because \lim_{x \to 0} \frac{\sin(x)}{x} = 1\right]$$

$$\therefore \frac{dy}{dx} = -\sin(x)$$

or) $x^2 - 2$.

➢ **Solution:-**

Let $y = x^2 - 2 \quad \cdots\cdots\cdots (i)$

Let δx *be the increment of* x *and* δy*, the corresponding increment of* y.

$$y + \delta y = (x + \delta x)^2 - 2 \quad \cdots\cdots\cdots (ii)$$

On Subtracting (i) and (ii)

$$\delta y = (x + \delta x)^2 - 2 - x^2 + 2$$

$$= x^2 + 2x\,\delta x + \delta x^2 - x^2$$

$$= 2x\,\delta x + \delta x^2$$

$$\frac{\delta y}{\delta x} = \frac{2x\delta x + \delta x^2}{\delta x}$$

Taking limit $\delta x \to 0$, by first principle method

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{2x\delta x + \delta x^2}{\delta x}$$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} 2x + \delta x$$

$$\frac{dy}{dx} = 2x + 0$$

$$\therefore \frac{dy}{dx} = 2x$$

19) *Find* $\frac{dy}{dx}$ (*Any one*):

a) $y = tan(5x^2 + 6)$

➢ **Solution:-**

$$y = tan(5x^2 + 6) \cdots\cdots\cdots\cdots (i)$$

Differenting (i) with to x, we get

$$\frac{dy}{dx} = \frac{d}{dx}(tan(5x^2 + 6)$$

$$\frac{dy}{dx} = \frac{d \tan (5x^2 + 6)}{d(5x^2 + 6)} \times \frac{d(5x^2 + 6)}{dx}$$

$$\frac{dy}{dx} = \sec^2 (5x^2 + 6) \times 10x$$

$$\therefore \frac{dy}{dx} = 10x \,. \sec^2 (5x^2 + 6)$$

b) $xy = log(x^2 + y^2)$

➢ **Solution:-**

$$xy = log(x^2 + y^2) \cdots\cdots\cdots\cdots (i)$$

Differenting (i) with to x, we get

$$\frac{d(xy)}{dx} = \frac{d\log(x^2+y^2)}{dx}$$

$$x\frac{d(y)}{dx} + y\frac{d(x)}{dx} = \frac{d\log(x^2+y^2)}{d(x^2+y^2)} \times \frac{d(x^2+y^2)}{dx}$$

$$x\frac{dy}{dx} + y = \frac{1}{(x^2+y^2)} \times \left[\frac{dx^2}{dx} + \frac{dy^2}{dx}\right]$$

$$x\frac{dy}{dx} + y = \frac{1}{(x^2+y^2)} \times \left[2x + \frac{dy^2}{dy} \times \frac{dy}{dx}\right]$$

$$x\frac{dy}{dx} + y = \frac{1}{(x^2+y^2)} \times \left[2x + 2y \times \frac{dy}{dx}\right]$$

$$x\frac{dy}{dx} + y = \frac{2x}{(x^2+y^2)} + \frac{2y}{(x^2+y^2)} \times \frac{dy}{dx}$$

$$\frac{dy}{dx}\left[x - \frac{2y}{(x^2+y^2)}\right] = \frac{2x}{(x^2+y^2)} - y$$

$$\frac{dy}{dx}\left[\frac{x^3 + x\,y^2 - 2y}{(x^2+y^2)}\right] = \frac{2x - x^2y - y^3}{(x^2+y^2)}$$

$$\frac{dy}{dx}(x^3 + x\,y^2 - 2y) = 2x - x^2y - y^3$$

$$\therefore \boldsymbol{\frac{dy}{dx} = \frac{2x - x^2y - y^3}{(x^3 + x\,y^2 - 2y)}}$$

20) Integrate : (any <u>ONE</u>)

a) $\int (2x+3)\sqrt{3x+1}\,dx$

➢ **Solution:-**

$$\textbf{Let } I = \int (2x+3)\sqrt{3x+1}\,dx$$

$$= \int 2\left(x+\frac{3}{2}\right)\sqrt{3x+1}\,dx$$

$$= \int \frac{2}{3}\left(3x+\frac{9}{2}\right)\sqrt{3x+1}\,dx$$

$$= \int \frac{2}{3}\left(3x+1+\frac{9}{2}-1\right)\sqrt{3x+1}\,dx$$

$$= \frac{2}{3}\int (3x+1)^{\frac{3}{2}}dx + \left(\frac{9}{2}-1\right)\sqrt{3x+1}\,dx$$

$$= \frac{2}{3}\left[\int (3x+1)^{\frac{3}{2}}dx + \frac{7}{2}\times\sqrt{3x+1}\,dx\right]$$

$$= \frac{2}{3}\left[\frac{(3x+1)^{\frac{3}{2}+1}}{3.\left(\frac{3}{2}+1\right)} + \frac{7}{2}\times\frac{(3x+1)^{\frac{1}{2}+1}}{3.\left(\frac{1}{2}+1\right)}\right]+C$$

$$= \frac{2}{3}\left[\frac{(3x+1)^{\frac{5}{2}}}{3.\left(\frac{5}{2}\right)} + \frac{7}{2}\times\frac{(3x+1)^{\frac{3}{2}}}{3.\left(\frac{3}{2}\right)}\right]+C$$

$$= \frac{2}{3}\left[\frac{2}{5}\,\frac{(3x+1)^{\frac{5}{2}}}{3.} + \frac{7}{2}\times\frac{2}{3}\frac{(3x+1)^{\frac{3}{2}}}{3.}\right]+C$$

$$\therefore\ \frac{4}{45}(3x+1)^{\frac{5}{2}} + \frac{28}{18}(3x+1)^{\frac{3}{2}}+C$$

b) $\int \sin^3 x \cos x \, dx$

➢ **Solution:-**

$$I = \int \sin^3 x \cos x \, dx$$

$$Let\ y = \sin x\ , dy = \cos x\ dx$$

$$I = \int y^3 . dy$$

$$I = \frac{y^4}{4} + c$$

$$\therefore I = \frac{\sin^4 x}{4} + c$$

c) $\int x \log x \, dx$

➢ **Solution:-**

$$\int \underset{\text{ii}}{x} \quad \underset{\text{i}}{\log x} \, dx$$

Using integration by part,

$$= \log x \int x dx - \int \left(\frac{d \log x}{dx} \int x dx \right) dx$$

$$= \log x . \frac{x^2}{2} + \int \frac{1}{x} \frac{x^2}{2} dx$$

$$= logx\,.\frac{x^2}{2} + \int \frac{x}{2}dx$$

$$= logx\,.\frac{x^2}{2} + \frac{x^2}{4} + c$$

$$\therefore \int x\, logx dx = logx\,.\frac{x^2}{2} + \frac{x^2}{4} + c$$

–The End –

Engineering Mathematics I_(Engg. All) 1^{st} Sem

(2080/81 Old) Question Paper Solution.

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1. a) Evaluate: $\lim\limits_{x \to 0} \dfrac{\tan 2x - \sin 2x}{x^3}$

➢ **Solution:-**

$$\lim_{x \to 0} \frac{\tan 2x - \sin 2x}{x^3}$$

When $x \to 0$ *the given function takes the form* $\left(\frac{0}{0}\right)$ *so,*

$$\lim_{x \to 0} \frac{\frac{\sin 2x}{\cos 2x} - \sin 2x}{x^3}$$

$$\lim_{x \to 0} \frac{\sin 2x - \sin 2x.\cos 2x}{x^3.\cos 2x}$$

$$\lim_{x \to 0} \frac{\sin 2x\,(1 - \cos 2x)}{x^3.\cos 2x}$$

$$\lim_{x \to 0} \frac{\sin 2x\,(2\ \sin^2 x)}{x^3.\cos 2x}$$

$$\lim_{x \to 0} \frac{\tan 2x\,.\,2\ \sin^2 x}{x^3}$$

$$2.2 \lim_{x \to 0} \frac{\tan 2x}{2x} \cdot \lim_{x \to 0} \left(\frac{\sin x}{x}\right)^2$$

$$= 4.1.1$$

$$= \mathbf{4}$$

b) Test the continuity or discontinuity at $x = 2$ if a function is defined by $f(x) = \begin{cases} 2x + 5 & for \quad x \leq 2 \\ 3x + 1 & for \quad x > 2 \end{cases}$

➢ **Solution:**

Left hand limit at $x = 2 = \lim_{x \to 2^-} f(x)$

$$= \lim_{x \to 2^-} (2x + 5)$$

$$= 2(2) + 5$$

$$= \mathbf{9}$$

Right hand limit at $x = 2 = \lim_{x \to 2^+} f(x)$

$$= \lim_{x \to 2^+} (3x + 1)$$

$$= 3(2) + 1$$

$$= \mathbf{7}$$

Function value at $f(1) = 2$

$$f(1) = 2(2) + 5 = 9$$

Since,

$$\lim_{x \to 2^-} f(x) = f(1) \neq \lim_{x \to 2^+} f(x)$$

Hence,

The function Discontinues *at* $x = 2$.

2. a) If the quadratic equation $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root, then prove that $a + b + c = 0$ or $a = b = c$.

➢ **Solution:**

Let α be a common root of the given equation. So,, It must satisfy:

$$a\alpha^2 + b\alpha + c = 0$$

$$b\alpha^2 + c\alpha + a = 0$$

Solving for $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha^2}$ by cross multiplication Method we get,

$$\frac{\alpha^2}{ab - c^2} = \frac{\alpha}{bc - a^2} = \frac{1}{ac - b^2}$$

from first two ratio,

$$\frac{\alpha^2}{ab - c^2} = \frac{\alpha}{bc - a^2} \Rightarrow \alpha = \frac{\boldsymbol{ab - c^2}}{\boldsymbol{bc - a^2}} \cdots\cdots (\boldsymbol{i})$$

from last two Ratio:

$$\frac{\alpha}{bc - a^2} = \frac{1}{ac - b^2} \Rightarrow \boldsymbol{\alpha} = \frac{\boldsymbol{bc - a^2}}{\boldsymbol{ac - b^2}} \cdots\cdots (\boldsymbol{ii})$$

From (i)and (ii),

$$\frac{\boldsymbol{ab - c^2}}{\boldsymbol{bc - a^2}} = \frac{\boldsymbol{bc - a^2}}{\boldsymbol{ac - b^2}}$$

or, $\quad (ab - c^2)(ac - b^2) = (bc - \alpha^2)^2$

or, $\quad a^2bc - ab^3 - ac^3 + b^2c^2 = b^2c^2 - 2a^2bc + a^4$

or, $\quad a^4 + ab^3 + ac^3 - 3a^2bc = 0$

or, $\quad a(a^3 + b^3 + c^3 - 3abc) = 0$

As $a \neq 0$

or, $(a^3 + b^3 + c^3 - 3abc) = 0$

$[\because a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)]$

or, $\quad (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

Either, $\boldsymbol{a + b + c = 0}$ *Proved.*

or, $\quad a^2 + b^2 + c^2 - ab - bc - ca = 0$

or, $\quad a^2 + b^2 + c^2 - ab - bc - ca = 0$

or, $\quad 2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca = 0$

or, $(a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (a^2 - 2ca + c^2) = 0$

or, $\quad (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

For LHS=RHS , a=b=c **As Sum of Perfect square is not Zero**

or, $\boldsymbol{a = b = c}$ **Proved.**

Hence, for one common root of the equations either

$\boldsymbol{a + b + c = 0}$ **or** $\boldsymbol{a = b = c}$

b) If $a^x = b^y = c^z$ and a, b, c are GP then prove that x, y ,z are in HP.

➢ **Refer to the solution 2079 New of Q. No 15 on Page 113.**

3. a) In any triangle ABC, If $a^4 + b^4 + c^4 - 2c^2(a^2 + b^2) = 0$; prove that $\angle c = 45°$ or $135°$.

➢ **Refer to the solution 2076 of Q. No 1 (a) on Page 3.**

b) If $cos^{-1}x + cos^{-1}y + cos^{-1}z = \pi$, prove that $x^2 + y^2 + z^2 + 2xyz = 1$.

➢ **Solution:**

Given,

Let $cos^{-1}x = A,$

$cos^{-1}y = B,$

$cos^{-1}z = C,$

Then, $\cos A = x, \cos B = y, \cos C = z$

Now,

$$cos^{-1}x + cos^{-1}y + cos^{-1}z = \pi$$

$$A + B + C = \pi$$

$$or,\ A + B = \pi - C$$

Taking **Cos** on both side,

$$\cos(A + B) = \cos(\pi - C)$$

$$or,\ \cos A\ \cos B - \sin A\ \sin B = -\cos C$$

$$\because\ \cos(A + B) = \cos A\ \cos B - \sin A\ \sin B$$

$$or,\ x\,y - \sin A\ \sin B = -z$$

$$\because\ \sin x = \sqrt{1 - (\cos x)^2}$$

$$or,\ x\,y - \sqrt{1 - (\cos A)^2}.\sqrt{1 - (\cos B)^2} = -z$$

$$or,\ xy - \sqrt{1 - x^2}.\sqrt{1 - y^2} = -z$$

$$or,\ xy + z = \sqrt{1 - x^2}\sqrt{1 - y^2}$$

Squaring both sides, we get

$$(xy+z)^2 = (1-x^2)(1-y^2)$$

$$or,\ \ x^2y^2 + 2xyz + z^2 = 1 - x^2 - y^2 + x^2y^2$$

$$\therefore\ \ \boldsymbol{x^2 + y^2 + z^2 + 2xyz = 1}$$

OR) Find the solution of $\tan\theta + \tan 2\theta = \tan 3\theta$.

➢ **Solution:**

Given ,

$$\tan\theta + \tan 2\theta = \tan 3\theta$$

$$\tan\theta + \tan 2\theta - \tan 3\theta = 0$$

$$or,\ \ \tan\theta + \tan 2\theta - \tan(\theta + 2\theta) = 0$$

$$or,\ \ \tan\theta + \tan 2\theta - \left(\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\,.\tan 2\theta}\right) = 0$$

$$or,\ \ (\tan\theta + \tan 2\theta) - \left(\frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\,.\tan 2\theta}\right) = 0$$

$$or, \frac{(\tan\theta + \tan 2\theta)(1 - \tan\theta\,.\tan 2\theta) - (\tan\theta + \tan 2\theta)}{1 - \tan\theta\,.\tan 2\theta} = 0$$

$$or, (\tan\theta + \tan 2\theta)(1 - \tan\theta\,.\tan 2\theta) - (\tan\theta + \tan 2\theta) = 0$$

$$or, (\tan\theta + \tan 2\theta) - (\tan\theta + \tan 2\theta).(\tan\theta\,.\tan 2\theta)$$
$$- (\tan\theta + \tan 2\theta) = 0$$

$$or, -(\tan\theta + \tan 2\theta).(\tan\theta\,.\tan 2\theta) = 0$$

$$or, (\tan\theta + \tan 2\theta).(\tan\theta\,.\tan 2\theta) = 0$$

$or, (\tan\theta + \tan 2\theta)(\tan\theta . \tan 2\theta) = 0$

Either,

$\tan\theta = 0$	$or,$	$\tan 2\theta = 0$
$\tan\theta = tan\ 0^\circ$		$\tan 2\theta = tan\ 0^\circ$
$So,\ \theta = 0$		$So,\ 2\theta = 0$
$\theta = n\pi + 0$		$2\theta = n\pi + 0$
$\therefore\ \theta = n\pi$		$\therefore\ \theta = \frac{n\pi}{2}$

$or,\quad \tan\theta + \tan 2\theta = 0$

$\tan 2\theta = -tan\ \theta$

$\tan 2\theta = tan\ (-\theta)$

$2\theta = (-\theta)$

$2\theta = n\pi + (-\theta)$

$2\theta + \theta = n\pi$

$3\theta = n\pi$

$\therefore\ \theta = \frac{n\pi}{3}$

Hence,

The Possible Solution of $\theta = \left(n\pi,\ \frac{n\pi}{2},\ \frac{n\pi}{3}\right)$

4) Find the derivative by using definition :

a) $y = e^{ax+b}$

➢ **Solution:-**

$$y = e^{ax+b}$$

Let δx *be the increment of* x *and* δy, *the corresponding increment of* y. So,

$$y + \delta y = e^{a(x+\delta x)+b}$$

$$\delta y = e^{a(x+\delta x)+b} - y$$

$$\delta y = e^{ax+a\delta+b} - e^{ax+b}$$

$$\delta y = e^{ax+b}.e^{a\delta x} - e^{ax+b}$$

$$\delta y = e^{ax+b}.(e^{a\delta x} - 1)$$

Dividing both sides by δx we get,

$$\frac{\delta y}{\delta x} = \frac{e^{ax+b}.(e^{a\delta x} - 1)}{\delta x}$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} \frac{e^{ax+b}.(e^{a\delta x} - 1)}{\delta x}$$

$$\frac{dy}{dx} = e^{ax+b} \cdot \lim_{\delta x \to 0} \frac{e^{a\delta x} - 1}{a\delta x}.a$$

$$\frac{dy}{dx} = e^{ax+b}.1.a$$

$$\therefore \frac{dy}{dx} = a.e^{ax+b}$$

b) $cos^2 x$.

➢ **Solution:-**

Let $y = cos^2 x$

Let δx *be the increment of* x *and* δy, *the corresponding increment of* y.

$$y + \delta y = \cos^2 (x + \delta x)$$

On Subtracting,

$$\delta y = \cos^2(x + \delta x) - \cos^2 x$$

$$\delta y = [\cos(x + \delta x) - \cos x][\cos(x + \delta x) + \cos x]$$

$$= [-2\, sin\left(\frac{(x + \delta x) + x}{2}\right).sin\left(\frac{(x + \delta x) - x}{2}\right)].[2\, cos\left(\frac{(x + \delta x) + x}{2}\right).cos\left(\frac{(x + \delta x) - x}{2}\right)]$$

$$= [-2\, sin\left(x + \frac{\delta x}{2}\right).sin\left(\frac{\delta x}{2}\right)].[2\, cos\left(x + \frac{\delta x}{2}\right).cos\left(\frac{\delta x}{2}\right)]$$

$$\delta y = \left[-2\, sin\left(x + \frac{\delta x}{2}\right).sin\left(\frac{\delta x}{2}\right)\right]\left[2cos\left(x + \frac{\delta x}{2}\right).cos\left(\frac{\delta x}{2}\right)\right]$$

$$\delta y = -4\, sin\left(x + \frac{\delta x}{2}\right).cos\left(x + \frac{\delta x}{2}\right) sin\left(\frac{\delta x}{2}\right).cos\left(\frac{\delta x}{2}\right)$$

Taking limit $\delta x \to 0$, by first principle

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$$

$$= \lim_{\delta x \to 0} - 4 \sin\left(x + \frac{\delta x}{2}\right).\cos\left(x + \frac{\delta x}{2}\right) \sin\left(\frac{\delta x}{2}\right).\cos\left(\frac{\delta x}{2}\right)$$

$$= 4[\lim_{\delta x \to 0} \frac{-.\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}] \times \frac{1}{2} \times [\lim_{\delta x \to 0} \sin\left(x + \frac{\delta x}{2}\right).\cos\left(\frac{\delta x}{2}\right).\cos\left(x + \frac{\delta x}{2}\right)]$$

$$\frac{dy}{dx} = 4.(-1).\frac{1}{2} \times \sin\left(x + \frac{0}{2}\right).\cos\left(\frac{0}{2}\right).\cos\left(x + \frac{0}{2}\right)$$

$$\therefore \frac{dy}{dx} = -2\sin(x).\cos(x) \qquad \left[\because \lim_{x \to 0} \frac{\sin(x)}{x} = 1\right]$$

5) *Find* $\frac{dy}{dx}$ *(Any one)*:

i) $x^3 + y^3 = 3xy$

➢ **Solution:**

Given, $x^3 + y^3 = 3xy$

Differentiating both sides with respect to x, we have

$$\frac{d}{dx}(x^3 + y^3) = \frac{d}{dx}(3xy)$$

$$or, \quad \frac{dx^3}{dx} + \frac{dy^3}{dy}.\frac{dy}{dx} = 3x.\frac{dy}{dx} + 3y\frac{dx}{dx}$$

$$or, \quad 3x^2 + 3y^2\frac{dy}{dx} = 3x.\frac{dy}{dx} + 3y$$

$$or, \quad (3y^2 - 3x)\frac{dy}{dx} = 3y - 3x^2$$

$$or, \quad \frac{dy}{dx} = \frac{3y - 3x^2}{3y^2 - 3x} = \frac{y - x^2}{y^2 - x}$$

$$\therefore \frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$$

b) $x = a^2 \tan t, \;\; y = 2a \sec t$

➢ **Refer to the solution 2080 Old of Q. No 5 (ii) on Page 125.**

6) Integrate : (<u>Any One</u>)

a) $\int \frac{1}{x^2\sqrt{x^2 - a^2}} dx$

➢ **Solution:-**

Let $x = a \sec\theta$. Then,

$$dx = a \sec\theta \tan\theta \; d\theta$$

and $\sqrt{x^2 - a^2} = \sqrt{a^2 \sec^2\theta - a^2} = a \tan\theta$

Now,

$$= \int \frac{1}{x^2\sqrt{x^2 - a^2}} dx$$

$$= \int \frac{a \sec\theta \tan\theta \; d\theta}{a^2 \sec^2\theta \,.\, a \tan\theta} \, d\theta$$

$$= \frac{1}{a^2} \int \frac{1}{\sec\theta} \, d\theta$$

$$= \frac{1}{a^2}\int \cos\theta \; d\theta$$

$$= \frac{\sin\theta}{a^2} + c$$

$$= \frac{1}{a^2} \cdot \frac{\sqrt{x^2 - a^2}}{x} + c$$

$[\because x = a\sec\theta \Rightarrow \sec\theta = \frac{x}{a} = \frac{h}{b}$

$P = \sqrt{h^2 - b^2} = \sqrt{x^2 - a^2}$

$\because \sin\theta = \frac{p}{h} = \frac{\sqrt{x^2 - a^2}}{x}$]

b) $\displaystyle\int_1^2 \frac{\sin(\log x)}{x}\, dx$

➢ **Solution:-**

$$Let\ I = \int_1^2 \frac{\sin(\log x)}{x}\, dx,$$

$$Let\ y = \log x$$

$When\ x = 1, y = \log 1 = 0$

$When\ x = 2, y = \log 2$

Differentiating [$y = \log x$ **]both sides ,we get**

$$dy = \frac{1}{x}\, dx$$

Now,

$$I = \int_1^{\log 2} \sin y \cdot \frac{1}{x} \cdot dx = \int_1^{\log 2} \sin y \; dy$$

$$= [-\cos y]_0^{\log 2}$$

$$= -\cos \log 2 - (-\cos 0)$$

$$= -\cos(\log 2) + 1$$

$$I = 1 - \cos(\log 2)$$

7) Find the coefficient x^5 in the expansion of $\left(x^2 - \frac{1}{x}\right)^{10}$.

➢ **Solution:-**

Given, $\left(x^2 - \frac{1}{x}\right)^{10}$

General terms in the above expansion is given by

$$t_{r+1} = c(10, r).(x^2)^{10-r}\left(-\frac{1}{x}\right)^r$$

$$= c(10, r).(x^2)^{10-r}.(-1)^r x^{-r}$$

$$= c(10, r).x^{20-2r}.(-1)^r x^{-r}$$

$$= c(10, r).x^{20-3r}.(-1)^r$$

Now,

The coefficient x^5 in the expansion, we have

$$20 - 3r = 5$$

$$20 - 5 = 3r$$

$$15 = 3r$$

$$\Rightarrow \boldsymbol{r = 5}$$

Hence,

$$t_{5+1} = t_6 = c(10,5)\ .x^{20-3.5}.(-1)^5$$

$$= \frac{10!}{(10-5)!\,5!}.\boldsymbol{x^5}.-1$$

$$t_6 = 252\,.\,\boldsymbol{x}^5.-1 = -252.\,\boldsymbol{x}^5$$

Hence, Coefficient of $\boldsymbol{x}^5 = \mathbf{-2}$

8)Find the sum to n terms $1.\,2^2 + 2.\,3^2 + 3.\,4^2 + \cdots$.

➢ **Solution:-**

Let t_n be the general term

$$\boldsymbol{t_n = n.\,(n+1)^2 = n\left(n^2 + 2n + 1\right) = n^3 + 2n^2 + n}$$

Let s_n be the required sum then

$$s_n = \sum_{k=1}^{n} \boldsymbol{t_n} = \sum_{k=1}^{n} (\boldsymbol{n^3 + 2n^2 + n})$$

$$s_n = \sum_{k=1}^{n} n^3 + 2.\sum_{k=1}^{n} n^2 + \sum_{k=1}^{n} n$$

$$s_n = (1^3 + 2^3 + 3^3 + \cdots n\ terms) + \ 2.\,(1^2 + 2^2 + 3^2 + \cdots n\ terms) + (1 + 2 + 3 + \cdots n\ terms\ ) \cdots\cdots\cdots (i)$$

We know that, $1^3 + 2^3 + 3^3 + \cdots n\ to\ terms = (\frac{\boldsymbol{n(n+1)}}{\mathbf{2}})^2$

$$(1^2 + 2^2 + 3^2 + \cdots n\ to\ terms) = \frac{\boldsymbol{n(n+1)(2n+1)}}{\mathbf{6}}$$

$$(1 + 2 + 3 + \cdots n\ to\ terms) = \frac{\boldsymbol{n(n+1)}}{\mathbf{2}}$$

$$s_n = (\frac{n(n+1)}{2})^2 + 2.\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} s_n$$

$$= (\frac{\boldsymbol{n(n+1)}}{\boldsymbol{2}})^2 + 2.\frac{\boldsymbol{n(n+1)}(2n+1)}{\boldsymbol{2}.3} + \frac{\boldsymbol{n(n+1)}}{\boldsymbol{2}}$$

$$= \frac{\boldsymbol{n(n+1)}}{\boldsymbol{2}} [\frac{n(n+1)}{2} + 2.\frac{(2n+1)}{3} + 1]$$

$$= \frac{n(n+1)}{2} [\frac{n(n+1)}{2} + \frac{(4n+2)}{3} + 1]$$

$$= \frac{n(n+1)}{2} [\frac{3n(n+1) + 2(4n+2)}{6} + 1]$$

$$= \frac{n(n+1)}{2} [\frac{3n(n+1) + 2(4n+2) + 6}{6}]$$

$$= \frac{n(n+1)}{2} [\frac{3n^2 + 3n + 8n + 4 + 6}{6}]$$

$$= \frac{n(n+1)}{2} [\frac{3n^2 + 11n + 10}{6}]$$

$$\therefore \ \boldsymbol{S_n} = \frac{\boldsymbol{n(n+1)(3n^2 + 11n + 10)}}{\boldsymbol{12}}$$

9) A committee of 5 is to be formed out of 6 gentlemen and 4 ladies, In how many ways this can be done when at most 2 ladies are included ?

➢ **Refer to the solution 2078 of Q. No 5 on Page 35.**

10) Find the equation of sides of an equilateral triangle whose vertex is $(-1, 2)$ and base is $y = 0$.

➢ **Solution:-**

Let ABC be an equilateral triangle whose vertex is $A(-1,2)$. The sides BA and AC with outside each make angle of $60°$ and $120°$ respectively with $x - axis$.

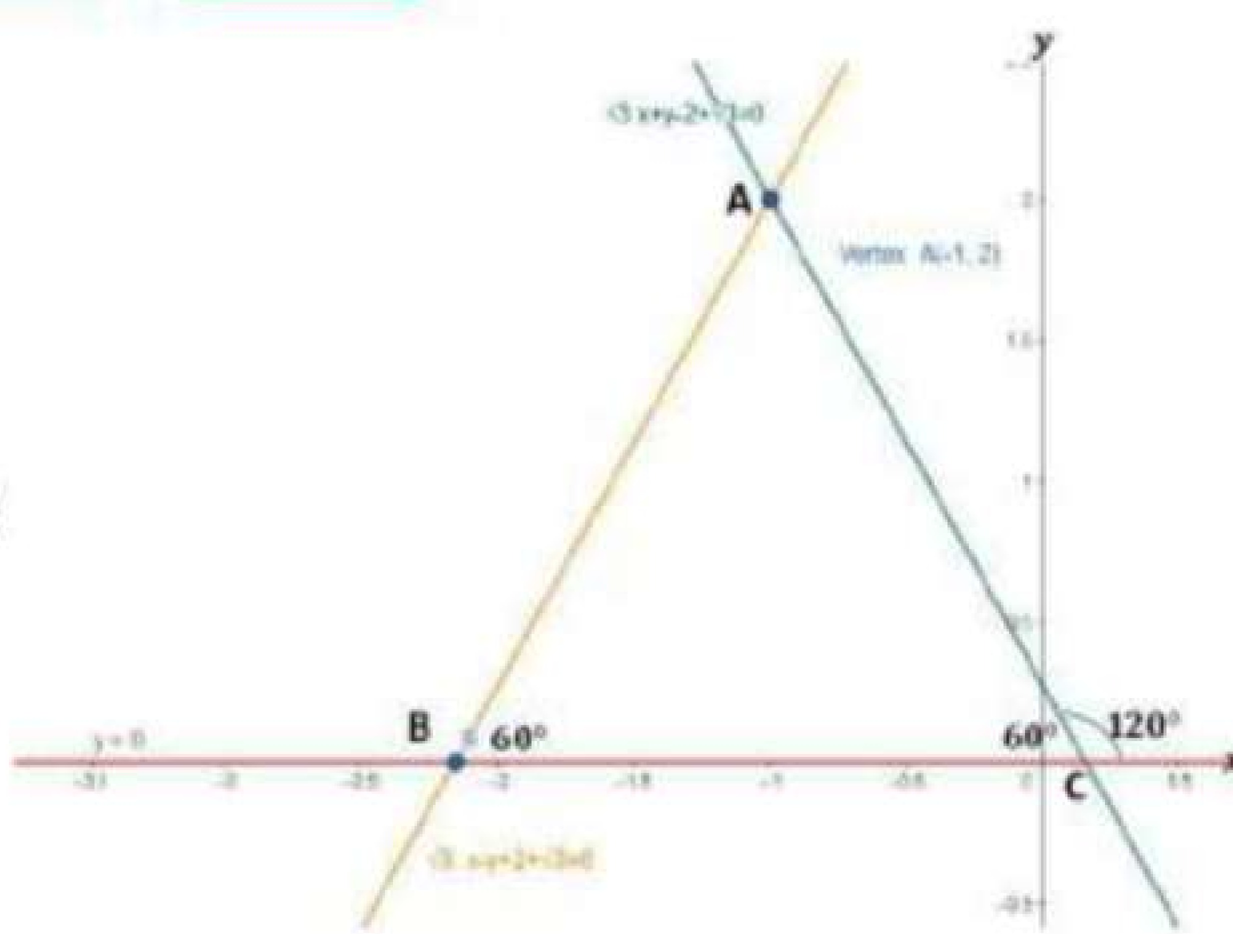
Then, the slope of line $AC = \tan 120° = -\sqrt{3}$
and slope of BA $= tan\, 60° = \sqrt{3}$.

Equation of AC is

$$y - 2 = -\sqrt{3}(x + 1)$$

$$or,\ y - 2 = -\sqrt{3}x - \sqrt{3}$$

$$\therefore\ \boldsymbol{\sqrt{3}x + y - 2 + \sqrt{3} = 0}$$



Equation of BA is

$$y - 2 = \sqrt{3}(x + 1)$$

$$or,\ y-2=\sqrt{3}\,x+\sqrt{3}$$

$$\therefore\ \boldsymbol{\sqrt{3}\,x-y+2+\sqrt{3}=0}$$

Hence, Required equation of sides of ΔABC are

$$\boldsymbol{\sqrt{3}x+y-2+\sqrt{3}=0\ \ and\ \sqrt{3}\,x-y+2+\sqrt{3}=0}$$

11) If P is the length of perpendicular dropped from origin on the line $\frac{x}{a}+\frac{y}{b}=1$ then, Prove that : $\frac{1}{a^2}+\frac{1}{b^2}+\frac{2}{p^2}$.

➢ **Solution:**

Given:

$$\frac{x}{a}+\frac{y}{b}=1\quad or,\quad \frac{x}{a}+\frac{y}{b}-1=0$$

If p be the length of perpendicular draw from origin on the given lines, then

$$p=\frac{|0+0-1|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2}}$$

$$p=\frac{|-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}}}$$

$$\Rightarrow p^2=\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}}$$

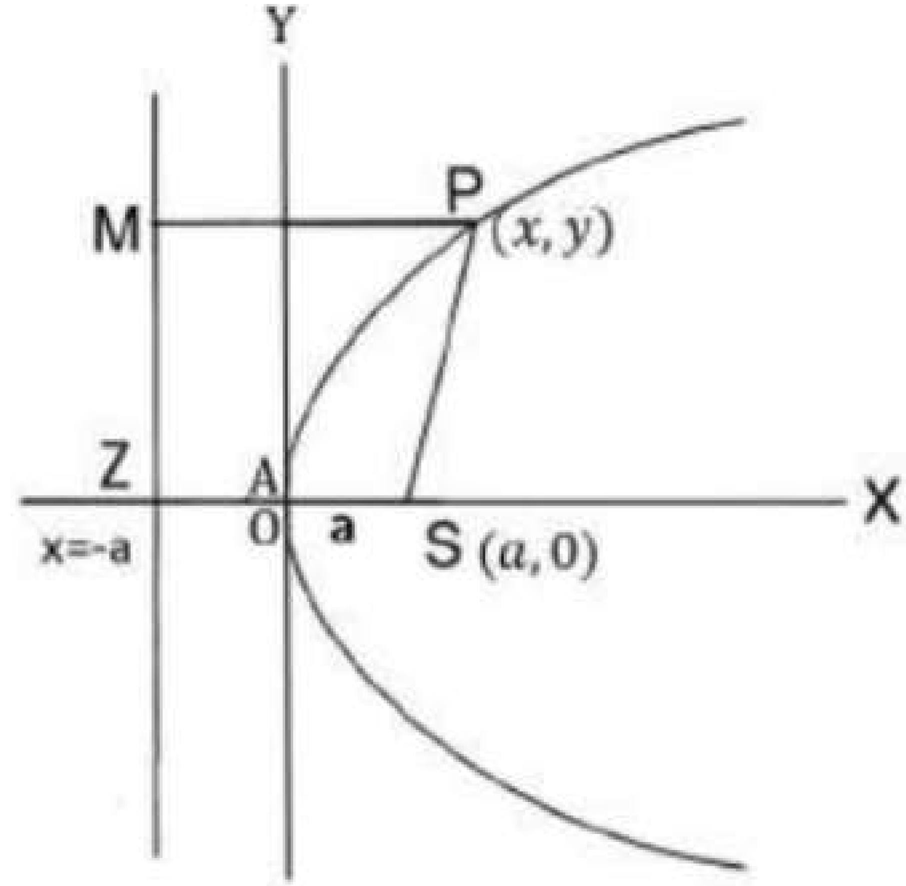
$$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$$

$$\Rightarrow \boldsymbol{\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{p^2}}$$

Proved

12) Find the equation of the parabola in the standard form $y^2 = 4ax$

➢ **Solution:**



Let S be focus and ZM, the directrix of the parabola. SZ is drawn perpendicular to ZM. Let A be the middle point of SZ, so that $\mathrm{SA} = \mathrm{AZ}$. Then A is the vertex and ZAS is the axis of the parabola.

Take the vertex A at the origin, the focus S on the $x - axis$ so that the axis of the parabola is the $x - axis$ and the directrix is parallel to the $y - axis$.

Let $\mathrm{AS} = a$ .Thus the coordinates of Z, A and S are Respectively $(-a, 0)$, $(0, 0)$ and $(a, 0)$, and the equation of the directrix is $x + a = 0$.

Let $\mathrm{P}(x, y)$ be any point on the parabola. Join PS and draw PM perpendicular to ZM.

Then, $\quad \mathrm{PS} = \mathrm{PM}.$

$\Rightarrow \quad \mathrm{PS}^2 = \mathrm{PM}^2$

$$or, (x - a)^2 + (y - 0)^2 = \left(\frac{x + a}{\sqrt{1}}\right)^2$$

$$y^2 = (x + a)^2 - (x - a)^2$$

$$y^2 = (x + a + x - a)(x + a - x + a)$$

$$y^2 = (2x)(2a)$$

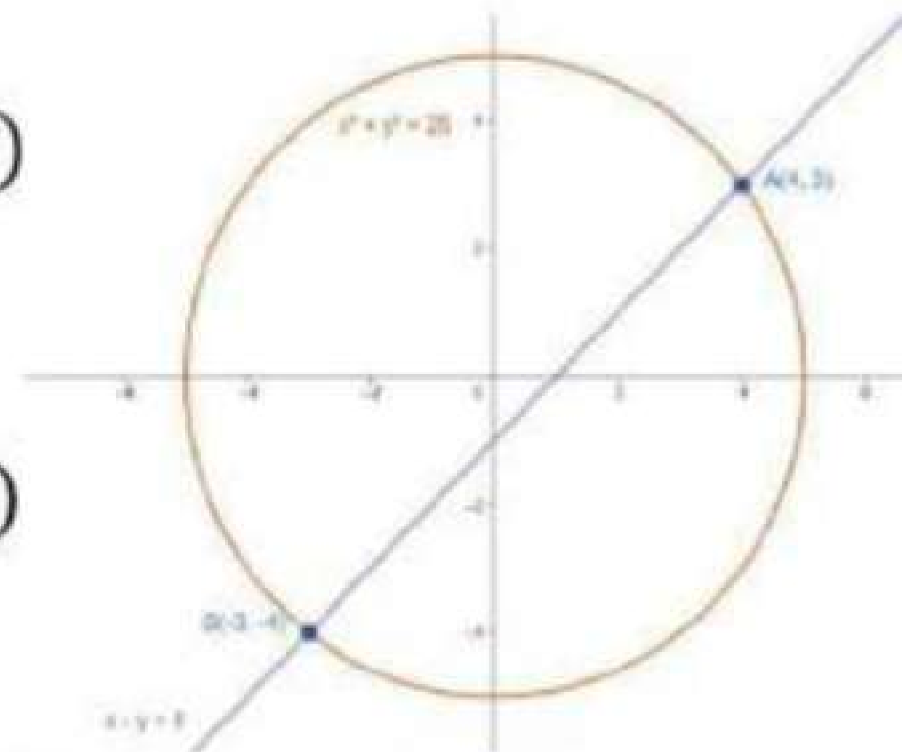
$$\boxed{\Rightarrow \boldsymbol{y^2 = 4ax.}}$$

13) Find the coordinates of the point of intersection of the line $x - y = 1$ and the circle $x^2 + y^2 = 25$.

➢ **Solution:**

Given:

The circle is $x^2 + y^2 = 25$ ……… (i)



The line is $x - y = 1$

or, $y = 1 + x$ ……… (ii)

From (i) and (ii) we get

$$x^2 + (1 + x)^2 = 25$$

$$\Rightarrow 2x^2 + 2x - 24 = 0$$

$$\Rightarrow x^2 + x - 12 = 0$$

$$\Rightarrow (x - 3)(x + 4) = 0$$

$$\therefore x = 3, -4$$

At When $x = 3$

$y = 1 + x$

$\Rightarrow y = 1 + 3$

$\therefore y = 4$

At When $x = -4$

$y = 1 + x$

$\Rightarrow y = 1 + (-4)$

$\therefore y = -3$

Hence,

The points of intersection are: $A(3, 4)$ ***and*** $B(-4, -3)$

14) If $\frac{\log x}{y-z} = \frac{\log y}{z-x} = \frac{\log z}{z-y}$, prove that $x^x . y^y . z^z = 1$

➢ **Refer to the solution 2078 of Q. No 12 (or) on page 41.**

–The End –

** **"If you find a mistake in Question/Answer, Kindly take a Screenshot and email to info@arjun00.com.np "**

Engineering Mathematics I_(Engg. All) 1^{st} Sem

(2080 Scholarship) Question Paper Solution.

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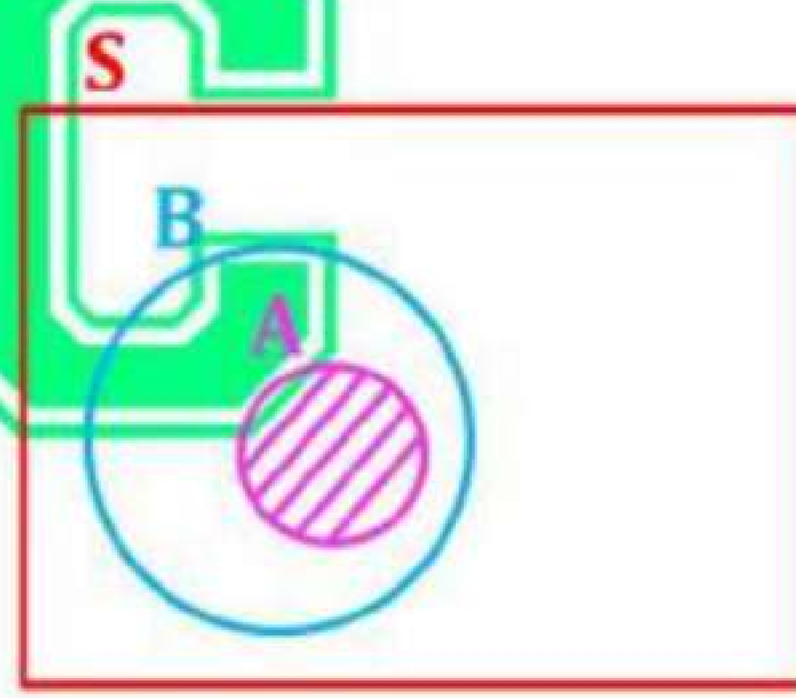
1. a) If $n(A) = 37$, $n(B) = 50$ *and* $A \subset B$ *then, find* $n(A \cup B)$ *and,* $n(A \cap B)$

➢ **Solution:**

Given,

$n(A) = 37$, $n(B) = 50$ *and* $A \subset B$ *then,*

Find $n(A \cup B)$ *and,* $n(A \cap B)$.



$A \subset B$ *i.e* A *is subset of* B

From figure,

So, $n(A \cup B) = n(B) = 50$

So, $n(A \cap B) = n(A) = 37$

b) $\log_a x^2 - 2\log_a \sqrt{x} = \log_a x$

➢ **Solution:**

$$LHS = log_a x^2 - 2\, log_a \sqrt{x}$$

$$= 2\log_a x - 2\log_a x^{\frac{1}{2}}$$

$$= 2\log_a x - 2 \times \frac{1}{2}\log_a x$$ By power rule, $\because \log a^n = n \log a$

$$= 2\log_a x - \log_a x$$

$$= \log_a x$$ **R.H.S proved**

2. a) Solve: $Cos5\theta = Cos3\theta$

➢ **Solution:**

$$Cos5\theta = Cos3\theta$$

$$Cos5\theta - Cos3\theta = 0$$

$$-2\,\mathrm{Sin}\,\frac{5\theta + 3\theta}{2}\,\mathrm{Sin}\,\frac{5\theta - 3\theta}{2} = 0$$

$$-\mathrm{Sin}\,4\theta\,\mathrm{Sin}\,\theta = 0$$

$$\mathrm{Sin}\,4\theta\,\mathrm{Sin}\,\theta = 0$$

Either,

$\mathrm{Sin}\,4\theta = 0$ *OR* $\mathrm{Sin}\,\theta = 0$

$4\theta = n\pi$ $\therefore \theta = n\pi$

$$\therefore \theta = \frac{n\pi}{4}$$

Hence,

$$\theta = \frac{n\pi}{4},\ n\pi$$

b) Prove that : $sin\,(2sin^{-1}x) = 2x\sqrt{1 - x^2}$

➢ **Refer to the solution 2080/81 New of Q. No 2 (b) on Page 137.**

3. a) Prove that: $\mathbf{sinA + sinB + sinC = \frac{S}{R}}$

➢ **Solution:**

$We\ Know\ that, \frac{\sin A}{a} = \frac{Sin\ B}{b} = \frac{Sin\ C}{c} = \frac{1}{2R}$

$So,\ Sin\ A = \frac{a}{2R},\ Sin\ B = \frac{b}{2R},\ Sin\ C = \frac{c}{2R}$

LHS: $Sin\ A + Sin\ B + Sin\ C$

Substituting the value of sinA, sinB, SinC.

$$= \frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R}$$

$$= \frac{a+b+c}{2R}$$

Since, semi perimeter (s) $= \frac{a+b+c}{2}$

$so, = \frac{S}{R}$ **R. H. S Proved**

b) Evaluate: $\mathbf{\frac{Lt}{x \to 2} \frac{x^2 - 5x + 6}{x^2 - x - 2}}$.

➢ **Solution:**

$$\frac{Lim}{x \to 2} \frac{x^2 - 5x + 6}{x^2 - x - 2}$$

$When\ x = 2\ Then, limit\ takes\ the\ form\ \left(\frac{0}{0}\right)\ So,$

$$\frac{Lim}{x \to 2} \ \frac{x^2 - 5x + 6}{x^2 - x - 2}$$

$$\frac{Lim}{x \to 2} \ \frac{x^2 - 3x - 2x + 6}{x^2 - 2x + x - 2}$$

$$\frac{Lim}{x \to 2} \ \frac{x(x-3) - 2(x-3)}{x(x-2) + 1(x-2)}$$

$$\frac{Lim}{x \to 2} \ \frac{(x-2)(x-3)}{(x-2)(x+1)}$$

$$= \frac{Lim}{x \to 2} \ \frac{x-3}{x+1}$$

$$= \frac{2-3}{2+1}$$

$$\therefore \ \frac{-1}{3}$$

4. a) Find $\frac{dy}{dx}$; *when* $y = \frac{1}{\sqrt[3]{x^2 - 2x + 1}}$.

➢ **Solution:**

$$y = \frac{1}{\sqrt[3]{x^2 - 2x + 1}} = \frac{1}{(x^2 - 2x + 1)^{\frac{3}{2}}} = (x^2 - 2x + 1)^{-\frac{3}{2}}$$

Differentiating both side w.r.t 'x' we get,

$$\frac{dy}{dx} = \frac{d}{dx} \ (x^2 - 2x + 1)^{-\frac{1}{2}}$$

Using chain rule,

$$\frac{dy}{dx} = \frac{d(x^2-2x+1)^{-\frac{3}{2}}}{d\,(x^2-2x+1)} \times \frac{d(x^2-2x+1)}{dx}$$

$$\frac{dy}{dx} = -\frac{3}{2}\ (x^2-2x+1)^{-\frac{3}{2}-1} \times (2x-2)$$

$$\therefore \boldsymbol{\frac{dy}{dx} = -\frac{3}{2}\ (x^2-2x+1)^{-\frac{5}{2}} \times (2x-2)}$$

b) Integrate: $\boldsymbol{\int \frac{2x+3}{x+1}\,dx}$

➢ **Solution:**

$$Let, I = \int \frac{2x+3}{x+1}\,dx$$

$$= \int \frac{2(x+\frac{3}{2})}{x+1}\,dx$$

$$= \int \frac{2[(x+1)-\frac{3}{2}-1]}{x+1}\,dx$$

$$= \int \frac{2[(x+1)-\frac{5}{2}]}{x+1}\,dx$$

$$= \int 2\,dx - 2 \times \frac{5}{2}\int \frac{1}{x+1}\,dx$$

$$\therefore \mathbf{I = 2x - 5\log(x+1) + C}$$

5. a) Integrate: $\int sin^2x\,dx$

➢ **Solution:**

$$Let, I = \int sin^2x\,dx$$

$$\int sin^2x\,dx \qquad \left[\because sin^2x = \frac{1-\cos 2x}{2}\right]$$

$$= \int \frac{1-\cos 2x}{2}dx$$

$$= \int \frac{1}{2}dx - \frac{1}{2}\int \cos 2x\,dx$$

$$\therefore \mathbf{I} = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

b) Find the sum of the series: $1 + 4 + 7 + 10 + \cdots$ to 40 terms.

➢ **Solution:**

Given,

$$\mathbf{1 + 4 + 7 + 10 + \cdots\ to\ 40\ terms}$$

$$n = no\ of\ terms = 40$$

Since, common difference $(d) = a_2 - a_1 = a_3 - a_2 = \cdots = 3$

Since, series is in AP

And first term $(a) = 1$

So, Sum to n terms formula $S_n = \frac{n}{2}(2a + (n-1)d)$

$\therefore$ **Sum of series** $= S_{40} = \frac{40}{2}(2 \times 1 + 39 \times 3) = 2380$

6. a) In how many ways can eight people be seated in a row of eight seats if two people insist on sitting next to each other?

➢ **Solution:**

Two people insist on sitting together next to each other so consider as **one**

So,7 people can be arranged in $P(7,6)$ ways

1 2	3	4	5	6	7	8

Also, two people can be arranged in 2 ways

Hence, The required Number of ways =

$$P(7,6) \times 2 = \frac{7!}{(7-6)!} \times 2 = \frac{7!}{1!} \times 2 = \mathbf{10080} \; \boldsymbol{ways.}$$

b) Expand $(2a + b)^5$ by the binomial theorem

➢ **Solution:**

Binomial theorem,

$$(a+x)^n = C(n,0)a^n x^0 + C(n,1)a^{n-1}x^1 + C(n,2)a^{n-2}x^2 + \cdots + C(n,n)a^0 x^n$$

so,

$$(\boldsymbol{2a + b})^5 = C(5{,}0).(2a)^5.b^0 + C(5{,}1).(2a)^4.b^1 + C(5{,}2).(2a)^3.b^2 + C(5{,}3).(2a)^2.b^3 + C(5{,}4).(2a)^1.b^4 + C(5{,}5).(2a)^0.b^5$$

$$(2a+b)^5 = 1.32a^5 + 5.16a^4.b + 10.8a^3.b^2 + 10.4a^2.b^3 + 5.2a.b^4$$
$$+1.b^5$$

$$\therefore \boldsymbol{(2a+b)^5 = 32a^5 + 80a^4b + 80a^3b^2 + 40a^2b^3 + 10ab^4 + b^5}$$

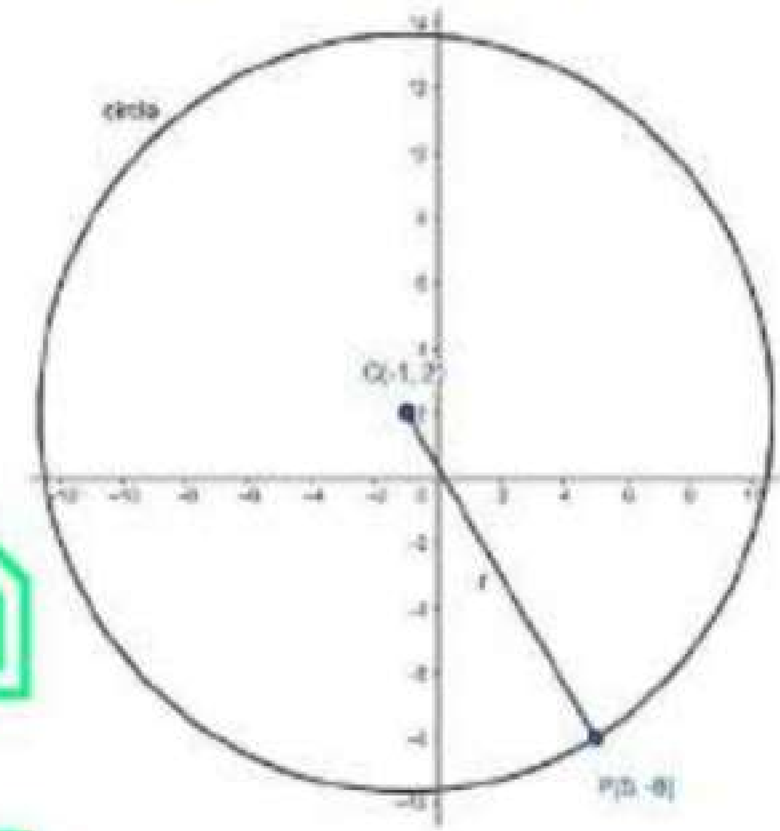
7. a) Find the equation of the circle with center at the point $(-1, 2)$and passing throught the point $(5, -8)$

➢ **Solution:**

Given,

Let Equation of the circle is

$$(x-h)^2 + (y-k)^2 = r^2$$



Center of circle $C(h, k) = \boldsymbol{C(-1, 2)}$

And Circle passes through point $\boldsymbol{P(5, -8)}$

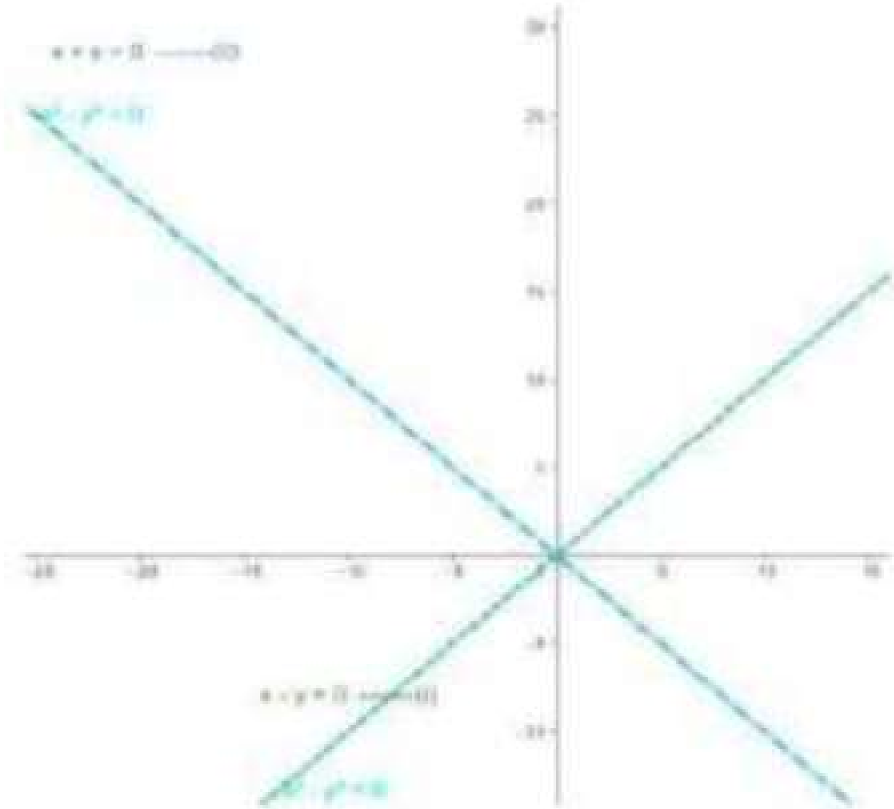
$$\boldsymbol{r = \text{radius of circle}} = PC = \sqrt{(-1-5)^2 + (2+8)^2}$$
$$= \sqrt{(-6)^2 + (10)^2}$$
$$= \sqrt{36+100}$$
$$= \sqrt{136}$$

Hence, Eqn of circle is

$$(x+1)^2 + (y-2)^2 = (\sqrt{136})^2$$

$\boldsymbol{(x+1)^2 + (y-2)^2 = 136}$ **is the required Equation of Circle.**

b) Find the single equation represented by the lines $x - y = 0$ and $x + y = 0$.



➢ **Solution:**

Given Equation,

$$x - y = 0 \cdots\cdots\cdots\cdots (i)$$

$$x + y = 0 \cdots\cdots\cdots\cdots (ii)$$

To find the single equation *i.e.* pair of straight line.

So, Multiplying eqn (i) and (ii) we get

$$(x-y)(x+y)=0$$

$\therefore x^2 - y^2 = 0$ **is the required pair of straight line (single equation)**

8) Let $f: R \longrightarrow R$ *and* $g: R \longrightarrow R$ be defined by $f(x) = x^2 + 1$ and $g(x) = x^5$. Find a) $fog(x)$ b) $gof(x)$ c) $f^{-1}(x)$

➢ **Solution:**

Given, $f(x) = x^2 + 1$ *and* $g(x) = x^5$

We have,

$$(\boldsymbol{a})\ \boldsymbol{fog}(\boldsymbol{x}) = f\{g(x)\} = f\{x^2\} = (x^2)^5 + 1 = x^{10} + 1$$

$$(\boldsymbol{b})\ \boldsymbol{gof}\ (\boldsymbol{x}) = g\{f(x)\} = g\{x^2 + 1\} = (x^2 + 1)^5$$

(c) $\boldsymbol{f^{-1}(x)}$

$Let\ y = f(x) = x^2 + 1$

Interchanging $\boldsymbol{x}$ and $\boldsymbol{y}$ we get,

$or,\ x = y^2 + 1$

$or,\ y^2 = x - 1$

$or,\ y = (x-1)^{\frac{1}{2}}$

$\boldsymbol{or,\ f^{-1}(x) = (x-1)^{\frac{1}{2}}\ x \in R}$

OR) Prove that: $x^{logy-logz} \times y^{logz-logz} \times z^{logx-logy} = 1$

➢ **Solution:**

L.H.S $x^{logy-logz} \times y^{logz-logx} \times z^{logx-logy}$

Since, using log properties $\boldsymbol{e^{logx} = x}$

$= x^{logy-logz} \times y^{logz-logx} \times z^{logx-logy}$

$= e^{log(x^{logy-logz} \times y^{logz-logx} \times z^{logx-logy})}$

$= e^{[logx^{logy-logz} + \log y^{logz-logx} + log\ z^{logx-logy}]}$

By product rule, $\because$ $\boldsymbol{log\ ab = log\ a + logb}$

$= e^{(logy-logz)\log x + (\log z - \log x)\log y + (\log x - logy) log\ z]}$

By power rule, $\because$ $\boldsymbol{log\ a^n = n\ log\ a}$

$= e^{[(\log x.logy - \log x.logz) + (\log y.\log z - \log y.\log x) \times (log\ z\log x - log\ zlogy))]}$

$= e^0$

$= 1$ <u>**R.H.S Proved.**</u>

9) Solve: $\sqrt{3}\,sinx - cosx = \sqrt{2}\ for\ 0 \le x \le 2x$.

➢ **Solution:**

$$\sqrt{3}\sin x - \cos x = \sqrt{2}$$

Dividing both side by $\sqrt{(\sqrt{3})^2 + 1} = 2$

$$\frac{\sqrt{3}\sin x - \cos x}{2} = \frac{\sqrt{2}}{2}$$

$$or, \frac{\sqrt{3}}{2}\sin x - \frac{1}{2}\cos x = \frac{1}{\sqrt{2}}$$

$$or,\quad \cos\frac{\pi}{6}.\sin x - \sin\frac{\pi}{6}.\cos x = \frac{1}{\sqrt{2}}$$

$$\sin\left(x - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}$$

$\because Sin\ x = \sin\theta$ General solution is $x = n\pi + (-1)^n\theta$

Hence, $\boldsymbol{x - \frac{\pi}{6} = n\pi + (-1)^n\frac{\pi}{4}}$ **is General Solution.**

for this range : $0 \le x \le 2\pi$

$\boldsymbol{n = 0}, x - \frac{\pi}{6} = 0 + (-1)^0\frac{\pi}{4}$ $x = \frac{\pi}{4} + \frac{\pi}{6}$ $x = \frac{5\pi}{12}$ $= \frac{5\pi}{12}$ *lie in the range*	$\boldsymbol{n = 1}, x - \frac{\pi}{6} = \pi + (-1)^1\frac{\pi}{4}$ $x = \pi - \frac{\pi}{4} + \frac{\pi}{6}$ $x = \pi - \frac{\pi}{12}$ $= \frac{11\pi}{12}$ *lies in the range*

10) If $tan^{-1}x + tan^{-1}y + tan^{-1}z = \frac{\pi}{2}$, prove that $xy + yz + zx = 1$.

➢ **Solution:**

Given,

$$tan^{-1}x + tan^{-1}y + tan^{-1}z = \frac{\pi}{2}$$

$$or, tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{2} - tan^{-1}z$$

$$or, \frac{x+y}{1-xy} = cot\ tan^{-1}z$$

$$or, \frac{x+y}{1-xy} = cot\ cot^{-1}\frac{1}{z}$$

$$or, \frac{x+y}{1-xy} = \frac{1}{z}$$

$$or, xz + yz = 1 - xy$$

$$\therefore \mathbf{xy + yz + zx = 1}$$

11) A function $f(x)$ be defined as follows.

$$f(x) = \begin{cases} 2x+3 & for\ x < 1 \\ 4 & for\ x = 1 \\ 6x-1 & for\ x > 1 \end{cases}$$

Is $f(x)$ continuous at $x = 1$? If not, how can you make it continuous ?

➢ **Solution:**

Left hand limit at x = 1 $= \lim_{x \to 1^-} f(x)$

$= \lim_{x \to 1^-} (2x+3)$

$= 2 \times 1 + 3$

$= 5$

Right hand limit at x = 1 $= \lim_{x \to 1^+} f(x)$

$= \lim_{x \to 1^+} 6x - 1$

$= 6 \times 1 - 1$

$= 5$

Function value at $f(1) = 4$

$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) \neq f(1)$

Hence,

Given, The function Discontinuous *at x* = 1.

To make it continuous :

$$f(x) \text{ be defined by } f(x) = \begin{cases} 2x+3 & for\ x < 1 \\ 5 & for\ x = 1 \\ 6x-1 & for\ x > 1 \end{cases}$$

12) Find $\frac{dy}{dx}$ from first principle of $y = \sqrt{tanx}$

➢ **Refer to the solution 2079 New of Q. No 12 on Page 61.**

13) Evaluate: $\int \frac{dx}{1-\cos x}\,dx$

➢ **Solution:**

$$Let, I = \int \frac{dx}{1-\cos x}$$

$$As, 1-\cos x = 2\sin^2 x$$

Integral becomes,

$$I = \int \frac{dx}{2\sin^2 x}$$

$$I = \frac{1}{2}\int \frac{1}{\sin^2 x}\,dx$$

$$= \frac{1}{2}\int \frac{1}{\sin^2 x} \times \frac{Sec^2 x}{Sec^2 x}\,dx$$

$$= \frac{1}{2}\int \frac{Sec^2 x}{tan^2 x}\,dx \qquad \left[\because Sin^2 x \times \sec^2 x = \frac{Sin^2 x}{cos^2 x} = tan^2 x\right]$$

$$= \frac{1}{2}\int \frac{dy}{y^2}$$

$$I = \frac{1}{2} \times \int y^{-2}\,dy \qquad \begin{bmatrix} \because Let\ y = \tan x \\ dy = sec^2 x\, dx \end{bmatrix}$$

$$I = \frac{1}{2} \times \frac{y^{-2+1}}{-2+1} + C$$

$$I = -\frac{1}{2y} + c = -\frac{1}{2tanx} + c$$

$$\therefore I = -\frac{Cot\,x}{2} + c$$

14) Find $\frac{dy}{dx}$ when $x^2 + y^2 = \text{Sin}(xy)$

➢ **Solution:**

$$x^2 + y^2 = sin(xy) \cdots\cdots (i)$$

Differentiating (i) with to x, we get

$$\frac{d\log(x^2+y^2)}{dx} = \frac{d\,sin(xy)}{dx}$$

$$\frac{d\log(x^2+y^2)}{d(x^2+y^2)} \times \frac{d(x^2+y^2)}{dx} = \frac{dsin(xy)}{dxy} \times \frac{d(xy)}{dx}$$

$$\frac{1}{(x^2+y^2)} \times \left[\frac{dx^2}{dx} + \frac{dy^2}{dx}\right] = \cos(xy)\left[x\frac{d(y)}{dx} + y\frac{d(x)}{dx}\right]$$

$$\frac{1}{(x^2+y^2)} \times \left[2x + \frac{dy^2}{dy} \times \frac{dy}{dx}\right] = \cos(xy)\ (x\frac{dy}{dx} + y)$$

$$\frac{1}{(x^2+y^2)} \times \left[2x + 2y \times \frac{dy}{dx}\right] = \cos(xy)\ (x\frac{dy}{dx} + y)$$

$$\frac{2x}{(x^2+y^2)} + \frac{2y}{(x^2+y^2)} \times \frac{dy}{dx} = \cos(xy)\ (x\frac{dy}{dx} + y)$$

$$\frac{2x}{(x^2+y^2)} + \frac{2y}{(x^2+y^2)} \times \frac{dy}{dx} = \cos(xy)\,.\,x\frac{dy}{dx} + \cos(xy)\,.\,y$$

$$\left[\frac{2y}{(x^2+y^2)} - \cos(xy)\,.\,x\right] \times \frac{dy}{dx} = \cos(xy)\,.\,y + \frac{2x}{(x^2+y^2)}$$

$$\left[\frac{2y - \cos(xy)\,.\,x(x^2+y^2)}{x^2+y^2}\right] \times \frac{dy}{dx} = \frac{(x^2+y^2).\cos(xy)\,.\,y + 2x}{x^2+y^2}$$

$$[2y - \cos(xy).x(x^2 + y^2)] \times \frac{dy}{dx} = (x^2 + y^2).\cos(xy).y + 2x$$

$$\therefore \frac{dy}{dx} = \frac{(x^2 + y^2).\cos(xy).y + 2x}{2y - \cos(xy).x(x^2 + y^2)}$$

15) Find the Sum of 4 + 44 + 444 + ...to n terms.

➢ **Solution:-**

Let s_n be the required sum then

$$s_n = 4 + 44 + 444 + 4444 + \cdots to\ n\ terms$$

$$= 4(1 + 11 + 111 + 1111 + \cdots to\ n\ terms)$$

$$= \frac{4}{9}[9 + 99 + 999 + \cdots . to\ n\ terms]$$

$$= \frac{4}{9}[(10 - 1) + (100 - 1) + (1000 - 1) + \cdots . to\ n\ terms]$$

$$= \frac{4}{9}[10 + 10^2 + 10^3 + \cdots to\ n\ terms]$$

$$-(1 + 1 + 1 + \cdots to\ n\ terms)$$

$$= \frac{4}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right] \qquad \left[\because s = \frac{a(r^n - 1)}{(r - 1)}\right]$$

$$= \frac{4}{9}\left\{\frac{10^{n+1} - 10}{9} - n\right\}$$

$$\therefore S_n = \frac{4}{9}\left\{\frac{10^{n+1} - 10}{9} - n\right\}$$

16) Find the sum of series : 1+(1+2)+(1+2+3)+... to n terms

➢ **Solution:-**

S=1+(1+2)+(1+2+3)+...+(1+2+3+4+...to n terms)

General Terms, $t_n = 1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$

Let S be the Sum of series,

$$S = \sum_{n=1}^{n} t_n = \sum_{n=1}^{n} \frac{n(n+1)}{2}$$

$$S = \sum_{n=1}^{n} \frac{n^2}{2} + \sum_{n=1}^{n} \frac{n}{2}$$

$$S = \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{n(n+1)}{2\times 2}$$

$$S = \frac{n(n+1)}{4}\left[\frac{(2n+1)}{3} + 1\right]$$

$$S = \frac{n(n+1)}{4}\left[\frac{(2n+1)+3}{3}\right]$$

$$S = \frac{n(n+1)}{4}\left[\frac{2n+4}{3}\right]$$

$$S = \frac{n(n+1)}{4} \times \frac{2(n+2)}{3}$$

$$\therefore \mathit{Sum\ of\ Series} = \frac{\boldsymbol{n(n+1)(n+2)}}{\boldsymbol{6}}$$

17) A person has 12 friends of whom 8 relatives. In how many ways can he/she invite 7 friends so that 5 of them may be relatives?

➢ **Solution:-**

Here, no of relatives $= 8$

And no of non $-$ relatives $= 12 - 8 = 4$

To find: The no of ways can he/she invite 7 friends so that 5 are relatives

$\boldsymbol{i.e}$ selection of 5 relatives and 2 non $-$ relatives

Possible Ways can invite = Selection of 5 relatives out of 8

$\times$ Selection of 2 non $-$ relatives out 4

$$= C(8,5) \times C(4,2)$$

$$= C(8,5) \times C(4,2)$$

$$= \frac{8!}{(8-5)!\,5!} \times \frac{4!}{(4-2)!\,2!}$$

$$= 56 \times 6$$

$$= \boldsymbol{336\ ways}$$

Hence, 336 *ways* can he/she invite 7 friends so that 5 of them may be relatives

Question wrong

Prove that the three lines $x + 3y = 5$, $2x - y = 3$ and $7x + 5y - 17 = 0$ are concurrent. Also, Find the point of concurrency.

Correct Question:

18) Prove that the three lines $x + 3y = 5$, $2x - y = 3$ & $7x + 5y - 19 = 0$ are concurrent. Also, Find the point of concurrency.

➢ **Solution:-**

Given lines,

$$x + 3y = 5 \cdots\cdots\cdots\cdots (i)$$
$$2x - y = 3 \cdots\cdots\cdots\cdots (ii) \times 3$$
$$7x + 5y - 19 = 0 \cdots\cdots\cdots\cdots (iii)$$

Multiplying eqn$(ii)$by 2 and Adding with eq(i) we get,

$$x + 3y = 5$$
$$+ \quad 6x - 3y = 9$$
$$7x = 14 \Rightarrow \boldsymbol{x = 2}$$

$from(i), 2 + 3y = 5 \Rightarrow 3y = 3 \Rightarrow \boldsymbol{y = 1}$

Hence point of intersect of line (i) and (ii) $is(\mathbf{2}, \mathbf{1})$

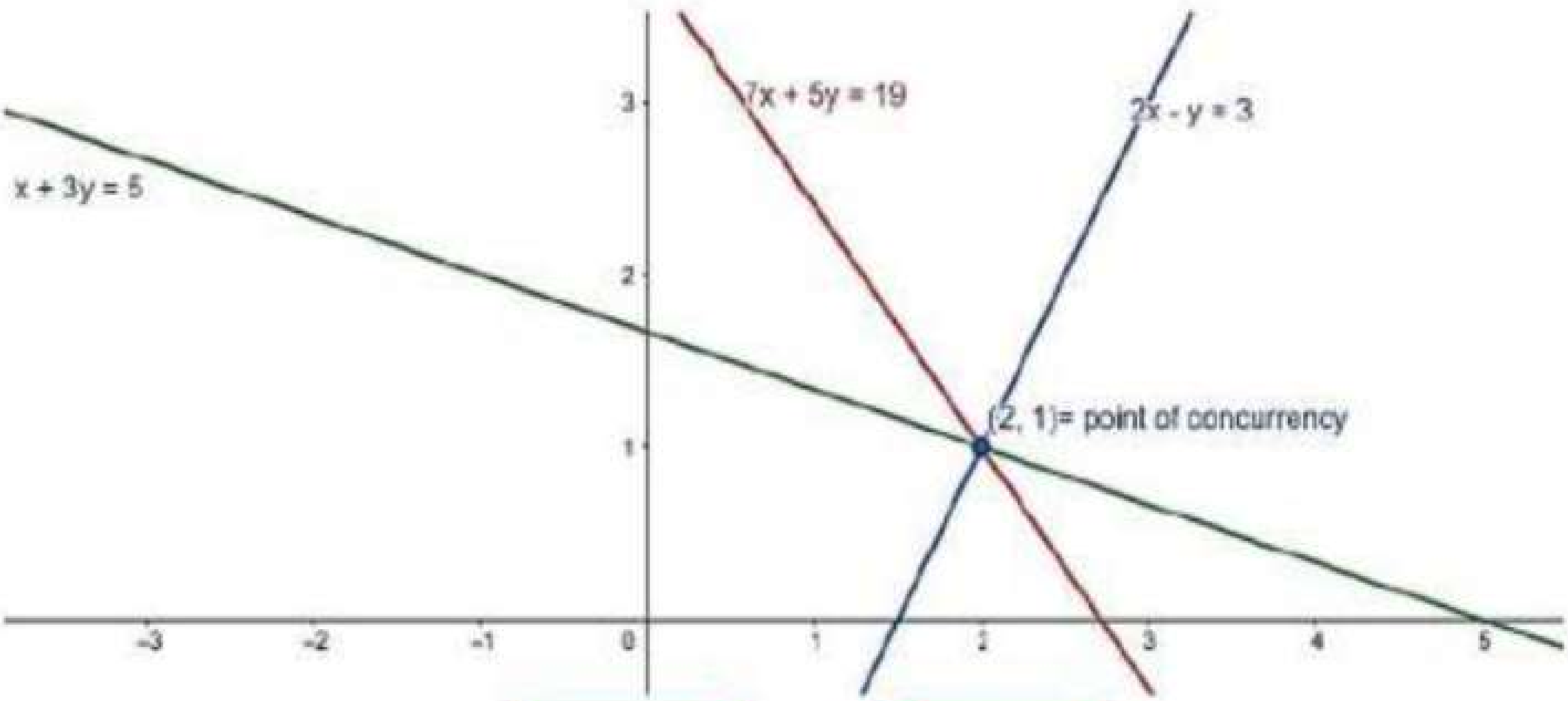
For Concurrent, the point of intersection must satisfy the line(iii)

$i.e\ 7.2 + 5.1 - 19 = 0$

$$14 + 5 - 19 = 0$$

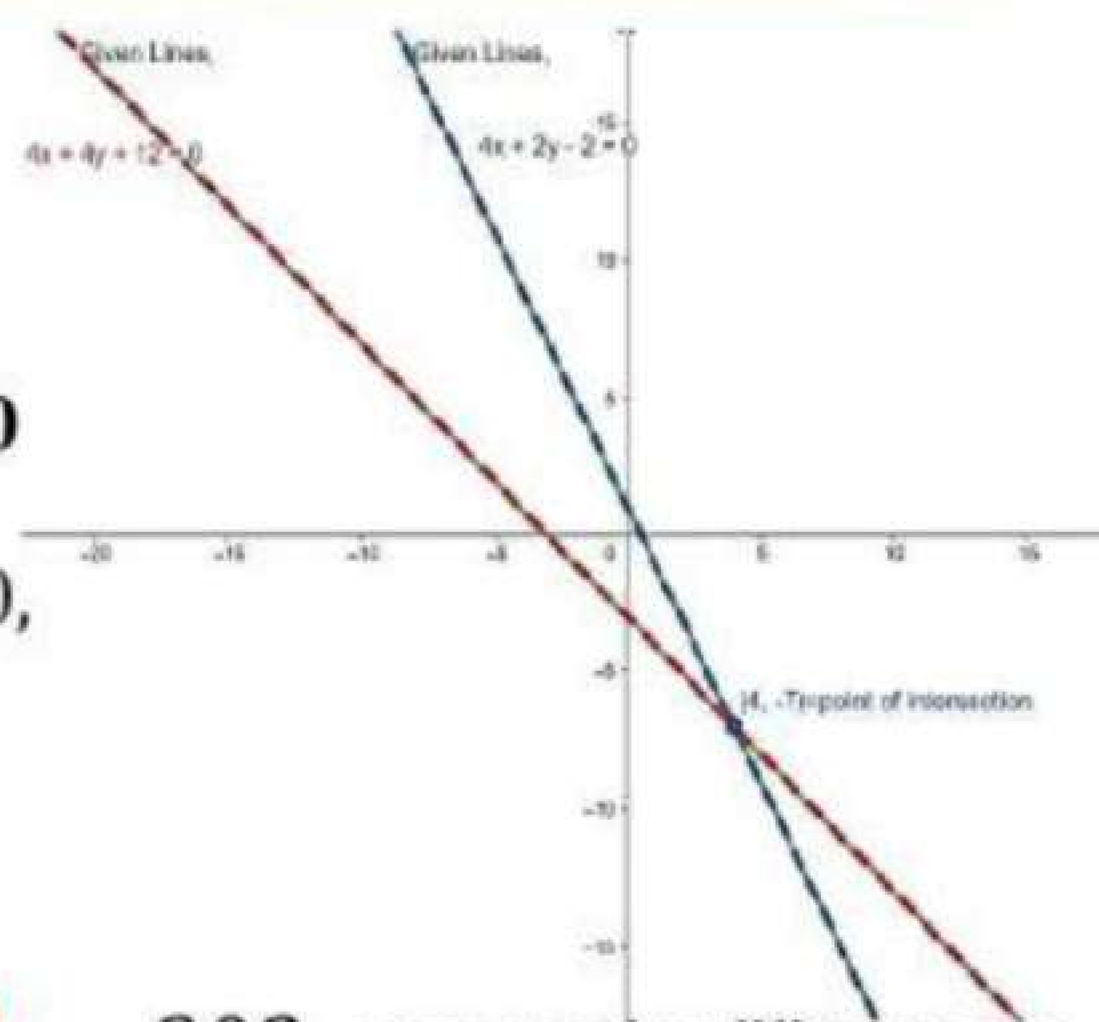
$0 = 0$ Hence, satisfied.

Hence, three lines are concurrent Point of concurrency $= (2, 1)$



19) Find the equations of two lines represented by the equation. $2x^2 + 3xy + y^2 + 5x + 2y - 3 = 0$, Also find the angle between them.

➢ **Solution:-**



Given lines,

$$\mathbf{2x^2 + 3xy + y^2 + 5x + 2y - 3 = 0}$$

$$2x^2 + x(3y + 5) + y^2 + 2y - 3 = 0,$$

Which is quadratic in x so,

$$x=\frac{-(3y+5)\pm\sqrt{(3y+5)^2-4.2(y^2+2y-3)}}{2.2}$$

$$x=\frac{-(3y+5)\pm\sqrt{9y^2+30y+25-8(y^2+2y-3)}}{2.2}$$

$$x=\frac{-(3y+5)\pm\sqrt{y^2+14y+49}}{2.2}$$

$$x=\frac{-(3y+5)\pm\sqrt{(y+7)^2}}{4}$$

$$x=\frac{-(3y+5)\pm(y+7)}{4}$$

$Taking + ve\ sign, x=\dfrac{-(3y+5)+(y+7)}{4}$

$4x=-2y+2$

$\mathbf{4x+2y-2=0} \cdots\cdots (i)$

$Let\ m_1\ slope\ of\ line\ then, \mathbf{m_1}=\dfrac{-coeff.of\ x}{coeff\ of\ y}=\dfrac{-4}{2}=\mathbf{-2}$

$Taking - ve\ sign, x=\dfrac{-(3y+5)-(y+7)}{4}$

$4x=-4y-12$

$\mathbf{4x+4y+12=0} \cdots\cdots\cdots\cdots (ii)$

$Let\ m_2\ slope\ of\ line\ then, \mathbf{m_2}=\dfrac{-coeff.of\ x}{coeff\ of\ y}=\dfrac{-4}{4}=\mathbf{-1}$

To find: point of intersection Solving eqn (i) *and eqn* (ii),

$$\begin{array}{r} 4x+2y-2=0 \\ -4x+4y+12=0 \\ \hline -2y-14=0 \\ \Rightarrow y=-7 \end{array}$$

OR, Solve Eqn (i)and (ii)
By calculator Directly,
$(x, y) = (4, 7)$

and from (i), $4x+2.(-7)-2=0 \Rightarrow 4x=16 \Rightarrow \boldsymbol{x=4}$

Hence, Point of intersection $= (\mathbf{4}, \mathbf{-7})$

To find , Let θ be angle between two lines then,

$$Tan\theta = \pm\frac{m_1-m_2}{1+m_1.m_2} = \pm\frac{-2-(-1)}{1+(-2).(-1)} = \pm\frac{-1}{3} = \pm\frac{1}{3}$$

$$\therefore \boldsymbol{\theta = tan^{-1}\left(\pm\frac{1}{3}\right)}$$

The angle between two lines is $\pm \tan^{-1}\left(\pm\frac{1}{3}\right)$

20) Find the equation of tangent and normal to the circle $\boldsymbol{x^2+y^2-3x+10y-15=0}$ *at* $\mathbf{(4, 11)}$.

➢ **Solution:-**

Given,

$$x^2+y^2-3x+10y-15=0$$

To find slope of tangent:

Differentiating both side w.r.t to 'x' we get,

$$\frac{d(\boldsymbol{x^2+y^2-3x+10y-15})}{dx} = 0$$

$$2x + \frac{dy^2}{dy} \times \frac{dy}{dx} - 3 + 10\frac{dy}{dx} = 0$$

$$2x + 2y \times \frac{dy}{dx} - 3 + 10\frac{dy}{dx} = 0$$

$$(2y + 10)\frac{dy}{dx} = 3 - 2x$$

$$\frac{dy}{dx} = \frac{3 - 2x}{2y + 10}$$

$At\ (4, -11)$ slope of tangent (m) $= \left|\frac{dy}{dx}\right|_{at\ (4,-11)} = \frac{3 - 2.4}{2. -11 + 10}$

$$m = \frac{-5}{-12} = \frac{\mathbf{5}}{\mathbf{12}}$$

Eqn of tangent having slope m and point$(4, -11)$ *is*

$$y + 11 = m(x - 4)$$

$$y + 11 = \frac{5}{12}(x - 4)$$

$$12y + 132 = (5x - 20)$$

$$\mathbf{5x - 12y - 152 = 0}$$

which is eq^n of tangent.

Since, Normal and tangent are perpendicular Hence,

Slope of Normal, $S_N \times m = -1$

$$S_N \times \frac{5}{12} = -1$$

$$S_N = \frac{-12}{5}$$

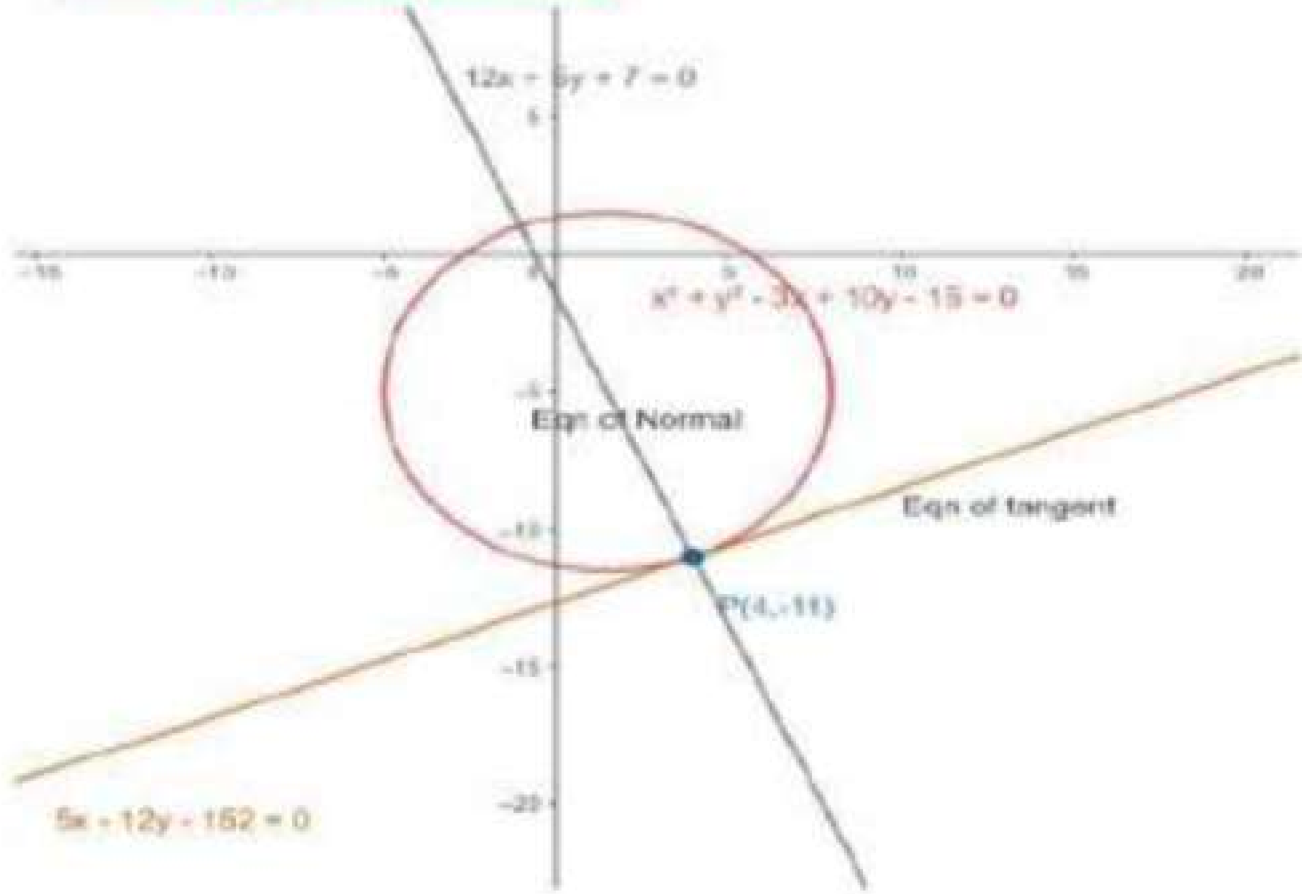
Eqⁿ of Normal at $(4, -11)$

$$y + 11 = \frac{-12}{5}(x - 4)$$

$$5y + 55 = -12x + 48$$

$$\mathbf{12x + 5y + 7 = 0}$$

Which is eqⁿ of Normal.



–The End –

**** "If you find a mistake in Question/Answer, Kindly take a Screenshot and email to info@arjun00.com.np "**