(Unit and Dimension)

Unit

The system of measurement which is used to denoted the particular quantity. for example: The unit of mass is kilogram, the unit of length is meter.

There are two types of unit.

(1) Fundamental unit

- Those unit which are independ in other units.
- There are seven fundamental unit.

S.N.	Quantity	Fundamental unit
1	Mass	kg
2	Length	m
3	time	sec
4	temperature	K
5	Luminous intensity	Candela (Cd)
6	Amount of substance	mole
7	Electric Current	Ampere (A)

(2) Derived unit

- Those unit which unit depend upon other fundamentals unit are called derived unit. Example:- The derived unit of speed is m/s or ms^{-1}.

List of Physical quantity and their derieved units.

(a) Speed

From the definition,

$$Speed = \frac{distance}{time\ taken}$$

But, The unit of distance = m
The unit of time = sec

$$So,\ speed = \frac{m}{sec}$$

$$speed = ms^{-1}.$$

(b) Velocity

From the definition,

$$velocity = \frac{displacement}{time\ taken}$$

But, The unit of displacement = m
The unit of time = sec

So,

$$velocity = \frac{m}{sec}$$

$$velocity = ms^{-1}$$

(c) acceleration

from the definition,

$$acceleration = \frac{velocity}{time\ taken}$$

But, The unit of velocity = m/s
The unit of time = ~~m~~ sec

So,

$$acceleration = \frac{m/s}{s}$$

$$acceleration = ms^{-2}$$

(e) force

from the definition,

froce = mass x acceleration

But, The unit of mass = kg
The unit of acceleration = m/s2

So,

$$\text{velocity} = \frac{m}{sec}$$

$$\text{velocity} = ms^{-1}$$

ⓒ acceleration

from the definition,

$$\text{acceleration} = \frac{\text{velocity}}{\text{time taken}}$$

But, The unit of velocity = m/s
The unit of time = ~~m~~ sec

So,

$$\text{acceleration} = \frac{m/s}{s}$$

$$\text{acceleration} = ms^{-2}$$

ⓔ force

from the definition,

froce = mass x acceleration

But, The unit of mass = kg
The unit of acceleration = m/s2

$$force = kg \times m/s^2$$
$$force = kgms^{-2}$$

(e) **Area**

from the definition,

$$Area = length \times breadth$$

But, The unit of length and breadth = m.

So,
$$Area = m \times m$$
$$= m^2$$

(f) **Volume**

from the definition,

$$Volume = length \times breadth \times height$$

But, The unit of length, breadth and height = m

So,
$$Volume = m \times m \times m$$
$$volume = m3$$

(g) Density

from the definition,

$$Density = \frac{mass}{volume}$$

But, The unit of mass = kg
The unit of volume = m^3

So,

$$Density = \frac{kg}{m^3}$$

$$Density = kgm^{-3}$$

(h) Pressure

from the definition,

$$Pressure = \frac{force}{Area}$$

But, The unit of force = kgm/s^2
The unit of area = m^2

So, $$Pressure = \frac{kgm/s^2}{m^2}$$

$$Pressure = kgm^{-1}s^{-2}$$

(i) **Work**

from the definition,

work = force x distance

But, The unit of force = kgm/s^2
The unit of distance = m

So,
$$work = kgm/s^2 \times m$$
$$= kgm^2s^{-2}$$

(j) **Power**

from the definition, ~~of~~

$$Power = \frac{work}{time\ taken}$$

But, The unit of work = kgm^2/s^2
The unit of time = s

$$So,\ Power = \frac{kgm^2/s^2}{s}$$

$$power = kgm^2s^{-3}$$

Dimensional Quantity

- Those physical quantity which can be express in term of mass, length & ~~pr~~ time is called dimensional quantity. Example: Force has the dimensional of $[M^1L^1T^{-2}]$

Dimensionless Quantity

- Those physical quantity which cannot be express in term of mass, length & time is called dimensionless quantity. For example: solid angle has unit steradian but has no dimension.

Dimensional formula

- A formula which express the mass, length and time of any physical quantity in term of M, L and T is called dimensional formula.

It is written as

$x = [M^a L^b T^c]$ [∵ when a, b & c are constent]

Dimensional formulas:

① Speed

from the definition,

$$speed = \frac{distance}{time\ taken}$$

Dimension of distance $= [M^0 L^1 T^0]$
Dimension of time $= [M^0 L^0 T^1]$

$$SPEED = [M^0 L^1 T^{-1}]$$

(11) **Velocity**

From the definition

$$velocity = \frac{displacement}{time\ taken}$$

Dimension of displacement $= [M^0 L^1 T^0]$
Dimension of time $= [M^0 L^0 T^1]$

So,

$$Velocity = \frac{[M^0 L^1 T^0]}{[M^0 L^0 T^1]}$$

$$Velocity = [M^0 L^1 T^{-1}]$$

(iii) Acceleration

from the definition,

$$\text{acceleration} = \frac{\text{velocity}}{\text{time taken}}$$

Dimension of velocity $= [M^0L^1T^{-1}]$
Dimension of time $= [M^0L^0T^1]$
So,

$$\text{acceleration} = \frac{[M^0L^1T^{-1}]}{[M^0L^0T^1]}$$

$$\text{acceleration} = [M^0L^1T^{-2}]$$

(iv) Force

from the definition,

Force = mass x acceleration

Dimension of mass $= [M^1L^0T^0]$
Dimension of acceleration $= [M^0L^1T^{-2}]$
So,
Force $= [M^1L^0T^0] \times [M^0L^1T^{-2}]$
Force $= [M^1L^1T^{-2}]$

(v) Area

from the definition

Area = length x breadth

Dimension of length and breadth = $[M^0L^1T^0]$

So,

Area = $[M^0L^1T^0]$ $[M^0L^1T^0]$

Area = $[M^0L^2T^0]$

(vi) Volume

from the definition,

volume = length x breadth x height

Dimension of length, breadth and height = $[M^0L^1T^0]$

So,

volume = $[M^0L^1T^0]$ $[M^0L^1T^0]$ $[M^0L^1T^0]$

volume = $[M^0L^3T^0]$

(vii) Work

from the definition,

work = force x distance

Dimension of force = $[M^1L^1T^{-2}]$

Dimension of distance $= [M^0L^1T^0]$

So,

work $= [M^1L^1T^{-2}][M^0L^1T^0]$

work $= [M^1L^2T^{-2}]$

(viii) Pressure / stress

from the definition,

$$\text{Pressure} = \frac{\text{force}}{\text{Area}}$$

Dimension of force $= [M^1L^1T^{-2}]$

Dimension of Area $= [M^0L^2T^0]$

So,

$$\text{Pressure} = \frac{[M^1L^1T^{-2}]}{[M^0L^2T^0]}$$

Pressure $= [M^1L^{-1}T^{-2}]$

(ix) Density

from the definition,

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

Dimension of mass $= [M^1L^0T^0]$

Dimension of volume $= [M^0L^3T^0]$

So, Density $=$ ~~[M L M~~ $[M^1L^{-3}T^0]$

(x) Power

From the definition,

$$Power = \frac{work}{time\ taken}$$

Dimension of work $= [M^1L^2T^{-2}]$
Dimension of time $= [M^0L^0T^{1}]$

So,

$$Power = \frac{[M^1L^2T^{-2}]}{[M^0L^0T^1]}$$

$$Power = [M^1L^2T^{-3}]$$

(xi) Gravitational Constant

from Netwon law of Gravitation:

$$F = \frac{GMm}{r^2}$$

$$G = \frac{Fr^2}{Mm}$$

Dimension of force $= [M^1L^1T^{-2}]$
Dimension of radius $= [M^0L^1T^0]$
Dimension of M, m $= [M^1L^0T^0]$

$$G = \frac{[M^1L^1T^{-2}][M^0L^2T^0]}{[M^1L^0T^0][M^1L^0T^0]}$$

$$G = \frac{[M^1L^3T^{-2}]}{[M^2L^0T^0]}$$

$$G = [M^{-1}L^3T^{-2}]$$

(xii) Specific heat capacity.

from heat equation,

$$Q = msdt$$

$$\text{or } s = \frac{Q}{mdt}$$

Dimension of heat $= [M^1L^2T^{-2}]$
Dimension of Mass $= [M^1L^0T^0]$
Dimension of $dt = [M^0L^0T^0K]$

$$s = \frac{[M^1L^2T^{-2}]}{[M^1L^0T^0]\,K}$$

$$s = [M^0L^2T^{-2}K^{-1}]$$

(xiii) Thermal Conductivity:

for thermal conductivity, we can find the dimensional formula using the equation,

$$\frac{\Delta Q}{\Delta t} = KA \frac{\Delta T}{\Delta x}$$

$$K = \frac{\Delta Q}{\Delta t} \cdot \frac{\Delta x}{\Delta T} \cdot \frac{1}{A}$$

Dimension of energy = $[M^1L^2T^{-2}]$
Dimension of time = $[M^0L^0T^1]$
Dimension of length = $[M^0L^1T^0]$
Dimension of temperature = $[M^0L^0T^0K]$
Dimension of Area = $[M^0L^2T^0]$

$$K = \frac{[M^1L^2T^{-2}]}{[M^0L^0T^1]} \times \frac{[M^0L^1T^0]}{[M^0L^0T^0K]} \times \frac{1}{[M^0L^2T^0]}$$

$$= \frac{[M^1L^3T^{-2}]}{[M^0L^2T^1K]}$$

$$K = [M^1L^{1}T^{-3}K^{-1}]$$

(xiv) **Co-efficient of viscocity**

from the definition,

$$\text{Co-efficent of viscocity} = \frac{F.r}{A.v}$$

Dimension of force $= [M^1L^1T^{-2}]$
Dimension of radius $= [M^0L^1T^0]$
Dimension of Area $= [M^0L^2T^0]$
Dimension of velocity $= [M^0L^1T^{-1}]$

So,

$$\text{co-efficent of viscocity} = \frac{[M^1L^1T^{-2}][M^0L^1T^0]}{[M^0L^2T^0][M^0L^1T^{-1}]}$$

$$= \frac{[M^1L^2T^{-2}]}{[M^0L^3T^{-1}]}$$

$$= [M^1L^{-1}T^{-1}]$$

Dimensional equation

- Those equation which give the relation between fundamental unit & derived unit is called dimensional equation.

Application of Dimensional equation

① To check the correctness of a physical relation.

Check the relation $v^2 = u^2 + 2as$ by dimensional method?

Note: If dimension of LHS is equal to dimension of RHS the relation is dimensionally correct.

Solution,

$v^2 = u^2 + 2as$

Dimension of LHS = Dimension of v^2

$= [M^0L^1T^{-1}]^2$

$= [M^0L^2T^{-2}]$

Dimension of RHS = Dimension of $u^2 + 2as$

$= [M^0L^1T^{-1}]^2 + 2[M^0L^1T^{-2}][M^0L^1T^0]$

$= [M^0L^2T^{-2}] + 2[M^0L^2T^{-2}]$

$= 3[M^0L^2T^{-2}]$

Since, 3 is dimensionless constant. So, it is omitted.

$= [M^0L^2T^{-2}]$

Since, the dimension of LHS = dimension of RHS, So the relation is dimensionally correct.

Check the correctness of physical relation. $v = \sqrt{\frac{2GM}{R}}$

Dimension of LHS = Dimension of v
$= [M^0L^1T^{-1}]$

Dimension of RHS = Dimension of $\sqrt{\frac{2GM}{R}}$

$$= \sqrt{\frac{2[M^{-1}L^3T^{-2}][M^1L^0T^0]}{[M^0L^1T^0]}}$$

$$= \sqrt{2[M^0L^2T^{-2}]}$$

$$= [M^0L^1T^{-1}]\sqrt{2}$$

Since, $\sqrt{2}$ is dimensionless constent. So it's omitted.
$= [M^0L^1T^{-1}]$

Since, The dimension of LHS = dimension of RHS
Ther Relation is dimensionally correct.

To change the system of unit form one to another.

If M_1, L_1 and T_1 be the mass, length and time in SI system. n_1 be its number. If Q_1 be the physical quantity in SI system

$$Q_1 = n_1 [M_1^a L_1^b T_1^c]$$ {∵ where: a, b and c are constant}

Again, If Q_2 be the physical quantity in CGS system and n_2 be the number then,

$$Q_2 = n_2 [M_2^a L_2^b T_2^c]$$

If the physical quantity is identical then

$$Q_1 = Q_2$$

or $n_1 [M_1^a L_1^b T_1^c] = n_2 [M_2^a L_2^b T_2^c]$

or $n_1 = n_2 \left[\frac{M_2}{M_1}\right]^a \left[\frac{L_2}{L_1}\right]^b \left[\frac{T_2}{T_1}\right]^c$

Similary,

$$n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

Convert 5N into dynes

Soln

SI System	CGS System
$n_1 = 5$	$n_2 = ?$
$M_1 = 1\,kg$	$M_2 = 1\,gm$
$L_1 = 1\,m$	$L_2 = 1\,cm$
$T_1 = 1\,sec$	$T_2 = 1\,sec$

But, the dimension of newton is $[M^1L^1T^{-2}]$ - (i)
Also the dimension of SI system $(Q_1) = n_1[M_1^a L_1^b T_1^c]$ - (ii)

Comparing (i) and (ii) then

$a = 1,\ b = 1,\ c = -2$

As we know the relation

or $n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

or, $n_2 = 5 \left[\frac{1kg}{1gm}\right]^1 \left[\frac{1m}{1cm}\right]^1 \left[\frac{1sec}{1sec}\right]^{-2}$

or, $n_2 = 5 \left[\frac{1000\cancel{gm}}{1\cancel{gm}}\right] \times \left[\frac{100\cancel{cm}}{1\cancel{cm}}\right] \times 1$

or, $n_2 = 5 \times 10^5$ dynes

App Used to derive some physical relation.

Prove that centripetal force $(F) = \frac{mv^2}{r}$ by dimensional method.

Solution

The centripeted force depends upon

① mass

$f \propto m^a$ —(i)

② velocity

$f \propto v^b$ —(ii)

③ radius

$f \propto r^c$ —(iii)

Combining ①, ② and ③ then

or, $f \propto m^a v^b r^c$

or, $f = k m^a v^b r^c$ — (iv)

Writing down the dimension of LHS and RHS then

or, $[M^1L^1T^{-2}] = k[M^1L^0T^0]^a [M^0L^1T^{-1}]^b [M^0L^1T^0]^c$

or $[M^1L^1T^{-2}] = k[M]^a [L]^{b+c} [T]^{-b}$

Comparing the power of LHS and RHS then

$a = 1$	$b + c = 1$
$-b = -2$	$2 + c = 1$
$\Rightarrow b = 2$	$c = -1$

Putting the value of a, b, c in equation (iv)

$$F = K m^1 v^2 r^{-1}$$

$$F = \frac{K m v^2}{r}$$

In SI system $K = 1$

So,

$$F = \frac{m v^2}{r}$$

The density of Gold is 19.3 gm/cc express in SI system

SI system	CGS system
$n_1 = ?$	$n_2 = 19.3$
$M_1 = 1\ kg$	$M_2 = 1\ gm$
$L_1 = 1\ m$	$L_2 = 1\ cm$
$T_1 = 1\ sec$	$T_2 = 1\ sec$

But, the dimension of gm/cc is $[M^1 L^{-3} T^0]$ – (i)

Also the dimension of CGS system $(Q_2) = n_2 [M_2^a L_2^b T_2^c]$

Comparing (i) and (ii)

$a = 1,\ b = -3,\ c = 0$

As we know that,

$$n_1 = n_2 \left[\frac{M_2}{M_1}\right]^a \left[\frac{L_2}{L_1}\right]^b \left[\frac{T_2}{T_1}\right]^c$$

$$= 19.3 \left[\frac{1\,gm}{1\,kg}\right]^1 \left[\frac{1\,cm}{1\,m}\right]^{-3} \left[\frac{1\,sec}{1\,sec}\right]^0$$

$$= 19.3 \left[\frac{1\,\cancel{kg}}{1000 \times 1\,\cancel{kg}}\right] \left[\frac{1\,m}{1\,cm}\right]^3$$

$$= 19.3 \left[\frac{1}{1000}\right] \left[\frac{1\,\cancel{m}}{\frac{1}{100}\,\cancel{m}}\right]^3$$

$$= 19.3 \times \frac{1}{1\cancel{000}} \times 1000\cancel{000}$$

$$= 19300\ kg/m^3$$

Vector

Physical quantity are classifed as:-

① Scalar quantity

Those physical quantity which have magnitude only but no direction is called scalar quantity. They can be repre sented by simple english alphabet. They can be added, subracted, multipied or divided following the law of algebra. for eg:- mass, speed, work etc.

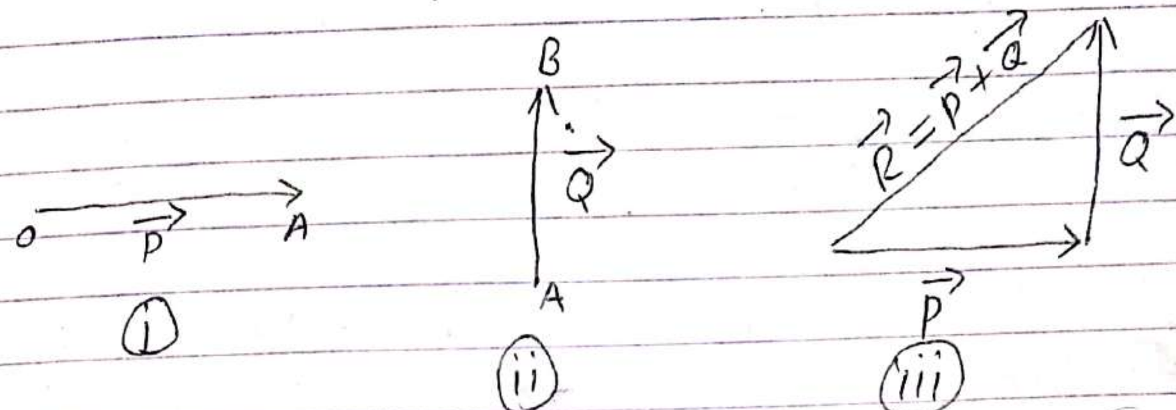
(II) Vector quantity

Those physical quantity which have magnitude well as direction is called vector quantity. They can be represente by bold english alphabet or by english alphabet with arrow on its arrow head.

They can't be added, subtracted, multipled according to the law of algebra but can be represented vectorically. for eg: displacement, velocity etc.

Vector addition

tail to tip method

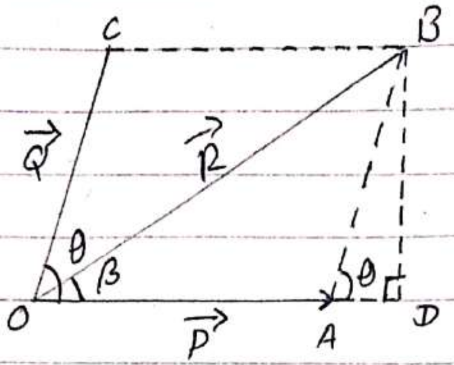


In this law of vector addition the tail of second vector is put upon the tip of first vector any order than the other line join the tail to tip in reverse order represents its addition and result.

Parallelogram law of vector addition

Statement

If two vectors active simultaneously of a point are represented in magnitude and direction by two side at a parallelogram then the third side of a starting from the same point are represent in magnetude and direction gives the resultant of these vector.



Let $\vec{P}$ & $\vec{Q}$ be two vectors starting from point O and $\vec{R}$ be its resultant. Here, θ be the angle between $\vec{P}$ and $\vec{Q}$ and β be the angle between $\vec{R}$ and $\vec{P}$.

In figure,

$OA = \vec{P}$
$OC = \vec{Q}$
$OB = \vec{R}$

Here the point A extended to meet the line coming from B and D.

In right angle triangle ΔOBD

or $OB^2 = OD^2 + BD^2$
or $OB^2 = (OA^2 + AD)^2 + BD^2$ —(i)

In Right angle triangle let $\triangle ABD$

or, $\sin\theta = \frac{BD}{AB}$

or, $BD = AB\sin\theta$

or $BD = \vec{Q}\sin\theta$ — (ii)

Also,

or, $\cos\theta = \frac{AD}{AB}$

or, $AD = AB\cos\theta$

or $AD = \vec{Q}\cos\theta$ — (iii)

from eqⁿ (i), (ii) and (iii) then

or, $R^2 = (\vec{P} + \vec{Q}\cos\theta)^2 + (\vec{Q}\sin\theta)^2$.

or, $R^2 = (\vec{P})^2 + 2\vec{P}\vec{Q}\cos\theta + (\vec{Q}\cos\theta)^2 + (\vec{Q}\sin\theta)^2$

or $R^2 = P^2 + 2\vec{P}\vec{Q}\cos\theta + (\vec{Q})^2(\cos^2\theta + \sin^2\theta)$

or $R^2 = P^2 + 2\vec{P}\vec{Q}\cos\theta + Q^2$

or $R^2 = P^2 + Q^2 + 2\vec{P}\vec{Q}\cos\theta$

or, $\vec{R} = \sqrt{P^2 + Q^2 + 2\vec{P}\vec{Q}\cos\theta}$ — (iv)

This eqⁿ (iv) gives the resultant of those vectors for direction;

or, $\tan\beta = \frac{BD}{OD}$

or $\tan\beta = \frac{BD}{OA + AD}$

or, $\tan\beta = \frac{\vec{Q}\sin\theta}{(\vec{P} + \vec{Q}\cos\theta)}$

This gives the direction between two vectors.

Special cases

Case 1

If $\theta = 0°$ ie two vectors are moving in the same direction, then

or $\vec{R} = \sqrt{P^2 + Q^2 + 2\vec{P}\vec{Q}\cos 0°}$

or $\vec{R} = \sqrt{P^2 + Q^2 + 2\vec{P}\vec{Q}}$

or $\vec{R} = \sqrt{(\vec{P} + \vec{Q})^2}$

or $\vec{R} = \vec{P} + \vec{Q}$

Case 2

If $\theta = 90°$ ie two vectors are movingp in perpenidicular direction

or, $\vec{R} = \sqrt{P^2 + Q^2 + 2\vec{P}\vec{Q}\cos 90}$

or, $\vec{R} = \sqrt{P^2 + Q^2 + 0}$

or, $\vec{R} = \sqrt{P^2 + Q^2}$

Case 3

If $\theta = 180°$ ie. two vectors are moving in opposite direction.

$\vec{R} = \sqrt{P^2 + Q^2 + 2\vec{P}\vec{Q}\cos 180}$

$\vec{R} = \sqrt{P^2 + Q^2 - 2\vec{P}\vec{Q}}$

$\vec{R} = \sqrt{(\vec{P} - \vec{Q})^2}$

$\vec{R} = \vec{P} - \vec{Q}$

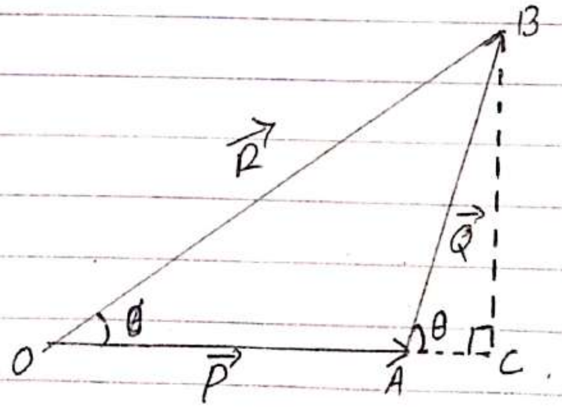
Triangle law of vector addition

Statement of Triangle law

If 2 vectors acting simultaneously on a body are represented both in magnitude and direction by 2 sides of a triangle taken in an order then the resultant (both magnitude and direction) of these vectors is given by 3rd sides of that triangle taken in opposite order.

Derivation of the law

Consider two vectors $\vec{P}$ and $\vec{Q}$ acting on a body and represented both in magnitude and direction by sides OA and AB respectively of a triangle OAB. Let θ be the angle between $\vec{P}$ and $\vec{Q}$. Let $\vec{R}$ be the resultant of vectors $\vec{P}$ and $\vec{Q}$. Then, according to triangle law of vector addition, side OB represents the resultant of $\vec{P}$ and $\vec{Q}$.



So, ~~with~~ we have,

$$\vec{R} = \vec{P} + \vec{Q}$$

Now, expand A to C and draw BC perpendicular to OC.

From triangle OCB,

or $OB^2 = OC^2 + BC^2$

or $OB^2 = (OA + AC)^2 + BC^2$ .... (i)

In triangle ACB,

$$\cos\theta = \frac{AC}{AB}$$

or, $AC = AB\cos\theta$

or, $AC = \vec{Q}\cdot\cos\theta$

Also,

$$\sin\theta = \frac{BC}{AB}$$

or. $BC = AB\sin\theta$

or, $BC = \vec{Q}\sin\theta$

Magnitude of resultant:

Subsituting value of AC and BC in (i), we get,

$$\vec{R}^2 = (\vec{P} + \vec{Q}\cos\theta)^2 + (\vec{Q}\sin\theta)^2$$
$$R^2 = (\vec{P})^2 + 2.\vec{P}.\vec{Q}\cos\theta + (\vec{Q}\cos\theta)^2 + (\vec{Q}\sin\theta)^2$$
$$R^2 = P^2 + 2\vec{P}.\vec{Q}\cos\theta + (\vec{Q})^2(\cos^2\theta + \sin^2\theta)$$
$$R^2 = P^2 + 2\vec{P}.\vec{Q}\cos\theta + Q^2$$
$$\vec{R} = \sqrt{P^2 + Q^2 + 2\vec{P}.\vec{Q}\cos\theta}$$

which is the magnitude of resultant.

Direction of resultant:

Let θ be the angle made by resultant R with P. Then,

or $$\tan\theta = \frac{BC}{OC} = \frac{BC}{OA + AC}$$

or $$\tan\theta = \frac{\vec{Q}\sin\theta}{\vec{P} + \vec{Q}\cos\theta}$$

which is the direction of resultant.

Multiplication of vector

Multiplication of vector can be classifed.

① **Scalar or dot product:**

The multiplication between two vectors which gives scalar quantity is called scalar product. In other words, the scalar or dot product of magnitude of two vectors and cosine of the angle between them. If $\vec{A}$ and $\vec{B}$ be two vectors having magnitide A and B and If θ be the angle between them the scalar or dot product between $\vec{A}$ and $\vec{B}$ is:

$$\vec{A}\,\vec{B} = |A|\,|B|\cos\theta$$

Properties of Scalar product :-

(i) In satisfy commutative law

$$\vec{A}.\vec{B} = \vec{B}.\vec{A}$$

(ii) In satisfy distribute law

$$\vec{A}.(\vec{B}+\vec{C}) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$$

(iii) It is maximum when $\theta = 0°$

$$\vec{A}.\vec{B} = |A|\,|B| = AB$$

(iv) It is minimum when $\theta = 90°$

$$\vec{A}.\vec{B} = 0$$

(v) The scalar product of a vector with itself is the square of the magnitude of given vector.

$$\vec{A}.\vec{A} = A^2$$

(vi) $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$

and $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

Vector or Cross product

The multiplication between two vectors which gives a vector quantity is called vector or cross product.

In other words:

It is define as the product of magnitude of two vectors and sin of the angles between them.

If $\vec{A}$ and $\vec{B}$ are two given vectors having magnitude A and θ be the angle between them, then the cross or vector product between two vectors is given by:

$$\vec{A} \times \vec{B} = |A|\,|B| \sin\theta\, \hat{n}$$

when ; $\hat{n}$ is the unit vector ~~period~~ ~~perpeniducal~~ perpendicular to both A and B and the direction of $\hat{n}$ is given by Right hand screw rule.

Properties of vector or cross product:

① It does not obey commutative law

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

But; $|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}|$

⑪ It obeys distributive law.

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

(iii) The ~~vet~~ vector product of a vector with itself become zero.

ie. $\vec{A} \times \vec{A} = 0$

(iv) It is maximum when $\theta = 90°$

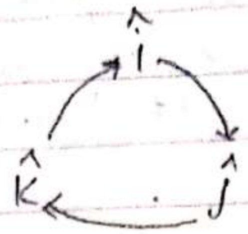
ie. $\vec{A} . \vec{B} = AB$

(v) It is minimum when $\theta = 0°$ or $180°$

ie. $\vec{A} \times \vec{B} = 0$

(vi) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$

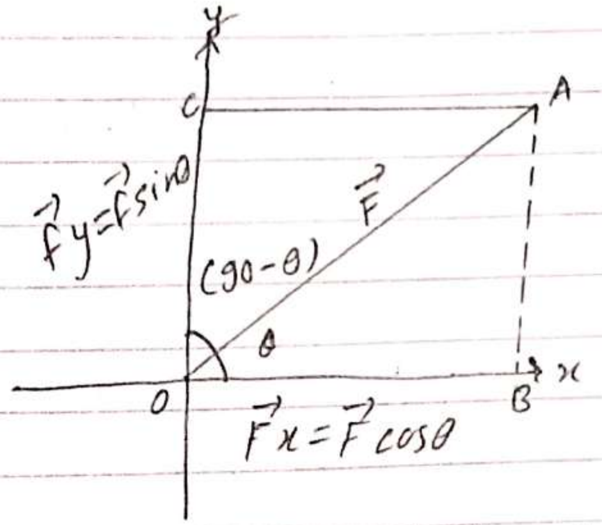
But $\hat{i}\times\hat{j} = \hat{k}$
$\hat{j}\times\hat{k} = \hat{i}$
$\hat{k}\times\hat{i} = \hat{j}$



Also $\hat{j}\times\hat{i} = -\hat{k}$
$\hat{k}\times\hat{j} = -\hat{i}$
$\hat{i}\times\hat{k} = -\hat{j}$

Resolution of vectors:-

The process of splitting of vectors into its components is resolution of vectors.



Let $\vec{F}$ be the resultant vector which is divided into two components ~~that itself~~ ie $\vec{F}x$ along the x direction and $\vec{F}y$ along the y direction

In Right angle triangle OAB then

$$\cos\theta = \frac{OB}{OA}$$

or $OB = OA\cos\theta$

or $\vec{F}x = \vec{F}\cos\theta$

Similarly in Right angle triangle OAC then

$$\cos(90^\circ - \theta) = \frac{OC}{OA}$$

or $\sin\theta = \frac{OC}{OA}$

or, $OC = OA\sin\theta$

or, $\vec{F}y = \vec{F}\sin\theta$

Can the sum of two equal vectors be equal to either of this vectors.

Let $\vec{P}$ and $\vec{Q}$ be two vectors and $\vec{R}$ be its resultant vectors. Then from parallelogram law of vector addition.

$$R^2 = P^2 + Q^2 + 2\vec{P}\vec{Q}\cos\theta$$

By question

$$\vec{R} = \vec{P} = \vec{Q}$$

$$R^2 = P^2 = Q^2$$

or: $P^2 = P^2 + P^2 + 2P^2 \cos\theta$

or $\cancel{P^2} = \cancel{P^2}(1+1+2\cos\theta)$

or $1 = 2+2\cos\theta$

or $1 = 2(1+\cos\theta)$

or $\frac{1}{2} = 1+\cos\theta$

or $\cos\theta = \frac{1}{2} - 1$

or, $\cos\theta = -\frac{1}{2}$

$\cos\theta = \cos 120^\circ$

$\theta = 120^\circ$

If the scalar product between two vectors is equal to the magnitude of vector ~~triangle~~ product what is the angle between?

or $\vec{A}\cdot\vec{B} = AB\sin\theta$

or $\cancel{AB}\cos\theta = \cancel{AB}\sin\theta$

or $1 = \frac{\sin\theta}{\cos\theta}$

or $1 = \tan\theta$

or $\tan\theta = \tan 45^\circ$

or $\theta = 45^\circ$

Equation of Motion:

A. body moves with intial velocity (u) accelerate with an acceleration (a) and then the equation of motion are:

① $v = u + at$

② $s = ut + \frac{1}{2}at^2$

③ $v^2 = u^2 + 2as$

* **Effection of motion under gravity**

a) When a body moves under the action of gravity then the eq^n of motion changes as:

① $v = u + gt$

② $h = ut + \frac{1}{2}gt^2$

③ $v^2 = u^2 + 2gh$

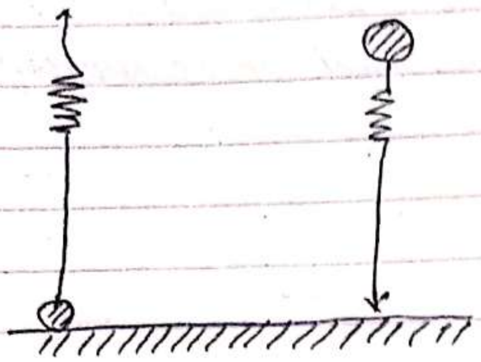
b) for the body projected against the gravity, then the eq^n of motion changes as:

① $v = u - gt$

② $h = ut - \frac{1}{2}gt^2$

③ $v^2 = u^2 - 2gh$

Is it possible that the acceleration is not zero but the velocity become zero? Explain



Yes, it is possible that the acceleration is not zero but the velocity became zero. It can be explained by two cases.

Case 1

When the body is projected upward, at first the velocity of a body goes on discreasing but when the body reaches the maximum height the velocity of a body becomes zero but it acceleration is not equal to zero and equal to the acceleration due to gravity ie $a = -g$.

Case 2

Similarly, when the body is drop from the height (h), at intially its intial velocity is zero but the acceleration of the body be equal to g.

Q A stone is droped from the top of a cliff and one second later, a second stone is dropped vertically downward with the velocity of 20 ms^{-1}, How far below the top of cliff will second stone pass the first?

Soln

for the first stone

initial velocity $(u) = 0\ ms^{-1}$
acceleration $(a) = 10\ ms^{-2}$
time taken $(T) = t$
let x be the height at which the 2nd stone pass the 1st stone.

from eqn of motion;

$$h = ut + \frac{1}{2}gt^2$$

$$x = 0 \times t + \frac{1}{2}gt^2$$

$$x = \frac{1}{2} \times 10 \times t^2$$

$$x = 5t^2 \quad -(i)$$

for 2nd stone

initial velocity $(u) = 20ms^{-1}$
acceleration $(a) = 10ms^{-1}$
time taken $(T) = (t-1)$ sec

from eqn of motion;

or $h = ut' + \frac{1}{2}gt'^2$

or $x = u(t-1) + \frac{1}{2}g(t-1)^2$

or $x = 20(t-1) + 5(t-1)^2$

or $x = 5(t-1)\{4 + t - 1\}$

or $x = 5(t-1)(t+3)$

or $\cancel{5}t^2 = \cancel{5}(t-1)(t+3)$ $[\because x = 5t^2$ from eqn (i)$]$

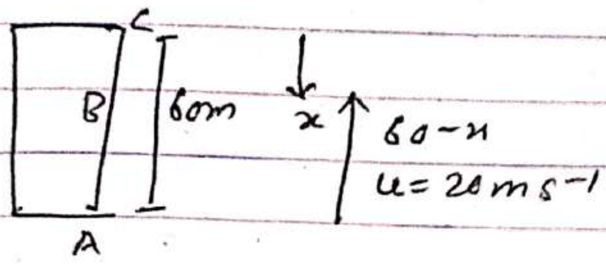
or $\cancel{t^2} = \cancel{t^2} + 3t - t - 3$
or $0 = 2t - 3$
or $3 = 2t$
or $\frac{3}{2} = t$

or $t = 1.5$ sec

Putting t value in eqn (i)

$x = 5 \times (1.5)^2$
$= \cancel{5 \times 3 = 15\,m}$ $5 \times 2.25 = 11.25\,m$

(Q) A stone is dropped from the top of the tower 60m height at the same time another body it project vertically upward from the ground with the velocity of $20ms^{-1}$, find when and where will they meet togther?



let x be the from point C at which stone meet together

for the stone projected downward;

initial velocity $(u) = 0ms^{-1}$
acceleration $(a) = 10ms^{-2}$

from eqⁿ of motion

or $h = ut + \frac{1}{2}gt^2$

or $x = 0 \times t + \frac{1}{2}gt^2$

or $x = 5t^2$ —(i)

for the stone projected upward;

initial velocity = $20 ms^{-1}$
acceleration = $-10 m/s^{-2}$

from eqn the eqn of motion

or $h = ut + \frac{1}{2}gt^2$

or $60 - x = 20t - 5t^2$

or $60 - \cancel{5t^2} = 20t - \cancel{5t^2}$ [$\because x = 5t^2$ from eqn (i)]

or $\frac{60}{20} = t$

or $t = 3$ sec

Putting t in eqn (i) then

$x = 5 \times 3^2$
$= 5 \times 9$
$= 45 m$ below the top.

Laws of Motion

force:

The pull or push or lift or any external agency which tend to change the state of rest or motion of body is called force.

* I^{st} law of motion:

It states that, "Every body in the universe tends to come in the state of rest or motion ~~at~~ unless any external force is applied on it." It is also called law of inertia.

for eg:

① An athletic runs for a certain distance before taking a long jump.

② The leaves of tree fall down when it is shaken

* Momentum

The product of mass and velocity of a moving body is called momentum. It is vector quantity and SI unit is $kg\,ms^{-1}$ and its dimension is $[MLT^{-1}]$. If m be the mass and 'v' be the velocity of moving body, then the momentum is defined by,

$$\boxed{P = mv}$$

for example:

(i) A cricketer prefer heavy bat to get more momentum.

(ii) A person gets hurt when he hit the hugh rock.

② Newton second law of motion

It states that, "The rate of change in momentum of a body is directly proportional to the force applied on the body."

If P is the momentum applied on a body at a time (t), then the rate of change in momentum be $\frac{dP}{dt}$ and if 'F' be the force ~~appit~~ applied on that body, then according to Newton 2nd law of motion;

or, $\frac{dP}{dt} \propto f$

or, $f \propto \frac{dP}{dt}$

or, $F = \frac{k\,dP}{dt}$

Where, k = proportinallity constant

If "m" be the mass and "v" be the velocity of moving body momentum (P) = mv such that

$$F = \frac{k\,d(mv)}{dt}$$

Keeping mass constant; then

$$F = \frac{k\,m\,dv}{dt}$$

$$F = kma$$

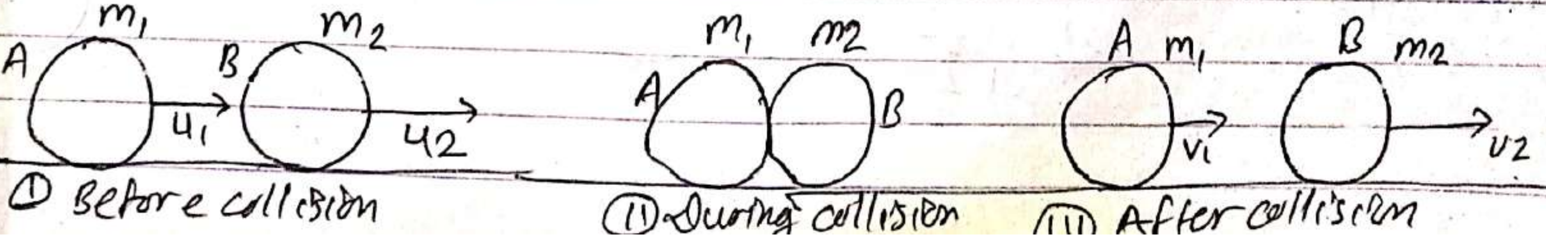
Where: a = acceleration produced in moving body
In SI system; k = 1 so; the eqn ① transforms to

$$F = ma$$

This is called Newton 2nd law of motion.

Principle of conversation of Linear momentum

It states that if no externel force is appilied on a body then total linear momentum before collision equal to total linear momentum after collision.



① Before collision (II) During collision (III) After collision

Let us consider two bodies A and B having masses m_1 and m_2 moves with initial velocities u_1 and u_2 as shown in figure (i) after sometime, these two bodies collide with one another as shown in ~~bodies~~ figure (ii). At last, after collision, two bodies moves with final velocity v_1 and v_2 as shown in figure (iii).

Initial momentum of body A $= m_1 u_1$
Initial momentum of body B $= m_2 u_2$
final momentum of body A $= m_1 v_1$
final momentum of body B $= m_2 v_2$

Change in momentum of body A $= m_1 v_1 - m_1 u_1$
$= m_1 (v_1 - u_1)$

Similarly change in momentum of body B $= m_2 v_2 - m_2 u_2$
$= m_2 (v_2 - u_2)$

If F_A be the force exerted by body A on B and F_B be the force exerted by body B on A.

Then: $F_A = \dfrac{\text{mass of B} \times \text{change in velocity}}{\text{time taken}}$

$= \dfrac{\text{change in momentum of Body B}}{\text{time}}$

$= \dfrac{m_2 (v_2 - u_2)}{\Delta t}$ —— (a)

Similarly; $F_B = \frac{m_1(v_1 - u_1)}{\Delta t}$ — (b)

From Newton 3rd law of motion;

or $F_A = -F_B$

or $\frac{m_2(v_2 - u_2)}{\Delta t} = \frac{m_1(v_1 - u_1)}{\Delta t}$

or $m_2v_2 - m_2u_2 = -m_1v_1 + m_1u_1$

or $m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$ — (c)

The eqn (c) gives the relation for the principle of conservation of linear momentum.

* Do you know?

If two bodies m_1 and m_2 stick / ~~coll~~ coalese/move together then the final velocity $v_1 = v_2 = v$ such that from the principle of ~~conser~~ conservation of linear momentum, then

or $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

or $m_1u_1 + m_2u_2 = m_1v + m_2v$ $[\because v_1 = v_2 = v]$

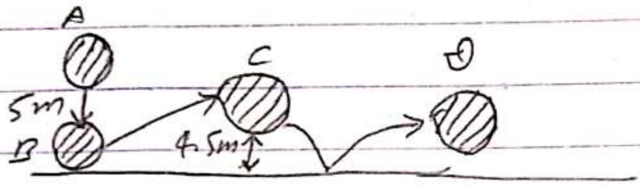
or $v(m_1 + m_2) = m_1u_1 + m_2u_2$

or $v = \frac{(m_1u_1 + m_2u_2)}{(m_1 + m_2)}$

This expression give the idea for common velocity (v) if two bodies move together.

* Numerical zone :-

① A rubber ball of mass 400 gm falls from a height of 5m and rebounds to a height of 4.5m find the impulse and average force between the ball and ground if the time during which they are in contact was 0.2 secs.



Solution

Case I for the body dropped:-

mass of ball (m) = 400 gm = 0.4 kg
initial velocity (u) = 0 ms^{-1}
height (h) = 5m
acceleration (a) = g = 10 ms^{-2}
final velocity (v) = ?

we have;

or $v^2 = u^2 + 2as$

or $v^2 = 0^2 + 2 \times 10 \times 5$

or $v^2 = 100$

or $v^2 = 10^2$

or $v = 10 ms^{-1}$

Case II : After rebounding

Final velocity $(v) = 0$
initial velocity $(u) = ?$
height attained $(h) = 4.5m$
acceleration due to gravity $= -10 m/s^2$

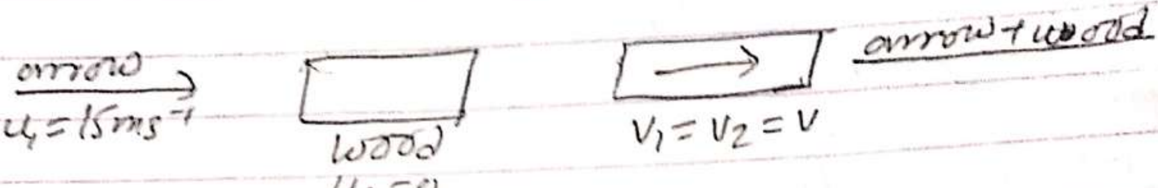
We know that,
or $v^2 = u^2 + 2ah$
or $0^2 = u^2 - 2 \times 10 \times 4.5$
or $0 = u^2 - 90$
or $90 = u^2$
or $u = \sqrt{90}$
or $u = 9.48\ ms^{-1}$

Now, $F = ma$
$= \frac{m(v-u)}{\Delta t}$
$= \cancel{mt}\ \frac{\cancel{0.4}^{2} \times (10 - 9.48)}{\cancel{0.2}}$
$= 2 \times 0.52$
$= 1.04 N$

~~Imp~~ Impulse $= F \times \Delta t$
$= 1.04 \times 0.2$
$= \cancel{2.08}\ 0.208\ kgm/s$

⊛ An arrow of mass 100 gm each shot into a block of wood of mass 400 gm lying at rest on the smooth surface. If the arrow is travelling horitizonally at the velocity of $15 ms^{-1}$ calculate the common velocity after impact.

Solution,



for arrow

$u_1 = 15 ms^{-1}$

$v_1 = v$

$m_1 = 0.1 kg$

for wood

$u_2 = 0$

$v_2 = v$

$m_2 = 0.4 kg$

From the principal of conversation of linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_1 u_1 + m_2 u_2 = m_1 v + m_2 v$$

$$v = \frac{(m_1 u_1 + m_2 u_2)}{(m_1 + m_2)}$$

$$v = \frac{(0.1 \times 15 + 0.4 \times 0)}{(0.1 + 0.4)}$$

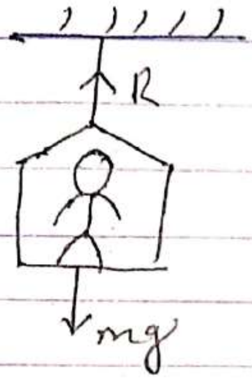
$$v = \frac{1.5}{0.5}$$

$$v = 3 ms^{-1}$$

Application of Newton 2nd law of motion:-

⊛ There are different cases that can be studied in free body diagram of lift.

① **When lift is at rest**

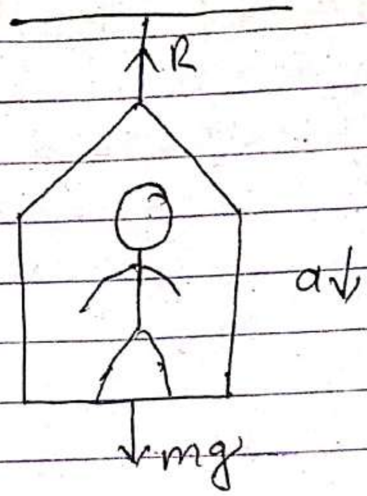


Suppose a man having mass (m) standing upon the lift which is at rest Here; the net force acting upon him is zero
i.e. $\Sigma F = 0$

If R be the normal reaction of rope acting upon lift, then under ~~stand~~ sustained equilibrium;

$$\text{or } R + (-mg) = 0$$
$$\text{or, } R - mg = 0$$
$$\text{or } R = mg$$

① **When the lift is accelerating downward**



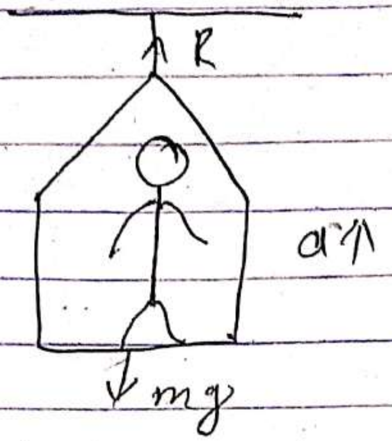
Suppose a man having mass (m) standing upon the lift which is accelerating downward with an acceleration (a), then from Newton 2nd law

$$\Sigma f = ma$$

Under sustained equilibrium, then

or $mg - R = ma$
or $R + ma = mg$
or $R = mg - ma$
or $R = m(g-a)$

(iii) When the lift is accelerating upward.



Suppose, a man having mass standing upon the lift which is accelerationg upward with an acceleration (a), then from Newton 2nd law of motion;

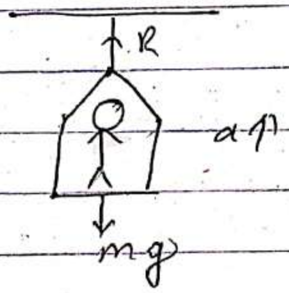
$$\Sigma F = ma$$

Under sustained equilibrium, then

$$R - mg = ma$$
$$\text{or, } R = ma + mg$$
$$\text{or, } R = m(g+a)$$

Q A lift moves up with a constant acceleration of $2 ms^{-1}$. Calculate the reaction of the floor on man of mass 50 kg standing in the lift. ($g = 9.8 ms^{-2}$)

Solution,

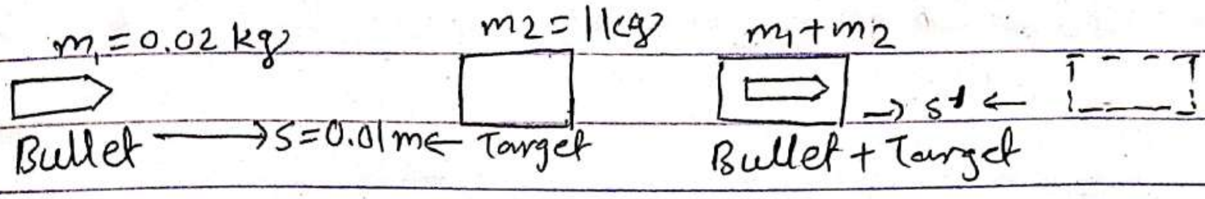


mass of man $(m) = 50$ kg
acceleration $(a) = 2 ms^{-2}$
acceleration due to gravity $(g) = 9.8 ms^{-2}$

For the lift accelerationg upward; then

$$R = m(g+a)$$
$$= 50(9.8+2)$$
$$= 50 \times 11.8$$
$$= 590 \text{ N}$$

Q A bullet of mass 0.02kg moving with a velocity of $100ms^{-1}$ is stop within 0.1m of target. find the average distance afforded by the target. (mass of target = 1 kg)



Solution,

initial velocity (u) = $10 ms^{-1}$
final velocity (v) = $0 ms^{-1}$
distance (s) = 0.1 m
we have;
$v^2 = u^2 + 2as$

or $\cancel{0^2 = 100^2 + 2a \cdot 0.1}$ $\cancel{2 \times 0.1}$ $0^2 = 100^2 + 2a \times 0.1$

or $\dfrac{-10000}{2 \times 0.1} = a$

or $a = -50000 m/s^2$

Also; from the principle of conservation of Linear momentum;
$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$

or $0.02 \times 100 + 1 \times 0 = 0.02 \times V + 1 \times V$

or $0.02 \times 100 = \cancel{V(1.02)}\ (1.02)V$

or $V = \dfrac{0.02 \times 100}{1.02}$

or $V = 1.96\ ms^{-1}$

Also; $v^2 = u^2 + 2as$

or $0^2 = 1.96^2 + 2 \times (-50000)s'$

or $2 \times 50000 s' = 1.96^2$

or $s' = \dfrac{3.84}{100000}$

$s' = 3.84 \times 10^{-5} m$

Rest — quiscent / Stillness / lacked aisical — static

Friction

The constant oppsing force produce on a body when a body slide ~~on~~ over the surface of another body. It is produce during to presence of rubness between two surfaces.

According to law of friction, the frictional force produce between two bodies is directly proportional to the normal reaction produce between them.

i.e. $F_r \propto R$

where F_r = frictional force

R = normal reaction

Types of friction

① ~~Stati~~ Static friction

The friction that appear in a body when the body is at rest is called static friction. And the force produce due to static friction is called force of static friction. And the force due to static friction is given by.

$$F_S = \mu_S R$$

where: μ_S = coefficient of static friction

Kinetic ~~Motion~~ friction

The friction that apper in a body when the body is at motion is called kinetic friction. And the force produce due to kinetic friction is called force of kinetic friction. And the force due to kinetic friction is given by

$$F_K = \mu_K R$$

where, μ_K = coefficent of kinetic friction.

Rolling friction

The friction that apper in a body when the body is at rolling state is called rolling friction. And the force produe due to rolling friction is called force of rolling friction. And the force due to rolling friction is given by,

$$F_R = \mu_R R$$

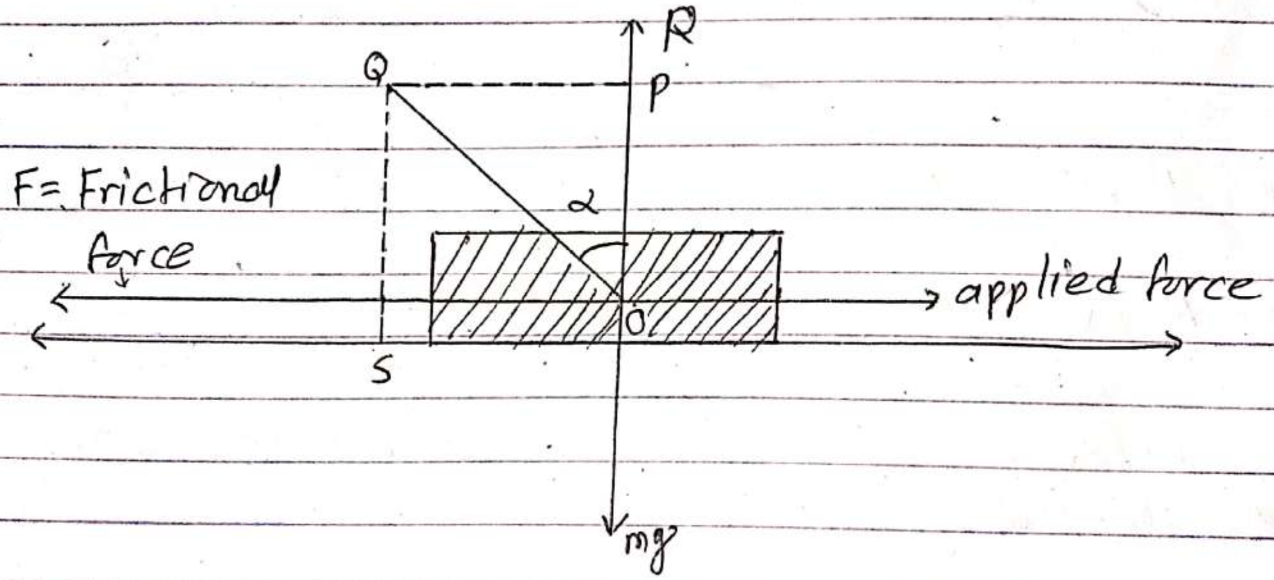
where, μ_R = coefficent of rolling friction.

* Co-efficient of friction:

The Ratio of frictional force to the normal reaction acting upon the body is called co-efficient of friction. It is denoted by μ and is defined by;

$$\mu = \frac{\text{frictional force}}{\text{Normal reaction}}$$

* Angle of friction:



The angle made by the resultant of frictional force with the normal reaction is called angle of reaction.

If 'R' denotes the normal reaction acting upon a body and 'F' denotes the frictional force which acts in the opposite direction of applied. Let α be the angle made by the resulant with the normal reaction (R).

Then, from figure,

In $\triangle OQP$

$\tan\alpha = \frac{PQ}{OP}$

$\tan\alpha = \frac{F}{R}$ ——— (1)

$\alpha = \tan^{-1}\left(\frac{F}{R}\right)$ ——— (11)

The eqn (11) gives the value of angle of friction.

From the definition of co-efficent of friction, then

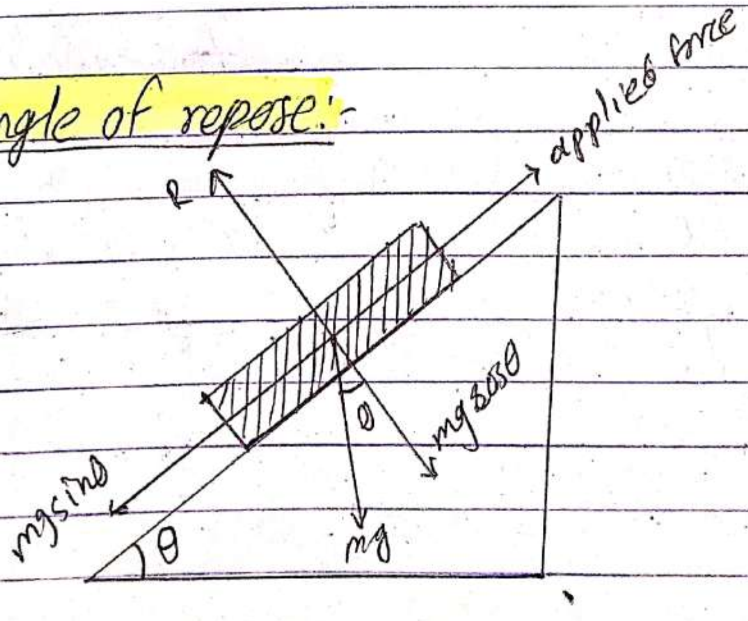
$\mu = \frac{F}{R}$ ——— (111)

From eqn (1) and (111) then

$\mu = \tan\alpha$

Thus, the coffêcient of friction is equal to the tangent of angle of friction.

Angle of repose:-



The angle made by an inclined plane with the horitzontal surface such that a body placed on this inclined plane just begin to slide is called angle of repose.

In given figure; R denotes the normal reaction, F be the limiting frictional force and θ be the angle made by the resultant with the normal reaction.

The normal reaction balanced the weight of the body which is represented by the component of force which is given by;

$$R = mg\cos\theta \quad \text{---} (1)$$

And the limiting frictional force (F) ~~at~~ acts in the opposite direction of applied force & is represented by;

$$F = mg\sin\theta \quad \text{---} (11)$$

Dividing eqn (11) by (1) then

$$\frac{F}{R} = \frac{\cancel{mg}\sin\theta}{\cancel{mg}\cos\theta}$$

$$\frac{F}{R} = \tan\theta$$

$$\tan\theta = \frac{F}{R} \quad \text{---} (111)$$

$$\theta = \tan^{-1}\left(\frac{F}{R}\right) \quad \text{---} (IV)$$

The eqn (iv) gives the value of angle of repose from the definition of coefficent of friction;

$$\mu = \frac{F}{R} \quad \text{--- (v)}$$

from eqn (iii) and (v) then

$$\mu = \tan\theta \quad \text{--- (vi)}$$

Thus, the ~~cof~~ coefficient of friction is equal to tangent of angle of repose.

Also, from the relation of coefficent of friction and angle of friction, then

$$\mu = \tan\alpha \quad \text{--- (vii)}$$

from eqn (vi) and (vii) then

$$\tan\theta = \tan\alpha$$
$$\theta = \alpha$$

Hence, the angle of repose is equal to angle of friction.

⊛ law of friction

(i) Frictional force acts in the opposite direction of motion.

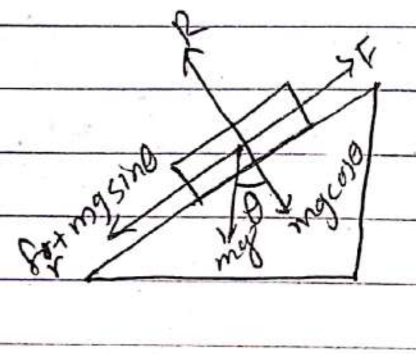
(ii) If normal reaction is constant, The frictional force acting upon at two surface is independent with the area at constant

(iii) Frictional force is directional proportional to the normal in static condition.

Do you know?

A. Case of Inclined plane.

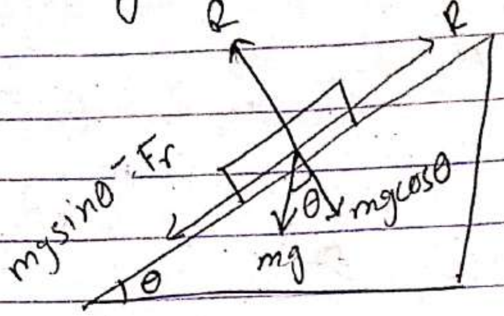
(i) When a body moves ups in an inclined plane



When a body moves up in an inclined plane; then total force (F) $= f_r + mg\sin\theta$

$$or,\ F = \mu R + mg\sin\theta$$
$$or,\ F = \mu mg\cos\theta + mg\sin\theta$$

(11) When a body move down in an inclined plane



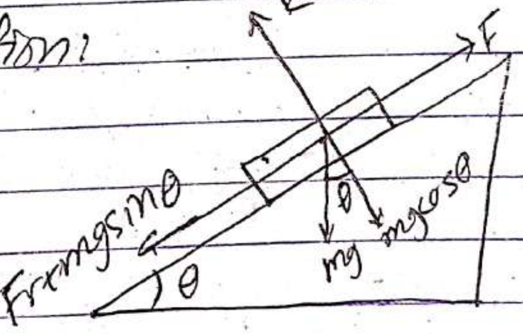
When a body moves down in an inclined plane; then total force $(F) = mg\sin\theta - f_r$

$$F = mg\sin\theta - \mu R$$
$$F = mg\sin\theta - \mu mg\cos\theta$$

Numerical

A slap of mass 10 kg is lie on plane inclined at 30° to the horizontal. find the least force which will pull the slab of upward. (Coefficient of friction = 0.2 and $g = 9.8\ m/s^2$

Solution:



mass (m) = 10 kg

coefficient of friction (μ) = 0.2

angle of repose (θ) = 30°

$g = 9.8\ ms^{-1}$

Least force which pull the slab upward is;

$F = F_r + mg\sin\theta$

or $F = \mu R + mg\sin\theta$

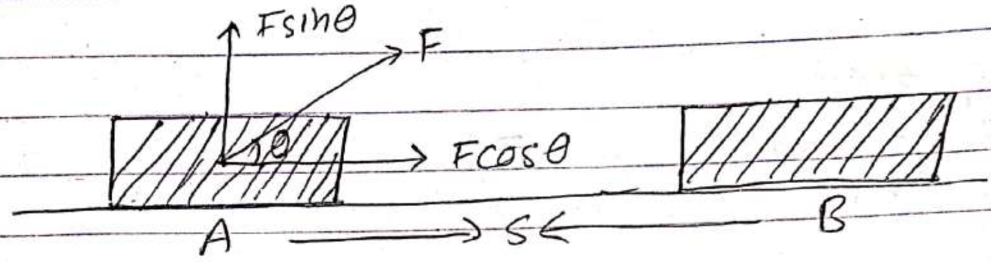
or $F = \mu mg\cos\theta + mg\sin\theta$

or $F = 0.2 \times 10 \times 9.8 \times \cos 30^\circ + 10 \times 9.8 \times \sin 30$

or $F = 65.68\ N$

WORK, ENERGY & POWER

Work: When certain force is applied on a body such that it cover or certain distance then work is said to be done. It is scalar quantity. It unit ~~in SI~~ Joule in SI sytem and ERG in CGS system. It dimension is $[M^1L^2T^{-2}]$.



Let, a body be displaced from position A to position B in horizontal direction by making an angle θ to the horizontal than the work done is given by:

Work done(W) = component of force along (AB) x displacement(AB)

$= F\cos\theta \times S$

$= FS\cos\theta$

$= \vec{F}.\vec{S}$

Therefore, work is also defined as the dot product of force and displacement so it is a scalar quantity.

It has three cases:

Case I

When θ is $0°$

then $W = FS\cos 0° = FS$

This means that work done is maximum when Force is applied in the same direction of displacement.

Case II: When θ is $90°$.

Then $W = FS\cos 90 = 0$

This means that work done is minimum when force is applied perpendicularly.

Case III:

When θ is $180°$.

Then $W = FS\cos 180 = -Fs$

This means that work done is negative when force is appiled in the opposite direction of displacement.

Energy

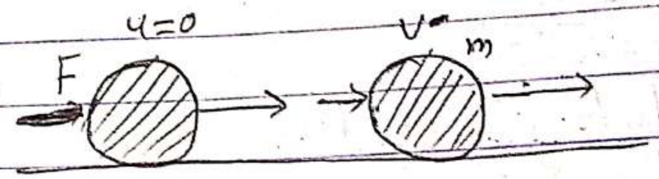
The capacity of doing work is called Energy. It is a scalar quantity. Its SI unit is Joule and CGS is ERG. It dimension is $[M^1L^2T^{-2}]$. Energy can be exist in various forms such as:

③ Mechanical, light, chemical, nuclear, heat, sound and so on.

Mechanical energy = The sum of kinetic energy and potential energy is called mechanical energy.
The kinetic energy and potential energy is explain below:

KINETIC ENERGY

Suppose a body of mass 'm' be start moving on a rough surface. Let the body has velocity 'v' at covering distance 's'. If 'a' be the acceleration produced in the body then, from equation of motion, we have:



$$v^2 = u^2 + 2as$$
$$v^2 = 0^2 + 2as$$
$$v^2 = 2as$$
$$s = \frac{v^2}{2a}$$

The K.E, orignally possessed by the body is equal to the work done against F, and hence,

or. $KE = F \times S$

or. $KE = m\cancel{a} \times \frac{v^2}{2\cancel{a}}$

$\therefore K.E. = \frac{1}{2} mv^2$

Potential Energy

Potential energy of a body is the work done to take the body to that height. Suppose a body of mass 'm' is raised to a distance 'h'. The force acting on a body is the gravitational pull of the earth (i.e) m*g which acts down ward to lift body above surface area of the earth. In this condition, we have to do the work aganist gravity.

$E_p = F * S$
$E_p = mg * h$
$E_p = mgh$

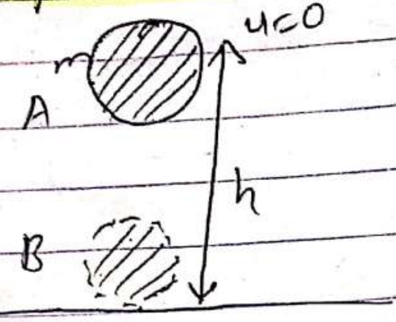
Hence, the work is stored up in the form of potential Energy.

Alternative method

The energy possessed by a body due to its position is called potential energy.

For example : (i) Energy of water in dam
(ii) Energy ~~of~~ due to stretched string.

* Measurement of potential energy:-



Let us consider a body having mass (m), initially at resut at height (h). When the body falls down it gains kinetic energy.

ie. $E_k = \frac{1}{2} mv^2$ — ①

under the condition

initial velocity (u) = 0
final velocity = v
acceleration due to gravity (a) = g
distance covered (s) = h

from the eqn of motion; then

or $v^2 = u^2 + 2as$

or $v^2 = 0^2 + 2gh$

or $v^2 = 2gh$ — ①

When the body reaches the ground, its kinetic energy is turned to potential energy.

i.e. $\varepsilon_k = \varepsilon_p$

$\varepsilon_p = \frac{1}{2} mv^2$

Thus, from eqn (11) then

or, $E_p = \frac{1}{2} m\, 2gh$

or, $E_p = mgh$

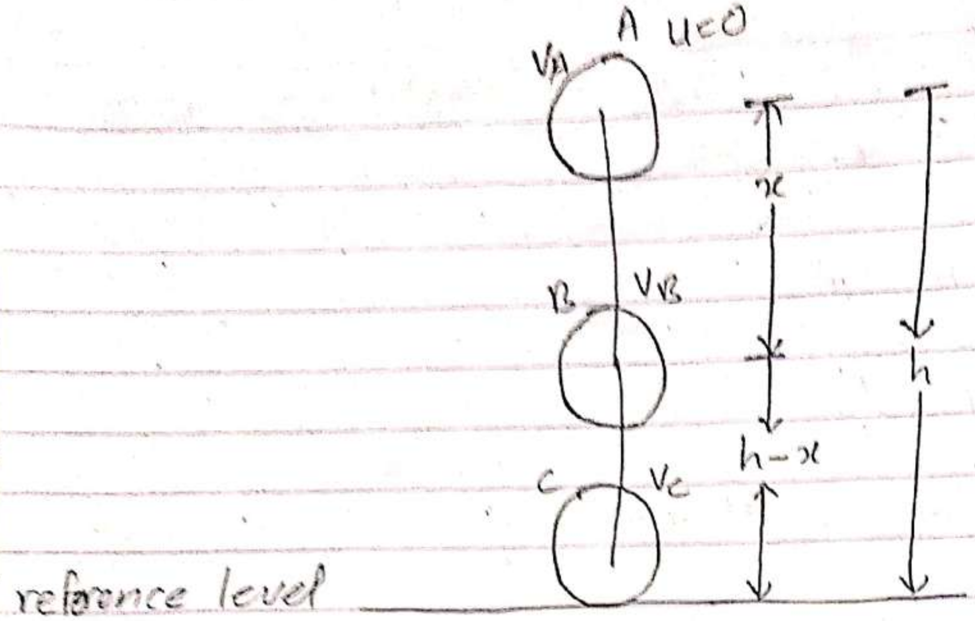
VVI.

* Principle of Conservation of Energy:-

STATEMENT:-

It state that, "Energy can neither be created nor be destroyed but can be changed from one from to another."

* Conservation of energy in freefall.



Let us consider a body of mass 'm' initially at rest at point A at a distance 'h' from the ground. After some time, this body reaches to the point B with velocity V_B covering distance 'x' and it is $(h-x)$ distance above the ground. At last the body reaches at point C with velocity 'V_C'.

At point A.

kinetic energy $(E_K) = \frac{1}{2} m v_A^2 = 0$

And potential energy $(E_p) = mgh$

Total energy $(T) = E_K + E_p$

$$= 0 + mgh$$
$$= mgh$$

At point B

Kinetic energy $(E_K) = \frac{1}{2} m v_B^2$ — ①

Under this condition,

distance covered $(s) = x$
initial velocity $(u) = 0$
final velocity $= v_B$
acceleration $(a) = g$

from eqn of motion;

or $v^2 = u^2 + 2as$
or $v_B^2 = 0 + 2gx$
or $v_B^2 = 2gx$ — (II)

from ① and (II) then;

$E_K = \frac{1}{2} m\, 2gx$

$E_K = mgx$ — (III)

And the potential energy,

or, $E_p = mg(h - x)$
or $E_p = mgh - mgx$ — (IV)

But, the total energy $(T) = E_K + E_p$

$$or,\ T = \cancel{mgr} + mgh - \cancel{mgr}$$

$$or,\ T = mgh$$

At point C

Kinetic energy $(E_K) = \frac{1}{2} m v_c^2$ —— (1)

Under this condition;

distance covered $(s) = h$

initial velocity $(u) = 0$

acceleration $(a) = g$

velocity $(v) = V_c$

from the eqn of motion, then

$$v^2 = u^2 + 2as$$

$$or,\ V_c^2 = 0 + 2gh$$

$$or,\ V_c^2 = 2gh \quad \text{—— (11)}$$

from eqn (1) and (11) then,

$$E_K = \frac{1}{2} m 2gh$$

$$E_K = mgh$$

But, at the refrence level, $h = 0$ so, potential energy $(E_p) = 0$.

Thus; Total energy (T) $= E_K + E_p$
$= mgh + 0$
$= mgh$

Thus, total energy at point A = total energy at point B = total energy at point C so, we say that total energy is conserved in free falling body.

Conservative force

If the work done by a force in moving a body from one point to another point is independent upon the path followed then such forces is said to be conservative force. In such force the total work done in the complete circular path is zero.

for ex: Gravitational force, force between the collision of atomic particles.

Non-conservative force

If the work done by a force in moving a body from one point to another point is dependent upon the path ~~the path~~ followed then such forces is said to be non-conservative force. In such force the total work done in the complete circular path is not zero.

for ex: frictional, force, collision between two heavy bodies.

Relation between kinetic energy and mometum:-

Let us consider a body of mass (m) moving with velocity (v) then kinetic energy and momentum of a body will be;

Kinetic energ $(E_k) = \frac{1}{2} mv^2$ — (i)

And;

momentum $(p) = mv$ — (ii)

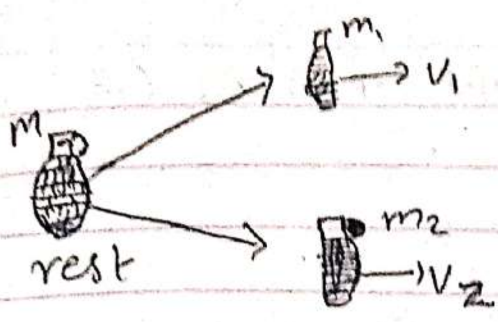
Then; from eq^n (i)

$$E_K = \frac{1}{2} \frac{m^2v^2}{m}$$

$$= \frac{(mv)^2}{2m}$$

$$E_K = \frac{p^2}{2m}$$

* Kinetic energy due to explosive force:-



Let us consider a heavy particles having (M) initially at rest. But when it explodes then it break into two fragments having masses m_1 and m_2 moving with velocities v_1 and v_2 as shown in figure.

Kinetic energy of a body of mass$(m_1) = E_1 = \frac{1}{2} m_1 v_1^2$

$$= \frac{m_1^2 v_1^2}{2m_1}$$

$$= \frac{(m_1 v_1)^2}{2m_1}$$

$$= \frac{p_1^2}{2m_1} \quad \text{—} (I)$$

Where; P_1 = momentum of fragment of mass (m_1) with velocity (v_2)

Similarly, Kinetic energy of a body of mass $(m_2) = E_2$

$$= \frac{P_2^2}{2m_2} \quad \text{—} (II)$$

where; P_2 = momentum of fragment of mass (m_2) with velocity (v_2).

Dividing (ii) by (i) then

$$\frac{E_2}{E_1} = \frac{\frac{P_2^2}{2m_2}}{\frac{P_1^2}{2m_1}}$$

or. $$\frac{E_2}{E_1} = \frac{m_1}{m_2} \frac{P_2^2}{P_1^2} \quad \text{— (iii)}$$

During the ~~e~~ explosion of bomb the linear momentum get conserve

ie $|P_1| = |P_2| = P$ — (iv)

From eqn (iii) and (iv) then

or. $$\frac{E_2}{E_1} = \frac{m_1}{m_2} \frac{P^2}{P^2}$$

or. $$\frac{E_2}{E_1} = \frac{m_1}{m_2}$$

In General, $E \propto \frac{1}{m}$

Therefore, the kinetic energy due to explosive forces is inversely proportional to mass.

Q. What will happen to the kinetic energy if momentum is ~~hoop half~~ halved?

(i) $\frac{E_K}{4}$ ✓

(ii) $\frac{E_K}{2}$

(iii) $\frac{E_K}{8}$

(iv) $\frac{E_K}{16}$

Solution:- The relation between kinetic and momentum is;

$$E_K = \frac{p^2}{2m}$$

By question; $p \rightarrow \frac{p}{2}$ then $E_K \rightarrow E_K'$ such that

$$E_K' = \frac{(\frac{p}{2})^2}{2m}$$

$$E_K' = \frac{p^2}{4.2m}$$

$$E_K' = \frac{E_K}{4}$$

* Work - Energy theorem

It state that, " The final kinetic energy of the body is equal to the sum ~~store~~ of work done on the body and it initial kinetic energy.

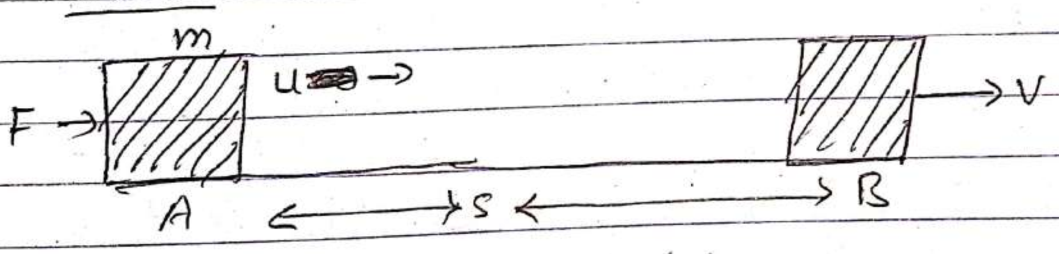
i.e. $(E_k)_F = W + (E_k)_i$

or $W = (E_k)_F - (E_k)_i$

where; $(E_k)_F$ = Final kinetic energy

$(E_k)_i$ = Initial kinetic energy

Proof:-



Let us consider a body of mass (m) initially at rest, such that ~~it gain velocity~~ it gain initialy velocity (u) when constant force (f) is applied on a body then it moves from A to B covering distance S and gain final velocity (v).

Then, work done by the force (f) is

$$W = FS \quad \text{—} \quad (1)$$

From Newton second law of motion

$F = ma$ —— (ii)

From eqn (i) and (ii) then,

$W = mas$ —— (iii)

From eqn of motion

$v^2 = u^2 + 2as$

$v^2 - u^2 = 2as$

$as = \frac{v^2 - u^2}{2}$ —— (iv)

From eq (iii) & (iv) then,

or $W = m\left(\frac{v^2 - u^2}{2}\right)$

or, $W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

or Work done = Final kinetic ~~Energy~~ Energy − Initial kinetic Energy.

ie. $W = (E_K)_F - (E_K)_i$

* Power

The rate of doing work is called power. It SI unit is watt or J/s. It is defined by:

$$\text{Power}(P) = \frac{\text{work}(w)}{\text{time taken}(t)}$$

$$\text{or, } P = \frac{w}{t}$$

$$\text{or, } P = \frac{Fd}{t}$$

$$\text{or, } P = Fv$$

This is the expression of Power in terms of velocity.

$$P = IV$$

Numerical zone

* What is the work done by a man carrying bag weighing 150 kg over his head when travels at a distance of 20 m in vertical direction & horizontal direction

mass of bag $(m) = 150$ kg
distance travel $(s) = 20$ m
we have:

Work done $(w) = Fs\cos\theta$

Here; the workdone depends upon the value of $\cos\theta$ in vertical and horizontal direction.

① In vertical direction:-

Here; $\theta = 0°$ So; Work done is;

$W = Fs\cos\theta$

$= Fs\cos 0°$

$= Fs$

$= mgh$

$= 50 \times 10 \times 20$

$= 10000\,J$

② In horizontal direction:-

Here; $\theta = 90°$ So; workdone is;

$W = Fs\cos 90°$

$W = Fs.0$

$W = 0$

(*) A stationary mass explodes into two frag ment of mass, 4 unit & 40 unit respectively. If the large mass has initial kinetic energy of 10 J. What is the initial kinetic energy of smaller mass?

Soln for smaller fragment

$m_1 = 4$ unit

$E_1 = ?$

for larger fragment

$m_2 = 40$ unit

$E_2 = 10\,J$

During the explosion of bomb, the kinetic energy is inversly proportional to mass.

ie. $E \propto \frac{1}{m}$

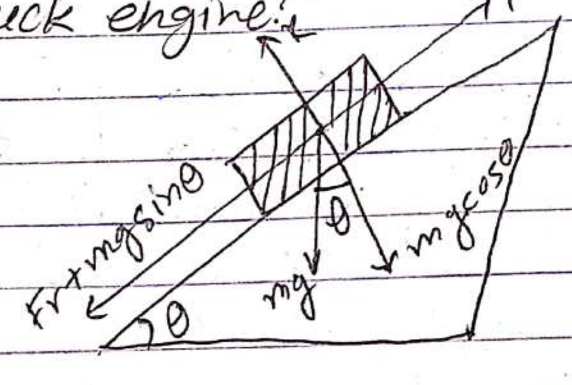
ie. $\frac{E_1}{E_2} = \frac{m_2}{m_1}$

or, $E_1 = \frac{m_2}{m_1} \times E_2$

or, $E_1 = \frac{40}{4} \times 10$

or, $E_1 = 100\,J$

(*) A truck of mass 3000 kg moves up and inclined plane at a constant speed of 36 Km/hr against of frictional force of 200 N. The inclined nation is such that it rasies 1m for every 15 m along the incline. Calculate the power that ~~tof~~ by truck engine?



mass (m) = 3000 kg
velocity (v) = 36 km/hr

$= \frac{\cancel{36} \times 10\cancel{00}}{\cancel{3600}}$

$= 10 \, m/s$

$\sin\theta = \frac{1}{15}$

acceleration due to gravity (g) = $10 ms^{-2}$

Total force needed to move the truck up in an inclined plane is;

$F = Fr + mg\sin\theta$

$= 200 + 3000 \times 1000 \times \frac{1}{15}$

$= 200 + 2000$

$= 2200 N$

(*) A body is sliding down or rough inclined plane which makes an angle of 30° with horizontal. Caculation the acceleration if co-effieent of friction (μ) is 0.25 and acceleration due to gravity is 9.8 m/s^2

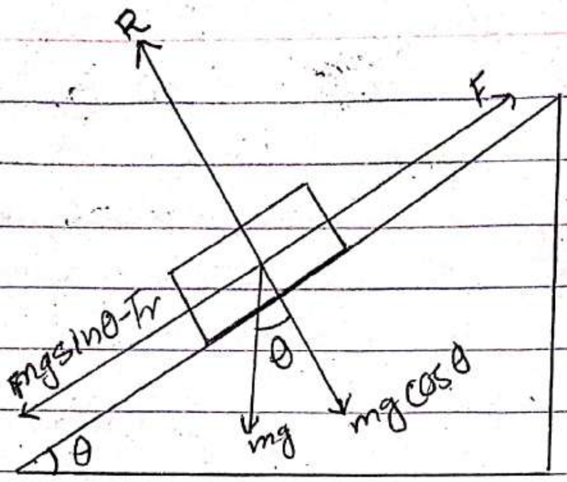
soln

co-efficent of friction (μ) = 0.25

acceleration due to gravity (a) = 9.8 m/s^2

Angle (θ) = 30°

acceleration (a) = ?



Total force is needed to move a body downward is:

or $F = mg\sin\theta - F_r$

or $F = mg\sin\theta - \mu R$

or $F = mg\sin\theta - \mu mg\cos\theta$

or $F = mg(\sin\theta - \mu \cos\theta)$

or $\cancel{m}a = \cancel{m}g\,(\sin 30 - \mu \cos 30)$

or $a = 9.8\left(\frac{1}{2} - 0.25 \times \frac{\sqrt{3}}{2}\right)$

or $a = 9.8 \times \frac{1}{2}\,(1 - 0.43)$

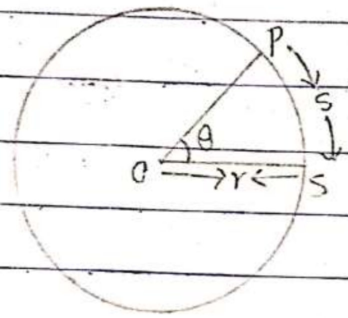
or $a = 4.9 \times 0.57$

or $a = 2.79\ m/s^2$

Circular Motion

Definition:

When a body moves in a circular path then such motion is called circular motion. A diagram to represent the circular motion is given below:-



Angular displacement:-

The angle travelled by a body in a regular interval of time is called angular displacement. It is denoted by 'θ'. It unit is radian. It is dimension less quantity.

From figure, Angular displacement (θ) = $\frac{\text{Arc length}}{\text{radius}}$

$$\text{or, } \theta = \frac{\text{arc } PS}{OS}$$

$$\text{or, } \theta = \frac{s}{r}$$

$\boxed{\omega = \text{omega}}$

Angular velocity:

The rate of change of angular displacement is called angular velocity. It is denoted by ω and its unit is radian/sec ($rad s^{-1}$). It's dimension is $[M^0L^0T^{-1}]$ and is defined by

$$\text{Angular velocity} = \frac{\text{angular displacement}}{\text{time taken}}$$

$$\text{or, } \omega = \frac{\theta}{t}$$

But, for $dt \to 0$ then $\omega = \frac{d\theta}{dt}$

Angular acceleration:-

The rate of change of angular velocity is called angular acceleration. It is denoted by α and it's unit is radian per sec^2 ($rad s^{-2}$) and ~~The~~ It's dimension is $[M^0L^0T^{-2}]$. It is defined by

$$\text{Angular acceleration } (\alpha) = \frac{\text{Angular velocity}}{\text{time taken}}$$

$$\text{or, } \alpha = \frac{\omega}{t}$$

But, for $dt \to 0$; then $\alpha = \frac{d\omega}{dt}$

Time period

The take taken by the particals to complete one revolution is called time period. It is denoted by 'T'.

Angular frequency:-

The number of complete revolution made by the particals per unit time is called angular frequency. It is donated by 'f' and It's unit is revolution per cycle or hertz. It is defined by:

$$\text{Angular frequency } (f) = \frac{1}{\text{time period } (T)}$$

$$\text{or, } f = \frac{1}{T}$$

Also; $f = \frac{n}{T}$; If no of revolutions is given.

Do you know?

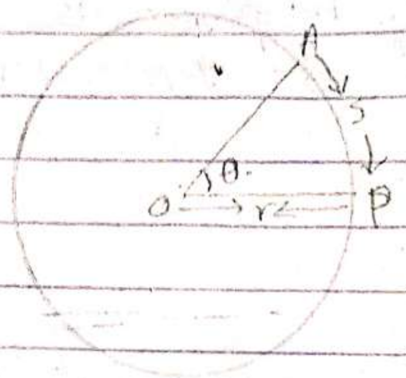
The relation between ω, T, f is given by

$$\omega = 2\pi f = \frac{2\pi}{T}$$

where; f = Angular frequency, T = time period

Imp

Relation between linear velocity and angular velocity



Let us consider, A body of mass (m) is moving in a circular path of center 'O' and radius 'r'. Let 's' be the arc length and 'θ' be the angle travelled by the body.

Then from the relation of angular displacement

or, $\theta = \frac{\text{arc } AP}{\text{radius } OP}$

or, $\theta = \frac{s}{r}$

or $s = \theta r$ —— ①

Differentiating eqn ① w.r.t. time taken (t), then

or, $\frac{ds}{dt} = \frac{d(\theta r)}{dt}$

or, $\frac{ds}{dt} = \frac{r d\theta}{dt} + \frac{\theta dr}{dt}$

If is kept constant, then $\frac{dr}{dt} \rightarrow 0$. So,

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

or, $v = r\omega$

or, $v = \omega r$

wher; $v = \frac{ds}{dt}$ = linear velocity

$\omega = \frac{d\theta}{dt}$ = angular velocity

Centripetal force

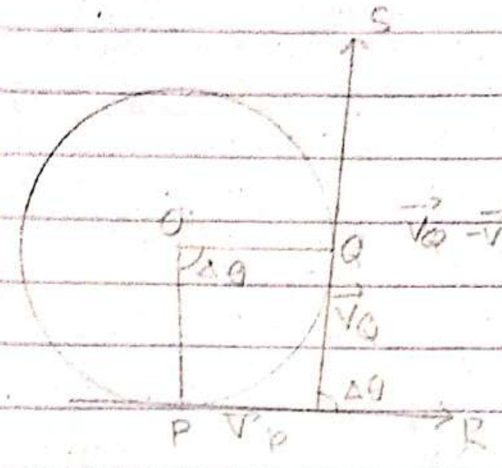
inward force which keep the moving in a circular path is called centripetal force. for example

(I) Planet revolving around the sun gets centripetal force by the gravitional force of attraction between the sun and planets.

(II) Electrons revolving round the nucleas gets centripetal force due to electrostatic force of attraction between electrons and neucles.

'IVI

* Expression for centripetal force :-



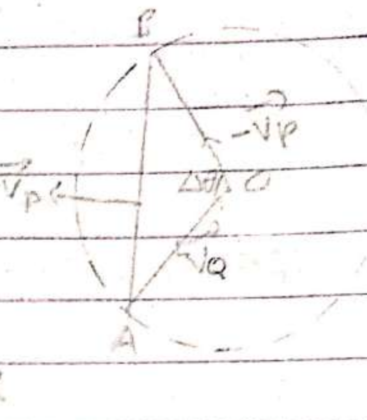


Fig (I) Fig (II)

Let us consider a body having mass (m) is moving uniformaly around the circular path of centre O & radius (r) as shown in figure (I). $\vec{V}_p$ & $\vec{V}_Q$ be the velocities at point P & Q in the direction of there tangents. Then the change in velocity when the body moves from P to Q in time interval "Δt" is given by:

$$\Delta v = \vec{V}_Q - \vec{V}_p$$

Similarly OA & OB represented $\vec{V}_Q$ & $\vec{V}_p$ in magnitude & $\vec{AB}$ give its direction then the side $\vec{AB}$ of triangle ΔOAB represent the change in velocity.

i.e $\Delta v = \vec{V}_Q - \vec{V}_p = \vec{AB}$

Now, the acceleration produce in time interval Δt is given by;

acceleration (a) = $\frac{\text{change in velocity}}{\text{time taken}}$

or, $a = \frac{\vec{V}_Q - \vec{V}_P}{\Delta t}$

or, $a = \frac{\Delta V}{\Delta t}$

or, $a = \frac{\vec{AB}}{\Delta t}$

or, $\vec{AB} = a \Delta t$ —— (1)

Now; from fig (11) then

$\Delta \theta = \frac{\vec{AB}}{OA} = \frac{\vec{AB}}{\vec{V}_Q}$

or, $\vec{AB} = \vec{V}_Q . \Delta\theta$ —— (11)

✓ Since, the body is moving uniformly

$|\vec{V}_Q| = |-\vec{V}_P| = v$

So; eqn (11) becomes

$\vec{AB} = v \Delta\theta$ —— (111)

from eqn (1) & (111) then,

$$\boxed{\alpha' = \frac{d\omega}{dt} = \frac{v^2}{r} = \omega^2 r}$$

or, $v\,\Delta\theta = a\,\Delta t$

or, $a = v.\frac{\Delta\theta}{\Delta t}$

for $\Rightarrow \Delta t \to 0$; then $\frac{\Delta\theta}{\Delta t} \to \frac{d\theta}{dt}$ which is equal to ω.

Thus; $a = v\omega$

or, $a = v.\frac{v}{r}$ $[\because v = \omega r]$

or $a = \frac{v^2}{r}$.

Now, the centripetal force is given by;

$F = ma$

$F = \frac{mv^2}{r}$ — (IV)

But, in terms of ω, the above eq^n reduced to;

$F = \frac{m(\omega r)^2}{r}$ $[\because v = \omega r]$

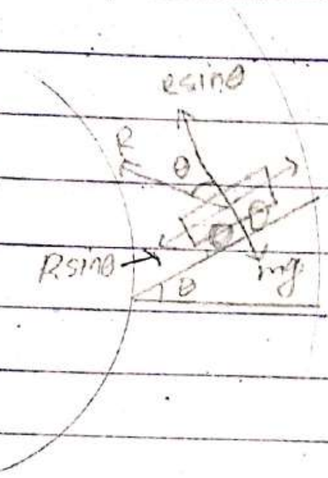
or, $F = \frac{m\omega^2 r^{\cancel{2}}}{\cancel{r}}$

or, $F = m\omega^2 r$ — (V)

The eqn (IV) & eqn (V) are the expression of centripetal force.

Application of circular motion:-

1) Motion of car in a banking curve path.



Let us consider a car having mass (m) is moving in a banked curved path of radius (r) with banking angle 'θ'. When the car moves in such curves path the centripetal force is independent of frictional force so the normal reaction R comes into play. Which is resolved in two component.

The vertical component of normal reaction (ie. Rcosθ) balanced the weight of the body:

ie. $R\cos\theta = mg$ —(1)

And The horizontal component of normal reaction ie. $R\sin\theta = \cancel{mv^2}$ provide necessary centripetal force; ie $R\sin\theta = \frac{mv^2}{r}$ —(11)

~~Divided~~ Dividing eqn (ii) by (i)

$$\frac{R\sin\theta}{R\cos\theta} = \frac{\frac{mv^2}{r}}{mg}$$

$$\tan\theta = \frac{v^2}{rg} \quad \text{——— (iii)}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) \quad \text{—— (iv)}$$

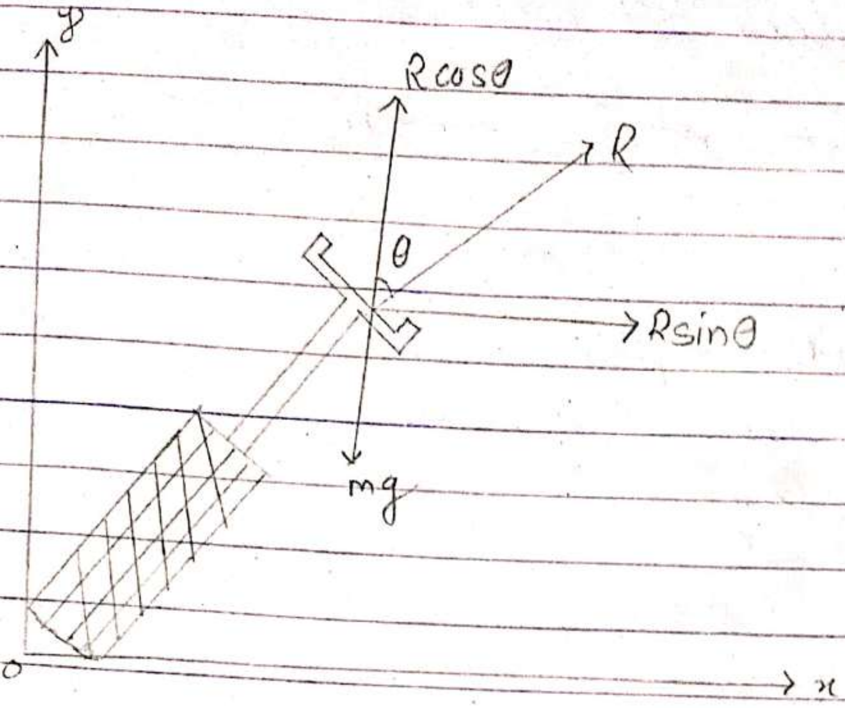
from eqn (iii)

$$v^2 = rg\tan\theta$$

or, $v = \sqrt{rg\tan\theta}$ —— (v)

The eqn (iv) gives banking angles while the eqn (v) gives the velocity of car in banking curves path.

Motion of cyslist in circular path.



The frictinol force between the tyre and the road cannot provide necessary centripetal force to move a cyclist in curve path so to provide to neccesary centripetal force a cyclist must inclined himself in a curve path in an inward direction.

Let us consider a cyclist of mass m is moving in a circulair path of radius 'r' with banking angle 'θ'. Here, the normal reaction comes into Play. which is divided into two component.

The vertical component of normal reaction balance the weight of the cyclist

ie. $R\cos\theta = mg$ — ①

And the horizontal component of normal reaction provide necessary centripetal force

i.e $R\sin\theta = \frac{mv^2}{r}$ —— (ii)

Dividing eqn (ii) by (i)

$$\frac{\cancel{R}\sin\theta}{\cancel{R}\cos\theta} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g}$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right) \quad \text{—— (iii)}$$

The equation (iii) gives inlination to the cyclist so that he may move in circular path.

Do you know?

we know that,

$$\tan\theta = \frac{v^2}{rg} \quad \text{—— (i)}$$

From the coffecient of friction;

$$\mu = \tan\theta \quad \text{—— (ii)}$$

from eq^n (i) & (ii) then

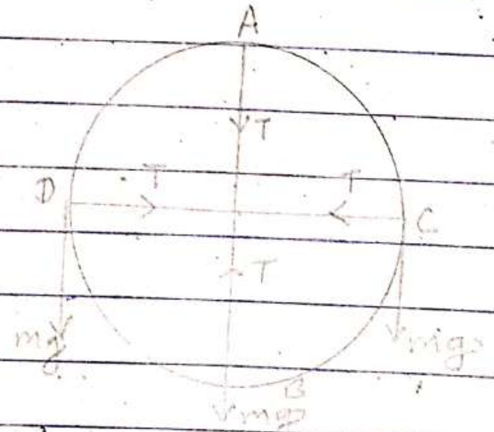
$$\mu = \frac{v^2}{rg}$$

$$v^2 = \mu r g$$

$$v = \sqrt{\mu r g}$$

Application no. 3

Motion in a vertical circle:-



Let us consider a body having mass m is moving in a vertical circle of radius 'r' with velocity 'v' as shown in figure. If 'T' be the tension acting upon the string then:

At point B

force at $B = T - mg$

or, $\frac{mv^2}{r} = T - mg$

or, $T = \frac{mv^2}{r} + mg$

or, $T = m\left(\frac{v^2}{r} + g\right)$

This the maximum tension produce at lower point of vertical circle.

- At point C or D:

Force at C or D = T

or, $\frac{mv^2}{r} = T$

or, $T = \frac{mv^2}{r}$

At point A;

Force at A = mg + T

or, $\frac{mv^2}{r} = mg + T$

or, $T = \frac{mv^2}{r} - mg$

or, $T = m\left(\frac{v^2}{r} - g\right)$

This is the minimum tension produce in upper point of the circle.

Q A motor cycle rider going 90 $kmhr^{-1}$ around a curve path with a radius of 100m must lean at an angle to the vertical. Find the angle at which he leans.

Hint/Gist

$$T\sin\theta = \frac{mv^2}{r} \quad \text{—(i)}$$

$$T\cos\theta = mg \quad \text{—(ii)}$$

Dividing (i) by (ii) then,

$$\frac{\cancel{T}\sin\theta}{\cancel{T}\cos\theta} = \frac{\frac{\cancel{m}v^2}{r}}{\cancel{m}g}$$

$$\tan\theta = \frac{v^2}{rg}$$

$v = 90\ km/hr = 25\ m/s$

$r = 100m$

$g = 10\ m/s^2$.

$$\tan\theta = \frac{(25)^2}{100 \times 10}$$

$$\tan\theta = \frac{625}{1000}$$

$$\tan\theta = 0.625$$

$$\theta = \tan^{-1}(0.625)$$

$\theta = 32^\circ$

Q An object moves in a circular path in radius of 2m with period of 5 sec. calculate the angular velocity and angular acceleration.

Given,

Time period = 5 sec
radius = 2m

We have

$$\omega = \frac{2\pi}{T}$$

$$= \frac{2 \times 3.14}{5}$$

$$= 1.256$$

Now; $\alpha = \omega^2 r$

$$= (1.256)^2 \times 2$$

$$= 1.57 \times 2$$

$$= 3.15$$

A card is tide to a pail of water and the pail is swung in a vertical circle of radius 1.0 m. What be the minimum speed at the highest point of the circle if no is spill from pail.

Given

radius(r) = 1m

acceleration due to gravity g) = 10m/s

In order to be not spill from pail, the condition must be;

$$\frac{mv^2}{r} = mg$$

$$v^2 = rg$$

$$v = \sqrt{rg}$$

$$v = \sqrt{1 \times 10}$$

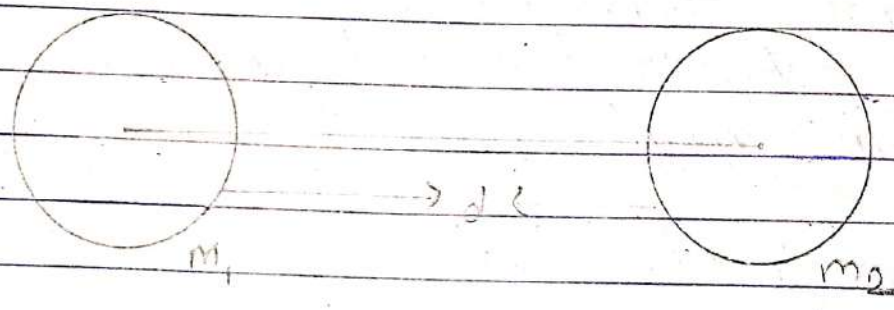
$$v = 3.16 m/s$$

Gravitation:

The mutual force of attraction between two bodies is called Gravitation.

Newton Law of Gravitation:-

It states that the force of attraction between two bodies is directly proportional to the product of masses and inversly proportional to the square of the distance between them



If m_1 and m_2 be two masses of different bodies seperated by distance (d) then according to Newton's law of gravitation.

$$F \propto m_1 m_2 \quad \text{—(i)}$$
$$F \propto \frac{1}{d^2} \quad \text{—(ii)}$$

Combining eqn (i) and eqn (ii) then,

$$F \propto \frac{m_1 . m_2}{d2}$$

$$F = \frac{G m_1 m_2}{d^2} \quad \text{— (iii)}$$

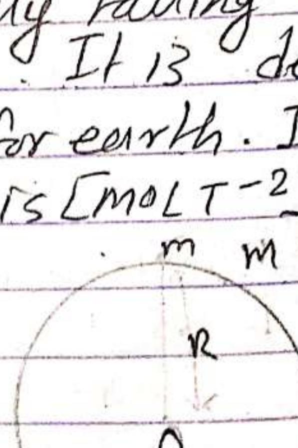
The eqn(ii) gives the gravitional force between two bodies. Here, the term G is called Gravitational constant and its value is $6.67 \times 10^{-11}\ Nm^2/kg^2$.

In eqn (ii); if $m_1 = m_2 = 1kg$ and $d = 1m$ then it becomes; $F = G$

Thus, the universal gravitational constant is the gravitational force between two unit masses seperated by unit distance apart

* <u>Acceleration due to gravity:-</u>

The acceleration produced on freely falling body acting towards the centre of earth. It is denoted by g and its value is $9.8 m/s^2$ for earth. Its unit is ms^{-2} and its dimension is $[M^0 L T^{-2}]$



Consider an object of mass (m) lying on the earth's surface. Let M and R be the mass and radius of earth. Then, the force of attraction between the earth and object having negligibe radius in comparision to earth's radius is given by Newton's law of gravitation.

$$F = \frac{GMm}{R^2} \quad \text{——(i)}$$

The weight of the body on earth's surface is

$$F = W = mg \quad \text{——(ii)}$$

From eqn (i) and eqn (ii) then

$$\therefore mg = \frac{GMm}{R^2}$$

$$\text{or,} \quad g = \frac{GM}{R^2}$$

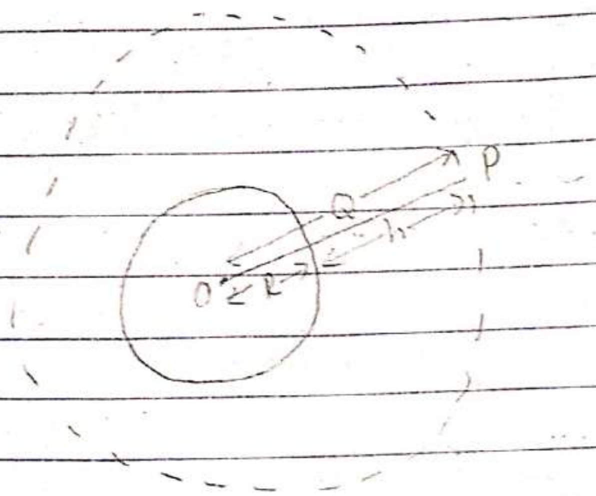
For constant G and M;

$$g \propto \frac{1}{R^2}$$

This relation shows that the acceleration produced on a freely falling body is independent of the mass of falling object.

Variation of acceleration due to gravity:-

a) Effect of 'g' due to height:-



Consider M and R be the mass and radius of the earth of centre O. Then, acceleration due to gravity at earth surface is given by:

$$g = \frac{GM}{R^2} \quad \text{—(i)}$$

Assume; P be the point at a height 'h' from the earth surface, then the new acceleration due to gravity at the point P is;

$$g' = \frac{GM}{(R+h)^2} \quad \text{—(ii)}$$

$$\boxed{(1+n)^{-n} = 1 - \frac{nx}{1!} + \frac{(nx)^2}{2!} + \frac{(nx)^3}{3!} + \cdots}$$

Dividing eqn (11) by eqn (1) then

$$\text{or, } \frac{g'}{g} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}}$$

$$\text{or, } \frac{g'}{g} = \left(\frac{R}{R+h}\right)^2$$

$$\text{or, } \frac{g'}{g} = \left(\frac{1}{\frac{R+h}{R}}\right)^2$$

$$\text{or, } \frac{g'}{g} = \frac{1}{\left(1+\frac{h}{R}\right)^2}$$

$$\text{or, } \frac{g'}{g} = \left(1+\frac{h}{R}\right)^{-2}$$

Using the Binomial expansion, then

$$\frac{g'}{g} = \left(1 - \frac{2h}{R}\right) + \cdots\cdots$$ (neglecting higher term)

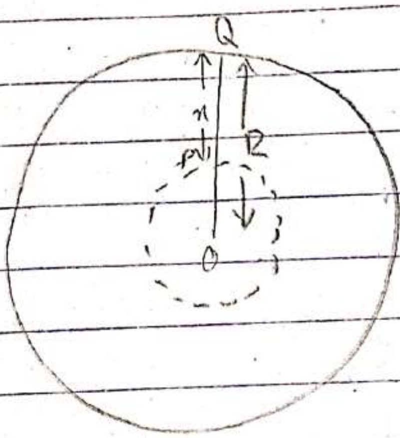
$$\text{or, } \frac{g'}{g} = \left(1 - \frac{2h}{R}\right)$$

Note: Density of the earth is invarient.

or, $g' = g\left(1 - \frac{2h}{R}\right)$

Thus, the value of acceleration due to gravity decrease when we move above the earth surface.

Effect of 'g' due to depth:-



Consider, M & R be the mass & radius of the earth of centre O. then; the acceleration due to gravity on the earth surface is;

$$g = \frac{GM}{R^2} \quad —①$$

When the body is at the point P at a distance x below the earth surface then the new acceleration due to gravity at the point P is given by;

$\boxed{\rho = 1010}$

$$g' = \frac{Gm'}{(R-x)^2} \quad \text{— (ii)}$$

Where; m' is the mass of sphere at point P.

Let 'ρ' be the density of earth.

Then; density $= \dfrac{\text{mass (m)}}{\text{volume (v)}}$

$$\text{or, } \rho = \frac{M}{V}$$

$$\text{or, } M = \rho \times V$$

But; volume of sphere (v) $= \frac{4}{3}\pi R^3$ — (iii)

Mass of sphere (M) $= \rho \frac{4}{3}\pi R^3$

$= \frac{4}{3}\pi \rho R^3$ — (iv)

Now; the eqn (i) reduces to;

$$g = \frac{G \frac{4}{3}\pi R^{\cancel{3}} \rho}{\cancel{R^2}}$$

$$g = \frac{4}{3}\pi \rho R G \quad \text{— (v)}$$

Similarly; volume of sphere at point P

$$= \frac{4}{3}\pi (R-x)^3$$

And; mass of sphere at point $P = \frac{4}{3}\pi (R-x)^3 \times \rho$

$$m' = \frac{4}{3}\pi \rho (R-x)^3 \quad \text{— (vi)}$$

Now; the eqn (ii) because of eqn (vi) becomes:

$$\text{or, } g' = \frac{G \frac{4}{3}\pi \rho (R-x)^3}{(R-x)^2}$$

$$\text{or, } g' = \frac{4}{3}\pi \rho G (R-x) \quad \text{— (vii)}$$

Dividing eqn (vii) by (v) then

$$\frac{g'}{g} = \frac{\cancel{\frac{4}{3}\pi \rho G}(R-x)}{\cancel{\frac{4}{3}\pi \rho G} R}$$

or $\frac{g'}{g} = \frac{R-x}{R}$

or $\frac{g'}{g} = \left(1 - \frac{x}{R}\right)$

or $g' = g\left(1 - \frac{x}{R}\right)$

Note: In the centre in the earth

$x = R$

$g' = g(1-1)$

$g' = 0$

But, mass is infinite & possitive

Thus, the acceleration due to gravity decreases below earth surface.

③

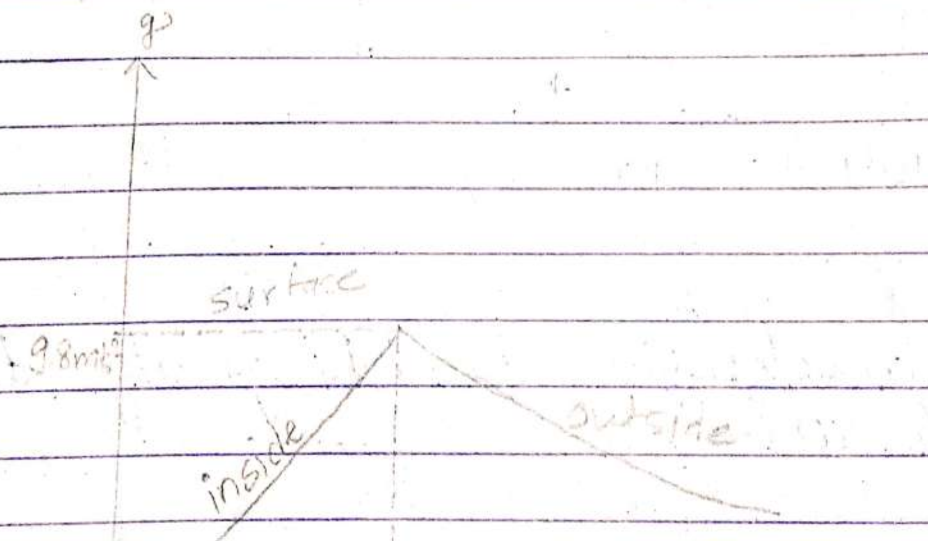
Variation of acceleration due to gravity at a distance from the centre of the earth.

The variation of acceleration due to gravity at a depth (x) and at a height 'h' above the earth of radius 'R' is given by;

$$g' = g\left(1 - \frac{x}{R}\right) \text{ and}$$

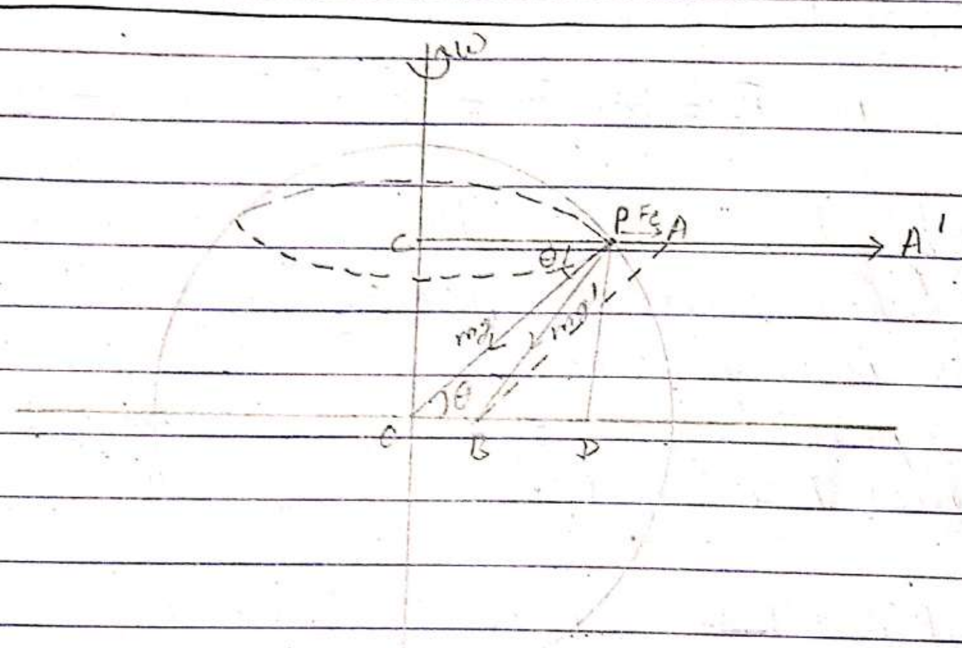
$$g' = g\left(1 - \frac{2h}{R}\right)$$

Where, g is the acceleration due to gravity on the earth surface



The above expression shows that the acceleration due to gravity is 0 at the center of the earth, maximum at the surface, Decreases linearly below the surface of the earth and decreases exponentially above the surface of the earth.

Variation of acceleration due to gravity due to rotation of the earth:-



Let us consider an object of mass 'm' at a point 'P' on the earth surface of centre O. If M and R be the mass and radius of the earth and ω be its angular velocity. here θ be the latitude of the body at point P

When the earth is not rotating, it weight $W = mg$ act toward the centre of the earth.

When the earth rotate with an angular velocity 'ω', on its axis, the object at P while also rotate about it's center C of radius R.

Where, $r = R\cos\theta$. So the object with experience a centrifugal force.

$$\text{ie. } F_c = m\omega^2 r = m\omega^2 R\cos\theta \quad \text{— (1)}$$

Here an object is under the action of two forces, Centrifugal force and its weight. The resultant of this two forces is represent by PB in figure which is the apperent weight mg'. Then, using parallelogram law of vector addition

or, $PB^2 = PO^2 + PA^2 + 2.PO.PA.\cos(180-\theta)$

or, $(mg')^2 = (mg)^2 + (m\omega^2 R\cos\theta)^2 + 2mg\, m\omega^2 R\cos\theta . -\cos\theta$

or, $m^2 g'^2 = m^2 g^2 + m^2\omega^4 R^2\cos^2\theta - 2m^2 g\,\omega^2 R\cos^2\theta$

or, $m^2 g'^2 = m^2 g^2 \left(1 + \frac{\omega^4 R^2\cos^2\theta}{g^2} - \frac{2\omega^2 R\cos^2\theta}{g}\right)$

or, $g'^2 = g^2 \left(1 + \frac{\omega^4 R^2\cos^2\theta}{g^2} - \frac{2\omega^2 R\cos^2\theta}{g}\right)$

$$g' = g\left(1 - \frac{2\omega^2 R\cos\theta}{g} + \frac{\omega^4 R^2 \cos^2\theta}{g^2}\right)^{1/2}$$

Since; $\frac{R\omega^2}{g}$ is small quantity. So, the term $\frac{\omega^4 R^2 \cos^2\theta}{g^2}$ can be neglected.

$$\therefore\ g' = g\left(1 - \frac{2\omega^2 R\cos^2\theta}{g}\right)^{1/2}$$

Expanding the above by binomial expension

$$\text{or, } g' = g\left(1 - \frac{1}{2}\,\frac{2\omega^2 R\cos^2\theta}{g}\right) + \text{- - - - -}$$

$$\text{or, } g' = g\left(1 - \frac{\omega^2 R\cos^2\theta}{g}\right)$$

$$\text{or, } g' = g - \omega^2 R\cos^2\theta$$

$\therefore$ This is our required expression.

Special cases

case I

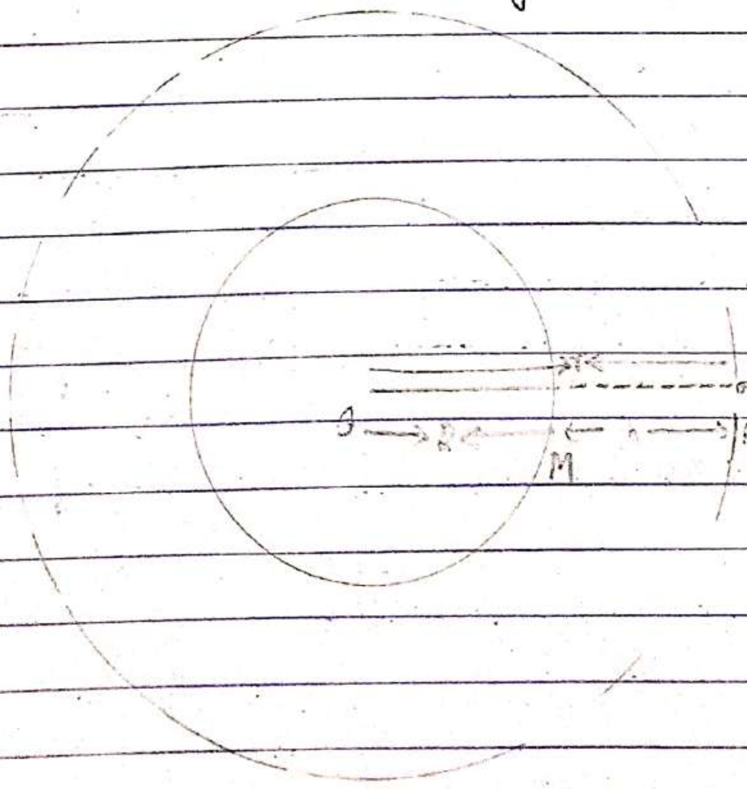
At pole, $\theta = 90$, then $g' = g$, Hence the value of g remain constant or unchange.

Case II

At equator, $\theta = 0$, then $g' = g - \omega^2 R$, Hence the value of g decreases due to the rotation of the earth.

Orbital Velocity

The velocity of a satellite to move in circular orbit is called orbital velocity.



Let M and R be the mass and radius of the earth of centre O. A satellite of mass m is at point P. Which is at the height h. from the earth surface.

For the satellite ~~moo~~ to move in circular orbit the necessary centripetal force is balance by the gravitational force of attraction between the earth and the satellite.

ie Centripetal force = Gravitational force

$$\frac{\cancel{m}v^2}{\cancel{r}} = \frac{GM\cancel{m}}{r^{\cancel{2}}}$$

$$\text{or, } v^2 = \frac{GM}{r}$$

$$\text{or, } v = \sqrt{\frac{GM}{r}} \quad \text{— (i)}$$

But, $r = R+h$. So;

$$v = \sqrt{\frac{GM}{(R+h)}} \quad \text{— (ii)}$$

At the surface of earth; $g = \frac{GM}{R^2}$ such that $GM = gR^2$. Hence; eqn (ii) becomes

$$v = \sqrt{\frac{gR^2}{(R+h)}} \quad \text{--- (iii)}$$

When the satellite is very close to earth; then $h \approx 0$
Hence; eqn (iii) becomes

$$v = \sqrt{\frac{gR^2}{R}}$$

or, $$v = \sqrt{gR} \quad \text{--- (iv)}$$

The eqn (i) (ii) (iii) and (iv) are the different relation of orbital velocity

Time period of satellite :-

The orbital velocity of satellite is:

$$v = \sqrt{\frac{GM}{R+h}}$$

But; $v = \omega r$. So;

or, $$\omega r = \sqrt{\frac{GM}{R+h}}$$

or, $$\omega = \sqrt{\frac{GM}{(R+h)}} \cdot \frac{1}{(R+h)}$$

or $\omega = \sqrt{\frac{GM}{(R+h)^3}}$

or, $\frac{2\pi}{T} = \sqrt{\frac{GM}{(R+h)^3}}$

or, $T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

At the surface of earth; $g = \frac{GM}{R^2}$ st $GM = gR^2$ implies

or, $T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$

When the satellite is very close to earth surface; $h \simeq 0$. Thus;

$$T = 2\pi\sqrt{\frac{R^3}{gR^2}}$$

$$T = 2\pi\sqrt{\frac{R}{g}}$$

This gives the expression for time period of satellite.

Energy of satellite

The orbital velocity of satellite is:

$$v = \sqrt{\frac{GM}{(R+h)}} \quad \text{— (1)}$$

The kinetic energy of satellite is:

$$E_k = \frac{1}{2} mv^2$$

$$= \frac{1}{2} m \left(\sqrt{\frac{GM}{(R+h)}}\right)^2$$

$$= \frac{GMm}{2(R+h)} \quad \text{— (11)}$$

But, the potential energy is Governed by Gravitational potential energy

$$E_p = -\frac{GMm}{(R+h)} \quad \text{— (111)}$$

Now, the total energy of satellite is:

$$T = E_k + E_p$$

$$= \frac{GMm}{2(R+h)} - \frac{GMm}{(R+h)}$$

$$T = \frac{-GMm}{2(R+h)}$$

This gives the total energy of the satellite in the orbit.

Geostationary Satellite and parking orbit

If a satellite moves in the same direction with equal time period that of the direction of rotation of the earth then such satellite are fixed at a single point are called geostationary satellite.

And the orbit in which they revolves are called parking orbit.

Since the time period of satellite is:

$$T = 2\pi\sqrt{\frac{(R+h)^3}{gR^2}}$$

For a geo-stationary satellite;

$T = 24\ hrs = 86400\ secs$
$R = 6.4 \times 10^6 m$
$g = 9.8\ ms^{-2}$

On substituting these values, we get

$h = 36000\ km$

This is the height of the parking orbit.

* Weightlessness

The ~~so~~ state of a body in which the apperant weight of the body become zero or the state the weighting machine ~~so~~ show the zero weight of the body is called weightlessness.

Special Cases

Case I : Weightlessness in freefall

If a body having mass 'm' falls downward in a lift then the reaction ~~action~~ acting on the lift. If the body falls freely then the above equation reduces ~~to~~. so the body experience weightlessness.

$R = mg - ma$
$R = m(g-a)$
$\cancel{g} \; a = g$
$R = m(g-g)$
$R = 0$

Case II : Weightless in Space

The necessary centripetal force for a satellite to move in the circular orbit is provided by the Gravitational force of attraction between the Earth and the satellite. Thus, the bodies inside the satellite does not do any work to move in circular orbit. So the reactional force on them is zero. Hence, the body experience weightlessness.

When $g = 0$ then $R = mg = 0$

This is the condition of weightlessness in space.

Q: An orbital satellite is revolving around the earth in a circular orbit at a height of 30 km above the earth surface. Find the orbital velocity and time period of satellite? ($g = 10\ m/s^2$, $R = 6370\ km$)

Soln

height $(h) = 30\ km$
radius of earth $(r) = 6370\ km = 6.37 \times 10^6\ m$
accn due to gravity $(g) = 10\ ms^{-2}$

radius of orbit $(r) = R+h$

$= 6370 + 30$

$= 6400\ km = 6.4 \times 10^6 m$

We have;

$$V = \sqrt{\frac{GM}{r}}$$

$$V = \sqrt{\frac{gR2}{(R+h)}}$$

$$V = \sqrt{\frac{10 \times (6.3 \times 10^6)^2}{6.4 \times 10^6}}$$

$$V = \sqrt{\frac{10 \times 6.3^2 \times 10^6}{6.4}}$$

$V = 7.9625 \times 10^3 m/s$

$V = 7.9625\ km/s$

Also; Time period of satellite is;

$$T = 2\pi \sqrt{\frac{(R+h)^3}{gR2}}$$

$$T = 2\pi \times \sqrt{\frac{(6.4 \times 10^6)^3}{10 \times (6.37 \times 10^6)^2}}$$

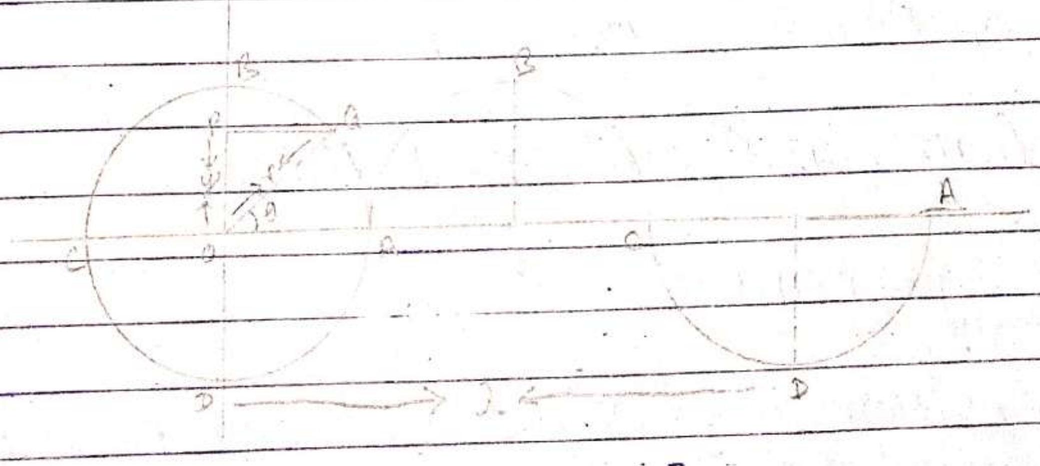
$$T = 2 \times 3.14 \times \sqrt{\frac{(6.4 \times 10^6)^3}{10 \times (6.37 \times 10^6)^2}}$$

$T = 5075$ secs

Simple Harmonic Motion

It is the to and fro motion of body in which the acceleration produces in a body is directly proportional to the displacment and is directed towards the mean position.

* Characteristics of Simple Harmonic Motion:



① Displacement

The displacement of a body vibrating in simple harmonic motion (S.H.M) is the distance of the body from its mean position.

If a body is moving in circular path of radius 'r' with angular velocity ω as shown in figure.

Now; In figure

In ΔOPQ;

$$\sin\theta = \frac{OP}{OQ}$$

or, $\sin\theta = \frac{y}{r}$

or, $y = r\sin\theta$ — ①

Also; $\omega = \frac{\theta}{t}$

or, $\theta = \omega t$ — (ii)

from eqn ① and eqn (ii) then

$$y = r\sin\omega t \quad \text{— (a)}$$

The equation (a) gives the displacement of a body vibrating in simple harmonic motion.

Special cases

① At mean position; $y=0$, (minimum displacement)

⑪ The displacement be maximum when $\sin\omega t$ is maximum and maximum value of $\sin\omega t$ is ± 1. So, at the extreme position the maximum displacement is;

$$y = \pm r$$

② Amplitude

The maximum value of displacement of a body vibrating in simple harmonic motion is called amplitude. And its value is equal to the radius of circular path.

③ Velocity :-

The rate of change of displacement of a body vibrating is simple harmonic motion is called velocity.

Since; displacement $(y) = r\sin\omega t$

And; the velocity $(v) = \frac{dy}{dt}$

$$= \frac{d(r\sin\omega t)}{dt}$$

$$= r\omega\cos\omega t \quad \text{—(I)}$$

Again; $v = r\omega\cos\omega t$

or, $v = \omega\sqrt{r^2\cos^2\omega t}$

or, $v = \omega\sqrt{r^2(1-\sin^2\omega t)}$

or, $v = \omega\sqrt{r^2 - r^2\sin^2\omega t}$

or, $v = \omega\sqrt{r^2 - (r\sin\omega t)^2}$

or, $v = \omega\sqrt{r^2 - y^2} \quad \text{—(II)}$

The equation (I) and (II) gives the velocity of a body vibrating in simple harmonic motion.

Special cases:-

a) At mean position; $y = 0$ then eq^n (II) gives

$$v = \omega\sqrt{r^2 - 0^2}$$

$$v_{max} = \omega r$$

This gives the maximum velocity at the mean position.

b) At extreme position; $y = r$ then eq^n (II) gives

or, $v = \omega\sqrt{r^2 - r^2}$

or, $v_{min} = 0$

This gives the minimum velocity at the extreme position.

* Acceleration

The rate of change in velocity of a body vibrating in simple harmonic motion is called acceleration.

Since; velocity $= \omega r \cos \omega t$.

And; the acceleration $(a) = \frac{dv}{dt}$

$= \frac{d\omega r \cos\omega t}{dt}$

$= \omega r \cdot -\sin\omega t \cdot \omega$

$= -\omega^2 r \sin\omega t$

$= -\omega^2 y$ — (1)

Here, the negative sign indicates that the acceleration is always directed toward the mean position, the eqn (1) shows that the acceleration is always directly proportional with the displacement. ie $a \propto y$

Phase

The state of body (position of body) vibrating in S.H.C is called phase of the body.

Wavelength (λ)

The linear distance travelled by a vibrating body in S.H.M is called wavelength. It is denoted by lamda (λ).

Time period

The time taken by a partical to complete one oscillation in S.H.M is called time period. It is denoted by T. Since; acceleration (a) = $\omega^2 y$ (taking magnitude only)

$$\text{or, } a = \omega^2 y$$

$$\text{or, } \omega^2 y = a$$

$$\text{or, } \omega^2 = \frac{a}{y}$$

$$\text{or, } \omega = \sqrt{\frac{a}{y}}$$

or, $\frac{2\pi}{T} = \sqrt{\frac{a}{y}}$

$$T = 2\pi\sqrt{\frac{y}{a}}$$

$$\text{Time period } (T) = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

Frequency

The number of complete oscillation made by the partical in one second is called frequency. It is denoted by "f" or "n".

Hence, frequency is the reciprocial of time period.

or, $f = \frac{1}{T}$

or, $f = \frac{1}{2\pi\sqrt{\frac{y}{a}}}$

or $f = \frac{1}{2\pi}\sqrt{\frac{a}{y}}$

$$\boxed{\text{Thus, frequency } (f) = \frac{1}{2\pi}\sqrt{\frac{\text{acceleration}}{\text{displacement}}}}$$

* Energy of a body executing or vibrating in S.H.M:-

A partical executing S.H.M has both kinetic energy as well as potential energy and their sum is total energy.

A. Potential energy:

Suppose, a particle of mass (m) executing S.H.M with an amplitude (r) and angular velocity (ω). If y is the displacement then the acceleration of a body vibrating in S.H.M is $a = -\omega^2 y$ — (i)

Where; negative sign indicates that it is directed toward the mean position.

The restoring force directed toward the mean position is given by;

$$F = ma$$

$$\text{or, } F = -m\omega^2 y \quad \text{— (ii)}$$

Now, the small amount of work done in displacing a body through a displacement dy against the force 'F' is given by:-

$$dw = -f\,dy$$
$$= -(-m\omega^2 y)\,dy$$
$$= m\omega^2 y\,dy \quad \text{— (iii)}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

The total amount of work done to displace the particle from mean position to the position of displacement y is obtain from integration

$$\text{or } W = \int_0^y dw$$

$$\text{or, } W = \int_0^y m\omega^2 y\,dy$$

$$\text{or } W = m\omega^2 \int_0^y y\,dy$$

$$\text{or } W = \frac{m\omega^2 y^2}{2}$$

$$\text{or, } W = \frac{1}{2} m\omega^2 y^2$$

The total work done on the particle will remain in the form of potential energy.

$$\text{ie } E_p = \frac{1}{2} m\omega^2 y^2 \quad \text{— (iv)}$$

B. Kinetic Energy = The kinetic energy of the particle moving with velocity (v) is given by:-

$$E_K = \frac{1}{2} mv^2 \quad \text{— (v)}$$

Road map: for E_p: a, F, dw, W   for E_k: v, E_k At last sum $E_p + E_k$

But; the velocity (v) of particle is;

$$v = \omega\sqrt{r^2 - y^2} \quad \text{— (VI)}$$

from eqn (V) and (VI) then

or, $E_k = \frac{1}{2} m [\omega\sqrt{r^2-y^2}]^2$

or, $E_k = \frac{1}{2} m\omega^2 (r^2 - y^2)$ — (VII)

The total energy of particle vibrating in SHM is obtained after adding eqn (IV) and eqn (VII)

Then; $T = E_p + E_k$

or, $T = \frac{1}{2} m\omega^2 y^2 + \frac{1}{2} m\omega^2 (r^2 - y^2)$

or, $T = \frac{1}{2} \cancel{m\omega^2 y^2} + \frac{1}{2} m\omega^2 r^2 - \frac{1}{2} \cancel{m\omega^2 y^2}$

or, $T = \frac{1}{2} m\omega^2 r^2$

Special cases:-

(i) At mean position, $y = 0$ then

$$E_K = \frac{1}{2} m\omega^2(r^2 - 0^2) = \frac{1}{2} m\omega^2 r^2$$

And;

$$E_p = \frac{1}{2} m\omega^2 y^2 = 0$$

Total Energy $(T) = E_K + E_p$

$$T = \frac{1}{2} m\omega^2 r^2$$

(ii) At the extreme position; $y = r$ then;

$$E_K = \frac{1}{2} m\omega^2 (r^2 - r^2) = 0$$

or, $E_p = \frac{1}{2} m\omega^2 y^2$

or, $E_p = \frac{1}{2} m\omega^2 r^2$

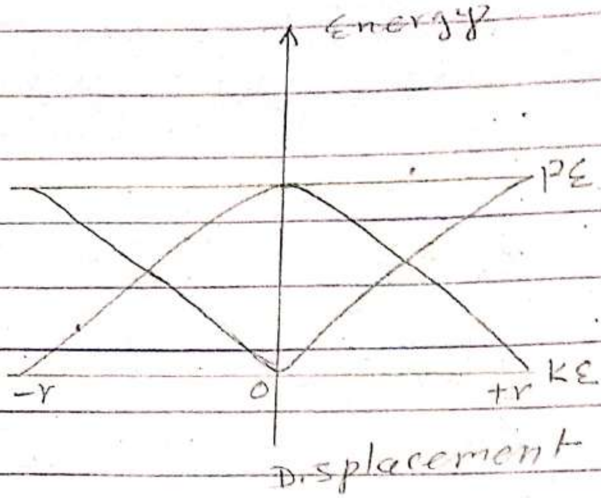
Thus; total energy $(T) = E_K + E_p$

$$T = 0 + \frac{1}{2} m\omega^2 r^2$$

$$T = \frac{1}{2} m\omega^2 r^2$$

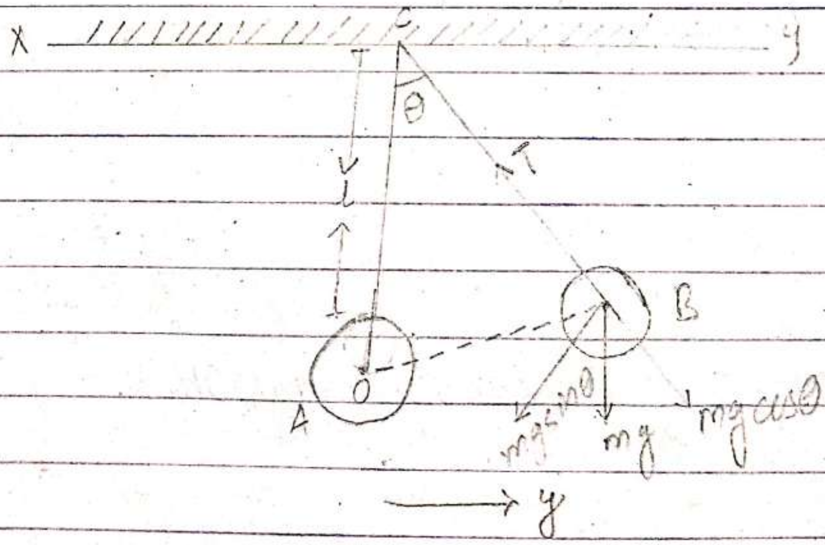
, the total energy is conserve in SHM.

The variation of kinetic energy and potential energy is a function in S.H.M



Simple pendulum

A heavy mass suspended from a rigid support by a weightless, inextensible and flexibe string which can vibrate about a fixed point is called simple pendulum. The point by which the string is suspended is called point of suspension.



Let us consider a pendulum of mass(m) which is suspended in a rigid support by a weightless, inextensible and flexible string of length (l) as shown in figure.

When the pendulum is displaed by an angle θ, then it reaches at the extreme position at point B.

from the figure;

$$\sin\theta = \frac{\text{arc OB}}{CB}$$

$$\sin\theta = \frac{y}{l}$$

for small angle θ, $\sin\theta \approx \theta$ st

$$\theta = \frac{y}{l} \quad \text{—(1)}$$

The weight of the body acting vertically downward, T is the tension acting in inward direction. Here, the weight of bob have two components. ie $mg\cos\theta$ acts along the string which balance the tension in string and the other component $mg\sin\theta$ acting along the perpendicular direction to the tension; which is the restoring force acting towards the mean position. So, the restoring force is; $f = -mg\sin\theta$ —(ii) where; negative sign indicates that the force is directed toward the mean position.

From Newton IInd Law of motion, then

$$F = ma \quad \text{--- (III)}$$

From eqn (II) and eqn (III) then

$$ma = -mg\sin\theta$$

$$\text{or, } a = -g\sin\theta$$

If θ is small, then $\sin\theta \simeq \theta$

$$\text{ie } a = -g\theta$$

$$a = -g\frac{y}{l}$$

$$a = -\left(\frac{g}{l}\right)y \quad \text{--- (IV)}$$

The eqn (IV) show that acceleration is directly proportional to the displacement and is directed toward the mean position

For time period;

The acceleration of a body in S.H.M is;

$$a = -\omega^2 y \quad \text{--- (v)}$$

eqn (iv) and eqn (v) then

$$\text{or, } \cancel{-}\omega^2 \cancel{y} = \cancel{-}\frac{g}{l}\cancel{y}$$

$$\text{or, } \omega^2 = \frac{g}{l}$$

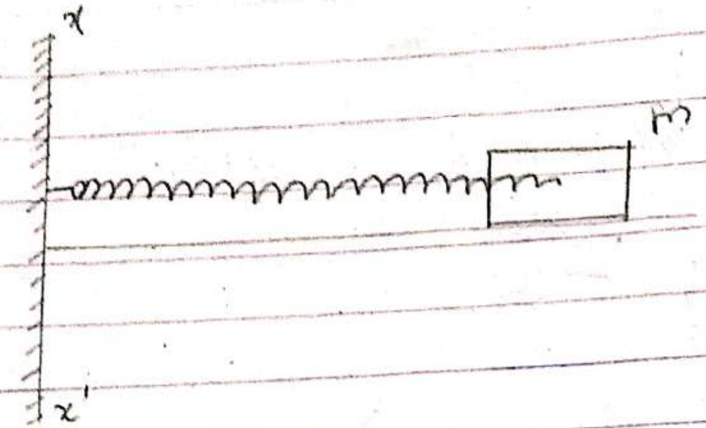
$$\text{or } \omega = \sqrt{\frac{g}{l}}$$

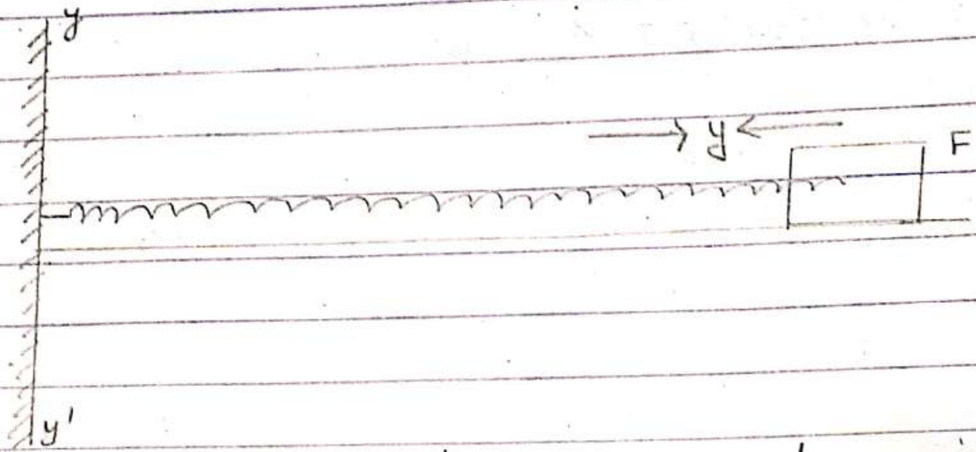
$$\text{or } \frac{2\pi}{T} = \sqrt{\frac{g}{l}}$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

This gives the time period of simple pendulum.

* <u>Mass Spring System (In Horizontal position)</u>





Let us consider a massless spring whose one end in fixed in rigid support and a body of mass 'm' is fixed to the another end : it kept in horizontal positionas shown in figure.

When the body is pulled by a distance 'y' and is released then the system vibrate in S.H.M from Hook's law; then

restoring force $\propto$ displacement

or, $f \propto y$

or, $F = -ky$

where; k = spring constant or force constant

From Newton IInd law of motion

$F = ma$ — (ii)

from eq (i) and (ii) then

$ma = -ky$

or $a = -\left(\frac{k}{m}\right) y$ — (iii)

The eqn (iii) shows that the acceleration is directly proportional to the displacement and the negative sign shows that it is directed toward the mean position. Hence, the motion of a spring in horizontal position in S.H.M in nature.

For time period:

The acceleration of a body vibrating in SHM is

$a = -\omega^2 y$ — (iv)

from eqn (iii) and (iv) then

$+\omega^2 \cancel{y} = +\left(\frac{k}{m}\right) \cancel{y}$

or $\omega^2 = \frac{k}{m}$

or $\omega = \sqrt{\frac{k}{m}}$

or $\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

$\therefore \quad T = 2\pi \sqrt{\frac{m}{k}}$

This is the time period of mass spring system in horizontal position.

* <u>Mass spring System in Vertical position:-</u>

Let us consider a massless spring whose one end is fixed to a rigid support at point A while at another end, mass(m) is attached and is kept in vertical position as if in figure.

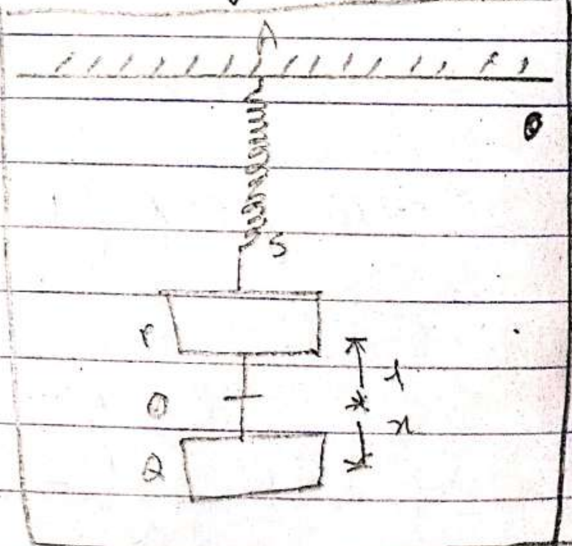
When the body is kept at the free end of spring; it gets stretched by a distance (l), then under the equilibrium position.

restoring force = weight of body

or $kl = mg$ — (i)

Where, k is spring constant or force constant.

When the body is pulled, then it get stretched by a distance (x) so under the Hook's laws



Restoring force $(F) = mg - k(l+x)$

or $F = mg - kl - kx$

or $F = mg - mg - kx$

or $F = -kx$ — (ii)

From Newton IInd law of motion; then

$F = ma$ — (iii)

But; from eqn (ii) and eqn (iii) then

$$ma = -kx$$
$$a = -\left(\frac{k}{m}\right)x \quad \text{---(iv)}$$

Thus, the eqn (iv) shows that the acceleration of the body is directly proportional to displacement and is directed towards the mean position. Thus, the motion of body is mass spring system in vertical position is simple harmonic in nature

For time period:-

The acceleration of a body vibrating in S.H.M $= -\omega^2 x$ — (v)

From eqn (iv) and (v) then,

or $\cancel{+}\omega^2 \cancel{x} = \cancel{+}\left(\frac{k}{m}\right)\cancel{x}$

or, $\omega^2 = \frac{k}{m}$

or, $\omega = \sqrt{\frac{k}{m}}$

or, $\frac{2\pi}{T} = \sqrt{\frac{k}{m}}$

or, $T = 2\pi\sqrt{\frac{m}{k}}$

This expression ~~the~~ gives the time period in mass spring system in vertical position.

Numerical zone

(*) A simple pendulum has effective length of 99.4 cm. Calculate the time period and taken time to complete 20 oscillation.

Solution

length (l) = 99.4 cm = 0.994 m

We have,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$= 2 \times 3.14 \times \sqrt{\frac{0.994}{10}}$$

$$= 2 \times 3.14 \times \sqrt{0.0994}$$

$$= 1.98 \text{ sec}$$

Time taken to complete 20 oscillation

$$= 20T$$
$$= 20 \times 1.98$$
$$= 39.6 \text{ sec}$$

(*) An object moving with simple harmonic motion has an amplitude of 0.02m and frequency of 20Hz. Calculate (i) accelerate at the mean and extremepoint (ii) velocity to that corresponding point

Solution,

Amplitude(a) $= 0.02m = r$

frequency $= 20 Hz$

(i) At mean position, $y = 0$

$V = \omega\sqrt{r^2 - y^2}$

$V = r \cdot \omega$

$V = r 2\pi f$

$V = 0.02 \times 2 \times \frac{22}{7} \times 20$

$V_{max} = 2.51 m/s$

$a_{min} = -\omega^2 y = 0$

(ii) at extreme position; $y = r$

$V = \omega\sqrt{r^2 - r^2}$

$V = \omega\sqrt{0}$

V[redacted] $= 0$

$a = \omega^2 r$

$= 4\pi^2 f^2 r$

$= 4 \times 9.8 \times 20^2 \times 0.02 = 313.6 m/s^2$

(*) A simple pendulum whose length is 1m oscillating 30 times per time in certain place. What is acceleration due to gravity?

Given,

$length(l) = 1m$

$frequency(f) = \frac{30}{1 min} = \frac{30}{60} = 0.5$ per sec

$Time\ period(T) = \frac{1}{f} = \frac{1}{0.5} = 2$ sec

We have

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\cancel{2} = \cancel{2} \times 3.14 \times \sqrt{\frac{1}{g}}$$

$$3.14 = \sqrt{g}$$

$$g = (3.14)^2$$
$$g = 9.8\ m/s^2$$

Rotation of Rigid bodies

* Moment of force or torque on a body:-

The product of force and perpendicular distance between the axis of rotation and line of axis of force is called moment. It SI unit is N-m and it dimension is $[ML^2T^{-2}]$.

Mathmetically it is define by;

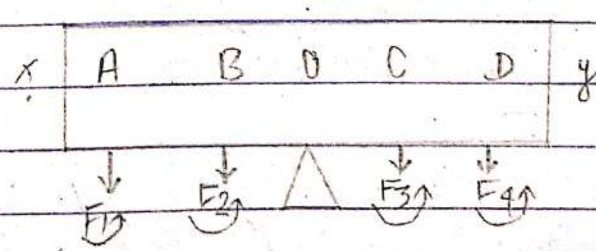
moment of force = force x perpendicular distance
$= F \times r$

* Principle of moment :-

It states that, " If a body is an equilibrium under the action of different forces then the sum of clockwise moment is equal to the sum of anticlockwise moment.

ie. Total clockwise moment = Total anti-clockwise moment

Proof:-



$\boxed{\tau = \text{Tau}}$

Let us consider a rectangular rigid bar XY which in equilibrium under the action of different force F_1, F_2, F_3 and F_4 acting at point A, B, C and D respectively as shown in figure.

Assume, F_1 and F_2 produce anti-clockwise moment while F_3 and F_4 produce clockwise moment.

Then;

Sum of clockwise moment $= F_3 \times OC + F_4 \times OD$

Similarly, Sum of anti-clockwise moment $= F_1 \times OA + F_2 \times OB$

Now, using principle of moment;

Sum of clockwise moment = Sum of anticlockwise moment

or, $F_3 \times OC + F_4 \times OD = F_1 \times OA + F_2 \times OB$

* Torque (τ)

The turning effect produce on a body about it's fix axis is called torque. It has the same magnitude to that of moment of force. It is denoted by (τ) and It is defined by;

$$\text{Torque} = F \times r$$

It's SI unit is N-m and has the dimension of $[ML^2T^{-2}]$ It is a vector quantity. In vector form

It can be written as

$$Torque\ (\tau) = Fr\sin\theta\,\hat{n}$$

① When $\theta = 90$; $\tau \to \tau_{max}$ such that $\tau_{max} = Fr$
② When $\theta = 0°$, $\tau \to \tau_{min}$ such that $\tau_{min} = 0$

⊛ Couple

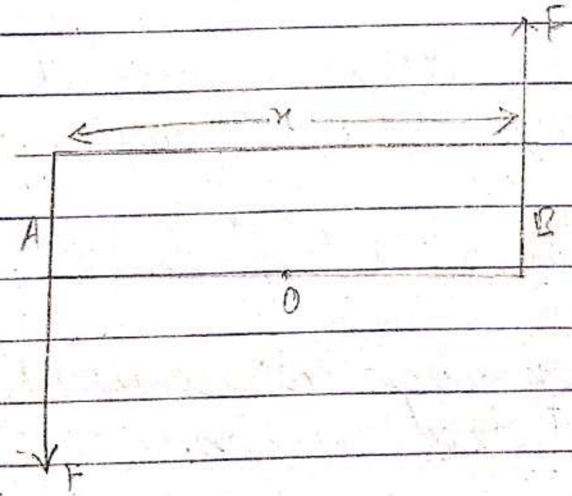
Two equal and oppisite parallel forces which line of action donot conside with each other is called Couple. The two force must have a same turning effect.

⊛ Moment of Couple

It is defined as the product of either of the forces and perpendicular distance between them. It is also called torque. Mathmetically, it is defined by;

$$Torque\ (\tau) = Force \times perpendicular\ distance$$
$$\tau = F \times x$$

Proof:



Let us consider a ~~rectangular~~ bar AB of length x in which the Force F is acting at ~~AB~~ point AB in opposite direction as shown in figure.

Here, the moment is produced in both the end of the bar which tends to rotate the bar in anticlockwise direction.

Now, the moment of couple = the sum of moment

$= F \times OA + F \times OB$

$= F \times (OA + OB)$

But; $(OA + OB) = x$

$\therefore$ Moment of couple $= F \times x$

* Forces in equilibrium

A rigid body will be in equilibrium, if a following conditions are satisfied

(a) First the vector sum of the forces acting on the body must be zero. ie $\Sigma f = 0$

~~(b)~~ This kind of condition is essential for the body to be in translational equilibrium.

(b) Second the net torque acting on the body must be zero. ie. $\Sigma \tau = 0$

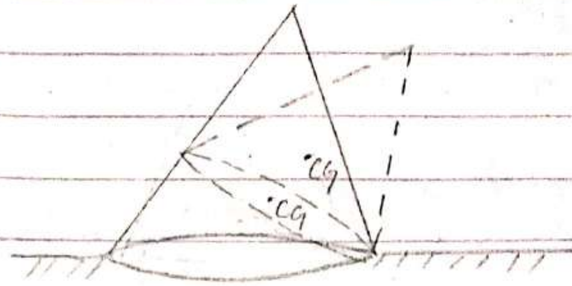
This kind of condition is essential for the body to be in rotational motion.

A body satisfing the condition of equilibrium is set to in perfect equilibrium.

Types of translational equilibrium

~~Types of translational~~

(i) Stable equilibrium:-



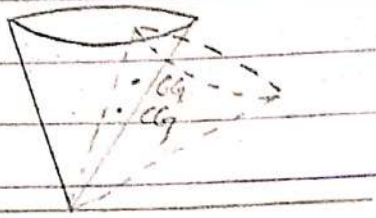
A body is in stable equilibrium if it returns to it's equilibrium position after it has been displace ~~light~~ slightly as shown in figure.

For this equilibrium, The centre of gravity lies at the lower position but the centre of gravity lies at upper position after it has been displaced.

(ii) Unstable Equilibrium

A body is in unstable equilibrium if it doesnot returns to it's equilibrium position and doesnot remain in the displaced position after it has been displace slightly as shown in figure.

For the unstable equilibrium the centre of gravity lies at upper position but the centre of gravity lies at lower position after it has been displaced.



(iii) Neutral Equilibrium

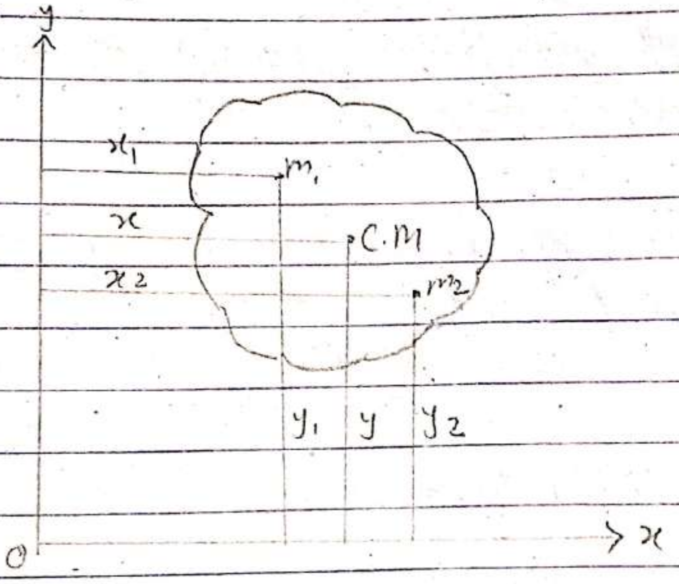
It body is in neutral equilibrium if it stays in the displaced position after it has been displaced slightly. as shown in figure.

In this equilibrium, the height of the centre of the gravity of the body remains at the same height from the base after it has been displaced slightly.

$$\Sigma m x = m_1 x_1 + m_2 x_2$$
$$\Sigma m y = m_1 y_1 + m_2 y_2$$
$$m = m_1 + m_2$$

* Centre of mass

The centre of mass of a body ~~of the t~~ is the define as the point where the hole mass of the body is suppose to be concantrated at centre of mass. An applied force produced linear acceleration but no rotation.



The x and y co-ordinates of centre of mass are;

$$(x, y) = \left(\frac{\Sigma m x}{M}, \frac{\Sigma m y}{M}\right)$$

These are the positron co-ordinate of centre of mass.

* Centre of gravity

It is a point where the hole weight of the body act and total gravitational torque of the body is zero.

The Characterstics of Centre of gravity of a body are:

(i) The position of centre of gravity depend upon the mass distrubeuted on the body.

(ii) The position of centre of gravity is independent of position and orientation.

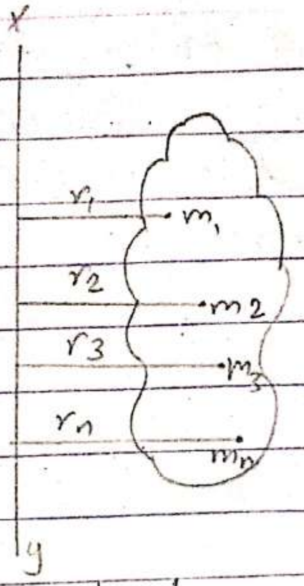
(iii) The centre of gravity lies within the body which may or may not contain mass.

(iv) The lower the position of centre of gravity in a body more is the stability.

* Moment of inertia

Moment of inertia of a rigid body about a particular axis is ~~depen~~ define as the sum of product of mass of all the particles contain in that body and square of their respective distance from the axis of rotation. It is scalar quantity. It's unit is kgm^2 in SI system and it's dimensional formula is $[ML^2T^0]$

-: Hyphes

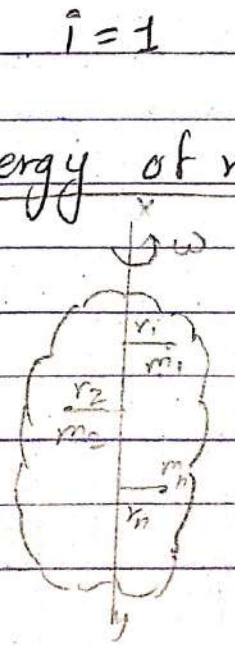


Assume, A rigid body consists of n-particles of masses $m_1, m_2, \ldots, m_n$ situated at distance $r_1, r_2, \ldots, r_n$ from the axis of rotation then the moment of inertia of the body about the axis of rotation is the sum of moment of inertia of all the particles about xy of which the body is made.

$$\text{Moment of Inertia}(I) = m_1 r_1^2 + m_2 r_2^2 + \cdots + m_n r_n^2$$

$$= \sum_{i=1}^{n} m_i r_i^2$$

* Kinetic Engergy of rotation of a body



Consider a rigid body consisting of n-particles of masses $m_1, m_2, m_3 --- m_n$ situated at distance $r_1, r_2, --- r_n$ respectively, then the moment of inertia of rigid body about the axis of rotation is;

$$I = \sum_{i=1}^{n} m_i r_i^2$$

Let the body rotates about an axis xy with an angular velocity (ω) and $v_1, v_2, v_3 --- v_n$ be the linear velocity of n-particles. Then; the total rotational kinetic energy of the given body is

$$E = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + --- + \frac{1}{2} m_n v_n^2$$

or, $$E = \frac{1}{2} m_1 (\omega r_1^2) + \frac{1}{2} m_2 (\omega r_2)^2 + --- + \frac{1}{2} m_n (\omega r_n)^2$$

or, $$E = \frac{1}{2} \omega^2 (m_1 r_1^2 + m_2 r_2^2 + --- + m_n r_n^2)$$

or, $$E = \frac{1}{2} \omega^2 \left(\sum_{i=1}^{n} m_i r_i^2 \right)$$

or, $$E = \frac{1}{2} \omega^2 I$$

$$\boxed{\text{or, } E = \frac{1}{2} I \omega^2}$$

Angular momentum

It is define as the product of linear momentum and distance between the object and axis of rotation. It is represented by L and is defined by;

Angular momentum (L) = Linear momentum x $\perp^r$ distance from the axis of rotation

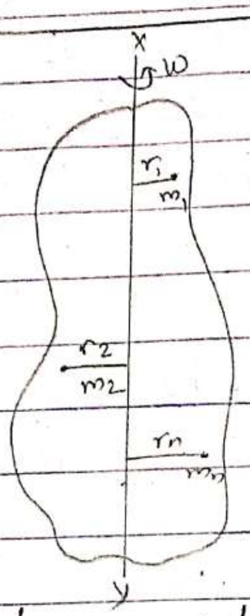
$$\text{or, } L = P \times r$$
$$\text{or, } L = mvr$$
$$\text{or, } L = mvr$$

Since; $v = \omega r$, then

$$\text{or } L = m\omega r r$$
$$\text{or } L = m\omega r^2$$

It unit in SI system is kgm^2/s and it dimension is $[ML^2T^{-1}]$

* Relation between angular momentum and moment of inertia:-



Consider a rigid body consisting of n-particles of masses $m_1, m_2, m_3 ---- m_n$ situated at distance $r_1, r_2, r_3 ----$ r_n respectively. Then the moment of inertia about the axis of rotation is;

$$I = m_1 r_1^2 + m_2 r_2^2 + ----- + m_n r_n^2 .$$

$$I = \sum_{i=1}^{n} m_i r_i^2 \text{ ——— } ①$$

Let the body rotates about an axis XY with an angular velocity (ω) and $v_1, v_2, v_3 --- v_n$ be the linear velocity of n-particles. Then; $v_1 = \omega_1, v_2 = \omega_2, ---- v_n = \omega r_n$. Thus the total angular momentum of the ~~body~~ given body is;

or $L = m_1 v_1 r_1 + m_2 v_2 r_2 + ---- + m_n v_n r_n$

or $L = m_1 (\omega r_1) r_1 + m_2 (\omega r_2) r_2 + ---- + m_n (\omega r_n) r_n$

or $L = m_1 \omega r_1^2 + m_2 \omega r_2^2 + ---- + m_n \omega r_n^2$

$$L = \omega(m_1r_1^2 + m_2r_2^2 + \ldots + m_nr_n^2)$$

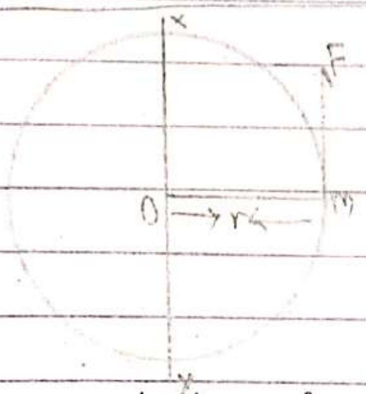
$$L = \omega\left(\sum_{i=1}^{n} m_ir_i^2\right)$$

or, $L = \omega I$

$L = I\omega$

Thus, the angular momentum is defined as the product of moment of inertia and angular momentum.

\# Relation between torque, moment of inertia and acceleration.



Let us consider a particle of mass 'm' rotating about an axis XY in a circular path of radius. Let 'F' be the tangencial force acting upon the body as shown in figure.

By the ~~definib~~ definition of Torque

$\tau = Fr$

$\tau = mar$

where; a is liner acceleration

Now the angular acceleration is;

$$\alpha = \frac{d\omega}{dt}$$

$$\alpha = \frac{d\frac{v}{r}}{dt}$$

$$\alpha = \frac{1}{r}\frac{dv}{dt}$$

$$\alpha = \frac{a}{r}$$

$$a = \alpha r \quad \text{— (11)}$$

From eqn (1) and (11) then;

$$\tau = m\alpha r r$$
$$\tau = m\alpha r^2$$
$$\tau = (mr^2)\alpha$$
$$\tau = I\alpha$$

Where, $I = mr^2$ is the moment of inertia of the particles about an axis XY.

* Law of Conseration of angular momentum

Statement: "If no external torque is applied upon the system rotating about its ~~one ax~~ own axis then the angular momentum of the system remain constant"

This the principle of conservation of angular momentum.

Proof:- The angular momentum of a rigid body about an axis with moment of Inertia 'I' & angular velocity 'ω' is;

$$L = I\omega$$

Differentiating both sides w.r.t time (t) then

$$\frac{dL}{dt} = I\frac{d\omega}{dt} = I\alpha \quad \text{--- (i)}$$

Where; angular acceleration $(\alpha) = \frac{d\omega}{dt}$

As we know that; Torque $(\tau) = I\alpha$ --- (ii)

From eqn (i) and eqn (ii) then

$$\frac{dL}{dt} = \tau$$

If no external torque is applied upon the system; then Torque $(\tau) = 0$

ie $\frac{dL}{dt} = 0$

or $dL = 0$

Integrating the above eqn

$\int dL = \int 0$

or L = constant

This can be written as;

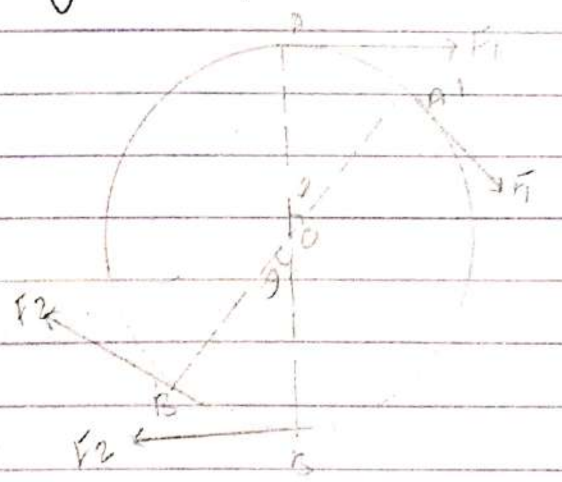
$L = I_1\omega_1 = I_2\omega_2 = \dots = I_n\omega_n$ = constant

In General;

$I\omega$ = constant

This proof the law of conservation of angular momentum.

<u>Work done by a couple</u>



Let us consider ~~of~~ a wheel of radius 'r' rotates through an angle 'θ'. Due to a couple of force acting tangentially in the rim as shown in figure. The torque acting on the wheel is given by;

$\tau = F \times AB$
$= F \times 2r$

Where; AB = diameter of wheel = $2r$

The work done by couple of Force 'F' is given by;

$$\text{or } W = F \times AA' + F \times BB'$$
$$\text{or, } W = F \times r\theta + F \times r\theta$$
$$\text{or, } W = F \times (r\theta + r\theta)$$
$$\text{or, } W = F \times (2r\theta)$$
$$\text{or, } W = (F \times 2r)\theta$$
$$\text{or, } W = \tau\theta$$

This gives the work done by a couple.

Note:
Do you know?

Translational motion	Rotational motion
distance (s)	θ
initial velocity (u)	ω_0
final velocity (v)	ω
acceleration (a)	α
mass (m)	I
Force (F)	τ
Work (w) = F.s	$W = \tau\theta$
kinetic Energy (E) = $\frac{1}{2}mv^2$	$KE = \frac{1}{2}I\omega^2$
linear momentum (p) = mv	$L = I\omega$

Q) A wheel which has a ~~momentum~~ moment of Inertia of 182.5 kgm^2 about an axis of rotation. If the wheel rotates at the rate of 1000 r.p.m. find the rotational Kinetic Energy of the way?

Soln

Moment of inertia (I) = 182.5 kgm^2

Rate of rotation (n) = 1000 rpm = $\frac{1000}{60}$ = ~~16.67~~ $revs^{-1}$

Angular velocity (ω) = $2\pi n = \frac{2 \times 22}{7} \times \frac{1000}{60}$

$= 104.76 \, rads^{-1}$

We have;

KE of rotation (V) = $\frac{1}{2} I \omega^2$

$= \frac{1}{2} \times 182.5 \times (104.76)^2$

$= 1 \times 10^6 J$

Equation of motion in Rotational Dynamics ~~Translational motion~~

Translation motion	Rotational motion
(i) $v = u + at$	$\omega = \omega_0 + \alpha t$
(ii) $s = ut + \frac{1}{2} at^2$	$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$
(iii) $v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$

Q) Wheel has a mass of 50 kg and radius of ~~dire~~ gyration 1m it is brought to rest of speed up 1800 rpm in 30 sec by a uniform redarding Torque. Find the value of Torque?

mass $(m) = 50\,kg$

Radius of gyration $(K) = 1m$

Final angular ~~velocity~~ speed $(\omega) = 0$

Initial angular speed $(\omega_0) = 2\pi f = 2 \times \frac{22}{7} \times \frac{1800}{60}$ rads^{-1}

$= 188.4\ rads^{-1}$

Time $(t) = 30\ sec$

we have ;

Moment of inertia $(I) = mk^2$

$= 50 \times 1^2$

$= 50\ kgm^2$

Also; $\omega = \omega_0 + \alpha t$

$\alpha = \frac{\omega - \omega_0}{t}$

$= \frac{0 - 188.4}{30}$

$= -6.28\ rads^{-2}$

There fore; Torque $(\tau) = I\alpha$

$= 50 \times -6.28$

$= -314\,Nm$

The magnitude of Torque $= 314\,Nm$

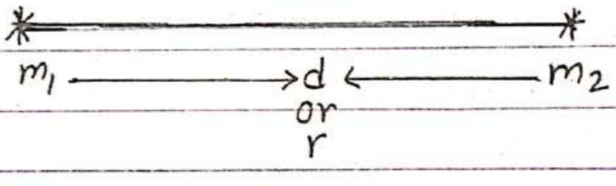
Magnetism

The properties of a magnetic by which it attract magnetic substances ~~is~~ is called magnetism. Magnet exhibt the property like

1. Attractive property
2. Repulsive property
3. Inductive property
4. Directional property

Coloumb's law of magnetism

This law states that "the force of attraction between two magnets is directly proportional to the square of distance between them,



If m_1 and m_2 be the pole strength and d be the distance between two poles. Then, according to the law of magnetism:

$$F \propto m_1 . m_2 \quad \text{—(i)}$$

$$F \propto \frac{1}{r^2} \quad \text{—(ii)}$$

Combining ① and ⑪ then

$$F \propto \frac{m_1 . m_2}{r^2}$$

$$F = \frac{k m_1 . m_2}{r^2}$$

where, k = proportionally constant and its value depend upon the nature of medium used.

$$F = \frac{m_1 . m_2}{r^2}$$

But, in SI system; $K = \frac{\mu_0}{4\pi}$, μ_0 = permeability of free space and its value is $4\pi \times 10^{-7} Hm^{-1}$

So, we can write

$$F = \frac{\mu_0}{4\pi} \frac{m_1 . m_2}{r^2}$$

This called coloumb's law of magnetism.

Unit pole

If. the pole strength of a magnet are equal which are seperate by a distance of one meter and the force experience by them is $10^{-7}N$, then coloumb's law can be written as

$$F = \frac{\mu_0}{4\pi} \frac{m_1 . m_2}{r^2}$$

By def^n; $F = 10^{-7}N$

$m_1 = m_2 = m$

$r = 1$

So, $\cancel{10^{-7}} = \frac{\cancel{4\pi} \times \cancel{10^{-7}} \times m^2}{\cancel{4\pi} \times 1^2}$

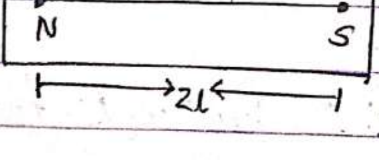
So $1 = m^2$

$m = \pm 1$

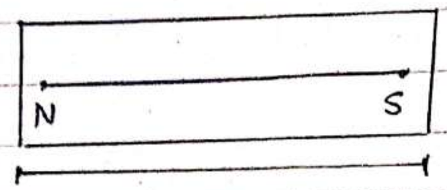
Here positive sign indicates north pole while the negative sign indicates south pole. The unit of pole strength is A.m or weber

Effective length

The distance between two poles of a magnet is called effective length. It is donated by $2l$.



Geometric length

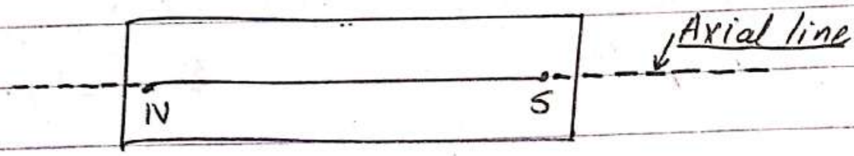


The distance between two ends of a magnet is called geometric length.

Experimentally, it has been found

$EL = 0.85 \times GL$

Magnetic axis

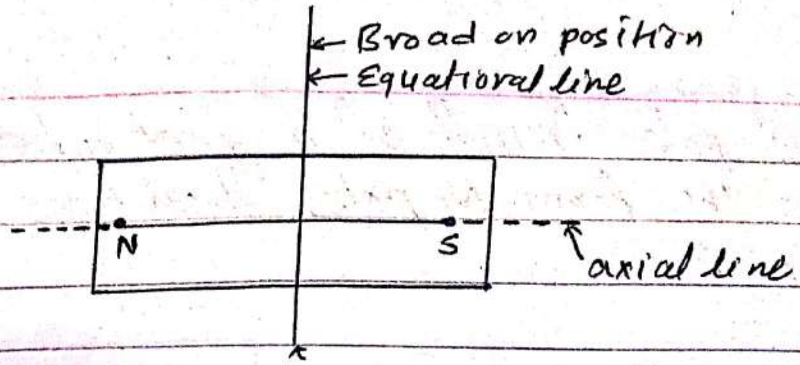


The line joining the two poles of a magnet is called magnetic axis.

When the magnetic axis is extended outward is called axial line.

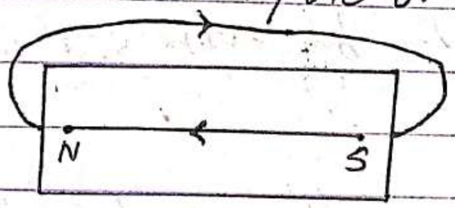
Equatorial line

A line perpendicular to the axial line which is the perpendicular bisector of magnetic axis.



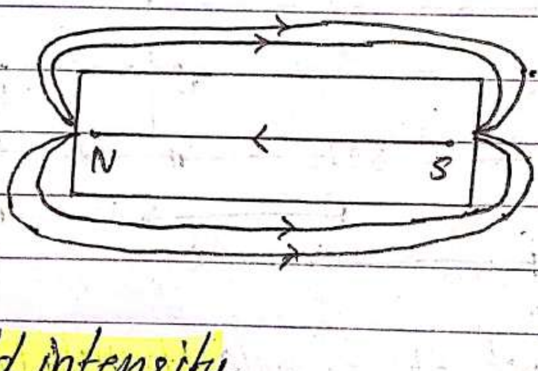
Magnetic movement

It is define as the product of the pole strength of a magnet which it effective length. It is ~~dento~~ denoted by $\vec{m}$ and defined by $\vec{M} = m \times 2l$ (It is vector quantity) and its direction is from south pole to north pole of a magnet.



Magnetic field

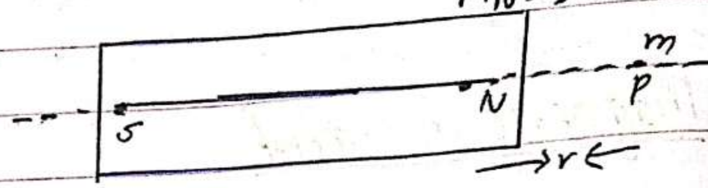
The region around a magnet within which we can feel it effect is called magnetic field.



Magnetic field intensity

The force experience by a unit north pole placed at that point is called magnetic field intensity. It is donated by B.

If m be the pole strength of magnet which is at point P at a distance 'r' from N-pole, then force experienced by unit N-pole is;



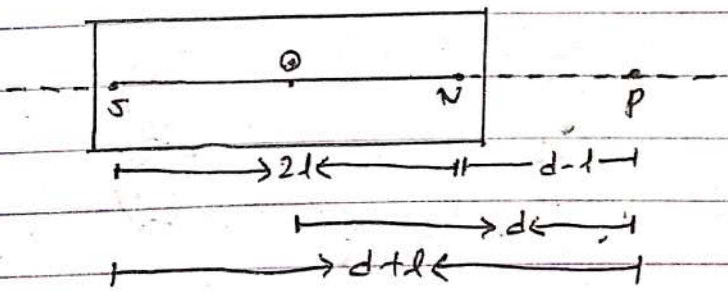
$$F = \frac{\mu_0 m . M_N}{4\pi r^2}$$

Then,

$$B = \frac{F}{M_N} = \frac{\mu_0 m}{4\pi r^2}$$

Its unit is N/Am. It can also be expressed in term of telsa (T) or weber/m².

⊛ Value of magnetic field intensity on the axial line



Let us consider a bar magnet SN of pole strength (m) and effective length (2l) as shown in figure.
Assume, P be the point at distance 'd' from the centre of the magnet. magnetic field intensity is to be calculated

from the figure

$OP = d$

$NP = OP - ON = d - l$

$SP = SO + OP$
$= d + l$

The magnetic field intensity at point P due to N-pole is

$$\vec{B_N} = \frac{\mu_0 m}{4\pi (NP)^2} \text{ along NP}$$

$$= \frac{\mu_0 m}{4\pi (d-l)^2} \quad \text{—(i)}$$

Similarly, the magnetic field intensity at point P due to S-pole is given by;

$$\vec{B_S} = \frac{\mu_0 m}{4\pi (SP)^2} \text{ along SP}$$

$$= \frac{\mu_0 m}{4\pi (d+l)^2} \quad \text{—(ii)}$$

Since, the maginitude of magnetic field intensity due to N-pole is greater than that of S-pole. So, the resultant magnetic field intensity ($\vec{B}$) is given by;

$$\vec{B} = \vec{B}_N - \vec{B}_S$$

$$\vec{B} = \frac{\mu_0 m}{4\pi(d-l)^2} - \frac{\mu_0 m}{4\pi(d+l)^2}$$

$$\vec{B} = \frac{\mu_0 m(d+l)^2 - \mu_0 m(d-l)^2}{4\pi(d^2-l^2)^2}$$

$$\vec{B} = \frac{\mu_0 m[\cancel{d^2} + 2dl + \cancel{l^2} - \cancel{d^2} + 2dl - \cancel{l^2}]}{4\pi(d^2-l^2)^2}$$

$$\vec{B} = \frac{\mu_0 m \, 4dl}{4\pi \cdot (d^2-l^2)^2}$$

$$\vec{B} = \frac{\mu_0 2d \cdot (m \times 2l)}{4\pi(d^2-l^2)^2}$$

$$\vec{B} = \frac{2\mu_0 \vec{M} d}{4\pi(d^2-l^2)^2} \quad \text{—(iii)}$$

for short magnet ; $d >>> l$, then $(d^2-l^2)^2 \rightarrow d^4$

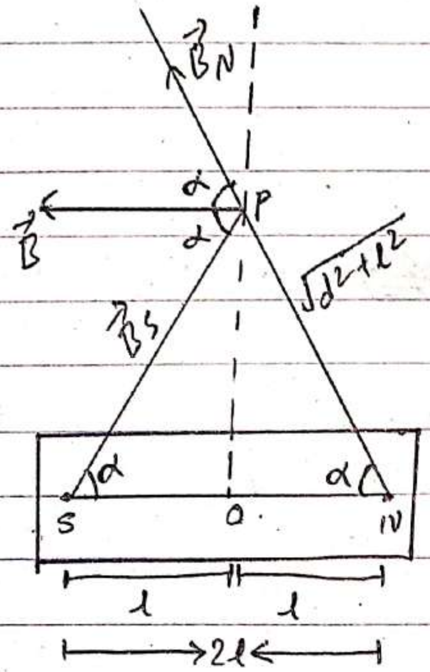
$$\vec{B} = \frac{2\mu_0 \vec{M} d}{4\pi d^4}$$

$$\vec{B} = \frac{2\mu_0 \vec{M}}{4\pi d^3}$$

Summary,

- First find N-pole
- Second find S-pole
- North pole > South pole
- find the resultant ($\vec{B}$)

(*) Value of magnetic field intensity on the equatorial line (Broad Side on position)



Let us consider bar magnet SN of effective length 2l and pole strength m as shown in figure.
Assume, P be the point at a distance d from the centre of the magnet where the value of magnetic field intensity is to be determined from the figure.

$SO = ON = l$
$OP = d$
$SP = NP = \sqrt{d^2 + l^2}$

Now, the magnetic field intensity at point P due to N-pole is;

$$\vec{B}_N = \frac{\mu_0 m}{4\pi (NP)^2} \text{ along } NP$$

$$= \frac{\mu_0 m}{4\pi (\sqrt{d^2+l^2})^2}$$

$$= \frac{\mu_0 m}{4\pi (d^2+l^2)} \quad \text{--- (i)}$$

Similarly, the magnetic field intensity at point P due to S-pole is;

$$\vec{B}_S = \frac{\mu_0 m}{4\pi (SP)^2} \text{ along } SP$$

$$= \frac{\mu_0 m}{4\pi (\sqrt{d^2+l^2})^2}$$

$$= \frac{\mu_0 m}{4\pi (d^2+l^2)} \quad \text{--- (ii)}$$

Since the magnitude of two vector equal so the resultant field intensity can be calculated from the parallelogram law of vector addition.

$$\text{or } \vec{B} = \sqrt{B_N^2 + B_S^2 + 2\vec{B}_N\vec{B}_S\cos 2\alpha}$$

$$\text{or, } \vec{B} = \sqrt{B_N^2 + B_N^2 + 2B_N^2\cos 2\alpha}$$

$$\text{or, } \vec{B} = \sqrt{2B_N^2 + 2B_N^2 \; (1+\cos 2\alpha)}$$

$$\text{or } \vec{B} = \sqrt{2B_N^2(\cancel{1} + 2\cos^2\alpha - \cancel{1})}$$

$$\text{or } \vec{B} = \sqrt{4B_N^2\cos^2\alpha}$$

$$\text{or, } \vec{B} = 2B_N\cos\alpha$$

$$\text{or, } \vec{B} = 2\frac{\mu_0 m}{4\pi(d^2+l^2)} \cdot \frac{l}{(d^2+l^2)^{1/2}}$$

$$\text{or } \vec{B} = \frac{\mu_0(m\times 2l)}{4\pi(d^2+l^2)^{3/2}}$$

$$\text{or } \vec{B} = \frac{\mu_0\vec{M}}{4\pi(d^2+l^2)^{3/2}} \quad \text{—(iii)}$$

For short magnet

$d >>> l$ then

$$\text{or } \vec{B} = \frac{\mu_0\vec{M}}{4\pi(d^2)^{3/2}}$$

$$\text{or } \vec{B} = \frac{\mu_0\vec{M}}{4\pi d^3}$$

A Bar magnet of magnetic length 10cm has a magnetic movement of 1.2 ampere m^2 Calculated the magnetic intensity at a point 20 cm from each pole.

magnetic length $(2l) = 10\,cm = 0.1m$

$l = 0.05m$

Magnetic moment $(M) = 1.2\ Am^2$

distance from each pole $(\sqrt{d^2+l^2}) = 20\,cm$

$= 0.2m$

Magnetic field intensity $(\vec{B}) = ?$

This is the case broad side on position.

So, magnetic field intensity is;

or $\vec{B} = \dfrac{\mu_0 \vec{M}}{4\pi (d^2+l^2)^{3/2}}$

or $\vec{B} = \dfrac{\cancel{4\pi} \times 10^{-7} \times 1.2}{\cancel{4\pi}\ (0.2)^3}$

$\vec{B} = 1.5 \times 10^{-5}\ T$

or $\vec{B} = \dfrac{1.2 \times 10^{-7}}{0.008}$

$\vec{B} = \dfrac{\overset{0.3}{\cancel{1.2}} \times 10^{-7} \times 10^3}{\cancel{8}\,2}$

A bar magnetic has a length 8 cm. The magnetic field intensity as a distance of 5cm from both ends of the magnet is 4×10^{-6} T ~~and~~ calculate the pole strength.

magnetic length $(2l) = 8cm = 0.08 m$

$l = 0.04 m$

pole strength $(m) = ?$

distance from each pole $(\sqrt{d^2+l^2}) = 5cm$

$= 0.05 m$

magnetic field intensity $= 4\times10^{-6}$ T

This is the case broad side on position

So; magnetic field intensity is;

$$\vec{B} = \frac{\mu_0 \vec{M}}{4\pi(d^2+l^2)^{3/2}}$$

$$\vec{M} = \frac{\vec{B}\,4\pi(d^2+l^2)^{3/2}}{4\pi\times10^{-7}}$$

$$\vec{M} = \frac{4\times10^{-6}\times(0.05)^3}{10^{-7}}$$

$$\vec{M} = 4\times10^{-6+7}\times125\times10^{-6}$$

$$\vec{M} = 500\times10^{-5}$$

$$\vec{M} = 5\times10^{-3}$$

$$\vec{M} = m\times2l$$

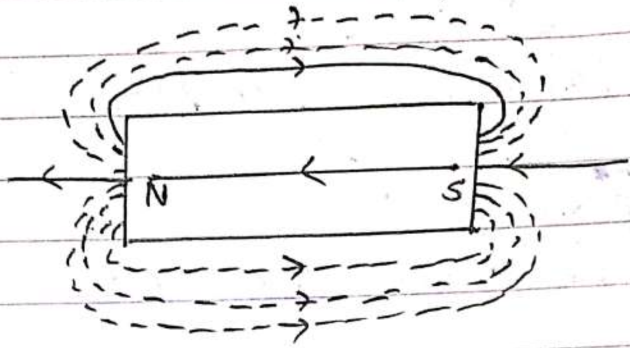
$$m = \frac{\vec{M}}{2l}$$

$$= \frac{5\times10^{-3}}{0.08}$$

$$= 0.0625$$

Magnetic lines of force

It is a path that a unit pole would take if it is allowed to move freely the direction of magnetic lines of force start form north pole to south pole of a magnet.



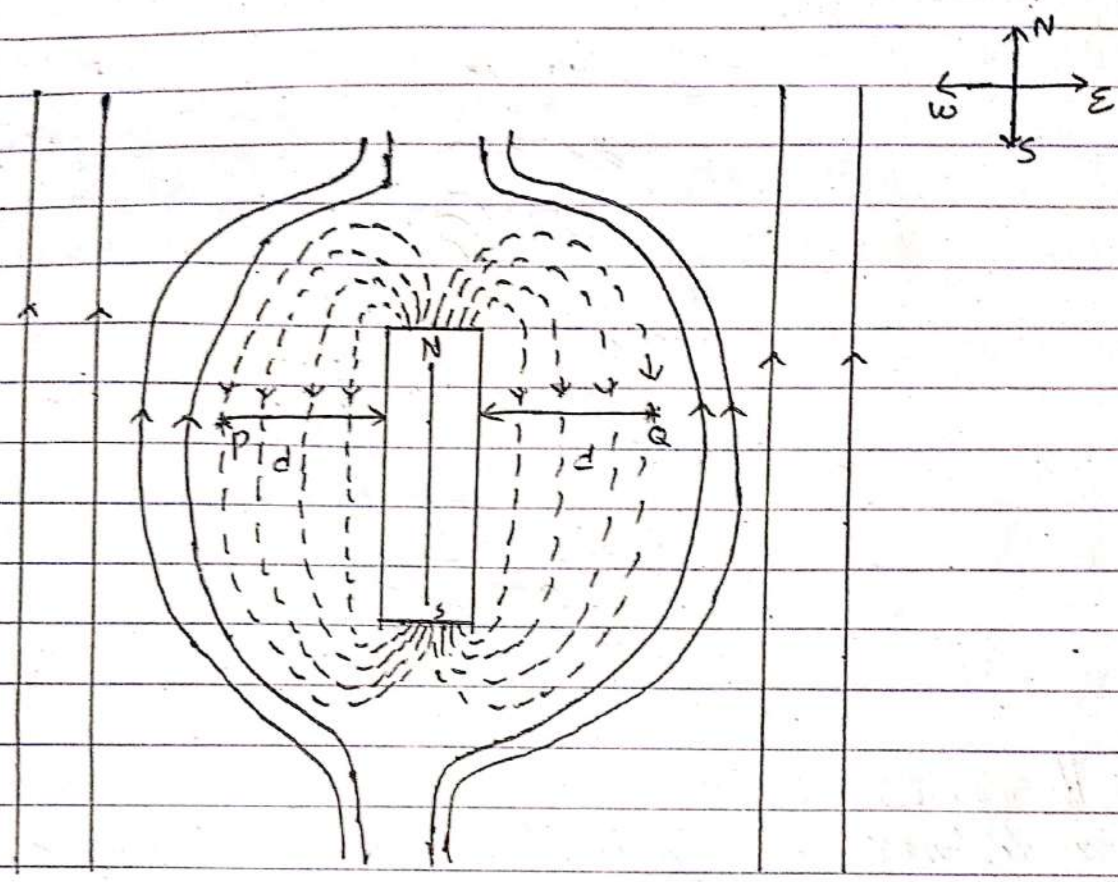
Properities of magnetic lines of force

They are closed and continous curved. They start from N-pole and terminate in S-pole. They are always held normally to the ~~magnet~~ tangent. They never intersect each other.

The neutral point

It is a point at while the field due to the magnet is completely neutralized. By the horitozontal component of earthes magnetic field. At neutral point a compass is unable to show proper direction because the value of $\vec{B}$ and $\vec{H}$ are equal and opposite ie $\vec{B} \simeq \vec{H}$.

A. Bar manget placed with its north pole pointing towards geographical north:-



In this position, the neutral points are observed on equatrial line at equal machine d from the central of magnet. In figure P and Q represents the neutral points.
If $\vec{B}$ be the magnetic field intensity and $\vec{H}$ be the earth's magnetic field intensity, then at neutral points $\vec{B} = \vec{H}$ such that

$$\vec{B} = \vec{H} = \frac{\mu_0 \vec{M}}{4\pi (d^2 + l^2)^{3/2}}$$

$$\vec{B} = \vec{H} = \frac{2\mu_0 \vec{M} d}{4\pi (d^2 - l^2)^2}$$

Numerical

A magnetic decoils of moment 1.44 Am^2 is placed horitzontally with north pole pointing towards north. find the position of neutral point assuming a very short magnet (Horitzonal component of earth magnetic field = 0.3×10^{-4} T)

Solution

- Magnetic moment $(\vec{m}) = 1.44\ Am^2$
Horitzontal component of earth magnetic field $(\vec{H}) = 0.35 \times 10^{-4}$ T
$\mu_0 = 4\pi \times 10^{-7}\ TmA^{-1}$

By question the N-pole of magnet is pointing towards geographical north. So, it is the case of Broad side on position and we have;

$$\vec{B} = \frac{\mu_0 \vec{m}}{4\pi (d^2 + l^2)^{3/2}}$$

For a short, $d >> l$

$$\vec{B} = \frac{\mu_0 \vec{m}}{4\pi d^3}$$

At neutral point $\vec{B} = \vec{H}$.

So, $\vec{H} = \frac{\mu_0 \vec{M}}{4\pi d^3}$

So, $d^3 = \frac{\mu_0 \vec{M}}{4\pi \vec{H}}$

So, $d^3 = \frac{\cancel{4\pi} \times 10^{-7} \times 1.44}{\cancel{4\pi} \times 0.35 \times 10^{-4}}$

So, $d^3 = 4.114 \times 10^{-3}$

So, $d = \sqrt{4.114 \times 10^{-3}}$

So, $d = 1.6 \times 10^{-7}$

So, $d = 0.16 m$

Tangent law

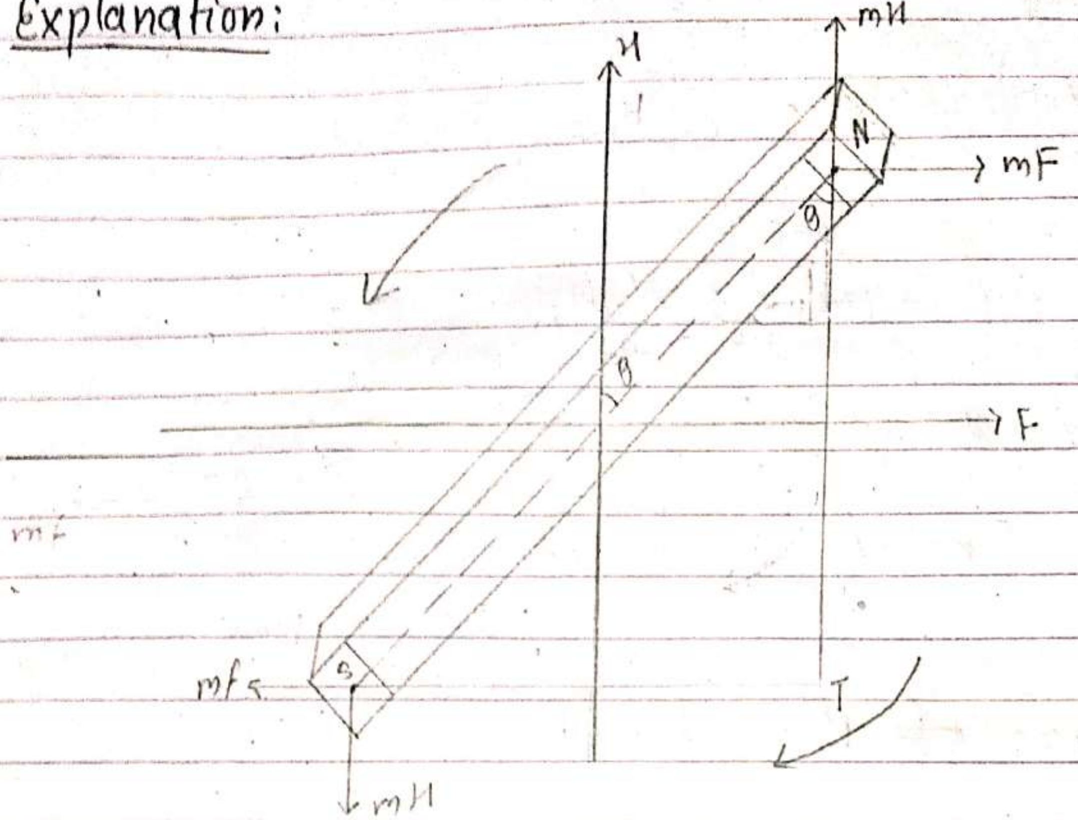
A. Statement :- This law state that, "If a magnetic needle is suspended freely in ~~a~~ two uniform perpendicular magnetic field. F and H then it rest along the resultant of these two field making an angle θ. such that,

$F \propto H$

or, $F = H \tan\theta$

where θ = Angle made by the axis of the rotation with a field H.

B. <u>Explanation:</u>



Let us take a bar magnet. NS suspended freely in uniform perpendicular magnet field H and F such that it come to rest by making an 'θ' with the field H. Then clearly, mf and mH be the two forces acting upon the two fields F and H respectively as shown in figure.

The moment of the couple due to field $H = mH \times ST$ — (1)
This tends to rotate the target in anticlockwise ~~motion~~ direction.
Again, the moment of the couple due to the field $F = mf \times NT$ — (ii)
This tends to rotate the magnet in clockwise direction.

from the principle of moment, we get;

$$mH \times ST = mf \times NT$$
$$F = H \times \frac{ST}{NT} \quad \text{— (iii)}$$

From the figure;

$$\tan\theta = \frac{ST}{NT} \quad \text{--- (iv)}$$

from eqn (iii) and (iv) then;

$$F = H \tan\theta$$

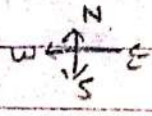
This verified Tangent law in magnetism.

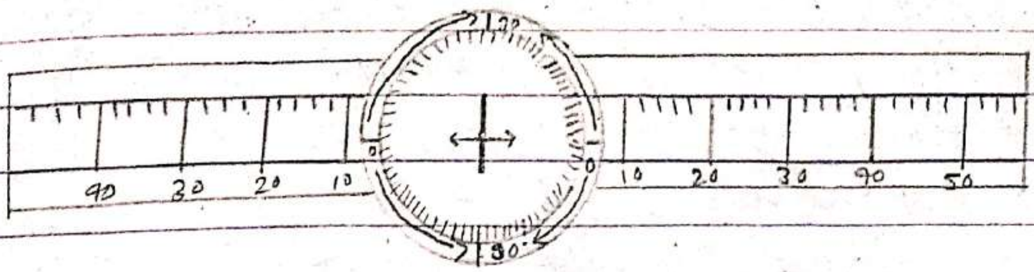
* Deflection ~~of~~ Magnetometer:

It is an instrument which is used for measuring and comparing magnetic moments and intensity of magnetizin-g field. It works on the principle of tanget law.

* Construction :-

It consists of a small compass needle pivoted at the centre of graduated circular scale. These graduations are marked from 0-90° in each quadrant. The light Al-pointer is attached at right angle to the compass needle at the centre. These are enclosed in a cylindrical box called magnetometer box. The magnetometer box is kept in a wooden flat having two long arms. A metre scale is fitted on two arms.



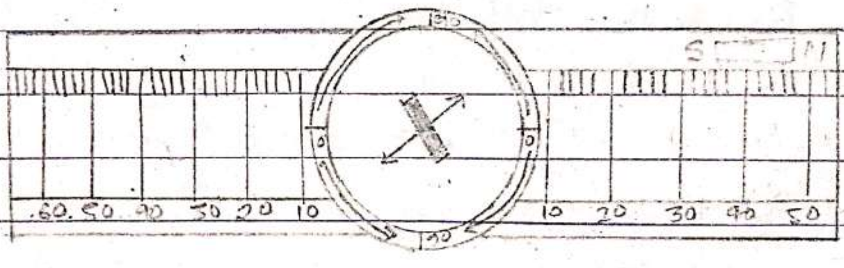


In order to study the magnetic moment of two magnets, the two position are determined. They are.

① ~~TAN~~ Tan-A position
⑪ Tan-B position

A. Tan-A position:-

In this position, the magnetometer is placed with its arms along the east-west direction and the magnet is placed in one of its arms symmetrically pointing its N-pole towards east or west direction. Here, the magnetometer box is rotated in such a way that the Al-pointer reads 0-0.



Suppose 'd' be the distance from the centre of magnet to the centre of needle. Let M be its magnetic moment and $2l$ be its effective length. Since, the needle is on end-on position, then the magnetic field intensity at such position is,

$$B = \frac{2\mu_0 M d}{4\pi(d^2-l^2)^2} \quad \text{——— (i)}$$

Since, two magnetic fields B and H are perpendicular to each other and the needle will be deflected by an angle θ. Then applying tangent law,

$$B = H\tan\theta \quad \text{——— (ii)}$$

from eqn (i) and (ii) them

$$H\tan\theta = \frac{2\mu_0 M d}{4\pi(d^2-l^2)^2}$$

$$M = \frac{4\pi H\tan\theta\,(d^2-l^2)^2}{2\mu_0 d}$$

This is the magnetic moment on Tan A position. For a short again, $d >\!>\!> l$. then

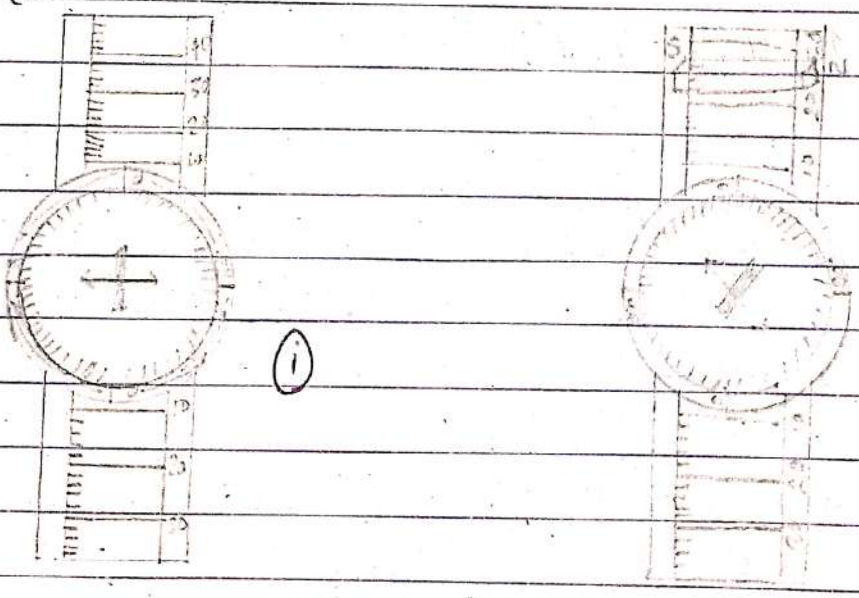
$$M = \frac{4\pi H\tan\theta\, d^3}{2\mu_0}$$

B. Tan-B position:-

In this position the magnetometer is placed with its arms pointing along north-south direction and the bar magnet is placed on one of its arm ~~point~~ perpendicular to it pointing its north pole towards east or west direction. as soon in figure: Here, the magnetometer is rotate such that the aluminium pointer reads 90°-90°

Since the needle is in broadside position of magnet then the magnetic field intensity due to the magnetic at it needle with B.

$$B = \frac{\mu_0 M}{4\pi(d^2+l^2)^{3/2}} \quad \text{——} \quad (1)$$



(i) (ii)

Suppose 'd' be the distance from the centre of magnet to the centre of needle. Let M be its magnetic moment and 2l be its effective length. Since; the needle is on ~~end~~ broadside-on position, then the magnetic field intensity at such position is.

~~Since, two magnetic fields B and H are perpen~~

$$B = \frac{\mu_0 M}{4\pi(d^2+l^2)^{3/2}}$$

~~This is the magnetic~~

Since, two magnetic fields B and H are perpendicular each other and the needle will be deflected angle θ. then applying tangent law;

$$B = H \tan\theta \quad \text{—— (ii)}$$

from eqn (i) and (ii) then

$$H \tan\theta = \frac{\mu_0 M}{4\pi(d^2+l^2)^{3/2}}$$

$$M = \frac{4\pi H \tan\theta (d^2+l^2)^{3/2}}{\mu_0}$$

This is the magnetic moment on Tan B position

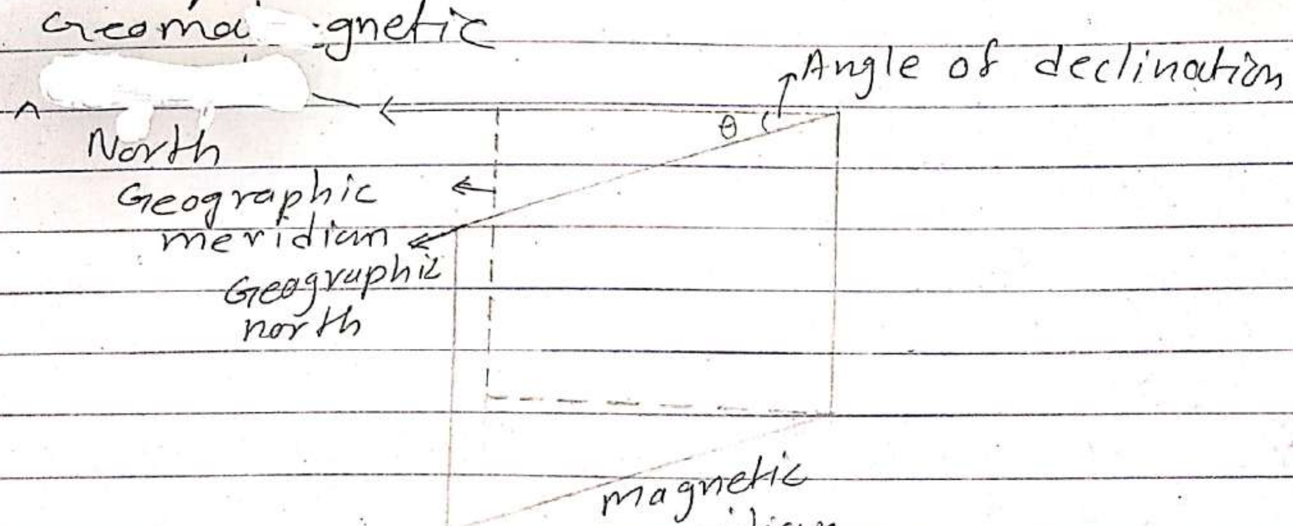
For a short magnet, $d >>> l$ then

$$M = \frac{4\pi H \tan\theta\, d^3}{\mu_0}$$

Earth- Magnetism

Angle of declination

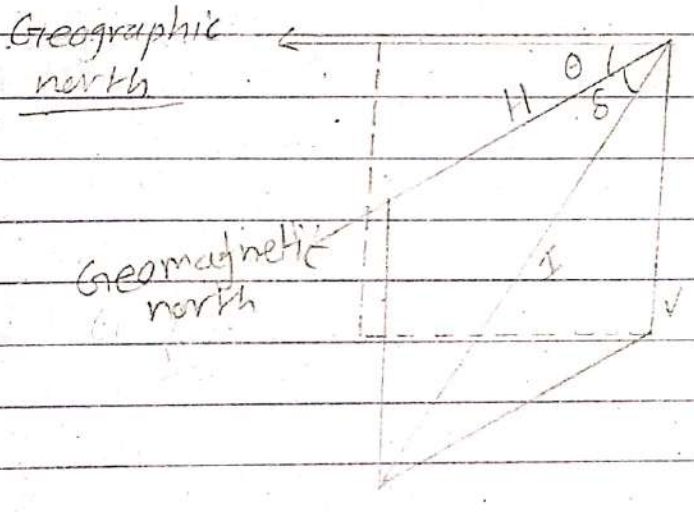
The vertical line passing through the axis of a needle suspended freely through its center of gravity is called magnetic meridain. Similarly the vertically line passing through the line joining geographical north and south pole is called geographic meridian. The angle between the magnetic meridian and geographic meridian is called angle of declination. It is denoted by θ. Ant it's valeue varies from place to place.



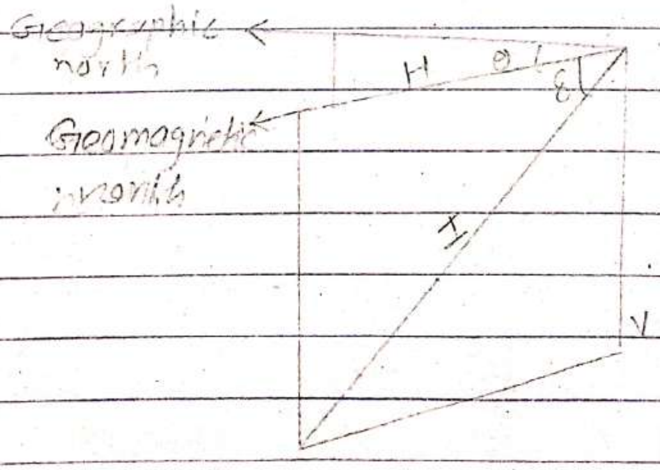
Angle of dip

It is defined as the angle between the intensity of a earth magnetic field and its horizonatal direction in the magnetic meridian. It is donated by δ. It value varies at the earth surface from zero at the geomagnetic equator to the maximum of 90° at the geo-magnetic poles. A compass needle doesnot work at the geomagnetic poles. Because the ~~horit~~ horizontal component at that place is 0.

ie $H = I \cos 90°$
$= 0$



(III) <u>Horizontal component of earth magnetic field:-</u>



The horizontal component of earth magnetic field, H is the component of earth magnetic field along the horizontal direction in the magnetic meridian.

$H = I \cos\delta$ - ① And verticall compunet $V = I \sin\delta$ - ⑪

Let, I be the intensity of earth's magnetic field at a place and H and V be its horizontal and vertical components respectively. In the magnetic meridian, the horizontal component is;

Dividing eqn ⑪ by ① then

$$\frac{I \sin\delta}{I \cos\delta} = \frac{V}{H}$$

$$\text{or, } \tan\delta = \frac{V}{H} \quad \text{—(III)}$$

Squaring and adding ① and ⑪ then

$$H^2 + V^2 = I^2\cos^2\delta + I^2\sin^2\delta$$

$$\text{or, } H^2 + V^2 = I^2(\cos^2\delta + \sin^2\delta)$$

$$\text{or, } H^2 + V^2 = I^2 \quad \text{—(iv)}$$

These are the equation (III) and (iv) Relate V, H, δ, I at place

The vertical and horizontal component are 0.2×10^{5} T & 0.34×10^{-5} T respectively. And calculate the angle of ~~declination~~ ~~depth~~ dip and total ~~flocks~~ flux density (I) at that place

Solution,

~~at~~ angle of dip $(\delta) = ?$

Total flux density $(I) = ?$

Vertical component $(V) = 0.2 \times 10^{-5}$ T

Horizontal component $(H) = 0.34 \times 10^{-5}$ T

From the relation

or, $\tan\delta = \dfrac{V}{H}$

or, $\tan\delta = \dfrac{0.2 \times \cancel{10^{-5}}}{0.34 \times \cancel{10^{-5}}}$

or, $\tan\delta = 0.59$

or $\delta = \tan^{-1}(0.59)$

or $\delta = 30.54°$

Also,

$H = I\cos\delta$

or, $I = \dfrac{H}{\cos \cancel{30.54} \delta}$

$I = \dfrac{0.34 \times 10^{-5}}{\cos 30.54}$

$I = 3.95 \times 10^{-6}$ T

Magnetic properties of materials:-

i) Magnetic field Intensity (H):-

The field intensity which produces magnetaization in a substance is called magnetic field intensity. It is denoted by H. and its unit is A/m or J/T.

ii) Intensity of Magnetization:-

The magnetic moment developed ~~that loft~~ per unit volume is called intensity of magetization. It is denoted by I and is defined by

$$I = \frac{M}{V}$$

In term of pole strength; Magnetic moment $(M) = m \times 2l$ and Volume $(V) = A \times 2l$

$$I = \frac{m \times 2l}{A \times 2l}$$

$$I = \frac{m}{A} \; ; \; m = \text{pole strength}$$

It's unit is ampere per m (A/m) or weber m^{-2}.

(iii) Magnetic Induction:

The magnetic Induction produce inside the specimen is the total number of magnetic field lines crossing the unit crossonal area where an area is ~~field~~ held perpendicular to the field lines. It is also known as magnetic flux density. It is denoted by B and It's unit is Tesla or weberm^{-2}.

The magnetic induction inside the magnetic specimen is the sum of magnetic flux density in vaccum and magnetic flux density in medium

ie $B = B_0 + B_M$

or, $B = \mu_0 H + \mu_0 I$

or $B = \mu_0 (H + I)$

where, H = magnetic field intensity
I = Intensity of magnetization

(iv) Magnetic Permeability (μ)

It is a measured of how permeable a material each for the passes of ~~l~~ magnetic lines of force through it. It is experimentally found that the magnetic induction inside the specimen B directably

~~prot~~ proportional to magnetizing field intensity.

$$B \propto H$$
$$B = \mu H$$
$$\mu = \frac{B}{H}$$

where; μ = magnetic permeability

Relative Permeability (μ_r)

It is define as the ratio of permeablity of medium to the permeability of free space. It is denoted by μ_r and its define by $\mu_r = \frac{\mu}{\mu_0}$

Magnetic susceptibility (χ) $\to$ kie

The intensity of magnetization is directional proportional to the magnetization field intensity.

ie $I \propto H$

or, $I = \chi H$

or $\chi = \frac{I}{H}$

These relation holds for para and diamagnetic materials. Here; χ = constant is called magnetic susceptibility which is defined as the ratio of intensity of magnetization to the magnetizating field intensity.

Relation between χ and μ_r

Let, B be the magnetic induction and H be the magnetizating field intensity and I be the intensity of magnetization then we have.

$$B = \mu_0 (H + I) \quad \text{— (i)}$$

And; $B = \mu H$ — (ii)

from eqn (i) and (ii)

$$\mu H = \mu_0 (H + I)$$

$$\frac{\mu}{\mu_0} = \frac{H+I}{H}$$

$$\frac{\mu}{\mu_0} = 1 + \frac{I}{H}$$

Since, $\mu_r = \frac{\mu}{\mu_0}$ and $\chi = \frac{I}{H}$ so; the above term becomes

$$\therefore \mu_r = 1 + \chi$$

* Classification of Magnetic materials

On the basis of behaviour of materials in a magnetizing field the magnetic materials are classifed as

(i) Diamagnetic Substance

Those magnetic substance which when place in magnetic field are weakly magnetize in a direction opposite to that of the applied field is called diamagnetic substances.

for example: Bismuth, Cu, Zn, Au, H_2O etc.

(ii) Paramagnetic Substance

Those substance which when placed in a magnetic field are weakly magnetized in the direction of applied field is called paramagnetic substance.

for example:- Al, Sb, $CuSO_4$, crown glass, etc.

(iii) Ferromagnetic substance

Those substance which when placed in a magnetic field are strongly magnetize in the direction of applied field is called ferromagnetic substance.

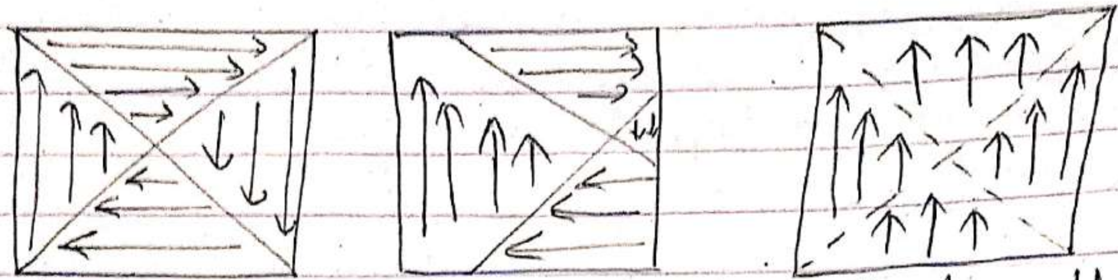
for example: Fe, Co, Ni, etc

* Properties of Diamagnetic, Paramagnetic & Ferromagnetic materials.

Property	Diamagnetic	Paramagnetic	Ferromagnetic
1) In Presence of magnet	feebly repelled by a magnet	feebly attracted by a magnet	strongly attracted by a magnet
2) In magnetizing field.	Weakly magnetized in the opposite direction of applied.	Weakly magnetized in the direction of applied field	Strongly magnetized in the direction of applied field
3) A rod of material freely suspened in magnetic field	It comes to rest in the perpendicular direction of applied field	It comes to rest along the direction of applied field.	It comes to rest along the direction of applied field.
4) Susceptibility (χ)	has small -ve value	has small +ve value	has large +ve value
5) Relative Permeability	<1	>1	has very large value
6) Example	Au, Cu, H_2O etc.	Sb, Al, $CuSO_4$ etc	Fe, Co, Ni

VI

* Domain theory of ferromagnetism.



a) In absence of H   b) In weak H   c) In strong H

In a ferromagnetic substances their are reason of atoms over which the magnetic moment is oriented or aligned in a particular direction called domains. A typical domain contains 10^{17} to 10^{21} atoms and occupies a volume of $10^{-12} - 10^{-8} m^3$.

In absence of external magnetic field H, the domains of ferromagnetic substances are randomly oriented so that the net magnetic moment is 0. As shown in figure a.

In presence of weak magnetic field (H) the magnetic domain are distract among them. Some of the magnetic domains lie along the direction of magnetizing field as shown in figure b.

In presence of strong magnetic field, All of the magnetic domains are oriented along the direction of external magnetic field. Hence, at last the substance act as a magnet.

The ferromagnetic substance losses its magnetic properties with the increase in temperature. Above a certain temperature, the substance losses its ferromagnetism and this temperature is called Curie temperature.
for eg: 1043K for Fe.

Hysteresis:-

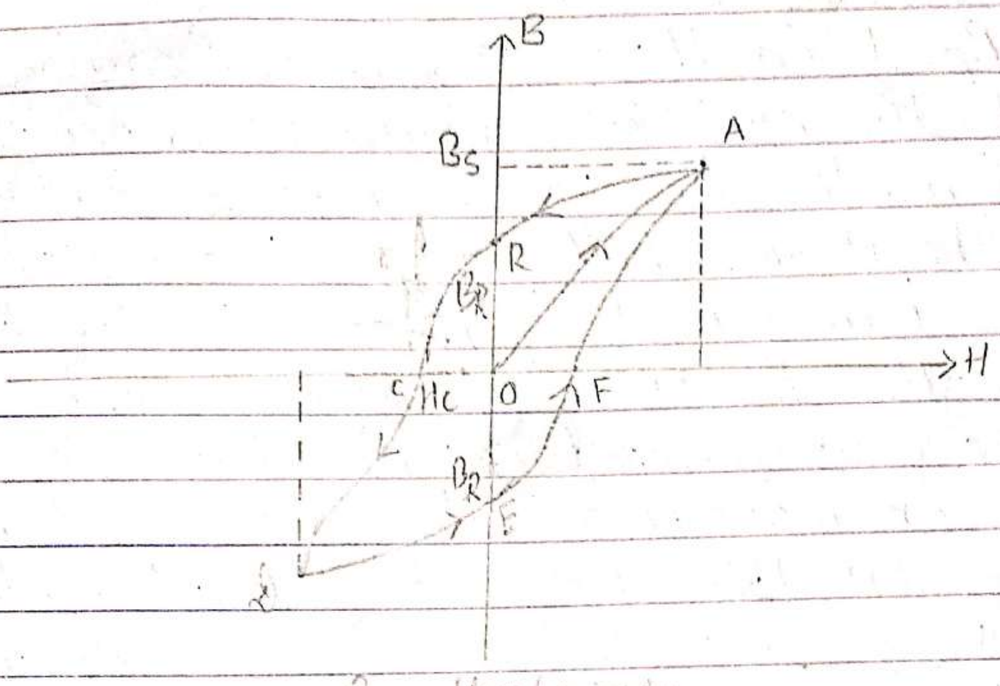


Fig: Hysteresis

- It is the phenomenon of lag of intensity of magnetization behind the magnetizating field

Let us take a ferromagnetic material (Sn, Fe, Ni) and apply external field (B_0) and place it into a solenoid

Let us compare the field B with magnetic intensity H. When H is increase from zero, the magnetization and Hence, the total field B in the material increases along the curve OA and become maximum at point A with a value of B_s as shown in figure. Now the rod is saturated at point A & the further increase in the value of H doesnot change the value of B. This is possible when the magnetic domain of the rod aligned with H. when H is brought back to zero the field B doesnot retrace its path along OA but it travel along the curve AR and it magnetized with the value of B_R. When H_c is reserved the magnetic moment of the material reoriented until the value of B reaches to zero. at point c. On further increasing the value of H the sample reaches to the B which is the point of saturation in opposite direction. On returning H to zero the material is permanently magnetize at point E. On further increasing the value of H in orginal direction the curve DEFA is traced. These behaviour of material is called hysteresis and the close curve ARCDEFA is called hysteresis loop.

This loop shows that the intensity of magnetization lag behind the magnetizing field (B_0 or H).

Raw optics

Definition: The branch of physics that deals with the study of light and vision is called optics.

Reflection at plane surface:

* **Reflection:** ~~When~~

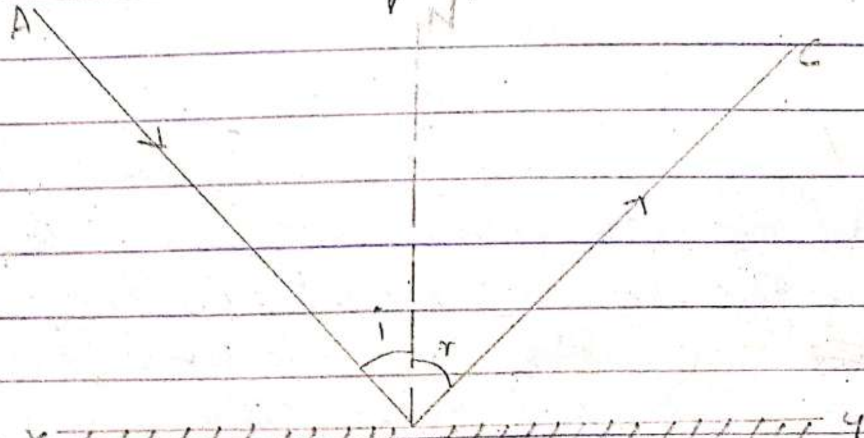
When a Ray of light strike at the point of incidence, then it is reflected back to the same medium which is called reflection.

* **Law of Reflection:**

There are mainly two law of reflection they are

1) The incidence ray, the normal ray and the reflected ray all lies on the same plane.

2) The angle of incidence is equal to the angle of reflection at the point of incidence.



From the figure AB= incidence ray, BN= normal ray, BC= reflected ray, XY = plane mirror,
∠ABN= Angle of incidence = ∠i
∠CBN = Angle of reflection = ∠r

Real and virtual image

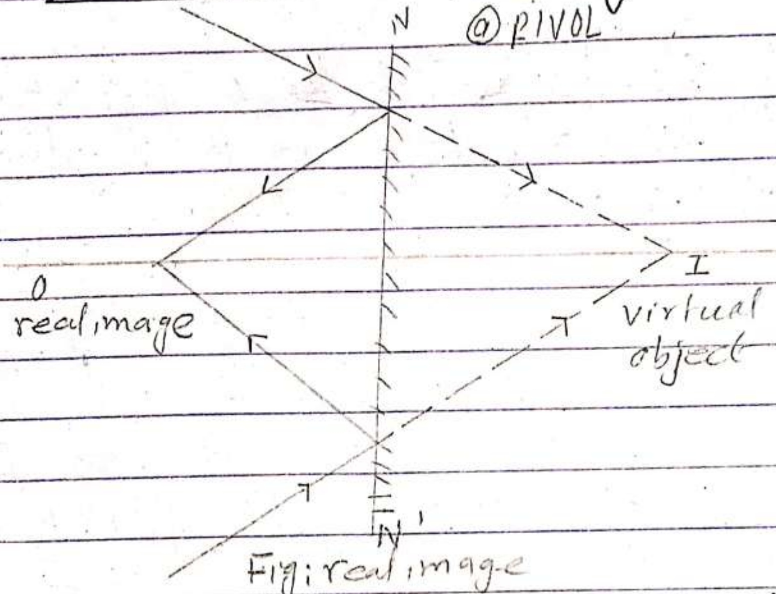


Fig: real image

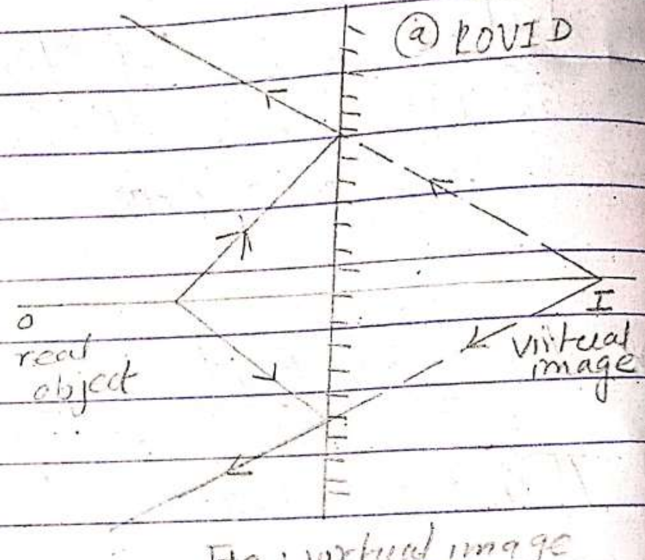


Fig: virtual image

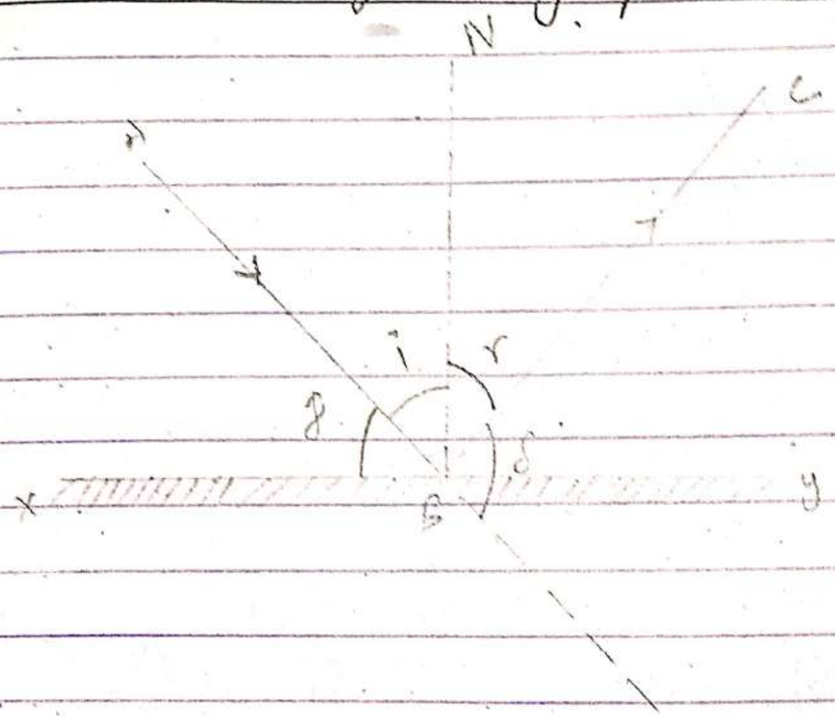
The image which is formed by the actual intersection of light rays is known as real image. It can be observe by naked eyes and can projected on screen. The image form is inverted and can be easily produced by concave mirror and convex lens.

However, the image which is formed by the virtual intersection of light ray is known as virtual image. The image formed is erect and cannot be projected on screen. The image formed by convex mirror and concave lens are generally virtual.

* Nature image formed by plane mirror

- The plane mirror produces virtual image behind it.
- The image formed is erect.
- The image distance is equal to the object distance.
- The size of image is equal to the size of object.
- The image formed is lateral erally inverted.

De viation of light by plane mirror



The bending of light from its initial position is known as deviation of light. And the angle between the direction of incidence ray and reflected ray is called angle of deviation. It is denoted by δ.

Consider An incidience AB fall upon reflecting surface xy at B so that it is reflected along BC. Let NB be the normal ray and the line ABD lies in a straight line.

From figure;

$\angle XBN = \angle XBA + \angle ABN$

or $90 = g + i$ — (1)

where; g is glancing angle.

Also; $\angle YBN = \angle CBY + \angle CBN$

or $90 = \angle CBY + r$ — (11)

from eqn (1) and (11) then

$g + i = \angle CBY + r$

But $\angle i = \angle r$

or $g + \cancel{i} = \angle CBY + \cancel{r}$

or $\angle CBY = g$ — (111)

Now; the angle of deviation is;

$\delta = \angle CBD$

or $\delta = \angle CBY + \angle YBD$

or $\delta = \angle CBY + \angle ABX \quad [\because \angle YBD = \angle ABX]$

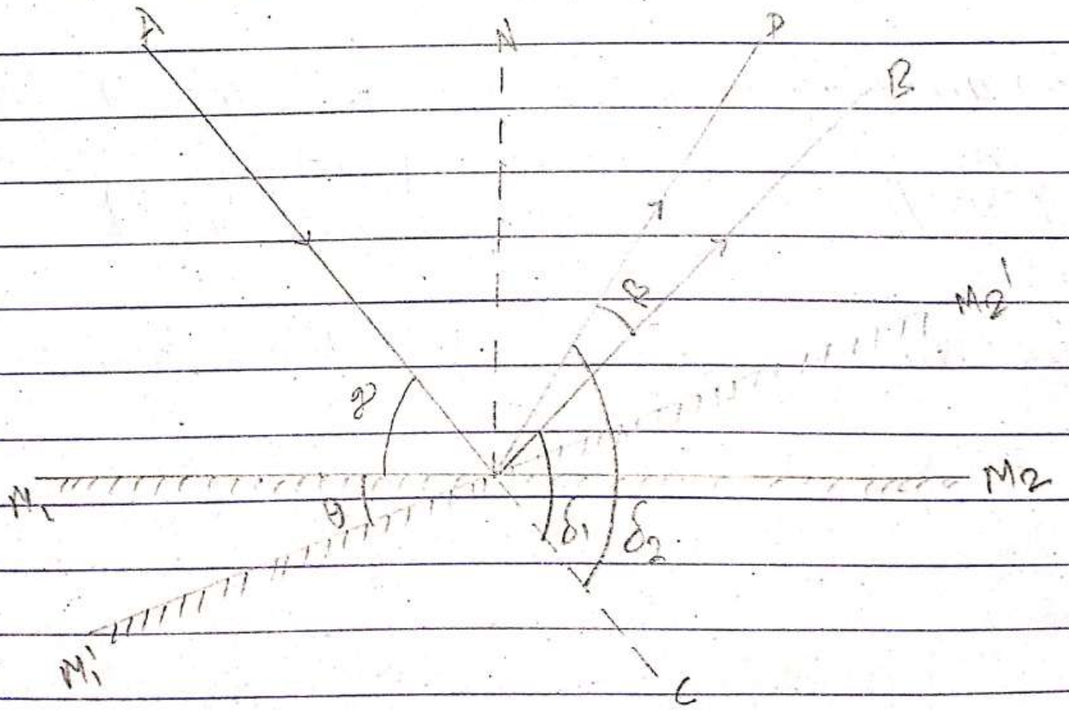
or $\delta = g + g$

or $\delta = 2g$

Thus, the angle of deviation produce by plane mirror is equal to twice of glancing angle.

Note:
The angle made by the incidience ray with plane mirror is called glancing angle. It is denote by g.

⊛ Deviation of reflected ray by rotated mirror:-



Consider a ray of light AO is incidenct on plane mirror M_1M_2 at point O and gets reflected along OB.
Let g be the glancing angle; then the angle of deviation is given by;

$$\delta_1 = \angle BOC = 2g \quad \text{--- (i)}$$

Let the plane mirror M_1M_2 is rotated by an angle 'θ' so that it reaches to new position of $M_1'M_2'$ and is reflected along OD st the new angle of deviation become

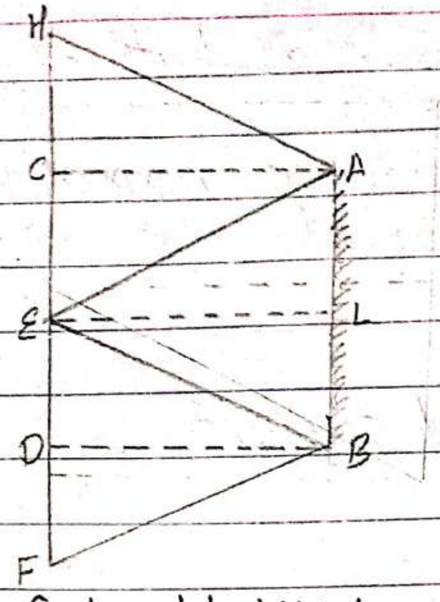
$$\delta_2 = \angle DOC = 2(g+\theta) \quad \text{--- (ii)}$$

Now; the angle of deviation of reflected ray is;

$$\beta = \angle BOD = \angle DOC - \angle BOC$$
$$\text{or } \beta = \delta_2 - \delta_1$$
$$\text{or } \beta = 2(g+\theta) - 2g$$
$$\text{or } \beta = 2\theta$$

Hence, if mirror is rotated by an angle 'θ' then the reflected ray will be rotated by an angle of 2θ. This is called the law of rotation of light.

* Size of the mirror required to see the full image of person.



Consider, A person of height 'HF' is standing infront of plane mirror 'AB' as shown in figure.
A ray of light from the head of a person travell ainlong HA and strike of a point A of a plane mirror and it reflected along AE to the eye 'E'. Similarly, the ray of light from the foot 'F' from the person travells along FB and strike the point B of the plane mirror and is reflected along BE to the eye 'E'.

Here; AC is the perpindicular bisector of HE and BD is the perpindicular bisector of EF and ~~EH~~ ~~A~~EL is the perpindicular bisector of ~~a~~ AB. ~~Hence~~
Hence, the size of the mirror.

$$AB = AL + LB$$

$$AB = C\cancel{D}E + ED$$

$$AB = \frac{1}{2} HE + \frac{1}{2} Ef$$

$$AB = \frac{1}{2} (HE + Ef)$$

or $A = \dfrac{HF}{2}$

Thus, the size of mirror is half of the size of person.

(Q) A ray of light incidence normally on a plane mirror. What are the value of glancing angle and angle of deviation?

Reflection at curved surface

(*) Spherical mirror

This mirror are the part of spherical reflecting surface. If the reflection take place on con~~vex~~cave surface then it is called concave mirror. And if reflection on convex surface then it is called convex mirror.

~~Convex~~ Concave mirror produces the real and virtual image while Convex mirror produces virtual image.

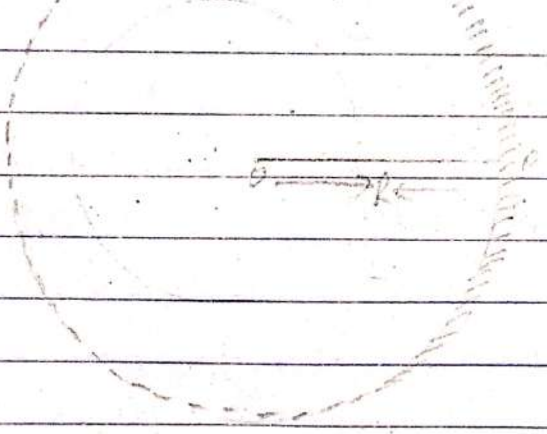


fig: concave mirror

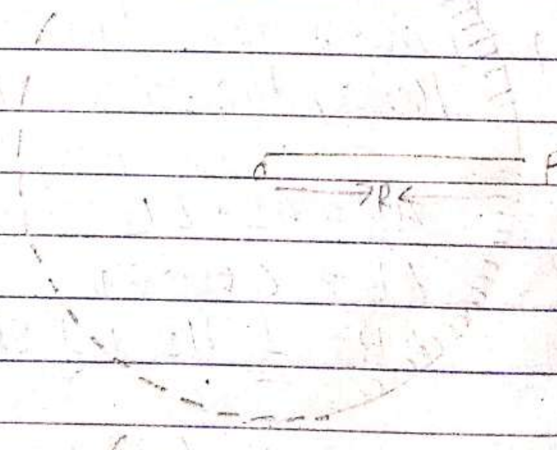


fig: convex mirror

TERMS CONNETED TO SPHERICAL MIRROR

1) Center of curvature:- The centre of curvature of a spherical mirror is the centeral point of the hallow sphere of which the mirror is a part.

2) Radius of curvature:- The radius of curvature of a spherical mirror is the radius of the hallow sphere of which mirror is a part, in other words, it is the distance between the centre of curvature and pole of a mirror. It is represented by letter R.

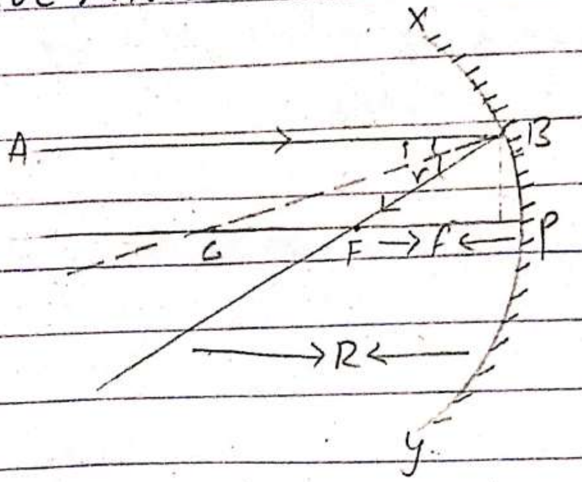
3) Pole:- The pole of spherical mirror is the centre or middle point of a spherical mirror. It is represented by letter P.

4) Principle axis:- Principle axis of a spherical mirror is the line passing through the centre of curvature and its pole. In other words, it is the line joining the pole and centre of curvature of a mirror.

5) Secondary axis:- Any straight line other than the principal axis passing through the centre of a spherical mirror is refereed to as secondary axis. It is usually represented by letter SS'.

6) Aperture of mirror – Aperture of a mirror is the portion of a mirror from which reflection of light takes place. In other words it is the maximum size of a mirror. It is usually represented by letter M and M'.

* Relation between radius of curvature and focal length of concave mirror:-



Consider a concave mirror of apperture XY. A ray AB is incident upon the concave mirror at point B and is reflected to pass through the principal focus (F). Here, BC is the normal drawn to principal axis.

From figure;

$CP = R =$ Radius of curvature
$FP = f =$ focal length

From the laws of reflection of light, then,
$\angle ABC = \angle CBF$ (Being $\angle i = \angle r$) — (i)

Since; $AB \parallel CP$, then
$\angle ABC = \angle BCF$ (Being alternate $\angle$) — (ii)

From eqn (i) and eqn (ii) then
$\angle BCF = \angle CBF$

Thus; ΔBCF is an isoceles triangle.

$\therefore CF = BF$

But; if the apperture of mirror is very small, then B lies very close to P such that $BF \approx FP$

Thus; $CF \doteq BF = FP$

From figures

$$CP = CF + FP$$
$$CP = FP + FP$$
$$R = f + f$$
$$R = 2f$$
$$f = \frac{R}{2}$$

Thus; The focal length of concave mirror is half the radius of curvature.

Imp Mirror Formula:-

The relation between object distance, image distance, and focal length of mirror is called ~~a~~ mirror formula.

The following assumption and sign convection are used to derived mirror formula.

/

/

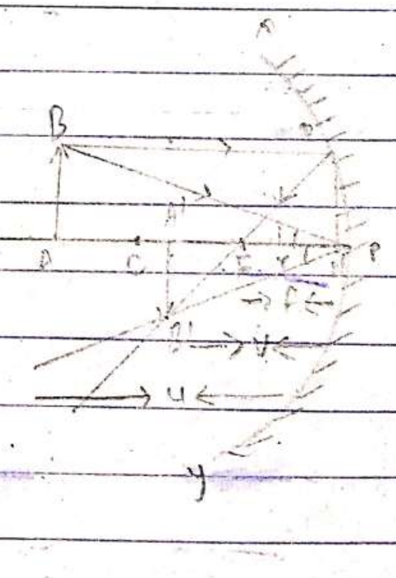
a) ~~Agus~~Assumption

- The apperture of the mirror should be very small.
- The object should be the point object and must be placed on principle axis.
- The light ray should not be incident obliquely.

b) Sign Convention

- All the distance are measure from the point of ~~a~~ mirror.
- The distance of real object and real image are taken as ~~of~~ positive while that of virtual object and image are taken as negative.
- The focal length and radius of curvature for concave mirror is taken as positive while those of convex mirror is taken as negative.

Mirror formula of concave mirror:-



Consider an apperture XY of concave mirror. An object AB is placed beyond C which image A'B' is formed between C and F. The image formed is real and inverted. Let us draw DN perpendicular to AP st DN=AB

From figure;

Object distance $= u = AP$
image distance $= v = A'P$
focal length $= f = FP$
Now; In ΔABP and $\Delta A'B'P$

$\angle BAP = \angle B'A'P$ (Being $90°$)
$\angle BPA = \angle A'PB'$ (Being $\angle i = \angle r$)
$\angle ABP = \angle A'B'P$ (Being remaining $\angle$s)

Thus; ΔABP and $\Delta A'B'P$ are similar.

Thus; $\frac{A'B'}{AB} = \frac{A'P}{AP}$ —— ①.

Simarly; $\Delta A'B'F$ and ΔDNF are also similar

Thus; $\frac{A'B'}{DN} = \frac{A'F}{FN}$

But; $DN = AB$

$$\frac{A'B'}{AB} = \frac{A'P - FP}{FN} \quad \text{—(11)}$$

From eqn (1) and (11) then

$$\frac{A'P}{AP} = \frac{A'P - FP}{FN}$$

If the apperture XY of mirror is very small, then $FN \approx FP$

$$\text{so, } \frac{A'P}{AP} = \frac{A'P - FP}{FP}$$

$$\text{or } \frac{v}{u} = \frac{v - f}{f}$$

$$\text{or, } vf = vu - uf$$

$$\text{or } vf + uf = vu$$

$$\text{or } f(v+u) = vu$$

$$\text{or, } \frac{v+u}{vu} = \frac{1}{f}$$

$$\text{or } \frac{\cancel{v}}{\cancel{v}u} + \frac{\cancel{u}}{v\cancel{u}} = \frac{1}{f}$$

$$\text{or } \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

This the formula for concave mirror.

* Linear magnification:-

It is define as the ratio of size of the image to the object. It is denoted by 'm' and is define by;

Linear magnification (m) = $\frac{\text{size of image}}{\text{size of height}}$

In figure; size of image = A'B'
size of object = AB

or $m = \frac{A'B'}{AB}$ —(iv)

But, from relation (1), then.

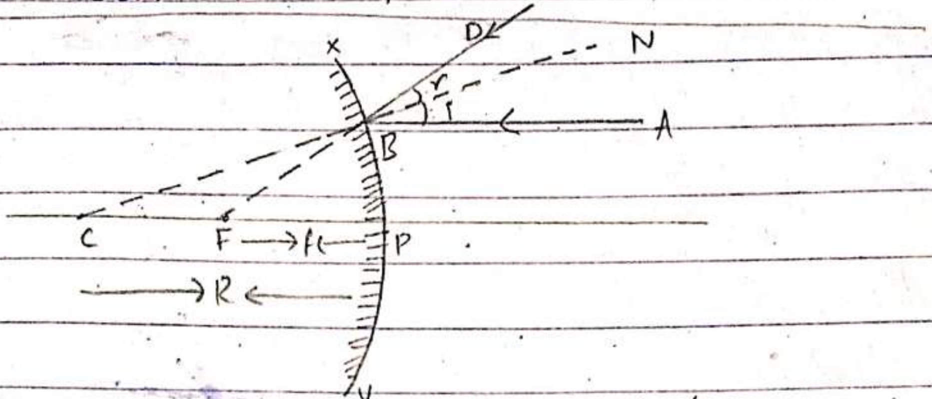
$\frac{A'B'}{AB} = \frac{A'P}{AP} = \frac{v}{u}$ —(v)

Thus; $m = \frac{A'B'}{AB} = \frac{A'P}{AP} = \frac{v}{u}$

$\therefore m = \frac{v}{u}$

Thus; linear magnification is define as the ratio of ~~###~~ image ~~to~~ distance to the object distance.

Relation between R and f on convex mirror:-



Consider a convex mirror of apperture XY. An incident ray AB is incident on it and is reflected along BD which appears to pass through the principal focus. Here; CN is drawn normal to the mirror.

From figure

Radius of curvature $= R = CP$
focal length $= f = FP$

Also; $\angle ABN = \angle NBD$ $(\angle i = \angle r)$ — (I)
$\angle ABN = \angle FCB$ — (II) (Being corresponding angle)

$\angle CBF = \angle NBD$ — (III) (Being VOA)

From eqn (I) and (II) then
$\angle NBD = \angle FCB$ — (IV)
from eqn (III) and (IV). then

$\angle CBF = \angle FCB$ — (V)

Thus, ΔFCB an isoceles Δ. So;

$FC = FB$

If the apperture of mirror is very small then B lies ~~very~~ lies very close to P at

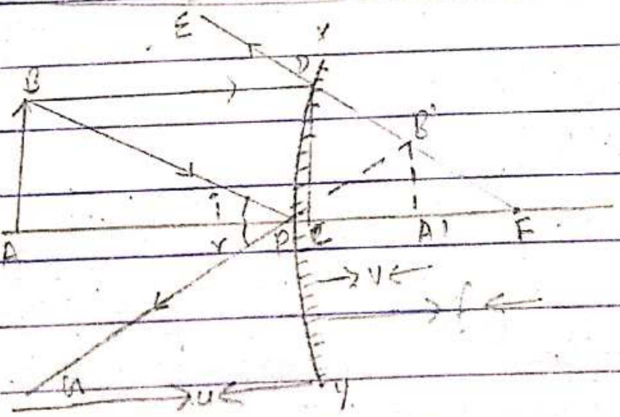
~~so~~ st $FB \approx FP$

Now; the radius of curvature is

$$R = CP$$
$$= FC + FP$$
$$= FB + FP$$
$$= FP + FP$$
$$= 2FP$$
$$R = 2f$$
$$f = R/2$$

Thus; the focal length of convex mirror is half of the radius of curvature.

Mirror formula for convex mirror :-



Consider a convex mirror of apperture XY and object AB lie on principle axis an incident ray BD is incident at point D and is travelled/ reflected along DE which appears ~~th~~ to pass through principle axis ~~a~~F as shown in figure. Similarly another ray BP incident on convex mirror and is reflected ~~around~~ PG. This reflected ray ~~mid~~ meet at point B' ~~show~~ so that the virtual image A'B' is formed.

Let us Draw DC perpendicular to principle axis so that;

from figure

Object distance = AP = u

image distance = A'P = -v

focal length = PF = -f

Also;

$\angle APB = \angle APG$ (Being $\angle i = \angle r$) — (I)

and $\angle APG = \angle B'PA'$ (Being V.O.A) — (II)

from eqn (I) and (II) then

$\angle APB = \angle B'PA'$ — (III)

In ΔAPB and $\Delta A'PB'$ then

$\angle APB = \angle B'PA'$ (from eqn (III))

$\angle BAP = \angle PA'B'$ (Being 90°)

$\angle ABP = \angle A'B'P$ (Remaining angle)

Thus; ΔAPB and $\Delta A'PB'$ are similar:

$$\therefore \frac{A'B'}{AB} = \frac{A'P}{AP} \quad \text{— (iv)}$$

Similarly; ΔCDF and $\Delta A'B'F$ are also similar.

$$\therefore \frac{A'B'}{CD} = \frac{A'F}{CF}$$

But; $CD = AB$ then

$$\frac{A'B'}{AB} = \frac{A'F}{CF}$$

$$\text{or } \frac{A'B'}{AB} = \frac{CF - CA'}{CF} \quad \text{— (v)}$$

From eqn (iv) and (v) then;

$$\frac{A'P}{AP} = \frac{CF - CA'}{CF}$$

If the apperture of the mirror is very small, then D lies close to P st $CF \simeq PF$ and $CA' \approx PA'$

$$\frac{A'P}{AP} = \frac{PF - PA'}{PF}$$

$$\frac{-v}{u} = -\frac{f-(-v)}{-f}$$

$$\frac{\cancel{-}v}{u} = \frac{-f+v}{\cancel{-}f}$$

$$\frac{v}{u} = \frac{-uf+uv}{f}$$

$$vf = -uf+uv$$
$$uf+vf = uv$$
$$f(u+v) = uv$$
$$\frac{1}{f} = \frac{u+v}{uv}$$

$$\frac{1}{f} = \frac{\cancel{u}}{\cancel{u}v} + \frac{\cancel{v}}{u\cancel{v}}$$

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

This is known as mirror formula.

Linear Magnification

It is define as the ratio of size of image to the size of object. It is denoted by m and is ~~denoted by m~~ defined by

$$\text{linear magnification } (m) = \frac{\text{size of image}}{\text{size of object}}$$

In figure; size of image = $A'B'$
size of object = AB

Thus; $m = \frac{A'B'}{AB}$ — (VI)

But, from eqn (IV) then

$$\frac{A'B'}{AB} = \frac{A'P}{AP} = \frac{-v}{u} \quad \text{— (VII)}$$

Thus; from eqn (VI) and eqn (VII) then

$$m = \frac{-v}{u}$$

This shows that linear magnification is negative for convex mirror.

Use of mirror:-

S. NO	Plane Mirror	Concave Mirror	Convex mirror
1	As a looking mirror	Reflecting Telescope	Driving Mirror
2	As a reflector in interferometer	Dental Mirror	Save view at point
3.	Periscope	Shaving mirror	Anti-shop lifting devices

(a) Which mirror is use as shaving or makeup mirror and why?

=) Concave mirror is use as shawing or makeup mirror as its forms an ~~erect~~ erect and magnified ~~~~ image ~~~~ of object when place at pole and focces of the mirror.

~~Shaw~~

shaw when our face is located between pole and focus of the mirror then an erect and magnified image will be formed.

(b) Why are Con~~~~vex used in cars for rarer ~~~~ view?

=) For the same apperture of the mirror the field of view will be maximum ~~~~ for concave mirror as compare ~~~~ to that plane or concave mirror so convex mirror are used in car ~~~~ for rarer view.

* Sign Convention:-

A) For concave mirror

Real Image	Virtual Image
$u \rightarrow +ve$	$u \rightarrow +ve$
$v \rightarrow +ve$	$v \rightarrow -ve$
$f \rightarrow +ve$	$f \rightarrow +ve$
$m \rightarrow +ve$	$m \rightarrow -ve$

B) For convex mirror

Image is virtual

- $u \rightarrow +ve$
- $v \rightarrow -ve$
- $f \rightarrow -ve$
- $m \rightarrow -ve$

Numerical Atmosphere:-

Q) An Object is placed in front of a concave mirror and the distance between object and image is 4 cm. If the magnification is 2; Calculate the focal length of mirror.

Soln

magnification (m) = 2

Let u and v be the object and image distances.

By question;

$$u + v = 4\,cm \quad —(I)$$

Also; $m = \frac{v}{u} = 2$

or, $v = 2u \quad —(II)$

Plugging (II) in (I) then

$$u + 2u = 4$$
$$3u = 4$$
$$u = \frac{3}{4}\,cm$$

Also; from (II) then

$$v = 2u = \frac{8}{9}\,cm$$

From mirror formula;

or, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

or, $\frac{1}{f} = \frac{1}{\frac{4}{3}} + \frac{1}{\frac{8}{3}}$

or $\frac{1}{f} = \frac{3}{4} + \frac{3}{8}$

or $\frac{1}{f} = \frac{6+3}{8}$

or, $f = \frac{8}{9}$ cm //

The image obtain by a concave mirror is erect and 3 times the size of object the focal length of mirror is 20 cm. Calculate the image and object distance?

Given:

Let u and v be the object and image respectively,

By question $m = \frac{v}{u} = \frac{I}{O} = -3$

$v = -3u$ — ①

Here; -ve sign is due to erect image.

from mirror formula; then

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{20} = \frac{1}{u} - \frac{1}{3u}$$

$$\frac{1}{20} = \frac{2}{3u}$$

$$3u = 40$$

$$u = \frac{40}{3} \text{ cm}$$

From eqn ① then

$$v = -3u$$

$$= -\cancel{3} \times \frac{40}{\cancel{3}}$$

$$= -40 \text{ cm}$$

Q. A concave mirror forms ~~an~~ a real image in a screen which is 2 times of size of object then Then Object and screen are moves until an image 3 times the size of object. If the shift of the 20 cm. Determine the shift of the object and focal length?

Soln

1st case:-

let u and v be the object and image distance respectively. such that the magnification is

$$m = \frac{v}{u} = 2$$

or, $v = 2u$

From mirror formula, then

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{2u}$$

$$\frac{1}{f} = \frac{3}{2u} \quad \text{—(1)}$$

focal length is near change

2nd case

If object and screen are moved until the image is three times of the size of object.

i.e. $m = 3$

Assume; u' and v' be the new object and image distances.

i.e $\frac{v'}{u'} = 3$

or $v' = 3u'$ — (ii)

By question;

$v' = 2u + 20$ — (iii)

From (ii) and (iii) then

$3u' = 2u + 20$

$u' = \frac{(2u+20)}{3}$

From mirror formula;

$\frac{1}{f} = \frac{1}{u'} + \frac{1}{v'}$

$$\frac{1}{f} = \frac{1}{\frac{2u+20}{3}} + \frac{1}{2u+20}$$

$$\frac{1}{f} = \frac{4}{2u+20}$$

$$\frac{3}{2u} = \frac{4}{2u+20}$$

$$6u + 60 = 8u$$

$$60 = 2u$$

$$u = 30 \text{ cm}$$

Now; $\frac{1}{f} = \frac{3}{2u}$

$$f = \frac{2 \times 30}{3}$$

$$f = 20 \text{ cm}$$

Also; $u' = \frac{2u+20}{3}$

$$u' = \frac{60+20}{3}$$

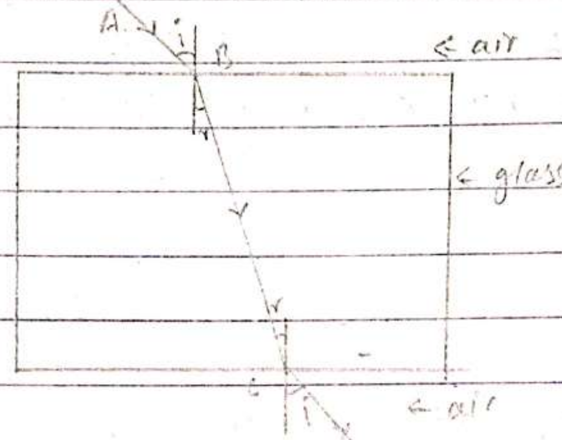
$$u' = \frac{80}{3} = 26.67 \text{ cm}$$

shift in distance = $u - u'$
$= 30 - 26.67$
$= 3.33$

Refraction through plane surfaces

Refraction :-

The phenomenon of bending of light when it passes from denser to rarer medium is called refraction. The medium where light travels faster is called rarer medium and the medium where light travel slower is called denser medium.



The main cause of refraction is the change in speed of light when passing from rarer to denser medium and vice versa

Laws of Refraction

1) The incident ray, normal ray and refracted ray all lie on the same plane at the same point.

2) For 2 given media, the ratio of sin of angle of incidence to the angle of refraction is constant for a given media. This is called Snell's law

If 'i' be the angle of incidence and 'r' be the angle of refraction; then

$$\frac{\sin i}{\sin r} = \text{constant}$$

or $${}_a\mu_b = \frac{\sin i}{\sin r} = \text{constant}$$

where; ${}_a\mu_b$ = Refractive index of denser medium (b) w.r.t rarer medium (a).

Refractive index

It is define as the ratio of sin of angle of incidence to the angle of refraction is constant for a given 2 media. It is denoted by μ.

If 'i' be the angle of incidence and 'r' be the angle of refraction; then

$$\frac{\sin i}{\sin r} = \text{constant}$$

or $${}_a\mu_b = \frac{\sin i}{\sin r} = \text{constant} \quad \text{—(1)}$$

where; ${}_a\mu_b$ = Refractive index of denser medium (b) ~~to~~ w.r.t rarer medium (a).

According to ~~wabe~~ wave theory of light the refractive index of medium two w.r.t to medium 1 is defined as the velocity of light in medium one to the velocity of light in medium two.

i.e. $${}_1\mu_2 = \frac{\text{Velocity of light in medium 1 } (v_1)}{\text{Velocity of light in medium 2 } (v_2)}$$

or $${}_1\mu_2 = \frac{v_1}{v_2} \quad \text{—— (11)}$$

If 'λ_1' and 'λ_2' be the wavelength in medium one and medium two then; $v_1 = \lambda_1 f$ and $v_2 = \lambda_2 f$

~~Thus~~ Thus; $${}_1\mu_2 = \frac{\lambda_1 f}{\lambda_2 f} = \frac{\lambda_1}{\lambda_2}$$

When light travels from vacuum to any denser medium is called absolute refractive index. In other word, the ratio of velocity of light in vaccum to the velocity in the light in medium is called absolute refractive index.

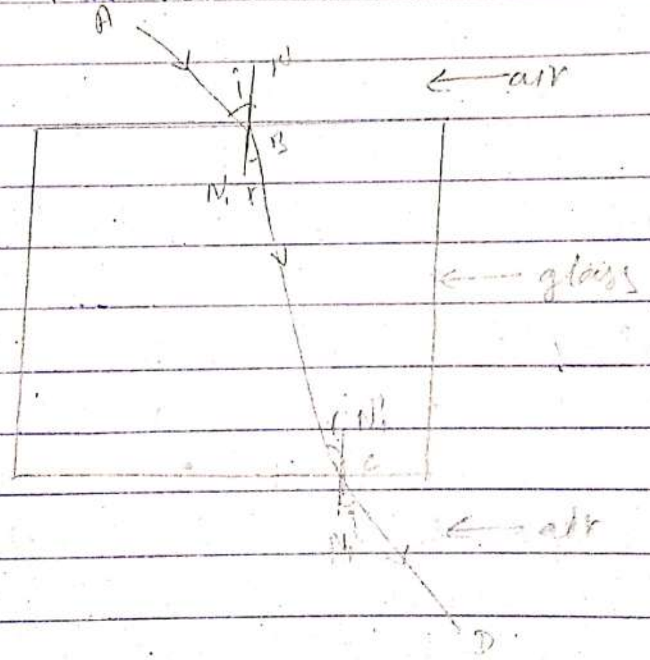
i.e. $$\text{Absolute R.I } (\mu) = \frac{\text{velocity of light in vaccum } (c)}{\text{velocity of light in media } (v)}$$

$$\mu = \frac{c}{v}$$

Note: frequency of light remain unchange.

R | λ_{max}, v_{max}, f = constant, μ_{min}
O
Y
G
B
I
V ↓ λ_{min}, V_{min}, f = constant, μ_{max}

* Relation between two refractive index or Reversibility of light:-



Let us consider an incident ray AB incident at point B with angle of incidence 'i'. It is refracted along BC with angle of refraction (r) and then finally emerged out along CD. Here, the angle of incidence is equal to angle of emergence.

The refractive index of glass w.r.t air is;

$$ {}_a\mu_g = \frac{\sin i}{\sin r} \quad \text{—(I)} $$

Similarly, the refractive index of air w.r.t glass is

$$ {}_g\mu_a = \frac{\sin r}{\sin i} \quad \text{—(II)} $$

Multiplying eqn (I) and eqn (II) then

or $$ {}_a\mu_g \cdot {}_g\mu_a = \frac{\cancel{\sin i}}{\cancel{\sin r}} \cdot \frac{\cancel{\sin r}}{\cancel{\sin i}} $$

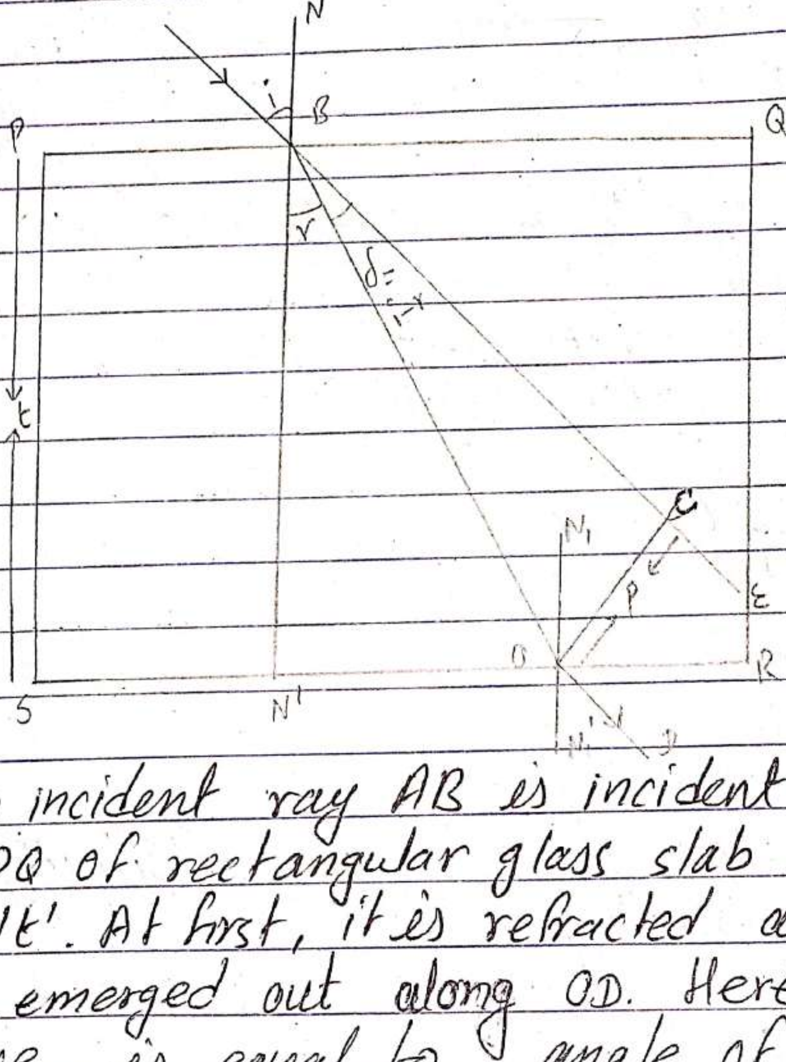
or $$ {}_a\mu_g \cdot {}_g\mu_a = 1 $$

$$ {}_a\mu_g = \frac{1}{{}_g\mu_a} $$

This verifies the principle of reversibility of light in case of refraction.

* ## Lateral shift :-

Statement:- The perpendicular distance between the direction of the incident ray and emergent ray. where the incident ray and emergent ray are parallel to each other is called lateral shift. It is denoted by 'p'.



Consider an incident ray AB is incident at B on surface PQ of rectangular glass slab PQRS of thickness 't'. At first, it is refracted along BO and is finally emerged out along OD. Here; the angle of incidence is equal to angle of emergence. Similarly, incid~~ence~~ent ray and emergent ray are parallel to one another. The perpendicular distance OC is drawn from O to C on BE which is called lateral shift. It is denoted by 'p' ie OC = P

from figure

$\angle ABN$ = angle of incidence = i
$\angle OBN'$ = angle of refrection = r
$\angle OBC$ = angle of deviation = δ

Also; $\angle ABN = \angle CBN'$ (Being V.OA)
or, $\angle ABN = \angle CBO + \angle OBN'$
or $\angle i = r + \delta$
or, $\delta = (i - r)$ — (1)

In $\triangle CBO$;
$$\sin(i-r) = \frac{OC}{OB}$$

$$OC = OB \sin(i-r)$$
$$p = OB \sin(i-r) \text{ — (ii)}$$

Also; In OBN';

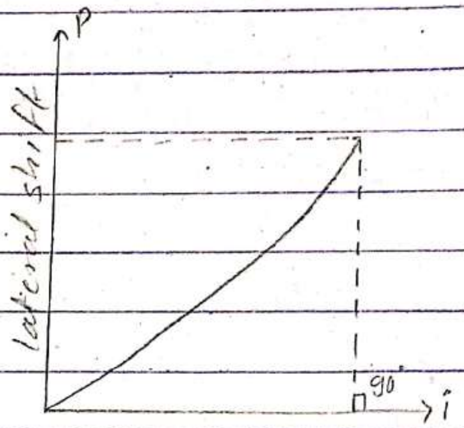
$$\cos r = \frac{BN'}{OB}$$

or $$OB = \frac{BN'}{\cos r} = \frac{t}{\cos r} \text{ — (iii)}$$

From eqn (ii) & (iii) then

$$p = \frac{t}{\cos r} \sin(i-r)$$

This gives the expression of Lateral shift. The value of lateral shift depend upon the thickness of glass slab and angle of incidence 'i'. The variation ~~verition~~ of angle of incidence with thickness of glass slab 't'



angle of incidence

Special cases:-

(i) When $i = 0°$ (normal incidence)

$$p = \frac{t}{\cos 0} \sin 0° = 0$$

Thus, the lateral shift is zero.

(ii) When $i = 90$ (Grazing incidence)

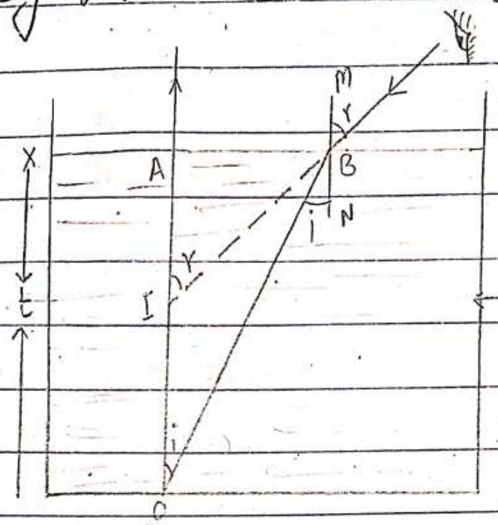
or, $p = t \frac{\sin(90 - r)}{\cos r}$

or, $p = t \frac{\cancel{\cos r}}{\cancel{\cos r}}$

or, $p = t$

Real and Apperant depth

When an object is seen through a medium which is denser then air. Then the depth of the object seen to be less then to be actual depth. actual depth of the object is called real depth. And the depth viewed by th observer is called apperant depth.



Consider an object O is placed at the bottom of transparent meduim of thickness 't'. A ray OA from O incident on point A on reflecting surface XY and passes along AD without any deviation. Similarly, another ray OB incident at point B and is refracted away from normal along the path BC. An object appears to be a position. I when seen through the air. Thus, a virtual image is formed at point I.

From Figure;
OA = real depth
AI = apperant depth

OI = shift in depth

Let 'i' be the angle of incidence and 'r' be the angle of refraction.

So, $\angle IOB = \angle OBN = i$ (Being Alternate ∠)
$\angle AIB = \angle MBC = r$ (Being corresponding ∠s)

The refractive index of air w.r.t water is;

$${}_{w}\mu_{a} = \frac{\sin i}{\sin r} \quad \text{--- (i)}$$

Similarly; the ~~surface~~ refractive index of water w.r.t air is;

$${}_{a}\mu_{w} = \frac{1}{{}_{w}\mu_{a}} = \frac{\sin r}{\sin i} \quad \text{--- (ii)}$$

From ΔAOB

$$\sin i = \frac{AB}{OB} \quad \text{--- (iii)}$$

Also; in ΔAIB;

$$\sin r = \frac{AB}{IB} \quad \text{--- (iv)}$$

Putting eqn (iii) and (iv) in eqn (ii) then

$${}_{a}\mu_{w} = \frac{\frac{AB}{IB}}{\frac{AB}{OB}}$$

or, $_a\mu_w = \frac{OB}{IB}$ — (v)

for small angle of incidence, $OB \approx OA$ and $IB \approx IA$ then;

$\mu = {}_a\mu_w = \frac{OA}{IA}$

or, $IA = \frac{t}{\mu}$ — (vi)

Now; the shift in depth is;

or $d = OI = OA - IA$

or, $d = t - \frac{t}{\mu}$

or, $d = t\left(1 - \frac{1}{\mu}\right)$

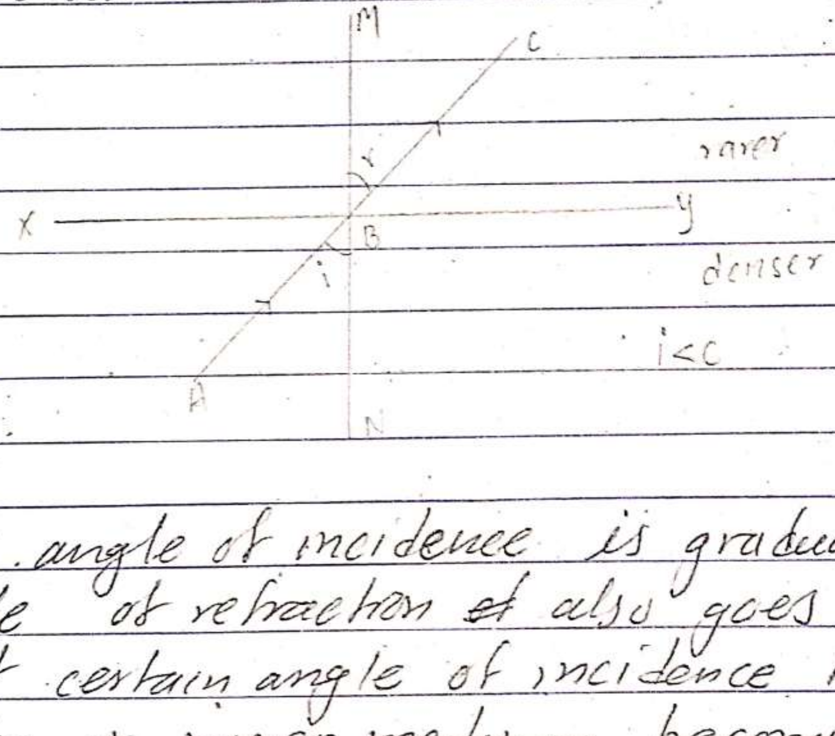
This gives the apperant displacement of an objet when viewed by the observe through air.

(Q) A stick partially deep in water ~~time~~ seen to be bent why?

It is due to refraction of light when ray of light travell from denser medium (water) to rarer medium (air) it bents away from normal. the refracted ray when reaches to our eye it seen the virtual image formed. ~~have~~ Hence, a stick partially deepled in water appears to be bend.

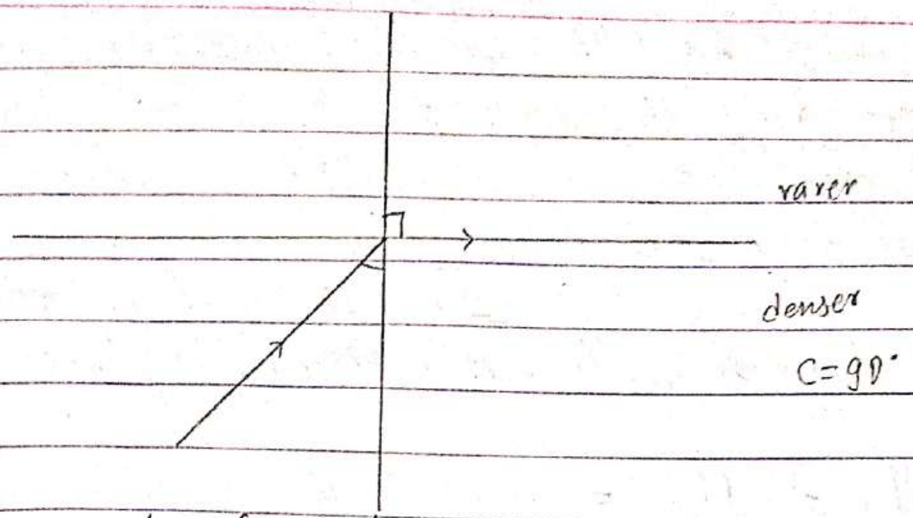
x Critical angle and total internal reflection:-

When light passes from denser to the rarer medium it bends away from the normal because of the change in speed of light when it moves from denser to rarer medium.

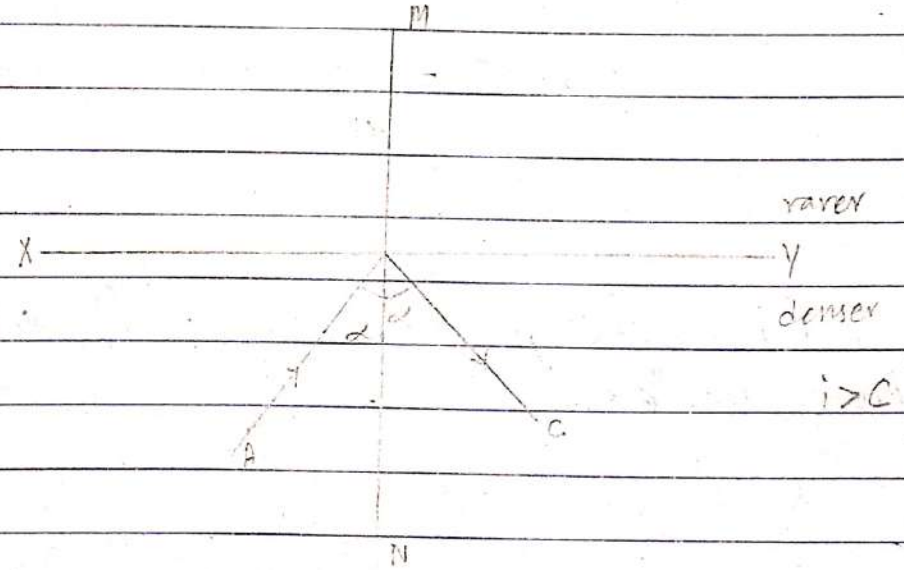


When the angle of incidence is gradually increased the angle of refraction ~~of~~ also goes on increasing then at certain angle of incidence the angle of refraction in rarer medium becomes 90°.

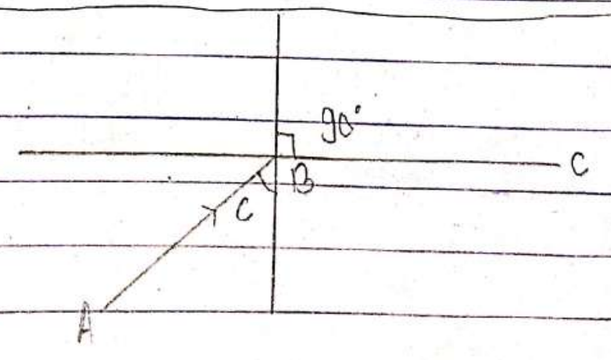
These particular angle of incidence in denser medium for which the angle of refraction in ~~den~~ rearer medium becomes 90° is called crictial angle. It is denoted by 'c'.



When the angle of incidence is ~~slightly~~ greater than critical angle 'c' then the light is reflected in the denser medium which is known as total internal reflection.



(*) Relation between critical angle and total internal reflection.



Let us consider AB be an incidence ray and BC be the reflected ray. The ray passes from denser to the rarer medium where angle of refraction is 90° and angle of incidence is c.

If μ be the refractive index of denser medium with rarer medium then

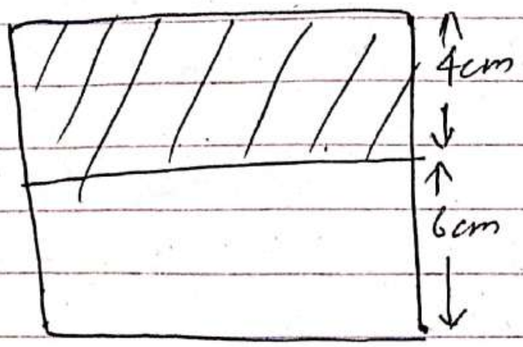
$${}_r\mu_d = \frac{\mu_d}{\mu_r} = \mu = \frac{\sin 90}{\sin c}$$

$$\boxed{\text{or } \mu = \frac{1}{\sin c}}$$

$$\text{or } \sin c = \frac{1}{\mu}$$

This is the ~~ref~~ relationship between ~~crit~~ critical angle and refractive index.

* What is the apperent position of an object below a rectunglar block of 6 cm thick. If a layer of 4cm water is on the top of the glass. If the refractive index is 3/2 and the water is 4/3.



Soln,

Thickness of water layer (t_1) = 4cm
Thickness of gless layer (t_2) = 6cm
Refractive index of water (μ_w) = 4/3
Refractive index of glass (μ_g) = 3/2

The displacement of an object is;

$$d = t\left(1 - \frac{1}{\mu}\right)$$

$$d = t\left(1 - \frac{1}{\mu_w}\right)$$

$$= 4\left(1 - \frac{3}{4}\right)$$

$$= 4 \times \frac{1}{4} = 1\,cm$$

For glass; $d_2 = t_2\left(1 - \frac{1}{\mu_g}\right)$

$$= 6\times\left(1 - \frac{2}{3}\right)$$

$$= 6\times\frac{1}{3}$$

$$= 2\,cm$$

total displacement $(d) = d_1 + d_2$

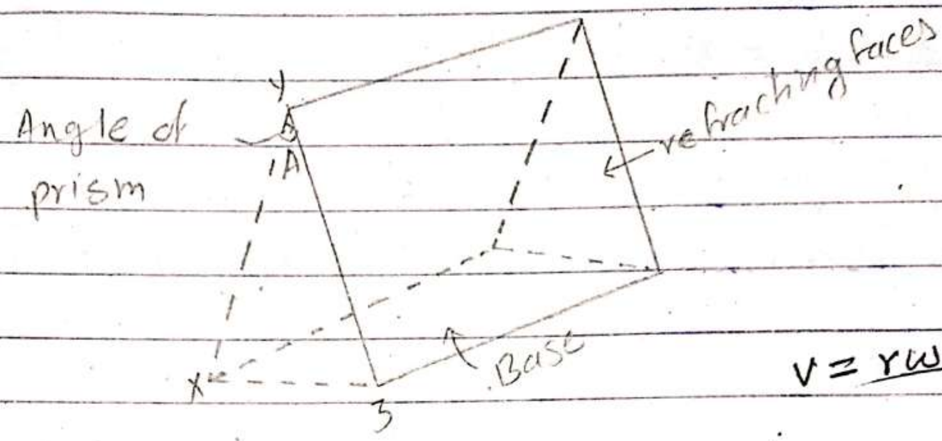
$$= 1\,cm + 2\,cm$$

$$= 3\,cm$$

Refraction through prism

Prism:

A prism is a transparent, refracting medium, bounded by two plane surface meeting each other along a straight edge. It consist of 3 ~~ract~~ rectangular faces and 2 traingluar faces. And the angle between two refractive faces is called angle of prism. It is denoted by 'A'.



$v = r\omega\cos\cancel{\omega t}$

$v = \omega\sqrt{r^2 - y^2}$

$y = 0$

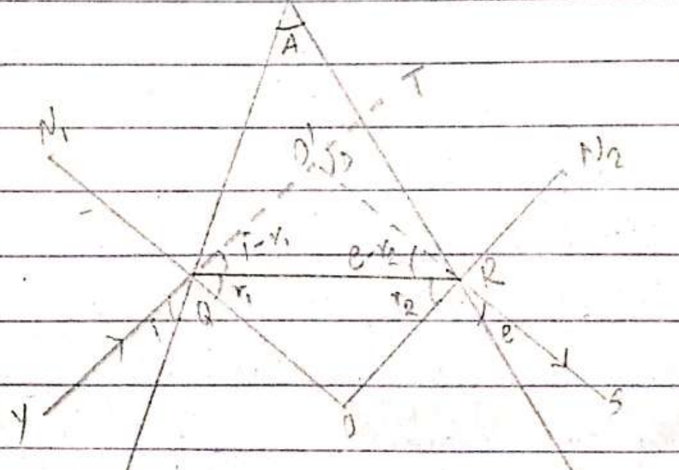
$v = r^2 - r^2$

0

$y = r$

$v = \omega r$

⊛ Refraction through a glass prism:-



Consider XYZ be the principle section of prism where 'A' is the angle of prism. PQ is an incidence ray which is incidence on face XY at Q with angle of incidence 'i'. It is refracted along QR with angle of refraction r_1 again the refracted ray emerged out along RS with angle of emergence 'e'. Here 'r_2' is the angle of refraction in Second face of ~~the prism~~ XY of prism. The angle between the direction of incidence ray and emergent ray is called angle of deviation.

ie $\angle TO'R = D$

Here, N_1O and N_2O are 2 normal drawn ~~to~~ on face XY and YZ as shown in figure. ~~From fig~~

From figure;

or $\angle TO'R = \angle O'QR + \angle O'RQ$

or $\angle TO'R = i - r_1 + e - r_2$

or $\angle TO'R = (i+e) - (r_1+r_2)$ —— (i)

Similarly, $\triangle OQR$;

or $\angle QOR + \angle OQR + \angle ORQ = 180°$

or $\angle QOR + r_1 + r_2 = 180°$ —— (ii)

Again; in quadrilateral ORYQ;

$\angle QOR + \angle QYR = 180°$

or $\angle QOR + A = 180°$ —— (iii)

From eqn (II) and eqn (III) then

$$\angle QOR + A = \angle QOR + r_1 + r_2$$

$$A = r_1 + r_2 \quad \text{— (IV)}$$

From eqn (I) and (IV) then;

$$D = \angle TO'K = (i+e) - A$$

$$D = (i+e) - A \quad \text{— (V)}$$

This is the expression for deviation produce by a prism.

Minimum deviation

When the angle of incidence is increase from zero, the angle of deviation decreases at first and then the angle of deviation ~~rapidly~~ increases rapidly {becomes minimum} as shown in figure. Thus, the minimum value of angle of incidence at which the angle of deviation becomes minimum is called angle of ~~minim~~ minimum deviation. In the case of minimum deviation,

(I) $i = e$

(II) $r_1 = r_2 = r$

From eqn IV, then

$$A = r_1 + r_2$$

$$A = r + r$$

$$A = 2r$$

$$r = \frac{A}{2}$$

Also from eqn (iv) then

$$D = i+e-A$$

In minimum deviation; $D \to D_m$ and $i=e$ st

$$D_m = i+i-A$$

$$D_m = 2i-A$$

$$2i = A+D_m$$

$$i = \frac{A+D_m}{2} \quad \text{— (vii)}$$

For 1st face XY, the Refractive index of material of prism is:

$$_a\mu_g = \mu = \frac{\sin i}{\sin r_1}$$

$$\text{or } \mu = \frac{\sin i}{\sin r}$$

$$\text{or } \mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\sin \frac{A}{2}}$$

This is the required relation between refractive index of material of prism, angle of prism and angle of minimum deviation.

⊛ Deviation through small angled prism:-

Let XYZ be the principal section of prism. PQ be ~~the~~ an incident ray which is incident at point Q with angle of incidence (i). Here; the ray is reflected along QR and is finally emerged out along RS with angle of emergence (e). Here; r_1 and r_2 be the angle of refraction on face XY and YZ respectively.

From Figure; the refractive index of material of prism on face XY and YZ is;

$$\mu = \frac{\sin i}{\sin r_1} \quad \text{—— (I)}$$

And,

$$\mu = \frac{\sin e}{\sin r_2} \quad \text{—— (II)}$$

For small angle of incidence, the angle of refraction on face XY is small. ie $\sin i \simeq i$ and $\sin r_1 \simeq r_1$. Thus the eqn (I) reduced to;

or $\mu = \frac{i}{r_1}$

or $i = \mu r_1$ —— (III)

Similarly, along the refracting face YZ;

$$\mu = \frac{e}{r_2}$$

or, $e = \mu r_2$ —— (IV)

The deviation produced by a prism is;

or, $D = i + e - A$

or, $D = \mu r_1 + \mu r_2 - A$

or, $D = \mu(r_1 + r_2) - A$

or, $D = \mu A - A$ $[\because A = r_1 + r_2]$

$\boxed{\text{or, } D = A(\mu - 1)}$ — (v)

The eqn (v) gives the deviation produced by small angle prism.

Numerical zone:

① Calculated the refractive index of glass prism having angle of prism 60° which the makes the angle of minimum deviation of 36°?

Soln

Angle of prism (A) = 60°
Angle of minimum deviation (D_m) = 36°
Refractive index (μ) = ?

We have;

$$\mu = \frac{\sin\left(\frac{A + D_m}{2}\right)}{\sin\frac{A}{2}}$$

$$\mu = \frac{\sin\left(\frac{60 + 36°}{\frac{60}{2}}\right)}{\sin\left(\frac{60}{2}\right)}$$

$\mu = \dfrac{\sin 48}{\sin 30}$

$\mu = 1.48611$

Imp

② The Refractive index of a glass prism is 1.62. If the angle of minimum deviation is 42°; What would be the angle of prism.

Soln R.I of glass prism (μ) = 1.62

angle of minimum deviation (D_m) = 42°

Angle of prism (A) = ?

We have,

or $\mu = \dfrac{\sin\left(\frac{A+D_m}{2}\right)}{\sin\frac{A}{2}}$

or $1.62 \times \sin\frac{A}{2} = \sin\left(\frac{A+42}{2}\right)$

or, $1.62 \times \sin\frac{A}{2} = \sin\left(\frac{A}{2}+21\right)$

or $1.62 \times \sin\frac{A}{2} = \sin\frac{A}{2}\cos 21 + \cos\frac{A}{2}\sin 21$

or $1.62 \times \sin\frac{A}{2} - \sin\frac{A}{2}\cos 21 = \cos\frac{A}{2}\sin 21$

or $\sin\frac{A}{2}(1.62 - \cos 21) = \cos\frac{A}{2}\sin 21$

or $\dfrac{\sin\frac{A}{2}}{\cos\frac{A}{2}} = \dfrac{\sin 21}{(1.62 - \cos 21)}$

or $\tan\frac{A}{2} = 0.52$

or $\frac{A}{2} = 27.47$

or $A = 2 \times 27.47$

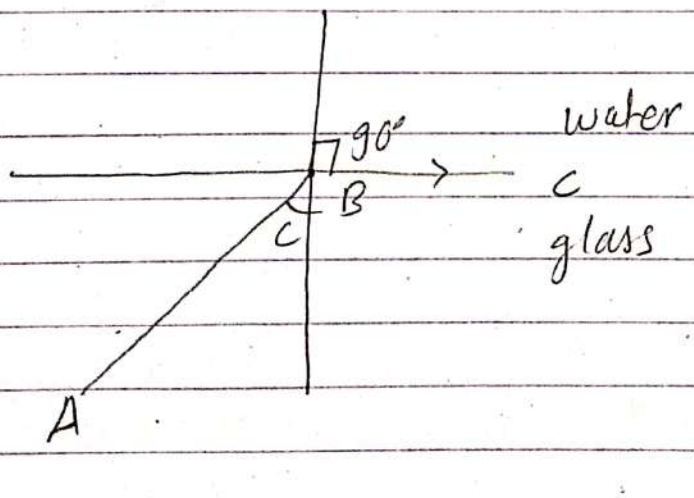
or $A = 54.94$

③ The refractive index of glass and water are 3/2 and 4/3 respectively. Calculate the critical angle in glass water interface.

R.I of water ($_a\mu_w$) = $4/3$

R.I of glass ($_a\mu_g$) = $3/2$

let 'c' be the critical angle.



For glass - water interface.

$$g\mu w = \frac{\sin c}{\sin 90°}$$

$$\text{or } g\mu a \cdot a\mu w = \frac{\sin c}{1}$$

$$\text{or } \frac{a\mu w}{a\mu g} = \sin c$$

$$\frac{4/3}{3/2} = \sin c$$

$$\sin c = \frac{8}{9}$$

$$c = \sin^{-1}(8/9)$$

$$c = 62.73'$$

Lenses:

A Lens is a transparent medium bounded by one or two curve surfaces. Generally, lens is made up of plastic or glass. It is divided ~~it~~ into two groups;

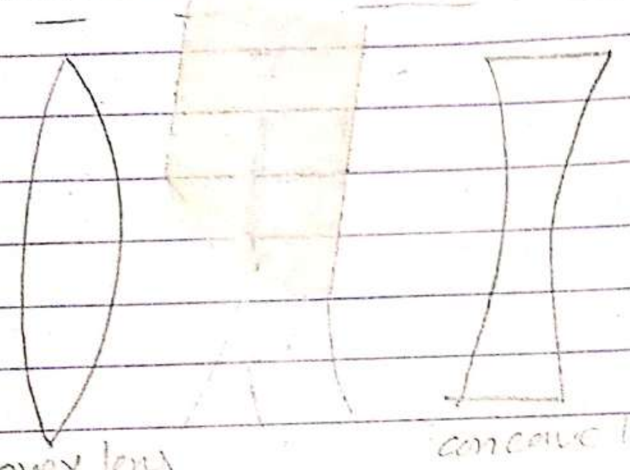
① Convex lens (or converging lens)

A convex lens is thicker is thicker at the center & thinner at the edges. This lens converses a parallel beam of light at a single point after refraction So, it is called converging lens.

It ~~movid~~ produce real image.

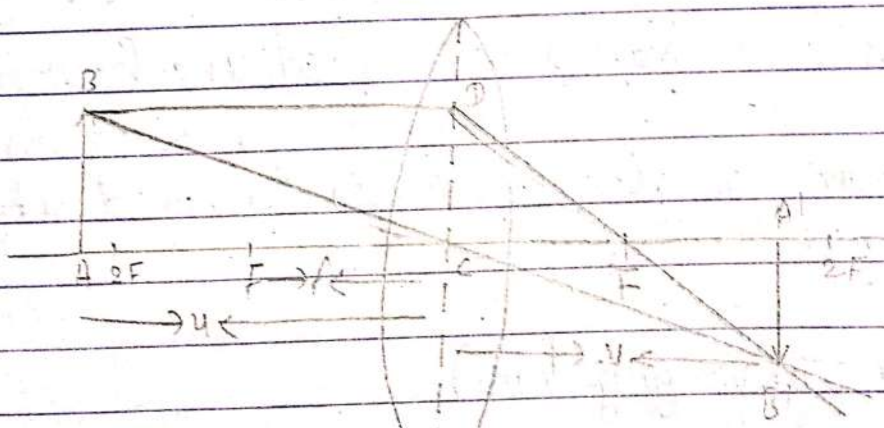
(b) Concave lens (or diverging lens):

Concave lens is thinner at the middle ~~at~~ and thickness at the edges. This lens diverses a parallel beam of light after refracting so it is called diverging lens. ~~The~~ It produce virtual image.



convex lens   concave lens

* Lens formula for convex Lens:-



Let us take a convex lens of focal length 'F' and optical center 'C'. Assume AB be an object place normally on the principle axis of the lens as shown in figure and its image is formed at A'. Here, A'B' is the real image of the object AB. ~~F~~

From the figure:-

Object distance = AC = u
image distance = A'C = v
focal length = CF = f

Now; In ΔABC and $\Delta A'B'C$, then

$\angle BAC = \angle B'A'C$ (~~Beig~~ Being 90°).
$\angle BCA = \angle B'CA'$ (Being V.O.A)
$\angle ABC = \angle A'B'C$ (Being Remaining angle)

Thus; ΔABC and $\Delta A'B'C$ are similar.

$$\frac{A'B'}{AB} = \frac{A'C}{AC}$$ (Being ~~C.S.S.T~~ Corresponding Side of same triangle)

Similarly; In ΔCDF and $\Delta A'B'F$ then

$$\frac{A'B'}{CD} = \frac{A'F}{CF}$$

But: $CD = AB$

$$\frac{A'B'}{AB} = \frac{A'F}{CF} \quad \text{--- (ii)}$$

From eqn (i) and (ii) then

$$\frac{A'C}{AC} = \frac{A'F}{CF}$$

or $$\frac{A'C}{AC} = \frac{CA' - CF}{CF}$$

$$\frac{v}{u} = \frac{v-f}{f}$$

$$vf = uv - uf$$

$$vf + uf = uv$$

$$f(v+u) = uv$$

$$f = \frac{uv}{v+u}$$

$$\frac{1}{f} = \frac{v+u}{uv}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

This is the lens formula for convex lens.

Magnification:

It is define as the ratio of size of image to the size of object. It is denote by 'm' and its define by;

$$\text{Magnification}(m) = \frac{\text{size of image}(A'B')}{\text{size of object}(AB)}$$

$$= \frac{A'B'}{AB} \quad \text{--- (III)}$$

But; from eqn (I) then

$$\frac{A'B'}{AB} = \frac{A'C}{AC} = \frac{v}{u} \quad \text{--- (IV)}$$

~~Fro~~ from eqn (III) and (IV) then

$$m = \frac{A'B'}{AB} = \frac{v}{u}$$

$$m = \frac{v}{u}$$

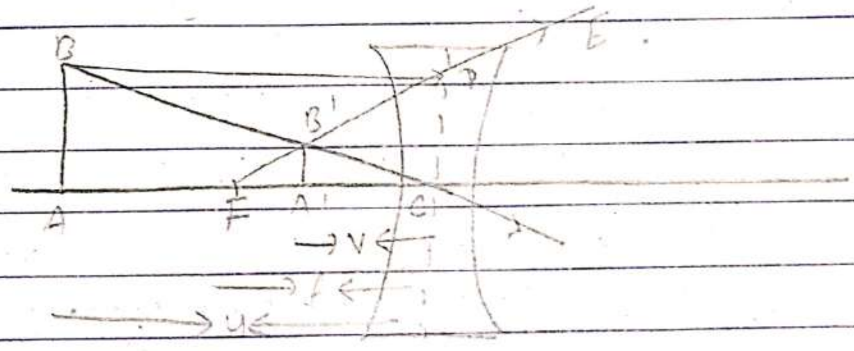
So; magnification can ~~so~~ also be define as the ratio of size of image distance to the object distance

Sign Convencation of convex lens:

(i) Every distance are measured from the optical center
(ii) The focal length of convex lens is positive.
(iii) The object distance is taken as positive for real object and negative for virtual object.

(iv) The image distance is taken as positive for real object and negative for virtual object.

Lens formula for concave lens:



Consider a thin concave lens of focal length 'f' and optical center 'c' as shown in figure. Let 'AB' be an object place normally on the principal axis. The two refracted ray DE and BC appears to meet at B' so that A'B' is the virtual image of the object formed by concave lens.

From the figure:

Object distance = $CA = u$
image distance = $CA' = -v$
focal length = $FC = -f$

In ΔABC and $\Delta A'B'C$ then;

$\angle BAC = \angle B'A'C$ (Being $90°$)
$\angle BCA = \angle B'CA'$ (Being common angle)
$\angle ABC = \angle A'B'C$ (Being remaining angle)

$\therefore \Delta ABC$ and $\Delta A'B'C$ are similar.

$$\frac{A'B'}{AB} = \frac{CA'}{CA} \text{ --- } (I)$$

Similarly; ΔCDF and $\Delta A'B'F$ are also similar.

$$\frac{A'B'}{CD} = \frac{FA'}{FC}$$

But; $CD = AB$

Then; $$\frac{A'B'}{AB} = \frac{FA'}{FC} \text{ --- } (II)$$

From eqn (I) and (II) then.

$$\frac{CA'}{CA} = \frac{FA'}{FC}$$

or, $$\frac{CA'}{CA} = \frac{FC - CA'}{FC}$$

$$\frac{-v}{u} = \frac{-f \cancel{+v} - (-v)}{-f}$$

$$\frac{\cancel{-}v}{u} = \frac{-f+v}{\cancel{-}f}$$

$$vf = -uf + uv$$

$$vf + uf = uv$$

$$f(v+u) = uv$$

$$f = \frac{uv}{v+u}$$

$$\frac{1}{f} = \frac{v}{uv} + \frac{u}{uv}$$

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

This is the lens formula for concave lens.

Magnification

It is define as the ratio of size of image to the size of object is called magnification. It is denoted by m and its define by;

$$\text{Magnification}(m) = \frac{\text{size of length}}{\text{size of object}}$$

In above figure; size of image $= A'B'$ and size of object $= AB$

$$\text{i.e. } m = \frac{A'B'}{AB} \quad \text{——(III)}$$

But: from eqn ① then

$$\frac{A'B'}{AB} = \frac{CA'}{CA} = \frac{-v}{u} \quad \text{——(IV)}$$

From eqn (III) and (IV) then

$$m = \frac{-v}{u}$$

Thus, the magnification of concave lens is negative and is define as the ratio of image distance to object distance.

MIQ

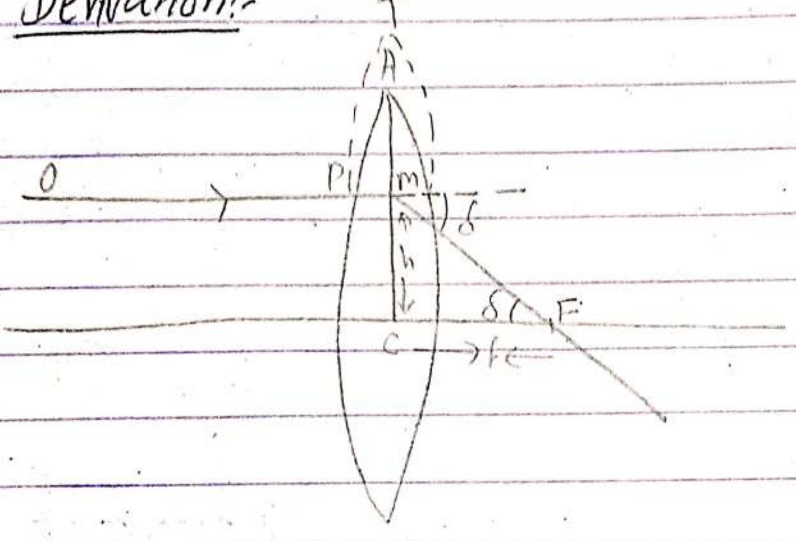
Lens Maker formula:-

This gives the relation between focal length, radius of curvature of surface and refractive index of materials.

(A) <u>Assumption:-</u>

① The apparture of the lens should be small.
② The thickness of the lens should be very small so that the optical center lie very close to the surface.
③ The deviation produce by the lens is similar to the deviation produce by small prism. ie. $\delta = A(\mu-1)$
④ The incidence and the refracted ray make small ~~with~~ angle with the principle axis.

(B) <u>Derivation:-</u>



Let us consider a ray OP is incident on a convex lens parallel to the principale axis at a height. Here, 'F' be the focal length of the lens. After refraction the imergent ray MF passing through the focus 'F' and 'δ' is the angle of deviation.

In $\triangle MCF$; then

$$\tan\delta = \frac{MC}{CF}$$

But; $MC = h$ and $CF = f$ then

$$\tan\delta = \frac{h}{f}$$

For small angle of incidence; $\tan\delta \simeq \delta$

st

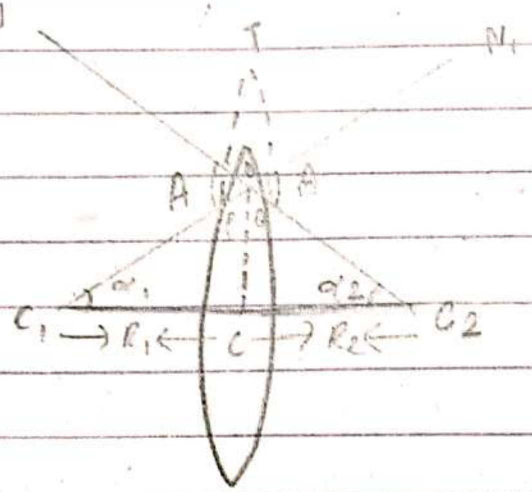
$$\delta = \frac{h}{f} \quad \text{— (i)}$$

Let PT and TQ be the two tangent draw on the surface of the lens which meet at point 'T' making small angle. Since, Lens acts as small angle prism then Angle of deviation $(\delta) = A(\mu-1)$ — (ii)

From eqn (i) and (ii) then

$$A(\mu-1) = \frac{h}{f}$$

$$\frac{1}{f} = \frac{A}{h}(\mu-1) \quad \text{— (iii)}$$



Let 'C_1' and 'C_2' be the centre of curvature of 1st and 2nd surface respectively.

Similarly, 'R_1' and 'R_2' be the radius of curvature here 'N_1''C_1' & 'N_2' 'C_2' are two normals. which makes an angle 'α_1' and 'α_2' to the principle axis.

Since, the angle between the lines is equal to the angle between the normals. So,

$\angle PTQ = \angle N_2OC_1 = \angle N_1OC_2 = A$

In $\Delta O'CC_1$ and $\Delta O'CC_2$

$$\tan\alpha_1 = \frac{h}{R_1} \quad \text{and} \quad \tan\alpha_2 = \frac{h}{R_2}$$

from small angled prism; then

$$\tan\alpha_1 \approx \alpha_1 = \frac{h}{R_1} \quad \text{and} \quad \tan\alpha_2 \approx \alpha_2 = \frac{h}{R_2}$$

from the figure;

$$\angle N_1O'C_2 = \angle O'C_1C + \angle O'C_2C$$

$$A = \alpha_1 + \alpha_2$$

$$A = \frac{h}{R_1} + \frac{h}{R_2}$$

$$A = h\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{A}{h} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \text{ —— (iv)}$$

Putting the value of (iv) on eqn (iii) then

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

This is the required relation of lens maker formula.

(Q) Does the focal length of a lens change when it is immersed in water?

=) When a lens of refractive index '${}_a\mu_g$' is immersed in water of refractive index '${}_a\mu_w$' then the lens maker formula becomes:

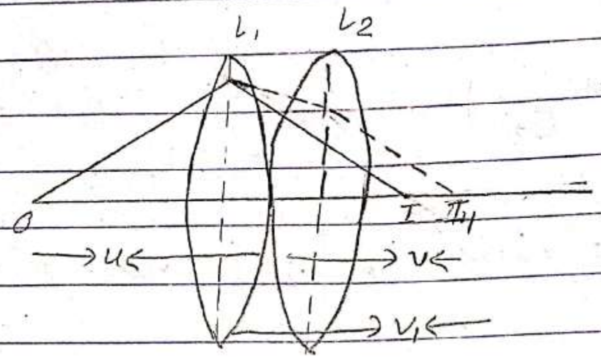
$$\frac{1}{f} = ({}_w\mu_g - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

or $$\frac{1}{f} = ({}_w\mu_a \, {}_a\mu_g - 1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

or, $$\frac{1}{f} = \left(\frac{{}_a\mu_g}{{}_a\mu_w} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Since; ${}_a\mu_g > {}_a\mu_w$ so; the focal length of lens is increased when immersed in water.

* Combination of two lens in contact:



Consider two thin convex lens 'L_1' and 'L_2' of focal length 'f_1' and 'f_2' placed in contact as shown in figure. Let 'O' be the point object and I be the final image formed by combined lens which is real.

In the absence of lens 'L_2' the lens 'L_1' converges the rays at 'I' as shown in figure.

For Lens L_1

object distance = u

image distance = v'

using lens formula; then

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v'} \quad \text{— ①}$$

In the presence of 'L_2' the image at 'I'' act as virtual object so, A conversing beam on 'L_2' forms real image at I.

For lens L_2

object distance $= -v'$
image distance $= v$

using lens formula then;

$$\frac{1}{f_2} = \frac{1}{-v'} + \frac{1}{v} \quad \text{—} \ (ii)$$

Adding eqn (i) and (ii) then;

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \cancel{\frac{1}{v'}} - \cancel{\frac{1}{v'}} + \frac{1}{v}$$

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{u} + \frac{1}{v} \quad \text{—} \ (iii)$$

For combine Lens;
object distance $= u$
image distance $= v$

using lens formula; then

$$\frac{1}{F} = \frac{1}{u} + \frac{1}{v} \quad \text{---} \quad (iv)$$

~~Form~~ from eqn (iii) and (iv) then

$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2}$$

Defect of vision

* Accomodation of ~~to~~ vision:

The ability of eye to bring the object lying at various distance to focus on retina is called accomodation of eye.

* Near point and far point:-

The nearest point upto which an object can be seen clearly by the eye is called near point.

And, the distance of nearest point from eye is called least distance of ~~district~~ distinct vision which 25 cm for eye.

The most distant person upto which an eye can see clearly is called far point. It is at infinity for normal eye.

* Range of vision:

The range of vision for normal eye is 25 cm to infinity.

* Persistance of vision:

A particular property of eye by virtue of which the impression or signal of image persist on retina upto $\frac{1}{10}$th of a second after the object has been removed is called persistance of vision.

④ Defect of vision:

The inability of eye to see an object clearly due to the presence of abnormality is called defect of vision. There are two defect of vision. They are:-

① Myopia or short sightness
② Hypermetropia or long sightness

Myopia or short sightness:-

It is a defect of vision in which a person can see nearby object clearly but cannot see the distant object object clearly. It is a young age defect. In this defect, the rays of light from distant object are focused inform of retina as shown in fig a.

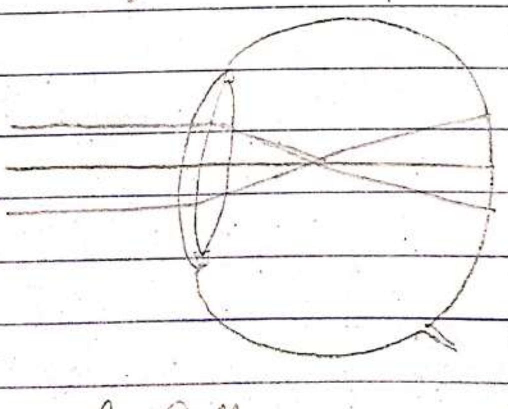


fig (a) Myopia

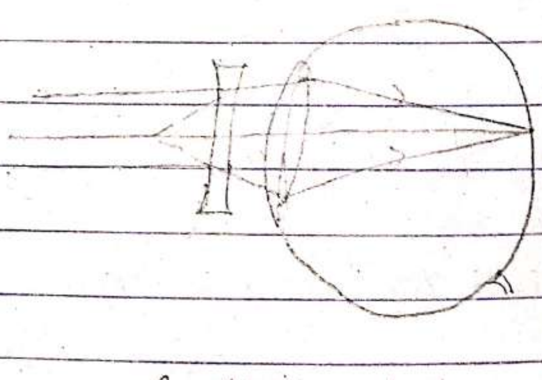


fig (b) Corrected eye

Causes:
1) Elongation of eye ball.
2) Decrease in focal length of eye lens.

Remedy:- This defect can be removed by using concave lens is suitable focal length so that the image is formed on media.

For concave lens.

Let 'x' be the distance of far point from the eye where virtual image is formed.

Here;
Object distance$(u) = \infty$
image distance $(v) = -x$
focal length $= f$

using the lens formula then,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $$\frac{1}{f} = \frac{1}{\infty} + \frac{1}{-x}$$

or, $F = -x$

Here the myopia eye can be corrected by using diversing lens of focal length 'f' ~~be~~ equal to the distance between eye lens and far point.

ii) Hypermetropia or long sightness: It is a defect of vision in which a person can see the distant object but cannot see near object. In this defect the rays of light of from distant object are focused behind the retina as shown in fig. It is common for old people.

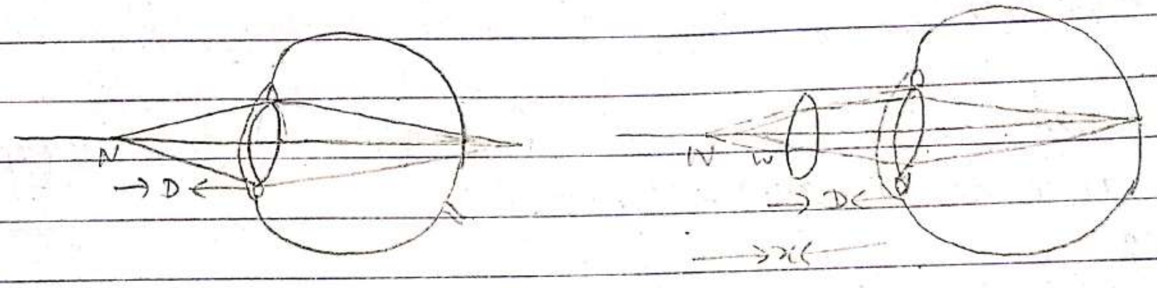


fig (a) Hypermetropia

fig (b) corrected eye

Causes:

* Contraction of eyeball
* Increase in focal length of eye lens.

Remedy:

This defect can be corrected by using convex lens of suitable focal length which bring the back to retina as shown in figure 'b'.

For convex lens

Let 'x' be the distance of near point from the eye which virtual image is formed and 'D' be the least distance of distinct vision.

Object distance $(u) = D$
image distance $(v) = -x$
focal length $= f$

from lens formula, then

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or $$\frac{1}{f} = \frac{1}{D} + \frac{1}{-x}$$

or $$\frac{1}{f} = \frac{x-D}{xD}$$

or $$f = \frac{xD}{x-D}$$

Hence; the hypermetropic eye can be corrected by using focal length equal to image distance greater than object distance.

Angular Magnification:-

It is define as the ratio of the angle substended by the final image at the eye to the angle substended by the object ~~image~~ to the eye. In other words, It may be define as the angular size of an image to the angular size of the corresponding object. It is denoted by 'M'.

If 'α' be the angle substended by the object at the eye and 'β' be the angle substended by the final image at the eye. then;

$$\text{Angular magnification} = \frac{\beta}{\alpha}$$

It is unit less quantity and has no dimension.

Simple microscope:-

A simple microscope is an ~~object~~ optical instruchment which is used to view near by object clearly. The convex lens of short focal length can be used as simple microscope. It is also known as magnifying glass / magnifier.

Principle of simple microscope:-

It works on the principle that when an object is placed between principle focus and the optical centre of the lens then an erect, virtual & highly magnified Image is form toward the same side of the lens as shown in figure below.

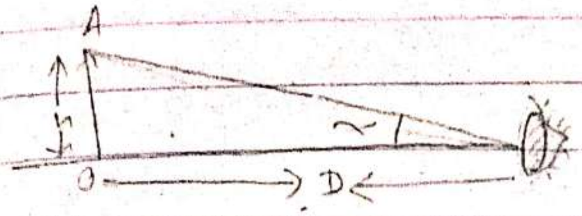


fig (a) visual angle with unaided angle

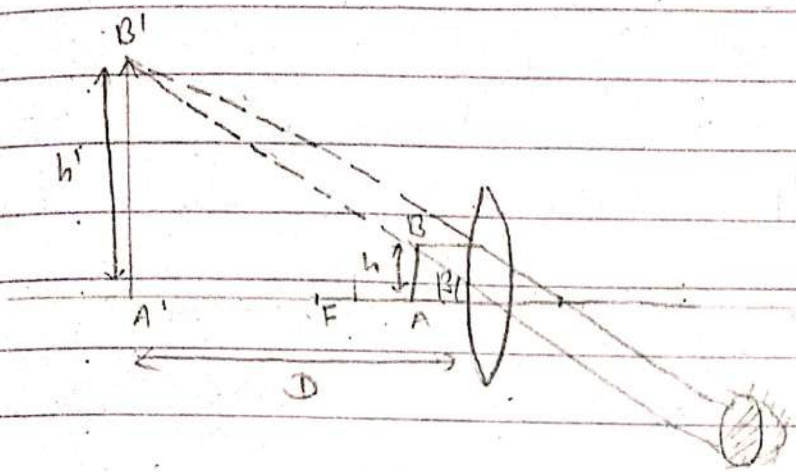


fig (b) Image formed by simple microscope at near point.

let us consider an object of height 'h' is placed infront of eye at least distance of distant vision then the visual angle substended by an object to the eye is;

$$\tan\alpha = \frac{h}{D}$$

From small angle $\tan\alpha \simeq \alpha$ such that $\alpha = \frac{h}{D}$ — (1)

Now consider an object AB is placed between principle focus 'F' and optical centre 'c' of convex lens of focal length 'f' so that virtual image A'B' is formed at least distance of distaint vision then the visual angle substended by final image to the eye is;

$$\beta = \frac{h'}{D} \quad \text{— (II)}$$

Thus, the angular magnification of simple microscope is;

$$M = \frac{\beta}{\alpha}$$

$$= \frac{\frac{h'}{D}}{\frac{h}{D}}$$

$$= \frac{h'}{h} \quad \text{——— (III)}$$

Using lens formula, then

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\frac{v}{f} = \frac{v}{u} + \frac{v}{v}$$

$$\frac{v}{f} = \frac{v}{u} + 1$$

$$\frac{v}{u} = \left(\frac{v}{f} - 1\right) \quad \text{—— (IV)}$$

Since; $\frac{h'}{h} = \frac{v}{u}$ = linear magnification of lens;

So; $M = \left(\frac{v}{f} - 1\right)$

But; $v = -D$. So, the above eqn becomes;

$$m = -\left(\frac{D}{f} + 1\right)$$

This relation gives the angular magnification of simple microscope when the final image is formed as least distance of distinct vision.

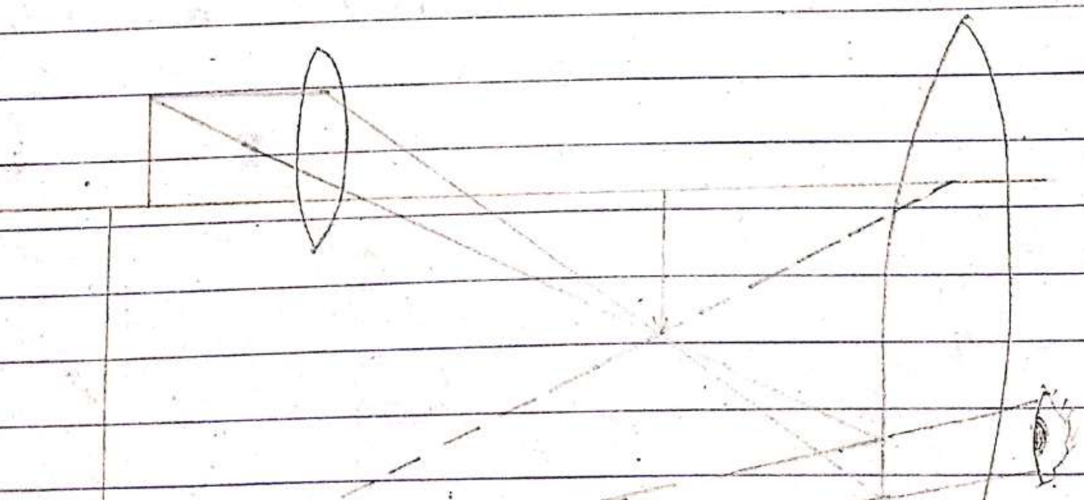
However, if the final image is formed at infinity then the ~~focal~~ object must be placed on the focus of the lens. So the angular magnification becomes;

or, $$M = \frac{\frac{h}{f}}{\frac{h}{D}}$$

or, $$M = \frac{D}{f}$$

Compound Microscope:
The compound microscope is an optical instrument which is used to obtain highly magnified image of an exteremly small object. It consists of two converging lens place coaxially i.e. objective lens lying close to the object and eye lens lies near to the eye and has larger focal length.

Principle of compound microscope:
When a small object is placed beyond the focal length of objective lens then a real, inverted & magnified image is produces on the other side of the lens. This image acts on object for the eye lens. The eye is so adjusted such that the final image is ~~forme~~ formed at the least distance of distinct vision.



Let us consider an object 'AB' of height 'h' is placed ~~through~~ beyond the focus of objective lens then a real, inverted & magnified image 'A'B'' is formed at another side of objective. This image act as an object for eye lens. This image A'B' lies between principle focus of eye piece and optical centre C_2. so that the final image A''B'' which is virtual, erect and highly magnified is formed at least distance of distinct vision.

When an object is placed at least distance of distinct vision then the visual angle substended by the object at the eye is;

$$\alpha = \frac{h}{D} \quad \text{— (1)}$$

Also; The visual angle substended by the final image at the eye piece is

$$\beta = \frac{h_2}{D} \quad \text{— (11)}$$

Hence, the angular magnification become;

$$\text{or } M = \frac{\beta}{\alpha}$$

$$\text{or } M = \frac{\frac{h_2}{D}}{\frac{h}{D}}$$

$$\text{or } M = \frac{h_2}{h}$$

or $M = \frac{h_2}{h_1} \times \frac{h_1}{h}$

or $M = M_e \times M_o$ — (iii)

where; M_e and M_o are angular magnification for objective lens and eye piece.

For eye piece;

Object distance$(u) = u_e$
image distance$(v_e) = -D$
focal length $= f_e$

Using lens formula, then

$$\frac{1}{f_e} = \frac{1}{u_e} + \frac{1}{v_e}$$

or $$\frac{v_e}{f_e} = \frac{v_e}{u_e} + \frac{v_e}{v_e}$$

or $$\frac{v_e}{u_e} = \left(\frac{v_e}{f_e} - 1\right)$$

or $$\frac{v_e}{u_e} = -\left(\frac{D}{f_e} + 1\right)$$

or $$M_e = -\left(\frac{D}{f_e} + 1\right) \text{ — (iv)}$$

Also for objective lens;

object distance $= u_0$
image distance $= v_0$
focal length $= f_0$

Using lens formula; then;

or $\frac{1}{u_0} + \frac{1}{v_0} = \frac{1}{f_0}$

or, $\frac{v_0}{u_0} + \frac{v_0}{v_0} = \frac{v_0}{f_0}$

or $\frac{v_0}{u_0} = \frac{v_0}{f_0} - 1$

or $M_0 = \left(\frac{v_0}{f_0} - 1\right)$ — (v)

Thus; from eqn (iv) and (v) on eqn (iii) then

or $M = -\left(\frac{D}{f_e} + 1\right)\left(\frac{v_0}{f_0} - 1\right)$

This is the required expression for angular magnification of compound microscope.

However, if the final image is formed at infinity then the angular magnification becomes,

$$M = \frac{\beta}{\alpha}$$

$$M = \frac{\frac{h_1}{fe}}{\frac{h}{D}}$$

$$M = \frac{h_1}{h} \cdot \frac{D}{fe}$$

But; $\frac{h_1}{h} = \frac{v_0}{u_0}$

Thus above eqn becomes.

$$M = \frac{v_0}{u_0} \cdot \frac{D}{fe}$$

(*) An object is placed 15 cm from a ~~diverging~~ converging lens of focal length 10 cm. Calculate the image distance & magnification?

Soln

Object distance (u) = 15cm
focal length (f) = 10cm
image distance (v) = ?

From lens formula, then

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{15}$$

$$\frac{1}{v} = \frac{3-2}{30}$$

$$\frac{1}{v} = \frac{1}{30}$$

$$v = 30 \text{ cm}$$

Also, Magnification (m) $= \frac{v}{u} = \frac{30}{15} = 2$

In case of diverging lens;

$u = 15$
$v = ?$
$f = -10\text{ cm}$

Using lens formula;

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$-\frac{1}{10} = \frac{1}{15} + \frac{1}{v}$$

$$\frac{1}{v} = -\left(\frac{1}{10} + \frac{1}{15}\right)$$

$$\frac{1}{v} = -\left(\frac{3+2}{30}\right)$$

$$\frac{1}{v} = -\left(\frac{\cancel{5}}{\cancel{30}\,6}\right)$$

$$v = -6\text{ cm}$$

Also; Magnification $= \frac{v}{u}$

$$= -\frac{\cancel{6}\,2}{\cancel{15}\,5} = -0.4$$

Note:-

Sign convention

(i) For convex lens (converging)

$u = +ve$

$v = +ve$

$f = +ve$

$m = +ve$

(ii) For concave lens (diverging)

$u = +ve$

$v = -ve$

$f = -ve$

$m = -ve$

(*) A diverging lens of 12cm focal length produces a virtual image whose linear dimension are 1/3 that of the object. Determine the position of object and image?

Soln

Let u and v be object and image distance respectively.

Magnification (m) = $-\frac{1}{3}$

or $\frac{v}{u} = -\frac{1}{3}$

or $v = -\frac{u}{3}$ —— ①

focal length (f) = -12cm

Using lens formula, then;

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

or, $\frac{1}{-12} = \frac{1}{u} + \frac{1}{\frac{-u}{3}}$

or, $-\frac{1}{12} = \frac{1}{u} - \frac{3}{u}$

or, $+\frac{1}{12} = \frac{+2}{u}$

or $u = 24$ cm

So; $v = -\frac{u}{3}$

$= -\frac{24}{3}$

$= 8\ cm$

* What is the power of spectaleles required by a hypermetropia eye whose near point is 125 cm

soln

near point (D) = object distance (u) = 25 cm
image distance (v) = $-\cancel{15}$ 125 cm

Using lens formula;

or, $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

or, $\frac{1}{f} = \frac{1}{25} - \frac{1}{125}$

or, $\frac{1}{f} = \frac{5-1}{125}$

or $\cancel{\frac{1}{f}}\ f = \frac{125}{4}$

or $f = 31.25\ cm$

$= 0.3125\ m$

The power of lens is;

$$P = \frac{1}{f}$$

$$= \frac{1}{0.3125}$$

$$= 3.2D$$

(*) **Photometry:** It is the branch of physics that deals with the pratical measurement of light

Total radiant flux: Total invisible energy radiated by source is called radiant energy and the total radiant energy per unit time is called total radiant flux: It's SI unit is watt.

Total luminous flux: The total visible energy radiated by source is called luminous energy and the luminous energy radiated by source per unit time is called luminous flux. It is donated by ϕ and is defined by;

$$\phi = \frac{\text{Luminous energy}}{\text{Time taken}}.$$

Its unit is Lumen in SI system.

* Luminous efficiency:

The ratio of total luminous flux to the total radiant flux of a source is called luminous efficiency. It's SI unit is lumen/watt. It is denoted by η and is defined by;

or, $\eta = \frac{\text{Total luminous flux } (\phi)}{\text{Total radiant flux } (R)} \times 100\%$

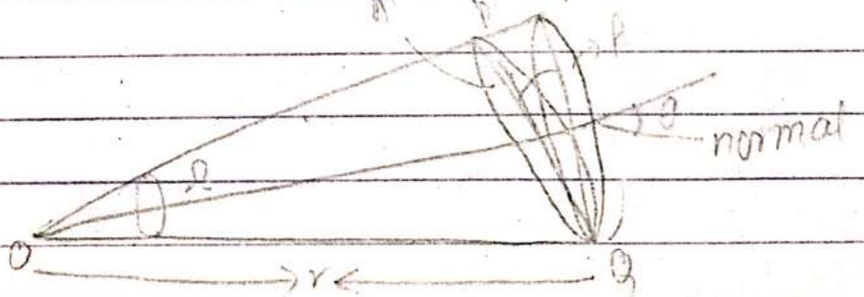
or, $\eta = \frac{\phi}{R} \times 100\%$

* Solid Angle (Ω):-

It is the angle subtended at point by the area of a surface. It measure the divergense of a cone. It is denoted by Ω.

Mathimatically, It is define as the ratio of the area of a surface to the square of distance of the point to the surface i.e. $\Omega = \frac{A}{r^2}$ — ①

This equation is valid only when the normal to the surface is parallel to the radius.



If normal to the surface makes an angle 'θ' with the radius then eqn ① becomes:

$$\Omega' = \frac{A'}{r^2} = \frac{A\cos\theta}{r^2}$$

In case of ~~special~~ spherical symmetry then eqn ① becomes,

or, $\Omega = \frac{4\pi r^2}{r^2}$

or, $\Omega = 4\pi$

Its unit is steradian.

(*) Luminous intensity: The Luminous flux emmitted by a source per unit solid angle is called luminous intensity. It is denoted by I and Its SI unit is Candela (Cd).

Mathematically,

$$\text{Luminous intensity } (I) = \frac{\text{Luminous flux } (\phi)}{\text{Solid angle } (\Omega)}$$

$$I = \frac{\phi}{\Omega}$$

The solid angle substended at the centre of the sphere is;

$$\Omega = 4\pi$$

So; the above eqn reduced to:

or, $I = \dfrac{\phi}{4\pi}$

$$\boxed{\text{or, } \phi = 4\pi I}$$

Illuminance:-
The total luminous flux incidence normally on unit Area of surface is called illuminance or intensity of illuminance. It is denoted by 'E' and its SI unit is lux or lumen/m^2 or Phot in CGS system. It is defined by;

$$\text{Illuminance } (E) = \frac{\text{luminous flux } (\phi)}{\text{Area } (A)}$$

$$\text{or, } E = \frac{\phi}{A} \quad \text{——— (1)}$$

For spherical symmetry, then

$$\left.\begin{array}{l} \phi = \cancel{EI}\ 4\pi I \\ \text{and } A = 4\pi r^2 \end{array}\right\} \text{——— (11)}$$

Thus; From eqn (i) & (ii) then

$$E = \frac{\cancel{4\pi} I}{\cancel{4\pi} r^2}$$

or, $$\boxed{E = \frac{I}{r^2}}$$

(*) Why does illuminance of a surface decreases as it moves away from the light source?

⇒ From the definition of illuminance:

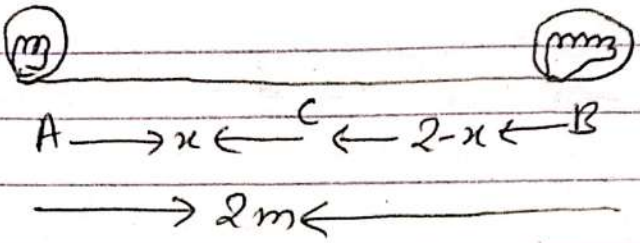
$$E = \frac{I}{r^2}$$

In General

$$E \propto \frac{1}{r^2}$$

So, when the distance from source increases then the illuminance of the surface decreases.

(*) Two electric bulbs of 16 candela and 64 candela are 2 m apart. Find the position of the line joing them at which the illuminance are equal?



<u>For Bulb A</u>

$I_1 = 16$ candela

$r_1 = x$

<u>For Bulb B</u>

$I_2 = 64$ candela

$r_2 = 2-x$

By question; illuminance of two source are equal

$$E_1 = E_2$$

or $$\frac{I_1}{r_1^2} = \frac{I_2}{r_2^2}$$

or, $$\frac{\cancel{16}}{x^2} = \frac{\cancel{64}\,4}{(2-x)^2}$$

or $$4 - 4x + x^2 = 4x^2$$

or, $$3x^2 + 4x - 4 = 0$$

or $$3x^2 + 6x - 2x - 4 = 0$$

or, $$3x(x+2) - 2(x+2) = 0$$

or $$(x+2)\,(3x-2) = 0$$

Either $3x - 2 = 0$ or $x + 2 = 0$

$x = \frac{2}{3}$ m $\quad$ $x = -2$ m

Here, x can never be -ve. So; $x = \frac{2}{3}$ m from bulb A.