CTEVT, DIPLOMA, QUESTION & SOLUTION

⇐ SECOND EDITION ⇒

Engineering Mathematics- II

(for Diploma I Yrs. II Part)

Second Semester

(Engineering All)

By

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S.No	Exam Year,	Month	
1)	2077	Chaitra	Regular/Back
2)	2078	Magh	Regular/Back
3)	2079	Falgun	Regular/Back
4)	2079	Falgun	R/B__(Old + Very Old)
5)	2080	Mangsir	R/B__(Old + Very Old)
6)	2080	Mangsir	Regular (New)

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Engg. Mathematics –II__(Engg. All) $2^{nd\ Sem}$

(2077) Question Paper Solution.

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1. a) If $\vec{a}$ and $\vec{b}$ are unit vectors and θ be the angle between them prove that $\frac{1}{2}|\vec{a}-\vec{b}| = \sin\frac{\theta}{2}$.

➢ **Solution:-**

To prove $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a}-\vec{b}|$

We have, $|\vec{a}-\vec{b}|^2 = (\vec{a}-\vec{b})^2$

$$= (\vec{a})^2 - 2\vec{a}.\vec{b} + (\vec{b})^2$$

$$= 1 - 2\cos\theta + 1 \qquad [\because \vec{a} = 1 \text{ and } \vec{b} = 1, \text{Unit Vector}]$$

$$= 2 - 2\cos\theta$$

$$= 2(1-\cos\theta)$$

$$= 2.2\sin^2\frac{\theta}{2}$$

$$= 4\sin^2\frac{\theta}{2}$$

$$|\vec{a}-\vec{b}| = 2\sin\frac{\theta}{2}$$

$$\therefore \frac{1}{2}|\vec{a}-\vec{b}| = \sin\frac{\theta}{2} \text{ Proved.}$$

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b) State De Moivre's theorem. Use it to find the cube roots of unity.

➢ De Moivre's Theorem

For any real number n,

$$[r(\cos\theta + i\sin\theta]^n = r^n(\cos\theta + i\sin n\theta)$$

To Find the cube roots of unity:

Let $z^3 = 1$, so that we need to find z.

Writing in polar form, we have

$$z = \cos 0° + i\sin 0°$$

$$z^3 = \cos 0° + i\sin 0°$$

$$= \cos n\,360° + i\sin n\,360°$$ general polar form.

$$\therefore\ z = (\cos n\,360° + i\sin n\,360°)^{\frac{1}{3}}$$

$$= \cos\frac{n\,360°}{3} + i\ \sin\frac{n\,360°}{3}$$ $\because\ [r(\cos\theta + i\sin\theta]^n = r^n(\cos\theta + i\sin n\theta)$

$$= \cos n\,120° + I\ \sin n\,120°,$$

where n = 0, 1, 2

For n = 0, $z = \cos 0° + i\sin 0° = 1 + i\,0$

$$= \mathbf{1}.$$

For n = 1, $z = \cos 120° + i\sin 120° = 1 + i\,\frac{\sqrt{3}}{2}$

$$= \frac{\mathbf{1 + i\sqrt{3}}}{\mathbf{2}}$$

For n = 2, $z = \cos 240° + i\sin 240° = -\frac{1}{2} - i\,\frac{\sqrt{3}}{2}$

$$= \frac{\mathbf{-1 - i\sqrt{3}}}{\mathbf{2}}$$

Hence, required roots are : $\mathbf{1,\ \frac{1 + i\sqrt{3}}{2},\ \frac{-1 - i\sqrt{3}}{2}}$

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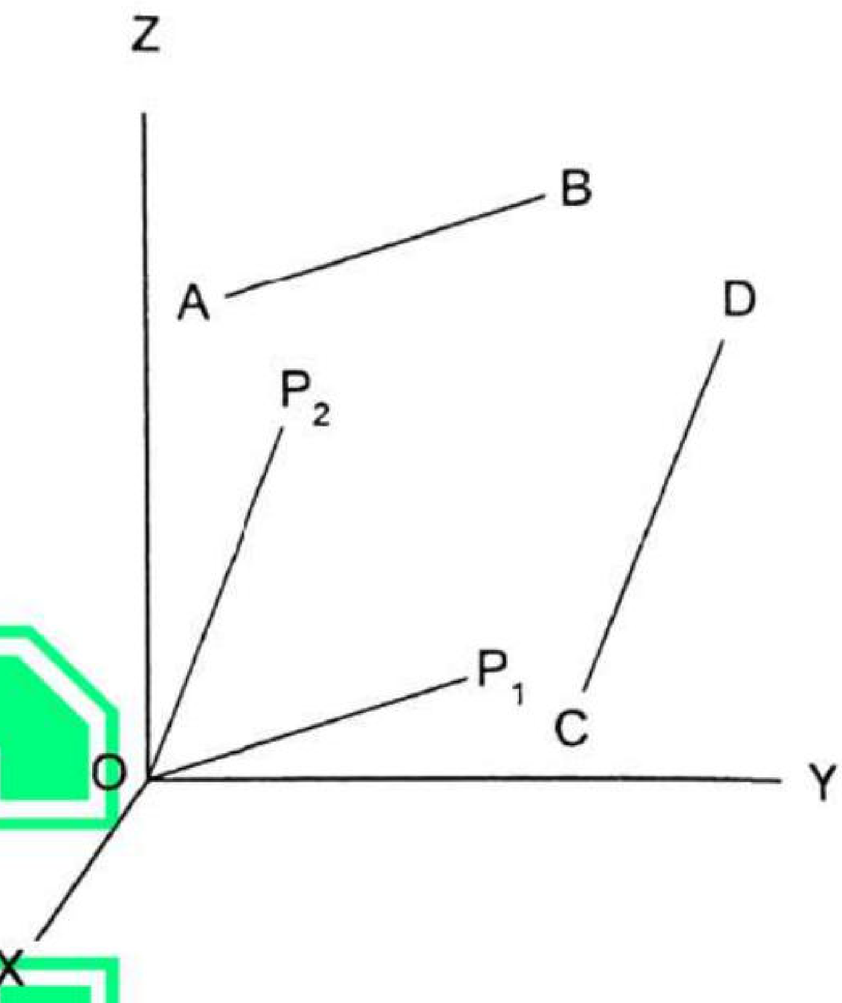
2. a) Define direction cosine. Find the angle between the lines whose direction cosines are given by l_1, m_1, n_1 and l_2, m_2, n_2 .

- If a line AB makes angles α, β, γ with the directions x-, y-, z-axis respectively, then $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called the **direction cosines**.

- To find the angle between two lines with direction cosine (l_1, m_1, n_1) and (l_2, m_2, n_2)



Let (l_1, m_1, n_1) and (l_2, m_2, n_2) be the direction cosines of two given lines AB and CD respectively. Let OP_1 and OP_2 be the lines through the origin O parallel to the lines AB and CD respectively so that the angle between them is the same as the angle between the lines AB and CD. Let this angle be θ. Also the direction cosines of OP_1 and OP_2 are respectively (l_1, m_1, n_1) and (l_2, m_2, n_2). Let the coordinates of P_1 and P_2 be (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively.

If $OP_2 = r_2$ the projection of OP_2 on OP_1 is $r_2 \cos\theta$.

Also the projection of OP_2 on OP_1 is equal to $l_1x_2 + m_1y_2 + n_1z_2$.

Therefore we have,

$$r_2 \cos\theta = l_1x_2 + m_1y_2 + n_1z_2$$

or, $$\cos\theta = l_1\frac{x_2}{r_2} + m_1\frac{y_2}{r_2} + n_1\frac{z_2}{r_2}$$ $[\because x_2 = l_2r_2, y_2 = m_2r_2, z_2 = n_2r_2]$

$$\cos\theta = \boldsymbol{l_1 l_2 + m_1m_2 + n_1n_2}$$

Which gives the angle θ.

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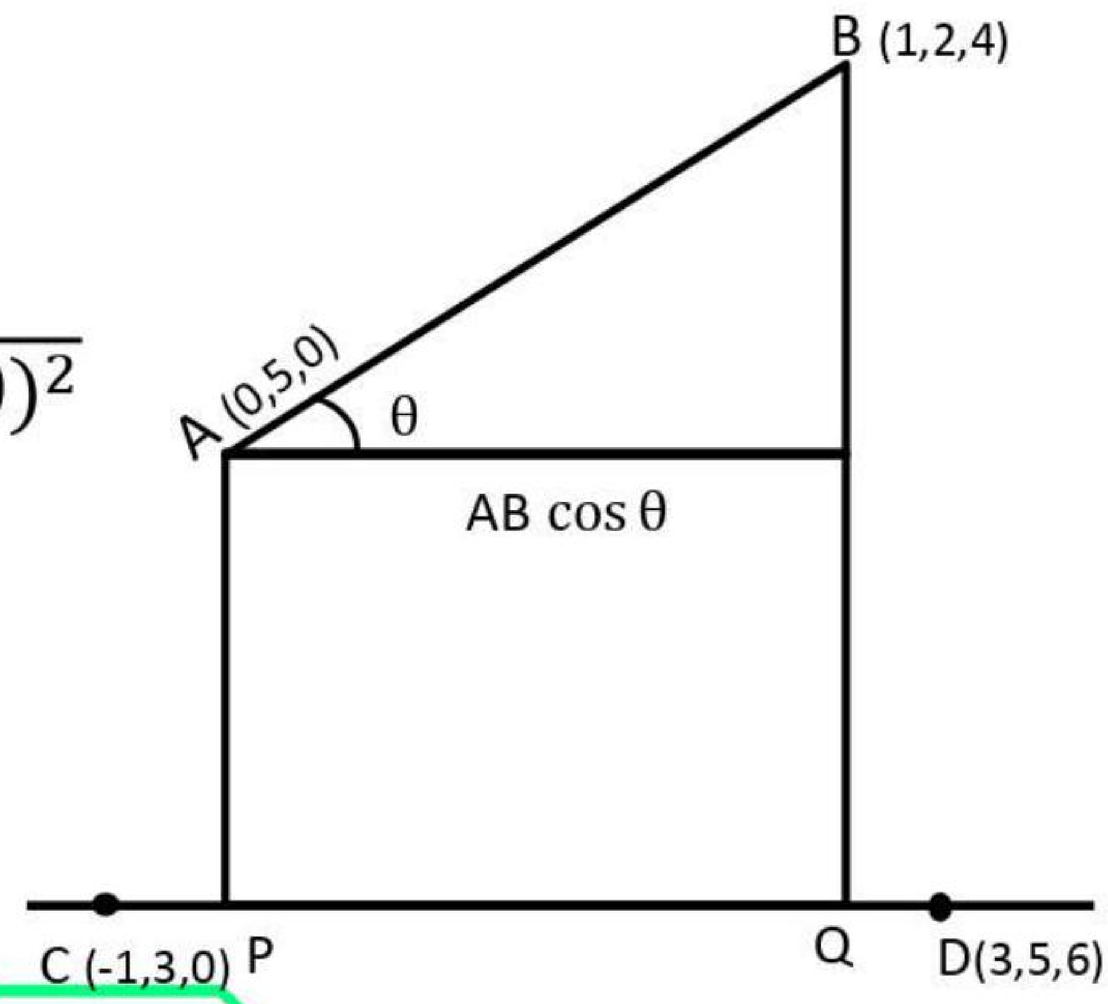
b) Find the projection of the line AB on CD if the co- ordinates of the points A, B, C, and D are (0, 5, 0) (1, 2, 4), (-1, 3, 0) and (3, 5, 6) respectively.

➢ **Solution:-**



$$AB = \sqrt{(1-0)^2 + (2-5)^2 + (4-0)^2}$$

$$AB = \sqrt{26} \text{ unit}$$

Projection of AB on CD = AB cos θ

As θ be angle between line AB & CD

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}\sqrt{{a_2}^2 + {b_2}^2 + {c_2}^2}}$$

For AB	**&**	**For CD**
$a_1 = 1 - 0 = 1$		$a_2 = 3 - (-1) = 4$
$b_1 = 2 - 5 = -3$		$b_2 = 5 - 3 = 2$
$c_1 = 4 - 0 = 4$		$c_2 = 6 - 0 = 6$

$$\cos\theta = \frac{1 \times 4 + (-3) \times 2 + 4 \times 66}{\sqrt{1^2 + (-3)^2 + 4^2}\ \sqrt{4^2 + 2^2 + 6^2}}$$

$$\mathbf{\cos\theta = \frac{22}{\sqrt{26}\sqrt{56}}}$$

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Projection of AB on CD = AB $\cos\theta$

$$= \sqrt{26} \times \frac{22}{\sqrt{26}\sqrt{56}}$$

$$= \mathbf{2.9398}$$

3. a) Solve by Cramer's rule or inverse matrix method of the equation $x + y - z = 3, 2y + z = 10$ and $5x - y - 2z = -3$.

- **Solution,**

Given,

$x + y - z = 3$
$2y + z = 10$
$5x - y - 2z = -3$

$$\text{Let } \mathbf{D} = \begin{vmatrix} 1 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & -1 & -2 \end{vmatrix} = 1\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 5 & -2 \end{vmatrix} - 1\begin{vmatrix} 0 & 2 \\ 5 & -1 \end{vmatrix}$$

$$= 1(-4+1) - 1(0-5) - 1(0-10)$$

$$= -3 + 5 + 10$$

$$= \mathbf{12}$$

$$\mathbf{D_1} = \begin{vmatrix} 3 & 1 & -1 \\ 10 & 2 & 1 \\ -3 & -1 & -2 \end{vmatrix} = 3\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix} - 1\begin{vmatrix} 10 & 1 \\ -3 & -2 \end{vmatrix} - 1\begin{vmatrix} 10 & 2 \\ -3 & -1 \end{vmatrix}$$

$$= 3(-4+1) - 1(-20+3) - 1(-10+6)$$

$$= -9 + 17 + 4$$

$$= \mathbf{12}$$

$$\mathbf{D_2} = \begin{vmatrix} 1 & 3 & -1 \\ 0 & 10 & 1 \\ 5 & -3 & -2 \end{vmatrix} = -1\begin{vmatrix} 10 & 1 \\ -3 & -2 \end{vmatrix} - 3\begin{vmatrix} 0 & 1 \\ 5 & -2 \end{vmatrix} - 1\begin{vmatrix} 0 & 10 \\ 5 & -3 \end{vmatrix}$$

$$= 1(-20+3) - 3(-0-5) - 1(-0+50)$$

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$$= -17 + 15 + 50$$
$$= 48$$

$$\mathbf{D_3} = \begin{vmatrix} 1 & 1 & 3 \\ 0 & 2 & 10 \\ 5 & -1 & -3 \end{vmatrix} = -1\begin{vmatrix} 2 & 10 \\ -1 & -3 \end{vmatrix} - 1\begin{vmatrix} 0 & 10 \\ 5 & -3 \end{vmatrix} + 3\begin{vmatrix} 0 & 2 \\ 5 & -1 \end{vmatrix}$$
$$= 1(-6+10) - 1(-0-50) + 3(-0+10)$$
$$= 4 + 50 - 30$$
$$= \mathbf{24}$$

By using Cramer's rule, We have

$$x = \frac{D_1}{D} = \frac{12}{12} = \mathbf{1},\ y = \frac{D_2}{D} = \frac{48}{12} = \mathbf{4},\ z = \frac{D_3}{D} = \frac{24}{12} = \mathbf{2}$$

Hence, $\boldsymbol{x = 1, y = 4, z = 2.}$

- *Row equivalent matrix method, See the Solu. 2078 of 1(b) page 26.*

b) Find the local Maxing and local Minima of the function $f(x) = 2x^3 - 3x^2 - 36x$ Also, find the point of inflection.

- **Solution**

Given equation $f(x) = 2x^3 - 3x^2 - 36x.$

$$f'(x) = 6x^2 - 6x - 36 \ \ and$$
$$f''(x) = 12x - 6$$

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For maximum and minimum value of $f(x)$, We have

$$f'(x) = 0$$

$$or,\ 6x^2 - 6x - 36 = 0$$

$$or,\ x^2 - x - 6 = 0$$

$$or,\ x^2 - 3x + 2x - 6 = 0$$

$$or,\ x(x-3) + 2(x-3) = 0$$

$$or,\ (x-3)(x-2) = 0$$

$$\therefore\ \boldsymbol{x = 3, -2.}$$

When $\boldsymbol{x = 3}$, $f''(x) = 12(3) - 6 = 36 > 0$. So f(x) has a maximum value at $x = 3$ and the maximum value is given by

$$= 2.(3)^3 - 3.(3)^2 - 36.3$$

$$= \mathbf{-81.}$$

When $\boldsymbol{x = -2}$, $f''(x) = 12(-2) - 6 = -36 < 0$. So $f(x)$ has a minimum value at $x = 4$ and the value is given by

$$= 2.(-2)^3 - 3.(-2)^2 - 36(-2)$$

$$= \mathbf{44}$$

For point of inflection, $f''(x) = 0$

$$or, 12x - 6 = 0$$

$$or, x = \frac{6}{12}$$

$$\therefore\ \boldsymbol{x = \frac{1}{2}}$$

$\therefore$ local minimum value at x = 3 and value = −81

$\therefore$ local minimum value at x = −2 and value = 44

$\therefore$ point of inflection at $x = \frac{1}{2}$

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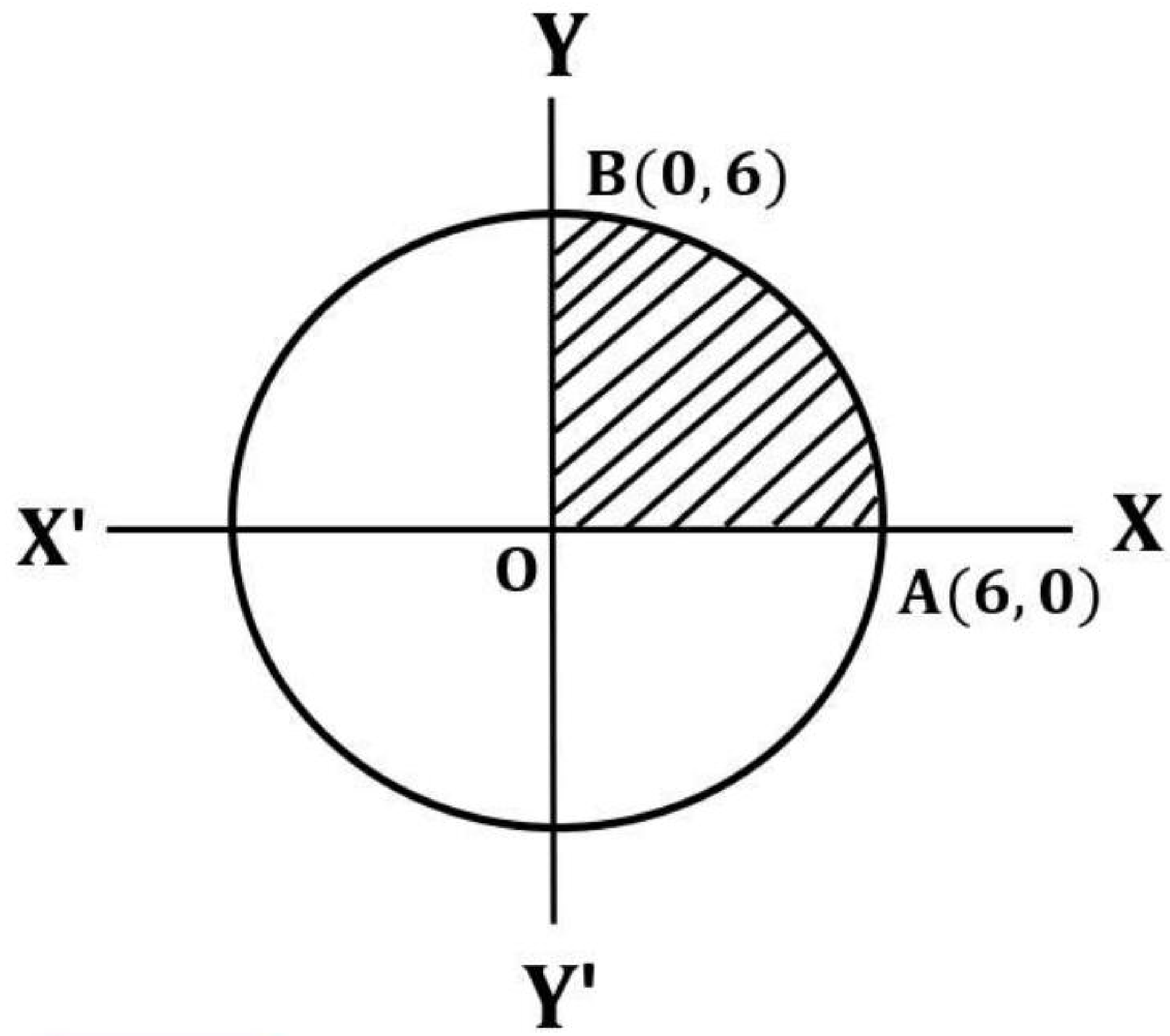
4. Find the area of the circle $x^2 + y^2 = 36$ using method of integration.



➢ **Solution**

Given Equation,

$$x^2 + y^2 = 36$$

$$\Rightarrow y = \sqrt{36 - x^2}$$

Its center is (0,0) and radius 6.

Circle is symmetrical about both $x - axis$ and $y - axis$.

Total area can be obtained by Finding $4 \times$ Area of OAB which lies between the ordinates $x = 0, x = 6$.

Hence,

Total area of the circle $\mathrm{A} = 4 \times area\ of\ \mathrm{OAB}$

$$A = 4\int_0^6 y\, dx$$

$$= 4\int_0^6 \sqrt{36 - x^2}\, dx$$

$$let\ x = 6\sin\theta\ ,\quad dx = 6\cos\theta\, d\,\theta$$

When, $x = 0 \rightarrow \theta = 0 \quad \& \quad x = 6 \rightarrow \theta = \frac{\pi}{2}$

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$$A = 4\int_0^{\frac{\pi}{2}} \sqrt{36 - 36\,sin^2\theta}\ .6\cos\theta\, d\,\theta$$

$$= 4\int_0^{\frac{\pi}{2}} 6\cos\theta\ .\ 6\cos\theta\, d\,\theta$$

$$= 4 \times 36 \int_0^{\frac{\pi}{2}} \cos^2\theta\, d\,\theta$$

$$= 4 \times 36 \times \int_0^{\frac{\pi}{2}} \frac{1 + \cos 2\,\theta\, d\,\theta}{2}$$

$$= \frac{4 \times 36}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 2 \times 36\left(\frac{\pi}{2} + 0\right)$$

$$= 2 \times 36 \times \frac{\pi}{2}$$

$$= 36\pi\, unit^2$$

∴ Area of the circle is $36\pi\, unit^2$ using method of integration.

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5. A class consists of 60 boys and 40 girls. It two students are chosen at random, what will be the probability that.

i) Both are Boys ii) One Boy and one Girl

➢ Solution

$$n(B) = 60,\ n(G) = 40,$$
$$n(S) = 60 + 40 = 100$$

Now,

When two students are chosen randomly then,

Possible outcomes ,n= selection of 2 std out of 100

$$= c(100,2)$$
$$= \frac{100!}{(100-2)!\ 2!}$$
$$= 4950$$

i) P(Both are Boys)=?

M= no. of favorable case.

$$= \text{selection of 2 boys out of 60}$$
$$= c\,(60,2)$$
$$= \frac{60!}{(60-2)!\ 2!}$$
$$= 1770$$

Hence, P(both are boys) = $\frac{M}{n} = \frac{c(60,2)}{c(100,2)}$

$$= \frac{1770}{4950}$$
$$= \frac{\mathbf{59}}{\mathbf{165}}$$

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ii) One Boy and one Girl

m= no. of favorable case.

= selection of 1 boy out of 60 and selection of 1 girl out of 40

$$= C(60,1) \times C(40,1)$$

$$= \frac{60!}{(60-1)!\ 1!} \times \frac{40!}{(40-1)!\ 1!} = 60 \times 40 = 2400$$

$$\therefore \text{P(One Boy and one Girl)} = \frac{m}{n} = \frac{2400}{4950} = \frac{16}{33}$$

6. Prove that: $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$

➢Solution

$$\text{L.H.S.} = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

Operating,

$$R_1 \to R_1 - R_2 \quad \& \quad R_2 \to R_2 - R_3$$

$$= \begin{vmatrix} 0 & x-y & x^2-y^2 \\ 0 & y-z & y^2-z^2 \\ 1 & z & z^2 \end{vmatrix}$$

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$$= \begin{vmatrix} 0 & x-y & (x-y)(x+y) \\ 0 & y-z & (y-z)(y+z) \\ 1 & z & z^2 \end{vmatrix}$$

Taking Common $(x-y)$ from R_1 , $(y-z)$ from R_2

$$(x-y)\,(y-z) \begin{vmatrix} 0 & 1 & x+y \\ 0 & 1 & y+z \\ 1 & z & z^2 \end{vmatrix}$$

Expanding along C_1,

$$=(x-y)\,(y-z).1 \begin{vmatrix} 1 & x+y \\ 1 & y+z \end{vmatrix}$$

$$=(x-y)\,(y-z)\,[y+z-(x+y)]$$

$$= (\boldsymbol{x}-\boldsymbol{y})\,(\boldsymbol{y}-\boldsymbol{z})\,(\boldsymbol{z}-\boldsymbol{x})$$

Proved.

7. Show that by using vector method the angle between two diagonals of a cube is $\theta = \cos^{-1}\left(\frac{1}{3}\right)$.

➢ Solution:

Let O be the origin and let OA = OB = OC = a. Then the coordinates of O, A, B, C, M and P are (0, 0, 0), (a, 0, 0), (0, a, 0), (0, 0, a), (a, a, 0) and (a, a, a) respectively.

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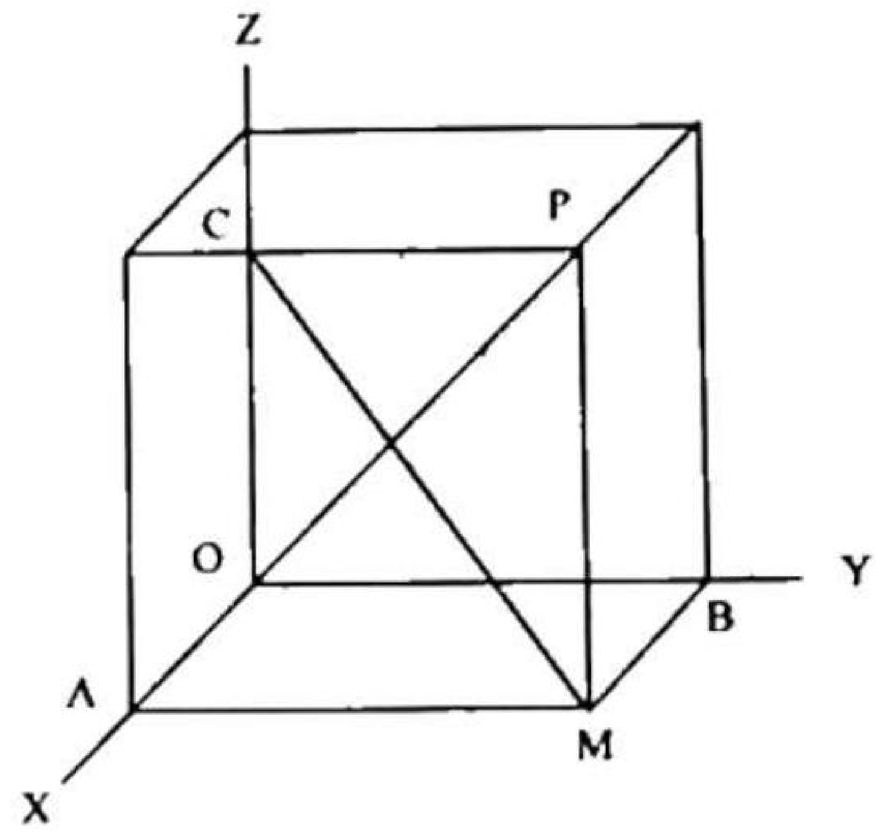

$Now,\ \overrightarrow{OP} = (a, a, a)$

$And,\ \overrightarrow{CM} = (a - 0, a - 0, 0 - a)$

$= (a, a, -a)$

$Again,$

$$\overrightarrow{OP}.\overrightarrow{CM} = (a, a, a).(a, a, -a)$$
$$= a^2 + a^2 - a^2$$
$$= a^2$$

$$\left|\overrightarrow{OP}\right| = \sqrt{a^2 + a^2 + a^2}$$
$$= a\sqrt{3}$$

$$\text{And}\left|\overrightarrow{CM}\right| = \sqrt{a^2 + a^2 + a^2}$$
$$= a\sqrt{3}$$

If be θ the angle between the two diagonals OP and CM, then

$$\cos\theta = \frac{\overrightarrow{OP}.\overrightarrow{CM}}{\left|\overrightarrow{OP}\right|\left|\overrightarrow{CM}\right|}$$
$$= \frac{a^2}{(a\sqrt{3})\,(a\sqrt{3})}$$
$$= \frac{1}{3}$$

$$\therefore \boldsymbol{\theta = \cos^{-1}\left(\frac{1}{3}\right)}$$

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8. If w be the cube root of unity prove that:

$$(1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8) = 9$$

➢ **Solution,**

LHS,

$$= (1-\omega)(1-\omega^2)(1-\omega^4)(1-\omega^8)$$

$$= (1-\omega)(1-\omega^2)(1-\omega^3\omega)(1-\omega^3\omega^3\omega^2)$$

$$= (1-\omega)(1-\omega^2)(1-1\times\omega)(1-1\times1\times\omega^2)$$

$$= (1-\omega)^2(1-\omega^2)^2$$

$$= (1-2\omega+\omega^2)(1-2\omega^2+\omega^4) \quad [\because \omega^2\times\omega=\omega^3=1]$$

$$= (-\omega-2\omega)(1-2\omega^2+\omega^3\omega)$$

$$= (-3\omega)(1-2\omega^2+1.\omega)$$

$$= (-3\omega)(-\omega^2+2\omega^2)$$

$$= (-3\omega)(-3\omega^2)$$

$$= 9\omega^3$$

$$= 9\times1$$

$$= \mathbf{9}$$

Proved.

9. Maximize and minimize $F = 34x + 6y$ subject to

$$x+y \le 6, x+y \ge 1, 1 \le x \le 3.$$

➢ **Solution,**

The boundary lines of the given inequalities are

$$x+y=6 \quad \cdots\cdots\cdots\cdots (i)$$

$$x+y=1 \quad \cdots\cdots\cdots\cdots (ii)$$

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$x = 1$

$x = 3$

From equation (i)

$x + y = 6$	
x	y
0	6
6	0

So, $(0,6)$ & $(6,0)$ are passes through equation (i)

From equation (ii)

$x + y = 1$	
x	y
1	0
0	1

So,

$(0,1)$ & $(1,0)$ are passes through equation (ii)

Also,

From graph the vertices B , C & D are calculated by solving equation

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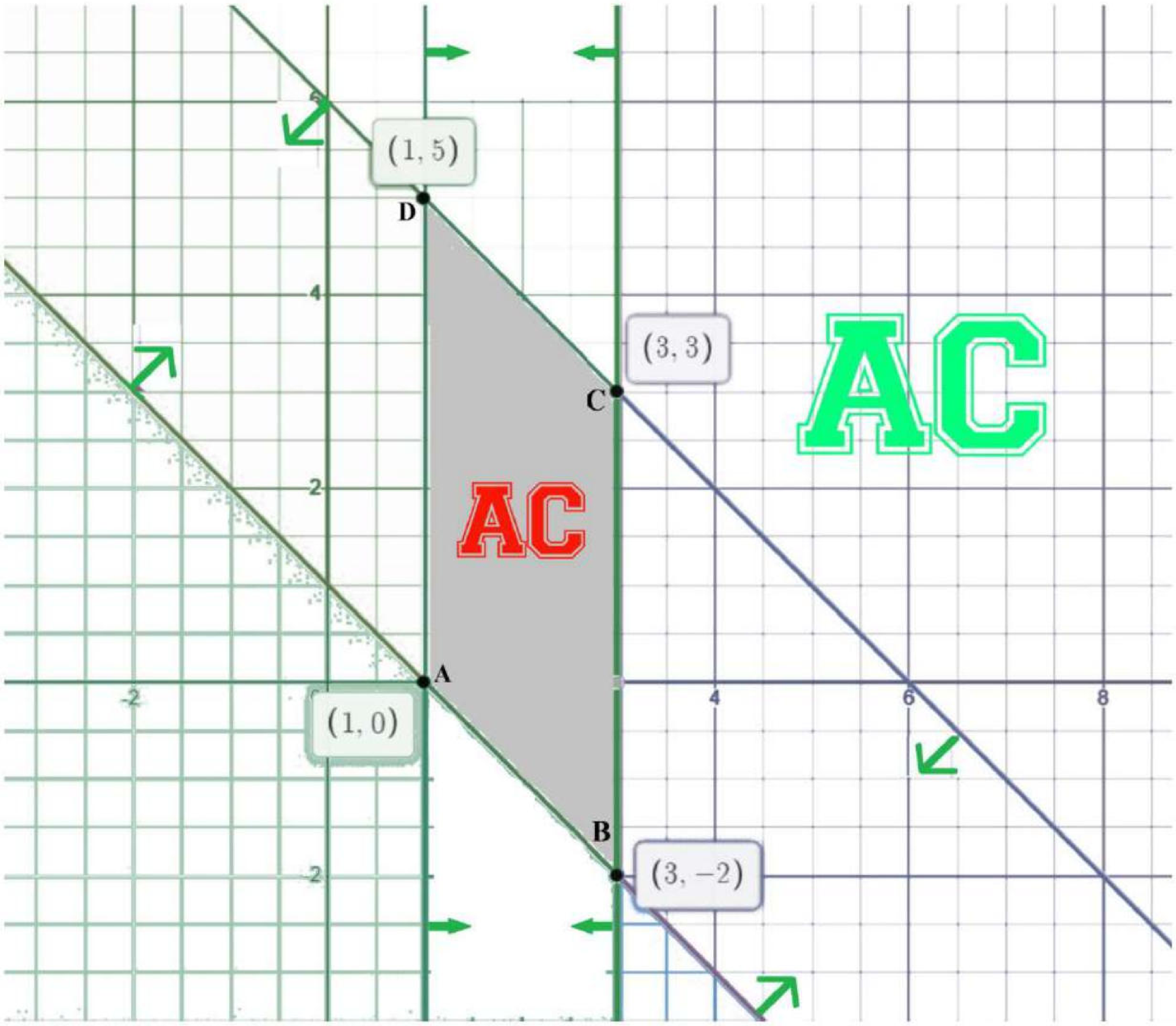

For vertices B,

Solving eqn , $x = 3 \; and \; x + y = 1$

B$(3, -2)$

For Vertices C,

Solving eqn $x = 3 \; and \; x + y = 6$

C$(3,3)$

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For Vertices D,

Solving eqn x=1 and 1+y=6

D(1,5)

Hence, The Vertices of ABCD are A(1,0) ,B(3,-2) ,C(3,3) and D(1,5).

Vertex	$f = 34x + 6y$
A(1,0)	**34 (MIN)**
B(3,-2)	90
C(3,3)	**120 (MAX)**
D(1,5).	64

Hence, Maximum Value $= 120$ at $c(3,3)$

Minimum Value $= 34$ at $A(1,0)$

10. Find the equation of the plane through the points $(1, 2, 1), (2, 2, 2)$ and $(0, 1, 0)$.

➢**Solution,**

Equation of the plane through $(1, 2, 1)$

$$a(x-1) + b(y-2) + c(z-1) = 0 \cdots\cdots\cdots (i)$$

Since,

Equation (i) Passes through (2,2,2) & (0,1,0)

So,

$$a + 0.b + c = 0 \cdots\cdots\cdots (ii)$$

$$-a - b + c = 0 \cdots\cdots\cdots (iii)$$

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Solving, (ii) & (iii) by cross multiplication method,

$$\frac{a}{(0+1)} = \frac{b}{-(1-1)} = \frac{c}{1-0}$$

$$or, \frac{a}{-1} = \frac{b}{0} = \frac{c}{1} = K \ \ (say)$$

$$a = -K, \qquad b = 0, \qquad c = K$$

Equation (i) becomes,

$$-K(x-1) + 0(y-2) + K(z-1) = 0$$

$$or, \ K(-x+1) + K(z-1) = 0$$

$$or, \ -x + 1 + z - 1 = 0$$

$$\boxed{\therefore \ \boldsymbol{z - x = 0}}$$

Which is Eq^n of plane.

11. Find standard deviation and coefficient of variation (CV) of the data given below.

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	4	6	8	10	4	3

➢ **Solution**

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$Class$	(f)	Mid value (x)	$a = 25$ $\mathrm{d} = x - \mathrm{a}$	$\mathrm{h} = 10$ $d' = \frac{(x-a)}{h}$	d'^2	fd'	fd'^2
$0-10$	4	5	-20	-2	4	-8	16
$10-20$	6	15	-10	-1	1	-6	6
$20-30$	8	25(a)	0	0	0	0	0
$30-40$	10	35	10	1	1	10	10
$40-50$	4	45	20	2	4	8	16
$50-60$	3	55	30	3	9	9	27
$\mathrm{N} = \mathbf{35}$						$\sum fd' = \mathbf{13}$	$\sum fd'^2 = \mathbf{75}$

Now,

$$(\overline{\mathrm{X}}) = \mathrm{a} + \frac{\sum fd'}{\mathrm{N}} \times \mathrm{h}$$

$$= 25 + \frac{13}{35} \times 10$$

$$\therefore (\overline{\mathrm{X}}) = 28.71$$

we have, $(\sigma) = \sqrt{\frac{\sum fd'^2}{\mathrm{N}} - \left(\frac{\sum fd'}{\mathrm{N}}\right)^2} \times \mathrm{h}$

$$= \sqrt{\frac{75}{35} - \left(\frac{13}{35}\right)^2} \times 10$$

$$\therefore (\sigma) = 14.16$$

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$$\mathbf{C.V} = \frac{\sigma}{\bar{X}} \times 100\%$$

$$= \frac{14.16}{28.71} \times 100\%$$

$$\therefore \text{C.V} = 49.32\ \%$$

12. Find the regression equation of y on x from the following data:

X	2	4	6	8	10	12
Y	5	6	13	16	23	24

Also, estimate the value of Y when $\mathbf{X = 5}$.

➢ **Solution**

x	y	x^2	xy
2	5	4	10
4	6	16	24
6	13	36	78
8	16	64	128
10	23	100	230
12	24	144	288
$\sum x = 42$	$\sum y = 87$	$\sum x^2 = 364$	$\sum xy = 758$

$\boldsymbol{Here}, n = 6$

$$\bar{x} = \frac{\sum x}{n} = \frac{42}{6} = \mathbf{7}\ , \bar{y} = \frac{\sum y}{n} = \frac{87}{6} = \mathbf{14.5}$$

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$$\boldsymbol{b_{yx}} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} = \frac{6 \times 758 - 42 \times 87}{6 \times 364 - (42)^2} = \frac{894}{420} = \mathbf{2.128}$$

Hence, the regression y on x is ,

$$y - \overline{y} = b_{yx}(x - \overline{x})$$

$$\text{or, } y - 12.83 = 2.128\,(x - 7)$$

$$\text{or, } y - 12.83 = 2.128\,x - 11.9$$

Therefore, $y = 2.128\,x + 0.93$

∴ The regression Equation y on x is ,

$$\boldsymbol{y = 2.128\,x + 0.93}$$

When $y = 5$ then,

$$\boldsymbol{y = 2.128 \times 5 + 0.93 = 11.57}$$

13. Find the correlation coefficients between x and y of the following data:

X	2	3	6	5	10	15	12	20	11
Y	19	17	17	10	17	15	13	14	12

➢ **Solution**

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X	Y	X^2	Y^2	XY
2	19	4	361	38
3	17	9	289	51
6	17	36	289	102
5	10	25	100	50
10	17	100	289	170
15	15	225	225	225
12	13	144	169	156
20	14	400	196	280
11	12	121	144	132
$\sum X = 84$	$\sum Y = 134$	$\sum X^2 = 1064$	$\sum Y^2 = 2062$	$\sum XY = 1204$

Here, $n = 9$

$$r = \frac{n\sum XY - \sum X \,.\, \sum Y}{\sqrt{n\sum X^2 - (\sum X)^2}\,\sqrt{n\sum Y^2 - (\sum Y)^2}}$$

$$r = \frac{9 \times 1204 - 84 \times 134}{\sqrt{9 \times 1064 - (84)^2}\,\sqrt{9 \times 2062 - (134)^2}}$$

$$r = \frac{-420}{\sqrt{2520}\,\sqrt{602}}$$

$$\therefore \boldsymbol{r = -0.340}$$

-The End-

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Engg. Mathematics –II__(Engg. All) $2^{nd\ Sem}$

(2078) Question Paper Solution.

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1.a) Construct a 3×3 matrix whose elements are given by

$$a_{ij} = 3i - 2j$$

➢ **Solution,**

Let A $= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a 3×3 matrix.

By Question $a_{ij} = \boldsymbol{3i - 2j}$

Thus,

$a_{11} = 3.1 - 2.1 = 1$

$a_{12} = 3.1 - 2.2 = -1$

$a_{13} = 3.1 - 2.3 = -3$

$a_{21} = 3.2 - 2.1 = 4$

$a_{22} = 3.2 - 2.2 = 2$

$a_{23} = 3.2 - 2.3 = 0$

$a_{31} = 3.3 - 2.1 = 7$

$a_{32} = 3.3 - 2.2 = 5$

$a_{33} = 3.3 - 2.3 = 3$

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Hence, the required matrix A is given by

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\therefore \quad A = \begin{bmatrix} 1 & -1 & -3 \\ 4 & 2 & 0 \\ 7 & 5 & 3 \end{bmatrix}$$

b) Solve using row equivalent matrix method or Cramer's rule.

$$\boldsymbol{x + y - z = 3}$$
$$\boldsymbol{2y + z = 10}$$
$$\boldsymbol{5x - y - 2z = -3}$$

- **Solution,**

Row equivalent matrix method,

$$x + y - z = 3$$
$$2y + z = 10$$
$$5x - y - 2z = -3$$

The augmented matrix is

$$\begin{bmatrix} 1 & 1 & -1 & : & 3 \\ 0 & 2 & 1 & : & 10 \\ 5 & -1 & -2 & : & -3 \end{bmatrix}$$

$$R_3 \to R_3 - 5R_1 \quad \begin{bmatrix} 1 & 1 & -1 & : & 3 \\ 0 & 2 & 1 & : & 10 \\ 0 & -6 & 3 & : & -18 \end{bmatrix}$$

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$$R_3 \to \frac{1}{3} R_3 \quad \left[\begin{array}{ccc:c} 1 & 1 & -1 & 3 \\ 0 & 2 & 1 & 10 \\ 0 & -2 & 1 & -6 \end{array}\right]$$

$$R_3 \to R_2 + R_3 \quad \left[\begin{array}{ccc:c} 1 & 1 & -1 & 3 \\ 0 & 2 & 1 & 10 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

$$R_3 \to \frac{R_3}{2} \quad \left[\begin{array}{ccc:c} 1 & 1 & -1 & 3 \\ 0 & 2 & 1 & 10 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$R_2 \to R_2 - R_3 \quad \left[\begin{array}{ccc:c} 1 & 1 & -1 & 3 \\ 0 & 2 & 0 & 8 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$R_2 \to \frac{1}{2} R_2 \quad \left[\begin{array}{ccc:c} 1 & 1 & -1 & 3 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$R_1 \to R_3 + R_1 \quad \left[\begin{array}{ccc:c} 1 & 1 & 0 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$$R_1 \to R_1 - R_2 \quad \left[\begin{array}{ccc:c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

Hence,

$$\boldsymbol{x = 1, y = 4, Z = 2\ .}$$

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OR

Using Cramer's rule

$$x + y - z = 3$$
$$2y + z = 10$$
$$5x - y - 2z = -3$$

$$D = \begin{vmatrix} 1 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & -1 & -2 \end{vmatrix} = 1(-4+1) - 1(-5) - 1(-10)$$
$$= -3 + 5 + 10$$
$$= \mathbf{12}$$

$$D_1 = \begin{vmatrix} 3 & 1 & -1 \\ 10 & 2 & 1 \\ -3 & -1 & -2 \end{vmatrix} = 3(-4+1) - 1(-20+3) - 1(-10+6)$$
$$= -9 + 17 + 4$$
$$= \mathbf{12}$$

$$D_2 = \begin{vmatrix} 1 & 3 & -1 \\ 0 & 10 & 1 \\ 5 & -3 & -2 \end{vmatrix} = 1(-20+3) - 3(-5) - 1(-50)$$
$$= -17 + 15 + 50$$
$$= \mathbf{48}$$

$$D_3 = \begin{vmatrix} 1 & 1 & -3 \\ 0 & 2 & 10 \\ 5 & -1 & -3 \end{vmatrix} = 1(-6+10) - 1(-50) - 3(0-10)$$
$$= 4 + 50 - 30$$
$$= \mathbf{24}$$

Now,

$$x = \frac{D_1}{D} = \frac{12}{12} = 1$$

$$y = \frac{D_2}{D} = \frac{48}{12} = 4$$

$$z = \frac{D_3}{D} = \frac{24}{12} = 2$$

Hence, $x = 1\,, y = 4, z = 2.$

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2. a) If Z and W are two complex numbers, prove that

$$|z| + |w| \geq |z + w|$$

➢ ***Proof***

Let $z = a + ib$ and $w = c + id$, so that $z + w = a + c + i(b + d)$, then

$|z| = \sqrt{a^2 + b^2}$, $|w| = \sqrt{c^2 + d^2}$

Also, $|z + w| = \sqrt{(a + c)^2 + (b + d)^2}$

Now, $|z| + |w| \geq |z + w|$ will be true

If $\sqrt{a^2 + b^2} + \sqrt{c^2 + d^2} \geq \sqrt{(a + c)^2 + (b + d)^2}$

i.e. $a^2 + b^2 + c^2 + d^2 + 2\sqrt{(a^2 + c^2) + (b^2 + d^2)} \geq (a + c)^2 + (b + d)^2$

i.e. $\sqrt{(a^2 + c^2) + (b^2 + d^2)} \geq ac + bd$

i.e. $(a^2 + c^2) + (b^2 + d^2) \geq a^2c^2 + b^2d^2 + 2abcd$

i.e. $a^2d^2 + b^2c^2 \geq 2abcd$

i.e. $a^2d^2 + b^2c^2 - 2abcd \geq 0$

i.e. $(ad - bc)^2 \geq 0$, which is true all real number a, b, c, d

$\therefore$ $|z| + |w| \geq |z + w|$

b) State and prove De-Moivre's Theorem.

➢ **Solution,**

De-Moirve's Theorem

Statement:- If n is any positive integer,

$$[r(\cos\theta + i\sin\theta)]^n = r^n\ (\cos n\theta + i\sin n\theta).$$

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Proof.

When $n = 1$,

$$[\,r(\cos\theta + i\sin\theta)]^1 = r^1\ (\cos 1.\theta + i\sin 1.\theta).$$

$$\Rightarrow\quad r(\cos\theta + i\sin\theta) = r\ (\cos\theta + i\sin\theta)$$

Which is true for $n = 1$

When $n = 2$,

$$[\,r(\cos\theta + i\sin\theta)]^2 = r^2\ (\cos\theta + i\sin\theta)^2.$$

$$\Rightarrow\quad = r^2(\cos^2\theta + 2\cos\theta\,.\,i\sin\theta + i^2\sin^2\theta)$$

$$= r^2(\cos^2\theta + i\sin 2\theta - \sin^2\theta)$$

$$= r^2(\cos^2\theta - \sin^2\theta + i\sin 2\theta)$$

$$= r^2(\cos 2\theta + i\sin 2\theta)$$

$[\because \sin 2\theta = 2\sin\theta\cos\theta\,]$

$[\because \cos 2\theta = \cos^2\theta - \sin^2\theta\,]$

Which is true for $n = 2$

Hence,

It is also true for $n = 3, 4....$

Let say the theorem is true for $n = k$ so, Then, prove for $n = k + 1$

$$[\,r(\cos\theta + i\sin\theta)]^k = r^k\ (\cos k\theta + i\sin k\theta).\ \cdots\cdots(\boldsymbol{i})$$

Multiplying both by $r(\cos\theta + i\sin\theta)$ in eqn $(\boldsymbol{i})$ so, It become $k + 1$

$$[\,r(\cos\theta + i\sin\theta)]^{k+1} = r^{k+1}\ (\cos k\theta + i\sin k\theta).\ (\cos\theta + i\sin\theta)$$

$$= r^{k+1}\,[\cos(k\theta + \theta) + i\sin(\,k\theta + \theta)]$$ **[∵ by product of complex form]**

$$= r^{k+1}\,[\cos(k + 1)\theta + i\sin(\,k + 1)\theta]$$

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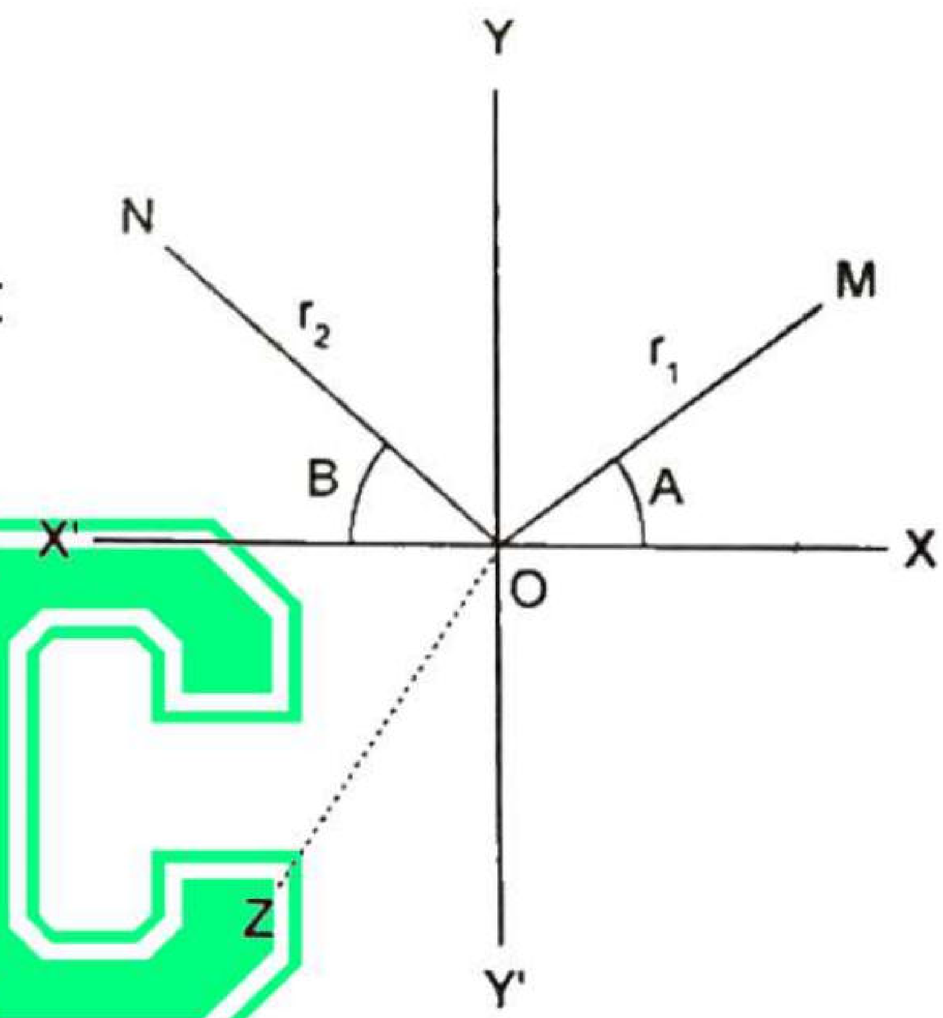
Which shows that the theorem is true for $n = k + 1$,whenever it is true for $n = k$. Continuing this way, The theorem is true for every positive integer n. Also it is true for real number n Hence, **Proved.**

3. a) Using vector method, prove that

$$Sin(A + B) = SinA.CosB + CosA.SinB$$

➢ **Solution,**



Let XOX' and YOY', the two mutually perpendicular straight lines represent $x - axis$ and $y - axis$ respectively.

Let $= \angle XOM = A$

and $\angle NOX' = B$

so that $\angle MON = \pi - (A + B)$

Let OM $= r_1$ and ON $= r_2$. Then the co-ordinates of M and N are

$$(r_1 \cos A, r_1 \sin A) \text{ and } (r_2 \cos(\pi - B), r_2 \sin(\pi - B))$$

So, position vector are:

So, $\overrightarrow{OM} = (r_1 \cos A, r_1 \sin A) = r_1 \cos A\ \vec{i} + r_1 \sin A\ \vec{j}$

and $\overrightarrow{ON} = (r_2 \cos(\pi - B), r_2 \sin(\pi - B))$

$= -r_2 \cos B\ \vec{i} + r_2 \sin B\ \vec{j}$

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$$\overrightarrow{OM} \times \overrightarrow{ON} = \begin{vmatrix} \vec{i} & \vec{J} & \vec{k} \\ r_1 \cos A & r_1 \sin A & 0 \\ -r_2 \cos B & r_2 \sin B & 0 \end{vmatrix}$$

Taking common r_1 *and* r_2 from R_2 and R_3,

$$= r_1 r_2 \begin{vmatrix} \vec{i} & \vec{J} & \vec{k} \\ \cos A & \sin A & 0 \\ -\cos B & \sin B & 0 \end{vmatrix}$$

$$= r_1 r_2 [\vec{i}(0-0) - \vec{J}(0-0) + \vec{k}(\cos A \sin B + \sin A \cos B)]$$

$$= r_1 r_2 \,(\cos A \sin B \sin A \cos B)\vec{k}$$

Magnitude of $\overrightarrow{OM} \times \overrightarrow{ON}$

$$\therefore \left|\overrightarrow{OM} \times \overrightarrow{ON}\right| = r_1 r_2 \,(\cos A \sin B + \sin A \cos B)$$

Since $\pi - (A + B)$ is the angle between OM and ON, So

$$\sin[\pi - (A + B)] = \frac{\left|\overrightarrow{OM} \times \overrightarrow{ON}\right|}{\left|\overrightarrow{OM}\right| \times \left|\overrightarrow{ON}\right|}$$

or, $$\sin(A + B) = \frac{r_1 r_2 \,(\cos A \sin B + \sin A \cos B)}{r_1 r_2}$$

Hence, $\sin(A + B) = \sin A \cos B + \cos A \sin B.$

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b) Find the area of the parallelogram determined by the vectors $\vec{i}+\vec{j}+\vec{k}$ and $2\vec{i}+3\vec{j}+\vec{k}$

➢ **Solution**

$$Let, \vec{a} = \vec{i}+\vec{j}+\vec{k} \ and \ \vec{b} = -2\vec{i}+3\vec{j}+\vec{k}$$

Then, $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ -2 & 3 & 1 \end{vmatrix}$

$$= (1-3)\vec{i} - (1+2)\vec{j} + (3+2)\vec{k}$$

$$= -2\vec{i} - 3\vec{j} + 5\vec{k}$$

$$|\vec{a} \times \vec{b}| = -2\vec{i} - 3\vec{j} + 5\vec{k}$$

$$= \sqrt{(-2)^2 + (-3)^2 + 5^2}$$

$$= \sqrt{38}$$

$\therefore$ *The required area of parallelogram* $= \sqrt{38}$ ***sq.units***

4. From the following table calculate the coefficient of correlation.

X	**4**	**8**	**10**	**2**	**6**
Y	**8**	**7**	**5**	**11**	**9**

➢ **Solution**

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		$\overline{X}=6$	$\overline{Y}=8$			
x	y	$X=x-\overline{X}$	$Y=y-\overline{Y}$	X	Y	XY
4	8	-2	-2	4	0	0
8	7	2	-1	4	1	-2
10	5	4	-3	16	9	-12
2	11	-4	3	16	9	-12
6	9	0	1	0	1	0
$\sum x=30$	$\sum y=40$			$\sum x^2=40$	$\sum y^2=20$	$\sum xy=-26$

$n=5$

Now,

$$\overline{X}=\frac{\sum x}{n}=\frac{30}{5}=6$$

$$\overline{Y}=\frac{\sum y}{n}=\frac{40}{5}=8$$

Using Product moment formula ,

$$\text{Coeff. of correlation, (r)}=\frac{\sum xy}{\sqrt{\sum x^2}\sqrt{\sum y^2}}$$

$$=\frac{-26}{\sqrt{40}\sqrt{20}}$$

$$\therefore \mathbf{r=-0.9192}$$

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5. Find the local maxima and minima and points of inflection

$$f(x) = 2x^3 - 9x^2 - 24x + 3$$

➢ **Solution**

Given equation $f(x) = 2x^3 - 9x^2 - 24x + 3$

$$f'(x) = 6x^2 - 18x - 24$$

$$f''(x) = 12x - 18$$

For local maximum and minimum value of $f(x)$, We have

$$f'(x) = 0$$

$$or,\ 6x^2 - 18x - 24 = 0$$

$$or,\ x^2 - 3x - 4 = 0$$

$$or,\ x^2 - 4x + x - 4 = 0$$

$$or,\ x(x-4) + 1(x-4) = 0$$

$$or,\ (x-4)(x-4) = 0$$

$$\therefore\ \boldsymbol{x = -1, 4}$$

When $x = -1$,

$f''(x) = 12(-1) - 18 = -30$ Which is $-\boldsymbol{ve}$ so $f(x)$ has a local maximum value at $x = -1$ and the value is given by

$$= 2.(1)^3 - 9.(-1)^2 - 24(-1) + 3$$

$$= 16$$

When $x = 4$,

$f''(x) = 12 \times 4 - 18 = -30$ Which is $+\boldsymbol{ve}$ so $f(x)$ has a local minimum value at $x = 4$ and the value is given by

$$= 2.(4)^3 - 9.(4)^2 - 24(4) + 3$$

$$= -109$$

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For point of inflection, $f''(x) = 0$

$$or, 12x - 18 = 0$$

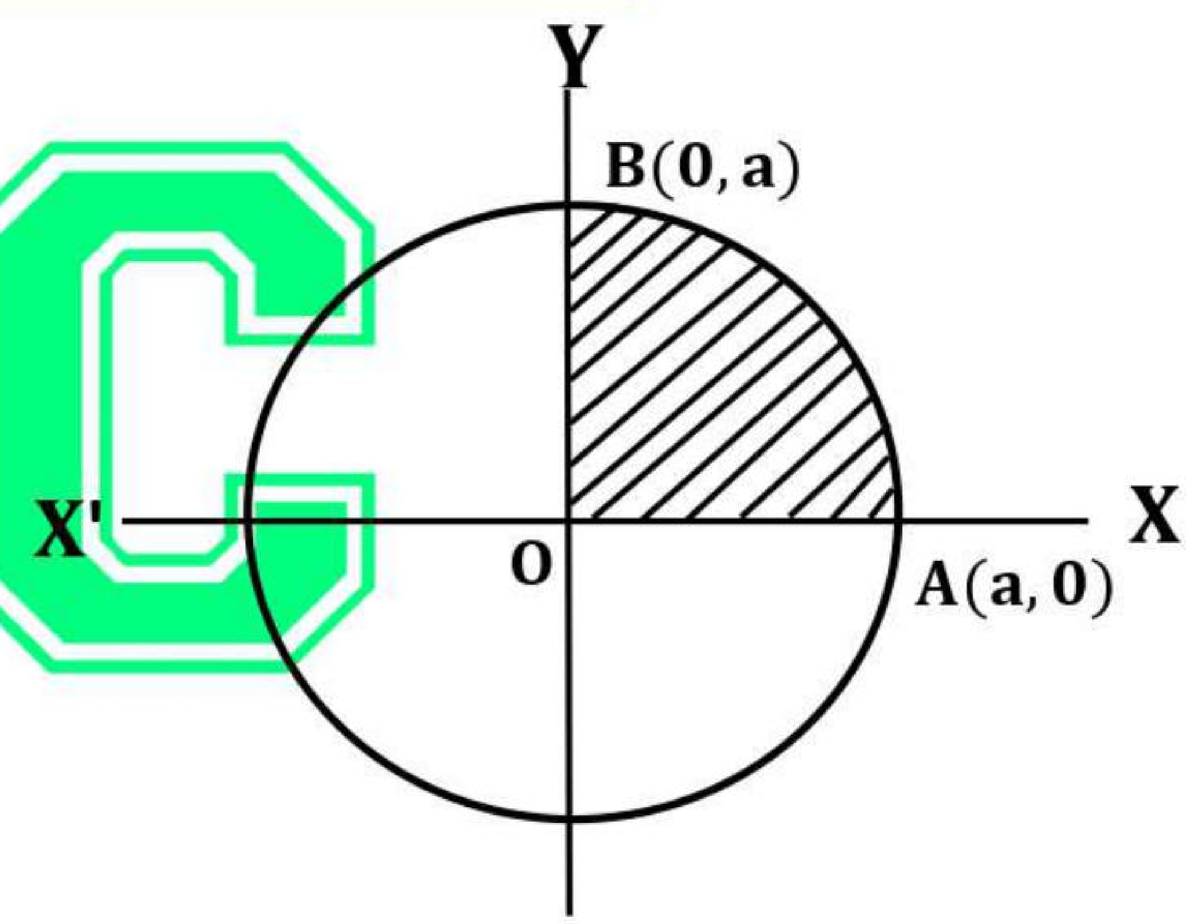
$$or, x = \frac{18}{12}$$

$$\therefore x = \frac{3}{2}$$

Hence, $x = \frac{3}{2}$ is the point of inflection.

6. Find the area of a circle $x^2 + y^2 = a^2$.

➢ **Solution**



Given Circle, $x^2 + y^2 = a^2$

Its center is (0,0) and radius a.

The $x - axis$ and $y - axis$ divide the circle into 4 equal sectors, one of the sectors is OAB which lies between the ordinates $x = 0, x = a$.

Hence, area of the circle

$A = 4 \times$ area of Sector OAB

$$= 4\int_0^a \sqrt{a^2 - x^2}\, dx$$

$$= 4\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_0^a$$

$$\left[\because \int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$$

$$= 4\left\{\left(\frac{a}{2}\sqrt{a^2 - a^2} + \frac{a^2}{2}\sin^{-1}\frac{a}{a}\right) - \frac{a^2}{2}\sin^{-1} 0\right\}$$

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$$= \frac{4a^2}{2}\sin^{-1} 1$$

$$= \frac{4a^2}{2} \cdot \frac{\pi}{2}$$

$$= \boldsymbol{\pi a^2\ sq.units.}$$

7. Calculate mean, standard deviation and C.V. from the following data:

Age	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of student	7	12	24	10	7

➢ **Solution**

Age	No. of Stud. (f)	Mid value (x)	$d = x - a$, $a = 25$	$d' = \frac{(x-a)}{h}$, $h = 10$	d'^2	fd'	fd'^2
0 – 10	7	5	−20	−2	4	−14	28
10 – 20	12	15	−10	−1	1	−12	12
20 – 30	24	25(a)	0	0	0	0	0
30 – 40	10	35	10	1	1	10	10
40 – 50	7	45	20	2	4	14	28
N = **60**						$\sum fd' = -2$	$\sum fd'^2 = 78$

Now,

$$(\bar{X}) = a + \frac{\sum fd'}{N} \times h$$

$$= 25 + \frac{-2}{60} \times 10$$

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$$\therefore (\bar{X}) = 24.67$$

$$\textbf{Now}, (\sigma) = \sqrt{\frac{\sum fd'^2}{N} - \left(\frac{\sum fd'}{N}\right)^2} \times h$$

$$= \sqrt{\frac{78}{60} - \left(\frac{-2}{60}\right)^2} \times 10$$

$$\therefore (\sigma) = 11.396$$

$$\textbf{C.V} = \frac{\sigma}{\bar{X}} \times 100\%$$

$$= \frac{11.396}{24.67} \times 100\%$$

$$\therefore C.V = 46.197\ \%$$

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8. If the volume of the expanding cube is increasing at the rate $24\ \mathrm{cm}^3/\mathrm{min}$, how fast is the surface area increasing when The surface area is $216\ \mathrm{cm}^2$?

➢ **Solution**

Let $l, S\ \&\ V$ be the length, surface area and volume of expanding cube respectively.

Then by question

$$\frac{dV}{dt} = 24\ cm^3/min$$

$$and\quad \frac{dS}{dt} = ?\quad at\ S = 216\ cm^2$$

Now,

$$S = 6\,l^2$$

$$\text{or, } 216 = 6\,l^2$$

$$\text{or, } l^2 = 36$$

$$l = 6\ cm$$

Volume is given by,

$$V = l^3$$

Differentiating both side w.r.t 't'

$$\frac{dV}{dt} = \frac{d(l^3)}{dt}$$

$$or, \frac{dV}{dt} = \frac{d(l^3)}{dl} \times \frac{dl}{dt}$$

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$$or, \frac{dV}{dt} = 3l^2 \times \frac{dl}{dt}$$

$$24 = 3 \times 6^2 \frac{dl}{dt}$$

$$or, \frac{dl}{dt} = \frac{24}{3 \times 6^2} = \frac{2}{9} \ cm/min$$

$$Now, S = 6\, l^2$$

Differentiating both side w.r.t 't'

$$\frac{dS}{dt} = 6\ \frac{dl^2}{dt}$$

$$\frac{dS}{dt} = 6\ \times \frac{dl^2}{dl} \times \frac{dl}{dt}$$

$$or, \frac{dS}{dt} = 6\ \times 2l \times \frac{dl}{dt}$$

$$or, \frac{dS}{dt} = 6 \times 2 \times 6 \times \frac{2}{9}$$

$$\therefore\ \boldsymbol{\frac{dS}{dt} = 16\ cm^2/min}$$

Hence,

The surface area is increasing at the rate of $16\ cm^2/min$ when Surface area is $216\ cm^2$

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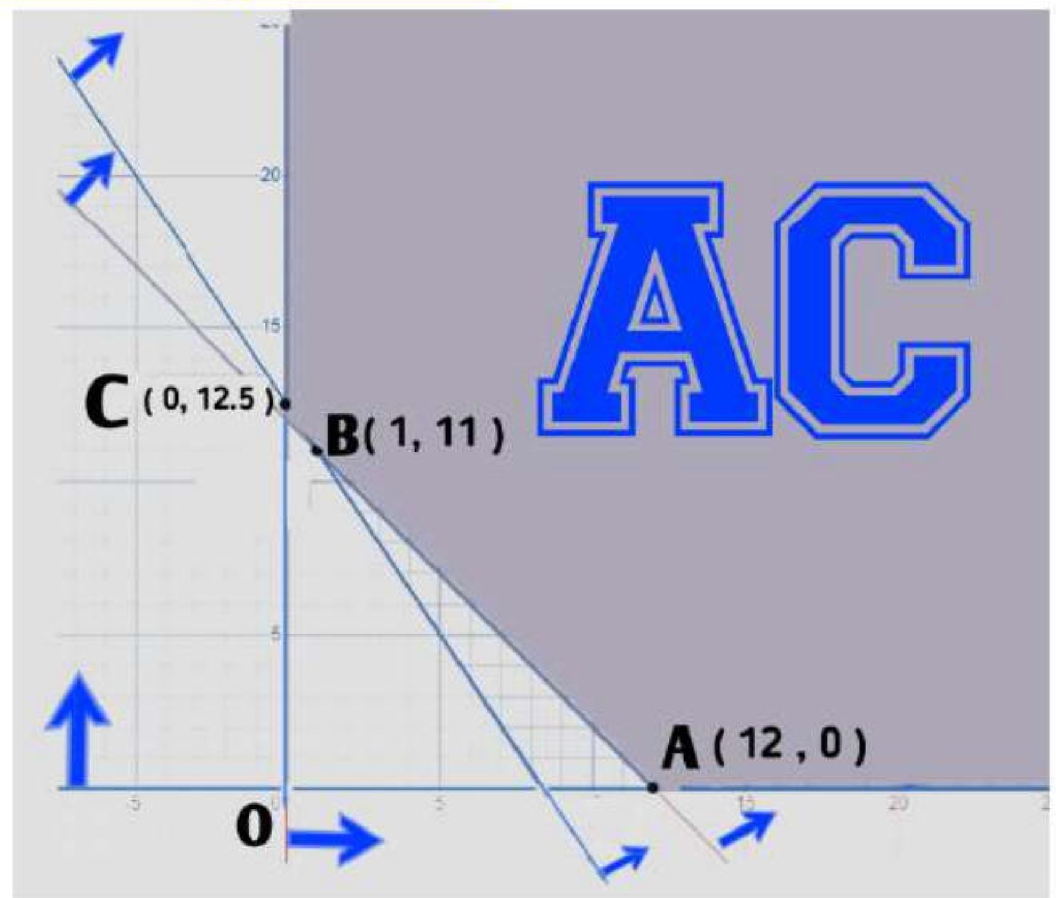
9. Maximize and minimize $z = 12x + 3y$ subject to $x + y \geq 12, 3x + 2y \geq 25$ and $x, y \geq 0$.

➢ **Solution**



The given objective function is

$$Z = 12x + 3y$$

and given construction are

$$x + y \geq 12$$
$$3x + 2y \geq 25$$
$$x \geq 0, y \geq 0$$

The boundary lines corresponding to the given inequalities are

$$x + y = 12$$
$$3x + 2y = 25$$
$$x = 0, y = 0$$

Now, drawing the graph represented by each equation and, we get the shaded region given in graph. The shaded region in the graph is the feasible region. The vertices of common points are $A(12, 0)$,$C(0,12)$ and To find the coordinate of B, we get $x = 1$ and $y = 1$.

Here, the shaded region is unbounded above, so only the minimum value exists.

Now, evaluating the value of objective function $Z = 12x + 3y$ at the vertices points, we get

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$Vertex\ (x, y)$	$Value\ of\ Z = 12x + 3y$
$A(12,0)$	144
$B(1,11)$	45
$C(0,12.5)$	$\mathbf{37.5}$

$\therefore$The minimum value of $Z = 37.5$ at (0,12.5)

10. A coin is tossed 5 times. Find the probability of getting

i) exactly 2 head ii) no head

➢**Solution**

$p =$ Prob. of getting head in one tossed = $\frac{1}{2}$

$$q = \text{Prob. of getting tail} = 1 - p = 1 - \frac{1}{2}$$

$n =$ no. of trails$= 5$

we have,

$$\text{P(r)} = \text{Prob. of getting } \boldsymbol{r} \text{ head in } \boldsymbol{n} \text{ trails}$$

$$\text{P(r)} = \text{C(n, r)}\ \text{p}^r q^{n-r}\ \cdots\cdots\cdots\cdots (\boldsymbol{i})$$

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Case – I,

Exactly 2 head $i.e, \quad r = 2$

From (i)

$$P(2) = C(5,2)\, p^2 q^{5-2}$$

$$P(2) = \frac{5\times4}{2}\left(\frac{1}{2}\right)^2 . \left(\frac{1}{2}\right)^3$$

$$P(2) = 10 \times \frac{1}{4} \times \frac{1}{8}$$

$$= \frac{10}{32}$$

∴Prob. of getting exactly 2 heads $= \frac{5}{16}$

Case – II,

No head $i.e, \quad r = 0$

From (i)

$$P(0) = C(5,0)\, p^0 q^{5-0}$$

$$P(0) = 1 \times 1. \left(\frac{1}{2}\right)^5$$

$$P(0) = \frac{1}{32}$$

∴ Prob. of getting no heads $= \frac{1}{32}$

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11. Prove that:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

➢ Solution

$$\text{LHS} = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$$

[Applying $C_2 \longrightarrow C_2 - C_1,\ C_3 \longrightarrow C_3 - C_1$]

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & b^3-a^3 & c^3-a^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^3 & (b-a)(b^2+ba+a^2) & (c-a)(c^2+ca+a^2) \end{vmatrix}$$

[Taking $b-a$ as a common from C_2 and $c-a$ from C_3]

$$= (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^3 & b^2+ba+a^2 & c^2+ca+a^2 \end{vmatrix}$$

$$= (b-c)(c-a).1\begin{vmatrix} 1 & 1 \\ b^2+ba+a^2 & c^2+ca+a^2 \end{vmatrix}$$

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$$= (b-c)(c-a)(c^2+ca+a^2-b^2-ba-a^2)$$
$$= (b-c)(c-a)(c^2-b^2+ca-ba)$$
$$= (b-c)(c-a)(c-b)(c+b+a)$$
$$= (a-b)(b-c)(c-a)(a+b+c)$$

Proved.

12. Find the equation of the plane through the intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ and perpendicular to the plane $4x + 5y - 3z = 8$.

➢ **Solution**

Given, $4x + 5y - 3Z - 8 = 0 \cdots\cdots (i)$

The eq^n of plane through the intersection of the plane $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ is given by

$$x + y + z - 6 + \lambda(2x + 3y + 4z + 5) = 0$$

$Where, \lambda\ is\ Constant$

$$(2\lambda + 1)x + (3\lambda + 1)y + (4\lambda + 1)\,z + 5\lambda - 6 = 0 \cdots\cdots (ii)$$

Since, Plane (ii) is Perpendicular to the plane (i) so,

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

So,

$$(2\lambda + 1)3 + (3\lambda + 1)5 + (4\lambda + 1) - 3 = 0$$

$$or,\ \ 8\lambda + 4 + 15\,\lambda + 5 - 12\lambda - 3 = 0$$

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$or, 11\,\lambda + 6 = 0$

$\therefore \lambda = \frac{-6}{11}$

Eq^n of plane is $x + y + z - 6 + \frac{-6}{11}(2x + 3y + 4z + 5) = 0$

$\Longrightarrow -x - 7y - 13z - 96 = 0$

$\therefore x + 7y + 13z + 96 = 0$ **Which is required eq^n of plane.**

13. A chance that A, B and C can solve a problem is $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ respectively. Find the probability that the problem will be solved.

➢ **Solution**

$$P(A) = \frac{1}{3}, P(B) = \frac{1}{4}, P(C) = \frac{1}{5}$$

Problem will be solved mean it is solved by at least one of them $i.e$,

$P(A \cup B \cup C) = ?$

We have, $P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= P(A) + P(B) + P(C) - P(A).P(B) - P(B).P(C) - P(C).P(A) + P(A).P(B).P(C)$

[$\because$ A, B, C are independent events, So, $P(A \cap B) = P(A).P(B)$ $etc.$]

$= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{3}\cdot\frac{1}{4} - \frac{1}{4}\cdot\frac{1}{5} - \frac{1}{5}\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{4}\cdot\frac{1}{5}$

$= \frac{3}{5}$

-The End-

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Engg. Mathematics –II__(Engg. All) 2nd Sem

(2079 New) Question Paper Solution.

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1 a) Form an equation, whose roots are thrice the roots of $3x^2 - 8x - 4 = 0$

➢**Solution**

Let α, β are roots of $3x^2 - 8x - 4 = 0$

Then,

$$\alpha + \beta = \frac{-(-8)}{3} = \frac{8}{3}$$

and,

$$\alpha\beta = \frac{-4}{3}$$

By question,

To find whose Eq^n having thrice root,

i.e. $\alpha' = 3\alpha$ & $\beta' = 3\beta$

The quadratic eq^n is given by

$$x^2 - (sum\ of\ root)x + product\ of\ roots = 0$$

$$or, x^2 - (\alpha' + \beta')x + \alpha'\beta' = 0$$

$$or, x^2 - 3(\alpha + \beta)x + 3\alpha \times 3\beta = 0$$

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$$or, x^2 - 3x(\alpha + \beta) + 9\alpha\beta = 0$$

$$or, x^2 - 3x \times \frac{8}{3} + 9 \times \frac{-4}{3} = 0$$

$$\Rightarrow 3x^2 - 24x - 36 = 0$$

Which is required eq^n.

b) If $a + ib = \sqrt{\frac{1+i}{1-i}}$, prove that $a^2 + b^2 = 1$.

➢ **Solution**

Given,

$$a + ib = \sqrt{\frac{1+i}{1-i}}$$

Taking modulus both side,

$$|a + ib| = \left|\sqrt{\frac{1+i}{1-i}}\right|$$

$$or, \sqrt{a^2 + b^2} = \sqrt{\left|\frac{1+i}{1-i}\right|} \qquad \left[\because |a + ib| = \sqrt{a^2 + b^2}\right]$$

$$or, \sqrt{a^2 + b^2} = \sqrt{\frac{|1+i|}{|1-i|}} \qquad \left[\because \left|\frac{C}{D}\right| = \frac{|C|}{|D|} \text{ by properties}\right]$$

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$$or, \sqrt{a^2+b^2} = \sqrt{\frac{\sqrt{1^2+1^2}}{\sqrt{1^2+(-1)^2}}}$$

$$\left[\because \quad |a+ib| = \sqrt{a^2+b^2} \quad |\mathbf{1-1.i}| = \sqrt{\mathbf{1^2+(-1)^2}} \right]$$

$$or, \sqrt{a^2+b^2} = \sqrt{\frac{\sqrt{2}}{\sqrt{2}}}$$

$$or, \sqrt{a^2+b^2} = \sqrt{1}$$

Squaring on both side,

$$\therefore \boldsymbol{a^2+b^2=1}$$

Proved.

2. a) If $\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 2 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ 3 & 1 \\ 1 & 2 \end{pmatrix}$ find the matrix of $\mathbf{AB-3I}$ and where $\mathbf{I}$ is a unit of matrix of order 2×2.

➢Solution

Given,

$$\mathrm{A} = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 2 & 1 \end{pmatrix} \text{ and } \mathrm{B} = \begin{pmatrix} 2 & 3 \\ 3 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\mathbf{AB-3I} = ?$$

Now,

$$\mathrm{AB} = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 1 \\ 1 & 2 \end{pmatrix}$$

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$$AB = \begin{pmatrix} 1\times 2 - 2\times 3 + 3\times 1 & 1\times 3 - 2\times 1 + 3\times 2 \\ -1\times 2 + 2\times 3 + 1\times 1 & -1\times 3 + 2\times 1 + 1\times 2 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 7 \\ 5 & 1 \end{pmatrix}$$

To find,

$$\mathbf{AB - 3I}$$

$$= \begin{pmatrix} -1 & 7 \\ 5 & 1 \end{pmatrix} - 3\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

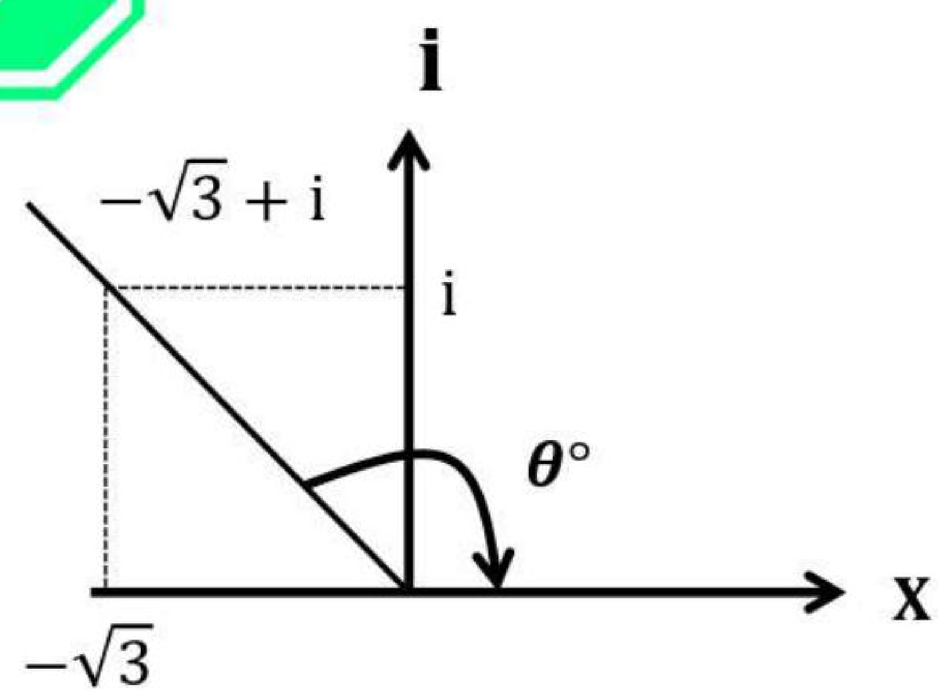
$$= \begin{pmatrix} -1-3 & 7-0 \\ 5-0 & 1-3 \end{pmatrix}$$

$$\therefore \mathbf{AB - 3I} = \begin{pmatrix} -4 & 7 \\ 5 & -2 \end{pmatrix}$$

b) Express the complex number $-\sqrt{3} + i$ polar form.

➢ **Solution**

Let $-\sqrt{3} + \mathrm{i} = \mathrm{r}(\cos\theta + \mathrm{i}\sin\theta)$



Comparing, we get

$$r\cos\theta = -\sqrt{3} \cdots\cdots\cdots (i)$$

$$r\sin\theta = 1 \qquad \cdots\cdots\cdots (ii)$$

Squaring both and add,

$$r^2\cos^2\theta + r^2\sin^2\theta = 4$$

$$r^2(\cos^2\theta + \sin^2\theta) = 4$$

$$r^2 = 4$$

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$r = 2 \qquad (+\text{ve only})$

Now,

Dividing (ii) by (i)

$$\tan\theta = \frac{1}{-\sqrt{3}}$$

$$\tan\theta = \tan^{-1}(-30°)$$

$$\theta = -30°$$

$$\theta = 180 - 30° \qquad [\because \textit{lies on } 2^{nd} \textit{ quad.}]$$

$$\theta = 150°$$

Hence,

$$-\sqrt{3} + \mathrm{i} = 2(\cos 150° + \mathrm{i}\sin 150°)$$

3 a) Fine equation of tangent and normal at point (4, 6) of the parabola y^2 = 9x .

➢ **Solution**

$y^2 = 9x$

The eq^n of tangent of parabola at (4,6) given by

$$y\,y_1 = \frac{9}{2}(x + x_1)$$

$$y \times 12 = 9(x + 4)$$

$$12y = 9x + 36 \cdots (i)$$ Which is eq^n of tangent.

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Slope of tangent (S_T), differentiating (i) $\left|\frac{dy}{dx}\right| = \frac{9}{12} = \frac{3}{4}$

Since, Normal and tangent are perpendicular Hence,

Slope of Normal, $S_N \times S_T = -1$

$$S_N \times \frac{3}{4} = -1$$

$$S_N = \frac{-4}{3}$$

Eq^n of Normal at (4,6)

$$y - 6 = \frac{-4}{3}(x - 4)$$

$$3y - 18 = -4x + 16$$

$$3y = -4x + 16 - 18$$

$$3y = -2x - 2,$$

Which is eq^n of Normal.

b. Find the center, vertices, eccentricity and length of latus rectum of the ellipse, $25x^2 + 4y^2 = 100$.

➢ **Solution**

Given eq^n of ellipse.

$$25x^2 + 4y^2 = 100$$

$$\frac{25x^2 + 4y^2}{100} = 1$$

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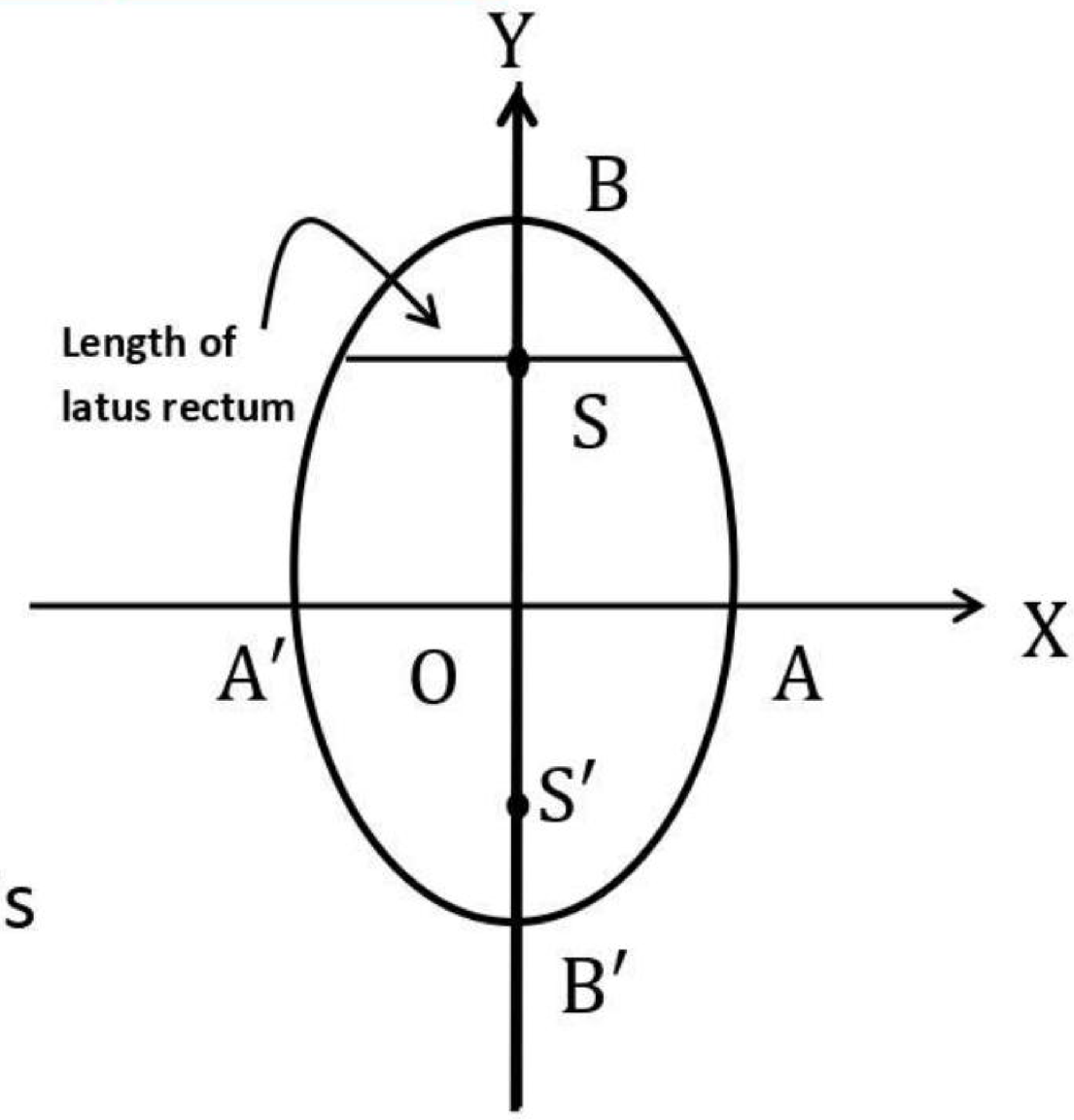
$$\frac{x^2}{4}+\frac{x^2}{25}=1$$



Comparing with $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$a^2=4,\quad b^2=25$$

$$\Rightarrow a=2,\qquad b=5$$

Since, $b>a$ So, the major $axis$ is along $y-axis$

and

Center of ellipse $(0,0)$

Vertices $=(0,\pm b)=(0,\pm 5)$

$$\text{Eccentricity}, e=\sqrt{1-\frac{a^2}{b^2}}$$

$$e=\sqrt{1-\frac{2}{25}}$$

$$e=\sqrt{\frac{21}{25}}$$

$$e=\frac{\sqrt{21}}{5}$$

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Length of lactum rectum$= \frac{2a^2}{b} = \frac{2\times4}{5} = \frac{8}{5}\ unit.$

**Note :- if ask Focus , eq^n of directrix Then,

For focus $S(0, \pm \text{b e}) = \left(0, \pm 5 \times \frac{\sqrt{21}}{5}\right)$

$= \left(0, \pm\sqrt{21}\right)$

Eq^n of directrix $\Rightarrow y = \pm\frac{b}{e} = \pm\frac{5}{\frac{\sqrt{21}}{5}} = \pm\frac{\mathbf{25}}{\sqrt{\mathbf{21}}}$

4. a) Find the ratio in which zx − plane divides the segment Joining $(2, 4, 8)$ *and* $(1, 6, 7)$.

➢**Solution :-** *Let* λ: 1 be ratio in which zx − plane divides the segment joining (2,4,8) & (1,6,7)

In zx plane, y coordinate is zero

$$y = \frac{\lambda y_2 + y_1}{\lambda + 1}$$

$$or, \qquad 0 = \frac{6\lambda + 4}{\lambda + 1}$$

$$or, \qquad 6\lambda + 4 = 0$$

$$\lambda = \frac{-4}{6} = \frac{-2}{3}$$

Hence,

$\frac{-2}{3}$: 1 be ratio in Which Divides the Segment joining (2,4,8)&(1,6,7)

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b) Find angle between two planes: $3x + 2y - 6z = 7$ and $2x + 3y + 2z = 5$

➢ **Solution,**

Let 'θ' be angle between two planes,

$$3x + 2y - 6z = 7 \cdots\cdots\cdots\cdots (i)$$

$$2x + 3y + 2z = 5 \cdots\cdots\cdots\cdots (ii)$$

Direction ratios of plane (i)

$$(a_1, b_1, c_1) = (3,\ \ 2, -6)$$

Direction ratios of plane (ii)

$$(a_2, b_2, c_2) = (2,\ \ 3,\ \ 2)$$

$$Cos\,\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{{a_1}^2 + {b_1}^2 + {c_1}^2}\sqrt{{a_2}^2 + {b_2}^2 + {c_2}^2}}$$

$$Cos\,\theta = \frac{3 \times 2 + 2 \times 3 - 6 \times 2}{\sqrt{3^2 + 2^2 + (-6)^2}\sqrt{2^2 + 3^2 + 2^2}}$$

$$Cos\,\theta = \frac{12 - 12}{\sqrt{9 + 4 + 36}\sqrt{4 + 9 + 4}}$$

$$Cos\,\theta = \frac{0}{\sqrt{49}\sqrt{17}} = 0$$

$$\theta = \cos^{-1}(0)$$

$$\therefore\ \boldsymbol{\theta = \frac{\pi}{2}}$$

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5. a) If $\vec{a} = (2, -1), \vec{b} = (-2, -3)$, find $2\vec{a} + 3\vec{b}$. its magnitude and unit vector along $2\vec{a} + 3\vec{b}$.

➢ **Solution**

$$\vec{a} = (2, -1) = 2\vec{i} - \vec{j}$$

$$\vec{b} = (-2, -3) = -2\vec{i} - 3\vec{j}$$

$$2\vec{a} + 3\vec{b} = 2(2\vec{i} - \vec{j}) + 3(-2\vec{i} - 3\vec{j})$$

$$= 4\vec{i} - 2\vec{j} - 6\vec{i} - 9\vec{j})$$

$$= -2\vec{i} - 11\vec{j}$$

Magnitude, $|2\vec{a} + 3\vec{b}| = \sqrt{(-2)^2 + (-11)^2} = 5\sqrt{5}$

Unit vector along $2\vec{a} + 3\vec{b} = \frac{2\vec{a}+3\vec{b}}{|2\vec{a}+3\vec{b}|}$

$$= \frac{-2\vec{i} - 11\vec{j}}{5\sqrt{5}}$$

$$= \frac{-2}{5\sqrt{5}}\vec{i} - \frac{11}{5\sqrt{5}}\vec{j}$$

$\therefore$ Unit vector along $2\vec{a} + 3\vec{b} = \left(\frac{-2}{5\sqrt{5}}, \frac{11}{5\sqrt{5}}\right)$

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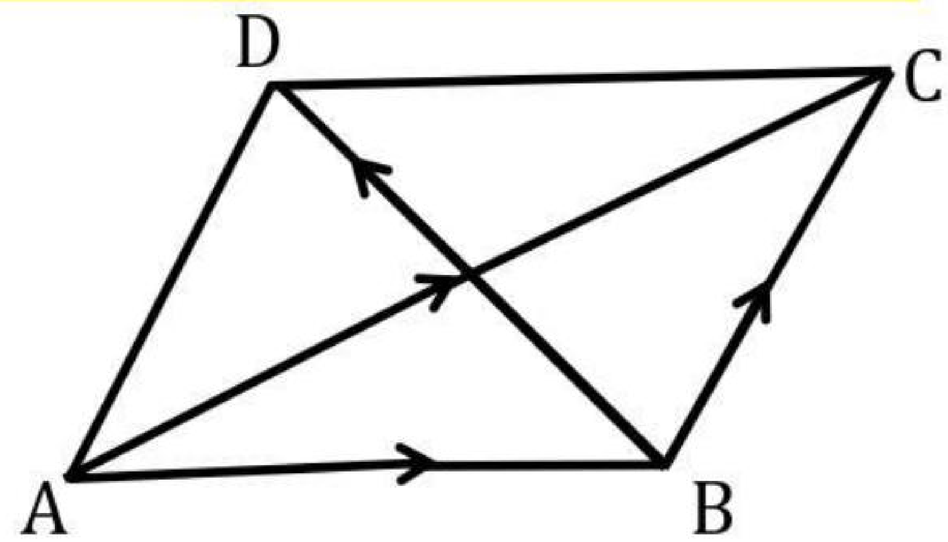
b) If AC and BD are diagonals of a parallelogram ABCD. Prove that $\overrightarrow{AB} + \overrightarrow{DC} = \overrightarrow{AC} + \overrightarrow{DB}$.



➢ **Solution**

To Prove: $\overrightarrow{AB} + \overrightarrow{DC} = \overrightarrow{AC} + \overrightarrow{DB}$

In $\Delta\, ABC$,using triangle law of vector addition

$$\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB} \cdots\cdots\cdots (i)$$

In Δ BCD. using triangle law of vector addition

$$\overrightarrow{DC} = \overrightarrow{DB} + \overrightarrow{BC} \cdots\cdots\cdots (ii)$$

Adding eq^n $(\boldsymbol{i})$ & $(\boldsymbol{ii})$

$$\overrightarrow{AB} + \overrightarrow{DC} = \overrightarrow{AC} + \overrightarrow{DB} + (\overrightarrow{CB} + \overrightarrow{BC})$$

$$(\because\ \overrightarrow{BC} = -\overrightarrow{CB})$$

$$\overrightarrow{AB} + \overrightarrow{DC} = \overrightarrow{AC} + \overrightarrow{DB} + (-\overrightarrow{BC}) + \overrightarrow{BC}$$

$$\boldsymbol{\overrightarrow{AB} + \overrightarrow{DC} = \overrightarrow{AC} + \overrightarrow{DB}}$$

Proved.

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6. a) Find unit vector perpendicular to each vectors :

$$\vec{i} + 3\vec{j} - 4\vec{k} \text{ and } 2\vec{i} + \vec{j} - \vec{k}.$$

➢ Solution

$$\vec{i} + 3\vec{j} - 4\vec{k} \text{ and } 2\vec{i} + \vec{j} - \vec{k}$$

Unit vector that is Perpendicular To both vector is

$$\hat{N} = \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|}$$

Let, $\vec{a} = \vec{i} + 3\vec{j} - 4\vec{k}$

$\vec{b} = 2\vec{i} + \vec{j} - \vec{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 3 & -4 \\ 2 & 1 & -1 \end{vmatrix}$$

$$= \vec{i}(-3+4) - \vec{j}(-1+8) + \vec{k}(1-6)$$

$$= \vec{i} - 7\vec{j} - 5\vec{k}$$

$$\vec{a} \times \vec{b} = \boldsymbol{\vec{i} - 7\vec{j} - 5\vec{k}}$$

$$\left|\vec{a} \times \vec{b}\right| = \sqrt{1^2 + (-7)^2 + (-5)^2} = \sqrt{75} = \mathbf{5\sqrt{3}}$$

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Now,

$$\hat{N} = \frac{\vec{a} \times \vec{b}}{\left|\vec{a} \times \vec{b}\right|}$$

$$\hat{N} = \frac{\vec{i} - 7\vec{j} - 5\vec{k}}{5\sqrt{3}}$$

$$\therefore \hat{N} = \frac{1}{5\sqrt{3}}\vec{i} - \frac{7}{5\sqrt{3}}\vec{j} - \frac{5}{5\sqrt{3}}\vec{k}$$

6. Find the first and third quartile from: 2, 5 ,7, 10, 20, 16.

➢ **Solution**

Given,

2 ,5 ,7 ,10 , 20 ,16

Arranging in ascending order,

2 ,5 ,7 ,10 , 16 ,20

$\mathbf{n = 6}$

Now,

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First Quartile,

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} item$$

$$Q_1 = \left(\frac{7}{4}\right)^{th} item$$

$$Q_1 = 1.75\ ^{th} item$$

$$Q_1 = 1\ ^{th} item + 0.75\ \ (2^{nd} item - 1^{th} item)$$

$$Q_1 = 2 + 0.75\ \ (5 - 2)$$

$$\boldsymbol{Q_1 = 4.25}$$

Third Quartile,

$$Q_3 = 3\left(\frac{n+1}{4}\right)^{th} item$$

$$Q_3 = 3 \times \frac{7}{4}^{\ th} item$$

$$Q_3 = 5.25\ ^{th} item$$

$$Q_3 = 5^{th} item + 0.25\ (6^{th}\ item - 5^{th}\ item)$$

$$Q_3 = 16 + 0.25\ (20 - 16)$$

$$\boldsymbol{Q_3 = 17}$$

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7. a) Find standard deviation for data 12, 14, 16, 18, 20.

➢ **Solution**

x	$x-\bar{x}$	$(x-\bar{x})^2$
12	-4	16
14	-2	4
16	0	0
18	2	4
20	4	16
$\sum x = 80$		$\sum(x-\bar{x})^2 = 40$

$$n = 5$$

Now,

$$\bar{\mathbf{x}} = \frac{\sum x}{n} = \frac{80}{5} = \mathbf{16}$$

$$\boldsymbol{S.D\ (\sigma)} = \sqrt{\frac{\sum(x-\bar{x})^2}{n}}$$

$$(\sigma) = \sqrt{\frac{40}{5}}$$

$$(\boldsymbol{\sigma}) = \mathbf{2.828}$$

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$$R_3 \to \frac{1}{17} R_3 \quad \begin{bmatrix} 1 & 1 & 1 & : & 4 \\ 0 & 1 & 5 & : & 5 \\ 0 & 0 & 1 & : & 1 \end{bmatrix}$$

$$R_2 \to R_2 - 5R_3 \quad \begin{bmatrix} 1 & 1 & 1 & : & 4 \\ 0 & 1 & 0 & : & 0 \\ 0 & 0 & 1 & : & 1 \end{bmatrix}$$

$$R_1 \to R_1 - R_2 \quad \begin{bmatrix} 1 & 0 & 1 & : & 4 \\ 0 & 1 & 0 & : & 0 \\ 0 & 0 & 1 & : & 1 \end{bmatrix}$$

$$R_1 \to R_1 - R_3 \quad \begin{bmatrix} 1 & 0 & 1 & : & 3 \\ 0 & 1 & 0 & : & 0 \\ 0 & 0 & 1 & : & 1 \end{bmatrix}$$

Hence,

$$\boldsymbol{x = 3\ , y = 0,\ Z = 1\ .}$$

10) Using De-Moivre's theorem. Find the square roots of $2 + 2\sqrt{3}\,i$

➢ **Solution,**

$Let\ z = 2 + 2\sqrt{3}i$

Here, $x = 2\ and\ y = 2\sqrt{3}$

For z in polar form we have,

$$r = \sqrt{x^2 + y^2} = \sqrt{2^2 + \left(2\sqrt{3}\right)^2} = 4$$

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and, $\tan\theta = \frac{y}{x} = \frac{2\sqrt{3}}{2}$

$\tan\theta = \sqrt{3}$

or, $\theta = 60°$

So, $z = 4(\cos 60° + i\sin 60°)$

$= 4[\cos(60° + n\,360°) + i\sin(60° + n\,360°)]$

$\therefore\ \sqrt{z} = z^{1/2}$ **In general polar form**

$= \sqrt{4}\,[\cos(60° + n\,360°) + i\sin(60° + n\,360°)]^{1/2}$

$= \sqrt{4}\cos\left(\frac{60° + n\,360°}{2}\right) + i\sin\left(\frac{60° + n\,360°}{2}\right)$

$= 2[\cos(30° + n\,180°) + i\sin(30° + n\,180°)]$

Where , $n = 0, 1$

Now, Using De-Moivre's theorem

When $n = 0$, $z^{1/2} = 2(\cos 30° + i\sin 30°)$

$z^{1/2} = \boldsymbol{\sqrt{3} + i}$

When $n = 1$, $z^{1/2} = 2(\cos 210° + i\sin 210°)$

$z^{1/2} = -\sqrt{3} - i \ = -(\boldsymbol{\sqrt{3} + i})$

$\therefore$ **Square root of** $\mathbf{2 + 2\sqrt{3}i} = \pm(\boldsymbol{\sqrt{3} + i})$

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Vertex (x, y)	$G = 10x + 15y$
A(16,0)	160
B(12,4)	**180 (MAX)**
C(0,10)	150
O(0,0).	**0 (MIN)**

Thus, the maximum value $=$ 180 at $B(12, 4)$ and the minimum value $=$ 0 at (0, 0).

12. Prove that the line $lx + my + n = 0$ will touch the Parabola In $y^2 = 4ax$ if $l\mathrm{n} = \mathrm{am}^2$

➢**Solution,** $Given\ line: lx + my + n = 0$

$Parabola: y^2 = 4ax$

Eliminating x form the given line and parabola we have,

$$y^2 = 4a\left(\frac{-my-n}{l}\right)$$

$$\boldsymbol{y^2}l + 4am\boldsymbol{y} + 4an = 0 \cdots\cdots\cdots (i)$$

$which\ is\ Quadratic\ Eqn(Ay^2 + By + C = 0)\ in\ y$

The given line touches the given parabola if discriminant of (i) is Zero. i.e. $B^2 - 4AC = 0$

$$i.e.\ (4am)^2 - 4.l.4an = 0$$

$$16a^2m^2 - 16a\,l\mathrm{n} = 0$$

$$16a(am^2 - l\mathrm{n}) = 0$$

$$am^2 - l\mathrm{n} = 0$$

$\Rightarrow$ $\boldsymbol{l\mathrm{n} = am^2}$ **Proved.**

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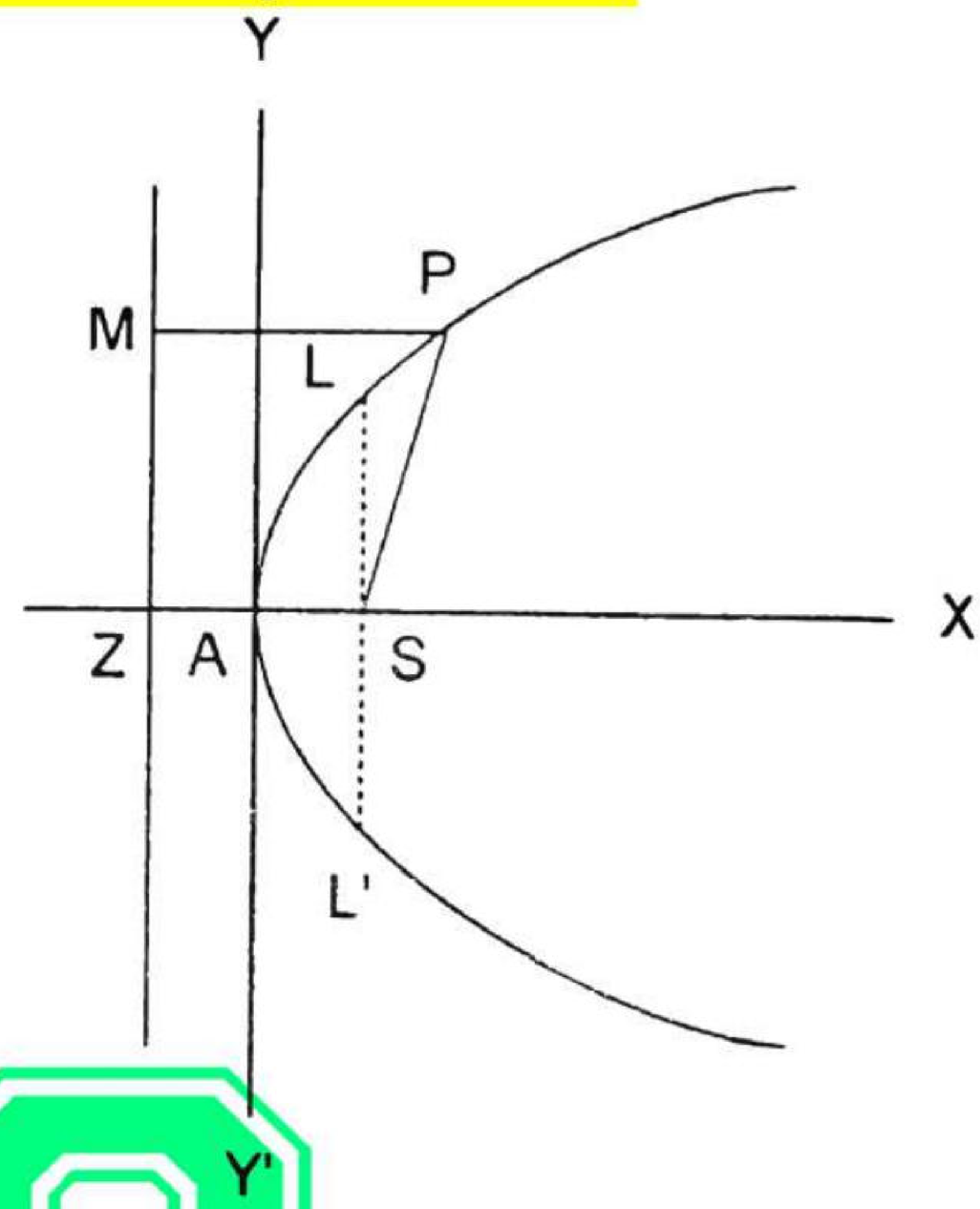
13) Establish the standard equation of parabola.

➢ **Solution,**



Let S be focus and ZM, the directrix of the parabola. SZ is drawn perpendicular to ZM. Let A be the middle point of SZ, so that SA $=$ AZ. Then A is the vertex and ZAS is the axis of the parabola.

To determine the equation of a parabola in the standard form, take the vertex A at the origin, the focus S on the $x - axis$ so that the axis of the parabola is the $x - axis$ and the directrix is parallel to the $y - axis$.

Let AS $= a$ .Thus the coordinates of Z, A and S are Respectively $(-a, 0)$, $(0, 0)$ and $(a, 0)$, and the equation of the directrix is $x + a = 0$.

Let $P(x, y)$ be any point on the parabola. Join PS and draw PM perpendicular to ZM.

Then, $\quad PS = PM.$

$\Longrightarrow \quad PS^2 = PM^2$

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Equation (i) Becomes,

$$-k(x-2)-5k(y+3)+6k(z-1)=0$$

$$or, k.\{-(x-2)-5(y+3)+6(z-1)=0$$

$$-x-5y+6z+2-15-6=\frac{0}{k}$$

$$-x-5y+6z-19=0$$

$$\Rightarrow x+5y-6z+19=0\,, which\ is\ Eqn.\ of\ plane.$$

16) Prove that the vectors: $5\vec{a}+6\vec{b}+7\vec{c}, 7\vec{a}-8\vec{b}+9\vec{c}$ and $3\vec{a}+20\vec{b}+5\vec{c}$ are coplanar.

➢ **Solution,**

Let $\overrightarrow{r_1}=5\overrightarrow{a}+6\overrightarrow{b}+7\overrightarrow{c}$, $\overrightarrow{r_2}=7\overrightarrow{a}-8\overrightarrow{b}+9\overrightarrow{c}$

and $\overrightarrow{r_3}=3\overrightarrow{a}+20\overrightarrow{b}+5\overrightarrow{c}$

If three vector are coplanar, one can expressed as the sum of the scalar multiples of the other two.

$$\overrightarrow{r_3}=x\,\overrightarrow{r_1}+y\,\overrightarrow{r_2} \qquad \text{where, } x \text{ and } y \text{ are scalars}$$

Then, $3\overrightarrow{a}+20\overrightarrow{b}+5\overrightarrow{c}$

$$=x\left(5\overrightarrow{a}+6\overrightarrow{b}+7\overrightarrow{c}\right)+y\left(7\overrightarrow{a}-8\overrightarrow{b}+9\overrightarrow{c}\right)$$

or, $\quad 3\overrightarrow{a}+20\overrightarrow{b}+5\overrightarrow{c}$

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$$= (5 + 7y\ )\overrightarrow{a} + (6x - 8y)\overrightarrow{b} + (7x + 9y)\overrightarrow{c}$$

Equating Coefficient of likes vectors,

$$5 + 7y = 3,\ 6x - 8y = 20 \text{ and } 7x + 9y = 5$$

Solving the first two equation , *we have*,

$$x = 2\ and\ y = -1$$

These values also satisfy the third equation. Hence form (i)

$$\overrightarrow{r_3} = 2\ \overrightarrow{r_1} - \overrightarrow{r_2}$$

$\therefore$ **The given vectors are coplanar.**

17) If $\vec{a} = 3\vec{i} + 4\vec{j}$ and $\vec{b} = \vec{i} - \vec{j} + \vec{k}$, show that $\vec{a} \times \vec{b}$ represents a vector which is perpendicular to both $\vec{a}\ and\ \vec{b}$.

➢Solution

$\vec{a} = 3\vec{i} + 4\vec{j}$ **and** $\vec{b} = \vec{i} - \vec{j} + \vec{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 4 & 0 \\ 1 & -1 & 1 \end{vmatrix}$$

$$= \vec{i}(4 - 0) - \vec{j}(3 - 0) + \vec{k}(-3 - 4)$$

$$\vec{a} \times \vec{b} = 4\vec{i} - 3\vec{j} - 7\vec{k}$$

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To show: $\vec{a} \times \vec{b}$ represents a vector which is perpendicular to both $\vec{a}$ *and* $\vec{b}$.

$$\overrightarrow{(\boldsymbol{a} \times \boldsymbol{b})}.\vec{\boldsymbol{a}} = (4\vec{i} - 3\vec{j} - 7\vec{k}).\,(\,3\vec{i} + 4\vec{j})$$

$$= 12 - 12$$

$$= \mathbf{0}$$

$i.e,\ \vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$

Also,

$$\overrightarrow{(\boldsymbol{a} \times \boldsymbol{b})}.\vec{\boldsymbol{b}} = (4\vec{i} - 3\vec{j} - 7\vec{k}).(\,\vec{i} - \vec{j} + \vec{k})$$

$$= 4 + 3 - 7$$

$$= \mathbf{0}$$

$i.e.\quad \vec{a} \times \vec{b}$ is perpendicular to both $\vec{b}$

Hence , $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ *and* $\vec{b}$.

18. Find the quartile deviation (Q.D.) and its coefficient from the following data :

Class	10-15	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	2	4	6	7	3	1	5

➢ **Solution**

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Class	Frequency (F)	C.F
10-15	2	2
15-20	4	6
20-25	6	12
25-30	7	19
30-35	3	22
35-40	1	23
45-45	5	28

$$N = 28$$

For Q_1:

$$Q_1 = \left(\frac{N}{4}\right)^{th} \text{item} = \frac{28}{4} = \boldsymbol{7^{th} item}$$

C.f is just greater than 7 and is 12. So, it lies between $20 - 25$.

$$\mathbf{L = 20, F = 6, C.f = 6, i = 5}$$

$$Q_1 = L + \frac{\frac{N}{4} - c.f}{f} \times i$$

$$= 20 + \frac{7 - 6}{6} \times 5$$

$$= 20 + 0.83$$

$$\mathbf{Q_1 = 20.83}$$

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For Q_3:

$$Q_3 = 3\left(\frac{N}{4}\right)^{th} \text{ item } = 3 \times 7 = \mathbf{21^{th} item}$$

C.f is just greater than 21 and is 22. So, it lies between 30-35.

$$\mathbf{L = 30,\ f = 3,\ C.f = 19,\ i = 5}$$

$$Q_3 = L + \frac{\frac{3N}{4} - c.f}{f} \times i$$

$$= 30 + \frac{21 - 19}{3} 5$$

$$= 30 + 3.33$$

$$\mathbf{Q_3 = 33.33}$$

$$\textbf{Coefficient of Q.D} = \frac{\mathbf{Q_3 - Q_1}}{\mathbf{Q_3 + Q_1}}$$

$$= \frac{33.33 - 20.83}{33.33 + 20.836}$$

$$= \frac{12.47}{54.16}$$

$$\therefore \textbf{Coefficient of Q.D} = \mathbf{0.230}$$

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19. By using product moment formula, calculate the coefficient of correlation:

Price (Rs.)	25	19	28	26	20	18	24	20	22	18
Sales (unit)	60	54	66	70	53	?	62	51	65	50

Where average sales is 59 units?

➢ Solution

Here,

$$\mathbf{n = 10}$$

Price (X)	25	19	28	26	20	18	24	20	22	18
Sales (Y)	60	54	66	70	53	**P**	62	51	65	50

$$\sum \boldsymbol{Y} = \mathbf{531} + \boldsymbol{P}$$

For Sales calculation P, Given $Avg. Sales\ \overline{\boldsymbol{Y}} = \mathbf{59}$

So $\bar{Y} = \frac{\sum Y}{n} = \frac{531+P}{10}$

$59 = \frac{531+P}{10}$

$590 = 531 + P$

$\Rightarrow \boldsymbol{P} = \mathbf{59}$

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x	y	$X=x-\bar{X}$	$Y=y-\bar{Y}$	X^2	Y^2	XY
25	60	3	1	9	1	3
19	54	-3	-5	9	25	15
28	66	6	7	36	49	42
26	70	4	11	16	121	44
20	53	-2	-6	4	36	12
18	59	-4	0	16	0	0
24	62	2	3	4	9	6
20	51	-2	-8	4	64	16
22	65	0	6	0	36	0
18	50	-4	-9	16	81	36
$\sum X = 220$	$\sum y = 590$			$\sum X^2 = 114$	$\sum Y^2 = 422$	$\sum XY = 174$

Now,

$$\bar{x} = \frac{\sum x}{n} = \frac{220}{10} = 22$$

$$\bar{y} = \frac{\sum y}{n} = \frac{220}{10} = 59$$

$$Coefficient\ of\ corelation(\boldsymbol{r}) = \frac{\sum XY}{\sqrt{X^2}\sqrt{Y^2}}$$

$$= \frac{174}{\sqrt{114}\sqrt{422}}$$

$\therefore$ **coefficient of corelation(r) = 0.793**

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20) A binomial distribution consists of 5 independent trials. If probabilities of 1 and 2 successes are respectively $\frac{1}{4}$ and $\frac{1}{3}$ find the probability of successes and failure in trial. Also find P(r = 3) .

➢ Solution

$$P(r) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$$

$$\Rightarrow \quad P(r = 1) = {}^{5}C_{1}\, p\, q^{5-1}$$

$$\Rightarrow \quad \frac{1}{4} = 5pq^{4} \;\cdots\cdots\cdots (i)$$

Again, $\quad P(r = 2) = {}^{5}C_{2}\, p^{2}\, q^{5-2}$

$$\Rightarrow \quad \frac{1}{3} = \frac{5 \times 4}{2} p^{2} q^{3} = 10p^{2}q^{3} \;\cdots\cdots\cdots (ii)$$

Form (i) and (ii)

$$\frac{1/4}{1/3} = \frac{5pq^{4}}{10p^{2}q^{3}}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{q}{2p}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{q}{p}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{1-p}{p} \qquad \because \boldsymbol{q = 1 - p}$$

$$\Rightarrow \quad 3p = 2 - 2p$$

$$\Rightarrow \quad 5p = 2$$

$$\therefore \quad p = \frac{2}{5} \quad \textbf{\textit{i.e Probability of successes in a trial}}$$

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$$Then, \qquad q = 1 - p = 1 - \frac{2}{5} = \frac{3}{5}$$

$$\text{Now,} \qquad p(r = 3) = {}^{5}C_3\, p^3\, q^2$$

$$= \frac{5 \times 4}{2}\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^2$$

$$= \frac{\mathbf{144}}{\mathbf{625}}$$

The End-

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Engg. Mathematics –II__(Engg. All) 2nd Sem

(2079 *Old + Very Old*) Question Paper Solution.

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1) Define vector product of two vectors. Find the area of triangle Whose two sides are determined by the vectors.

$$2\vec{i} - \vec{j} + \vec{k} \text{ and } 3\vec{i} + 4\vec{j} - \vec{k}$$

- If θ be the angle between $\vec{a}$ and $\vec{b}$ and $\vec{\eta}$ be unit vector normal to the plane of $\vec{a}$ and $\vec{b}$, then the vector product $\vec{a} \times \vec{b}$ is defined as

$$\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}| \sin\theta\, \vec{\eta}\ .$$

- **Second Part :**

$$Let, \vec{a} = \ \mathbf{2\,\vec{i} - \vec{j} + \vec{k}} \ and \ \vec{b} = \mathbf{3\vec{i} + 4\vec{j} - \vec{k}}$$

Then, $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix}$

$$= (1-4)\vec{i} - (-2-3)\vec{j} + (8+3)\vec{k}$$

$$= -3\vec{i} + 5\vec{j} + 11\vec{k}$$

$$|\vec{a} \times \vec{b}| = -3\vec{i} + 5\vec{j} + 11\vec{k}$$

$$= \sqrt{(-3)^2 + (5)^2 + 11^2} = \sqrt{\mathbf{155}}$$

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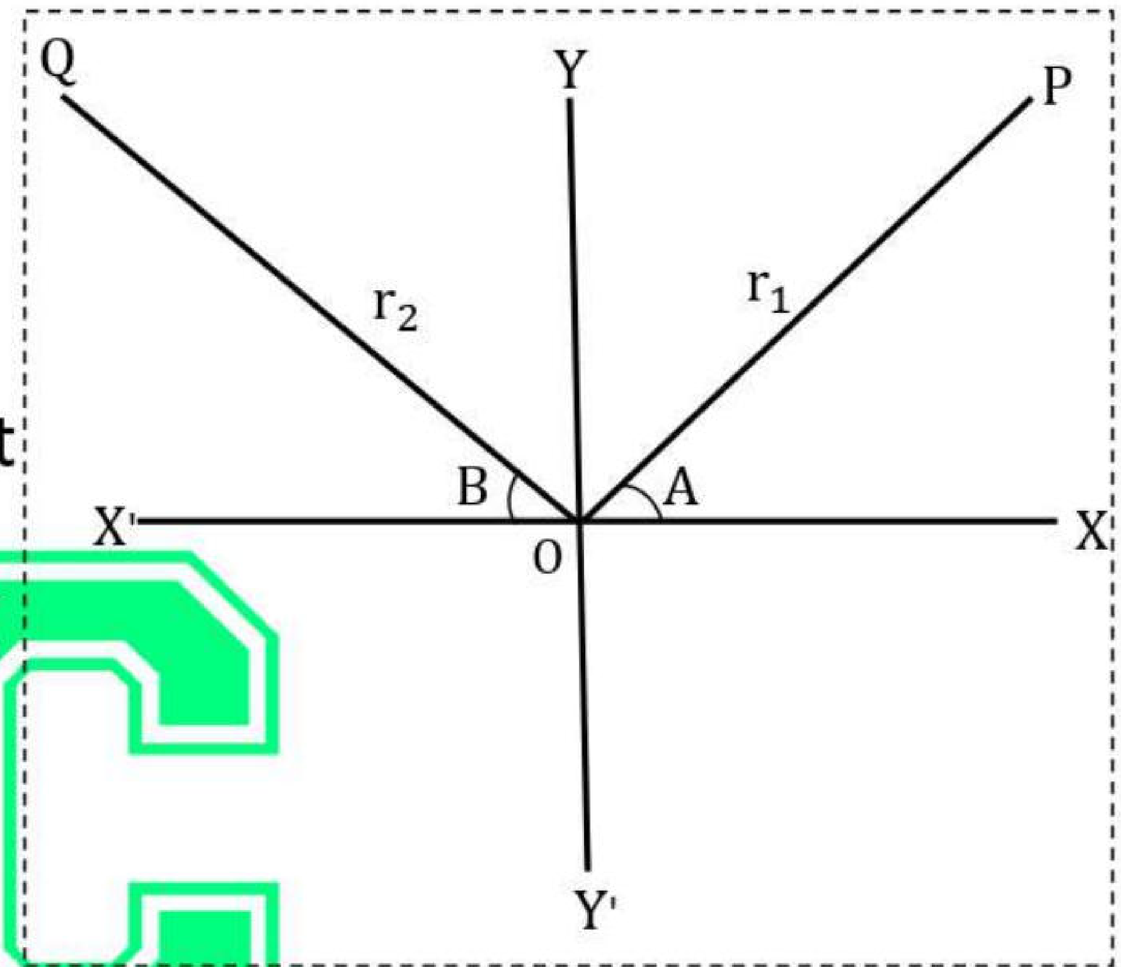
$$Area\ of\ triangle = \frac{1}{2} \times \left| \vec{a} \times \vec{b} \right| = \frac{\sqrt{155}}{2}$$

$$\therefore The\ required\ area\ of\ triangle = \frac{\sqrt{155}}{2} sq.units$$

2) Prove that $\mathbf{Cos\,(A+B) = CosA.CosB - SinA.SinB}$ by using vector method.



- Let XOX' and YOY' be two mutually perpendicular straight lines representing $x - axis$ and $y - axis$ respectively. Let $\angle XOP = A$ and $\angle QOX' = B$ so that $\angle POQ = \pi - (A + B)$. Also, let $OP = r_1$ and $OQ = r_2$. Then the coordinates of P and Q are $(r_1 \cos A,\ r_1 \sin A)$ and $(r_2 \cos(\pi - B), r_2 \sin(\pi - B)) = (r_2 \cos B,\ r_2 \sin B)$. So

$$\overrightarrow{OP} = (r_1 \cos A,\ r_1 \sin A) \quad \& \quad \overrightarrow{OQ} = (-r_2 \cos B,\ r_2 \sin B)$$

Now,

$$\begin{aligned}\overrightarrow{\mathbf{OP}} \cdot \overrightarrow{\mathbf{OQ}} &= (r_1 \cos A, r_1 \sin A) \cdot (-r_2 \cos B, r_2 \sin B) \\ &= -r_1 r_2 \cos A \cos B + r_1 r_2 \sin A \sin B \\ &= -r_1 r_2 (\cos A \cos B - \sin A \sin B)\end{aligned}$$

Since $\pi - (A + B)$ is the angle between $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, so

$$\cos[\pi - (A + B)] = \frac{\overrightarrow{OP} \cdot \overrightarrow{OQ}}{|\overrightarrow{OP}|\,|\overrightarrow{OQ}|}$$

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$$or, -\cos\ (A+B) = \frac{-r_1\ r_2(\cos A\ \cos B - \ \sin A\ \sin B)}{r_1\ r_2}$$

$\therefore$ $\mathbf{\cos\ (A+B) = \cos A \cos B - \sin A \sin B}$ **Proved.**

3) State De-Moivre's theorem. Use it to find the fourth roots of $\frac{-1}{2} + i.\frac{\sqrt{3}}{2}$.

➢ De-Moirve's Theorem

Statement:- If n is any positive integer,

$$[\,r(\cos\theta + i\sin\theta)]^n = r^n\ (\cos n\theta + i\sin n\theta).$$

➢ **Second Part :-**

$$\text{Let } z^4 = \frac{-1}{2} + i\ \frac{\sqrt{3}}{2} = r\ (\cos\theta + i\ \sin\theta)$$

Equating real and imaginary parts, we get

$$r\cos\theta = \frac{-1}{2}\ and\ r\sin\theta = \frac{\sqrt{3}}{2}$$

$$\text{This gives } r^2 = \left(\frac{-1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$

$or,\qquad r = 1$ (Positive value only)

$$\text{Also,}\quad \cos\theta = \frac{1}{-2} \text{ and } \sin\theta = \frac{\sqrt{3}}{2} \text{ giving } \theta = 120°$$

$$\therefore z^4 = \frac{-1}{2} + i\ \frac{\sqrt{3}}{2} = 1\ (\cos 120° + i\sin 120°)$$

$$= (\cos 120° + n\sin 120°) + i\sin(120° + n\ 360°) \text{ general form.}$$

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$$\therefore z^4 = [\cos(120° + n\sin 360°) + i\sin(120° + n\,360°)]^{\frac{1}{4}}$$

$$= \cos\left(\frac{120° + n\sin 120°}{4}\right) + i\sin\left(\frac{120° + n\sin 120°}{4}\right),$$

where n = 0, 1, 2, 3

For n = 0, $z = \cos 30° + \mathrm{i}\sin 30° = \frac{\sqrt{3}}{2} + \mathrm{i}\,\frac{1}{2}$

For n = 1, $z = \cos 120° + \mathrm{i}\sin 120° = \frac{-1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}$

For n = 2, $z = \cos 210° + \mathrm{i}\sin 210° = -\frac{\sqrt{3}}{2} + \mathrm{i}\,\frac{1}{2}$

For n = 3, $z = \cos 300° + \mathrm{i}\sin 300° = \frac{1}{2} - \frac{\sqrt{3}}{2}\,\mathrm{i}$

$\therefore$ **Required roots are** $\pm\left(\frac{\sqrt{3}}{2} + \mathrm{i}\,\frac{1}{2}\right), \pm\left(\frac{1}{2} - \mathrm{i}\frac{\sqrt{3}}{2}\right)$

4) If z and w are two complex numbers, prove that:

$$|z + w| \leq |z| + |w|$$

➢ **Proof**

Let $z = a + ib$ and $w = c + id$, so that $z + w = a + c + i(b + d)$, then

$$|z| = \sqrt{a^2 + b^2}, |w| = \sqrt{c^2 + d^2}$$

Also, $|z + w| = \sqrt{(a + c)^2 + (b + d)^2}$

Now, $|z + w| \leq |z| + |w|$ will be true

If $\sqrt{(a + c)^2 + (b + d)^2} \leq \sqrt{a^2 + b^2} + \sqrt{c^2 + d^2}$

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i.e. $(a+c)^2+(b+d)^2 \le a^2+b^2+c^2+d^2 + 2\sqrt{(a^2+c^2)+(b^2+d^2)}$

i.e. $ac+bd \le \sqrt{(a^2+c^2)+(b^2+d^2)}$

i.e. $a^2c^2+b^2d^2+2abcd \le (a^2+c^2)+(b^2+d^2)$

i.e. $2abcd \le a^2d^2+b^2c^2$

i.e. $0 \le 2abcd - a^2d^2+b^2c^2$

i.e. $0 \le (ad-bc)^2$ which is true all real number a, b, c, d

$\therefore \quad |\boldsymbol{z}+\boldsymbol{w}| \le |\boldsymbol{z}|+|\boldsymbol{w}|$

5) Maximize and minimize: $F = 9x + 20y$ subject to $y - x \ge 1$, $y - x \le 3, 2 \le x \le 5$.

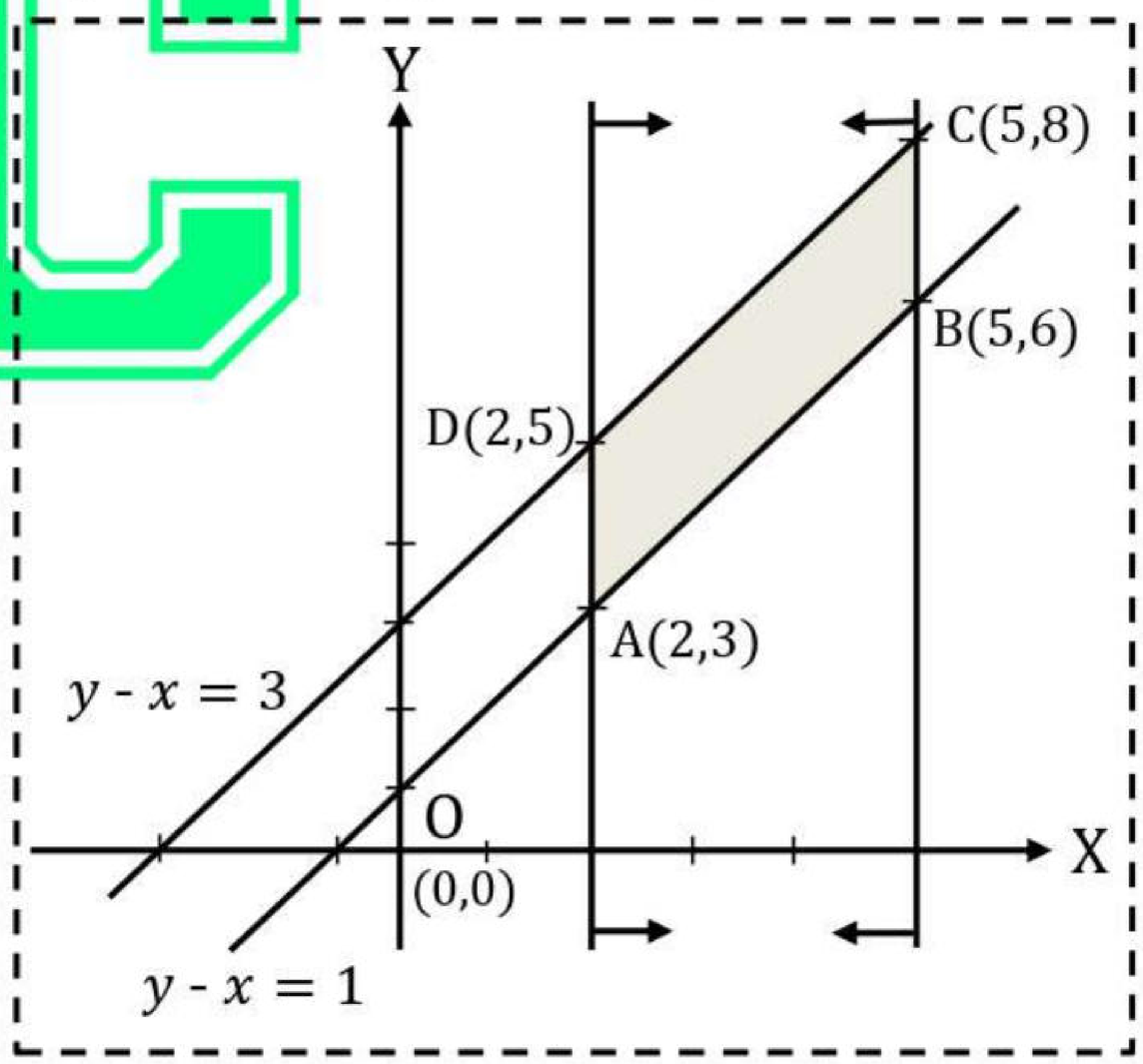
- The given objective function is $F = 9x + 20y$ and the given inequalities are $-x \ge 1,\ y - x \le 3,\ 2 \le x \le 5$.

The boundary of the given inequalities are $y - x = 1$

$y - x = 3$

$x = 2,\ \ x = 5$

Now, drawing the graph represented by each of these equations and shading the region satisfying the given inequalities, we get the following feasible region, shown in graph.



The shaded region ABCD in the graph is the feasible region. The vertices of A, B, C, D are $(2,3)$, $(5,6)$, $(5,8)$ and $(2,5)$ respectively. Now evaluating the value of given objective function $F = 9x + 20y$ at each vertices of the feasible region, which is shown in the following table:

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Vertices (x, y)	Value of $F = 9x + 20y$
A(2,3)	78
B(5,6)	165
C(5,8)	205
D(2,5)	118

Thus, the max-value of $F = 205$ at C(5,8) and min-value of $F = 78$ at A(2, 3).

6) Define transpose of a matrix. If the matrix:

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}, Show\ that\ (AB)^T = B^T A^T.$$

- A matrix obtained from a given matrix A by interchanging its rows and columns is known as the transpose of the given matrix and it is denoted by A' or A^T.

- **Second Part :**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}, \text{Show that } (AB)^T = B^T A^T$$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$$

$$\therefore \mathbf{(AB)^T} = \begin{bmatrix} \mathbf{19} & \mathbf{43} \\ \mathbf{22} & \mathbf{50} \end{bmatrix}$$

$$\text{Also, } B^T = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}, A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$\therefore B^T A^T = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 5+14 & 15+28 \\ 6+16 & 18+32 \end{bmatrix} = \begin{bmatrix} \mathbf{19} & \mathbf{43} \\ \mathbf{22} & \mathbf{50} \end{bmatrix}$$

Hence, $\mathbf{(AB)^T = B^T A^T}$**.** Proved

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7) Prove that: $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

➢ Solution:

$$\text{L.H.S.} = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

[Applying $R_1 \longrightarrow R_1 + R_2 + R_3$]

$$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

[Taking a+b+c as a common from R_1]

$$= (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ 2b & a-b-c & 0 \\ 2c & 0 & a-b-c \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1, C_3 \to C_3 - C_1$]

$$= (a+b+c).1 \begin{vmatrix} a-b-c & 0 \\ 0 & a-b-c \end{vmatrix}$$

$$= (a+b+c)(a-b-c)^2$$

$$= (a+b+c)(a+b+c)^2$$

$$= (a+b+c)^3$$

$\therefore$ **R.H.S = L.H.S Proved.**

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8) Solve by using Cramer's rule or Row equivalent matrix method.

$$5x - 3y = 20$$
$$2x + 3y = 8$$
$$x + y + z = 5$$

➢ **Solution, By using Cramer's rule**

Given, Re writing the Given equation

$$5x - 3y + 0.z = 20$$
$$2x + 3y + 0.z = 8$$
$$x + y + z = 5$$

(∵ Expanding along C3)

$$D = \begin{vmatrix} 5 & -3 & 0 \\ 2 & 3 & 0 \\ 1 & 1 & 1 \end{vmatrix} = 0 - 0 + 1(15 + 6) = \mathbf{21}$$

$$D_1 = \begin{vmatrix} 20 & -3 & 0 \\ 8 & 3 & 0 \\ 5 & -1 & 1 \end{vmatrix} = 0 - 0 + 1(60 - 24) = \mathbf{84}$$

$$D_2 = \begin{vmatrix} 5 & 20 & 0 \\ 2 & 8 & 0 \\ 1 & 5 & 1 \end{vmatrix} = 0 - 0 + 1(40 - 40) = \mathbf{0}$$

$$D_3 = \begin{vmatrix} 5 & -3 & 20 \\ 2 & 3 & 8 \\ 1 & 1 & 5 \end{vmatrix} = 20(2 - 3) - 8(5 + 3) + 5(15 + 6) = \mathbf{21}$$

Now,

$$x = \frac{D_1}{D} = \frac{84}{21} = \mathbf{4}, \quad y = \frac{D_2}{D} = \frac{0}{21} = \mathbf{0}, \quad z = \frac{D_3}{D} = \frac{21}{21} = \mathbf{1}$$

Hence, $x = 4$, $y = 0$, $z = 1$.

OR

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Row equivalent matrix method,

$$5x - 3y - 0.z = 20$$
$$2x + 3y = 8$$
$$x + y + z = 5$$

The augmented matrix is

$$\begin{bmatrix} 5 & -3 & 0 & : 20 \\ 2 & 3 & 0 & : 8 \\ 1 & 1 & 1 & : 5 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$
$$\begin{bmatrix} 1 & 1 & 1 & : 5 \\ 2 & 3 & 0 & : 8 \\ 5 & -3 & 0 & : 20 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$$\begin{bmatrix} 1 & 1 & 1 & : 5 \\ 0 & 1 & -2 & : -2 \\ 5 & -3 & 0 & : 20 \end{bmatrix}$$

$R_3 \to R_3 - 5R_1$
$$\begin{bmatrix} 1 & 1 & 1 & : 5 \\ 0 & 1 & -2 & : -2 \\ 0 & -8 & -5 & : -5 \end{bmatrix}$$

$R_1 \to R_1 - R_2$
$$\begin{bmatrix} 1 & 0 & 3 & : 7 \\ 0 & 1 & -2 & : -2 \\ 0 & -8 & -5 & : -5 \end{bmatrix}$$

$R_3 \to R_3 + 8R_2$
$$\begin{bmatrix} 1 & 0 & 3 & : 7 \\ 0 & 1 & -2 & : -2 \\ 0 & 0 & -21 & : -21 \end{bmatrix}$$

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$$R_3 \to -\frac{R_3}{21} \qquad \begin{bmatrix} 1 & 0 & 3 & :7 \\ 0 & 1 & -2 & :-2 \\ 0 & 0 & 1 & :1 \end{bmatrix}$$

$$R_2 \to R_2 + 2R_3 \qquad \begin{bmatrix} 1 & 0 & 3 & :7 \\ 0 & 1 & 0 & :0 \\ 0 & 0 & 1 & :1 \end{bmatrix}$$

$$R_1 \to R_1 - 3R_3 \qquad \begin{bmatrix} 1 & 0 & 0 & :4 \\ 0 & 1 & 0 & :0 \\ 0 & 0 & 1 & :1 \end{bmatrix}$$

Hence,

$$\boldsymbol{x = 4, y = 0, Z = 1\,.}$$

9) A closed cylindrical can is to be made so that its volume is 52 cm^3. Find the dimensions if the surface area is to be minimum.

➢ **Solution :**

Let r be the radius, h be the height, v be the volume and s be the Total surface area of the cylindrical can. Then,

$$v = \pi r^2 h \qquad or, 52 = \pi r^2 h \qquad \Rightarrow h = \frac{52}{\pi r^2}$$

$$and\ s = 2\pi r(r + h) \qquad \cdots\cdots\cdots (ii)$$

Taking h as a constant , substituting value of h in (ii),

$$s = 2\pi r\left(r + \frac{52}{\pi r^2}\right) = 2\pi r^2 + \frac{104}{r}$$

Now, $\dfrac{ds}{dr} = 4\pi r - \dfrac{104}{r^2}$ and $\dfrac{d^2s}{dr^2} = 4\pi + \dfrac{208}{r^3}$

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For minimum surface area S, $\frac{ds}{dr} = 0$

$$or, \quad 4\pi r - \frac{104}{r^2} = 0$$

$$or, \quad 4\pi r = \frac{104}{r^2}$$

$$or, \quad r^3 = \frac{104}{4\pi}$$

$$or, \quad r = \left(\frac{26}{\pi}\right)^{\frac{1}{3}}$$

At $r = \left(\frac{26}{\pi}\right)^{\frac{1}{3}}$, $\frac{d^2s}{dr^2} = \frac{208}{\frac{26}{\pi}} + 4\pi = \mathbf{12\pi > 0}\ (+ve)$

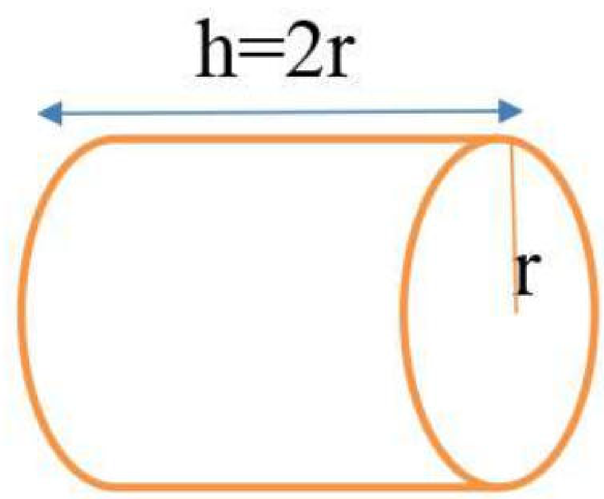
So, s is minimum when $\boldsymbol{r = \left(\frac{26}{\pi}\right)^{\frac{1}{3}}}$

Now, from (i) $52 = \pi r^2 h$

$$or, \quad h = \frac{52}{\pi r^2} = \frac{52.r}{\pi r^3} = \frac{52.r}{\pi.\frac{26}{\pi}} = \mathbf{2r}$$

Hence, for the minimum surface area, height of the cylinder = 2 radius

$i.e.\ h = 2r,\ where\ r = \left(\frac{26}{\pi}\right)^{\frac{1}{3}} cm. = 2.02cm$



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10) A spherical balloon is inflated at the rate of 10ft^3/sec. At what rate is the radius increasing when the radius is 1 ft?

➢ **Solution :**

Let **r** be the radius of the balloon and V be the volume in time t.

Then by question $\frac{dV}{dt} = 10\ \frac{\text{ft}^3}{\text{sec}}$, $r = 1\text{ ft}, \frac{dr}{dt} = ?$

We have,

$$\text{V} = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi .3r^2 \frac{dr}{dt}$$

$$or, \quad 10 = 4\pi \times 1^2 \times \frac{dr}{dt}$$

$$or, \quad \frac{10}{4\pi} = \frac{dr}{dt}$$

$$\therefore \quad \boldsymbol{\frac{dr}{dt} = \frac{10}{4\pi}\ \text{ft./sec}}$$

Hence, At the rate of $\frac{10}{4\pi}$ ft./sec ,the radius is increasing when radius is 1 ft.

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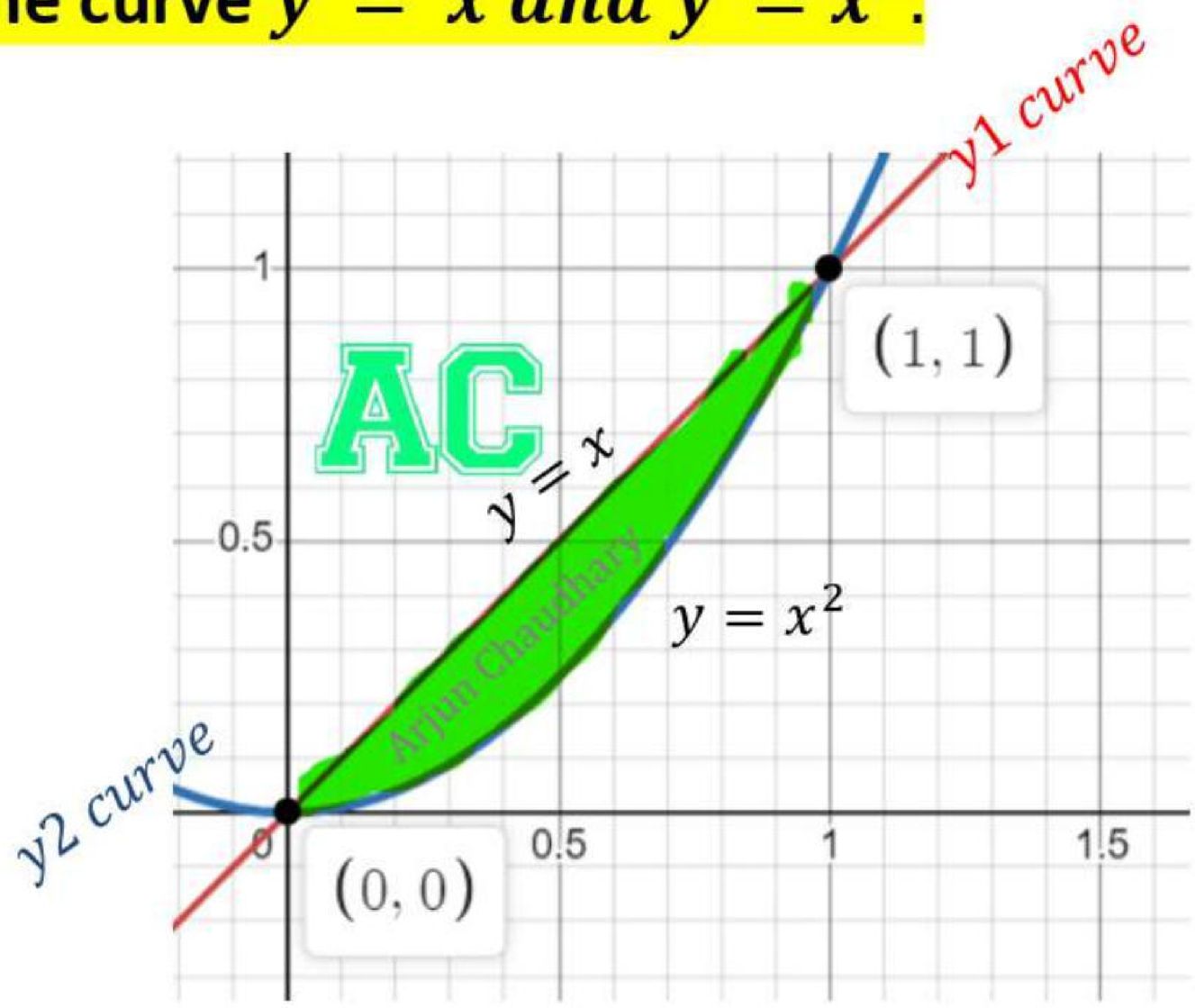
11) Find the area bounded by the curve $y = x$ and $y = x^2$.



➢ **Solution :**

Given,

$$y = x \quad \cdots\cdots\cdots\cdots (1)$$

$$y = x^2 \quad \cdots\cdots\cdots\cdots (2)$$

Solving we get,

$$x = x^2$$
$$x - x^2 = 0$$
$$x(1 - x) = 0$$
$$\Rightarrow \boldsymbol{x = 0, x = 1}$$

For, $x = 0, y = 0$

$x = 1, y = 1$ Hence, Points of intersection are (0,0) and (1,1)

Area bounded by the curve $= \int_{x_1}^{x_2} (y_1 - y_2)\,dx$

$$= \int_0^1 (x - x^2)\,dx$$

$$= \left| \frac{x^2}{2} - \frac{x^3}{3} \right|_0^1$$

$$= \left[\left(\frac{1}{2} - \frac{1}{3}\right) - 0\right]$$

$$= \frac{1}{6}\ sq.unit$$

Hence, Area bounded by the curve is $= \boldsymbol{\frac{1}{6}\ sq.unit}$

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12) Find the equation of the plane through (3, 4, 5) and parallel to the plane $x + y + z = 3$.

➢ **Solution :**

Let equation of pane parallel to plane $x + y + z - 3 = 0. is$

$x + y + z + k = 0 \cdots\cdots\cdots\cdots (1)$ where, K is constant

Since , plane 1 also passes through (3,4,5) so, it must satisfy the equation so,

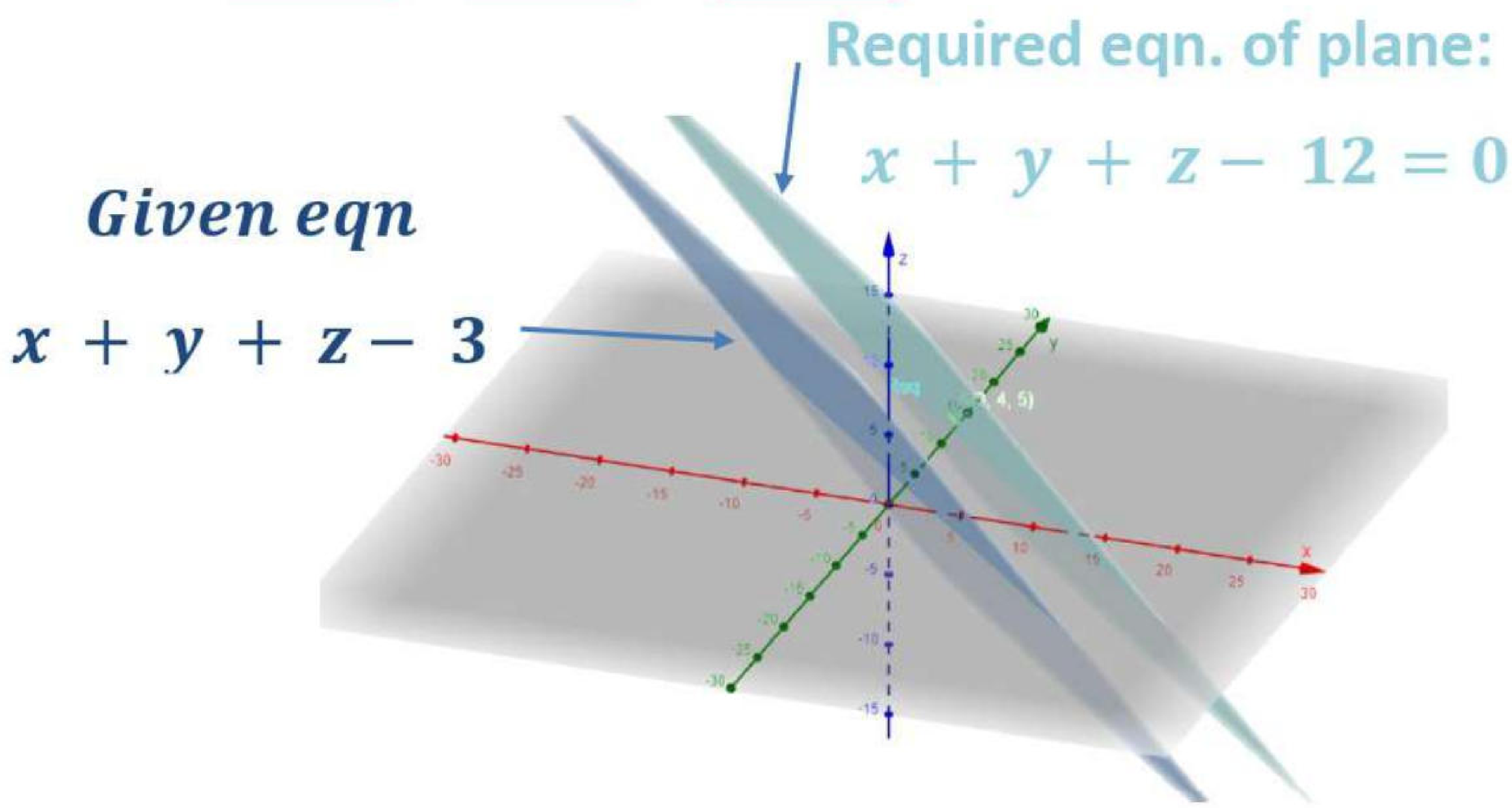
$$3 + 4 + 5 + k = 0$$

$$\Rightarrow k = -12$$

Hence, Required Equation of plane is

$$x + y + z - 12 = 0$$

AC



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13) Find the direction cosines of the line which is perpendicular to the lines with direction cosines proportional to 3, -1, 1 and -3, 2, 4.

➢ **Solution :**

Let l, m, n be the direction cosines of a line which is perpendicular to the lines having direction ratios $3, 1, 1$ and $-3, 2, 4$.

$$\therefore l \times 3 + m(-1) + n(1) = 0$$
$$l(-3) + m(2) + n(4) = 0$$

$$or, \quad \frac{l}{-4-2} = \frac{m}{-3-12} = \frac{n}{6-3}$$

$$or, \quad \frac{l}{-6} = \frac{m}{-15} = \frac{n}{3}$$

$$or, \quad \frac{l}{-2} = \frac{m}{-5} = \frac{n}{1} = \frac{\sqrt{l^2+m^2+n^2}}{\sqrt{(-2)^2+(-5)^2+(1)^2}}$$

$$or, \quad l = \frac{-2}{\sqrt{30}}, m = \frac{-5}{\sqrt{30}}, n = \frac{1}{\sqrt{30}}$$

$$\therefore \quad (\boldsymbol{l}, \boldsymbol{m}, \boldsymbol{n}) = \left(\frac{-2}{\sqrt{30}}, \frac{-5}{\sqrt{30}}, \frac{1}{\sqrt{30}}\right)$$

14) Calculate the quartile deviation (Q.D.) and its coefficient from the following data :

Class	10-15	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	2	4	6	7	3	1	5

➢ **Solution**

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Class	Frequency (F)	C.F
10-15	2	2
15-20	4	6
20-25	6	12
25-30	7	19
30-35	3	22
35-40	1	23
45-45	5	28

$$N = 28$$

For Q_1:

$$Q_1 = \left(\frac{N}{4}\right)^{th} \text{item} = \frac{28}{4} = \mathbf{7^{th} item}$$

C.f is just greater than 7 and is 12. So, it lies between $20 - 25$.

$$\mathbf{L = 20, F = 6, C.f = 6, i = 5}$$

$$Q_1 = L + \frac{\frac{N}{4} - c.f}{f} \times i$$

$$= 20 + \frac{7 - 6}{6} \times 5$$

$$= 20 + 0.83$$

$$\mathbf{Q_1 = 20.83}$$

For Q_3:

$$Q_3 = 3\left(\frac{N}{4}\right)^{th} \text{item} = 3 \times 7 = \mathbf{21^{th} item}$$

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C.f is just greater than 21 and is 22. So, it lies between 30-35.

$$\mathbf{L = 30,\ f = 3,\ C.f = 19,\ i = 5}$$

$$Q_3 = L + \frac{\frac{3N}{4} - c.f}{f} \times i$$

$$= 30 + \frac{21 - 19}{3} 5$$

$$= 30 + 3.33$$

$$\mathbf{Q_3 = 33.33}$$

$$\textbf{Coefficient of Q.D} = \frac{\mathbf{Q_3 - Q_1}}{\mathbf{Q_3 + Q_1}}$$

$$= \frac{33.33 - 20.83}{33.33 + 20.836}$$

$$= \frac{12.47}{54.16}$$

$$\therefore \textbf{Coefficient of Q.D} = \mathbf{0.230}$$

15) Calculate coefficient of correlation between the variables X and Y:

x	12	14	16	18	12	15	16	17
y	1	5	7	3	9	4	3	4

➢ **Solution**

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		$\overline{X} = 15$	$\overline{Y} = 4.5$			
x	y	$X = x - \overline{X}$	$Y = y - \overline{Y}$	X	Y	XY
12	1	-3	-3.5	9	12.25	10.5
14	5	-1	0.5	1	0.25	-0.5
16	7	1	2.5	1	6.25	2.5
18	3	3	-1.5	9	2.25	-4.5
12	9	-3	4.5	9	20.25	-13.5
15	4	0	-0.5	0	0.25	0
16	3	1	-1.5	1	2.25	-1.5
17	4	2	-0.5	4	0.25	-1
$\sum x = 120$	$\sum y = 36$			$\sum x^2 = 34$	$\sum y^2 = 44$	$\sum xy = -8$

$n = 8$

Now, $\overline{X} = \frac{\sum x}{n} = \frac{120}{8} = \mathbf{15}$ and $\overline{Y} = \frac{\sum y}{n} = \frac{3.6}{8} = \mathbf{4.5}$

Using Product moment formula ,

$$Coeff.\ of\ correlation, (r) = \frac{\sum xy}{\sqrt{\sum x^2}\ \sqrt{\sum y^2}}$$

$$= \frac{-8}{\sqrt{34}\ \sqrt{44}}$$

$$\therefore \mathbf{r = -0.2068}$$

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16) A committee of 5 is to be formed out of a group of 7 men and 6 women. Find the probability that in the committee there will be 3 men and 2 women.

➢ **Solution :**

Total People = 7 + 6 = 13

Total no of ways such that 5 people selected out of 13 = C(13,5)

$$i.e \text{ Possible outcomes}, n = \frac{13!}{8!\,5!} = \mathbf{1287\ ways}$$

No of ways such that 3 men out of 7 and 2 women out of 6 are selected in $= C(7,3) \times C(6,2)$

$$\text{No of favourable cases, m} = \frac{7!}{4!\ 3!} \times \frac{6!}{4!\ 2!}$$

$$= 35 \times 15$$

$$= \mathbf{525\ ways}$$

The Probability that in a committee of 5 formed such that 3 man & 2 women $= \frac{m}{n} = \frac{525}{1287} = \frac{175}{429} = 0.4079 = 40.79\%$

Hence, **40.79% probability that in the committee there will be 3 men and 2 women**

← The End →

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Engg. Mathematics –II__(Engg. All) $2^{nd\ Sem}$

(2080 *Old + Very Old*) Question Paper Solution.

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1. a) By using De-Moivres theorem find the cube root of i.

➢ **Solution :**

Let $z^3 = \mathrm{i} = 0 + 1\,.\mathrm{i} = \cos 90° + \mathrm{i}\sin 90°$

$$= \cos(90° + k.360°) + i\ \sin(90° + k.360°)$$

$$\therefore z = \{\cos(90° + k.360°) + i\ \sin(90° + k.360°)\}^{\frac{1}{3}}$$

$$= \cos\frac{90° + k.360}{3} + i\,sin\frac{90° + k.360}{3}$$

$$= \cos(30° + k.120°) + i\ \sin(30° + k.120°)$$

Where, $k = 0\,,1\,,2$. For $k = 0\,,1\,,2$, the roots are,

$$\cos 30° + \mathrm{i}\sin 30°, \cos 150° + \mathrm{i}\sin 150°,\ \cos 270° + \mathrm{i}\sin 270°$$

$$i.e., \frac{\sqrt{3}}{2} + \frac{1}{2}i,\quad -\frac{\sqrt{3}}{2} + \frac{1}{2}i, -i.$$

b) If w be a cube root of unity, prove that:

$$(2 + \omega + \omega^2)^3 + (1 + \omega - \omega^2)^8 - (1 - 3\omega + \omega^2)^4 = 1$$

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➢ **Solution,**

LHS,

$$= (2 + \omega + \omega^2)^3 + (1 + \omega - \omega^2)^8 - (1 - 3\omega + \omega^2)^4$$

$$= (1 + 1 + \omega + \omega^2)^3 + (1 + \omega - \omega^2)^8 - (1 - 3\omega + \omega^2)^4$$

$$[\because \mathbf{1 + \omega + \omega^2 = 0}]$$

$$= (1 + 0)^3 + (-\omega^2 - \omega^2)^8 - (\omega - 3\omega)^4$$

$$= 1 + (-2\omega^2)^8 - (-4\omega)^4$$

$$= 1 + 256\omega^{16} - 256\omega^{14}$$

$$= 1 + 256(\omega^3)^5\,\omega - 256\,\omega^3.\omega$$

$$[\because \boldsymbol{\omega^3 = 1}]$$

$$= 1 + 256\,\omega - 256\,\omega$$

$$= 1$$

$\therefore$ RHS = LHS , Proved.

2. a) Find the area of the traingles determined by the Vectors $\overrightarrow{2\,i} - \vec{j} + \vec{k}$ and $3\vec{i} + 4\vec{j} - \vec{k}$

➢ **Solution**

$Let, \vec{a} = \overrightarrow{\mathbf{2\,i}} - \vec{\boldsymbol{j}} + \vec{\boldsymbol{k}}$ and $\vec{b} = \mathbf{3\vec{i} + 4\vec{j} - \vec{k}}$

Then, $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix}$

$$= (1 - 4)\vec{i} - (-2 - 3)\vec{j} + (8 + 3)\vec{k}$$

$$= -3\vec{i} + 5\vec{j} + 11\vec{k}$$

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$$|\vec{a} \times \vec{b}| = -3\vec{\imath} + 5\vec{\jmath} + 11\vec{k}$$

$$= \sqrt{(-3)^2 + (5)^2 + 11^2} = \sqrt{155}$$

$$Area\ of\ triangle = \frac{1}{2} \times |\vec{a} \times \vec{b}| = \frac{\sqrt{155}}{2}$$

$$\therefore The\ required\ area\ of\ triangle = \frac{\sqrt{155}}{2} sq.units$$

b) Find the equation of the plane through the points (4, 5, 1), (3, 9, 4) and (-4, 4, 4).

➢ **Solution,**

Equation of the plane through (4,5, 1) is

$$a(x-4) + b(y-5) + c(z-1) = 0 \quad \cdots\cdots\cdots (i)$$

Since,

Equation (i) Passes through (3,9,4) & $(-4,4,4)$

So,

$$a(3-4) + b(9-5) + c(4-1) = 0$$

$$or, \quad -a + 4b + 3c = 0 \qquad \cdots\cdots\cdots (ii)$$

$$a(-4-4) + b(4-5) + c(4-1) = 0$$

$$or, \quad -8a + b + 3c = 0 \qquad \cdots\cdots\cdots (iii)$$

Solving, (ii) & (iii) by cross multiplication method,

$$\frac{a}{12+3} = \frac{b}{-24+1} = \frac{c}{1+32}$$

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$$or, \quad \frac{a}{15} = \frac{b}{-21} = \frac{c}{33} = K \ \ (say)$$

$$a = 15K, \qquad b = -21K, \qquad c = 33K$$

Equation (i) becomes,

$$15K(x-4) - 21K(y-5) + 33K(Z-1) = 0$$

$$or, \ 5(x-4) - 7(y-5) + 11(z-1) = 0$$

$$\therefore \ 5x - 7y + 11z - 4 = 0$$

Which is the required Eq^n of plane.

3. a) Solve by using Cramer's rule or Row equivalent matrix:

$$x + 2y - 3z = 9$$
$$2x - y + 2z = -8$$
$$3x - y - 4z = 3$$

➢ **Solution,** *Row equivalent matrix method,*

The augmented matrix is

$$\begin{bmatrix} 1 & 2 & -3 & : 9 \\ 2 & -1 & 2 & : -8 \\ 3 & -1 & -4 & : 3 \end{bmatrix}$$

$$R_2 \to R_2 - 2R_2, \ \ R_3 \to R_3 - 2R_3 \qquad \begin{bmatrix} 1 & 2 & -3 & : 9 \\ 0 & -5 & 8 & : -26 \\ 0 & -7 & 5 & : -24 \end{bmatrix}$$

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$$R_2 \to -\frac{1}{5}R_2 \qquad \begin{bmatrix} 0 & 2 & -3 & : & 9 \\ 0 & 1 & \frac{-8}{5} & : & \frac{26}{5} \\ 0 & -7 & 5 & : & -24 \end{bmatrix}$$

$$R_1 \to R_1 - 2R_2, R_3 \to R_3 - 7R_2 \qquad \begin{bmatrix} 1 & 0 & \frac{1}{5} & : & \frac{-7}{5} \\ 0 & 1 & \frac{-8}{5} & : & \frac{26}{5} \\ 0 & 0 & \frac{-31}{5} & : & \frac{62}{5} \end{bmatrix}$$

$$R_3 \to -\frac{5}{31}R_3 \qquad \begin{bmatrix} 1 & 0 & \frac{1}{5} & : & \frac{-7}{5} \\ 0 & 1 & \frac{-8}{5} & : & \frac{26}{5} \\ 0 & 0 & -1 & : & -2 \end{bmatrix}$$

$$R_1 \to R_1 - \frac{1}{5}R_3,\ R_2 \to R_1 + \frac{8}{5} + R_3 \qquad \begin{bmatrix} 1 & 0 & 0 & : & -1 \\ 0 & 1 & 0 & : & 2 \\ 0 & 0 & 1 & : & -2 \end{bmatrix}$$

Hence,

$$\boldsymbol{x = -1,\ y = 2,\ z = -2\ .}$$

Or

$$x + 2y - 3z = 9$$
$$2x - y + 2z = -8$$
$$3x - y - 4z = 3$$

Solving by Cramer's rule :

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$$\text{Let } D = \begin{vmatrix} 1 & 2 & -3 \\ 2 & -1 & 2 \\ 3 & -1 & -4 \end{vmatrix} = 1(4+2) - 2(-8-6) + (-3)(-2+3)$$
$$= 6 + 28 - 3$$
$$= \mathbf{31}$$

$$D_1 = \begin{vmatrix} 1 & 2 & -3 \\ -8 & -1 & 2 \\ 3 & -1 & -4 \end{vmatrix} = 9(4+2) - 2(32-6) + (-3)(8+3)$$
$$= 54 - 52 - 33$$
$$= \mathbf{-31}$$

$$D_2 = \begin{vmatrix} 1 & 9 & -3 \\ 2 & -8 & 2 \\ 3 & 3 & -4 \end{vmatrix} = 1(32-6) - 9(-8-6) + (-3)(-6+24)$$
$$= 26 + 126 - 90$$
$$= \mathbf{62}$$

$$D_3 = \begin{vmatrix} 1 & 2 & 9 \\ 2 & -1 & -8 \\ 3 & -1 & 3 \end{vmatrix} = 1(-3-8) - 2(6+24) + 9(-2+3)$$
$$= -11 - 60 + 9$$
$$= \mathbf{-62}$$

Now,

$$x = \frac{D_1}{D} = \frac{-31}{31} = \mathbf{-1}, \quad y = \frac{D_2}{D} = \frac{62}{31} = \mathbf{2}, \quad z = \frac{D_3}{D} = \frac{-62}{31} = \mathbf{-2}$$

Hence,

$$\boldsymbol{x = -1,\ y = 2,\ z = -2\ .}$$

b) Prove that:

$$\begin{vmatrix} b+c & a & b \\ c+a & c & d \\ a+b & b & c \end{vmatrix} = (a+b+c)(a-c)^2$$

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➢ **LHS** $= \begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$

$= \begin{vmatrix} 2(a+b+c) & a+b+c & a+b+c \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$ $(R_1 \to R_1 + R_2 + R_3)$

$= (a+b+c) \begin{vmatrix} 2 & 1 & 1 \\ c+a & c & a \\ a+b & b & c \end{vmatrix}$ (Taking a+b+c common from R_1)

$= (a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ a & c & a \\ a & b & c \end{vmatrix}$ $(C_1 \to C_1 - C_2)$

$= (a+b+c) \begin{vmatrix} 1 & 0 & 0 \\ a & c-a & 0 \\ a & b-a & c-a \end{vmatrix}$ $(C_2 \to C_2 - C_1, C_3 \to C_3 - C_1)$

$= (a+b+c) \begin{vmatrix} c-a & 0 \\ b-a & c-a \end{vmatrix}$

$= (a+b+c)\,[\,(c-a)^2 - 0\,]$

$= \boldsymbol{(a+b+c)\,(c-a)^2}$

$\therefore$ **RHS** $=$ **LHS** Proved.

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4) Sand is pouring from a pipe at the rate of 18 cm^3/sec. The falling sand forms a cone on the ground in such a way that the height of the cone is one-sixth of the radius of the base. How fast is height of the cone increasing when its height is 3cm?

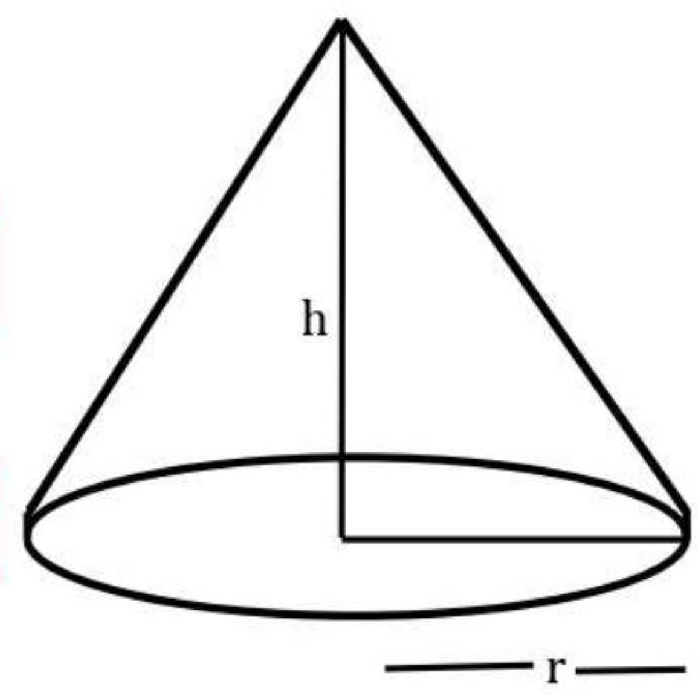
➢Solution,

Let r be the radius of the base, h be the height and v be the volume of the sand cone at time t.

By question,

$$h = \frac{1}{6} r$$

$$or, \quad r = 6h$$



$$\frac{dv}{dt} = \frac{18cm^3}{sec}$$

$$h = 3cm, \quad \frac{dh}{dt} = ?$$

We have, $v = \frac{1}{2}\pi r^2 h = \frac{1}{3}\pi(6h)^2 h = 12\pi h^3$

$$or, \quad \frac{dv}{dt} = \frac{d}{dt}12\pi h^3 = 12\pi\frac{dh^3}{dt} = 36\pi h^2\frac{dh}{dt}$$

$$or, \quad 18 = 36\pi \times (3)^2 \times \frac{dh}{dt}$$

$$\therefore \quad \boldsymbol{\frac{dh}{dt} = \frac{1}{18\pi}\ cm/sec.}$$

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5) A man who has got 144 metres of fencing material Wishes to enclose a rectangular garden. Find the maximum area he can enclose.

➢Solution,

Let x be the length and y be the breath of the garden. If P be perimeter and A be the area of the garden, then

$$P = 2(x + y)$$

$$or, \quad 144 = 2(x + y)$$

$$or, \quad x + y = 72 \quad \cdots\cdots\cdots (i)$$

$$and \quad A = xy \quad \cdots\cdots\cdots (ii)$$

Now, eliminating y between (i) and (ii), we have

$$A = x(72 - x)$$

$$\frac{dA}{dx} = 72 - 2x$$

$$and, \quad \frac{d^2A}{dx^2} = -2$$

For maximum area A,

$$\frac{dA}{dx} = 0$$

$$or, \quad 72 - 2x = 0$$

$$\therefore \quad x = 36$$

When x $= 36$ $y = 72 - 36 = 36$

$At\ x = 36$ $\frac{d^2A}{dx^2} = -2$ which is negative.

Hence, the area is maximum when $x = 36$ and $y = 36$ $and\ the$ $maximum\ area\ A = 36 \times 36 = 1296\ sq.mtrs$

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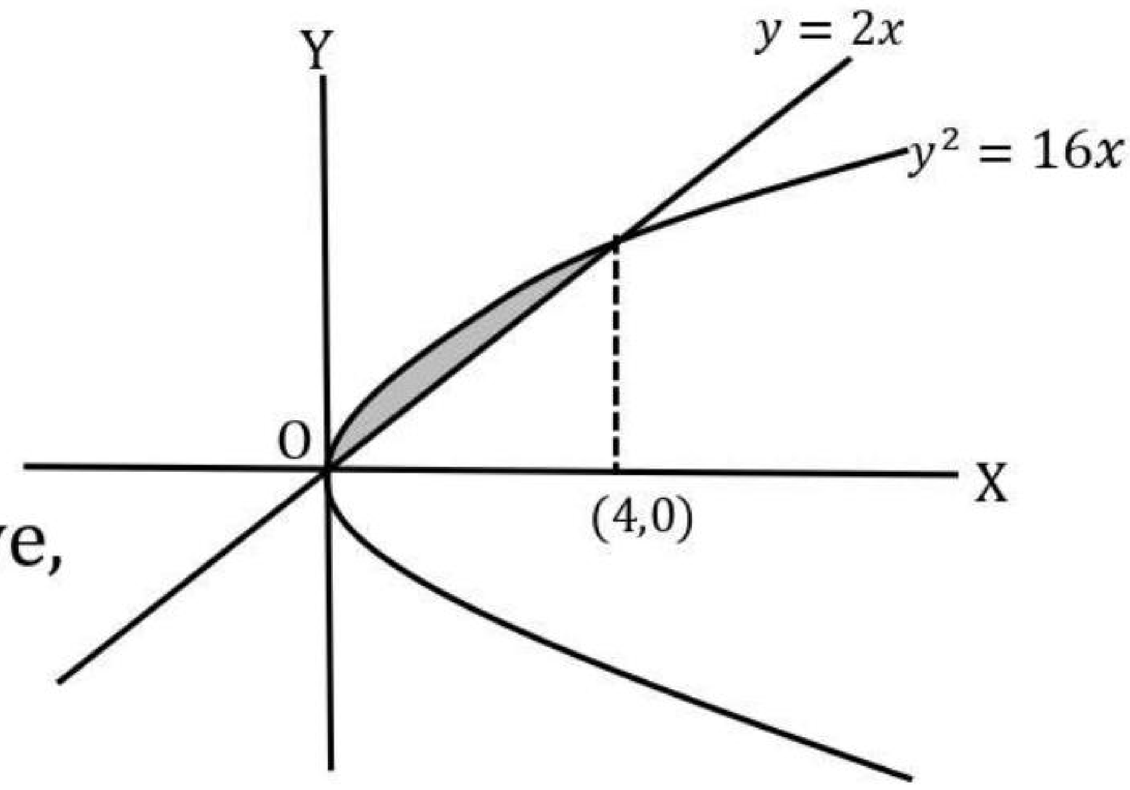
6) Find the Area bounded by the curve $y^2 = 16x$ and the line $y = 2x$.



➢ **Solution,**

Given curve and line are $y^2 = 16x$ and $y = 2x$ respectively.

Eliminating y between them, we have,

$$(2x)^2 = 16x$$

$$or, \quad x^2 = 4x$$

$$or, \quad x(x - 4) = 0$$

Either $x = 0$ or $x = 4$

$\therefore$ Ordinates are $x = 0$, $x = 4$.

Hence the required area, $A = \int_0^4 (y_1 - y_2)dx$, $where\ y_1 = 4\sqrt{x}$ $and\ y_2 = 2x$.

$$= \int_0^4 (y_1 - y_2)dx$$

$$= \int_0^4 \left(4\sqrt{x} - 2x\right)dx$$

$$= \left[\frac{8}{3}x^{\frac{3}{2}} - x^2\right]_0^4$$

$$= \frac{8}{3}4^{\frac{3}{2}} - 4^2$$

$$= \frac{64}{3} - 16$$

$$\therefore \boldsymbol{A = \frac{16}{3}\ sq.units}$$

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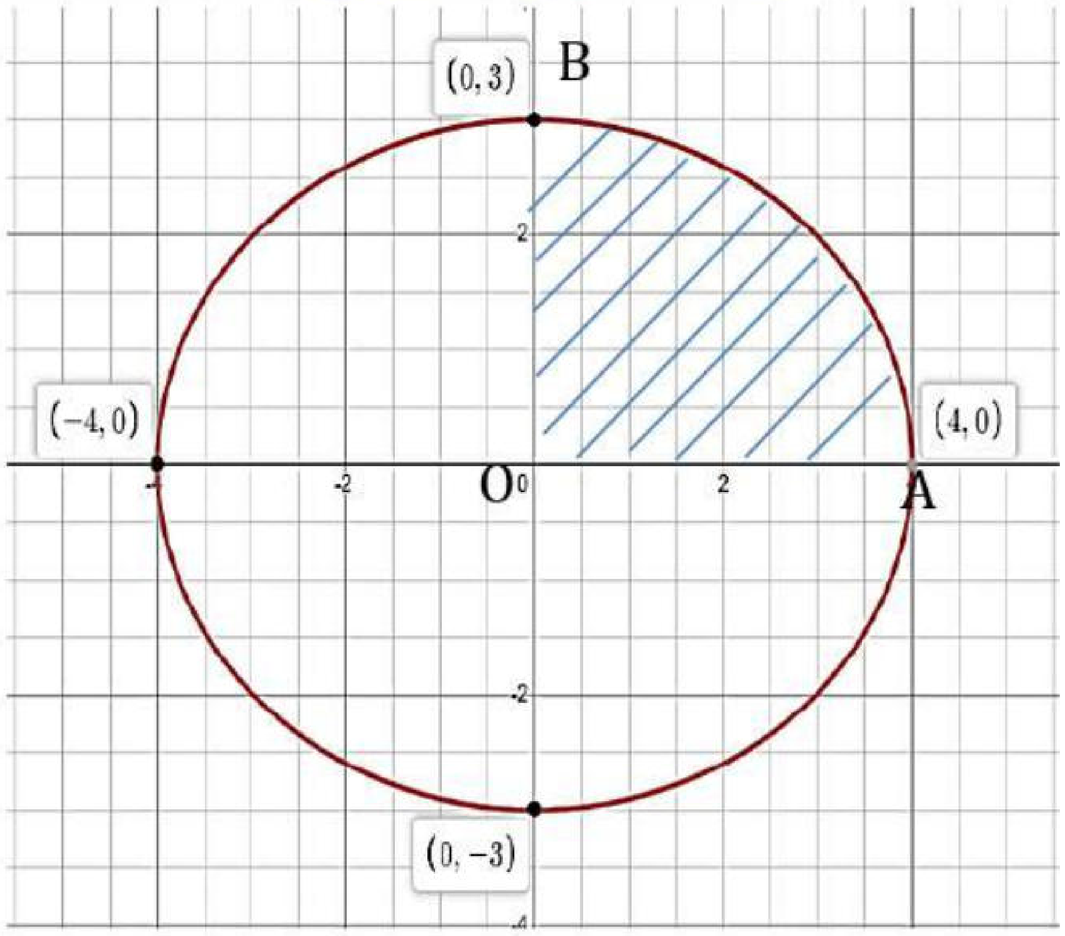
OR) Find the area bounded by the curve $\frac{x^2}{16}+\frac{y^2}{9}=1$



➢ **Solution,**

Given Equation,

$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

$$y^2=9(1-\frac{x^2}{16})$$

$$\Longrightarrow y=\frac{3}{4}\sqrt{16-x^2}$$

Its center is (0,0) and is symmetrical about both $x-axis$ and $y-axis$.

Total area Bounded by Curve = 4 × Area of OAB which lies between the ordinates $x=0, x=4$.

Hence, Total area of the Curve, A $=4\times$ *area of* OAB

$$A=4\int_0^4 y\,dx$$

$$=4\int_0^4 \frac{3}{4}\sqrt{16-x^2}dx$$

$$let\ x=4\sin\theta\ ,\quad dx=4\cos\theta\, d\,\theta$$

Limits, $x=0\rightarrow\theta=0$ & $x=4\rightarrow\theta=\frac{\pi}{2}$

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$$A = 4 \times \frac{3}{4}\int_0^{\frac{\pi}{2}} \sqrt{16 - 16\, sin^2\theta}\,.\,4\cos\theta\, d\theta$$

$$= 3\int_0^{\frac{\pi}{2}} 4\cos\theta\ .\ 4\cos\theta\, d\theta$$

$$= 3 \times 16 \int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

$$= 3 \times 16 \times \int_0^{\frac{\pi}{2}} \frac{(1+\cos 2\theta)d\theta}{2}$$

$$= \frac{3 \times 16}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 3 \times 8\left(\frac{\pi}{2} + 0\right)$$

$$= 24 \times \frac{\pi}{2}$$

$$= \mathbf{12\pi}\ \ \boldsymbol{sq.units}$$

∴ Area Bounded By the Curve is $12\pi\ \ sq.unit$

7) Prove that the line joining the points $(1, 2, 3)$ and $(-1, -2, -3)$ is perpendicular to the line joining the pints $(-2, 1, 5)$ and $(3, 3, 2)$.

- **Solution,**

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The direction ratios of the line joining the points (1, 2, 3) and (-1, -2, -3) are -2, -4, -6.

The direction ratios of the line joining the points (-2, 1, 5) and (3, 3, 2) are $3 + 2, 1 - 3, 2 - 5,$

$i.e.\ 5, \quad +2, \quad -3$

Now,

$$-2(5) - 4(+2) - 6(-3) = 0$$

or, $-10 - 8 + 18 = 0$

So, the lines joining the given points are perpendicular to each other.

8) Maximize and minimize F $= 9x + 7y$ subject to $x + 2y \leq 7$, $x - y \leq 4$, $x \geq 0$, $y \geq 0$.

- **Solution,**

$\boldsymbol{When\ x = 0, y = 0}$

$0 + 2.0 \leq 7 \Rightarrow 0 \leq 7$ (true) so, it is directed towards origin.

The boundary lines of the given inequalities are

$$\boldsymbol{x + 2y = 7} \quad \cdots\cdots\cdots\cdots (\boldsymbol{i})$$
$$\boldsymbol{x - y = 4} \quad \cdots\cdots\cdots\cdots (\boldsymbol{ii})$$

From equation (i)

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$x + 2y = 7$	
x	y
0	$\frac{7}{2}$
7	0

So, $C\left(0, \frac{7}{2}\right)$ & $(7,0)$ are passes through equation (i)

From equation (ii),

When, $x = 0, y = 0$

$0 \leq 4$ (true) so, it is directed towards origin.

$x - y = 4$	
x	y
0	4
4	0

So,$(0,4)$ & $A(4,0)$ are passes through equation (ii)

When, $x = 0, y = 0$

$0 \leq 6$ (true) so, it is directed towards origin.

Also, From graph the vertices B are calculated by solving equation

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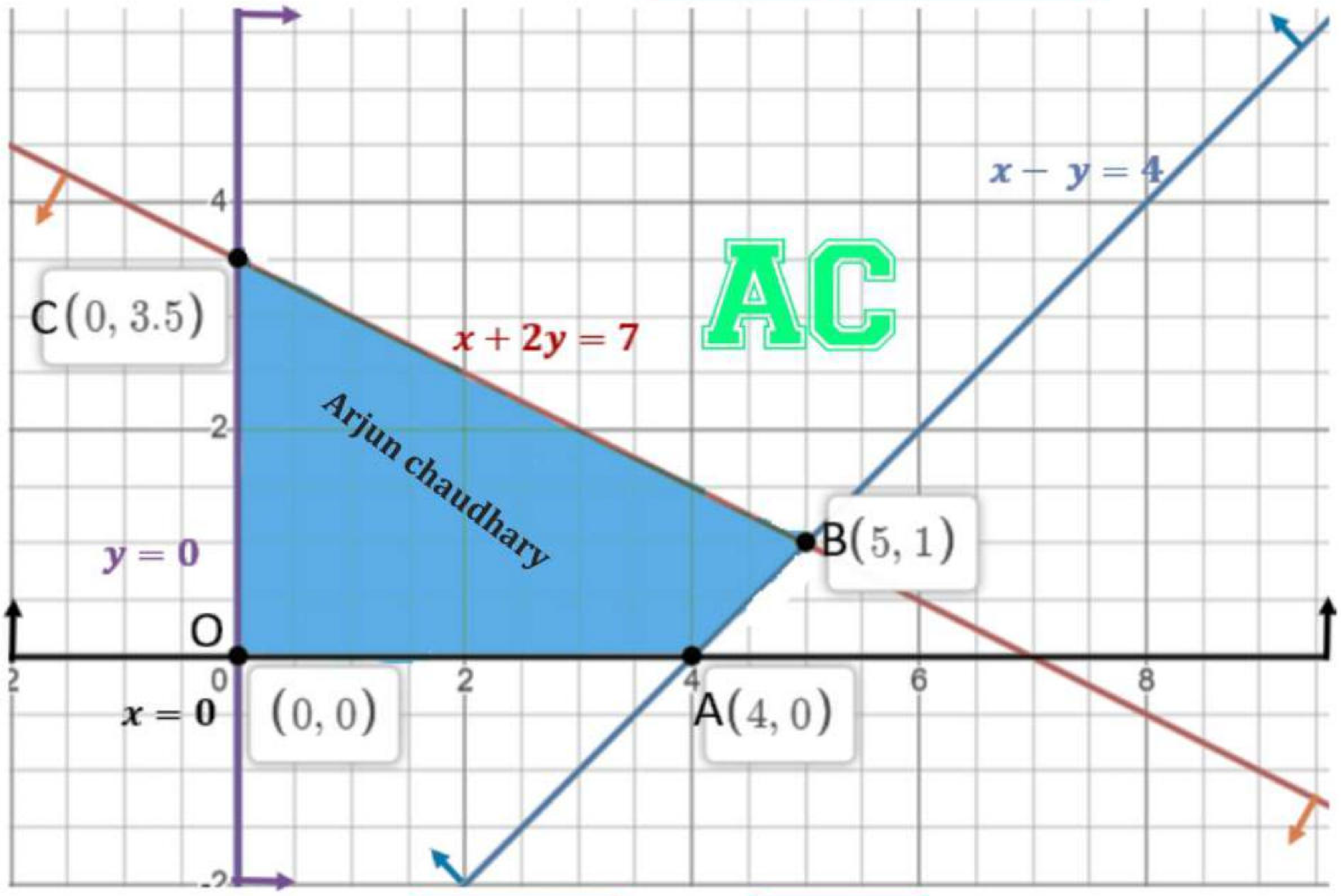

For Vertices **B**,

Solving equation (i) and (ii)

We get, B (5,1)

Hence, The Vertices of ABCD are O(0,0) , A(4,0) , B(5,1) and C(0,3.5).

Vertices	$F = 9x + 7y$
O(0,0)	$9 \times 0 + 7 \times 0 =$ **0 (MIN)**
A(4,0)	$9 \times 4 + 7 \times 0 = 36$
B(5,1)	$9 \times 5 + 7 \times 1 =$ **52 (MAX)**
C(0, 3.5)	$9 \times 0 + 7 \times 3.5 = 24.5$

Hence, Maximum Value = 52 at C(5,1)

Minimum Value = 0 at O(0,0)

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9) A committee of 3 is to be formed from 6 men and 4 women. What is the probability that all three are women?

➢ **Solution,**

The selection of member in the committee can made as follows.

men(6)	women (4)	Selection of 3
0	3	$C(6,0) \times C(4,3)$

The Probability that all three are women.

$$= C(6,0) \times C(4,3)$$

$$= \frac{6!}{6!\,0!} \times \frac{4!}{1!\,3!}$$

$$= 1 \times 4$$

$$= \mathbf{4}$$

10) A company that produces 10% of its products are defective among such 6 products find the probability that one is defective.

➢ **Solution**

$$P = \text{Prob. of getting defective product} = \frac{10}{100} = \frac{\mathbf{1}}{\mathbf{10}}$$

$$q = \text{Prob. of getting tail} = 1 - p = 1 - \frac{1}{10} = \frac{\mathbf{9}}{\mathbf{10}}$$

$$n = 6$$

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we have,

$$P(r) = \text{Prob. of getting } \boldsymbol{r} \text{ defective product in } \boldsymbol{n} \text{ products}$$

$$P(r) = C(n, r)\, p^r q^{n-r} \;\cdots\cdots\cdots\cdots (\boldsymbol{i})$$

To find : probability that one is defective, $r = 1$

From $(\boldsymbol{i})$

$$P(r = 1) = C(6,1)\, p^1 q^{6-1}$$

$$P(r = 1) = 6 \times \left(\frac{1}{10}\right)^1 . \left(\frac{9}{10}\right)^5$$

$$P(r = 1) = 0.3543 = 35.43\%$$

$\therefore$ **Probability that one is defective** $= \mathbf{35.43\%}$

AC

11) Calculate the quartile deviation (Q.D.) and its coefficient from the following data :

Class	10-15	15-20	20-25	25-30	30-35	35-40	40-45
Frequency	2	4	6	7	3	1	5

➢ **Solution**

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Class	Frequency (F)	C.F
10-15	2	2
15-20	4	6
20-25	6	12
25-30	7	19
30-35	3	22
35-40	1	23
45-45	5	28

$$\mathbf{N = 28}$$

For Q_1:

$$Q_1 = \left(\frac{N}{4}\right)^{th} \text{item} = \frac{28}{4} = \boldsymbol{7^{th} item}$$

C.f is just greater than 7 and is 12. So, it lies between $20 - 25$.

$$\mathbf{L = 20, F = 6, C.f = 6, i = 5}$$

$$Q_1 = L + \frac{\frac{N}{4} - c.f}{f} \times i$$

$$= 20 + \frac{7 - 6}{6} \times 5$$

$$= 20 + 0.83$$

$$\mathbf{Q_1 = 20.83}$$

For Q_3:

$$Q_3 = 3\left(\frac{N}{4}\right)^{th} \text{item} = 3 \times 7 = \boldsymbol{21^{th} item}$$

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C.f is just greater than 21 and is 22. So, it lies between 30-35.

$$\mathbf{L = 30,\ f = 3,\ C.f\ = 19,\ i = 5}$$

$$Q_3 = L + \frac{\frac{3N}{4} - c.f}{f} \times i$$

$$= 30 + \frac{21 - 19}{3} 5$$

$$= 30 + 3.33$$

$$\mathbf{Q_3 = 33.33}$$

$$\textbf{Coefficient of Q.D} = \frac{\mathbf{Q_3 - Q_1}}{\mathbf{Q_3 + Q_1}}$$

$$= \frac{33.33 - 20.83}{33.33 + 20.836}$$

$$= \frac{12.47}{54.16}$$

$$\therefore \textbf{Coefficient of Q.D} = \mathbf{0.230}$$

12) Find the regression equation of y on x from the following data Also, estimate the value of y when $x = 5$.

X	2	4	6	8	10	12
Y	5	6	13	16	23	24

➢ **Solution,**

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x	y	x^2	xy
2	5	4	10
4	6	16	24
6	13	36	78
8	16	64	128
10	23	100	230
12	24	144	288
$\sum x = 42$	$\sum y = 87$	$\sum x^2 = 364$	$\sum xy = 758$

Here, $n = 6$

$$\bar{x} = \frac{\sum x}{n} = \frac{42}{6} = \mathbf{7}, \bar{y} = \frac{\sum y}{n} = \frac{87}{6} = \mathbf{14.5}$$

$$\boldsymbol{b_{yx}} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2} = \frac{6 \times 758 - 42 \times 87}{6 \times 364 - (42)^2} = \frac{894}{420} = \mathbf{2.128}$$

Hence, the regression y on x is ,

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$\text{or, } y - 12.83 = 2.128\,(x - 7)$$

$$\text{or, } y - 12.83 = 2.128\,x - 11.9$$

Therefore, $y = 2.128\,x + 0.93$

$\therefore$ The regression Equation y on x is , $y = 2.128\,x + 0.93$

When $y = 5$ then, $y = 2.128 \times 5 + 0.93 = \mathbf{11.57}$

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13) Show that the vectors: $\vec{i}+\vec{j}+\vec{k}$, $2\vec{i}+3\vec{j}-\vec{k}$, $-\vec{i}-2\vec{j}+2\vec{k}$ are co-planar.

➢ **Solution**

$$\vec{r_1} = \vec{i}+\vec{j}+\vec{k}$$

$$\vec{r_2} = 2\vec{i}+3\vec{j}-\vec{k}$$

$$\vec{r_3} = -\vec{i}-2\vec{j}+2\vec{k}$$

To show: $\vec{r_1}, \vec{r_2}$ *and* $\vec{r_3}$ **are coplanar**

➢ If three vector are coplanar, one can expressed as the sum of the scalar multiples of the other two.

$$\vec{r_3} = x\vec{r_1} + y\vec{r_2} \qquad where, x\ and\ y\ are\ scalars$$

$$or, \qquad -\vec{i}-2\vec{j}+2\vec{k} = x(\vec{i}+\vec{j}+\vec{k}) + y(2\vec{i}+3\vec{j}-\vec{k})$$

$$-\vec{i}-2\vec{j}+2\vec{k} = (x+2y)\vec{i} + (x+3y)\vec{j} + (x-y)\vec{k}$$

Equating Coefficient of likes vectors,

$$(x+2y) = -1 \qquad \cdots\cdots\cdots\cdots (i)$$

$$x+3y = -2 \qquad \cdots\cdots\cdots\cdots (ii)$$

$$x-y = 2 \qquad \cdots\cdots\cdots\cdots (iii)$$

Solving eqn (i) *and* (ii) weget, $\boldsymbol{x = 1\ and\ y = -1}$

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Put ,$x = 1$ *and* $y = -1$ in eqn (iii)

$1 \times 1 - 1 \times -1 = 2$

$2 = 2$, satisfied.

Hence, Three vector are coplanar and its *can expresssed as*

$$\overrightarrow{r_3} = \overrightarrow{r_1} - \overrightarrow{r_2}$$

⇐ The End ⇒

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1. a) If the equation $x^2 + 2\,(k+2)x + 9k = 0$ has equal roots. Find k.

➢ **Solution**

By, question roots are equal so,

Discriminant is Zero $i.e,\ b^2 - 4ac = 0$

$$\{2(k+2)\}^2 - 4.1.9k = 0$$

$$or,\quad 4(k+2)^2 - 4.9k = 0$$

$$or,\quad (k+2)^2 - 9k = 0$$

$$or,\quad k^2 + 4k + 4 - 9k = 0$$

$$or,\quad k^2 - 5k + 4 = 0$$

Solving we get, $k = \frac{5 \pm \sqrt{25-16}}{2} = \frac{5 \pm 3}{2}$

Hence, the value of $k = 4\ or, 1$

b) Find the conjugate of the complex number $\frac{3+4i}{3-4i}$

➢ **Solution**

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Given,

Let $Z = \frac{3+4i}{3-4i}$ **to find** $\bar{Z} = ?$

$$Z = \frac{3+4i}{3-4i} \times \frac{3+4i}{3+4i}$$

$$Z = \frac{(3+4i)^2}{9-16i^2} = \frac{9+24i+16i^2}{9-16i^2}$$

$$= \frac{9+24i-16}{9+16} \qquad [\because\ i^2 = -1\ ]$$

$$= \frac{24i-7}{25}$$

$$Z = \frac{24i}{25} - \frac{7}{25}$$

$$= -\frac{7}{25} + \frac{24}{25}i$$

$$\bar{Z} = -\frac{7}{25} + \frac{24}{25}i$$

$$= -\frac{7}{25} - \frac{24}{25}$$

$if\ Z = a + bi$

$\because \bar{Z} = a + bi = a - bi$

2. a) If $A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$ find the matrix of $(AB)^T$

- **Solution**

Given,

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$$A = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$$

Now,

$$AB = \begin{pmatrix} 1 & -2 & 3 \\ -1 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ 3 & 1 \\ 1 & 2 \end{pmatrix}$$

$$AB = \begin{pmatrix} 2 & -1 \\ 0 & 3 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \times 3 - 1 \times -2 & 2 \times 1 - 1 \times 4 \\ 0 \times 3 + 3 \times -2 & 0 \times 1 + 3 \times 4 \end{pmatrix}$$

$$\mathbf{AB} = \begin{pmatrix} 8 & -2 \\ -6 & 12 \end{pmatrix}$$

To find,

$$(\mathbf{AB})^{\boldsymbol{T}}$$

$$= \begin{pmatrix} 8 & -2 \\ -6 & 12 \end{pmatrix}^{T}$$

$$= \begin{pmatrix} 8 & -6 \\ -2 & 12 \end{pmatrix}$$

$$\therefore (\mathbf{AB})^{\boldsymbol{T}} = \begin{pmatrix} \mathbf{8} & \mathbf{-6} \\ \mathbf{-2} & \mathbf{12} \end{pmatrix}$$

b) If w be the cube root of unity prove that:

$$(\mathbf{1} - \boldsymbol{\omega} + \boldsymbol{\omega}^{\mathbf{2}})(\mathbf{1} + \boldsymbol{\omega} - \boldsymbol{\omega}^{\mathbf{2}}) = \mathbf{4}$$

- **Solution**

Given,

$$\text{LHS} = (1 - \omega + \omega^2)(1 + \omega - \omega^2)$$

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$= 1 + \omega - \omega^2 - \omega - \omega^2 + \omega^3 + \omega^2 + \omega^3 - \omega^4$

$= 1 + \omega - \omega^2 - \omega - \omega^2 + \omega^3 + \omega^2 + \omega^3 - \omega^3.\omega$

$[\because \omega^2 \times \omega = \omega^3 = 1]$

$= 1 + \omega - \omega^2 - \omega - \omega^2 + 1 + \omega^2 + 1 - \omega$

$= 3 - \omega^2 - \omega$

$= 3 + 1 - 1 - \omega^2 - \omega$

$= 4 - (1 + \omega + \omega^2)$ $[\because 1 + \omega^2 + \omega = 0]$

$= 4 - 0$

$= \mathbf{4}$ Proved.

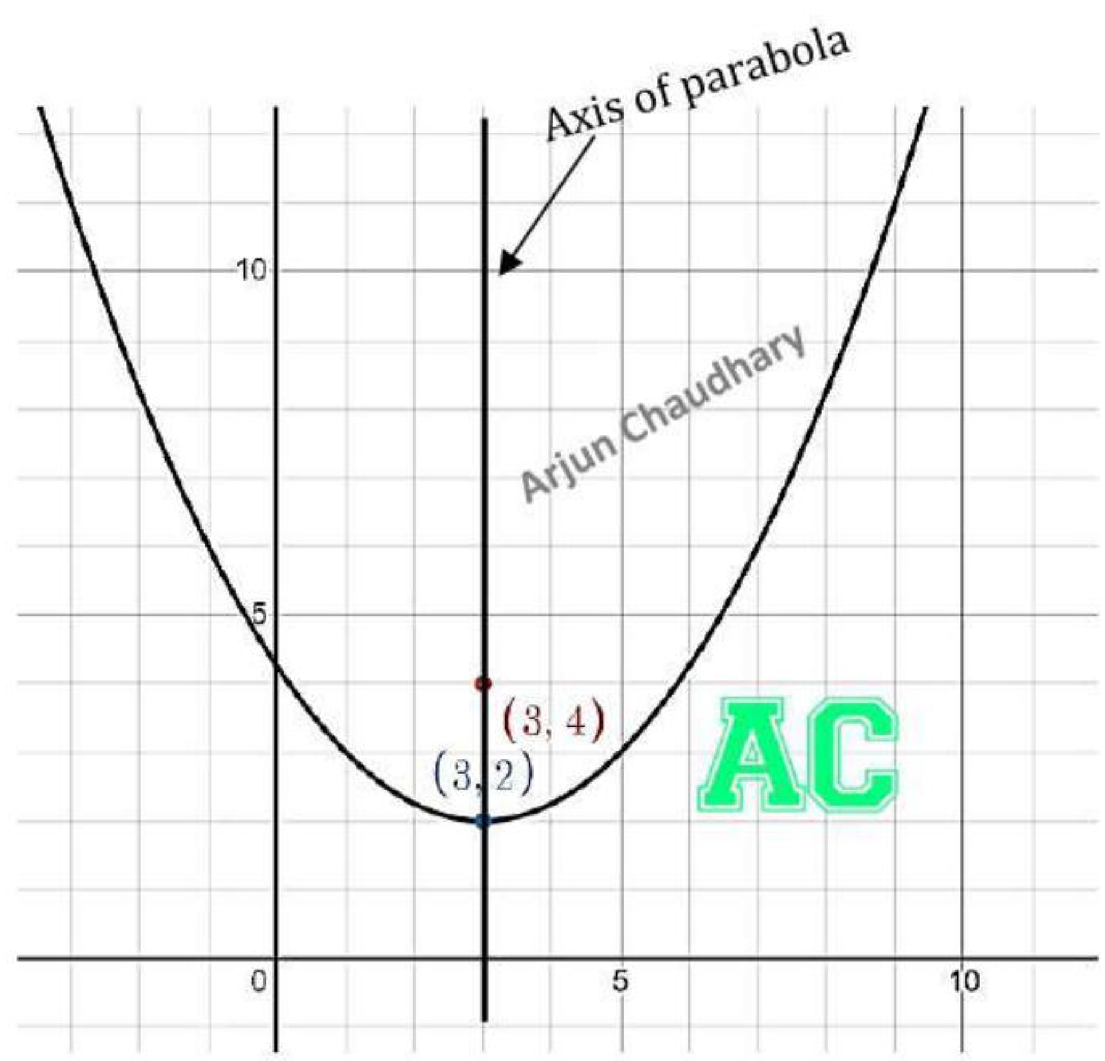
3. a) Find the equation of parabola vertex (3,2) and focus (3, 4).

➢ **Solution**

Since, X-coordinates and focus are equal so, axis of parabola is parallel to y-axis.

Here, Vertex $(h, k) = (3, 2)$ and focus $(h, k + b) = (3, 4)$

$\therefore$ $\mathbf{b = 2, h = 3}$ **and** $\mathbf{k = 2}$



Required equation of parabola is

$$(x - h)^2 = 4b(y - k)$$

$$or, (x - 3)^2 = 4 \times 2(y - 2)$$

$$or, x^2 - 6x + 9 = 8(y - 2)$$

$$\boldsymbol{or,\ x^2 - 6x - 8y + 25 = 0}$$

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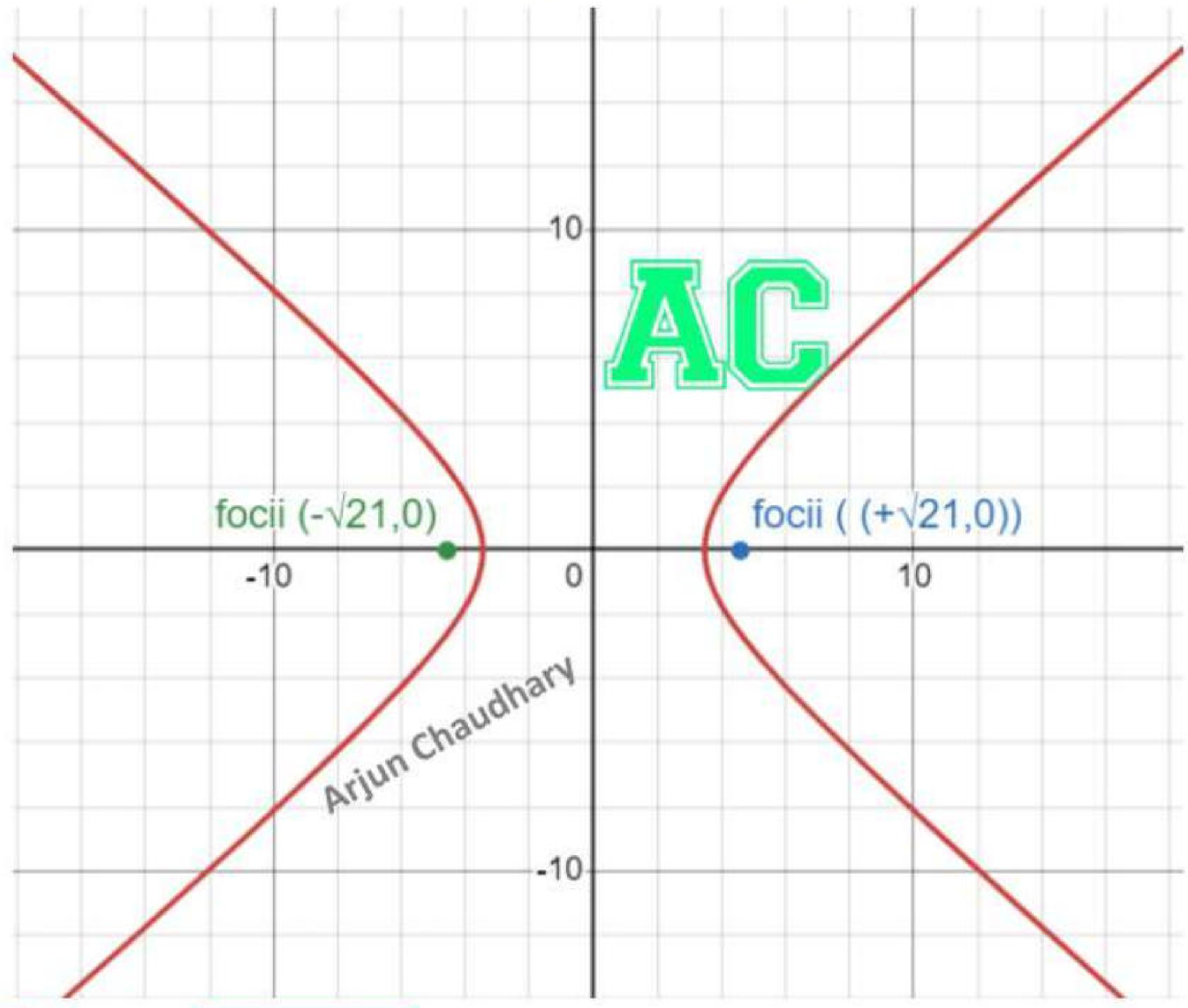
b) Find the foci of the hyperbola $3x^2 - 4y^2 = 36$.



➢ **Solution**

Given,

$$3x^2 - 4y^2 = 36$$

$$\frac{3x^2 - 4y^2}{36} = 1$$

$$\frac{x^2}{12} - \frac{y^2}{9} = 1$$

Comparing with $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$$a^2 = 12, \quad b^2 = 9$$

$$\Rightarrow a = 2\sqrt{3}, \qquad b = 3$$

Center of ellipse (0,0)

$$\text{Vertices} = (\pm\, a, 0) = \left(\pm\, 2\sqrt{3}, 0\right)$$

$$\text{Eccentricity}\,, e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{12}} = \sqrt{\frac{\mathbf{21}}{\mathbf{12}}}$$

$$\text{For focii} \quad S\,(\pm\, ae, 0) = \left(\pm 2\sqrt{3} \times \sqrt{\frac{21}{12}}, 0\right) = (\pm\sqrt{\mathbf{21}}, \mathbf{0})$$

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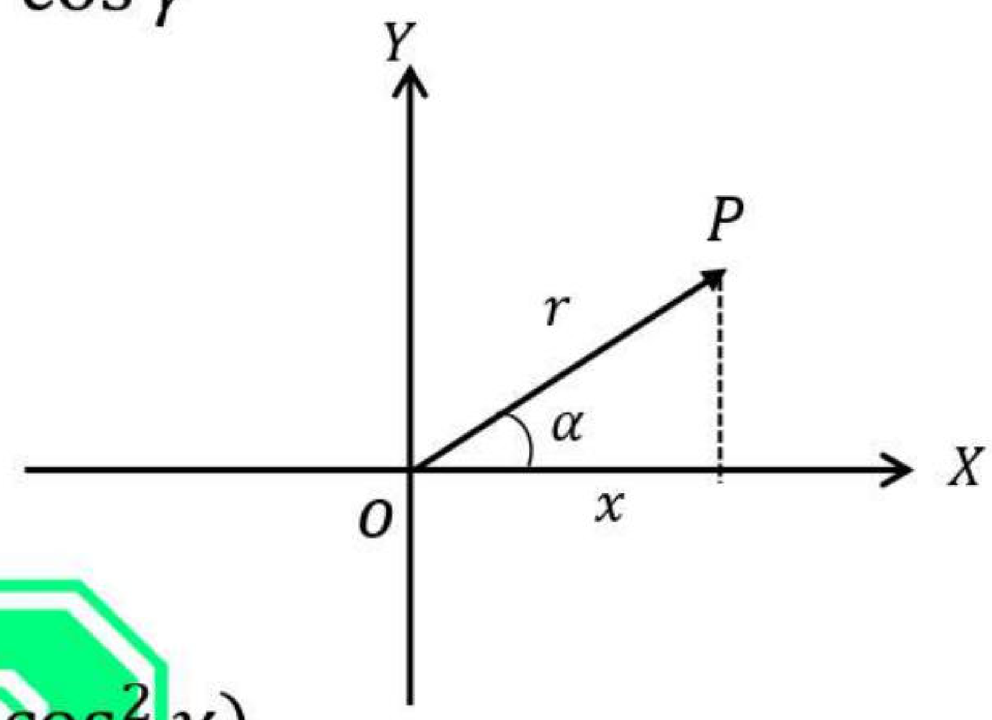
4. a) Find the direction cosines of a line which are equally Inclined to the axes.

➢ **Solution**

Let l, m, n are direction cosines in the x, y, z axes and α, β, γ are the angles made with d. c along respective axes $i.e,$

$$l = \cos\alpha\,, \mathrm{m} = \cos\beta \; and \; n = \cos\gamma$$



$\cos\alpha = \frac{x}{r}$ or, $x = r.\cos\alpha$

Similarly, $y = r.\cos\beta \; and \, z = r.\cos\gamma$

Squaring both sides, and adding

$$x^2 + y^2 + z^2 = r^2\,(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$$

As, $OP = r = \sqrt{x^2 + y^2 + z^2} \Longrightarrow r^2 = r^2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma)$

$$\boldsymbol{\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad or, [\, l^2 + m^2 + n^2 = 1\,]}$$

By question, equally inclined to the axes $i.e.,\;\; \alpha = \beta = \gamma$

$$or, \qquad \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$

$$or, \qquad 3\cos^2\alpha = 1$$

$$or, \qquad \cos^2\alpha = \frac{1}{3}$$

$$\Longrightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$$

Hence, $\cos\alpha = \cos\beta = \cos\gamma = \pm\frac{1}{\sqrt{3}}$, $\quad l = m = n = \pm\frac{1}{\sqrt{3}}$

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Alternative solution:

Let l, m, n *be d.c of lines making* α, β, γ *with postives axes respectively.*

We have relation , $\boldsymbol{l^2 + m^2 + n^2 = 1} \cdots\cdots\cdots\cdots (\boldsymbol{i})$

Also we have, $l = \cos\alpha$, $\mathrm{m} = \cos\beta$ *and* $n = \cos\gamma$

Equation (i) becomes,

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

By question, equally inclined to the axes $i.e.$, $\alpha = \beta = \gamma$

$$or, \quad \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$

$$or, \quad 3\cos^2\alpha = 1$$

$$or, \quad \cos^2\alpha = \frac{1}{3}$$

$$\Rightarrow \cos\alpha = \pm\frac{\mathbf{1}}{\sqrt{\mathbf{3}}}$$

Hence, $\cos\alpha = \cos\beta = \cos\gamma = \pm\frac{1}{\sqrt{3}}$

$$\boldsymbol{l = m = n = \pm\frac{1}{\sqrt{3}}}$$

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b) Find the distance between the parallel planes $3x + 2y - 6z = 1$ and $6x + 4y - 12z + 9 = 0$.

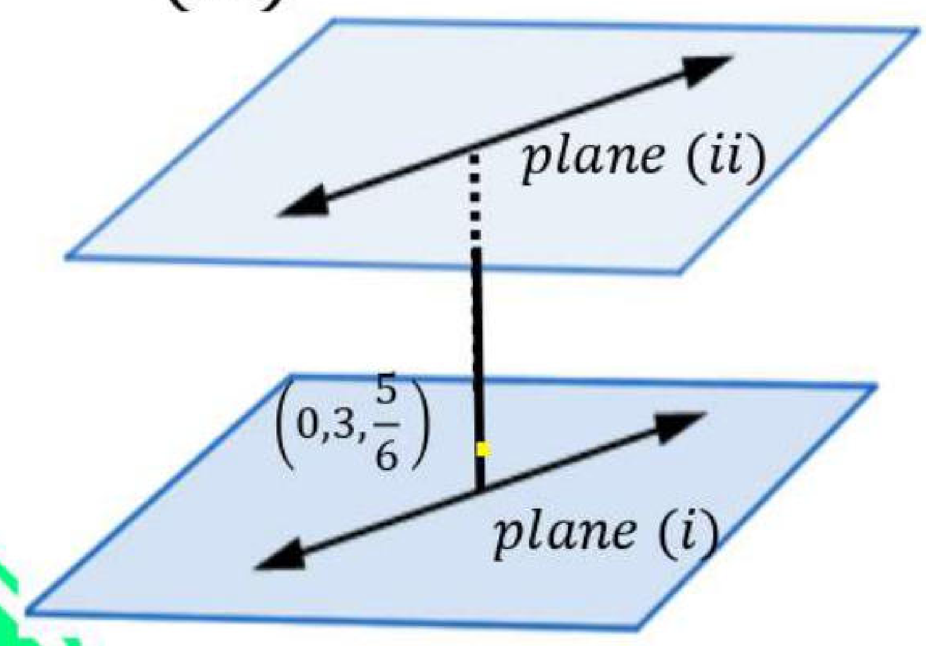
➢ **Solution**

Given lines,

$$3x + 2y - 6z - 1 = 0 \cdots\cdots\cdots\cdots (i)$$

$$6x + 4y - 12z + 9 = 0 \cdots\cdots\cdots\cdots (ii)$$

When $x = 0, y = 3$ in (i) Then,



$$3 \times 0 + 2 \times 3 - 6z - 1 = 0$$

$$z = \frac{5}{6}$$

$\therefore$ Point $\left(0,3\ ,\frac{5}{6}\right)$ lies on the plane (i)

Let 'P' be length of perpendicular drawn from point $\left(0,3\ ,\frac{5}{6}\right)$ lies on the plane $6x + 4y - 12z + 9 = 0$ is

$$P = \left|\frac{6 \times 0 + 4 \times 3 - 12 \times \frac{5}{6} + 9}{\sqrt{6^2 + 4^2 + (-12)^2}}\right|$$

$$P = \left|\frac{11}{\sqrt{196}}\right|$$

$$P = \left|\frac{11}{14}\right|$$

$$\therefore \mathbf{P = \frac{11}{14} = 0.785\ unit.}$$

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5. a) If $\vec{a} = 2\vec{i} + 3\vec{j}$ and $\vec{b} = -\vec{i} + \vec{j}$ find the unit vector along $2\vec{a} - 3\vec{b}$.

➢ **Solution**

Given,

$$2\vec{a} - 3\vec{b} = 2(2\vec{i} + 3\vec{j}) - 3(-\vec{i} + \vec{j})$$
$$= 4\vec{i} + 6\vec{j} + 3\vec{i} - 3\vec{j}$$
$$or,\ 2\vec{a} - 3\vec{b} = 7\vec{i} + 3\vec{j}$$

$$|2\vec{a} - 3\vec{b}| = \sqrt{7^2 + 3^2} = \sqrt{58}$$

Unit vector along $\widehat{2\vec{a} - 3\vec{b}} = \frac{7\vec{i}+3\vec{j}}{\sqrt{58}} = \frac{7}{\sqrt{58}}\vec{i} + \frac{3}{\sqrt{58}}\vec{j}$

b) If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$.Prove that a is perpendicular to $\vec{b}$.

➢ **Solution**

Given, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$

Squaring both side,

$$|\vec{a} + \vec{b}|^2 = |\vec{a} - \vec{b}|^2$$
$$(\vec{a} + \vec{b})^2 = (\vec{a} - \vec{b})^2$$
$$(\vec{a})^2 + 2\vec{a}\vec{b} + (\vec{b})^2 = (\vec{a})^2 - 2\vec{a}\vec{b} + (\vec{b})^2$$
$$a^2 + 2\vec{a}.\vec{b} + b^2 = a^2 - 2\vec{a}.\vec{b} + b^2$$
$$\Rightarrow 2\vec{a}.\vec{b} = -2\vec{a}\vec{b}$$
$$\Rightarrow 4\vec{a}.\vec{b} = 0$$
$$\vec{a}.\vec{b} = 0$$

Hence , $\vec{a}$ *is perpendicular to* $\vec{b}$

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6. a) Find the area of the triangle whose two sides are determined by the vectors $\overrightarrow{2\,i} - \vec{j} + \vec{k}$ and $3\vec{i} + 4\vec{j} - \vec{k}$.

➢ Solution

Given,

$$Let, \vec{a} = \overrightarrow{2\,i} - \vec{j} + \vec{k} \; and \; \vec{b} = 3\vec{i} + 4\vec{j} - \vec{k}$$

Then, $\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix}$

$$= (1-4)\vec{i} - (-2-3)\vec{j} + (8+3)\vec{k}$$

$$= -3\vec{i} + 5\vec{j} + 11\vec{k}$$

$$|\vec{a} \times \vec{b}| = -3\vec{i} + 5\vec{j} + 11\vec{k}$$

$$= \sqrt{(-3)^2 + (5)^2 + 11^2} = \sqrt{155}$$

$$Area\ of\ triangle = \frac{1}{2} \times |\vec{a} \times \vec{b}| = \frac{\sqrt{155}}{2}$$

$$\therefore The\ required\ area\ of\ triangle = \frac{\sqrt{155}}{2} sq.units$$

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b) Two coin are tossed simultaneously. Find the probability of getting at least one head

➢ **Solution**

$$P = \text{ Prob. of getting head in one tossed } = \frac{1}{2}$$

$$q = \text{Prob. of getting tail} = 1 - p = 1 - \frac{1}{2}$$

$$n = \text{ no. of trails} = 2$$

we have, $P(r) = \text{Prob. of getting } \boldsymbol{r} \text{ head in } \boldsymbol{n} \text{ trails}$

$$P(r) = C(n, r)\, p^r q^{n-r} \cdots\cdots\cdots\cdots (\boldsymbol{i})$$

To find : at least 1 head $i.e,\quad r \geq 1$

$$P(r \geq 1) = 1 - P(r < 1)$$

$$P(r \geq 1) = 1 - P(r = 0)$$

From $(\boldsymbol{i})$

$$P(r \geq 1) = 1 - C(2,0)\, p^0 q^{2-0}$$

$$P(r \geq 1) = 1 - 1 \times 1. \left(\frac{1}{2}\right)^2$$

$$P(r \geq 1) = 1 - \frac{1}{4} = \frac{3}{4}$$

$\therefore$ Prob. of getting at least one heads $= \boldsymbol{\frac{3}{4}}$

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7. a) If mean and median of the given data are 3.5 and 4.5. Find Mode.

➢ **Solution**

$$\text{Mean} = 3.5$$
$$\text{Median} = 4.5$$

We have relation,

$$\textbf{Mean} - \textbf{mode} = \textbf{3}(\textbf{mean} - \textbf{median})$$

or, $3.5 - \text{mode} = 3(3.5 - 4.5)$

or, $3.5 - \text{mode} = -3$

or, $\text{mode} = 3 + 3.5 = \mathbf{6.5}$

Hence, Mode $= 6.5$

b) If $n = 15, \sigma_x = 3.2, \sigma_y = 3.4$ and $\sum(x - \bar{x})(y - \bar{y}) = 122$. Find the correlation coefficient between the two variables.

➢ **Solution**

$n = 15$

$\sigma_x = 3.2$, $\sigma_y = 3.4$

$\sum(x - \bar{x})(y - \bar{y}) = 122$

$$\text{Coeff. of correlation, (r)} = \frac{\frac{1}{n}\sum(x - \bar{x})(y - \bar{y})}{\sigma_x \times \sigma_y}$$

$$(\text{r}) = \frac{\frac{1}{15} \times 122}{3.2 \times 3.4} = \mathbf{0.7475}$$

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8) Prove the quadratic equation $ax^2 + bx + c = 0$ have not more than two roots.

➢ **Solution**

The given quadratic equation is $\mathbf{ax^2 + bx + c = 0}$.

For, if possible, let α, β, γ be three different roots of quadratics equation;

$$ax^2 + bx + c = 0 \qquad \text{----------}(a \neq 0)$$

Then, since each of these values must satisfy the equation;

We have,

$$a\alpha^2 + b\alpha + c = 0 \qquad \cdots\cdots\cdots\cdots (i)$$
$$a\beta^2 + b\beta + c = 0 \qquad \cdots\cdots\cdots\cdots (ii)$$
$$a\gamma^2 + b\gamma + c = 0 \qquad \cdots\cdots\cdots\cdots (iii)$$

From equation (i) and (ii) by subtraction; we get,

$$a(\alpha^2 - \beta^2) + b(\alpha - \beta) = 0$$

Since $\alpha \neq \beta$, divide out by $\alpha - \beta$; Then,

$$a(\alpha + \beta) + b = 0$$

Similarly, from equation (ii) and (iii); we get

$$a(\beta + \gamma) + b = 0$$

Hence by subtraction; we get

$a(\alpha - \gamma) = 0$ which is impossible , since, by hypothesis $a \neq 0$, and α is not equal to γ.

Hence, there cannot be more than two different roots.

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9) Prove that :

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = (y-z)(z-x)(x-y)(xy+yz+zx)$$

➢ **Solution**

$$\text{LHS} = \begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix}$$

$$= \begin{vmatrix} x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \\ yz & zx-yz & xy-yz \end{vmatrix}$$

[Applying $C_2 \to C_2 - C_1,\ C_3 \to C_3 - C_1$]

$$= \begin{vmatrix} x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \\ yz & -z(y-x) & -y(z-x) \end{vmatrix}$$

$$= (y-x)(z-x)\begin{vmatrix} x & y-x & z-x \\ x^2 & y^2-x^2 & z^2-x^2 \\ yz & -z & -y \end{vmatrix}$$

[Taking $y-x$ as a common from C_2 and $z-x$ form C_3]

$$= (y-x)(z-x)\begin{vmatrix} x & 1 & 1 \\ x^2 & y+x & z+x \\ yz & -z & z-y \end{vmatrix}$$

[Applying $C_3 \to C_3 - C_2$]

$$= (y-x)(z-x)(z-y)\begin{vmatrix} x & 1 & 0 \\ x^2 & y+x & 1 \\ yz & -z & 1 \end{vmatrix}$$

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$$= (y-x)(z-x)(z-y)\begin{vmatrix} x & 1 & 0 \\ x^2-yz & x+y+z & 0 \\ yz & -z & 1 \end{vmatrix}$$

[Applying $R_2 \to R_1 - R_3$]

$$= (y-x)(z-x)(z-y).1\begin{vmatrix} x & 1 \\ x^2-yz & x+y+z \end{vmatrix}$$

$$= (y-x)(z-x)(z-y)(x^2+xy+zx-x^2+yz)$$

$$= (y-z)(z-x)(x-y)(xy+yz+zx)$$

$=$ RHS **Proved.**

Or

Solve by row equivalent matrix method or Cramer's rule:

$$x+y+z-1$$
$$x+2y+3z=4$$
$$x+3y+7z=13$$

➢ **Solution,**

Solving by Row equivalent matrix method,

The augmented matrix is

$$\begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 1 & 2 & 3 & : & 4 \\ 1 & 3 & 7 & : & 13 \end{bmatrix}$$

$$\xrightarrow{R_2 \to R_2 - 2R_1, R_3 \to R_3 - R_1} \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & 1 & 2 & : & 3 \\ 0 & 2 & 6 & : & 12 \end{bmatrix}$$

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$$R_1 \to R_1 - R_2\,, R_3 \to R_3 - 2R_1 \quad \begin{bmatrix} 1 & 0 & -1 & : & -2 \\ 0 & 1 & 2 & : & 3 \\ 0 & 0 & 2 & : & 6 \end{bmatrix}$$

$$R_3 \to \frac{1}{2}R_3 \quad \begin{bmatrix} 1 & 0 & -1 & : & -2 \\ 0 & 1 & 2 & : & 3 \\ 0 & 0 & 1 & : & 3 \end{bmatrix}$$

$$R_1 \to R_1 + R_3\,, R_2 \to R_2 - 2R_3 \quad \begin{bmatrix} 1 & 0 & 0 & : & 1 \\ 0 & 1 & 0 & : & -3 \\ 0 & 0 & 1 & : & 3 \end{bmatrix}$$

Hence,

$$\boldsymbol{x = 1,\ y = -3,\ z = 3}\ \ i.e., (1\,, -3\,, 3)$$

OR

Solving by Cramer' rule,

$$x + y + z - 1$$
$$x + 2y + 3z = 4$$
$$x + 3y + 7z = 13$$

We have,

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 7 \end{vmatrix} = 1(14-9) - 1(7-3) + 1(3-2)$$
$$= 5 - 4 + 1$$
$$= \mathbf{2}$$

$$D_1 = \begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 3 \\ 13 & 3 & 7 \end{vmatrix} = 1(14-9) - 1(28-39) + 1(12-26)$$
$$= 5 + 11 - 14$$
$$= \mathbf{2}$$

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$$D_2 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 3 \\ 1 & 13 & 7 \end{vmatrix} = 1(28-39) - 1(7-3) + 1(13-4)$$
$$= -11 - 4 + 9$$
$$= \mathbf{-6}$$

$$D_3 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 13 \end{vmatrix} = 1(26-12) - 1(13-4) + 1(3-2)$$
$$= 14 - 9 + 1$$
$$= \mathbf{6}$$

Now,

$$x = \frac{D_1}{D} = \frac{2}{2} = 1$$

$$y = \frac{D_2}{D} = \frac{-6}{2} = -3$$

$$z = \frac{D_3}{D} = \frac{6}{2} = 3$$

Hence,

$$\boldsymbol{x = 1,\ y = -3,\ z = 3}\ i.e., (\mathbf{1}, -\mathbf{3}, \mathbf{3})$$

10) Maximize $x = 3x + 5y$ subject to constraints $3x + 2y \leq 180, \leq 4, y \leq 6$ and $x, y \geq 0$.

➢ **Solution,**

When $x = 0, y = 0$
$\mathbf{3.0 + 2.0 \leq 18 \Rightarrow 0 \leq 18}$ **(true) so, it is directed towards origin.**

The boundary lines of the given inequalities are

$$\boldsymbol{3x + 2y = 18} \quad \cdots\cdots\cdots\cdots (\boldsymbol{i})$$

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From equation (i)

$3x + 2y = 18$	
x	y
0	9
6	0

So,C(0,9) & D(6,0) are passes through equation (i)

$$\boldsymbol{x \leq 4} \cdots\cdots\cdots\cdots (\boldsymbol{ii})$$

When, $x = 0, y = 0$

$0 \leq 4$ (true) so, it is directed towards origin.

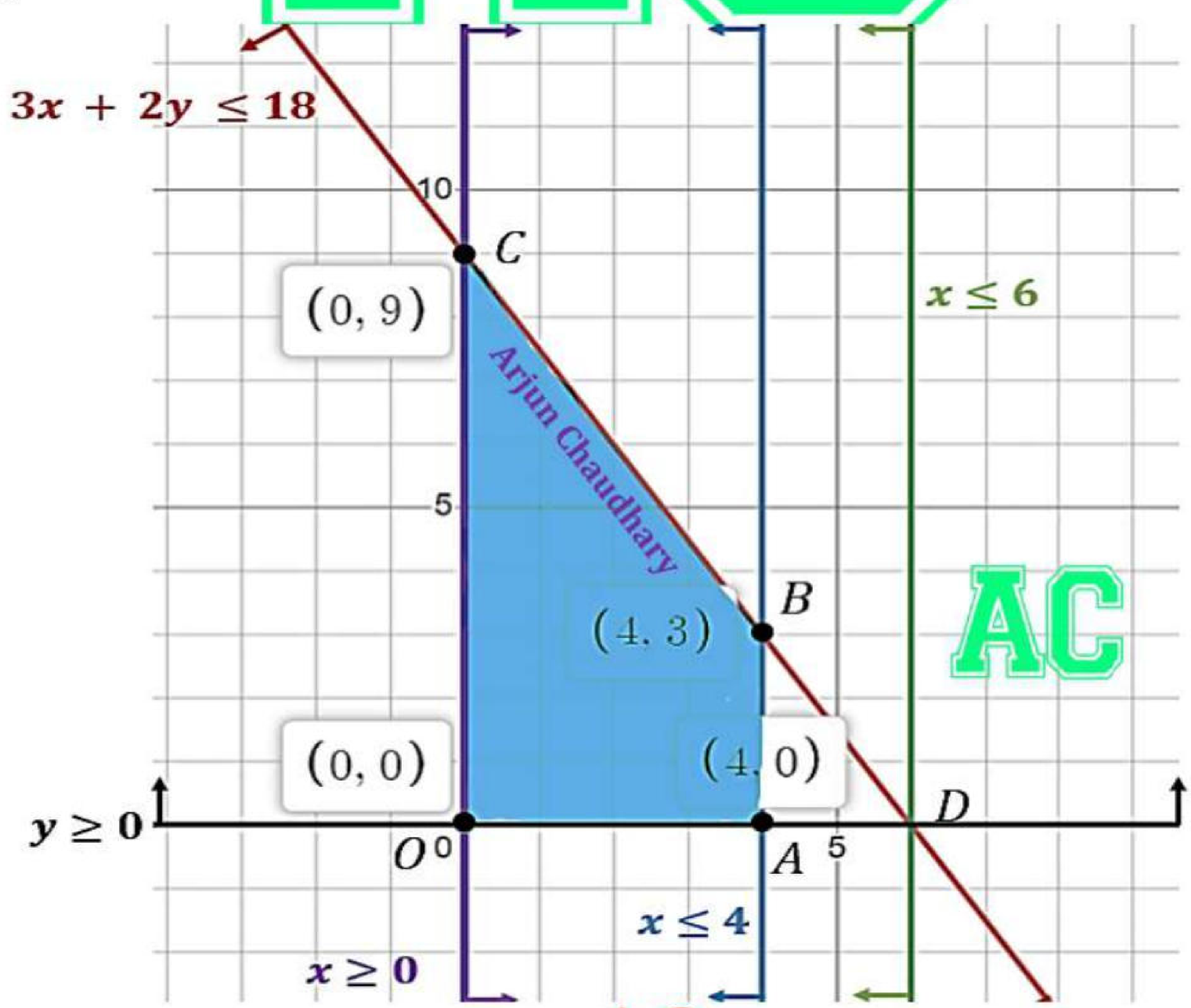
$$\boldsymbol{x \leq 6} \cdots\cdots\cdots\cdots (\boldsymbol{iii})$$

When, $x = 0, y = 0$

$0 \leq 6$ (true) so, it is directed towards origin.

Also,

From graph the vertices B & C are calculated by solving Equation.



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For Vertices B,

Solving equation $x = 4$ and $3x + 2y = 18$

B(4,3)

Hence, The Vertices of ABCD are O(0,0) ,A(4,0) ,B(4,3) and C(0,9).

Vertex	$f = 3x + 5y$
O(0,0)	**0(MIN)**
A(4,0)	12
B(4,3)	27
C(0,9).	**45(MAX)**

Hence, Maximum Value = 45 at c(3,3)

Minimum Value = 0 at A(0,0)

11) Using De-Moivre's theorem. Find the fourth roots of unity.

➢ **Solution,**

Let $z^4 = 1$

$or,\ \ z^4 = \cos 0° + i\ \sin 0° = \cos\ n\,360° + i\ \sin\ n\,360°$

$\therefore z = (\cos\ n\,360° + i\ \sin\ n\,360°)^{\frac{1}{4}}$, where $n = 0, 1, 2, 3.$

$$= \cos\left(\frac{n\,360°}{4}\right) + i\ \sin\left(\frac{n\,360°}{4}\right)$$

$$= \cos\ (n\,90°) + i\ \sin\ (n\,90°)$$

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For n = 0, $z = \cos 0° + \mathrm{i} \sin 0° = 1$

For n = 1, $z = \cos 90° + \mathrm{i} \sin 90° = \mathrm{i}$

For n = 2, $z = \cos 180° + \mathrm{i} \sin 180° = -1$

For n = 3, $z = \cos 270° + \mathrm{i} \sin 270° = -\mathrm{i}$

$\therefore$ **Required roots are 1, −1, i, −i**, $i.e,\ \pm 1,\ \pm i$

or) Define absolute value of a complex number. If z and w are two complex numbers, prove that: $|z + w| \leq |z| + |w|$

- Let $Z = a + bi$ be a complex number Then, Absolute value of complex number is denoted by $|Z|$ and given by

$$|Z| = \sqrt{a^2 + b^2}$$

$$|z + w| \leq |z| + |w|$$

$$\Rightarrow |z| + |w| \geq |z + w|$$

- **Proof**

Let $z = a + ib$ and $w = c + id$, so that $z + w = a + c + i(b + d)$, then

$$|z| = \sqrt{a^2 + b^2}, |w| = \sqrt{c^2 + d^2}$$

$Also,\ \ |z + w| = \sqrt{(a + c)^2 + (b + d)^2}$

$Now,\ \ |z + w| \leq |z| + |w|$ will be true

If $\sqrt{(a + c)^2 + (b + d)^2} \leq \sqrt{a^2 + b^2} + \sqrt{c^2 + d^2}$

$i.e, (a + c)^2 + (b + d)^2 \leq a^2 + b^2 + c^2 + d^2 + 2\sqrt{(a^2 + c^2) + (b^2 + d^2)}$

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$i.e.\ \ ac + bd \ \leq \sqrt{(a^2 + c^2) + (b^2 + d^2)}$

$i.e.\ \ a^2c^2 + b^2d^2 + 2abcd \ \leq (a^2 + c^2) + (b^2 + d^2)$

$i.e.\ \ 2abcd \ \leq \ a^2d^2 + b^2c^2$

$i.e.\ \ 0 \leq 2abcd - \ a^2d^2 + b^2c^2$

$i.e.\ \ 0 \leq (ad - bc)^2$ which is true all real number a, b, c, d

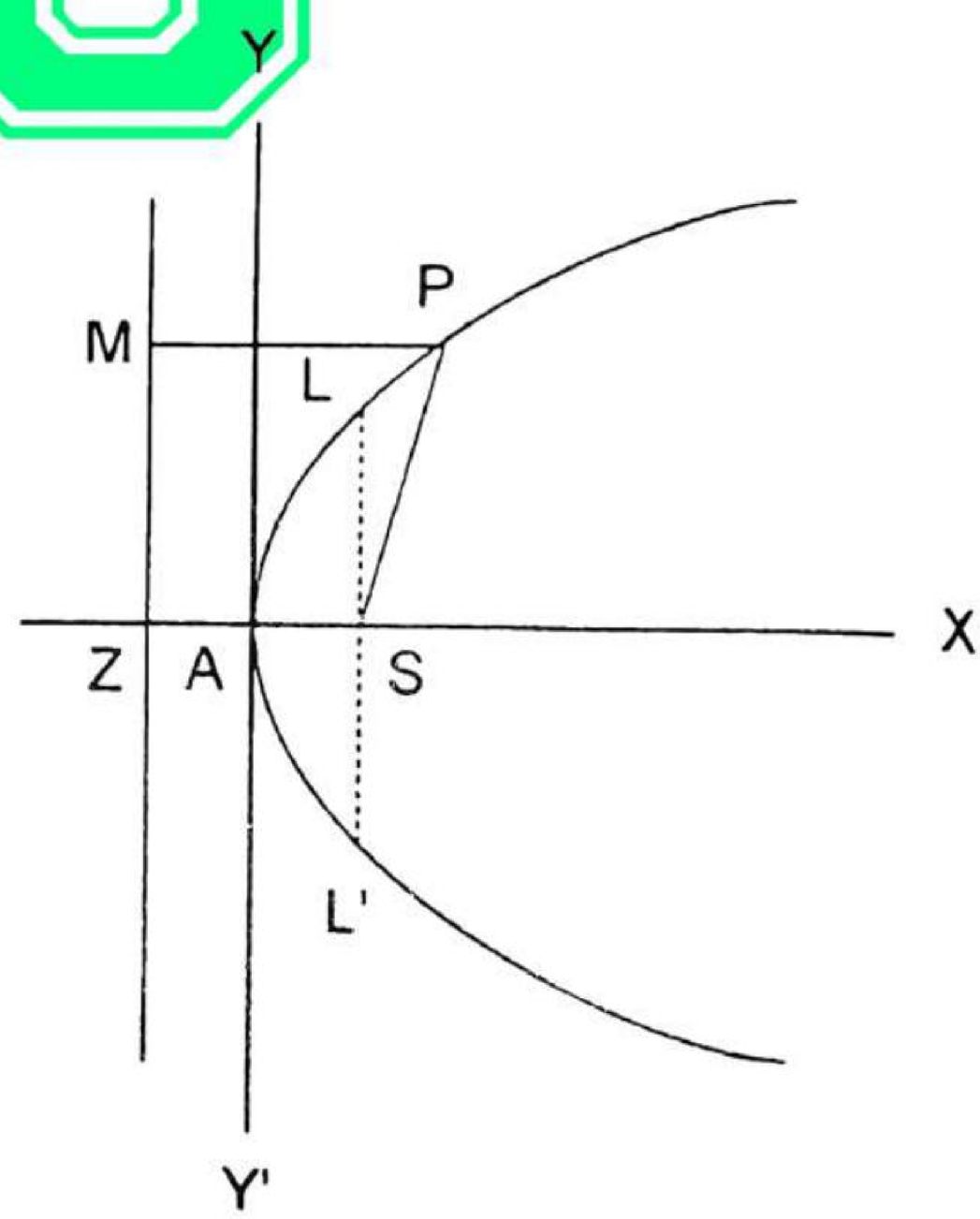
$\therefore \ \ |\boldsymbol{z} + \boldsymbol{w}| \leq |\boldsymbol{z}| + |\boldsymbol{w}|$

12) Define Parabola. Find the equation of the parabola in the standard form $y^2 \ = \ 4ax$.

- **Parabola :-** A plane curve generated by a point moving so that its distance from a fixed point is equal to its distance from a fixed line.

- 2nd Part :

Solution,



Let S be focus and ZM, the directrix of the parabola. SZ is drawn perpendicular to ZM. Let A be the middle point of SZ, so that SA $=$ AZ. Then A is the vertex and ZAS is the axis of the parabola.

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To determine the equation of a parabola in the standard form, take the vertex A at the origin, the focus S on the $x - axis$ so that the axis of the parabola is the $x - axis$ and the directrix is parallel to the $y - axis$.

Let $\mathrm{AS} = a$ .Thus the coordinates of Z, A and S are Respectively $(-a, 0)$, $(0, 0)$ and $(a, 0)$, and the equation of the directrix is $x + a = 0$.

Let $\mathrm{P}(x, y)$ be any point on the parabola. Join PS and draw PM perpendicular to ZM.

Then, $\mathrm{PS} = \mathrm{PM}$.

$\Rightarrow \quad \mathrm{PS}^2 = \mathrm{PM}^2$

$or, \quad (x - a)^2 + (y - 0)^2 = \left(\frac{x + a}{\sqrt{1}}\right)^2$

$or, \quad y^2 = (x + a)^2 - (x - a)^2$

$or, \quad y^2 = (x + a + x - a)(x + a - x + a)$

$\Rightarrow \boldsymbol{y^2 = 4ax}$.

13) Show that $x^2 + 4y^2 - 4x + 24y + 24 = 0$ represents the equation of an ellipse. Find center vertices focus and length of axes.

➢ **Solution**

Given,

$$x^2 + 4y^2 - 4x + 24y + 24 = 0$$
$$x^2 - 4x + 4 + 4y^2 + 24y + 20 = 0$$

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$$(x-2)^2 + 4(y^2+6y) + 20 = 0$$

$$(x-2)^2 + 4(y^2+2.3y+9) - 36 + 20 = 0$$

$$(x-2)^2 + 4(y+3)^2 - 16 = 0$$

$$(x-2)^2 + 4(y+3)^2 = 16$$

$$\frac{(x-2)^2 + 4(y+3)^2}{16} = 1$$

$$\frac{(x-2)^2}{16} + \frac{(y+3)^2}{4} = 1$$

Which is equation of ellipse.

Comparing with $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$$a^2 = 16, \quad b^2 = 4, h = 2, k = -3$$
$$\Longrightarrow a = 4, \qquad b = 2$$

Since, a > b So, the major *axis* is along $x - axis$

Center of ellipse $(h,k) = (2,-3)$
Vertices $= (h \pm a, \mathrm{k}) = (2 \pm 4,0)$ i.e $(6,0)$ &$(-2,0)$

$$\text{Eccentricity}, \mathrm{e} = \sqrt{1 - \frac{\mathrm{b}^2}{\mathrm{a}^2}} = \sqrt{1 - \frac{4}{16}} = \sqrt{\frac{12}{16}} = \frac{\sqrt{3}}{2}$$

For focii $S(\mathrm{h} \pm \mathrm{ae}, \mathrm{k}) = \left(2 \pm 4 \times \frac{\sqrt{3}}{2}, -3\right) = \left(\mathbf{2 \pm 2\sqrt{3}, -3}\right)$

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Lengths of axes,

$$Length\ of\ major\ axes = 2a = 2 \times 4 = \mathbf{8}$$

$$Length\ of\ minor\ axes = 2b = 2 \times 2 = \mathbf{4}$$

OR) Determine the equation of the hyperbola in the standard position with focus at (-7, 0) and eccentricity $\frac{7}{4}$.

➢ **Solution**

Given, focus = $(-7,0)$ and Eccentricity , e $= \frac{7}{4}$

Standard Equation of hyperbola,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Focus $(\pm ae, 0) = (-7,0)$

$$\Rightarrow\ -ae = -7\ \ \Rightarrow a.\frac{7}{4} = 7\ \ \Rightarrow \boldsymbol{a = 4}$$

$$\text{Eccentricity , e} = \sqrt{1 + \frac{b^2}{a^2}}\ \ \Rightarrow\ \ \frac{49}{16} = 1 + \frac{b^2}{16}$$

$$or,\ \ \frac{49}{16} - 1 = \frac{b^2}{16}\ or,\ \ \frac{33}{16} = \frac{b^2}{16}\ \Rightarrow b^2 = 33$$

Standard Equation of hyperbola,

$$\frac{x^2}{16} - \frac{y^2}{33} = 1$$

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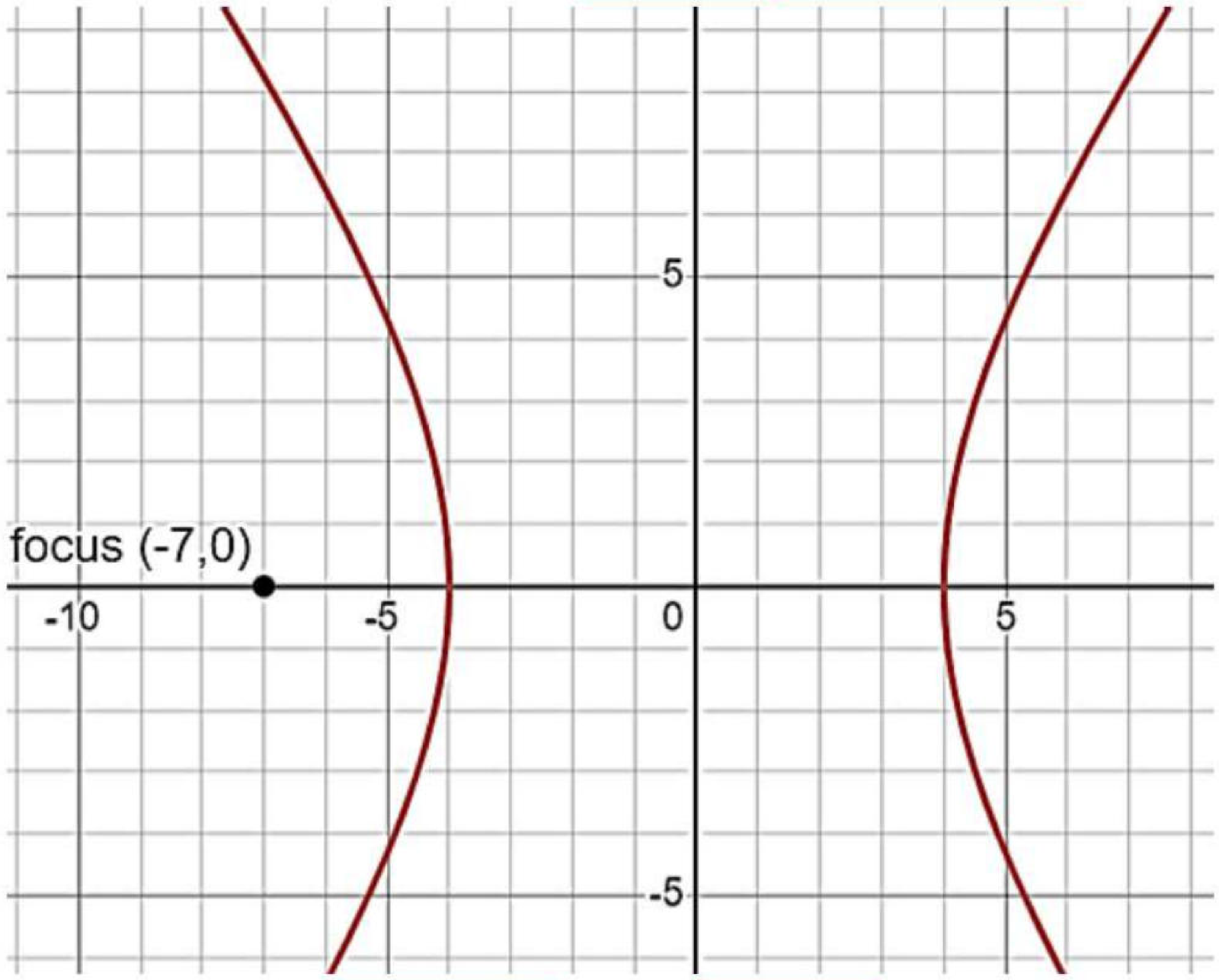

14) Find the equation of the plane through the points $(1, 2, 1), (2, 2, 2)$ and $(0, 1, 0)$.

➢ **Solution,**

Equation of the plane through $(1, 2, 1)$

$$a(x-1) + b(y-2) + c(z-1) = 0 \cdots\cdots\cdots (i)$$

Since,

Equation (i) Passes through (2,2,2) & (0,1,0)

So,

$$a + 0.b + c = 0 \cdots\cdots\cdots (ii)$$

$$-a - b + c = 0 \cdots\cdots\cdots (iii)$$

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Solving, (ii) & (iii) by cross multiplication method,

$$\frac{a}{(0+1)} = \frac{b}{-(1-1)} = \frac{c}{1-0}$$

$$or, \frac{a}{-1} = \frac{b}{0} = \frac{c}{1} = K \ \ (say)$$

$$a = -K, \qquad b = 0, \qquad c = K$$

Equation (i) becomes,

$$-K(x-1) + 0(y-2) + K(z-1) = 0$$

$$or, \ K(-x+1) + K(z-1) = 0$$

$$or, \ -x + 1 + z - 1 = 0$$

$$\boxed{\therefore \ \boldsymbol{z - x = 0}}$$

Which is Eq^n of plane.

15) Find the ratio in which the *yz* plane divides the line joining (4, 6, 7) and (-1,2,5). Also find the coordinates of the point in the yz.

➢ **Solution**

Let *yz-plane* divides the line joining points A(4, 6, 7) and B (− 1, 2, 5) in the ratio k:1 at P(x, y, z).

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In $yz\text{-}plane\ x = 0.$

$$\therefore\ 0 = \frac{k(-1) + 4}{k + 1}$$

$$or,\ \ k = 4$$

The ratio is $k{:}\,1,\ \ i.e,\ 4{:}\,1$

Hence, the point

$$\mathrm{P}(x, y, z) = \mathrm{P}\left(0, \frac{4.2 + 6}{5}, \frac{4.5 + 7}{5}\right)$$

$$= \mathrm{P}\left(\mathbf{0}, \frac{\mathbf{14}}{\mathbf{5}}, \frac{\mathbf{27}}{\mathbf{5}}\right)$$

OR) Find the angle between two lines whose direction cosines are given by $l + m + n = 0$ *and* $2lm - mn + 1nl = 0.$

- **Solution,**

Given Equation,

$$l + m + n = 0 \qquad \cdots\cdots\cdots (i)$$

$$or,\ \ m = -(l + n) = 0$$

$$or,\ \ \ \ l = -(n + m)$$

$$2lm - 2nl + mn = 0 \ \ \cdots\cdots\cdots (ii)$$

Eliminating **n** so, Subsitituting Value of $\boldsymbol{n}$ in eqn(ii),

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$$2lm - 2l \times -(l+m) + m \times -(l+m) = 0$$

$$or,\ \ 2lm + 2l^2 + 2lm - ml - m^2 = 0$$

$$or,\ \ 2\boldsymbol{l^2} + 3m\boldsymbol{l} - m^2 = 0$$

$$or,\ \ \frac{2\boldsymbol{l^2} + 3m\boldsymbol{l} - m^2}{m^2} = \frac{0}{m^2}$$ *Dividing by both side* $\boldsymbol{m^2}$

$$or,\ \ 2\left(\frac{\boldsymbol{l}}{\boldsymbol{m}}\right)^{\boldsymbol{2}} + 3\frac{\boldsymbol{l}}{\boldsymbol{m}} - 1 = 0,$$ which is qudratic in $\frac{\boldsymbol{l}}{\boldsymbol{m}}$

$$\frac{l}{m} = \frac{-3 \pm \sqrt{9 - 4.2.(-2)}}{2 \times 2}$$

$$\frac{l}{m} = \frac{-3 \pm \sqrt{25}}{4} = \frac{\boldsymbol{-3 \pm 5}}{\boldsymbol{4}}$$

Taking $+$ ve sign, $\frac{l}{m} = \frac{-3+5}{4} = \frac{1}{2} \Rightarrow \boldsymbol{l} = \frac{\boldsymbol{m}}{\boldsymbol{2}}$

Taking $-$ ve sign, $\frac{l}{m} = \frac{-3-5}{4} = -2 \Rightarrow \boldsymbol{l} = \boldsymbol{-2m}$

Substituting $\boldsymbol{l} = \frac{\boldsymbol{m}}{\boldsymbol{2}}$ **in eqn**$(\boldsymbol{i})$

$$\frac{m}{2} + m + n = 0 \quad \Rightarrow n = -\frac{3}{2}m \quad \Rightarrow \frac{n}{-3} = \frac{m}{2}$$

Equating $\boldsymbol{l} = \frac{\boldsymbol{m}}{\boldsymbol{2}}$ ***and*** $\frac{n}{-3} = \frac{m}{2}$

$$\Rightarrow \frac{n}{-3} = \frac{m}{2} = \frac{l}{1}$$

$$\Rightarrow \frac{l}{1} = \frac{m}{2} = \frac{n}{-3} = \frac{l^2 + m^2 + n^2}{\sqrt{1^2 + 2^2 + (-3)^2}}$$

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$$\Rightarrow \frac{l}{1} = \frac{m}{2} = \frac{n}{-3} = \frac{1}{\sqrt{14}} \qquad [\because l^2 + m^2 + n^2 = 1]$$

$$\Rightarrow l = \frac{1}{\sqrt{14}}$$

$$\Rightarrow m = \frac{2}{\sqrt{14}}$$

$$\Rightarrow n = \frac{-3}{\sqrt{14}}$$

Direction cosines of line (1)=($\boldsymbol{l_1, m_1, n_1}$) $= (\frac{\mathbf{1}}{\sqrt{\mathbf{14}}}, \frac{\mathbf{2}}{\sqrt{\mathbf{14}}}, \frac{\mathbf{-3}}{\sqrt{\mathbf{14}}})$

Again, Substituting $\boldsymbol{l = -2m}$ $\boldsymbol{in\ eqn(i)}$

$$-2m + m + n = 0 \quad \Rightarrow n = -m \quad \Rightarrow \frac{n}{-1} = \frac{m}{1}$$

Equating $\frac{l}{-2} = \frac{m}{1}$ *and* $\frac{n}{-1} = \frac{m}{1}$

$$\Rightarrow \frac{l}{-2} = \frac{m}{1} = \frac{n}{-1}$$

$$\Rightarrow \frac{l}{-2} = \frac{m}{1} = \frac{n}{-1} = \frac{l^2 + m^2 + n^2}{\sqrt{(-2)^2 + 1^2 + (-1)^2}}$$

$$\Rightarrow \boldsymbol{\frac{l}{-2} = \frac{m}{1} = \frac{n}{-1} = \frac{1}{\sqrt{6}}}$$

$$\Rightarrow l = \frac{-2}{\sqrt{6}}$$

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$$\Rightarrow m = \frac{1}{\sqrt{6}}$$

$$\Rightarrow n = \frac{-1}{\sqrt{6}}$$

Direction cosines of line (2)=$(\boldsymbol{l_2}, \boldsymbol{m_2}, \boldsymbol{n_2}) = (\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-1}{\sqrt{6}})$

Let $\boldsymbol{l_1}, \boldsymbol{m_1}, \boldsymbol{n_1}$ *and* $\boldsymbol{l_2}, \boldsymbol{m_2}, \boldsymbol{n_2}$ be direction cosines of two lines and $\boldsymbol{\theta}$ be the angle between them so,

$$\boldsymbol{\cos\theta = l_1.l_2 + m_1.m_2 + n_1.n_2}$$

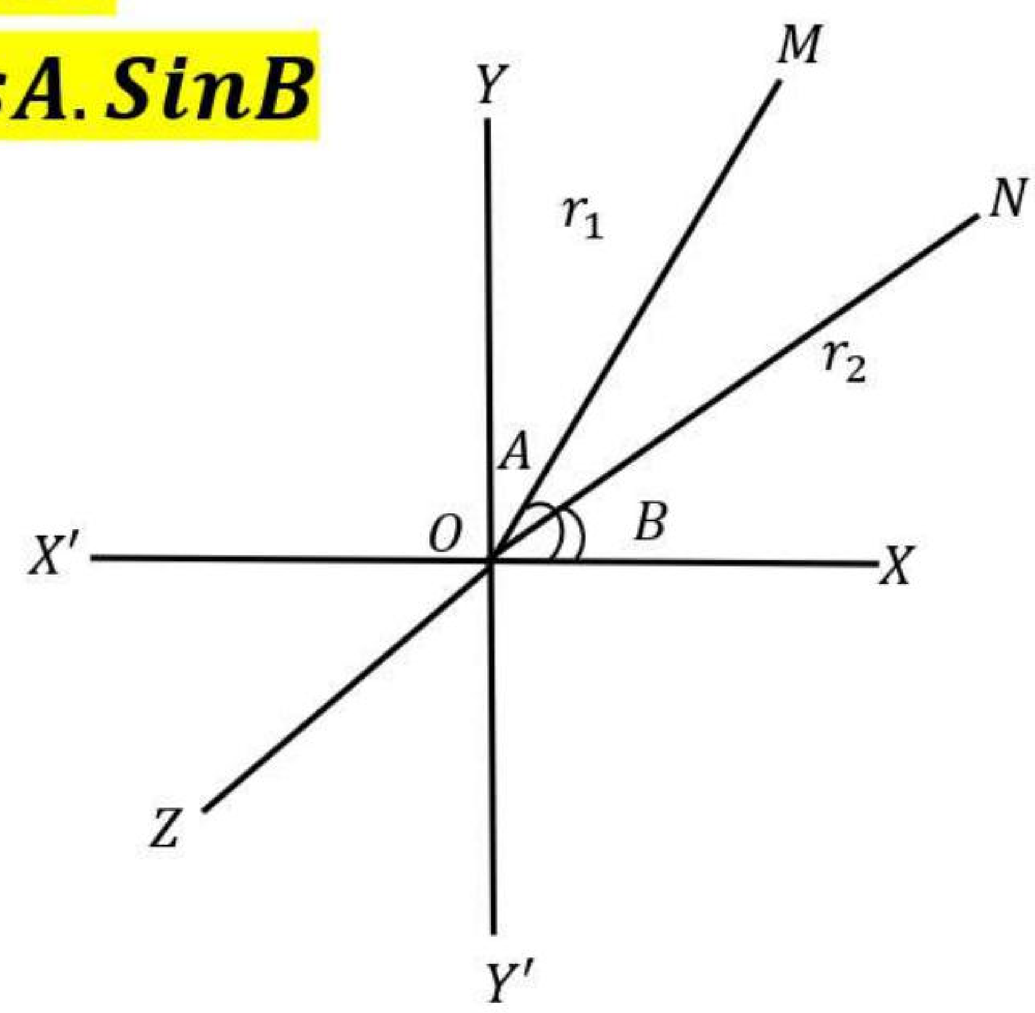
$$\cos\theta = \frac{1}{\sqrt{14}}.\frac{-2}{\sqrt{6}} + \frac{2}{\sqrt{14}}.\frac{1}{\sqrt{6}} + \frac{-3}{\sqrt{14}}.\frac{-1}{\sqrt{6}}$$

$$\cos\theta = \frac{\boldsymbol{-2+2+3}}{\boldsymbol{\sqrt{84}}} = \frac{\boldsymbol{3}}{\boldsymbol{\sqrt{84}}}$$

$$\theta = \boldsymbol{\cos^{-1}}(\frac{\boldsymbol{3}}{\boldsymbol{\sqrt{84}}})$$

16) Using vector method, prove that :

$$\boldsymbol{Sin(A-B) = SinA.CosB - CosA.SinB}$$



- **Solution,**

Let XOX' and YOY', the two mutually perpendicular straight lines represent $x-axis$ and $y-axis$ respectively.

Let $= \angle XOM = A$

and $\angle NOX' = B$

so that $\angle MON = (A-B)$

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Let OM $= r_1$ and ON $= r_2$. Then the co-ordinates of M and N are $(r_1 \cos A, r_1 \sin A)$ *and* $(r_2 \cos B, r_2 \sin B)$

So, position vector are:

So, $\overrightarrow{OM} = (r_1 \cos A, r_1 \sin A) = r_1 \cos A\ \vec{i} + r_1 \sin A\ \vec{j}$

and $\overrightarrow{ON} = (r_2 \cos B, r_2 \sin B)$

$$= r_2 \cos B\ \vec{i} + r_2 \sin B\ \vec{j}$$

$$\overrightarrow{OM} \times \overrightarrow{ON} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ r_1 \cos A & r_1 \sin A & 0 \\ r_2 \cos B & r_2 \sin B & 0 \end{vmatrix}$$

Taking common r_1 *and* r_2 from R_2 and R_3,

$$= r_1 r_2 \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \cos A & \sin A & 0 \\ \cos B & \sin B & 0 \end{vmatrix}$$

$$= r_1 r_2 [\vec{i}(0-0) - \vec{j}(0-0) + \vec{k}(\cos A \sin B - \sin A \cos B)]$$

$$= r_1 r_2 (\cos A \sin B - \sin A \cos B)\vec{k}$$

Magnitude of $\overrightarrow{OM} \times \overrightarrow{ON}$

$$\therefore |\overrightarrow{OM} \times \overrightarrow{ON}| = r_1 r_2 (\cos A \sin B - \sin A \cos B)$$

Since $(A - B)$ is the angle between OM and ON, So

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$$\sin(A-B)=\frac{|\overrightarrow{OM}\times\overrightarrow{ON}|}{|\overrightarrow{OM}|\times|\overrightarrow{ON}|}$$

or, $$\sin(A-B)=\frac{r_1r_2\,(\cos A\sin B-\sin A\cos B)}{r_1r_2}$$

Hence, $\sin(A-B)=\sin A\cos B-\cos A\sin B.$

17) Prove that the three vectors $\vec{a}-2\vec{b}+3\vec{c}$, $-2\vec{a}+3\vec{b}-4\vec{c}$ and $-\vec{b}+2\vec{c}$ *are coplanar*.

- **Solution,**

$$\vec{r_1}=\vec{a}-2\vec{b}+3\vec{c}$$
$$\vec{r_2}=-2\vec{a}+3\vec{b}-4\vec{c}$$
$$\vec{r_3}=-\vec{b}+2\vec{c}$$

To show: $\vec{r_1}, \vec{r_2}$ *and* $\vec{r_3}$ **are coplanar**

- **If three vector are coplanar, one can expressed as the sum of the scalar multiples of the other two.**

$\vec{r_3}=x\vec{r_1}+y\vec{r_2}$ where, x *and* y are scalars

$or, -\vec{b}+2\vec{c}=x(\vec{a}-2\vec{b}+3\vec{c})+y(-2\vec{a}+3\vec{b}-4\vec{c})$

$or\ -\vec{b}+2\vec{c}=(x-2y)\vec{a}+(-2x+3y)\vec{b}+(3x-4y)\vec{c}$

Equating Coefficient of likes vectors,

$$(x-2y)=0\cdots\cdots\cdots\cdots(i)$$
$$-2x+3y=-1\cdots\cdots\cdots\cdots(ii)$$
$$3x-4y=2\cdots\cdots\cdots\cdots(iii)$$

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Solving eqn (i) and (ii) $weget$, $\boldsymbol{x = 2\ and\ y = 1}$

Put , $x = 2\ and\ y = 1$ in eqn (iii)

$3 \times 2 - 4 \times 1 = 2$

$2 = 2,$ **satisfied**

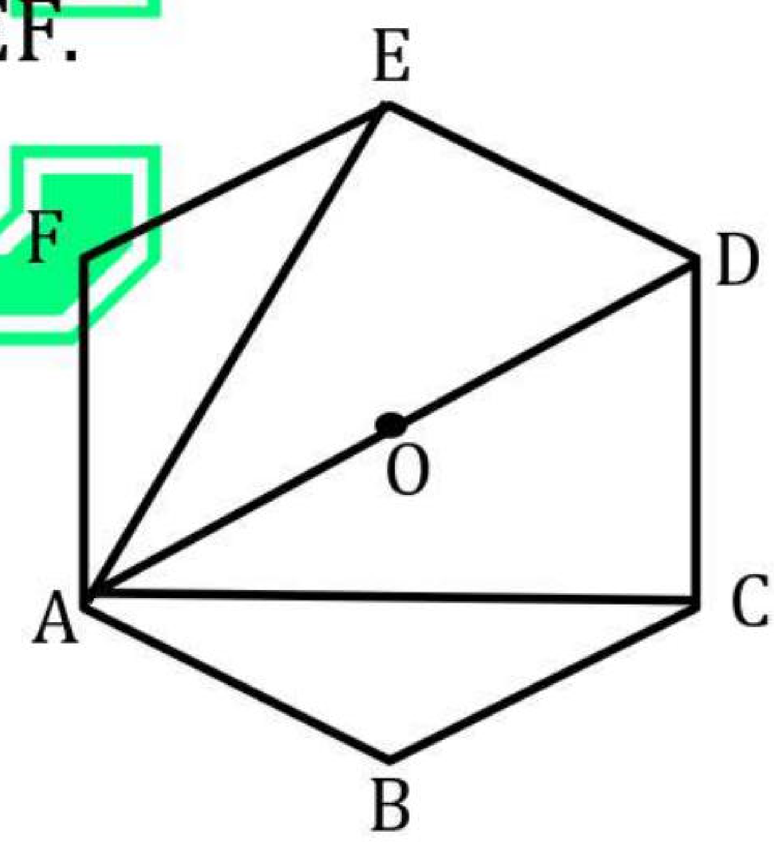
Hence, Three vector are coplanar and its can expresssed as

$$\overrightarrow{r_3} = 2\overrightarrow{r_1} + \overrightarrow{r_2}$$

Or) If $ABCDEF$ is a regular hexagon and O is its center, then show that: $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} = 6\overrightarrow{AO}$

➢ **Solution,**

If $ABCDEF$ is a regular hexagon, then we can suppose O is its center of the regular hexagon ABCDEF.



Now, $\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF}$

$= \overrightarrow{AD} + (\overrightarrow{AB} + \overrightarrow{AE})\ \overrightarrow{AC} + \overrightarrow{AF}$

$= \overrightarrow{AD} + (\overrightarrow{ED} + \overrightarrow{AE})\ \overrightarrow{AC} + \overrightarrow{AF}$

$= \overrightarrow{AD} + \overrightarrow{AD} + (\overrightarrow{AC} + \overrightarrow{CD})$

$= \overrightarrow{AD} + \overrightarrow{AD} + \overrightarrow{AD}$

$= 3\overrightarrow{AD}$

$= 3 \times 2\ \overrightarrow{AO}$ $\quad [\because \overrightarrow{AD} = 2\ \overrightarrow{AO}]$

$\therefore\ \boldsymbol{6\overrightarrow{AO}}$

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18) Calculate coefficient of variation from the following data:

Class	0-10	10-20	20-30	30-40	40-50
Frequency	2	4	6	7	3

➢ **Solution,**

$Class\ x$	f	$Mid\ value\ (M)$	$d' = \frac{m-a}{10}$ $(a = 25)$, $h = 10$	d'^2	Fd'	Fd'^2
0-10	2	5	-2	4	-4	8
10-20	4	15	-1	1	-4	4
20-30	6	25	0	0	0	0
30-40	7	35	1	1	7	7
40-50	3	45	2	4	6	12
	$N=22$			$\sum fd' = 5$		$\sum fd'^2 = 31$

So,

$$\bar{x} = a + \frac{\sum fd'}{N} \times h$$

$$= 25 + \frac{(5)}{22} \times 10 = \mathbf{27.27}$$

Mean $(\bar{x}) = 27.27$

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$$\textbf{Standard Deviation } (\sigma) = \sqrt{\frac{\sum f d'^2}{N} - \left(\frac{\sum f d'}{N}\right)^2} \times h$$

$$= \sqrt{\frac{31}{22} - \left(\frac{5}{22}\right)^2} \times 10$$

$$= \sqrt{1.41 - 0.052} \times 10$$

$$= 1.165 \times 10$$

$$= \mathbf{11.65}$$

$$\textbf{Coefficient of Variation} = \frac{\sigma}{\bar{x}} \times 100\%$$

$$= \frac{11.65}{27.27} \times 100$$

$$= \mathbf{42.72\%}$$

19) Calculate Karl Pearson's coefficient of correlation from the following data:

Age of Husband	23	22	24	23	26	27
Age of Wives	20	18	20	21	21	22

➢ **Solution,**

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Age of husband (x)	Age of Wives (y)	$x = X - \bar{x}$	$y = Y - \bar{y}$	xy	x^2	y^2
23	20	-1.17	-0.33	0.3861	1.3689	0.1089
22	18	-2.17	-2.33	5.0561	4.7089	5.4289
24	20	-0.17	-0.33	0.0561	0.0289	0.1089
23	21	-1.17	0.67	-0.7839	1.3689	0.4489
26	21	1.83	0.67	1.2261	3.3489	0.4489
27	22	2.83	1.67	4.7261	8.0089	2.7889
$\sum x = 145$	$\sum y = 122$			$\sum xy = 10.67$	$\sum x^2 = 18.83$	$\sum y^2 = 9.33$

$\mathbf{n = 6}$

Now,

$$\bar{X} = \frac{\sum x}{n} = \frac{145}{6} = 24.17\,, \quad \bar{Y} = \frac{\sum y}{n} = \frac{122}{6} = 20.33$$

Using Product moment formula ,

$$\textbf{Coeff. of correlation, (r)} = \frac{\sum xy}{\sqrt{\sum x^2}\ \sqrt{\sum y^2}}$$

$$= \frac{10.67}{\sqrt{18.83}\sqrt{9.33}}$$

$$\therefore \mathbf{r = 0.8050}$$

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20) A class consists of 40 boys and 60 girls. It two students are chosen at random, what will be the probability that. (a) Both are Boys (b) Both are Girls (C) One Boy and one Girl.

➢Solution

$n(B) = 40, \quad n(G) = 60,$

$n(S) = 60 + 40 = 100$

Now,

When two students are chosen randomly then, Possible outcomes , $n =$ selection of 2 std out of 100

$= c(100, 2)$

$= \frac{100!}{(100-2)!\ \ 2!}$

$= \mathbf{4950}$

(a) P (Both are Boys) = ?

$M =$ no. of favorable case.

$=$ selection of 2 boys out of 40

$= c\ (40,2)$

$= \frac{40!}{(40-2)!\ \ 2!}$

$= 780$

Hence, $P(\text{both are boys}) = \frac{M}{n} = \frac{c(40,2)}{c(100,2)}$

$= \frac{780}{4950}$

$= \frac{\mathbf{5}}{\mathbf{33}}$

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(b) P (Both are girls) = ?

$$M = \text{ no. of favorable case.}$$
$$= \text{ selection of 2 girls out of 60}$$
$$= c\ (60,2)$$
$$= \frac{60!}{(60-2)!\ \ 2!}$$
$$= 1770$$

Hence, $P(\text{both are girls}) = \frac{M}{n} = \frac{c(60,2)}{c(100,2)}$

$$= \frac{1770}{4950}$$
$$= \frac{\mathbf{59}}{\mathbf{165}}$$

(C) P (one boy and one girls) = ?

$$M = \text{ no. of favorable case.}$$
$$= \text{ selection of 1 girls out of 60 and selection of 1 boys out of 40}$$
$$= c\ (60,1) \times c\ (40,1)$$
$$= \frac{60!}{(60-1)!\ \ 1!} \times \frac{40!}{(40-1)!\ \ 1!}$$
$$= 60 \times 40$$
$$= 2400$$

$\therefore$ **P (one boy and one girls)** $= \frac{M}{n} = \frac{2400}{4950}$

$$= \frac{\mathbf{16}}{\mathbf{33}}$$

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OR) The incidence of occupations diseases in an industry is such that the workman have a 20% chance of suffering from it. What is the probability that out of six men, four or more will contact the disease?

➢ **Solution,**

$$P = \text{probability of Suffering from diseases } = 20\%$$

$$P = \frac{1}{5}$$

$$q = 1 - \frac{1}{5} = \frac{4}{5}$$

$$\mathbf{n = 6}$$

we have,

To find : $P(r \geq 4) = ?$

$$P(r) = C(n,r)\, p^r q^{n-r}$$

$$P(r \geq 4) = P(4) + P(5) + P(6) \quad \cdots\cdots\cdots (i)$$

$$\mathbf{P(4) = C\,(6,4).\left(\frac{1}{5}\right)^4 . \left(\frac{4}{5}\right)^{6-4}}$$

$$= \frac{6!}{(6-4)!4!}\ \frac{1}{5^4} \times \left(\frac{4}{5}\right)^2$$

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$$= 15 \times \frac{1}{625} \times \frac{16}{25}$$

$$= \frac{\mathbf{48}}{\mathbf{3125}}$$

$$\mathbf{P(5) = C\,(6,5).\left(\frac{1}{5}\right)^{5}.\left(\frac{4}{5}\right)^{6-5}}$$

$$= \frac{6!}{(6-5)!5!}\ \frac{1}{5^5} \times \frac{4}{5}$$

$$= 15 \times \frac{1}{3125} \times \frac{4}{5}$$

$$= \frac{\mathbf{24}}{\mathbf{15625}}$$

$$\mathbf{P(6) = C\,(6,6).\left(\frac{1}{5}\right)^{6}.\left(\frac{4}{5}\right)^{6-6}}$$

$$= \frac{6!}{(6-6)!\,6!}\ \frac{1}{5^6} \times \left(\frac{4}{5}\right)^{0}$$

$$= 1 \times \frac{1}{15625} \times 1$$

$$= \frac{\mathbf{1}}{\mathbf{15625}}$$

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$$\mathbf{P}(r \geq 4) = P(4) + P(5) + P(6)$$

$$= \frac{48}{3125} + \frac{24}{15625} + \frac{1}{15625}$$

$$= \frac{53}{3125}$$

$$= 0.01696$$

$$= \mathbf{1.696\%}$$

Hence, the probability that out of six men, four or more will contact the disease is 1.696%

$\Leftarrow$ ***The End*** $\Rightarrow$

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