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UNIT-1

Introduction

The word "statistics" seems to be derived from the Latin word 'status', Italian word 'statiste', German word 'statistik' and French word 'statistique', each of which means political state.

Definition

1) Horacesecrets
2) Croxton and cowden
3) Ya Lun chau

1) Horacesecrets -> Statistic may be defined as the aggregate of facts affected to the marked extent by a multiplicity of causes, numerically expressed, enumerated or estimated according to the reasonable standard of accuracy, collected in systematic manner for a predetermined purpose, and placed in relation to each other.

2) Croxton and cowden -> statistics may be defined as the science of collection, presentation, analysis and interpretation of numerical data. (CPAIN)

3) Ya lun chau -> statistics is the method of decision making in the face of uncertainity on the basis of numerical data and calculation.

*** Functions of statics

1) collection of data
2) Presentation of data
3) Analysis of data
4) Interpretation of data

** Limitations of statistics

1) statistics deals with quantitative data only however qualitative informations can be converted into quantitative data by the method of coding, scoring, ranking.

2) Statistics result are true in average.

3) Statistics deals with masses not for individuals. No statics is used for single observation.

4) Statistics results are subjected to certain error.

5) Statistics is only a means to draw a conclusion about masses.

6) Statistics can be misused in different way.

** Use of statistics in various Field

1) statistics in planning -> For planning of production, consumption, demand, supply, price and expenditure.

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2) Statistics in state -> collecting data related to manpower, crime income, death, birth etc, For military policy, financial policies.

3) Statistics in Mathematics -> Assumption, analysis and testing, application of mathematical tools.

4) Statistics in Economics -> Solution of various problems related to production, consumption, wealth, wages, prices, profit, saving, investment, poverty etc.

5) Statistics in business and management

i) Marketing decision
ii) Credit policy
iii) Accounting
iv) Inventory control (भोण्डार) -> (स्तोर)

6) Some other discipline

i) Statistics in industry -> production, supply, demand, expenditure
ii) statistics in insurance
iii) statistics in astronomy
iv) statistics in physical science
v) statistics in social science
vi) statistics in Bio-chemistry

Note :- R.A fisher is the real giant of statistics and called Father of statistics.

UNIT-2

Collection of data

Points to remember before collecting data

1) Scope of the study
2) Problem / statement of the problem
3) Sources of information.
4) Techniques of data collection
5) Unit of data collection.
6) Degree of accuracy

Types of data

1) Primary data -> The data which are originally collected by an investigator for first time for the purpose of statistical inquiry are known as primary data. For e.g :- Data collected by CBS, NRB (Nepal Rastra Bank) etc.

2) Secondary data -> The data which are not originally collected but obtained from published or unpublished sources are secondary data. For e.g :- Data taken from newspapers, data from internet etc.

Method of primary data collection

1) Direct personal interview
2) Indirect oral interview
3) Mailed questionnairs (प्रश्नावली)
4) Information through correspondance (पत्राचार)
5) Schedules through enumerators. (गणक) (कार्यपत्र)
6) Information through local agent.

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Choice of method of data collection

1) the nature of inquiry.
2) object and scope of inquiry.
3) Financial resources
4) Time factor
5) Accuracy needed

Sources/method of secondary data

a) Published sources
i) official publications and reports
ii) Semi-official publication
iii) Private publication

b) Unpublished sources
i) Records maintained by private form.
ii) Business interprises.
iii) Hospital records
iv) Research institutes etc.

Classification and Tabulation of data

Classification is the process of arranging data into different groups or classes according to their attributes (गुण) (characteristics- similar)

Tabulation is the systematic process of presentation of data in rows and columns according to their resemblence. Tabulation is done after classification.

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Types of classification

1> Geographical classification -> On the basis of place.

Example:-

Region	population
Terai	51.5%
Mountain	42.78%
Hill	6.73%

2> Chronological classification -> On the basis of time.

Example :-

BS	Import of Fuel (KL)
1928	100
1956	200
1990	250

3> Qualitative classification -> On the basis of quality.

4> Quantitative classification -> On the basis of quantity.

Parts of table

1> Table number
2> Title
3> Head notes
4> Caption of stub -> (row head) caption -> (column head)
5> Body of table
6> Foot note
7> Source note

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Table no.1.1

Head notes or prefactory note

Stub	Boys		Girls		Total
	Column Head	Column Head	Column Head	Column Head	
			Body		

footnotes

(Source : NRB Bulletin 2063 Asar)

Frequency

The number of times which a variate value occours is known as Frequency.

Frequency distribution

Frequency distribution is simply a table in which the collected data is classified and presented in different groups. It is of two types:-

A) Univariate Frequency distribution

i) Individual series → Individual series is a series where items are listed singly after observation as distinguished from listing them in group.

For e.g:-

Day	Sun	Mo	Tue	Wed	Thrus	Fri	Sat
Temp(°C)	32	31	35	26	25	30	25

ii) Discrete Series -> The series is formed by discrete variables.

for e.g:-

No. of goals	0	1	2	3	4
No. of Match	27	9	8	5	2

iii) Continous series -> The series formed by continous variable is called continous series.

For.eg:-

wages	100-200	200-300	300-400	400-500	500-600
frequency	25	10	7	3	1

B) Bivariate frequency distribution -> It is a frequency distribution in which there are two or more than two variate values.

1) The marks obtained by 30 students of a class are given below.

40 60 70 30 38 25 42 50 58 59 46 77 50 54 35
42 65 72 34 20 35 48 54 53 60 78 62 51 42 28

classify the above data taking a class interval of 10.

->

class Interval (C.I)	Tally Mark	Frequency
20-30	III	3
30-40	~~IIII~~	5
40-50	~~IIII~~ I	6
50-60	~~IIII~~ III	8
60-70	IIII	4
70-80	IIII	4

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2) In a survey it was found that 64 families bought milk in the following quantities in particular month.

19	9	39	23	16	17	28	24	10	28	14	34	33	29	11	37
16	22	19	6	18	20	18	20	7	24	23	22	23	13	26	30
22	12	14	24	7	25	10	21	18	20	25	5	26	36	11	13

8	21	31	23	15	21
15	32	17	12	27	
22	21	16	9	17	

Convert above data in a frequency distribution of classes 5-9, 10-14 etc.

->

Class Interval (C.I)	Tally Mark	Frequency
5 - 9		
10 - 14		
15 - 19		
20 - 24		
25 - 29		
30 - 34		
35 - 39		

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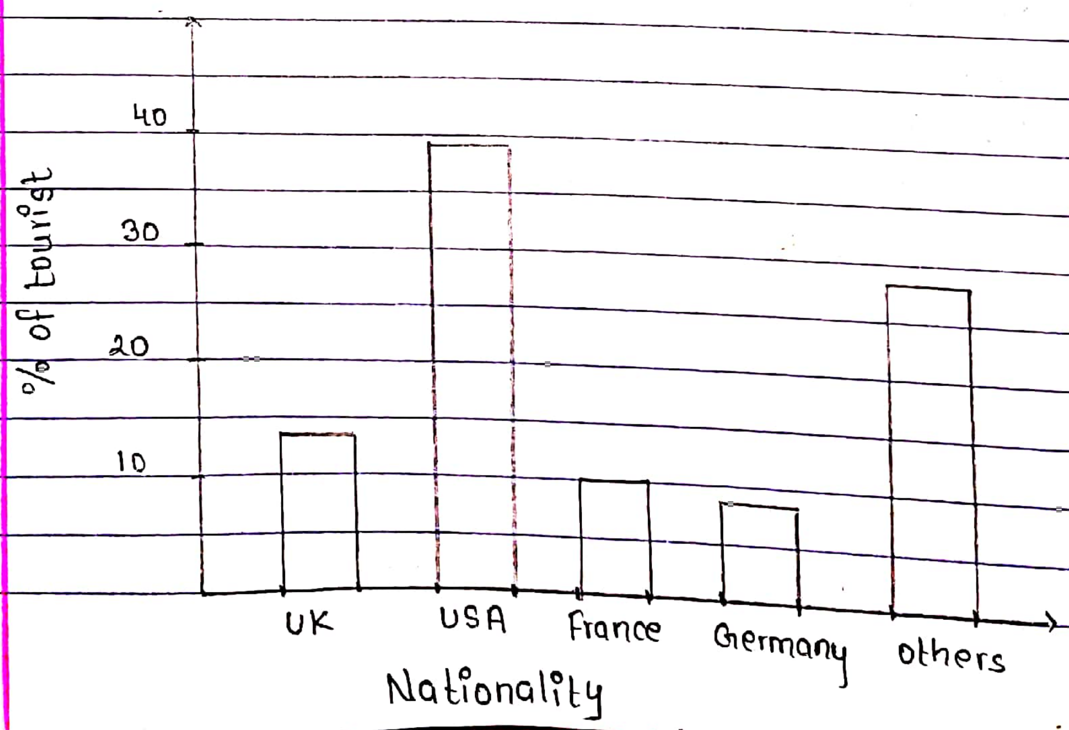
Representation of Data

1) Bar diagram -> It is one dimensional diagram and used for only one variable. It is simple and frequently used in statistics for comparing two or more values of single variable. It consists of set of equidistance rectangles of equal width. for e.g:- Following information gives the percentage of tourists by nationality in the year 2013 for the country Nepal.

Nationality	percentage of tourist
UK	13.2
USA	39.4
France	10.0
Germany	8.6
others	28.5

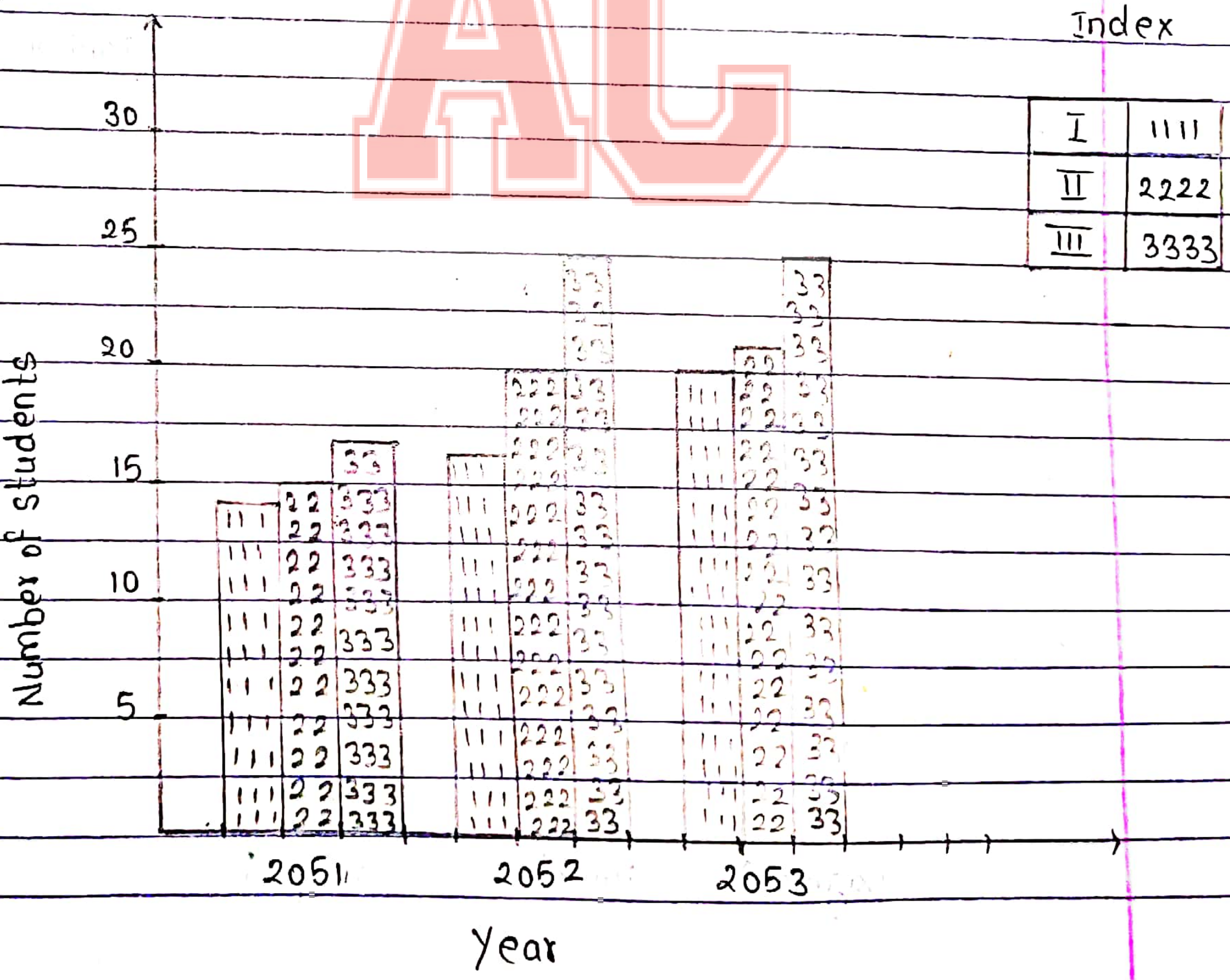
Represent the information by bar diagram.



2) Multiple Bar diagram -> Multiple bar diagram is also one dimensional diagram. when two or more values of variable are to be compared then such diagram is used. It's procedure is same to that of simple bar diagram but differentiate bars by different colours.

i) The following table shows the S.L.C result of certain school in a year. represent in by suitable diagram.

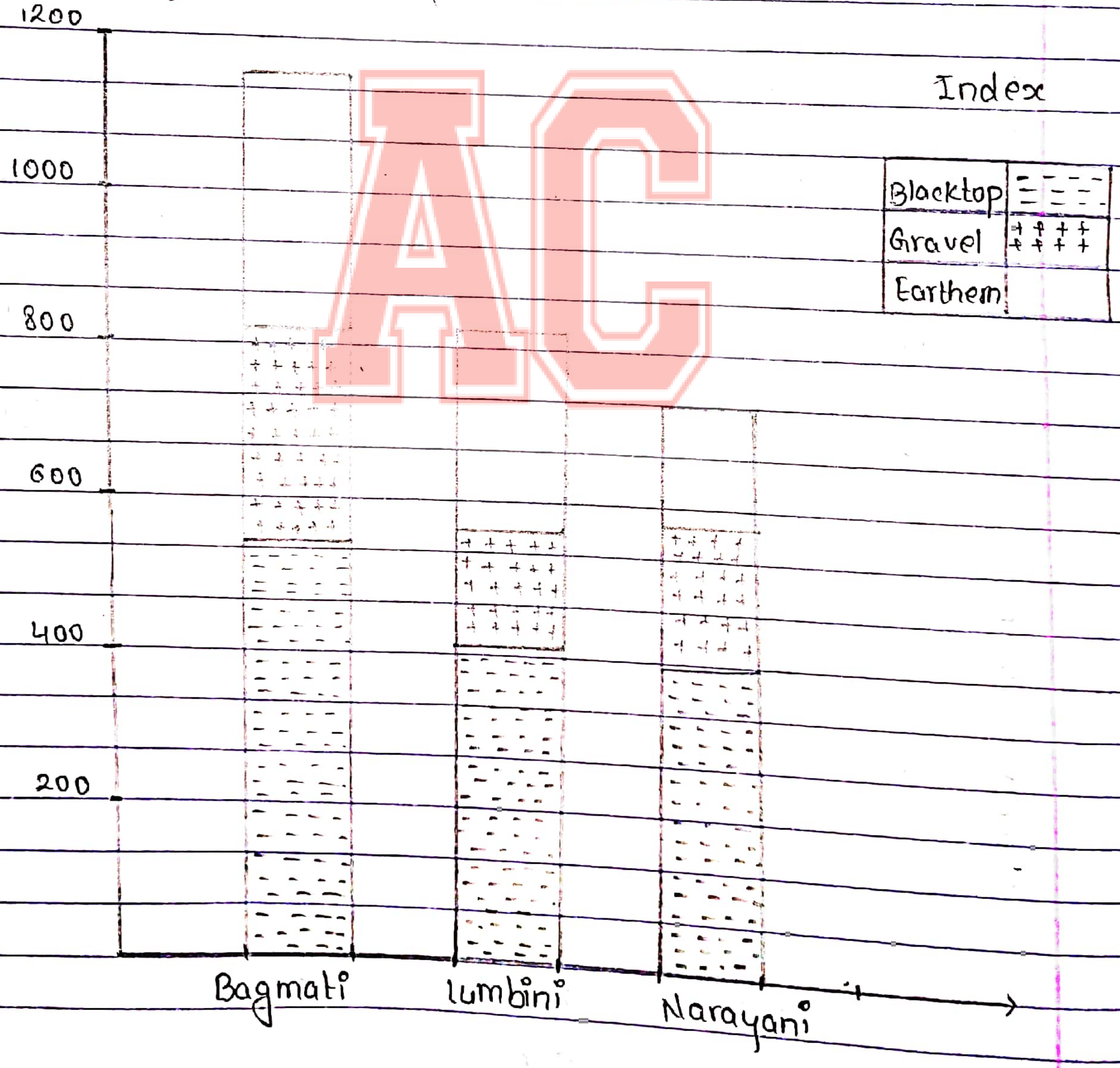
YEAR	I div.	II div	III div
2051	12	15	18
2052	17	20	25
2053	20	22	26



3) Sub-divided bar diagram → Sub-divided bar diagram is a diagram for two or more components of a total. such diagram is also used for comparision of different components with one another.

i)

Zone	Blacktop	Gravel	Earthern	Total
Bagmati	598	258	266	1122
Lumbini	428	90	308	826
Narayani	396	165	139	700



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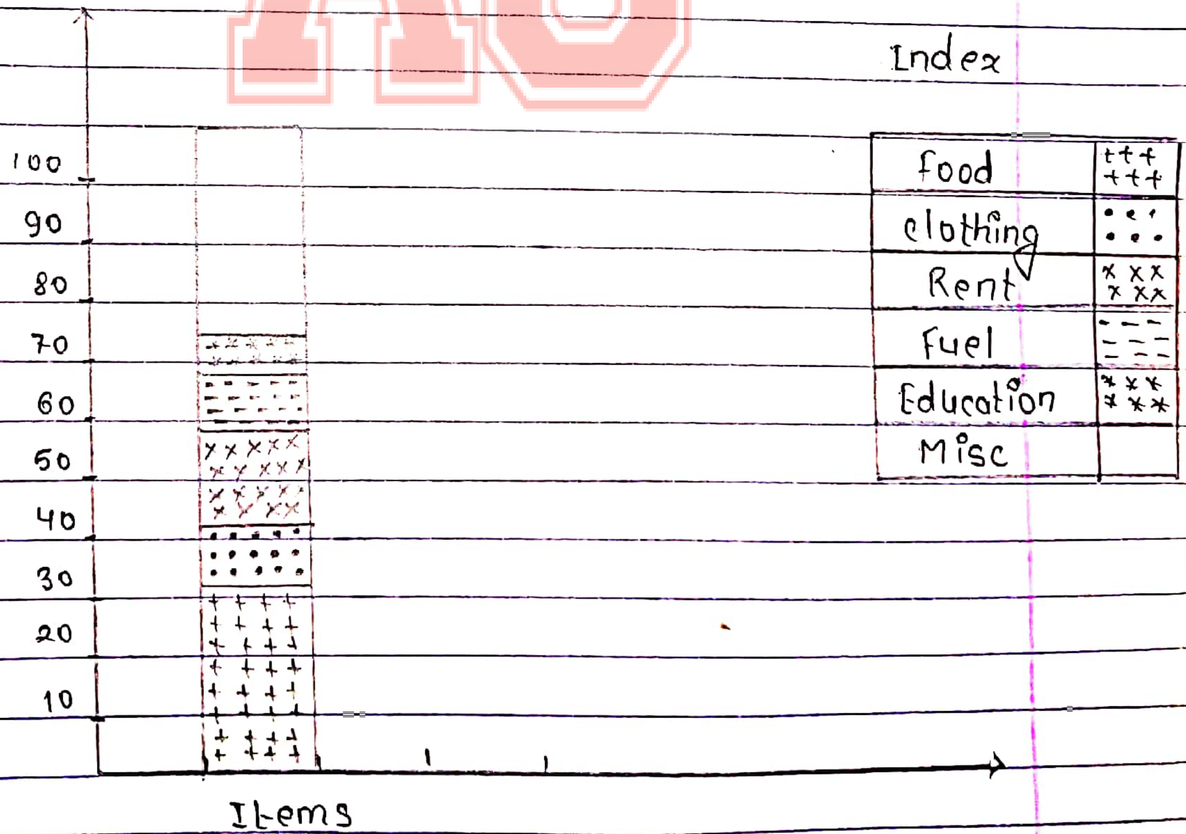
4) percentage bar diagram → Sub-divided bar diagram presented in the form of percentage is percentage bar diagram. It is also used for comparision.

i) Prepare a percentage bar diagram from following data.

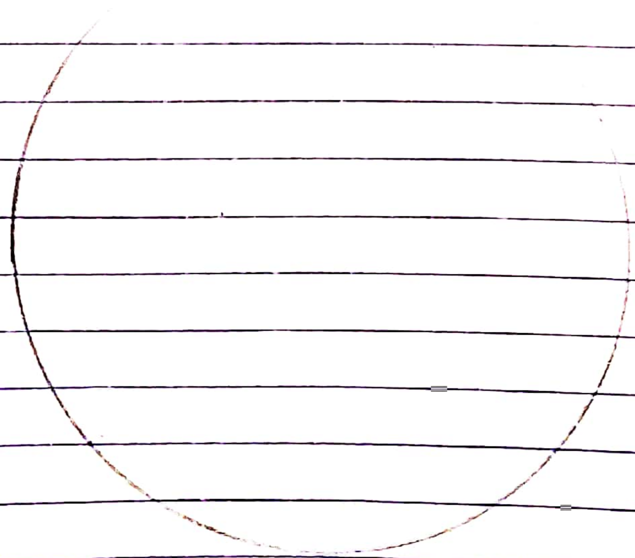
Items	Expenditure (Rs)	Percentage	cumulative %
Food	240	33.33	33.33
clothing	66	49.17	42.50
Rent	125	17.36	59.86
Fuel	57	7.92	67.78
Education	42	5.83	73.61
Misc.	190	26.39	100%
Total	720	100%	



5> Angular or pie diagram -> It is a diagram in the form of circle whose area represents the total value. The circle diagram dividing into different sectors by radial lines.

i> Prepare a circle/angular/pie diagram of following.

		Degree measurement
Food	60%	$60/100 \times 360^\circ = 216^\circ$
Rent	15%	$15/100 \times 360^\circ = 54^\circ$
clothing	10%	$10/100 \times 360^\circ = 36^\circ$
Education	8%	$8/100 \times 360^\circ = 28.8^\circ$
Health	5%	$5/100 \times 360^\circ = 18^\circ$
others	2%	$2/100 \times 360^\circ = 7.2^\circ$
Total	100%	



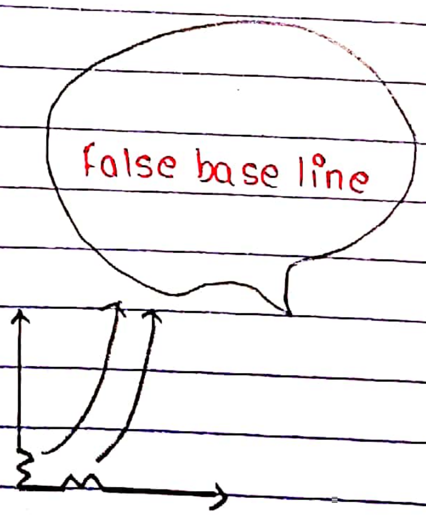
Histogram

Frequency distribution is graphical form is called Histogram. It consists the set of adjacent vertical rectangles on x-axis with base equal to the corresponding class interval and height of rectangle is porportional to the frequency of corresponding class.

Difference between bar diagram and Histogram

Bar diagram	Histogram
1) Bar diagram are one dimensional.	1) Histogram are two dimensional.
2) In Bar diagrams bars are seperated by certain gap.	2) Histogram are adjacent.
3) Bar diagram represents length (height only)	3) Histogram represent area (l×b).
4) Bar diagram represents cannot used to find mode.	4) Histograms are used to find mode.

Note:-

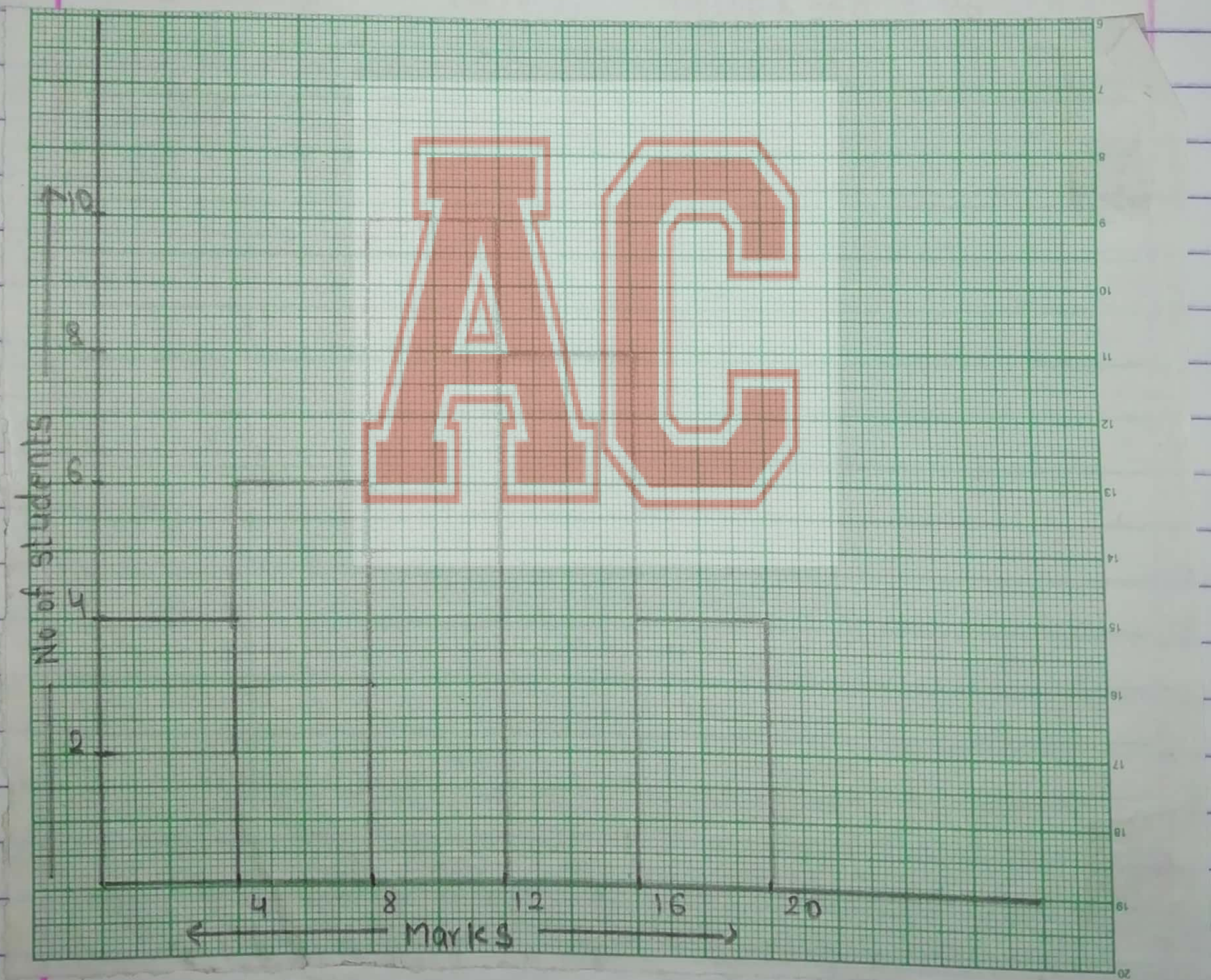


1) Draw a histogram of given data.

Marks	0-4	4-8	8-12	12-16	16-20
No. of students	4	6	10	8	4

→ Solution

Representing the data in Histogram as below.



2> Represent the Following data in Histogram.

Wages	10-15	15-20	20-25	25-30	30-40	40-60	60-80
No. of workers	7	20	27	15	24	20	8

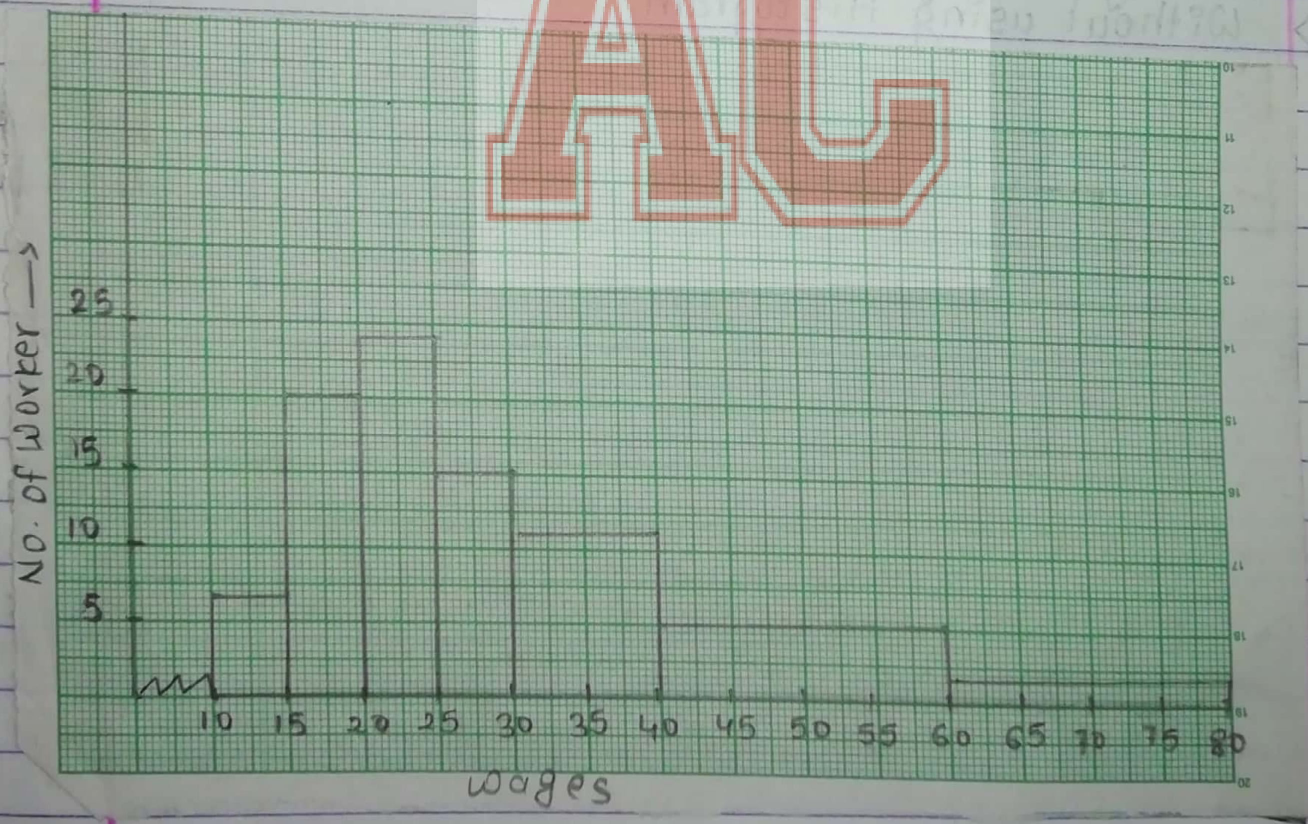
-> Solution,

This is the case of unequal frequency distribution of class. so Adjust of frequency must be done.

Here,

minimum class size is 5.

∵ The class size of 30-40 is double of minimum class size. so, frequency of this class must be divided by 2.



Frequency polygan

Frequency polygan is another method of graphical representation of frequency distribution when the distribution is discrete the frequency polygan is obtained by plotting the points with the value of variable as x-cordinate and corresponding frequency as y-cordinate and joining the points if frequency distribution is continous. frequency polygon can be drawn from following two wages.

1) Using Histogram
2) Without using Histogram

Note :- polygon -> Ruler used

Curve -> smooth free hand

Line graph

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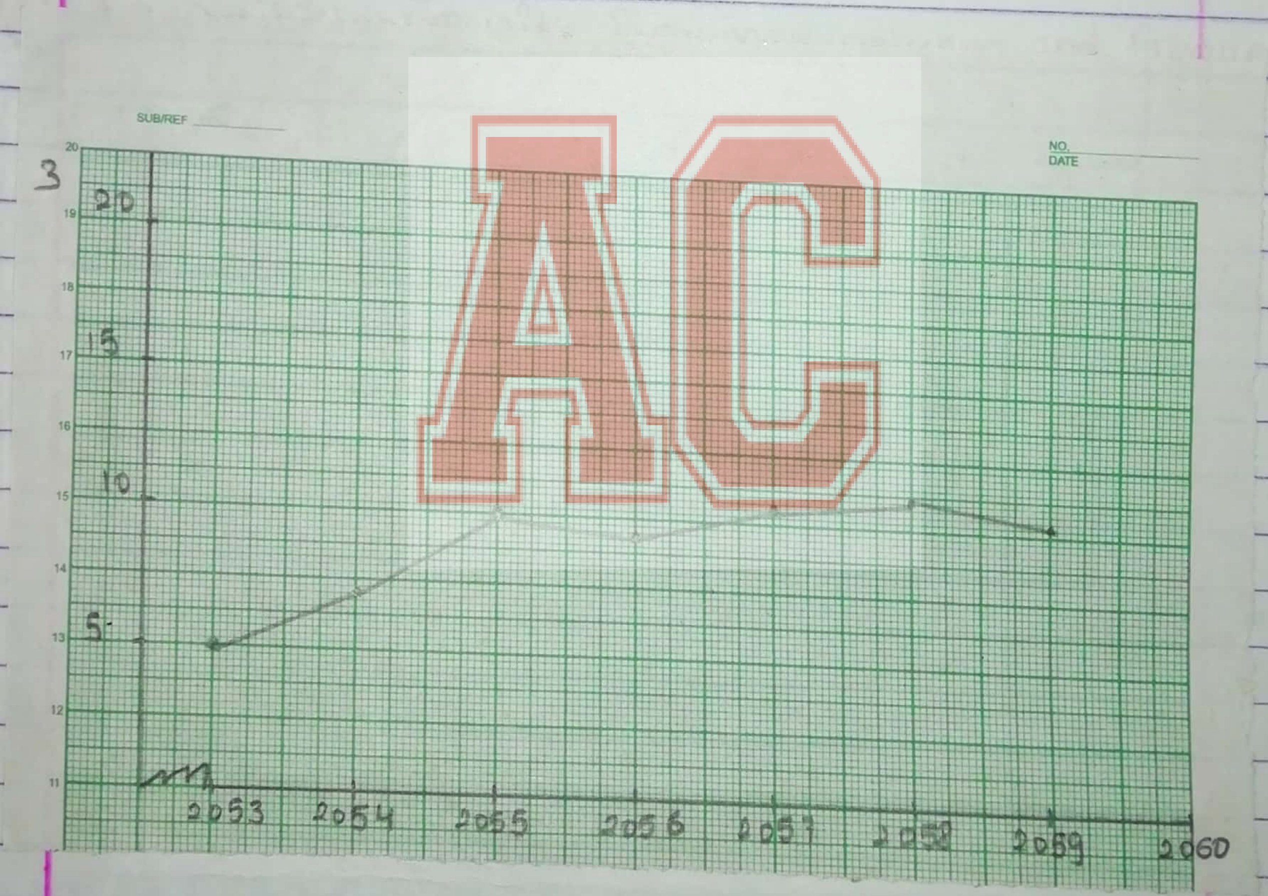
1) Draw frequency polygan of the data given.

Year	2053	2054	2055	2056	2057	2058	2059	2060
Sales(000)	5	9	10	7	12	14	11	20

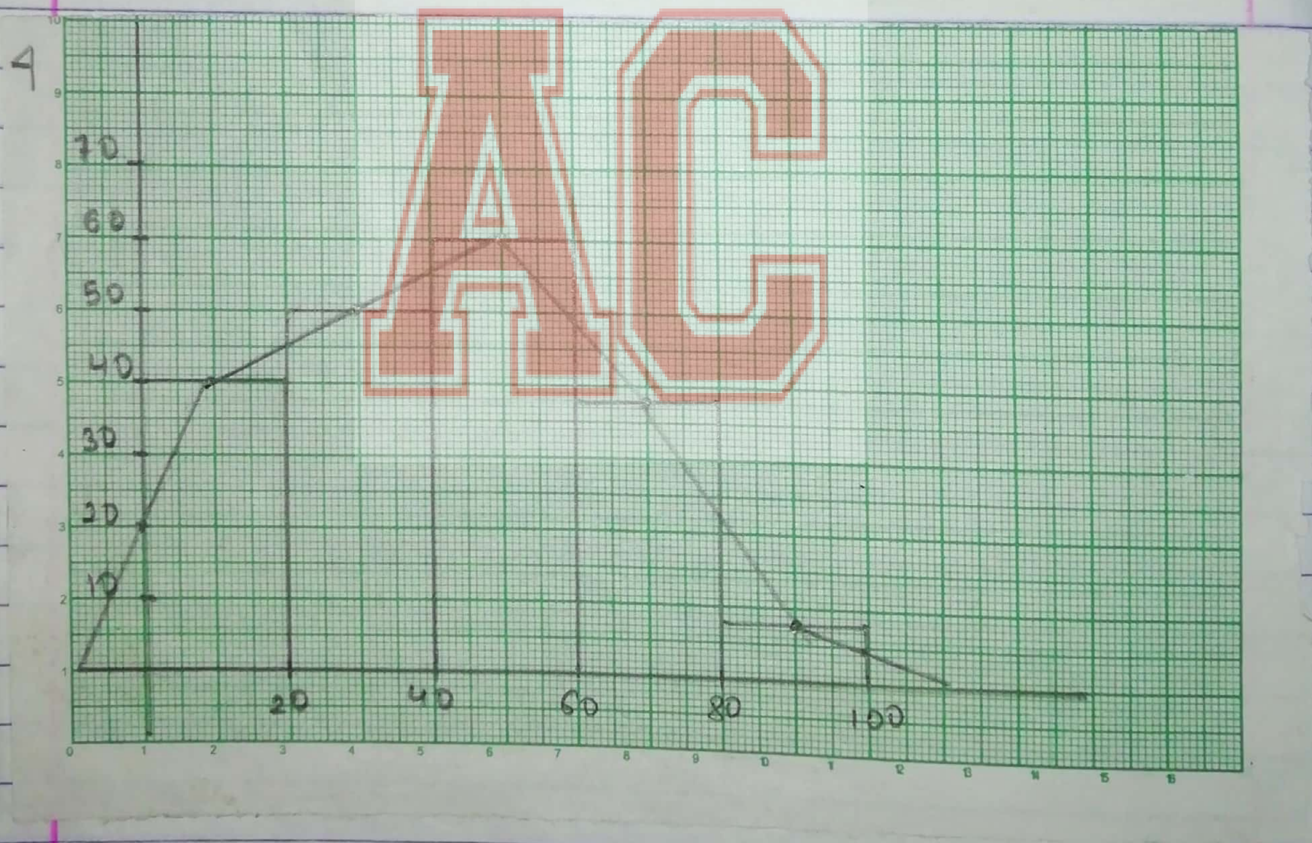
-> Given data is descrete frequency distribution.



2) Draw the histogram and the frequency polygon of the data.

Wages	0-20	20-40	40-60	60-80	80-100
No. of worker	40	51	64	38	7

Representing the above information by histogram



Steps to draw frequency polygon using Histogram

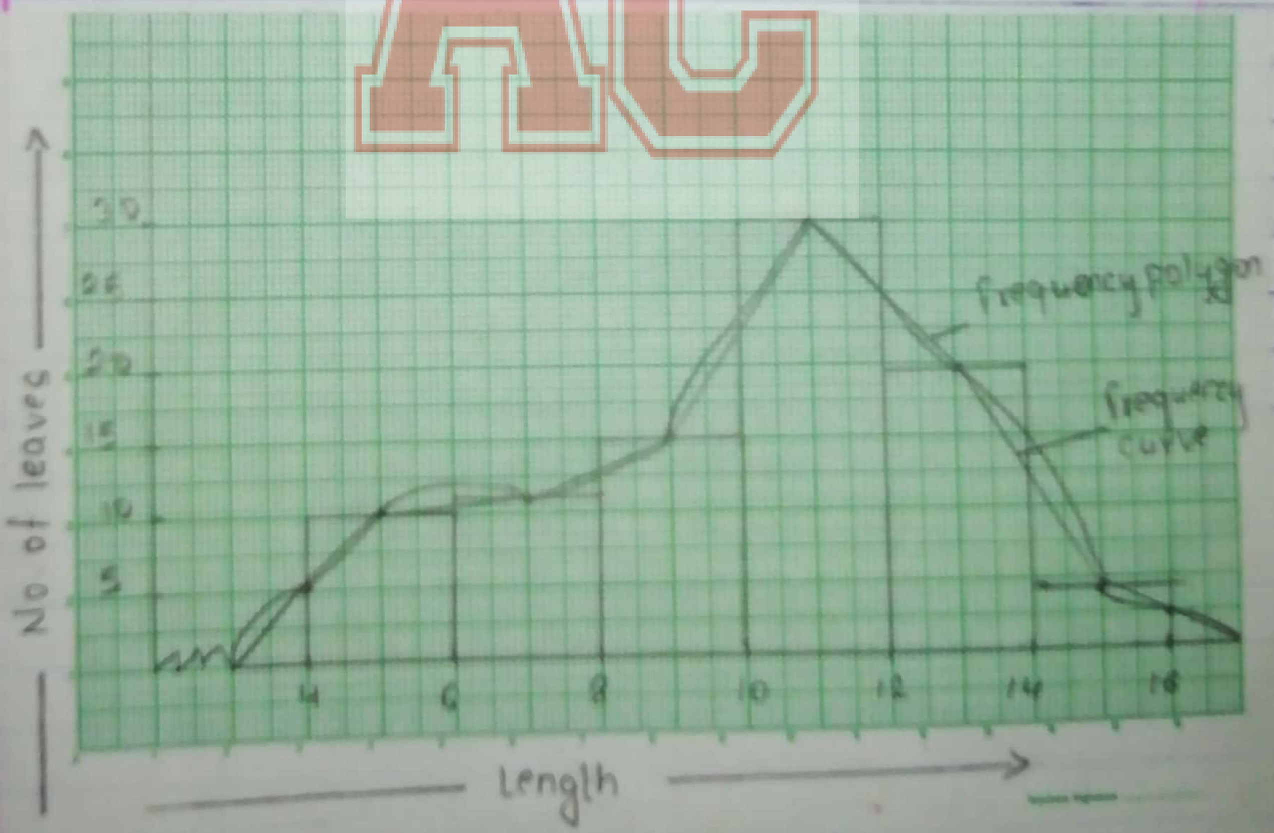
1) Draw a histogram
2) Find the mid-point of top of the rectangles.
3) Join the mid-points by straight line such that polygon must be closed

Frequency polygon

IMP

1) Draw a histogram with frequency polygon and frequency curve. Representing the following figure.

length (cm)	4-6	6-8	8-10	10-12	12-14	14-16
No. of leaves	10	[illegible]	16	30	18	8



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Ogive (cumalative frequency curve) -> Graphical representation of cumulative frequency distribution is called ogive they are of two types:-

1) less than ogive
2) More than ogive

i) Draw a less than ogive from the data given.

Length (cm)	0-10	10-20	20-30	30-40	40-50	50-60
No. of leaves	5	10	18	23	7	6

i) Find the number of leaves having length less than equal to 35 cm.
ii) Find the median.

First of all we prepare less than cumulative frequency distribution table.

Length (cm)	No. of leaves (f)	less than frequency	c.f
0-10	5	10	5
10-20	10	20	15
20-30	18	30	33
30-40	23	40	56
40-50	7	50	63
50-60	6	60	69
	N = 69		

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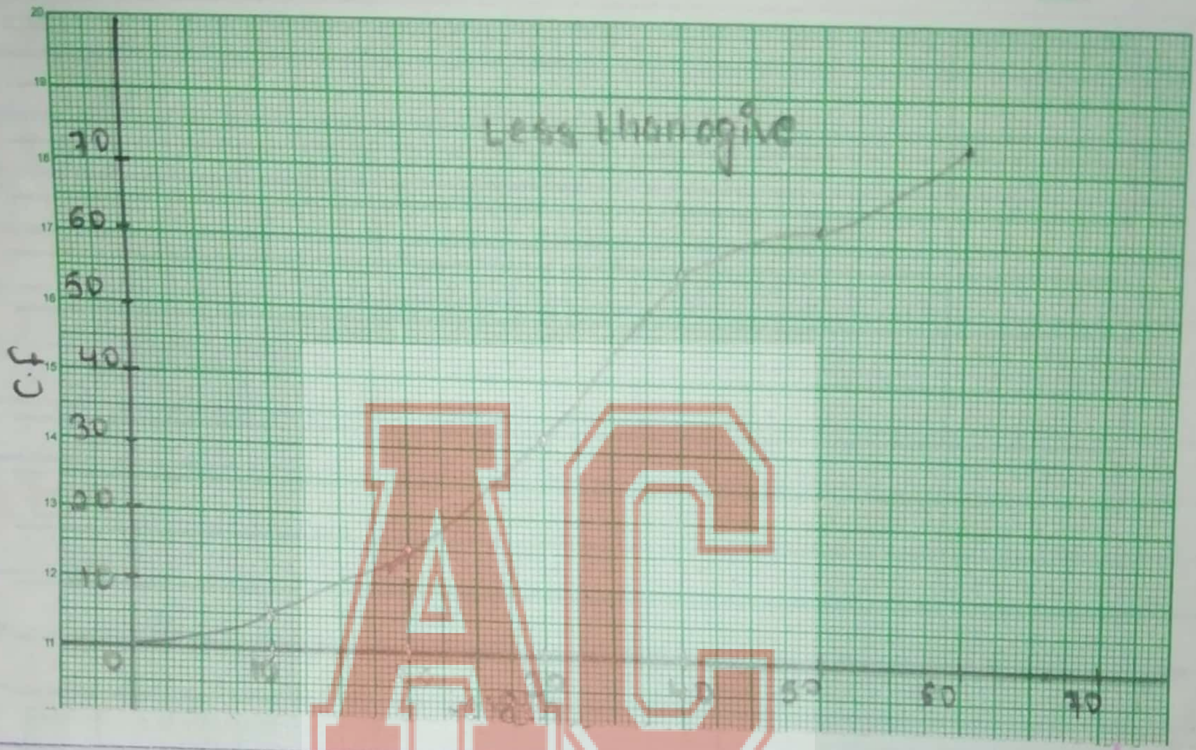

1) the number of leaves having length less than 35 are 44.

2) Here,

$$\frac{N}{2} = \frac{69}{2} = 34.5$$

$\therefore$ Median = 3

2) Prepare a more than frequency distribution ogive of the data given.

Length	0-10	10-20	20-30	30-40	40-50	50-60
No. of leaves	5	10	18	23	7	6

Length (cm)	No. of leaves (F)	more than frequency	c.f
0-10	5	0	69
10-20	10	10	64
20-30	18	20	54
30-40	23	30	36
40-50	7	40	13
50-60	6	50	6
	N = 69		

Note:- The intersection point of more than and less than ogive is median.

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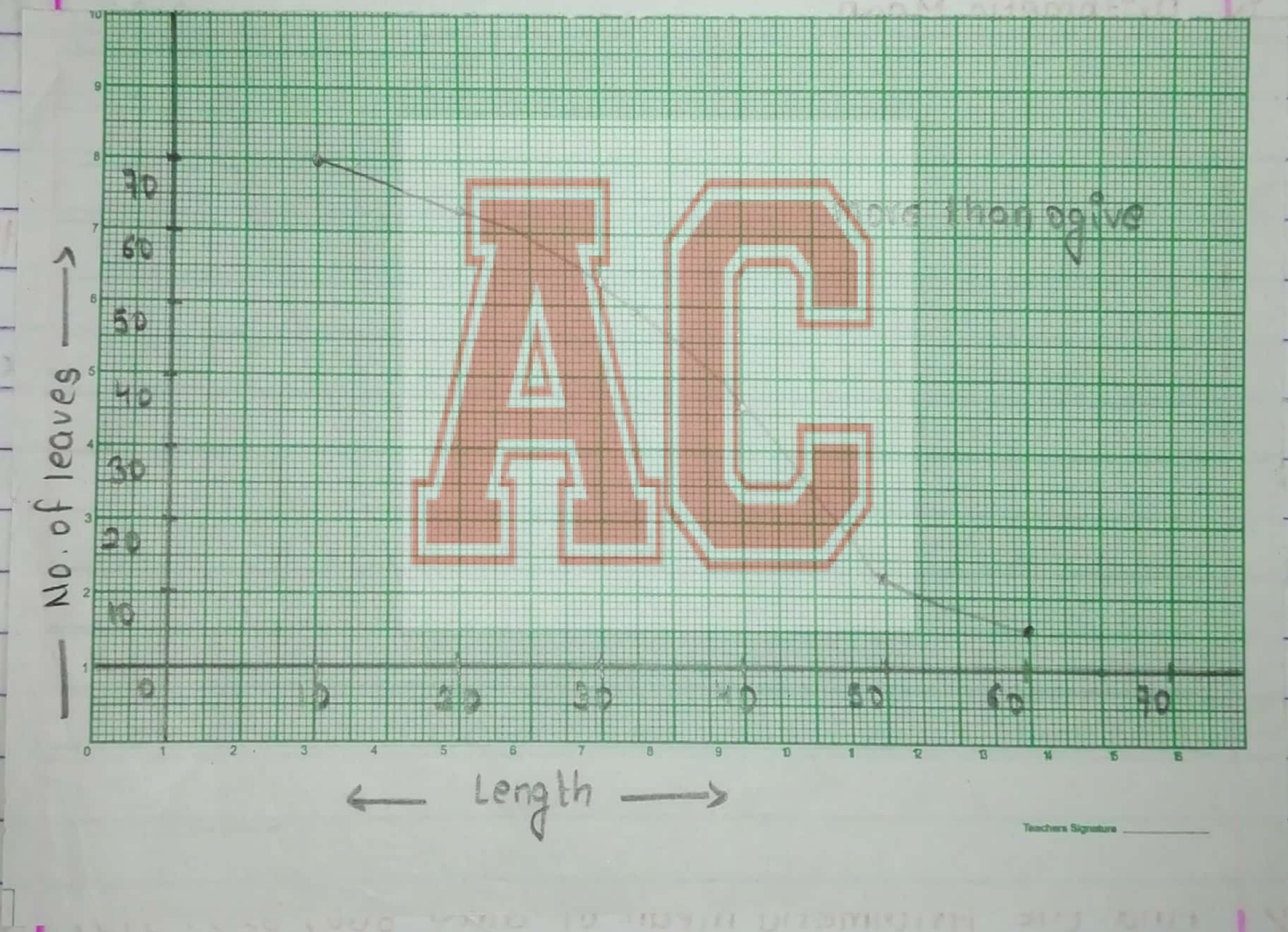

more than ogive
No. of leaves
Length
0
10
20
30
40
50
60
70
Teachers Signature

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Imp

Measurement of central tendency

1) Mean
2) Median
3) Mode

1) Mean
i) Arithmetic Mean

Some Formulas

	Direct	Shortcut	Step Deviation
Individual	$\bar{X} = \frac{\Sigma X}{N}$	$\bar{X} = a + \frac{\Sigma d}{N}$	$\bar{X} = a + \frac{\Sigma d'}{N} \times i$
Descrete	$\bar{X} = \frac{\Sigma FX}{N}$	$\bar{X} = a + \frac{\Sigma fd}{N}$	$\bar{X} = a + \frac{\Sigma fd'}{N} \times i$
Continous	$\bar{X} = \frac{\Sigma fm}{N}$	$\bar{X} = a + \frac{\Sigma fd}{N}$	$\bar{X} = a + \frac{\Sigma fd'}{N} \times i$

1) Find the Arithmetic mean of data 300, 325, 375, 400, 425, 500 (Individual series)

a) Use direct method
b) Use shortcut method
c) Use step deviation method

a) Here,

$\Sigma X = 300 + 325 + 375 + 400 + 425 + 500$

$= 2325$

$N = 6$

$\therefore$ Arithmetic mean $= \frac{\Sigma X}{N} = \frac{2325}{6} = 387.5$ #

b) Here,

Shortcut method

Let $a = 400$

a = assumed mean

d = deviation

X	$d = X - a$
300	-100
325	-75
375	-25
400	0
425	25
500	100
	$\Sigma d = -75$

$\therefore$ A.M $= a + \frac{\Sigma d}{N}$

$= 400 + \frac{(-75)}{6}$

$= 400 - 12.5$

$= 387.5$ #

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c) Step deviation method

$a = 400$

x	$d = x - a$	$d' = d/25$ (i)
300	-100	-4
325	-75	-3
375	-25	-1
400	0	0
425	25	1
500	100	4
	$\Sigma d = -75$	$\Sigma d' = -3$

Now,

$$AM = a + \frac{\Sigma d'}{N} \times i$$

$$= 400 + \frac{-3}{6} \times 25$$

$$= 400 - 12.5$$

$$= 387.5$$ #

2) Descrete

1) calculate mean (Arithmetic mean) of the data given by.

i) Direct method
ii) shortcut method
iii) Step Deviation

a) Direct Method

x	f	fx
5	4	20
6	6	36
7	12	84
8	10	80
9	8	72
	N=40	$\Sigma fx = 292$

$$\therefore AM = \frac{\Sigma fx}{N}$$

$$= \frac{292}{40}$$

$$= 7.3 \ \#$$

b) Shortcut method

Let $a = 7$

x	F	d = x - a	f.d
5	4	-2	-8
6	6	-1	-6
7	12	0	0
8	10	1	10
9	8	2	16
	N=40		$\Sigma fd = 12$

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$$\therefore AM = a + \frac{\Sigma fd}{N}$$

$$= 7 + \frac{12}{40}$$

$$= 7 + 0.3$$

$$= 7.3$$

c) Step deviation

$a = 7$

X	F	$d = x - a$	fd	$d' = d/i$
5	4	-2	-8	-2
6	6	-1	-6	-1
7	12	0	0	0
8	10	1	10	1
9	8	2	16	2
	N = 40			

$$\therefore AM = a + \frac{\Sigma fd'}{N} \times i$$

$$= 7 + \frac{12}{40} \times 1$$

$$= 7.3 \ \#$$

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2) If A.M is 7.3. find the missing frequency.

x	5	6	7	8	9
f	4	6	12	?	8

x	F	fx
5	4	20
6	6	36
7	12	84
8	?	$8x$
9	8	72
	$N = 30 + x$	$\Sigma fx = 212 + 8x$

$$\therefore AM = \frac{\Sigma fx}{N}$$

$$7.3 = \frac{212 + 8x}{30 + x}$$

or, $219 + 7.3x = 212 + 8x$

or, $219 - 212 = 8x - 7.3x$

or, $7 = 0.7x$

or, $x = \frac{7}{0.7}$

$\therefore x = 10$

#

3) Continous

1) Find the Arithmetic mean from the data given below.

C.I	15-25	25-35	35-45	45-55	55-65	65-75
f	4	11	19	14	0	1

a) Direct method

C.I	f	m	fm
15-25	4	20	80
25-35	11	30	330
35-45	19	40	760
45-55	14	50	700
55-65	0	60	0
65-75	1	70	70
	N = 49		$\Sigma fm = 1940$

$$\therefore \bar{X} = \frac{\Sigma fm}{N}$$

$$= \frac{1940}{49}$$

$$= 39.59$$ #

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b) Shortcut Method

C.I	f	m	let a = 40 d = m - a	fd
15-25	4	20	-20	-80
25-35	11	30	-10	-110
35-45	19	40	0	0
45-55	14	50	10	140
55-65	0	60	20	0
65-75	1	70	30	30
	N = 49			$\Sigma fd = -20$

$$\therefore AM = a + \frac{\Sigma fd}{N}$$

$$= 40 + \frac{(-20)}{49}$$

$$= 40 - \frac{20}{49}$$

$$= 40 - 0.41$$

$$= 39.59$$ #

c) Step deviation Method

C.I	F	m	Let a = 40 d = m - a	$d' = d/10$	fd'
15 - 25	4	20	-20	-2	-8
25 - 35	11	30	-10	-1	-11
35 - 45	19	40	0	0	0
45 - 55	14	50	10	1	14
55 - 65	0	60	20	2	0
65 - 75	1	70	30	3	3
	N = 49				$\Sigma fd' = -2$

Now,

$$AM = a + \frac{\Sigma fd'}{N} \times i$$

$$= 40 + \frac{(-2)}{49} \times 10$$

$$= 40 - \frac{2}{49} \times 10$$

$$= 40 - \frac{20}{49}$$

$$= 40 - 0.41$$

$$= 39.59$$ #

Geometric Mean

1) Individual series $G = \text{antilog}\left[\frac{\Sigma \log x}{N}\right]$

2) Discrete series $G = \text{antilog}\left[\frac{\Sigma f \cdot \log x}{N}\right]$

3) Continous series $G = \text{antilog}\left[\frac{\Sigma f \log x}{N}\right]$

Geometric mean of individual series

1) 300, 325, 375, 400, 425, 500

→ Here,

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x	log x
300	2.4771
325	2.5119
375	2.5740
400	2.6021
425	2.6284
500	2.6990
	$\Sigma \log x = 15.4925$

$$\therefore G.M(\bar{x}) = \text{antilog}\left[\frac{\Sigma \log x}{N}\right]$$

$$= \text{antilog}\left[\frac{15.4925}{6}\right]$$

$$= \text{antilog}\,[2.5821]$$

$$= 382.0322 \;\#$$

Descrete series

1)

x :	3	5	7	9
f :	20	40	30	10

x	f	log x	f log x
3	20	0.4771	9.5420
5	40	0.6990	27.9600
7	30	0.8451	25.3530
9	10	0.9542	9.5420
	N = 100		$\Sigma f.\log x = 72.3970$

$$\therefore GM(\bar{x}) = \text{antilog}\left[\frac{\Sigma f \log x}{N}\right]$$

$$= \text{antilog}\left[\frac{72.3970}{100}\right]$$

$$= \text{antilog}\,[0.723970]$$

$$= 5.2963$$ #

Continous series

1) Calculate Geometric mean of the given data.

Class	2-4	4-6	6-8	8-10
frequency	20	40	30	10

Solution,

calculation of Geometric mean

class	frequency	m	log m	F. log m
2-4	20	3	0.4771	9.5420
4-6	40	5	0.6990	27.9600
6-8	30	7	0.8451	25.3530
8-10	10	9	0.9542	9.5420
	N = 100			$\Sigma f.\log x = 72.3970$

$$\therefore GM(\bar{x}) = \text{antilog}\left[\frac{\Sigma f \log x}{N}\right]$$

$$= \text{antilog}\left[\frac{72.3970}{100}\right]$$

$$= \text{antilog}\,[0.723970]$$

$$= 5.2963$$

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Harmonic Mean

1) Individual series $(\bar{x}) = \frac{N}{\sum \frac{1}{x}}$

2) Discrete series $(\bar{x}) = \frac{N}{\sum f \times \frac{1}{x}} = \frac{N}{\sum \frac{f}{x}}$

3) Continous series $(\bar{x}) = \frac{N}{\sum \frac{f}{x}}$

0.0030

1) Calculate the Harmonic mean of the data.

Individual

x	$1/x$
300	0.0033
325	0.0031
375	0.0027
400	0.0025
425	0.0024
500	0.0020
N = 6	$\sum 1/x = 0.0160$

$\therefore H = \frac{N}{\sum 1/x}$

$= \frac{6}{0.0160}$

$= 375.00$ #

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Discrete

1) Calculate the H.M of the given data.

x :	3	5	7	9
F :	20	40	30	10

x	F	$\frac{1}{x}$	$f \times \frac{1}{x}$
3	20	0.3333	6.6660
5	40	0.2000	8.0000
7	30	0.1429	4.2870
9	10	0.1111	1.1100
	N = 100		$\Sigma f \cdot \frac{1}{x} = 20.0630$

$$\therefore HM = \frac{N}{\Sigma f \cdot \frac{1}{x}}$$

$$= \frac{100}{20.0630}$$

$$= 4.9843$$

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1) Calculate the H.M of the data below.

C.I	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
F	10	20	10	25	10	50	40	30

Here,

C.I	F	m	1/m	f/m
20-30	10	25	0.0400	0.4000
30-40	20	35	0.0286	0.5720
40-50	10	45	0.0222	0.2220
50-60	25	55	0.0182	0.4550
60-70	10	65	0.0154	0.1540
70-80	50	75	0.0133	0.6650
80-90	40	85	0.0118	0.4720
90-100	30	95	0.0105	0.3150
	N = 195			$\Sigma f/m = 3.2550$

Now,

$$H.M = \frac{N}{\Sigma F/m}$$

$$= \frac{195}{3.2550}$$

$$= 59.9078$$

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Imp

Median

1) Individual Data $M_d(Q_2) = \left(\frac{N+1}{2}\right)^{th}$ value

2) Descrete series $M_d(Q_2) = \left(\frac{N+1}{2}\right)^{th}$ value

3) Continous series $M_d = \frac{N}{2}^{th}$ value

$$M_d = L + \frac{(N/2 - C.f)}{F} \times h$$

1) Find the median.
Individual

Temp(°C)	27	32	39	42	44	44	46

→ Here,

$N = 7$

$\therefore Q_2 = \left(\frac{N+1}{2}\right)^{th}$ term

$= \left(\frac{7+1}{2}\right)^{th}$ term

$= 4^{th}$ term

$M_d = 42$

Data must be in the increasing order

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2) Descrete

Find the median.

Height (cm)	40	45	50	52	60	70
No. of plants	7	8	9	6	4	2

Height (x)	No. of plant	C.F
40	7	7
45	8	15
50	9	24
52	6	30
60	4	34
70	2	36
	N = 36	

Now,

$$M_d = \left(\frac{N+1}{2}\right)^{th} \text{ item}$$

$$= \left(\frac{37}{2}\right)^{th} \text{ item}$$

$$= 18.50^{th} \text{ item}$$

$$= 50$$

Imp

3) Continous

Find the median of the data given.

C.I	5-9	10-14	15-19	20-24	25-29
F	10	18	26	15	11

-> Given data is not a continous ; so, we make continous by subtracting factor from lower limit and adding factor to the upper limit.

i.e

$$\text{Factor} = \frac{\text{LL of preceding class} - \text{UL of this class}}{2}$$

$$= \frac{10-9}{2}$$

$$= 0.5$$

Now,

C.I	F	C.F
4.5 - 9.5	10	10
9.5 - 14.5	18	28
14.5 - 19.5	26	54
19.5 - 24.5	15	69
24.5 - 29.5	11	80
	N = 80	

Now,

$$Md = \left(\frac{N}{2}\right)^{th} \text{ value}$$

$$= \left(\frac{80}{2}\right)^{th} \text{ value}$$

$$= 40^{th} \text{ value}$$

Now using

$$Md = L + \frac{(N/2)^{th} - c.f}{f} \times h$$

$$= 14.5 + \frac{40-28}{26} \times 5$$

$$= 14.5 + \frac{12}{20} \times 5$$

$$= 14.5 + 2.30$$

$$= 16.8$$

~~Mode~~ IMP

Mode

Mode Frequently occuring item of frequency distribution is called mode. Mode can be calculated by Following methods.

1) By Inspection method
2) By grouping method
3) By graphical method
4) Empirical formula method

1) By inspection method -> In this method mode can be located / calculated by inspection.

i) Data may be individual data → Most frequently occuring item.

ii) Discrete data → Most frequently occuring item.

iii) Continous distribution → $M_0 = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

where, l = lower limit
f_1 = highest frequency
f_0 = preceding frequency (before f_1)
f_2 = After f_1
h = width of class

I) By Inspection

Example I

1) Find the mode of the distribution.
2, 5, 7, 3, 5, 5, 4, 5, 6

Values	Frequency
2	1
3	1
4	1
5	4
6	1
7	1

Here, most frequently occuring item is 5 which is repeated 4 times. Hence mode of the distribution is 5.

2) Find the mode of distribution.

Wages	100	150	200	300	400
No. of worker	20	25	30	24	12

Wages	No. of worker
100	20
150	25
200	30
300	24
400	12

From above table the most frequently occuring item is 200 which occurs 30 time hence mode is 200.

Imp
3) Find the mode of distribution.

Weight	30-40	40-50	50-60	60-70	70-80	80-90
frequency	18	37	45	27	15	8.

Since distribution is regular. so, mode can be calculated by method of inspection. Since most frequently occuring item lies in the interval. so, modal class is 50-60.

$\because l = 50, \; f_0 = 37, \; f_1 = 45, \; f_2 = 27, \; h = 10$

$$M_0 = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_1} \times h$$

$$= 50 + \frac{45 - 37}{2 \times 45 - 37 - 27}$$

$$= 50 + \frac{8 \times 10}{26}$$

$$= 53.07$$

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II) Grouping method

This method is used for discrete Frequency distribution, which is not regular,

x :	20	21	22	23	24	25	26	27	28	29
f :	6	9	4	2	10	8	7	5	1	3

Since, distribution is haphazard so, we use grouping method for the calculation of mode.

X	col_1	col_2	col_3	col_4	col_5	col_6
20	6	15				
21	9			19		
22	4	6	13		15	
23	2					16
24	10		12	20		
25	8	18			25	
26	7		15			20
27	5	12		13		
28	1		6		9	
29	3	4				

Analysis table

	20	21	22	23	24	25	26	27	28	29
					1					
					1	1				
						1	1			
				1	1	1				
					1	1	1			
						1	1	1		

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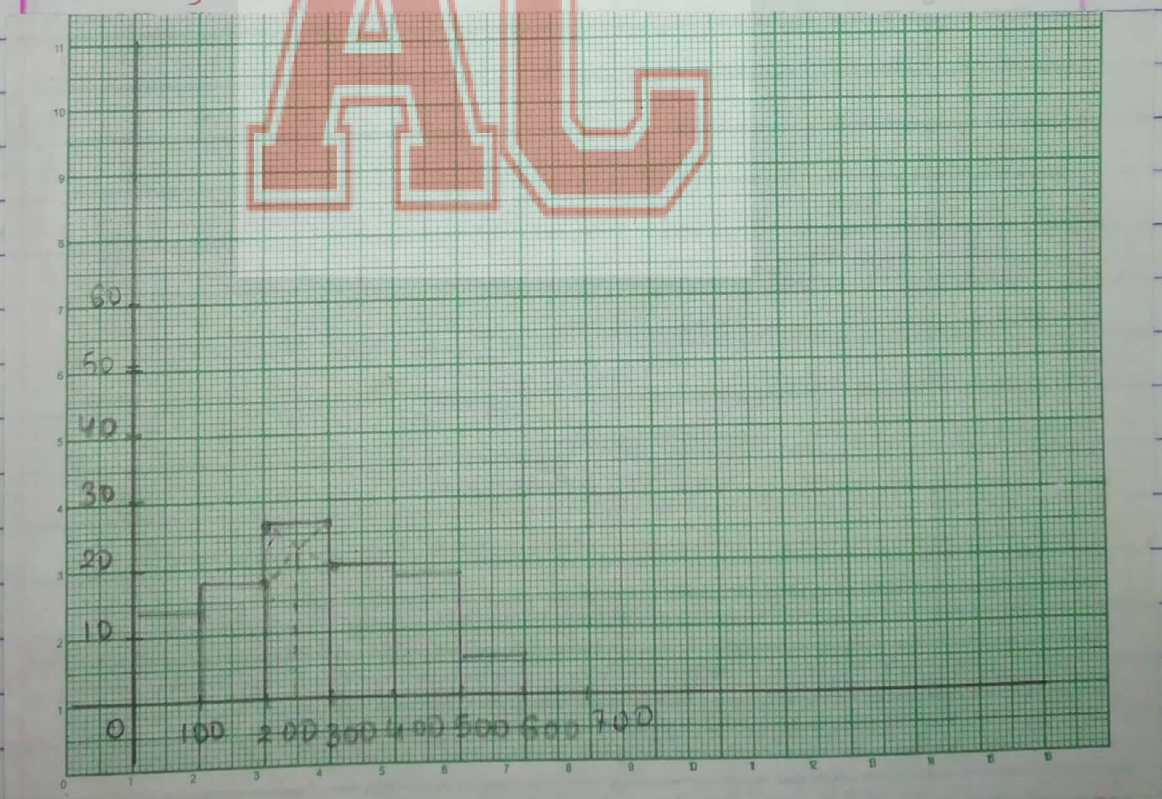
From analysis table we conclude that 25 repeat maximum number of times and hence mode is 25.

III) Graphical method

This method is used for continous distribution. In this method we draw a histogram and then calculated the mode. This method is alternative method of inspection method.

1) Calculate the mode graphically.

Quantity	0-100	100-200	200-300	300-400	400-500	500-600
Frequency	13	18	27	20	17	6



∴ Graphically mode of given distribution is 256.

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IV) Emphirical method

This method is used when there are more than one maximum repeated frequency.

The formula used for such calculation is

$$M_0 = 3Md - 2\bar{x}$$

1) Find the mode of given distribution.

5, 6, 7, 6, 7, 8, 6, 7, 9

(X) values	Frequency (F)	xf
5	1	5
6	3	18
7	3	21
8	1	8
9	1	9
	N = 9	$\Sigma x = 61$

Since most frequently occuring item are 6 and 7 both of which occurs 3 times. so the cannot have two modes for this we use empirical formula method.

Here,

$Md = \left(\frac{N+1}{2}\right)^{th}$ value

$= \cancel{3.5^{th} \text{ value}}$ 5^{th} value

$= 7$

and mean $(\bar{x}) = \frac{\Sigma x}{N}$

$= \frac{61}{9} = 6.78$

$\therefore$ Mode $(M_0) = 3Md - 2\bar{x}$

$= 3 \times 7 - 2 \times 6.178$

$= 7.44$

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Partition value

Partition value $(P_i) = \frac{iN}{K}$ for continous series

$= \frac{i(N+1)}{K}$ for Discrete and individual series

1) Find the $Q_1, Q_3, P_4, P_8, D_3, D_7$ of the following data.

900, 1000, 650, 1200, 860, 920

Arranging the data in ascending order.

650, 860, 900, 920, 1000, 1200

-> Now,

for $Q_i = \frac{i(N+1)}{K}$ th item

$Q_1 = \frac{1(6+1)}{4}$ th item

$= 1.75$th item

$\therefore Q_1 =$ first value + 0.75(second − first)

$= 650 + 0.75(860 - 650)$

$= 650 + 0.75 \times 210$

$= 807.5$

Now,

For $Q_3 = \frac{3(6+1)}{4}^{th}$ value

$= 5.25^{th}$ value

$\therefore Q_3 =$ fifth value $+ 0.25(6^{th} - 5^{th})$

$= 1000 + 0.25(1200 - 1000)$

$= 1000 + 0.25 \times 200$

$= 1050$

Again,

$P_4 = \frac{4(6+1)}{100}^{th}$ item

$= \frac{4 \times 7}{100}^{th}$ item

$= 0.28^{th}$ item

$= 0.28$ (first value $- 0$)

$= 0.28 \times 650$

$= 182$

Again,

$P_8 = \frac{8(6+1)}{100}^{th}$ item

$= \frac{8 \times 7}{100}^{th}$ item

$= 0.56^{th}$ item

$= 0.56$ (first value $- 0$)

$= 0.56 \times 650$

$= 364$

Again,

$D_3 = \frac{3(6+1)}{10}^{th}$ value

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$= \frac{3 \times 7}{10}$ th value

$= 2.1$th value

$= 2^{nd}$ value $+ 0.1(900 - 860)$

$= 860 + 0.1 \times 40$

$= 864$

Again,

$D_7 = \frac{7(6+1)}{10}$th item

$= \frac{7 \times 7}{10}$ th item

$= 4.9$th item

$= 920 + 0.9(1000 - 920)$

$= 920 + 0.9 \times 80$

$= 992$

Descrete

2) Find Q_2, D_5, P_{60} of the data given.

Monthly saving (Rs):	240	300	450	600	1000
No. of families :	25	16	20	24	15

Monthly saving	No. of families	C.F
240	25	25
300	16	41
450	20	61
600	24	85
1000	15	100
	N = 100	

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Now,

$Q_2 = \frac{2(100+1)}{4}$ th value

$= 50.5^{th}$ value

$= 450$

Again,

$D_5 = \frac{5(100+1)}{4}^{th}$ value

$= \left(\frac{5 \times 101}{4}\right)^{th}$ value

$= 50.5^{th}$ value

$= 450$

Again,

$P_{60} = \frac{60(100+1)}{100}^{th}$ value

$= \frac{60 \times 101}{100}$

$= 60.6^{th}$ value

$= 450$ #

Continous

3) Find Q_1, Q_3, P_1, P_{99}, D_1 and D_9 of the data given.

C.I	F	C.F
20-30	10	10
30-40	20	30
40-50	10	40

50 - 60	25	65
60 - 70	10	75
70 - 80	50	125
80 - 90	40	165
90 - 100	30	195
	N = 195	

$Q_1 = \left(\frac{1 \times 195}{4}\right)^{th}$ item

$= 48.75^{th}$ item

$= 50 - 60$

$\therefore l = 50, N = 195, c.f = 40, f = 25, h = 10$

$\therefore Q_1 = l + \frac{\left(\frac{N}{4} - c.f\right)}{f} \times h$

$= 50 + \frac{48.75 - 40}{25} \times 10$

$= 50 + \frac{8.75}{25} \times 10$

$= 50 + 3.5$

$= 53.5$

$Q_3 = \left(\frac{3 \times 195}{4}\right)^{th}$ item

$= 146.25^{th}$ value

$= 80 - 90$

$\therefore l = 80, N = 195, c.f = 125, f = 40, h = 10$

$\therefore Q_3 = 80 + \frac{146.25 - 125}{40} \times 10$

$= 80 + 5.31$

$= 85.31$ #

Again,

$P_1 = \left(\frac{1 \times 195}{100}\right)^{th}$ item

$= 1.95^{th}$ value

$= 20 - 30$

$\therefore L = 20, N = 195, c.f = 0, f = 10, h = 10$

$P_1 = L + \left(\frac{\frac{iN}{K} - c.f}{f}\right) \times 10$

$= 20 + \frac{1.95 - 0}{10} \times 10$

$= 20 + 1.95$

$= 21.95$

Again,

$P_{99} = \left(\frac{99 \times 195}{100}\right)^{th}$ value

$= \left(\frac{19305}{100}\right)^{th}$ value

$= 193.05^{th}$ value

$= 90 - 100$

$L = 90, c.f = 165, f = 30, h = 10$

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$$P_{99} = 90 + \frac{193.05 - 165}{30} \times 10$$

$$= 90 + 9.35$$

$$= 99.35$$

$$D_1 = \left(\frac{1 \times 195}{10}\right)^{th} \text{ value}$$

$$= \left(\frac{195}{10}\right)^{th} \text{ value}$$

$$= 19.50^{th} \text{ value}$$

$$= 30 - 40$$

$L = 30$, $C.F = 10$, $f = 20$, $h = 10$

$$D_1 = 30 + \frac{19.50 - 10}{20} \times 10$$

$$= 34.75$$

$$D_9 = \left(\frac{9 \times 195}{10}\right)^{th} \text{ value}$$

$$= 175.50^{th} \text{ value}$$

$$= 90 - 100$$

$L = 90$, $C.F = 165$, $f = 30$, $h = 10$

$$D_9 = 90 + \frac{175.5 - 165}{30} \times 10$$

$$= 93.50$$

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Measurement of dispersion

1) Range
2) Semi Quartile deviation
3) Mean deviation
4) standard deviation.
5) Lorent'z curve

1) Range

Range = L-S L = Largest value
S = smallest value

2) Coefficient of range = $\frac{L-S}{L+S}$

1) Find the range and coefficient of range of the data given below.

Day	S	M	T	W	Th	F	S
Temp(°c)	32	34	31	25	21	35	36

-> solution,
Largest value (L) = 36
smallest value (s) = 21

∴ Range = L - S
= 36 - 21
= 15

Now,

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Coefficient of Range $= \frac{L-S}{L+S}$

$= \frac{36-21}{36+21}$

$= 0.26$

2)

Height (cm) :	153	155	157	159	161	163
No. of student :	25	21	28	20	18	24

Here,

$L = 163$

$S = 153$

$\therefore R = L - S$

$= 163 - 153$

$= 10$

Now,

Coefficient of Range $= \frac{10}{316} = 0.03$ #

3)

C.I	F
0-4	7
4-8	7
8-12	10
12-16	15
16-20	7
20-24	6

$L = 24$

$S = 0$

$\therefore R = L - S$

$= 24 - 0 = 24$

Now, coeff. of Range $= \frac{24}{24}$

$= 1$

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?

Imp

i) From the information given below find.

i) which factory pays larger amount as daily wages.
ii) what is the average daily wages for the workers of two factory.

	Factory A	Factory B
No. of wage earners	250	200
Average daily wages	Rs 12	Rs 13.8

-> i) Here,

Given,

For A

$N_1 = 250$

$\bar{X}_1 = Rs\ 12$

Now,

$$\bar{X}_1 = \frac{\Sigma X_1}{N}$$

$$\text{or, } 12 = \frac{\Sigma X_1}{250}$$

$$\text{or, } \Sigma X_1 = Rs\ 3000$$

For B $N_2 = 200$

$\bar{X}_2 = 13.8$

Now,

$$\bar{X}_2 = \frac{\Sigma X_2}{N}$$

$$13.8 = \frac{\Sigma x_2}{200}$$

$$\Sigma X_2 = Rs\ 2760$$

∴ company A pays more than company B.

ii)

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ii) Average daily wage of the workers of both factories is

$$(\bar{x}) = \frac{N_1\bar{X}_1 + N_2\bar{X}_2}{N_1 + N_2}$$

$$= \frac{250 \times 12 + 200 \times 13.8}{250 + 200}$$

$$= \frac{3000 + 2760}{450}$$

$$= \frac{5760}{450}$$

$$= 12.8$$ #

Semi interquartile range / Quartile Deviation

$$Q.D. = \frac{Q_3 - Q_1}{2}$$

$$\text{coefficient of } Q.D. = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Note:- More the value of coefficient of Q.D. more will be variability.

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Imp

1) Find the Q.D of the data given below. (Individual)
25, 28, 32, 32, 36, 48, 44, 45, 50, 50

→ Arranging the data in ascending order

25, 28, 32, 32, 36, 44, 45, 48, 50, 50

Now,

$Q_1 = \frac{1(N+1)}{4}$

$= \frac{1 \times 11}{4}$

$= \frac{11}{4}$

$= 2.75$ item

$\therefore Q_1 = 28 + 0.75(32-28)$

$= 28 + 0.75 \times 4$

$= 31$

Similarly

$Q_3 = \frac{3(10+1)}{4}$

$= \frac{3 \times 11}{4}$

$= 8.25$

$\therefore Q_3 = 48 + 0.25(50-48)$

$= 48 + 0.25 \times 2$

$= 48.5$

$\therefore$ Quartile Deviation $Q.D = \frac{Q_3 - Q_1}{2}$

$= \frac{48.5 - 31}{2}$

$= 8.75$

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$\therefore$ coefficient of Q.D. $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

$= \frac{17.5}{79.5}$

$= 0.22$ #

Imp

2) You are given two variables A and B find which is more variable. (Discrete)

A

value	15	20	25	30	35	40	45
frequency	15	33	56	103	40	32	10

B

value	100	150	200	250	300	350	400
frequency	8	12	13	22	6	8	2

-> solution,

A			B		
value	frequency	c.f	value	frequency	c.f
15	15	15	100	8	8
20	33	48	150	12	20
25	56	104	200	13	33
30	103	207	250	22	55
35	40	247	300	6	61
40	32	279	350	8	69
45	10	289	400	2	71
	N=289			N=71	

Now,

Q_1 for A $= \frac{1(N+1)}{4}$ th item

$= \frac{289+1}{4}$

$= 72.5$ item

$\therefore Q_1 = 25$

and

$Q_3 = \frac{3(N+1)}{4}$ item

$= 217.50$ item

$\therefore Q_3 = 35$

Now,

coeff of Q.D. $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$

$= \frac{35-25}{35+25}$

$= \frac{10}{60}$

$= 0.17$

Now, Q_1 for B $= \frac{1(7+1)}{4}$ th item

$= 18$ item

$\therefore Q_1 = 150$

and $Q_3 = \left(\frac{3 \times 72}{4}\right)$ th item

$= \frac{216}{4}$ th item

$= 54$ th item

$\therefore Q_3 = 250$

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Now,

$$\text{coeff. of Q.D} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$$= \frac{250 - 150}{250 + 150}$$

$$= \frac{100}{400}$$

$$= 0.25$$

∵ coefficient of Q.D of B is greater than the coefficient of Q.D of A. Hence, B is more variable.

3) Continous

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C.I	0-10	10-20	20-30	30-40	40-50
F	5	8	15	16	6

Given data can be arranged as

C.I	F	C.F
0-10	5	5
10-20	8	13
20-30	15	28
30-40	16	44
40-50	6	50
	N=50	

Now,

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$$Q_1 = \left(\frac{N+1}{4}\right)^{th} \text{ item}$$

$$= \left(\frac{50+1}{4}\right)^{th} \text{ item}$$

$$= 12.75^{th} \text{ item}$$

here,

$l = 20$

$c.f = 13$

$f = 15$

$i = 10$

Now,

$$Q_1 = 20 + \frac{12.75 - 13}{15} \times 10$$

$$= 19.75$$

$$Q_3 = \left(\frac{3(N+1)}{4}\right)^{th} \text{ item}$$

$$= 3 \times 12.75^{th} \text{ item}$$

$$= 38.25^{th} \text{ item}$$

Here,

$l = 30$

$c.f = 28$

$f = 16$

$i = 10$

Now,

$$Q_3 = 30 + \frac{38.25 - 28}{16} \times 10$$

$$= 30 + 6.41$$

$$= 36.41$$

Now,

$$Q.D = \frac{Q_3 - Q_1}{2}$$

$$= \frac{36.41 - 19.75}{2}$$

$$= 16.66$$

$$\text{coefficient of } Q.D = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$$= \frac{36.41 - 19.75}{36.41 + 19.75}$$

$$= \frac{16.66}{56.16}$$

$$= 0.30$$

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Mean Deviation Imp

1) Individual series $M.D = \frac{\sum|X-\bar{x}|}{N}$

2) Discrete series $M.D = \frac{\sum f|X-\bar{x}|}{N}$

3) Continous series $M.D = \frac{\sum f|X-\bar{x}|}{N}$

4) Coefficient of $M.D = \frac{MD}{Mean}$

Note :- we can calculate mean deviation from mean $(\bar{x})$, from median, and Mode.

1) Find the mean deviation from both mean and median of the series.

40, 44, 60, 54, 62

-> Soln

Arranging the data in ascending order

40, 44, 54, 60, 62

Now,

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X	$X-\bar{X}$	$\lvert X-\bar{X}\rvert$	$X-Md$	$\lvert X-Md\rvert$
40	-12	12	-14	14
44	-8	8	-10	10
54	2	2	0	0
60	8	8	6	6
62	10	10	8	8
		$\Sigma\lvert X-\bar{X}\rvert=40$		$\Sigma\lvert X-M_d\rvert=38$

$\therefore$ Mean $(\bar{X}) = \frac{\Sigma X}{N} = \frac{260}{5} = 52$

(m.d)

$\therefore$ Mean deviation from Mean $= \frac{\Sigma\lvert X-\bar{X}\rvert}{N}$

$= \frac{40}{5}$

$= 8$

Again,

Median (Md) $= \left(\frac{N+1}{2}\right)^{th}$ item

$= \left(\frac{6}{2}\right)^{th}$ item

$= 3^{rd}$ item

$= 54$

(Md)

Mean deviation from median $= \frac{\Sigma\lvert X-Md\rvert}{N}$

$= \frac{38}{5}$

$= 7.6$

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$$\text{coefficient of MD} = \frac{MD}{\bar{X}}$$

$$= \frac{8}{52}$$

$$= 0.15$$

Again,

$$\text{coefficient of MD} = \frac{MD}{Md}$$

$$= \frac{8}{7.6}$$

$$= 0.14$$

2) Find the mean deviation from both mean and median of the Discrete series.

x :	10	11	12	13	14
f :	3	12	18	12	2

x	F	fx	Let a=12 x-a	|x-a|	f|x-a|	c.f	x-Md	|x-Md|	f|x-Md|
10	3	30	-2	2	6	3	-2	2	6
11	12	132	-1	1	12	15	-1	1	12
12	18	216	0	0	0	33	0	0	0
13	12	156	1	1	12	45	1	1	12
14	2	28	2	2	4	47	2	2	4
	N= 47	$\Sigma fx=562$			$\Sigma f\|x-a\|=34$				$\Sigma f\|x-Md\|=34$

$$\text{Mean } (\bar{X}) = \frac{\Sigma fx}{N} = \frac{562}{47} = 11.96$$

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$\therefore$ Mean deviation from mean $= \frac{\sum f|x-a|}{N}$

$= \frac{34}{47}$

$= 0.72$

coefficient of mean deviation $= \frac{MD}{Mean}$

$= \frac{0.72}{11.96}$

$= 0.06$

Now,

Median$(M_d) = \frac{(N+1)}{2}$th item

$= \left(\frac{48}{2}\right)$th item

$= 24$th item

$M_d = 12$

$f\|X-M_d\|$
6
12
0
12
4
$\sum f\|X-M_d\|$ $= 34$

Again,

Mean deviation from median $= \frac{\sum f|X-M_d|}{N}$

$= \frac{34}{47}$

$= 0.72$

$\therefore$ coefficient of MD $= \frac{MD}{M_d}$

$= \frac{0.72}{12}$

$= 0.06$ #

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3) Find the mean deviation from mean, median & mode of continous series.

Income	80 - 120	120 - 160	160 - 200	200 - 240	240 - 280
Frequency	30	25	20	15	10

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Standard Deviation(σ)

1) Individual series $\sigma = \sqrt{\frac{\Sigma(X-\bar{X})^2}{N}}$ (Direct method)

$$\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$ (shortcut method)

2) Descrete and continous $\sigma = \sqrt{\frac{\Sigma f(X-\bar{X})^2}{N}}$ (direct method)

$$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$$ (shortcutmethod)

3) coefficient of standard deviation S.D $= \frac{\sigma}{X}$

4) coefficient of variation $= \frac{\sigma}{\bar{X}} \times 100$

5) Variance $= (S.D)^2$

Note:- for consistency we compare C.V lesser than C.V more will be consistency.

1) Find the standard deviation and coefficient of standard deviation of the data & coefficient of variation (Direct)

100, 150, 200, 250, 400, 300, 350

-> Arranging data in ascending order

100, 150, 200, 250, 300, 350, 400

X	$X-\bar{X}$	$(X-\bar{X})^2$
100	-150	22500
150	-100	10000
200	-50	2500
250	0	0
300	50	2500
350	100	10000
400	150	22500
$\Sigma X = 1750$		$\Sigma(X-\bar{X})^2 = 70000$

mean $\therefore \bar{X} = \frac{\Sigma X}{N} = \frac{1750}{7} = 250$

$\therefore S.D(\sigma) = \sqrt{\frac{\Sigma(X-\bar{X})^2}{N}}$

$= \sqrt{\frac{70000}{7}}$

$= \sqrt{10000}$

$\sigma = 100$

coefficient of $S.D = \frac{\sigma}{\bar{X}} = \frac{100}{250} = 0.4$

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$$\text{coefficient of variation} = \frac{\sigma}{\bar{X}} \times 100$$

$$= 0.4 \times 100$$

$$= 40$$

2) Find the standard deviation & coefficient of standard deviation of the data and coefficient of variation. (shortcut)

32, 34, 31, 25, 21, 35, 36

→ Arranging data in ascending order
21, 25, 31, 32, 34, 35, 36

X	Let a = 32 d = x − a	d^2
21	−11	121
25	−7	49
31	−1	1
32	0	0
34	2	4
35	3	9
36	4	16
$\Sigma X = 214$	$\Sigma d = -10$	$\Sigma d^2 = 200$

$$\therefore \bar{X} = \frac{\Sigma X}{N} = \frac{214}{7} = 30.57$$

$$\sigma = \sqrt{\frac{\Sigma d^2}{N} - \left(\frac{\Sigma d}{N}\right)^2}$$

$$= \sqrt{\frac{200}{7} - \left(\frac{-10}{7}\right)^2}$$

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$= \sqrt{28.57 - 2.04}$

$= \sqrt{26.53}$

$= 5.15$

$\text{coeff. of (S.D)} = \frac{\sigma}{\bar{x}} = \frac{5.15}{30.57} = 0.17$

$\text{coeff. of variation} = \frac{\sigma}{\bar{x}} \times 100$

$= 0.17 \times 100$

$= 17$ #

3> Find the standard deviation & coefficient of standard deviation of the data and coefficient of variation by descrete series.

x	f	fx	Let $a=12$, $d=(x-a)$	fd	d^2	fd^2
10	3	30	-2	-6	4	12
11	12	132	-1	-12	1	12
12	18	216	0	0	0	0
13	12	156	1	12	1	12
14	2	28	2	4	4	8
	$N=47$	$\Sigma fx=562$		$\Sigma fd=-2$		$\Sigma fd^2=44$

$\therefore \bar{x} = \frac{\Sigma fx}{N} = \frac{562}{47} = 11.95$

$\sigma = \sqrt{\frac{\Sigma fd^2}{N} - \left(\frac{\Sigma fd}{N}\right)^2}$

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$$= \sqrt{\frac{44}{47} - \left(\frac{-2}{47}\right)^2}$$

$$= \sqrt{0.94 - (0.04)^2}$$

$$= \sqrt{0.92}$$

$$= 0.96$$

$\therefore$ coefficient of S.D $= \frac{\sigma}{\bar{x}} = \frac{0.96}{11.95} = 0.08$

$\because$ coefficient of variation $= \frac{\sigma}{\bar{x}} \times 100$

$= 0.08 \times 100$

$= 8$ #

4) Calculate the standard deviation of the data. (continous)

value	frequency
90 - 99	2
80 - 89	12
70 - 79	22
60 - 69	20
50 - 59	14
40 - 49	4
30 - 39	1

-> Given data can be arranged as

C.I	f	m	Let a=64.5 d = m - a	$d' = \frac{d}{10}$	fd'	d'^2	fd'^2
30-39	1	34.5	-30	-3	-3	9	9
40-49	4	44.5	-20	-2	-8	4	16
50-59	14	54.5	-10	1	-14	1	14
60-69	20	64.5	0	0	0	0	0
70-79	22	74.5	10	1	22	1	22
80-89	12	84.5	20	2	24	4	48
90-99	2	94.5	30	3	6	9	18
	N=75				$\Sigma fd' = 27$		$\Sigma fd'^2 = 127$

$$\therefore \bar{x} = a + \frac{\Sigma fd'}{N} \times i$$

$$= 64.5 + \frac{27}{75} \times 10$$

$$= 64.5 + 0.36 \times 10$$

$$= 64.5 + 3.6$$

$$= 68.10$$

$$\therefore \sigma = \sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)^2} \times h$$

$$= \sqrt{\frac{127}{75} - \left(\frac{27}{75}\right)^2} \times 10$$

$$= \sqrt{1.69 - (0.36)^2} \times 10$$

$$= \sqrt{1.5604} \times 10$$

$$= 12.49$$

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Coefficients of S.D $= \frac{\sigma}{\bar{X}} = \frac{12.49}{68.10} = 0.19$

coefficient of variation $= \frac{\sigma}{X} \times 100$

$= 0.19 \times 100$

$= 18.34$ #

Note:- varience is the square of S.D

$$\sigma^2 = \left(\sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)} \times 10\right)$$

$$\sigma = \left(\sqrt{\frac{\Sigma fd'^2}{N} - \left(\frac{\Sigma fd'}{N}\right)} \times 10\right)^2$$

$$\sigma = \frac{\Sigma fd'^2}{N} - \frac{\Sigma fd'}{N} \times 10$$

Lorent'z curve

lorent'z curve is graphical method for studying variations.

M.O. lorentz first use this method in the field of income and wealth.
Nowadays it is widely used in business to the distribution of profit, production and wages.

Steps

1) Cumulate the frequencies and express in percentage.
2) On x-axis start from 0-100 and take cumulative frequency%
3) On y-axis do same as 2 for next variate.
4) Join the (0,0) and (100,100) by a straight line.
5) Place all the co-ordinates next to the same graph and find the curve.

1) Represent the data graphically so as to bring out the inequilities in earning.

Earning	900	1000	1100	1200	1300	1400	1500	1600
A	33	30	24	21	18	12	9	3
B	32	22	15	12	10	6	2	1

Given data can be arranged as

Earning	A			B			Earning %	Earning %
	f	C.F	%	f	C.F	%		
900	33	33	22	32	32	32	900	9
1000	30	63	42	22	54	54	1900	19
1100	24	87	58	15	69	69	3000	30
1200	21	108	72	12	81	81	4200	42
1300	18	126	84	10	91	91	5500	55
1400	12	138	92	6	97	97	6900	69
1500	9	147	98	2	99	99	8400	84
1600	3	150	100	1	100	100	10,000	100

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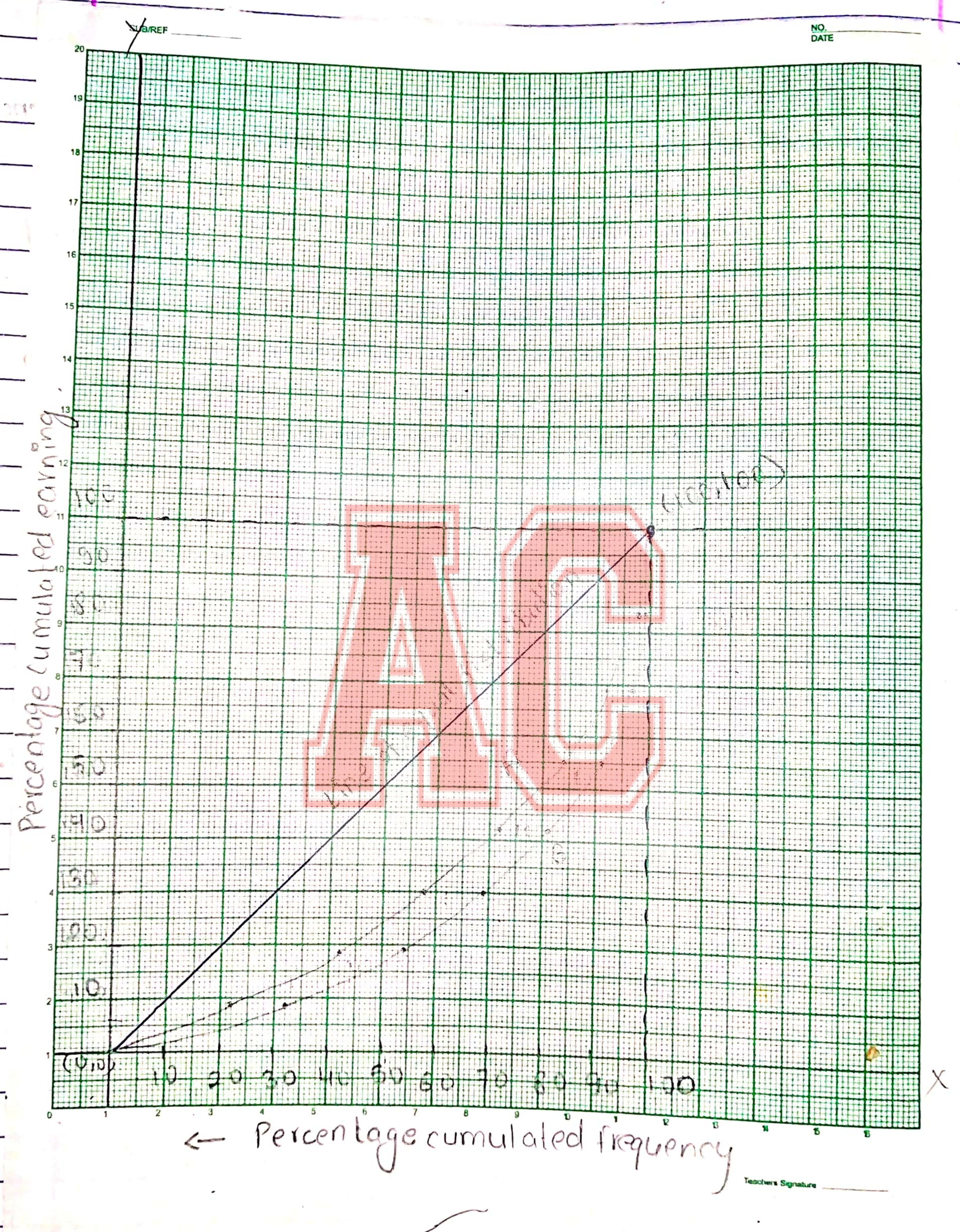

SUB/REF
NO.
DATE
Y
Percentage cumulated earning
100
90
80
70
60
50
40
30
20
10
(0,0)
(100,100)
10 20 30 40 50 60 70 80 90 100
X
← Percentage cumulated frequency
Teachers Signature

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Co-relation and regression

Co-relation -> (Two variables are said to have co-relation when they are so related that the change in the value of one variable is accompained by the change in another variable.) for example

1) amount of irrigation and amount of production are co-related.

2) Advertisement and expenditure are co-related.

Types of correlation

1) Positive and negative correlation -> If two variables vary in same direction is called positive correlation and if two variables vary in opposite direction then they are said to be negatively correlated.

Ex:- i) x : 10 20 25 50
y : 5 8 10 20
positive correlation

ii) x : 100 50 30 10
y : 8 5 3 2
positive correlation

iii) x : 10 20 25 50
y : 50 20 10 8
Negative correlation

iv) x : 100 50 30 10
y : 1 2 3 7
Negative correlation

2) Linear and Non-linear correlation -> If unit change in one variable causes the constant change in another variable such relation is linear corelation and if unit change in one variable has not constant change in another variable such relation is non-linear correlation.

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Example

1) x : 1 2 3 4 5
y : 25 40 55 70 85
Linear corelation

ii) x : 5 6 7 8 9
y : 500 600 800 900 1500
Non-linear corelation.

3) simple, multiple, partial correlation.

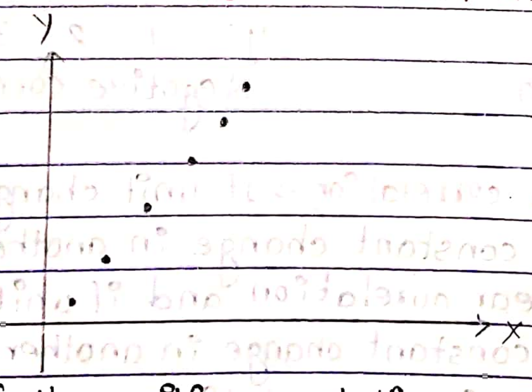
- Corelation between two variable is simple corelation.
- corelation between more than two variable is multiple corelation.
- corelation between two variables keeping other as a constant is partial correlation.

Methods of studying correlation (IMP)

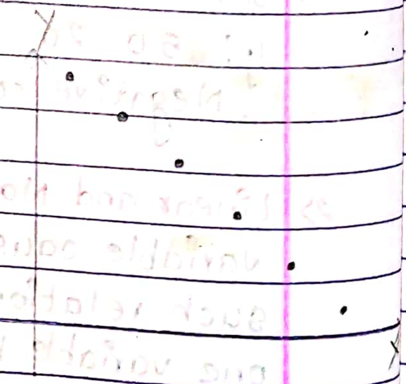
1) Scatter diagram
ii) Karl pearson's correlation coefficient.
iii) Spearman's rank corelation coefficient.

1) Scatter Diagram -> In this method two variables x & Y are plotted on a graph if the dots are closer and forms a straight line then there is heigher correlation. If dots are scattered than there is low correlation.

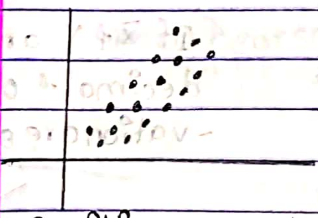
Some scatter diagram



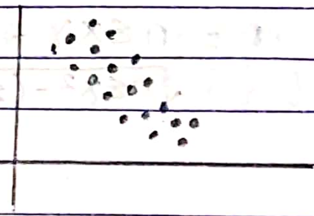
Perfectly positive correlation.



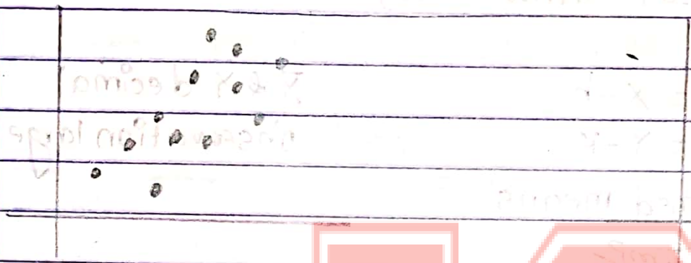
Perfectly negative correlati



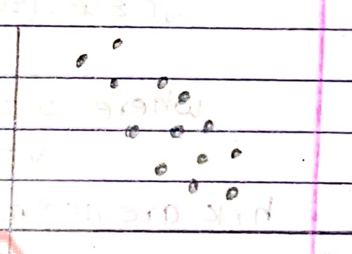
Positive correlation
(High degree)



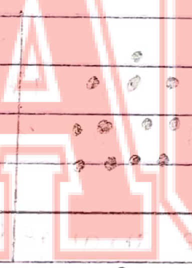
Negative correlation
(Low degree)



Low degree positive correlation



High degree Negative correlation



No correlation

2) Karl pearsons correlation coefficient imp

1) Correlation coefficient $(r) = \dfrac{\Sigma xy}{\sqrt{\Sigma x^2} \cdot \sqrt{\Sigma y^2}}$

where $x = (X - \bar{X})$
$y = (Y - \bar{Y})$

$= \dfrac{cov(X,Y)}{\sqrt{var(X)} \cdot \sqrt{var(Y)}}$

$\bar{X}$ & $\bar{Y}$ are not in decimal

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2) $$r = \frac{n\Sigma XY - \Sigma X \cdot \Sigma Y}{\sqrt{n\Sigma X^2 - (\Sigma X)^2} \cdot \sqrt{n\Sigma Y^2 - (\Sigma Y)^2}}$$

If $\bar{X}$ & $\bar{Y}$ are in decimal & observation are smaller

3) $$r = \frac{n\Sigma uv - \Sigma u.\Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2} \cdot \sqrt{n\Sigma v^2 - (\Sigma v)^2}}$$

where $u = x - h$

$v = Y - K$

h, K are assumed means

$\bar{X}$ & $\bar{Y}$ decimal observation large

Imp

1) Calculate Karl pearsons correlation coefficient of the data given.

X :	12	9	8	10	11	13	17
Y :	14	8	6	9	11	12	3

Given data can be arranged as

X	Y	XY	X^2	Y^2
12	14	168	144	196
9	8	72	81	64
8	6	48	64	36
10	9	90	100	81
11	11	121	121	121
13	12	156	169	144
17	3	51	289	9
$\Sigma X = 80$	$\Sigma Y = 63$	$\Sigma XY = 706$	$\Sigma X^2 = 968$	$\Sigma Y^2 = 651$

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$\therefore$ Karl pearsons correlation coeff. $(r) = \dfrac{n\sum XY - \sum X \cdot \sum Y}{\sqrt{n\sum X^2 - (\sum X)^2} \cdot \sqrt{n\sum Y^2 - (\sum Y)^2}}$

$$= \frac{7 \times 706 - 80 \times 63}{\sqrt{7 \times 968 - (80)^2} \cdot \sqrt{7 \times 651 - (63)^2}}$$

$$= \frac{4942 - 5040}{\sqrt{6776 - 6400} \cdot \sqrt{4557 - 3969}}$$

$$= \frac{-98}{\sqrt{376} \cdot \sqrt{588}}$$

$$= \frac{-98}{19.39 \times 24.24}$$

$$= \frac{-98}{470.18}$$

$r = -0.21$

Hence, There is weak negative correlation between variable.

Interpretation of (r)

Negative

1) If $r = -1$ => perfect -ve correlation.
2) If $r = -0.70$ => strong -ve correlation.
3) If $r = -0.50$ => moderate negative correlation.
4) If $r = 0$ => no correlation.
5) If $r = -0.30$ => weak -ve correlation.

Positive

1) If $r = 1$ => perfect +ve correlation.
2) If $r = 0.70$ => strong +ve correlation.
3) If $r = 0.50$ => moderate positive
4) If $r = 0.30$ => weak +ve correlation
5) If $r = 0$ => no correlation

25)

X:	12	9	8	10	11	3	17
Y:	14	8	6	9	11	12	3

Given data can be arranged as:

X	Y	$x = X - \bar{X}$	$y = Y - \bar{Y}$	xy	x^2	y^2
12	14	2	5	10	4	25
9	8	-1	-1	1	1	1
8	6	-2	-3	6	4	9
10	9	0	0	0	0	0
11	11	1	2	2	1	4
3	12	-7	3	-21	49	9
17	3	7	-6	-42	49	36
$\Sigma X = 70$	$\Sigma Y = 63$			$\Sigma xy = -44$	$\Sigma x^2 = 108$	$\Sigma y^2 = 84$

$$\bar{X} = \frac{\Sigma X}{N} = \frac{70}{7} = 10$$

$$\bar{Y} = \frac{\Sigma Y}{N} = \frac{63}{7} = 9$$

Now,

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2} \cdot \sqrt{\Sigma y^2}}$$

$$r = \frac{-44}{\sqrt{108} \cdot \sqrt{84}}$$

$$r = \frac{-44}{10.39 \cdot 9.16}$$

$$r = \frac{-44}{95.22}$$

$$r = -0.46$$

Hence there is moderate negative correlation between variables.

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3) Find the correlation coefficient between age & blood pressure given below.

Age(X) :	56	42	36	47	49	42	60	72	63	55
BP(Y) :	147	125	118	128	145	140	155	160	149	150

Given data can be arranged as

X	Y	u = X − h	v = Y − 140	uv	u^2	v^2
56	147	1	7	7	1	49
42	125	−13	−15	195	169	225
36	118	−19	−22	418	361	484
47	128	−8	−12	96	64	144
49	145	−6	5	−30	36	25
42	140	−13	0	0	169	0
60	155	5	15	75	25	225
72	160	17	20	340	289	400
63	149	8	9	72	64	81
55	150	0	10	0	0	100
$\Sigma X = 522$	$\Sigma Y = 1417$	$\Sigma u = -28$	$\Sigma v = 17$	$\Sigma uv = 1173$	$\Sigma u^2 = 1178$	$\Sigma v^2 = 1733$

$$r = \frac{n\Sigma uv - \Sigma u . \Sigma v}{\sqrt{n\Sigma u^2 - (\Sigma u)^2} \cdot \sqrt{n\Sigma v^2 - (\Sigma v)^2}}$$

$$r = \frac{10 \times 1173 - (-28) \times 17}{\sqrt{10 \times 1178 - (-28)^2} \cdot \sqrt{10 \times 1733 - (17)^2}}$$

$$r = \frac{12206}{\sqrt{11780 - 784} \cdot \sqrt{17330 - 289}}$$

$$r = \frac{12206}{13688.54}$$

$$r = 0.892 \; \#$$

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4) Find the co-relation coefficient from the data given below.

firms	1	2	3	4	5	6	7	8
sales	50	55	55	60	65	65	65	60
Expenses	11	13	14	16	16	15	15	14

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3) Rank correlation coefficient (spearman's rank correlation coefficient ρ)

In this method all observations are ranked from 1 to up of both the variables then the difference of rank is taken in a column and spearman's rank correlation coefficient can be calculated by

$$\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$$ (Rank's are not repeated).

$$\rho = 1 - \frac{6\left\{\sum d^2 + \frac{m(m^2-1)}{12} + \frac{m(m^2-1)}{12} + \dots\right\}}{n(n^2-1)}$$

1) find R from the data given.

Candidates	A	B	C	D	E	F
Rank of X	1	3	2	5	4	6
Rank of Y	2	1	3	6	4	5

Rank (X)	Rank (Y)	$d = R_1 - R_2$	d^2
1	2	-1	1
3	1	2	4
2	3	-1	1
5	6	-1	1
4	4	0	0
6	5	1	1
			$\sum d^2 = 8$

$$\therefore \rho = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 8}{(6^2-1)} = 0.77 \#$$

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2) Find the rank correlation coefficient of the data.

Year	1	2	3	4	5	6	7
Price	3.6	2.6	3.4	3.1	2.7	3.0	2.0
Rainfall	12	15	13	10	9	14	18

Price (X)	Rainfall (Y)	Rank of X (R_1)	Rank of Y (R_2)	$d = R_1 - R_2$	d^2
3.6	12	1	5	-4	16
2.6	15	6	2	4	16
3.4	13	2	4	-2	4
3.1	10	3	6	-3	9
2.7	9	5	7	-2	4
3.0	14	4	3	1	1
2.0	18	7	1	6	36
					$\Sigma d^2 = 86$

$$\therefore \rho = 1 - \frac{6 \Sigma d^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 86}{7(7^2-1)}$$

$$= 1 - \frac{516}{7 \times 48}$$

$$= 1 - 1.53$$

$$= -0.54 \#$$

3) Find the rank correlation coefficient.

Marks in accountancy (X) : 15 20 28 12 40 60 20 80

Marks in Mathematics (Y) : 40 30 50 30 20 10 30 60

X	Y	Rank of X (R_1)	Rank of Y (R_2)	$d = R_1 - R_2$	d^2
15	40	7	3	4	16
20	30	5.5	5	0.5	0.25
28	50	4	2	2	4
12	30	8	5	3	9
40	20	3	7	-4	16
60	10	2	8	-6	36
20	30	5.5	5	0.5	0.25
80	60	1	1	0	0
					$\Sigma d^2 = 81.5$

$$\therefore \rho = 1 - \frac{6\left\{\Sigma d^2 + \frac{m(m^2-1)}{12} + \frac{p(p^2-1)}{12}\right\}}{n(n^2-1)}$$

$$= 1 - \frac{6\left\{81.5 + \frac{2(2^2-1)}{12} + \frac{3(3^2-1)}{12}\right\}}{8(8^2-1)}$$

$$= 1 - \frac{6\{81.5 + 0.5 + 2\}}{540}$$

$$= 1 - 1$$

$$= 0$$

Permutation and combination

1) when all objects are different and placed in row.

$$^nP_r = P(n, r) = \frac{n!}{(n-r)!}$$

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2) when set of objects are repeated, if n objects are to be permutated in which 'p' of one kind 'q' of second kind 'r' of third kind then total number of permutation is $= \frac{n!}{p!q!r!}$

Q.N.1) Three persons enter a room and there are seven vacant seats in a line. Find in how many ways they can take their seats.

→ Given,

Total number of seats (n) = 7

Number of persons (r) = 3

∴ total number of ways $P(n,r) = \frac{n!}{(n-r)!}$

$= \frac{7!}{(7-3)!}$

$= \frac{7\times6\times5\times4!}{4!}$

$= 210$ ways

2) In how many ways the letter of the word "arrange" can be arranged?

→ Here,

total number of observation (n) = 7

(P) = 2

(q) = 2

∴ total number of ways $= \frac{n!}{p!q!} = \frac{7!}{2!2!}$

$= \frac{7\times6\times5\times4\times3\times\cancel{2!}}{2\times1\times\cancel{2!}}$

$= 1260$ ways

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3) In how many ways the number of three different can be formed from the letters 2, 3, 6, 8.

-> Since there are four digits:
Hundredth position can be choosen in 4 ways.
Tenth " " " " " 3 "
Ones " " " " " 2 "
∴ total ways = 4×3×2 = 24 ways.

Circular arrangement (n-1)!

Exercise I

1) A football stadium has four entrance gates and nine exists. In how many different ways can a man enter and leave the stadium.

-> Since there are four entrance gates and nine exit gates. A man can enter in the stadium 4 ways and leave the stadium in 9 ways therefore,
total number of ways = 4×9 = 36 ways.

2) There are 6 doors in a hotel. In how many ways can a guest enter the hotel and leave by different doors.

-> Since there are 6 doors in a hotel. A man can enter in hotel in 6 ways. A man can leave the hotel in 5 ways.
Total number of ways = 6×5 = 30 ways.

3) In how many ways can a man send 3 of his childrens in 7 different colleges of a certain term.

-> The choice for first children is 7. The choice for second children 6 ways and the choice for third is 5 ways.
∴ Total number of ways = 7×6×5 = 210 ways.

4) There are 5 roads between city A and B. In how many ways can a man go from a city to the other and return by a different road.

-> A man can go A to B city in 5 ways $P(5,1)$
A man can return to A from B = 4 ways $P(4,1)$
$\therefore$ total number of ways = $5 \times 4 = 20$ ways.

5) There are 5 main roads between A and B and 4 between B and C in how many ways a person drive from A to C and return without driving on the same road.

-> A man can go from A to B in 5 ways.
A man can go from B to C in 4 ways.

A man can return from C to B in 3 ways.
A man can return from B to A in 4 ways.
$\therefore$ total number of ways = $5 \times 4 \times 3 \times 4 = 240$ ways.

6) How many numbers of at least 3 different digits can be formed from the integers 1, 2, 3, 4, 5, 6

-> The ones place can be choosen in 6 ways.
The tens place can be choosen in 5 ways.
The hundredth place can be choosen in 4 ways.

H	T	O
4	5	6

Now,
total number of ways = $6 \times 5 \times 4 = 120$ ways.

7) How many number of 3 different digits less than 500 can be formed from the integars 1, 2, 3, 4, 5, 6.

-> The hundredth position can be choosen in 4 ways.
The tens position can be choosen in 5 ways.
The ones position can be choosen in 4 ways.

H	T	O
4	5	4

$\therefore$ total number of ways = $4 \times 5 \times 4 = 80$ ways.

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8) Of the numbers formed by using all the figures 1, 2, 3, 4, 5 one's how many of them are even.

-> The one's place can be choosen in 2 ways.
Ten's place can be choosen in 4 ways.
Hundredth place can be choosen in 3 ways.
Thousand place can be choosen in 2 ways.
Tenth thousand place can be choosen in 1 ways.
∴ Total ways = 2×4×3×2×1 = 48 ways.

9) How many numbers between 4000 and 5000 can be formed from the digits 2, 3, 4, 5, 6, 7.

-> Thousand place can be choosen in 1 ways.
Hundreth place can be choosen in 5 ways.
Ten's place can be choosen in 4 ways.
One's place can be choosen in 3 ways.
∴ Total number of ways = 1×5×4×3 = 60 ways.

T	H	T	O
1	5	4	3
4	(2,3,5,6,7)	(3,5,6,7)	(5,6,7)

10) How many different number of 3 digits can be formed from the integers 2, 3, 4, 5, 6? How many of them will be divisible by 5.

-> One's place can be choosen in 5 ways.
Ten's place can be choosen in 4 ways.
Hundredth place can be choosen in 3 ways.

H	T	O
3	4	5

∴ total number of ways = 5×4×3 = 60 ways

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To find the 3 digit divisible by 5.

One's place can be choosen in 1 ways.
Ten's place can be choosen in 4 ways.
Hundredth place " " " 3 "
∴ Total number of ways = 1×4×3 = 12 ways.

Exercise 2

1) Find the number of permutation of 5 different objects taken 3 at a time.

-> Soln

Number of objects (n) = 5
and taken item (r) = 3
total number of permutation $p(n,r) = \frac{n!}{(n-r)!}$

$= \frac{5!}{(5-3)!}$

$= \frac{5\times4\times3\times\cancel{2!}}{\cancel{2!}}$

$= 60$

2) If 3 person's enter a bus in which there are 10 vacant seats. find in how many ways they can sit?

-> Soln

total number of objects (n) = 10
Number of person (r) = 3
∴ total number of ways he can sit $p(n,r) = \frac{n!}{(n-r)!}$

$= \frac{10!}{(10-3)!}$

$= \frac{10!}{7!} = \frac{10\times9\times8\times\cancel{7!}}{\cancel{7!}} = 720$

Note $0! = 1$

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3) How many plates of vehicles consisting of 4 different digits can be made out of integers 4, 5, 6, 7, 8, 9.

-> Here,

total number of integers $(n) = 6$

Number of digits of plates of vehicles $(r) = 4$

$\therefore$ total number of ways $p(n,r) = \frac{n!}{(n-r)!}$

$$= \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6\times5\times4\times3\times\cancel{2!}}{\cancel{2!}} = 360 \text{ ways.}$$

4) In how many ways can four boys and 3 girl's be sitted in a row containing 7 seats.

a) If they may sit anywhere?

b) If the boys and girls must alternate?

c) If three girls are together?

-> Here,

a) There are 7 seats for 7 students. so they can sit

$$p(7,7) \text{ ways} = \frac{7!}{0!} = \frac{7!}{1!} = \frac{7\times6\times5\times4\times3\times2\times1!}{1!} = 5040$$

b) If boys and girls seat alternately

B -> G -> B -> G -> B -> G -> B

4 boys can sit in 4 seats

3 girls can sit in 3 seats.

$\therefore$ Total ways $= P(4,4) \times P(3,3)$

$$= \frac{4!}{(4-4)!} \times \frac{3!}{(3-3)!} = 144$$

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c) If 3 girls sit together than we assume 3 girls as one then,
there are 5 seats for 5 students.

Total ways $P(5,5) = \frac{5!}{(5-5)!}$

$= \frac{5!}{1!}$

$= \frac{5\times4\times3\times2\times1!}{1!}$

$= 120$

3 girls can sit in $P(3,3)$ ways $= \frac{3!}{1!}$

$= \frac{3\times2\times1!}{1!}$

$= 6$

$\therefore$ total number of ways $= 6\times120 = 720$ ways.

5) In how many ways can eight people be seated in a row of eight seats so that two particular persons are always together.

-> Since there are eight people and eight seats if two persons are always together then we assume these two as a one so they can be seated in 7 ways.

$= 7\times6\times5\times4\times3\times2\times1$

$= 5040$ ways

Two peoples can be seated in $2!$ ways.

$\therefore$ Total no. of ways $= 7!\times2!$

$= 5040\times2$

$= 10080$ ways

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6) Six different books are arranged on shelf. find the no. of different ways in which two particular books are

a) always together

b) not together (total - together)

→ solution,

a) Since there are six books to be placed on a shelf and two particular books always come together so we assume these two books as one. so they can be arranged in $5!$ ways

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$ ways

Two books can be arranged in $2!$ ways.

$\therefore$ Total no. of ways $= 5! \times 2!$

$= 120 \times 2$

$= 240$

b) Again,

six different books can be arranged $6!$ ways

$\therefore$ The no. of ways in which two particular books doesnot come together $= 6! - 240$

$= 720 - 240$

$= 480$ ways.

7) In how many ways can 4 red beads, 5 white beads, 3 blue beads can be arranged in a row.

→ Solution,

No. of red beads = 4

No. of white beads = 5

No. of blue beads = 3

Total no. of beads $= 4+5+3 = 12$

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$\therefore$ They can be arranged in a row $= \dfrac{n!}{p!q!r!}$

$= \dfrac{12!}{4! \times 5! \times 3!}$

$= \dfrac{479001600}{24 \times 120 \times 6}$

$= 27720$ ways

Imp

8) In how many ways can the letters of the following words be arranged.

a) ELEMENT

b) NOTATION

c) MATHEMATICS

d) MISSISSIPPI

→ Solution

a) Total letters(n) = 7

no. of E's (P) = 3

$\therefore$ Total no. of ways $= \dfrac{n!}{p!} = \dfrac{7!}{3!} = \dfrac{7 \times 6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}} = 840$ ways.

b) Total letters (n) = 8

no. of N's (p) = 2

no. of O's (q) = 2

no. of T's (r) = 2

$\therefore$ Total no. of ways $= \dfrac{n!}{p!q!r!} = \dfrac{8!}{2! \times 2! \times 2!}$

$= \dfrac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2!}{2! \cdot 2! \cdot 2!}$

$= 5040$ ways

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d) Total number of letter(n) = 11
No. of repetation S's (p) = 4
No. of repetation I's (q) = 4
No. of repetation P's (r) = 2

$\therefore$ Total no. of ways $= \frac{n!}{p!q!r!} = \frac{11!}{4!4!2!}$

$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4!}{4!4!2!}$

$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 2}$

$= \frac{1663200}{48} = 34650.$

9) How many numbers of 6 digit can be formed with the digits 2, 3, 2, 0, 3, 3 ?

→ Soln

Here, no. of digits (n) = 6
no. of 2's (p) = 2
no. of 3's (q) = 3

$\therefore$ Total no. of ways $= \frac{n!}{p!q!} = \frac{6!}{2! \times 3!} = 60$ ways.

10) How many ways 4 art students and 4 science student be arranged in a circular table.
a) If they may sit anywhere.
b) If they sit alternately.

→ Soln

a) Total no. of students (n) = 8
If they may sit anywhere they can be arranged in $(n-1)!$
$= (8-1)! = 7! = 7\times6\times5\times4\times3\times2\times1 = 5040$ ways.

b) 4 art students can occupy 4 sits & 4 science students can occuppy 4 sits so if they sit alternately they can be arranged in $(4-1)! \times 4!$
$= 3! \times 4!$
$= 3\times2\times1\times4\times3\times2\times1$
$= 144$ ways

11) In how many ways can 8 people be sited in a round table if 2 people insists in sitting next to each other?
-> Soln,
Since two people insists to sit next to each other so we assume these two as a one. Hence total no. of circular arrangement $= 6!$
$= 6\times5\times4\times3\times2\times1$
$= 720$ Ways
Two people can be sited in $2!$ ways
$\therefore$ total no. of ways $= 6! \times 2!$
$= 720\times2$
$= 1440$ ways.

12) In how many ways can 7 different coloured beads be made into a braslet?
NOTE :- In the case the braslet clockwise and anticlockwise
so total ways $= \frac{\text{---}}{2}$
-> soln

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No. of beads (n) = 7
No. of arrangement = $(n-1)! = (7-1)! = 6!$
Here,
distinction between clockwise and anticlockwise are same so total no. of arrangement = $\frac{6!}{2} = \frac{6\times5\times4\times3\times2\times1}{2} = 360$

13a) In how many ways 4 letters be posted in 6 letter boxes?
b) How many even no. of 3 digits can be formed when repeat-ation is allowed?
c) In how many ways 3 prizes be distributed among 4 students so that each student may receive any no. of prizes?
→ Soln
a) 4 letters be posted in 6 letter boxes in 6^4 ways
= 1296 ways

b) One's place can be choosen in 5 ways.
Ten's place can be choosen in 10 ways.
Hundred's place can be choosen in 9 ways.
∴ Total no. of ways = $5\times10\times9$
= 450 ways.

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14) In how many ways can the letters of the word 'MONDAY' be arranged? How many of these arrangements do not begin with M? How many begin with M and donot end with Y?

→ Soln:

The letters of the word MONDAY can be arranged $= 6!$
$= 720$ ways.

Now,
the no. of arrangement that begins with M are
$1 \times 5 \times 4 \times 3 \times 2 = 120$
∴ no. of arrangement that do not begin with M $= 720 - 120$
$= 600$ ways

Here,
No. of arrangements that begins with M and end with Y $= 4 \times 3 \times 2 \times 1$
$= 24$
∴ Total arrangements that begins with M and donot end with Y $= 120 - 24 = 96$ ways.

15) show that the number of ways in which the letter of the word:

a) 'COLLEGE' can be arranged so that 2 E's always come together is 360.

b) 'ARRANGE' can be arranged so that 2 R's donot come together is 900.

→ Soln.

a) we consider 2E's as one so that total no. of ways
$= \frac{6!}{2!} = 360$

proved

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b) There are 7 letters in the word 'ARRANGE' so they can be arrange in $\frac{7!}{2!2!}$ ways $= 1260$ ways.

If two R's come together then they can be arranged in $\frac{6!}{2!} = 360$ ways.

$\therefore$ Two R's do not come together $= 1260 - 360$
$= 900$ ways

proved

1) Arrangement -> Permutation
2) selection -> combinition
3) $c(n,r) \rightarrow \frac{n!}{(n-r)!\cdot r!}$

$c(5,5) \rightarrow \frac{5!}{(5-5)!\cdot 5!} = 1$

$c(5,0) \rightarrow \frac{5!}{(5-0)!\cdot 0!} = \frac{5!}{5!} = 1$

$c(10,9) \rightarrow \frac{10!}{(10-9)!\,9!} = \frac{10\times 9!}{1\times 9!} = 10$

$c(10,1) \rightarrow \frac{10!}{(10-1)!\,1!} = \frac{10\times 9!}{9!\times 1} = 10$

$c(8,1) = \frac{8!}{(8-1)!\,1!} = \frac{8\times 7!}{7!\times 1!} = 8$

Exercise

1) A boy puts his hand into a bag which contains 10 different coloured marbles and brings out 3. How many different results are possible?

-> Here,

No. of balls(n) = 10

No. of selected balls(r) = 3

Possible results = c(n,r)

= c(10,3)

$$c(10,3) = \frac{10!}{(10-3)! \times 3!}$$

$$= \frac{10 \times 9 \times 8 \times 7!}{7! \times 3!}$$

= 120

2) Find the number of ways in which a student can select 5 courses out of 8 courses. If 3 courses are compulsary in how many ways the selection be made?

-> Here,

Total no. of courses (n) = 8

No. of selected courses (r) = 5

No. of ways = c(n,r)

= c(8,5)

$$c(8,5) = \frac{8!}{(8-5)! \times 5!}$$

$$= \frac{8 \times 7 \times 6 \times 5!}{3 \times 2 \times 5!}$$

= 56

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If 3 courses are compulsary than selection of 2 be made from remaining 5 courses.
therefore total no. of ways $= C(5,2)$

$$= \frac{5!}{(5-2)! \, 2!}$$

$$= \frac{5 \times 4 \times 3!}{3! \times 2!}$$

$$= \frac{20}{2}$$

$$= 10$$

3) From at 10 persons in how many ways can a selection of 4 be made.
i) when 1 particular person is always included.
ii) when 2 particular person are always excluded.

-> Here,
total number of person $(n) = 10$
Person to be selected $(r) = 4$

i) when one particular person is always included then selection of 3 is to be made out of 9.
therefore total no. of ways $= C(9,3)$

$$= \frac{9!}{(9-3)! \times 3!}$$

$$= \frac{9 \times 8 \times 7 \times 6!}{6! \times 3 \times 2 \times 1}$$

$$= 84$$

ii) when 2 persons are also excluded than selection of 4 to be made out of 8.
Total no. of ways $= C(8,4)$

$$= \frac{8!}{(8-4)! \times 4!} = \frac{1680}{4 \times 3 \times 2 \times 1} = 70$$

4) A bag contains 8 white balls and 5 blue balls. In how many ways can 5 white balls and 3 blue balls be drawn.

-> Here,

No. of white ball = 8
No. of blue ball = 5
No. of white ball to be select = 5
No. of blue to be select = 3

then,

total no. of ways = $C(8,5) \times C(5,3)$

$$= \frac{8!}{(8-5)! \times 5!} \times \frac{5!}{(5-3)! \times 3!}$$

$$= \frac{336}{3\times 2} \times \frac{20}{2}$$

$$= 560$$

5) From a group of 11 men and 8 women, how many committees consisting of 3 men and two women are possible.

-> Soln.

No. of men = 11
No. of women = 8
No. of selected man = 3
No. of selected woman = 2

Now,

Total No. of committees = $C(11,3) \times C(8,2)$

$$= \frac{11!}{(11-3)! \cdot 3!} \times \frac{8!}{(8-2)! \, 2!}$$

$$= \frac{11\times 10\times 9\times \cancel{8!}}{\cancel{8!} \cdot 3!} \times \frac{8\times 7\times \cancel{6!}}{\cancel{6!} \cdot 2!}$$

$$= 165 \times 28$$

$$= 4620$$

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6> A person has got 12 acquaintances of whom 8 are relatives in how many ways can he invite 7 guests so that 5 of them may be relatives?

-> Here,

Total no. of acquaintances = 12

No. of relatives = 8

other than relatives = 12 - 8 = 4

If he has to invite 7 guests out of which 5 are relatives then 2 of 4 to be select also.

$\therefore$ Total no. of ways $= C(8,5) \times C(4,2)$

$= \frac{8!}{(8-5)! \times 5!} \times \frac{4!}{(4-2)! \times 2!}$

$= \frac{336}{6} \times \frac{24}{4}$

$= 336$

7> There are 10 electric bulbs of a shop out of which three are defectives. In how many ways can a selection of 6 be made so that 4 of them may be good bulbs?

-> Given,

total no. of bulbs = 10

Defective bulbs = 3

Good bulbs = (10 - 3) = 7

Bulbs to be selected = 6

selection of good bulbs = 4

selection of defective bulbs = 2 (6 - 4)

Now, total number of ways $= C(7,4) \times C(3,2)$

$= \frac{7!}{(7-4) \times 4!} \times \frac{3!}{(3-2)! \times 2!}$

$= \frac{7 \times 6 \times 5 \times 4!}{3! \times 4!} \times \frac{3!}{2!} = 105.$

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8) From 6 gentleman and 4 ladies a committee of 5 is to be formed? In how many ways can this be done so as to include at least 1 lady?

→ Given,

Number of gentle man = 6

Number of ladies = 4

Number of selection = 5

Number of ladies = at least one.

such committee can be formed in following ways.

i) Gentleman 4 and lady 1

ii) Gentleman 3 and ladies 2

iii) Gentleman 2 and ladies 3

iv) Gentleman 1 and ladies 4

$= C(6,4) \times C(4,1) + C(6,3) \times C(4,2) + C(6,2) \times C(4,3) + C(6,1) \times C(4$

$= 15 \times 4 + 20 \times 6 + 15 \times 4 + 6 \times 1$

$= 60 + 120 + 60 + 6$

$= 246$

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9) A candidate required to answer 6 out of 10 questions which are divided into 2 groups each containing 5 question and he is not permitted to attemp more than 4 from any group. In how many different ways can he make of his choice?

-> Here,

Total no. of questions = 10
No. of questions in each group = 5
No. of questions to be answered = 6
The selection of questions be made as below.
i) 2 from first and 4 from second.
ii) 3 from first and 3 from second.
iii) 2 from first and 2 from second.

Total no. of ways = $c(5,2) \times c(5,4) + c(5,3) \times c(5,3) + c(5,4) \cdot c(5,2)$

$= 10 \times 5 + 10 \times 10 + 5 \times 10$

$= 50 + 100 + 50$

$= 200$

10> A man has 5 friends in how many ways can he invite one or more of them toward dinner?

-> Here,

Total no. of friends = 5

He can invite his friends as below.

i> only 1 friend
ii> only 2 friends
iii> only 3 friends
iv> only 4 friends
v> only 5 friends

$\therefore$ Total number of ways $= c(5,1) + c(5,2) + c(5,3) + c(5,4) + c(5,5)$

$= 5 + 10 + 10 + 5 + 1$

$= 31$

11> From 4 mathematician, 6 statistician and five economist how many committees of 6 members can be formed so as to include two members from each category.

-> soln

No. of mathematician(n) = 4

No. of statistician (n) = 6

No. of Economist (n) = 5

No. of selected person from each(r) = 2

Now,

Total no. of commitees $= c(4,2) \times c(6,2) \times c(5,2)$

$= \frac{4!}{(4-2)!\,2!} \times \frac{6!}{(6-2)!\,2!} \times \frac{5!}{(5-2)!\,2!}$

$= \frac{4\times3\times2!}{2!\,.2!} \times \frac{6\times5\times4!}{4!\,.2!} \times \frac{5\times4\times3!}{3!\,.2!}$

$= 6 \times 15 \times 10$

$= 900$

Imp 12> How many commetees can be form from a set of 7 boys and five girls if each committees contains 4 boys and three girls?

-> Soln

No. of boys = 7

No. of girls = 5

No. of selected boys = 4

No. of selected girls = 3

Now,

total no. of commettees = $C(7,4) \times C(5,3)$

$$= \frac{7!}{(7-4)!\,4!} \times \frac{5!}{(5-3)!\,.3!}$$

$$= \frac{7\times6\times5\times4!}{3!\,.4!} \times \frac{5\times4\times3!}{2!\,.3!}$$

$$= 35 \times 10$$

$$= 350.$$

Binomial theorem

An expression of the form $ax+b$, $5m-3n$, $2x+5y$ etc are called binomial expression.

1) 1

2) 1 2 1

3) 1 3 3 1

4) 1 4 6 4 1

5) 1 5 10 10 5 1

for any positive $n>0$

$(a+b)^n = c(n,0)\,a^{n-0}b^0 + c(n,1)a^{n-1}b^1 + c(n,2)a^{n-2}b^2 + c(n,3)\,a^{n-3}b^3 + \ldots + c(n,r)\,a^{n-r}b^r + \ldots + c(n,n)\,a^{n-n}b^n$

1) Find the following using binomial theorem.

i) $(2x+3y)^7$

Using binomial theorem

$= c(7,0)\,(2x)^{7-0}.(3y)^0 + c(7,1).(2x)^{7-1}.(3y)^1 + c(7,2).(2x)^{7-2}\,3y^2 + c(7,3)\;2x^{7-3}.3y^3 + c(7,4)\,2x^{7-4}.3y^4 + c(7,5)\,2x^{7-5}.3y^5 + c(7,6)\,2x^{7-6}.3y^6 + c(7,7)\,2x^{7-7}.3y^7$

$= c(7,0)\,(2x)^7.3y^0 + c(7,1)\,2x^6.3y^1 + c(7,2).2x^5\,3y^2 + c(7,3).2x^4\,3y^3 + c(7,4)\,2x^3.3y^4 + c(7,5)\,2x^2.3y^5 + c(7,6)\,2x^1.3y^6 + c(7,7)\,2x^0\,3y^7$

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b) $(a+b)^7$

$= C(7,0)a^7 \cdot b^0 + C(7,1)a^6b^1 + C(7,2)a^5b^2 + C(7,3)a^4 \cdot b^3 + C(7,4)a^3b^4 + C(7,5)a^2b^5 + C(7,6)ab^6 + C(7,7)a^0b^7.$

c) $(2x-3y)^4$

$= C(4,0)(2x)^4 \cdot (-3y)^0 + C(4,1)(2x)^3 \cdot (-3y)^1 + C(4,2)(2x)^2 \cdot (-3y)^2 + C(4,4)(2x)^0(-3y)^4.$

d) $\left(\frac{x}{2} + \frac{2}{y}\right)^5$

$= C(5,0)\left(\frac{x}{2}\right)^5 \cdot \left(\frac{2}{y}\right)^0 + C(5,1)\left(\frac{x}{2}\right)^4 \cdot \left(\frac{2}{y}\right)^1 + C(5,2)\left(\frac{x}{2}\right)^3 \cdot \left(\frac{2}{y}\right)^2 + C(5,3)\left(\frac{x}{2}\right)^2 \cdot \left(\frac{2}{y}\right)^3 + C(5,4)\left(\frac{x}{2}\right)^1 \cdot \left(\frac{2}{y}\right)^4 + C(5,5)\left(\frac{x}{2}\right)^0 \cdot \left(\frac{2}{y}\right)^5.$

e) $\left(x^2 - \frac{1}{x^2}\right)^5$

$= C(5,0)(x^2)^5 \cdot \left(\frac{-1}{x^2}\right)^0 + C(5,1)(x^2)^4 \cdot \left(\frac{-1}{x^2}\right)^1 + C(5,2)(x^2)^3 \cdot \left(\frac{-1}{x^2}\right)^2 + C(5,3)(x^2)^2 \cdot \left(\frac{-1}{x^2}\right)^3 + C(5,4)x^2\left(\frac{-1}{x^2}\right)^4 + C(5,5)(x^2)^0 \cdot \left(\frac{-1}{x^2}\right)^5.$

$= 1 \cdot x^{10} - 5\frac{x^8}{x^2} + 10x^4 - 10\frac{1}{x^2} + 5 \cdot \frac{1}{x^6} - 1 \cdot \frac{1}{x^{10}}$

2) Find the stated term of the followings.

a) 7th term of $(2x+y)^{12}$.

Note:- General term of binomial expression is
$T_{r+1} = C(n,r)\, a^{n-r} b^r$.

Soln.

7th term $= T_7$
$= T_{6+1}$
$= C(12,6)\,(2x)^{12-6} y^6$.

b) 4th term of $\left(2x^2 + \frac{1}{x}\right)^8$

Soln

4th term $= T_4$
$= T_{3+1}$
$= C(8,3)\,(2x^2)^{8-3}\left(\frac{1}{x}\right)^3$
$= C(8,3)\, 2x^{10} \cdot \frac{1}{x^3}$
$= 56 \cdot 2x^7$

c) 5th term of $\left(x - \frac{2}{x}\right)^7$

Soln.

5th term $= T_5$
$= T_{4+1}$
$= C(7,4)\, x^{7-4} \cdot \left(-\frac{2}{x}\right)^4$
$= 35 \cdot x^3 \cdot -\frac{16}{x^4}$
$= 35 \cdot x^3 \cdot -\frac{16}{x^4}$

d) 11th term of $\left(\frac{2}{x}+\frac{y}{2}\right)^{50}$

Soln

11th term $= T_{11}$

$= T_{10+1}$

$= C(50,10)\left(\frac{2}{x}\right)^{40}\cdot\left(\frac{y}{2}\right)^{10}$.

3) General term of binomial expression.

Middle term

a) when n is even the number of terms in the expansion are odd then there is only one middle term and given by.

$$T\left(\frac{n}{2}+1\right)$$

b) when n is odd the number of terms in the expansion are given then there are two middle terms given by.

$$T\left(\frac{n+1}{2}\right) \text{ and } T\left(\frac{n+3}{2}\right)$$

Q.3 Find the general terms of the following.

a) $\left(x^2-\frac{1}{x}\right)^8$

we know that, general terms of binomial expansion be.

$T_{(r+1)} = c(n,r)\, a^{n-r} b^r$

$T_{(r+1)} = C(8,r)\,(x^2)^{8-r}\left(-\frac{1}{x}\right)^r$

$= C(8,r)\, x^{16-2r}\cdot(-1)^r\cdot\frac{1}{x^r}$

$= C(8,r)\, x^{16-3r}\,(-1)^r$

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b) $\left(\frac{a}{b}+\frac{b}{a}\right)^{2n+1}$

Soln

$T_{(r+1)} = C(n,r)\, a^{n-r}\, b^{r}$

$= C(2n+1,r)\left(\frac{a}{b}\right)^{2n+1-r} \cdot \left(\frac{b}{a}\right)^{r}$

c) $\left(x+\frac{1}{2x}\right)^{7}$

Soln

$T_{(r+1)} = C(n,r) \cdot a^{n-r} \cdot b^{r}$

$= C(7,r)\, x^{7-r} \cdot \frac{1}{2x^{r}}$

Q. Find the coeff. of x^5 in the expansion of $\left(x+\frac{1}{2x}\right)^{7}$.

Here,

General term of given expression is

$T_{r+1} = C(7,r)\,(x)^{7-r} \cdot \left(\frac{1}{2x}\right)^{r}$

$= C(7,r)\, x^{7-r} \cdot \left(\frac{1}{2}\right)^{r} \cdot \left(\frac{1}{x}\right)^{r}$

$= C(7,r)\, x^{7-r-r} \cdot \frac{1}{2^{r}}$

$= C(7,r)\, x^{7-2r} \cdot \frac{1}{2^{r}}$

To find the coefficient of x^5 we must have,

$7-2r = 5$

$2 = 2r$

$\therefore r = 1$

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$\therefore$ coeff. of x^5 is $\frac{1}{2}\, C(7,1)$

$= \frac{7}{2}$

Q. Find the coeff. x^2 in the expansion of $\left(x^3 + \frac{a}{x}\right)^{10}$.

Here,

General term of given expression is

$T_{(r+1)} = C(10,r)\,(x^3)^{10-r} \cdot \left(\frac{a}{x}\right)^r$

$= C(10,r)\, x^{30-3r}\, \frac{a^r}{x^r}$

$= C(10,r)\, x^{30-4r} \cdot a^r$

to find the coeff. of x^2 we must have,

$30 - 4r = 2$

$30 - 2 = 4r$

$r = 7$

$\therefore$ coeff. of x^2 is $a^7 \cdot C(10,7)$

$= a^7 \cdot 120$

Q. Find the term independent free from x.

a) $\left(x + \frac{1}{x}\right)^{2n}$

Here,

General term of given expression is

$T_{(r+1)} = C(2n,r)\, x^{2n-r}\, \frac{1}{x^r}$

$= C(2n,r)\, x^{2n-2r}$

to find the term independent (free) from x we must have,

$$2n - 2r = 0$$
$$n = r$$

$\therefore$ Required coeff $= C(2n, r)$
$= C(2n, n)$
$= \frac{2n!}{n!\,n!}$
$= \frac{2}{n!}$

b) $\left(x^2 + \frac{1}{x}\right)^{12}$

Here,

General term of given expression is

$T_{(r+1)} = C(12, r)\,(x^2)^{12-r} \cdot \left(\frac{1}{x}\right)^r$

$= C(12, r)\, x^{24-2r} \cdot \frac{1}{x^r}$

$= C(12, r)\, x^{24-2r-r}$

$= C(12, r)\, x^{24-3r}$

to find the term free from x we must have,

$$24 - 3r = 0$$
$$r = \cancel{24}\ 8$$

$\therefore$ Required coeff. $= C(12, \cancel{24}\,8)$

Q. find the middle term of the following.

a) $\left(x + \frac{1}{x}\right)^{18}$

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Here, 18 is odd number and hence the expansion has even number of terms. so, there is two middle term & given by.

$$T_{\frac{N+1}{2}} = T_{\frac{18+1}{2}}$$

$$= T_{\left(9+\frac{1}{2}\right)}$$

$$= C(18,9)\, x^9 \cdot \frac{1}{x^9}$$

$$= C(18,9)$$

And,

$$T_{\frac{N+3}{2}} = T_{\frac{18+3}{2}}$$

$$= T_{\left(9+\frac{3}{2}\right)}$$

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Probability

Mathematical or classical defination of probability.

Classical → The probability of happening of event E is denoted by P(E) and denoted by.

$$P(E) = \frac{m}{n} = \frac{\text{no. of favourable causes}}{\text{no. of equally likely causes}}$$

Statistical or Empirical definition of probability

1) Law of addition

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If A and B are mutually exclusive events then $P(A \cap B) = 0$

2) Law of multiplication

$$P(A \text{ and } B) = P(A) \times P(B)$$

probability always between 0 & 1.

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Exercise I

1) A die is thrown once, determine the probability of getting
a) An even number
b) A number ≥ 3
c) A number ≤ 4

→ Here,
Total no. of case $(n) = 6$

a) no. of Favourable cases $(m) = 3$

$$\therefore p(E) = \frac{m}{n} = \frac{3}{6} = \frac{1}{2}$$

b) no. of Favourable cases $(m) = 4$

$$\therefore P(E) = \frac{m}{n} = \frac{4}{6} = \frac{2}{3}$$

c) no. of favourable cases $(m) = 4$

$$\therefore P(E) = \frac{m}{n} = \frac{4}{6} = \frac{2}{3}$$

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27) A bag contains 9 red, 7 white and 4 black balls. A ball is drawn at random. Find the probability that

a) A white ball

b) Not a black ball

-> Given,

red balls = 9

white balls = 7

black balls = 4

total no. of balls (n) = 20

a) No. of favourable case (m) = 7

∴ Probability of white ball $P(W) = \frac{m}{n} = \frac{7}{20}$

b) Probability of black ball $P(B) = \frac{m}{n} = \frac{4}{20} = \frac{1}{5}$

Now,

Probability of not a black ball $P(\bar{B}) = 1 - \frac{1}{5}$

$= \frac{4}{5}$

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3) A card is drawn at random from a well shuffled deck of 52 cards. find the probability of being it.

a) A red card b) A heart

c) A spade d) A red 8, red 9 or red 10

e) An ace, a king or a queen

Here,

Total no. of cards (n) = 52

a) No. of red cards (m) = 26

$\therefore$ Probability of red card $P(R) = \frac{m}{n} = \frac{26}{52} = \frac{1}{2}$

b) No. of heart cards (m) = 13

$\therefore$ Probability of heart card $P(H) = \frac{13}{52} = \frac{1}{4}$

c) No. of spade cards (m) = 13

$\therefore$ Probability of spade card $P(S) = \frac{13}{52} = \frac{1}{4}$

d) No. of favourable case (m) = 2+2+2 = 6

$\therefore$ Probability of happening of event $P(E) = \frac{6}{52} = \frac{3}{26}$

e) No. of favourable case (m) = 4+4+4 = 12

$\therefore$ Probability of happening of event $P(E) = \frac{12}{52} = \frac{3}{13}$

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4) The number of students participating in different extra curricular activities on the ocasion of college day is listed below:

No. of extra curricular activities	No. of students
0	18
1	30
2	12
	total = 60

Find the probability that a student selected at random participate in
a) one activity
b) at least one activity

Here, Total no. of students $n(s) = 60$

Participating in 1 activity $n(m) = 30$

Participating in at least 1 activity $n(k) = 30+12$
$= 42$

a) ∴ Probability of happening event $P(E) = \frac{n(m)}{n(s)}$

$= \frac{30}{60} = \frac{1}{2}$

b) $P(E) = \frac{n(k)}{n(s)} = \frac{42}{60} = \frac{7}{10}$

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5> From 20 tickets marked from 1-20, one is drawn at random
a> odd no.
b> multiple of 4 or 5

Here,

total no. of tickets(n) = 20

a> No. of favourable case(m) = 10

$\therefore P(E) = \frac{m}{n} = \frac{10}{20} = \frac{1}{2}$

b> No. of favourable case(m) = 5+4-1 = 8

$\therefore P(E) = \frac{m}{n} = \frac{8}{20} = \frac{2}{5}$

6> In a computer course the probability that a student will grade A is 0.57. The proba-bility he will get grade B is 0.34.
a> what is the probability that the student will get grade A or B?
b> If there are only 3 grades A, B, C. what is the probability that the student will get grade c?

Note → Events are said to be independent, if $P(A \cap B) = P(A) \times P(B)$

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Given,

a) $P(A) = 0.57$

$P(B) = 0.34$

$P(A \text{ or } B) = ?$

Now,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.57 + 0.34 - 0$$
$$= 0.91$$

b) Probability of grade C $P(C) = 1 - 0.91$
$= 0.09$

7 a) $P(A) = 0.4$, $P(B) = 0.35$ and $P(A \cup B) = 0.55$ find $P(A \cap B)$. Are the events independent?

Here,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.4 + 0.35 - 0.55$$
$$= 0.20$$

For independent events

$$P(A \cap B) = P(A) \times P(B)$$
$$0.20 = 0.4 \times 0.35$$
$$0.20 = 0.14$$
$$\therefore P(A \cap B) \neq P(A) \times P(B)$$

Hence, not independent.

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b) A and B are two independent event with $P(A) = \frac{2}{3}$ and $P(B) = \frac{3}{5}$. Find $P(A \cup B)$.

Here,

$$P(A) = \frac{2}{3}$$

$$P(B) = \frac{3}{5}$$

Now,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{2}{3} + \frac{3}{5} - [P(A) \times P(B)]$$ [∵ Independent event]

$$= \frac{2}{3} + \frac{3}{5} - \left(\frac{2}{3} \times \frac{3}{5}\right)$$

$$= \frac{10+9}{15} - \frac{6}{15}$$

$$= \frac{13}{15}$$

Note -> Sample space :- Set of all Possible outcome

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8) Two coins are tossed simultaneously. find the sample space & find the probability that
i) Both are heads
ii) At least one head

Here,

Sample space (S) = {HH, HT, TH, TT}

i) Total outcome = 4
Favourable case (m) = 1
$\therefore$ P (both head) = $\frac{1}{4}$

ii) Favourable case (m) = 3
$\therefore$ P (at least one head) = $\frac{3}{4}$

Imp 9) A class consist of 60 boys, 40 girls. If 2 students are choosen at random, what is the probability that
a) Both are boys
b) Both are girls
c) one boy and one girl
d) 3 girl and 2 boy

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Given,

No. of boys = 60

No. of girls = 40

Total no. of student = 100

a) Probability of both boys $= \frac{C(60,2)}{C(100,2)}$

$$= \frac{\frac{60!}{(60-2)! \times 2!}}{\frac{100!}{(100-2)! \times 2!}}$$

$$= \frac{1770}{4950}$$

$$= 0.35$$

b) Probability of girls $= \frac{C(40,2)}{C(100,2)}$

$$= \frac{\frac{40!}{(40-2)! \times 2!}}{\frac{100!}{(100-2)! \times 2!}}$$

$$= \frac{780}{4950}$$

$$= 0.15$$

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c) P(one boy and one girl) $= \dfrac{60_{C_1} \times 40_{C_1}}{100_{C_2}}$

$$= \frac{\dfrac{60!}{(60-1)! \times 1!} \times \dfrac{40!}{(40-1)! \times 1!}}{\dfrac{100!}{(100-2)! \times 2!}}$$

$$= \frac{60 \times 40}{4950}$$

$$= 0.48$$

d) P(3 girl and 2 boy) $= \dfrac{40_{C_3} \times 60_{C_2}}{100_{C_5}}$

$$= \frac{\dfrac{40!}{(40-3)! \times 3!} \times \dfrac{60!}{(60-2)! \times 2!}}{\dfrac{100!}{(100-5)! \times 5!}}$$

$$= \frac{9880 \times 1770}{75287520}$$

$$= 0.23$$

10) An urn (थैलो) contain 8 white ball and 4 red ball. If two balls are drawn at random. Find the probability of one of each colour.

a) One of each colour
b) Both are same colour

Here,
No. of white balls (W) = 8
No. of red balls (R) = 4
Total no. of balls = 12

a) $P(\text{one of each color}) = \dfrac{C(8,1) \times C(4,1)}{C(12,2)}$

$$= \frac{\dfrac{8!}{(8-1)! \times 1!} \times \dfrac{4!}{(4-1)! \times 1!}}{\dfrac{12!}{(12-2)! \times 2!}}$$

$$= \frac{8 \times 4}{66}$$

$$= 0.48$$

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b) P(both are of same colour) = P(W,W) + P(R,R)

$$= P(W) \times P(W) + P(R) \times P(R)$$

$$= \frac{C(8,2)}{C(12,2)} + \frac{C(4,2)}{C(12,2)}$$

$$= \frac{\frac{8!}{(8-2)! \times 2!}}{\frac{12!}{(12-2)! \times 2!}} + \frac{\frac{4!}{(4-2)! \times 2!}}{\frac{12!}{(12-2)! \times 2!}}$$

$$= \frac{28}{66} + \frac{6}{66}$$

$$= \frac{34}{66}$$

$$= 0.51$$

11) Two dice are rolled once. what is the probability of getting

a) A total of 8

b) Total of 9 or 6

Here,

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Total no. of cases = 36 $(6)^2$

The set that makes the total sum 8 are

(2,6), (3,5), (4,4), (5,3), (6,2)

m = 5

a) $\therefore P(\text{getting sum } 8) = \frac{m}{n} = \frac{5}{36}$

b) $\therefore P(9 \text{ or } 6) = P(9) + P(6)$

$= \frac{4}{36} + \frac{5}{36}$

$= \frac{1}{9} + \frac{5}{36}$

$= \frac{1}{4}$

073

12) Suppose 3 people are selected random from a group of 7 man and 6 women. what is the probability that 2 men and 1 women are selected?

Here,

No. of men = 7

No. of women = 6

Total no. of people = 13

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$\therefore P(2 \text{ men and } 1 \text{ women}) = \dfrac{{}^7C_2 \times {}^6C_1}{{}^{13}C_3}$

$= \dfrac{\dfrac{7!}{(7-2)! \times 2!} \times \dfrac{6!}{(6-1)! \times 1!}}{\dfrac{13!}{(13-3)! \times 3!}}$

$= \dfrac{21 \times 6}{286}$

$= 0.44$

13) 4 cards are drawn from a well suffled deck of 52 cards.

a) find the probability that all 4 are spade

b) All 4 are black.

Here,

Total no. of cards = 52

Spade cards = 13

Black cards = 26

a) $P(4 \text{ spade cards}) = \dfrac{{}^{13}C_4}{{}^{52}C_4}$

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$$= \frac{715}{270725}$$

$$= 0.0026$$

b) $P(4 \text{ black cards}) = \frac{{}^{26}C_4}{{}^{52}C_4}$

$$= \frac{14950}{270725}$$

$$= 0.055$$

14) A problem in statistics is given to 3 student A, B, C whose chance of solving it is $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ respectively. Find the probability that the problem will be solved.

Here,

$P(A) = \frac{1}{3}$

$P(B) = \frac{1}{4}$

$P(C) = \frac{1}{5}$

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$P(\bar{A}) = 1 - \frac{1}{3} = \frac{2}{3}$

$P(\bar{B}) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(\bar{C}) = 1 - \frac{1}{5} = \frac{4}{5}$

P(problem will not solved) $= P(\bar{A}) \times P(\bar{B}) \times P(\bar{C})$

$= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$

$= \frac{2}{5}$

$\therefore$ P(problem will be solved) $= 1 - \frac{2}{5}$

$= \frac{3}{5}$

15) A lot contains 10 items of which 3 are defectives 3 items are choosen at random from the lot one after other without replacement. Find the probability that all 3 are defective.

Here,

Total no. of items = 10

Defective item = 3

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$P(1^{st}\text{ defective}) = \frac{3}{10}$

$P(2^{nd}\text{ defective}) = \frac{2}{9}$

$P(3^{rd}\text{ defective}) = \frac{1}{8}$

$\therefore P(\text{all defective}) = \frac{3}{10} \times \frac{2}{9} \times \frac{1}{8}$

$= \frac{6}{720}$

$= \frac{1}{120}$

16) A bag contains 6 white balls and 8 blue balls. 2 balls are randomly drawn from the bag one after without replacement. Find the probability that

a) 1st is white and 2nd is blue

b) Both are white

Here,

Total no. of balls = 14

No. of white balls = 6

No. of blue balls = 8

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a) P(1st white and 2nd blue) $= \frac{^6C_1}{^{14}C_1} \times \frac{^8C_1}{^{13}C_1}$

$= \frac{6}{14} \times \frac{8}{13}$

$= \frac{48}{182}$

$= 0.26$

b) P(both are white) $=$ P(w w)

$= P(W) \times P(W)$

$= \frac{C(6,2)}{C(14,2)}$

$= \frac{15}{91}$

$= 0.16$

Binomial Distribution

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1) Probability distribution of binomial distribution is

$$P(x=r) = c(n,r)\, p^r q^{n-r}$$

where,

p is success

q is failure

$\Rightarrow p+q=1$

2) $\text{Mean}(\bar{x}) = np$

3) $S.D = \sqrt{npq}$

4) $\sigma^2 = npq$

1) Find mean and S.D of B.D

i) $q = 0.50$ and $n = 40$

Here,

$q = 0.50$

$n = 40$

for p

$p+q=1$

$\Rightarrow p = 1-q$

$= 1-0.50$

$= 0.50$

$\therefore \text{Mean} = np$

$= 40 \times 0.50$

$= 20$

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$S.D = \sqrt{npq}$
$= \sqrt{40 \times 0.50 \times 0.50}$
$= 1.58$

ii) $P = 0.6$, $n = 50$

Here,

$p = 0.6$
$n = 50$

For q,

$p + q = 1$
$\Rightarrow q = 1 - p$
$= 1 - 0.6$
$= 0.4$

$\therefore$ Mean $= np$
$= 50 \times 0.4$
$= 20$

$\therefore S.D = \sqrt{npq}$

$= \sqrt{50 \times 0.6 \times 0.4}$

$= 1.69$

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2) A student obtained the following answer to a certain problem to him $\bar{x} = 2.4$, $S.D = \sqrt{3.2}$ for binomial distribution. comment on the result.

Given,

$$\bar{x} = 2.4$$

$$S.D = \sqrt{3.2}$$

we know that,

$$S.D = \sqrt{npq}$$

$$\sqrt{3.2} = \sqrt{npq}$$

$$3.2 = 2.4 \times q$$

$$q = 1.33 > 1$$

Hence Given statement is incorrect.