ASEN 5044

Final Project

Cooperative Air-Ground Robot Localization

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Part 1

Exercise 1

Assumptions and System Dynamics

The system consists of a UGV and UAV modeled as nonlinear kinematic systems.
UGV Dynamics:

$$\dot{\xi}_g = v_g \cos\theta_g, \quad \dot{\eta}_g = v_g \sin\theta_g, \quad \dot{\theta}_g = \frac{v_g}{L}\tan\phi_g$$

UAV Dynamics:

$$\dot{\xi}_a = v_a \cos\theta_a, \quad \dot{\eta}_a = v_a \sin\theta_a, \quad \dot{\theta}_a = \omega_a$$

The state vector is:

$$x = [\xi_g, \eta_g, \theta_g, \xi_a, \eta_a, \theta_a]^T, \quad u = [v_g, \phi_g, v_a, \omega_a]^T$$

State Transition Matrix (A)

The matrix $A = \frac{\partial f(x,u)}{\partial x}$ represents the partial derivatives of the dynamics with respect to the state variables.
UGV Contributions:

$$\hat{A}_{\text{UGV}} = \begin{bmatrix} 0 & 0 & -v_g \sin\theta_g \\ 0 & 0 & v_g \cos\theta_g \\ 0 & 0 & 0 \end{bmatrix}$$

UAV Contributions:

$$\hat{A}_{\text{UAV}} = \begin{bmatrix} 0 & 0 & -v_a \sin\theta_a \\ 0 & 0 & v_a \cos\theta_a \\ 0 & 0 & 0 \end{bmatrix}$$

Combined System:

$$A_{\text{comb}} = \begin{bmatrix} \hat{A}_{\text{UGV}} & 0 \\ 0 & \hat{A}_{\text{UAV}} \end{bmatrix} = \begin{bmatrix} 0 & 0 & -v_g \sin\theta_g & 0 & 0 & 0 \\ 0 & 0 & v_g \cos\theta_g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -v_a \sin\theta_a \\ 0 & 0 & 0 & 0 & 0 & v_a \cos\theta_a \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Control Input Matrix (B)

The matrix $B = \frac{\partial f(x,u)}{\partial u}$ represents the partial derivatives of the dynamics with respect to the control inputs.
UGV Contributions:

$$\hat{B}_{\text{UGV}} = \begin{bmatrix} \cos\theta_g & 0 \\ \sin\theta_g & 0 \\ \frac{\tan\phi_g}{L} & \frac{v_g}{L\cos^2\phi_g} \end{bmatrix}$$

UAV Contributions:

$$\hat{B}_{\text{UAV}} = \begin{bmatrix} \cos\theta_a & 0 \\ \sin\theta_a & 0 \\ 0 & 1 \end{bmatrix}$$

Combined System:

$$B_{\text{comb}} = \begin{bmatrix} \hat{B}_{\text{UGV}} & 0 \\ 0 & \hat{B}_{\text{UAV}} \end{bmatrix} = \begin{bmatrix} \cos\theta_g & 0 & 0 & 0 \\ \sin\theta_g & 0 & 0 & 0 \\ \frac{\tan\phi_g}{L} & \frac{v_g}{L\cos^2\phi_g} & 0 & 0 \\ 0 & 0 & \cos\theta_a & 0 \\ 0 & 0 & \sin\theta_a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Measurement Jacobians (H)

For relative range (r) and azimuth (θ_r), the measurement Jacobian H combines the derivatives of measurement equations with respect to the states:

$$H = \begin{bmatrix} \frac{\partial\theta_r}{\partial x} \\ \frac{\partial r}{\partial x} \end{bmatrix}$$

Key terms derived for $\frac{\partial\theta_r}{\partial x}$ and $\frac{\partial r}{\partial x}$ were:

$$\frac{\partial\theta_r}{\partial\xi_g} = \frac{\eta_a - \eta_g}{r^2}, \quad \frac{\partial\theta_r}{\partial\eta_g} = \frac{\xi_g - \xi_a}{r^2}, \quad \frac{\partial r}{\partial\xi_g} = \frac{\xi_g - \xi_a}{r}, \quad \frac{\partial r}{\partial\eta_g} = \frac{\eta_g - \eta_a}{r}$$

Summary of Jacobians

The Jacobian matrices computed for the dynamics and measurement models are:

- A: 6×6 state transition matrix.
- B: 6×4 control input matrix.
- H: Measurement Jacobian derived for range and azimuth.

Exercise 2

1. System Dynamics

The continuous-time (CT) nonlinear dynamics of the system are:

- **UGV Dynamics:**

$$\dot{\xi}_g = v_g\cos\theta_g, \quad \dot{\eta}_g = v_g\sin\theta_g, \quad \dot{\theta}_g = \frac{v_g}{L}\tan\phi_g$$

- **UAV Dynamics:**

$$\dot{\xi}_a = v_a\cos\theta_a, \quad \dot{\eta}_a = v_a\sin\theta_a, \quad \dot{\theta}_a = \omega_a$$

The state vector is:

$$x = [\xi_g, \eta_g, \theta_g, \xi_a, \eta_a, \theta_a]^T, \quad u = [v_g, \phi_g, v_a, \omega_a]^T$$

The nominal operating point is:

$$x_0 = [\xi_{g,0}, \eta_{g,0}, \theta_{g,0}, \xi_{a,0}, \eta_{a,0}, \theta_{a,0}]^T, \quad u_0 = [v_{g,0}, \phi_{g,0}, v_{a,0}, \omega_{a,0}]^T$$

2. Linearization

The system is linearized around the nominal operating point:

$$A = \left.\frac{\partial f(x,u)}{\partial x}\right|_{x_0,u_0}, \quad B = \left.\frac{\partial f(x,u)}{\partial u}\right|_{x_0,u_0}$$

From Question 1, the linearized matrices are:

$$A = \begin{bmatrix} 0 & 0 & -v_g \sin\theta_g & 0 & 0 & 0 \\ 0 & 0 & v_g \cos\theta_g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -v_a \sin\theta_a \\ 0 & 0 & 0 & 0 & 0 & v_a \cos\theta_a \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \cos\theta_g & 0 & 0 & 0 \\ \sin\theta_g & 0 & 0 & 0 \\ \frac{\tan\phi_g}{L} & \frac{v_g}{L\cos^2\phi_g} & 0 & 0 \\ 0 & 0 & \cos\theta_a & 0 \\ 0 & 0 & \sin\theta_a & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Discretization

To discretize the system with a sampling time $\Delta T = 0.1$, the discrete-time state-space equations are:

$$x_{k+1} = A_d x_k + B_d u_k$$

where:

$$A_d \approx I + A\Delta T, \quad B_d \approx B\Delta T$$

The discretized matrices are:

$$A_d \approx \begin{bmatrix} 1 & 0 & -v_g \sin\theta_g \Delta T & 0 & 0 & 0 \\ 0 & 1 & v_g \cos\theta_g \Delta T & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -v_a \sin\theta_a \Delta T \\ 0 & 0 & 0 & 0 & 1 & v_a \cos\theta_a \Delta T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad B_d \approx \begin{bmatrix} \cos\theta_g \Delta T & 0 & 0 & 0 \\ \sin\theta_g \Delta T & 0 & 0 & 0 \\ \frac{\tan\phi_g \Delta T}{L} & \frac{v_g \Delta T}{L\cos^2\phi_g} & 0 & 0 \\ 0 & 0 & \cos\theta_a \Delta T & 0 \\ 0 & 0 & \sin\theta_a \Delta T & 0 \\ 0 & 0 & 0 & \Delta T \end{bmatrix}$$

4. Observability and Stability

- **Observability:** The observability matrix O is:

$$O = \begin{bmatrix} C \\ CA_d \\ CA_d^2 \\ \vdots \\ CA_d^{n-1} \end{bmatrix}$$

  O must have full rank for the system to be observable. This depends on the measurement matrix C.

- **Stability:** Stability is determined by the eigenvalues of A_d. If all eigenvalues lie inside the unit circle ($|\lambda| < 1$), the system is stable.

5. Summary

- The system is linearized about the nominal operating point x_0, u_0, and discrete-time matrices A_d, B_d are derived for $\Delta T = 0.1$.
- Observability depends on the rank of O, and stability is determined by the eigenvalues of A_d.

Exercise 3

1. Objective

The goal is to simulate the linearized discrete-time (DT) dynamics and measurement models near the linearization point, compare them with the full nonlinear system dynamics, and validate the accuracy of the linearized model. This is done by:

- Simulating both the linearized DT model and the nonlinear model starting from a perturbed nominal initial condition.
- Assuming no process noise, measurement noise, or control input perturbations.
- Comparing the resulting states and measurements from both models through plots.

2. Simulation Setup

The nominal state $x_{\text{nom}}(t)$ is defined as a function of time for both the UGV and UAV:

$$x_{\text{nom}}(t) = \begin{bmatrix} \xi_{g,\text{nom}}(t) \\ \eta_{g,\text{nom}}(t) \\ \theta_{g,\text{nom}}(t) \\ \xi_{a,\text{nom}}(t) \\ \eta_{a,\text{nom}}(t) \\ \theta_{a,\text{nom}}(t) \end{bmatrix}$$

A small initial perturbation is added to the nominal state:

$$\delta x_0 = [0.1, 0.1, 0.05, 0.1, 0.1, 0.05]^T, \quad x_{\text{perturbed}}(0) = x_{\text{nom}}(0) + \delta x_0$$

3. Nonlinear Dynamics Simulation

The nonlinear system is described by the following state equations:

$$\dot{x} = f(x, u), \quad y = h(x)$$

where:

$$f(x, u) = \begin{bmatrix} v_g \cos\theta_g \\ v_g \sin\theta_g \\ \frac{v_g}{L}\tan\phi_g \\ v_a \cos\theta_a \\ v_a \sin\theta_a \\ \omega_a \end{bmatrix}, \quad h(x) = (\text{measurement model})$$

The nonlinear system is simulated using ‘ode45‘ in MATLAB with the perturbed initial condition $x_{\text{perturbed}}(0)$.

4. Linearized Dynamics Simulation

The discrete-time linearized system is given by:

$$\delta x_{k+1} = F_k \delta x_k + G_k \delta u_k, \quad \delta y_k = C_k \delta x_k$$

where:

$$F_k = I + A_k \Delta T, \quad G_k = B_k \Delta T$$

The Jacobian matrices A_k and B_k are computed as:

$$A_k = \frac{\partial f(x,u)}{\partial x}, \quad B_k = \frac{\partial f(x,u)}{\partial u}$$

The linearized model is simulated for the same perturbed initial condition δx_0 and compared against the nonlinear simulation.

5. Measurement Comparison

The measurement model is defined for range and bearing:

$$C_k = \begin{bmatrix} \frac{x_5 - x_2}{\sqrt{(x_5-x_2)^2+(x_4-x_1)^2}} & \frac{-(x_4-x_1)}{\sqrt{(x_5-x_2)^2+(x_4-x_1)^2}} & 0 & \frac{-(x_5-x_2)}{\sqrt{(x_5-x_2)^2+(x_4-x_1)^2}} & \frac{x_4-x_1}{\sqrt{(x_5-x_2)^2+(x_4-x_1)^2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Measurements δy_k from the linearized model are compared to the nonlinear measurements $y_{\text{nonlinear}}(t)$ computed directly from the nonlinear simulation.