Machine Learning with Python

Machine Learning Workshop @ MPSTME

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Machine Learning Research Topics

Recommendation Systems

Source: https://beckernick.github.io/matrix-factorization-recommender/ (https://beckernick.github.io/matrix-factorization-recommender/)

- We study low-rank matrix factorization for recommendations and use it on a **dataset of 1 million movie ratings** (from 1 to 5) available from the MovieLens (http://grouplens.org/datasets/movielens/) project.
- The MovieLens datasets were created collected by GroupLens Research at the University of Minnesota.

Matrix Factorization via Singular Value Decomposition

- Matrix factorization is the **breaking down of one matrix in a product of multiple matrices**. There are many different ways to factor matrices, but singular value decomposition is particularly useful for making recommendations.
- At a high level, SVD is an algorithm that decomposes a matrix R into the best lower rank (i.e. smaller/simpler) approximation of the original matrix R. Mathematically, it decomposes R into two unitary matrices and a diagonal matrix:

$$R = U\Sigma V^T$$

where R is users's ratings matrix, U is the user "features" matrix, Σ is the diagonal matrix of singular values (essentially weights), and V^T is the movie "features" matrix. U and V^T are unitary matrices.

- U represents how much users "like" each feature and V^T represents how relevant each feature is to each movie.
- To get the lower rank approximation, we take these matrices and keep only the top k features, which we think of as the underlying tastes and preferences vectors.

In [1]:
```
import pandas as pd
import numpy as np

ratings_list = [i.strip().split("::") for i in open('Data/ratings.dat', 'r').readlines()]
users_list = [i.strip().split("::") for i in open('Data/users.dat', 'r').readlines()]
movies_list = [i.strip().split("::") for i in open('Data/movies.dat', 'r').readlines()]
```

In [2]:
```
# Make dataframes for processing

ratings_df = pd.DataFrame(ratings_list,
                          columns = ['UserID', 'MovieID', 'Rating', 'Timestamp'],
                          dtype = int)
movies_df = pd.DataFrame(movies_list, columns = ['MovieID', 'Title', 'Genres'])
movies_df['MovieID'] = movies_df['MovieID'].apply(pd.to_numeric)
```

In [3]:
```
ratings_df.head()
```

Out[3]:

	UserID	MovieID	Rating	Timestamp
0	1	1193	5	978300760
1	1	661	3	978302109
2	1	914	3	978301968
3	1	3408	4	978300275
4	1	2355	5	978824291

In [4]:
```
movies_df.head()
```

Out[4]:

	MovieID	Title	Genres
0	1	Toy Story (1995)	Animation|Children's|Comedy
1	2	Jumanji (1995)	Adventure|Children's|Fantasy
2	3	Grumpier Old Men (1995)	Comedy|Romance
3	4	Waiting to Exhale (1995)	Comedy|Drama
4	5	Father of the Bride Part II (1995)	Comedy

These look good, but we want the format of our Ratings matrix to be **one row per user and one column per movie**.

We'll `pivot` `ratings_df` to get that and call the new variable `R`.

In [5]:
```
# Example of How-to Pivot?
df = pd.DataFrame({'foo': ['one','one','one','two','two','two'],
                   'bar': ['A', 'B', 'C', 'A', 'B', 'C'],
                   'tic': [1, 2, 3, 4, 5, 6]})
print(df)
df.pivot(index='foo', columns='bar', values='tic')
```

```
   foo bar  tic
0  one   A    1
1  one   B    2
2  one   C    3
3  two   A    4
4  two   B    5
5  two   C    6
```

Out[5]:

bar	A	B	C
foo			
one	1	2	3
two	4	5	6

In [6]:
```
R_df = ratings_df.pivot(index = 'UserID',
                        columns ='MovieID', values = 'Rating').fillna(0)
R_df.head()
```

Out[6]:

MovieID	1	2	3	4	5	6	7	8	9	10	...	3943	3944	3945	3946	3947	3948	3949	3950	3951	3952
UserID																					
1	5.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	2.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

5 rows × 3706 columns

In [7]:
```
R_df.tail()
```

Out[7]:

MovieID	1	2	3	4	5	6	7	8	9	10	...	3943	3944	3945	3946	3947	3948	3949	3950	3951	3952
UserID																					
6036	0.0	0.0	0.0	2.0	0.0	3.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6037	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6038	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6039	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6040	3.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

5 rows × 3706 columns

- Now, de-mean the data (normalize by each users mean) and convert it from a dataframe to a numpy array.

In [10]:
```
R = R_df.values

user_ratings_mean = np.mean(R, axis = 1)

# Make it zero-mean
R_demeaned = R - user_ratings_mean.reshape(-1, 1)
```

- All set. With our ratings matrix properly formatted and normalized, we are ready to do the singular value decomposition

Applying Singular Value Decomposition

Scipy and Numpy both have functions to do the singular value decomposition. Here, we will use the Scipy function `svds` because it let's us choose how many latent factors we want to use to approximate the original ratings matrix (instead of having to truncate it after).

In [11]:
```
from scipy.sparse.linalg import svds
U, sigma, Vt = svds(R_demeaned, k = 50)
```

- The Σ returned is just the values instead of a diagonal matrix.
- This is useful, but since we are going to leverage matrix multiplication to get predictions, we'll convert it to the diagonal matrix form.

In [12]:
```
print(sigma)
sigma = np.diag(sigma)
print(sigma)
```

```
[ 147.18581225  147.62154312  148.58855276  150.03171353  151.79983807
  153.96248652  154.29956787  154.54519202  156.1600638   157.59909505
  158.55444246  159.49830789  161.17474208  161.91263179  164.2500819
  166.36342107  166.65755956  167.57534795  169.76284423  171.74044056
  176.69147709  179.09436104  181.81118789  184.17680849  186.29341046
  192.15335604  192.56979125  199.83346621  201.19198515  209.67692339
  212.55518526  215.46630906  221.6502159   231.38108343  239.08619469
  244.8772772   252.13622776  256.26466285  275.38648118  287.89180228
  315.0835415   335.08085421  345.17197178  362.26793969  415.93557804
  434.97695433  497.2191638   574.46932602  670.41536276 1544.10679346]
[[ 147.18581225    0.            0.         ...    0.
     0.            0.        ]
 [   0.          147.62154312    0.         ...    0.
     0.            0.        ]
 [   0.            0.          148.58855276 ...    0.
     0.            0.        ]
 ...
 [   0.            0.            0.         ...  574.46932602
     0.            0.        ]
 [   0.            0.            0.         ...    0.
   670.41536276    0.        ]
 [   0.            0.            0.         ...    0.
     0.         1544.10679346]]
```

Making Predictions from the Decomposed Matrices

- We now have everything we need to make movie ratings predictions for every user.
- We can do it all at once by following the math and matrix multiply U, Σ, and V^T back to get the rank $k = 50$ approximation of R.
- We also need to add the user means back to get the actual star ratings prediction.

In [13]:
```
all_user_predicted_ratings = np.dot(np.dot(U, sigma), Vt) + user_ratings_mean.reshape(-1, 1)
all_user_predicted_ratings[:10,:3] # just checking
```

Out[13]:
```
array([[ 4.28886061,  0.14305516, -0.1950795 ],
       [ 0.74471587,  0.16965927,  0.33541808],
       [ 1.81882382,  0.45613623,  0.09097801],
       [ 0.40805697, -0.07296018,  0.03964241],
       [ 1.57427245,  0.02123904, -0.05129994],
       [ 2.08095434,  0.20089638,  0.25742654],
       [ 0.05056107,  0.14555134,  0.07143897],
       [ 0.81875979,  0.15726791,  0.69156809],
       [ 3.6672577 ,  0.12555919,  0.02581049],
       [ 4.51691241,  2.9681962 ,  1.25410159]])
```

Note:

- To put this kind of system into production, we'd want to create a training and validation set and optimize the number of latent features (k) by minimizing the Root Mean Square Error. Intuitively, the Root Mean Square Error will decrease on the training set as k increases (because we are approximating the original ratings matrix with a higher rank matrix).
- We could **create a training and validation set and optimize k by minimizing RMSE**
- We now will see some movie recommendations.

Making Movie Recommendations

- With the predictions matrix for every user, we can build a function to recommend movies for any user.
- All we need to do is **return the movies with the highest predicted rating that the specified user hasn't already rated**.
- Though we have not actually used any explicit movie content features (such as genre or title), we'll merge in that information to get a more complete picture of the recommendations.
- We'll also return the list of movies the user has already rated, for the sake of comparison.

In [14]:
```
preds_df = pd.DataFrame(all_user_predicted_ratings, columns = R_df.columns)
preds_df.head()
```

Out[14]:

MovieID	1	2	3	4	5	6	7	8	9	'
0	4.288861	0.143055	-0.195080	-0.018843	0.012232	-0.176604	-0.074120	0.141358	-0.059553	-0.1959
1	0.744716	0.169659	0.335418	0.000758	0.022475	1.353050	0.051426	0.071258	0.161601	1.56724
2	1.818824	0.456136	0.090978	-0.043037	-0.025694	-0.158617	-0.131778	0.098977	0.030551	0.73547
3	0.408057	-0.072960	0.039642	0.089363	0.041950	0.237753	-0.049426	0.009467	0.045469	-0.11137
4	1.574272	0.021239	-0.051300	0.246884	-0.032406	1.552281	-0.199630	-0.014920	-0.060498	0.45051

5 rows × 3706 columns

In [15]:
```
# Function for recommending movies to any user

def recommend_movies(predictions_df, userID, movies_df, original_ratings_df, num_recommendations=
5):

    # Get and sort the user's predictions
    user_row_number = userID - 1 # UserID starts at 1, not 0
    sorted_user_predictions = preds_df.iloc[user_row_number].sort_values(ascending=False) # UserID
starts at 1

    # Get the user's data and merge in the movie information.
    user_data = original_ratings_df[original_ratings_df.UserID == (userID)]
    user_full = (user_data.merge(movies_df, how = 'left', left_on = 'MovieID', right_on = 'MovieI
D').
                     sort_values(['Rating'], ascending=False)
                 )

    print('User {0} has already rated {1} movies.'.format(userID, user_full.shape[0]))
    print('Recommending highest {0} predicted ratings movies not already rated.'.format(num_recomm
endations))

    # Recommend the highest predicted rating movies that the user hasn't seen yet.
    recommendations = (movies_df[~movies_df['MovieID'].isin(user_full['MovieID'])].
         merge(pd.DataFrame(sorted_user_predictions).reset_index(), how = 'left',
               left_on = 'MovieID',
               right_on = 'MovieID').
         rename(columns = {user_row_number: 'Predictions'}).
         sort_values('Predictions', ascending = False).
                       iloc[:num_recommendations, :-1]
                      )

    return user_full, recommendations
```

In [16]:
```
already_rated, predictions = recommend_movies(preds_df, 837,
                                              movies_df, ratings_df, 10)
```

```
User 837 has already rated 69 movies.
Recommending highest 10 predicted ratings movies not already rated.
```

In [17]:
```
already_rated.head(10)
```

Out[17]:

	UserID	MovieID	Rating	Timestamp	Title	Genres
36	837	858	5	975360036	Godfather, The (1972)	Action|Crime|Drama
35	837	1387	5	975360036	Jaws (1975)	Action|Horror
65	837	2028	5	975360089	Saving Private Ryan (1998)	Action|Drama|War
63	837	1221	5	975360036	Godfather: Part II, The (1974)	Action|Crime|Drama
11	837	913	5	975359921	Maltese Falcon, The (1941)	Film-Noir|Mystery
20	837	3417	5	975360893	Crimson Pirate, The (1952)	Adventure|Comedy|Sci-Fi
34	837	2186	4	975359955	Strangers on a Train (1951)	Film-Noir|Thriller
55	837	2791	4	975360893	Airplane! (1980)	Comedy
31	837	1188	4	975360920	Strictly Ballroom (1992)	Comedy|Romance
28	837	1304	4	975360058	Butch Cassidy and the Sundance Kid (1969)	Action|Comedy|Western

In [18]:
```
predictions
```

Out[18]:

	MovieID	Title	Genres
516	527	Schindler's List (1993)	Drama|War
1848	1953	French Connection, The (1971)	Action|Crime|Drama|Thriller
596	608	Fargo (1996)	Crime|Drama|Thriller
1235	1284	Big Sleep, The (1946)	Film-Noir|Mystery
2085	2194	Untouchables, The (1987)	Action|Crime|Drama
1188	1230	Annie Hall (1977)	Comedy|Romance
1198	1242	Glory (1989)	Action|Drama|War
897	922	Sunset Blvd. (a.k.a. Sunset Boulevard) (1950)	Film-Noir
1849	1954	Rocky (1976)	Action|Drama
581	593	Silence of the Lambs, The (1991)	Drama|Thriller

- These look like good recommendations.
- It's also good to see that, though we didn't actually use the genre of the movie as a feature, the truncated matrix factorization features "picked up" on the underlying tastes and preferences of the user.
- W've recommended some film-noirs, crime, drama, and war movies - all of which were genres of some of this user's top rated movies.

===

Sentiment Analysis of Movie Reviews (NLP Application)

In [19]:
```
import pandas as pd
from sklearn.feature_extraction.text import CountVectorizer
from sklearn.feature_extraction.text import TfidfTransformer
from sklearn.pipeline import Pipeline
from sklearn.metrics import classification_report, accuracy_score
from sklearn.linear_model import SGDClassifier
```

In [20]:
```
sent_data = pd.read_csv('Data/movie_reviews.csv')
```

In [21]:
```
print(sent_data.head())
# Data has
# The sentiment: a binary (0/1) variable
# The text of the movie review: string
```

```
                                              review sentiment
0  One of the other reviewers has mentioned that ...  positive
1  A wonderful little production. <br /><br />The...  positive
2  I thought this was a wonderful way to spend ti...  positive
3  Basically there's a family where a little boy ...  negative
4  Petter Mattei's "Love in the Time of Money" is...  positive
```

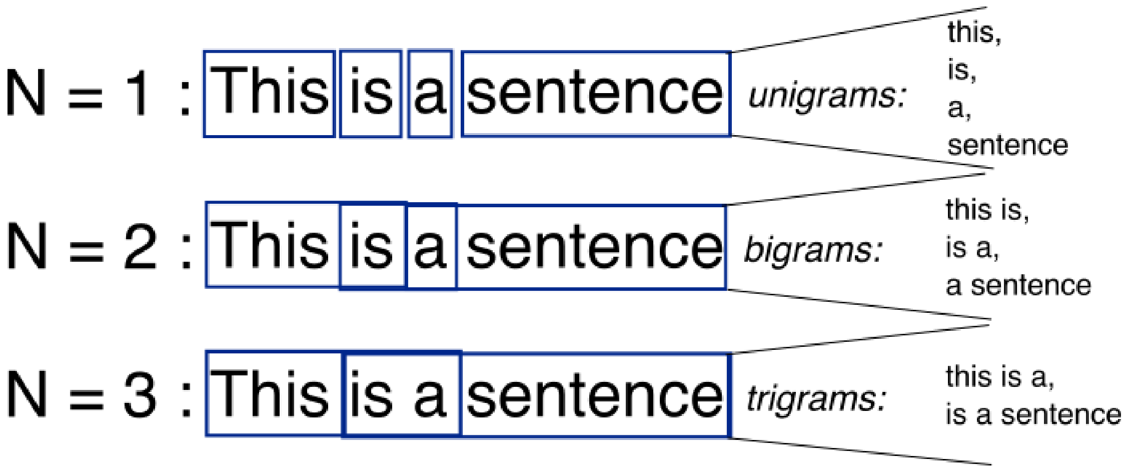
In [22]:
```
# prepare training and testing dataset (total 50,000 entries)
train_data, test_data = sent_data[:25000], sent_data[25000:]

X_train = train_data['review']
y_train = train_data['sentiment']

X_test = test_data['review']
y_test = test_data['sentiment']
```

N-Gram Models

- An n**-gram model** is a type of **probabilistic language model** for predicting the next item in such a sequence in the form of a $(n-1)$ order **Markov model**.
- n-gram models are widely used in probability, communication theory, computational linguistics, statistical natural language processing, computational biology, and data compression.
- "Gram" is derived from the Greek γράμμα, which means "letter". Hence, an n-gram is a combination of n letters: a 2-gram is a combination of two letters, and so on.

In [25]:
```
from IPython.display import Image
Image('Images/NGrams.png')
```

Out[25]:



Building a sentiment classifier: unigram features

- In order to classify text, we have to convert them into vectors.
- In scikit-learn, this task is very easy. We have only to pass dataset to CountVectorizer. It tokenizes text and convert tokenized text to frequency matrix. In addition, for a better operation, we compute weights for words where each weight gives the importance of the word. Such a weight is the tf-idf score.
- Let's start by building a **TFIDF matrix** with a **Pipeline**.

In [25]:
```
def build_pipeline():
    text_clf = Pipeline([('vect', CountVectorizer(min_df=1, stop_words='english', binary=True)),
                         ('tfidf', TfidfTransformer()),
                         ('clf', SGDClassifier(l1_ratio=0, n_jobs=-1, max_iter=1000)),
                         ])
    return text_clf
```

In [26]:
```
# Feed to classifier
text_clf = build_pipeline()
text_clf = text_clf.fit(X_train, y_train)
```

In [27]:
```
# Evaluate
y_pred = text_clf.predict(X_test)

print('Accuracy: {}'.format(accuracy_score(y_test, y_pred)))
print()
print(classification_report(y_test, y_pred))
```

```
Accuracy: 0.88948

              precision    recall  f1-score   support

    negative       0.90      0.88      0.89     12474
    positive       0.88      0.90      0.89     12526

   micro avg       0.89      0.89      0.89     25000
   macro avg       0.89      0.89      0.89     25000
weighted avg       0.89      0.89      0.89     25000
```

- About 89% accuracy. This is not bad. If we tune more parameters, we can reach a higher score.

Building a sentiment classifier: bigram features

In [28]:
```
def build_pipeline():
    text_clf = Pipeline([('vect', CountVectorizer(ngram_range=(1, 2), token_pattern=r'\b\w+\b', mi
n_df=1, stop_words='english', binary=True)),
                         ('tfidf', TfidfTransformer()),
                         ('clf', SGDClassifier(l1_ratio=0, n_jobs=-1, max_iter=1000)),
                         ])
    return text_clf
```

In [29]:
```
text_clf = build_pipeline()
text_clf = text_clf.fit(X_train, y_train)
```

In [30]:
```
y_pred = text_clf.predict(X_test)

print('Accuracy: {}'.format(accuracy_score(y_test, y_pred)))

print(classification_report(y_test, y_pred))
```

```
Accuracy: 0.89704
              precision    recall  f1-score   support

    negative       0.91      0.88      0.90     12474
    positive       0.88      0.91      0.90     12526

   micro avg       0.90      0.90      0.90     25000
   macro avg       0.90      0.90      0.90     25000
weighted avg       0.90      0.90      0.90     25000
```

- Improved to 90% accuracy using bigram features

===

Eigen-Faces (Applying PCA for Face Recognition)

[https://github.com/guruucsd/EigenfaceDemo (https://github.com/guruucsd/EigenfaceDemo)](https://github.com/guruucsd/EigenfaceDemo)

Performing PCA on Faces dataset

Let's apply PCA to a dataset consisting of face data.

In [32]:
```
import pickle
from sklearn.decomposition import PCA
from ipywidgets import interact, interactive, fixed, interact_manual
import ipywidgets as widgets
```

In [33]:
```
with open('Data/cafe.pkl', 'rb') as f:
    dataset = pickle.load(f, encoding='latin-1')
```

In [34]:
```
print(dataset.images.shape)
print(dataset.data.shape)
```

```
(80, 95, 60)
(80, 5700)
```

In [35]:
```
import matplotlib.pyplot as plt
%matplotlib inline

@interact
def plot_face(image_id=(0, dataset.images.shape[0]-1)):
    plt.imshow(dataset.images[image_id],cmap='gray')
    plt.title('Image Id = %d, Gender = %d' % (dataset.target[image_id], dataset.gender[image_id]))
    plt.axis('off')
```

Preprocessing

We'll center the data by subtracting the mean. The first axis (`axis=0`) is the `n_samples` dimension.

In [36]:
```
X=dataset.data.copy() # So that we won't mess up the data in the dataset

X_mean = X.mean(axis = 0,keepdims = True) # Mean for each dimension across sample (centering)

X_std = X.std(axis = 0,keepdims = True)

X -= X_mean

print(all(abs(X.mean(axis = 0))<1e-12)) # Are means for all dimensions very close to zero?
```

```
True
```

Then we perform SVD to calculate the projection matrix V. By default, `U,s,V=svd(...)` returns full matrices, which will return $n \times n$ matrix `U`, n-dimensional vector of singular values `s`, and $d \times d$ matrix `V`. But here, we don't really need $d \times d$ matrix `V`; with `full_matrices=False`, `svd` only returns $n \times d$ matrix for `V`.

In [37]:
```
from numpy.linalg import svd

U,s,Vt = svd(X, compute_uv = True, full_matrices = False)
print(str(U.shape))
print(str(s.shape))
print(str(Vt.shape))
```

```
(80, 80)
(80,)
(80, 5700)
```

We can plot how much each eigenvector in `V` contributes to the overall variance by plotting `variance_ratio` = $\frac{s^2}{\sum s^2}$. (Notice that `s` is already in the decreasing order.)

The `cumsum` (cumulative sum) of `variance_ratio` then shows how much of the variance is explained by components up to `n_components`.

In [38]:
```
variance_ratio = s**2/(s**2).sum()   # Normalized so that they add to one
len(variance_ratio)
```

Out[38]: 80

In [39]:
```
import numpy as np

@interact
def plot_variance_ratio(n_components=(1, len(variance_ratio))):
    n=n_components-1
    fig, axs = plt.subplots(1, 2, figsize=(12, 5))

    axs[0].plot(variance_ratio)
    axs[0].set_title('Explained Variance Ratio')
    axs[0].set_xlabel('n_components')
    axs[0].axvline(n, color='r', linestyle='--')
    axs[0].axhline(variance_ratio[n], color='r', linestyle='--')

    axs[1].plot(np.cumsum(variance_ratio))
    axs[1].set_xlabel('n_components')
    axs[1].set_title('Cumulative Sum')

    captured=np.cumsum(variance_ratio)[n]
    axs[1].axvline(n, color='r', linestyle='--')
    axs[1].axhline(captured, color='r', linestyle='--')
    axs[1].annotate(s='%f%% with %d components' % (captured * 100, n_components), xy=(n, capture
d),xytext=(10, 0.5), arrowprops=dict(arrowstyle="->"))
```

Since we're dealing with face data, each row vector of $\mathbf{V}$ is called an "eigenface".

The first "eigenface" is the one that explains a lot of variances in the data, whereas the last one explains the least.

In [40]:
```
image_shape = dataset.images.shape[1:] # (H x W)

@interact
def plot_eigenface(eigenface=(0, Vt.shape[0]-1)):
    v = Vt[eigenface]*X_std

    plt.imshow(v.reshape(image_shape), cmap='gray')
    plt.title('Eigenface %d' % (eigenface))
    plt.axis('off')
```

- Now let's try reconstructing faces with different number of principal components (PCs)!
- The transformed `X` is reconstructed by multiplying by the sample standard deviations for each dimension and adding the sample mean. For this reason, even for zero components, you get a face-like image!
- The rightmost plot is the "relative" reconstruction error (image minus the reconstruction squared, divided by the data standard deviations). White is where the error is close to zero, and black is where the relative error is large (1 or more). As we increase the number of PCs, we see the error mostly going to zero (white).

In [41]:
```
import math

@interact
def plot_reconstruction(image_id=(0,dataset.images.shape[0]-1), n_components=(0, Vt.shape[0]-1),
                        pc1_multiplier=widgets.FloatSlider(min=-2,max=2, value=1)):
    # This is where we perform the projection and un-projection
    Vn = Vt[:n_components]
    M = np.ones(n_components)
    if n_components > 0:
        M[0] = pc1_multiplier
    X_hat = np.dot(np.multiply(np.dot(X[image_id], Vn.T), M), Vn)

    # Un-center
    I = X[image_id] + X_mean
    I_hat = X_hat + X_mean
    D = np.multiply(I-I_hat,I-I_hat) / np.multiply(X_std, X_std)

    # And plot
    fig, axs = plt.subplots(1, 3, figsize=(10, 10))

    axs[0].imshow(I.reshape(image_shape), cmap='gray', vmin=0, vmax=1)
    axs[0].axis('off')
    axs[0].set_title('Original')

    axs[1].imshow(I_hat.reshape(image_shape), cmap='gray', vmin=0, vmax=1)
    axs[1].axis('off')
    axs[1].set_title('Reconstruction')

    axs[2].imshow(1-D.reshape(image_shape), cmap='gray', vmin=0, vmax=1)
    axs[2].axis('off')
    axs[2].set_title('Difference^2 (mean = %f)' % math.sqrt(D.mean()))

    plt.tight_layout()
```

Image Morphing

As a fun exercise, we'll morph two images by taking averages of the two images within the transformed data space. How is it different than simply morphing them in the pixel space?

In [42]:
```
def plot_morph(left=0, right=1, mix=0.5):
    # Projected images
    x_lft = np.dot(X[left], Vt.T)
    x_rgt = np.dot(X[right], Vt.T)

    # Mix
    x_avg = x_lft * (1.0-mix) + x_rgt * (mix)

    # Un-project
    X_hat = np.dot(x_avg[np.newaxis,:], Vt)
    I_hat = X_hat + X_mean

    # And plot
    fig, axs = plt.subplots(1, 3, figsize=(10, 10))

    axs[0].imshow(dataset.images[left], cmap='gray', vmin=0, vmax=1)
    axs[0].axis('off')
    axs[0].set_title('Left')

    axs[1].imshow(I_hat.reshape(image_shape), cmap='gray', vmin=0, vmax=1)
    axs[1].axis('off')
    axs[1].set_title('Morphed (%.2f %% right)' % (mix * 100))

    axs[2].imshow(dataset.images[right], cmap='gray', vmin=0, vmax=1)
    axs[2].axis('off')
    axs[2].set_title('Right')

    plt.tight_layout()

interact(plot_morph,
         left=widgets.IntSlider(max=dataset.images.shape[0]-1),
         right=widgets.IntSlider(max=dataset.images.shape[0]-1,value=1),
         mix=widgets.FloatSlider(value=0.5, min=0, max=1.0))
```

Out[42]:
```
<function __main__.plot_morph(left=0, right=1, mix=0.5)>
```