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Chaos Based Image Encryption using Latin Rectangle Scrambling

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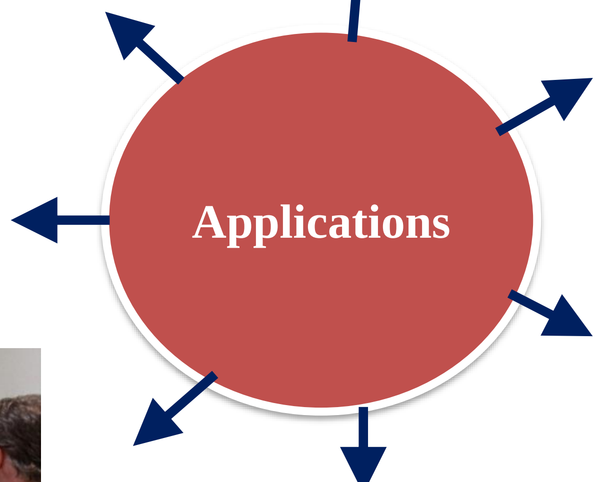
Problem: Image Encryption

Medical Imaging

Satellite Images

Military Images

Multimedia Security

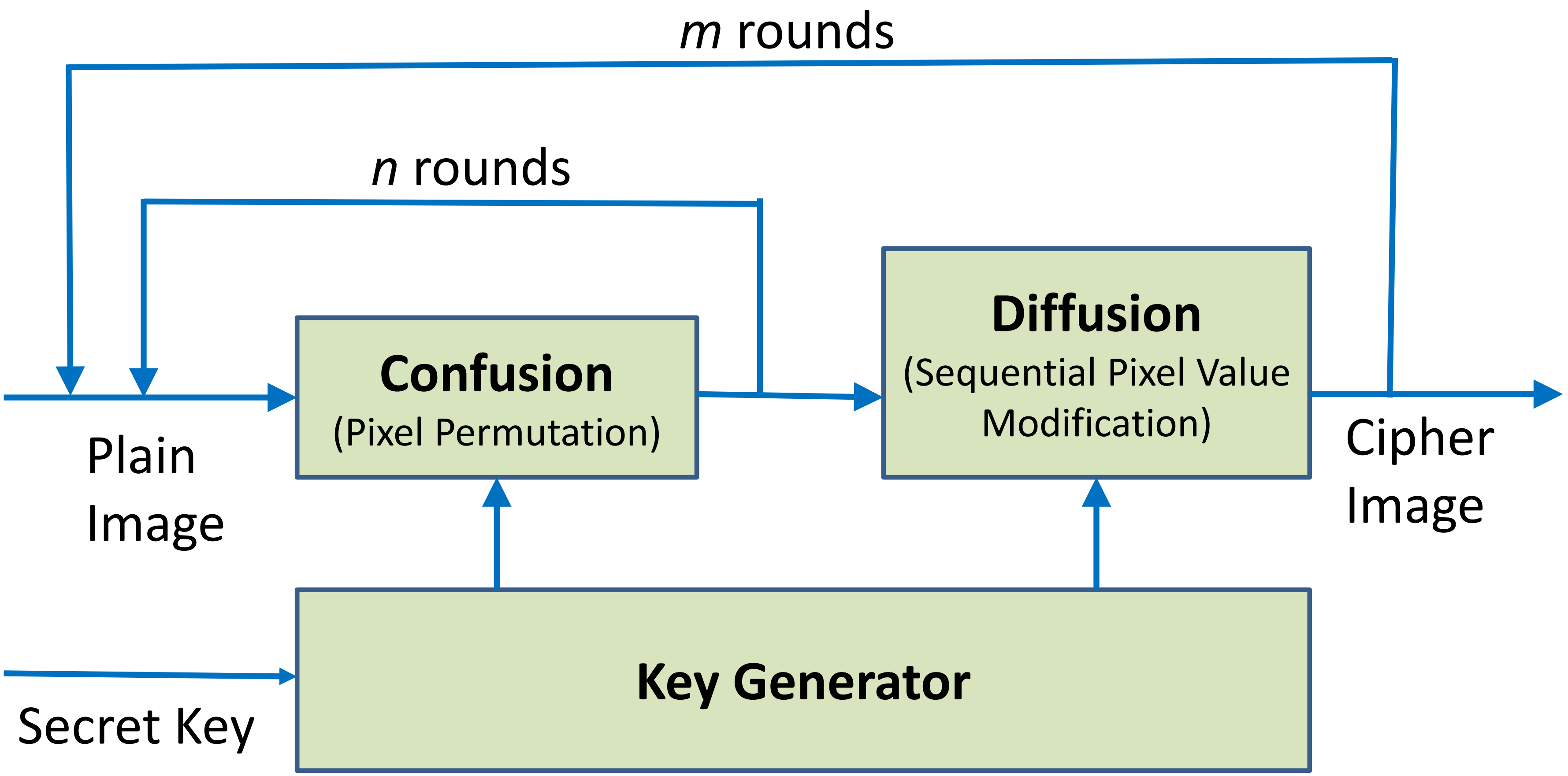


Confidential video conference

Pay-per-view TV

Aadhar card Enrolment

General Architecture

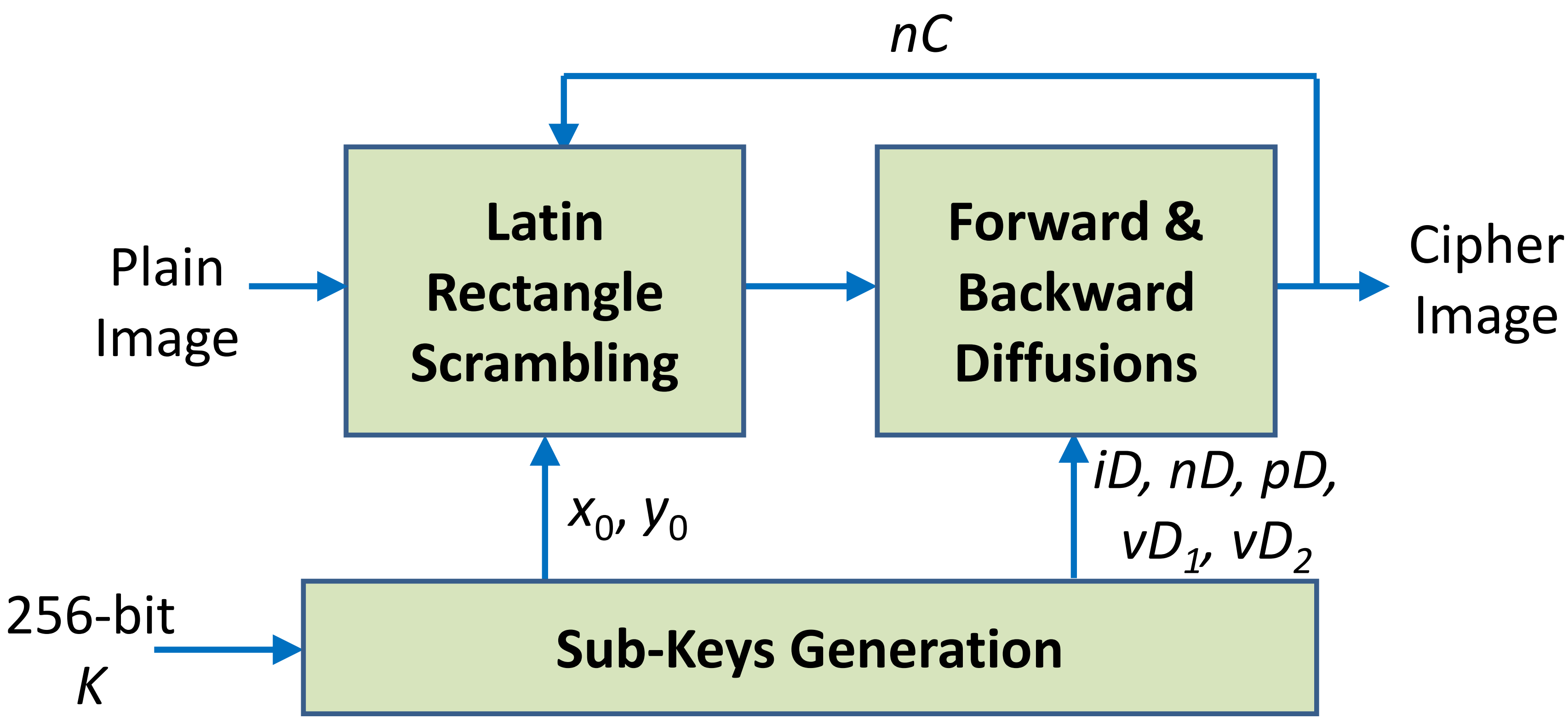


Related Work

Chaos: **Deterministic** model of **dynamical** system that yields **unpredictable** behavior

Chaos Based Solutions	Non-Chaos Based Solutions
Logistic Map [1]	Sudoku Matrix approach [4]
Baker Map [2]	Latin Square Image Cipher [5]
Arnold Map [3]	Selective Encryption [6]

Proposed Architecture



Latin Square (LS) / Latin Rectangle (LR)

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} \begin{bmatrix} \alpha & \beta & \delta \\ \delta & \alpha & \beta \\ \beta & \delta & \alpha \end{bmatrix} \qquad \begin{bmatrix} 1 & 3 & 2 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \\ 5 & 2 & 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} \alpha & \beta & \mu & \delta \\ \beta & \alpha & \delta & \mu \end{bmatrix}$$

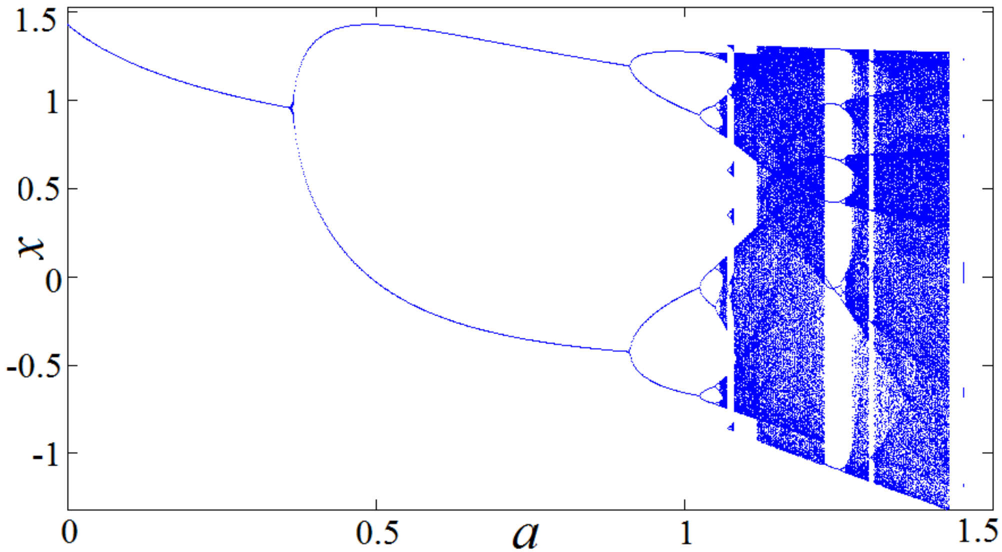
- *Latin Square* = *P* x *P* array such that each row and column contains every symbol from {1, 2, …, *P*} exactly once

- *Latin Rectangle* = *P* x *Q* array with elements occurring at most once in each row and column

Latin Rectangle Scrambling

- Using a discrete 2D Hénon chaotic map:

$$x_{n+1} = 1 - ax_n^2 + y_n$$
$$y_{n+1} = bx_n$$



Bifurcation Diagram

Chaotic when
a = 1.4, b = 0.3

Lyapunov Exponent = 0.4241

Latin Rectangle Scrambling

Algorithm I: Construction of Latin Square	
Input:	x_0, y_0 – initial conditions H, W – height and width of image to be scrambled
Out:	LS – Latin square of order max(H, W)
1.	$M = \max(H, W)$; $a = 1.4$; $b = 0.3$; $N_0 = 1000$
2.	for $n \leftarrow 1$ to $\mathbf{N_0+M}$ do $X(n), Y(n) \leftarrow HénonMap(x_0, y_0)$
3.	$X \leftarrow X(N_0+1: N_0+M)$; $Y \leftarrow Y(N_0+1: N_0+M)$
4.	$\boldsymbol{X \leftarrow 10^6 X - floor(10^6 X); Y \leftarrow 10^6 Y - floor(10^6 Y)}$
5.	$Q \leftarrow Sort(X)$; $F \leftarrow Sort(Y)$
6.	for $r \leftarrow 1$ to M do
7.	$LS(r, :) \leftarrow Q >> F(i)$
8.	end

Latin Rectangle Scrambling

Algorithm II: Latin Rectangle Scrambling

Input: I – plain image to be scrambled
LS – Latin square of order $M = \max(H, W)$

Out: SI – Scrambled image

1. If $\boldsymbol{H == W}$
2. for $i \leftarrow 1$ to H do $T(i,:) = I(i, LS(i,:)')$ % *Row*
3. for $j \leftarrow 1$ to W do $SI(:,j) = T(LS(:,j)', j)$ % *Column*
4. else if $\boldsymbol{H > W}$
5. for $j \leftarrow 1$ to W do $T(:,j) = I(LS(:,j)', j)$ % ***Column***
6. $LR \leftarrow \text{zeros}(H, W)$
7. for $i \leftarrow 1$ to M do
8. $r = LS(i,:)$; $\boldsymbol{r(r > W) = [\ ]}$; $LR(i,:) = r$; % ***LR***
9. for $i \leftarrow 1$ to H do $SI(i,:) = T(i, LR(i,:)')$ % ***Row***

Latin Rectangle Scrambling

10	3	50	130
4	15	77	137
15	54	102	125
75	113	113	112
146	126	97	110
168	115	89	104

(a)

4	5	6	2	1	3
5	6	1	3	2	4
6	1	2	4	3	5
3	4	5	1	6	2
1	2	3	5	4	6
2	3	4	6	5	1

(b)

4	5	6	2	1	3
5	6	1	3	2	4
6	1	2	4	3	5
3	4	5	1	6	2
1	2	3	5	4	6
2	3	4	6	5	1

(c)

75	126	89	137
146	115	50	125
168	3	77	112
15	113	97	130
10	15	102	110
4	54	113	104

(d)

4	2	1	3
1	3	2	4
1	2	4	3
3	4	1	2
1	2	3	4
2	3	4	1

(e)

137	126	75	89
146	50	115	125
168	3	112	77
97	130	15	113
10	15	102	110
54	113	104	4

(f)

(a) Plain Image (6 x 4),

(b) Latin Square (6 x 6),

(c) Partial Latin Square (6 x 4),

(d) Column Scrambling with (c),

(e) Latin Rectangle (6 x 4),

(f) Row Scrambling with (e)

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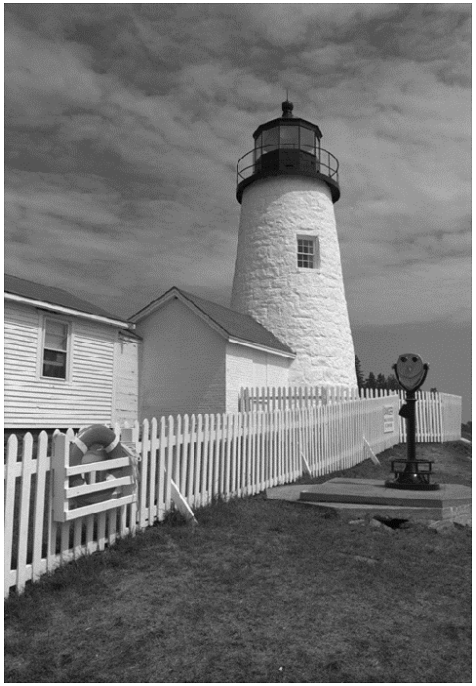
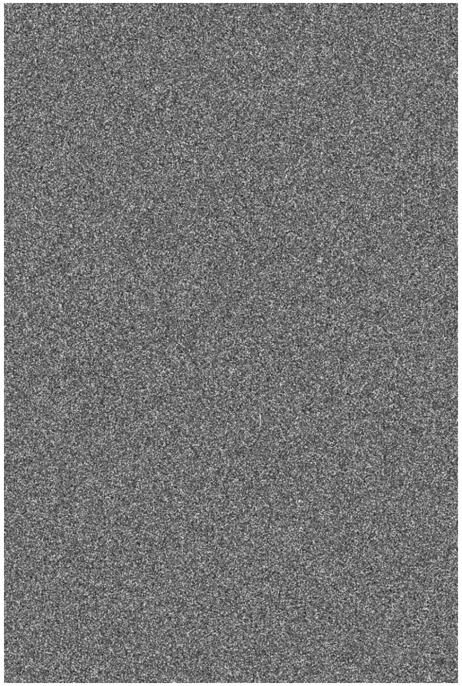
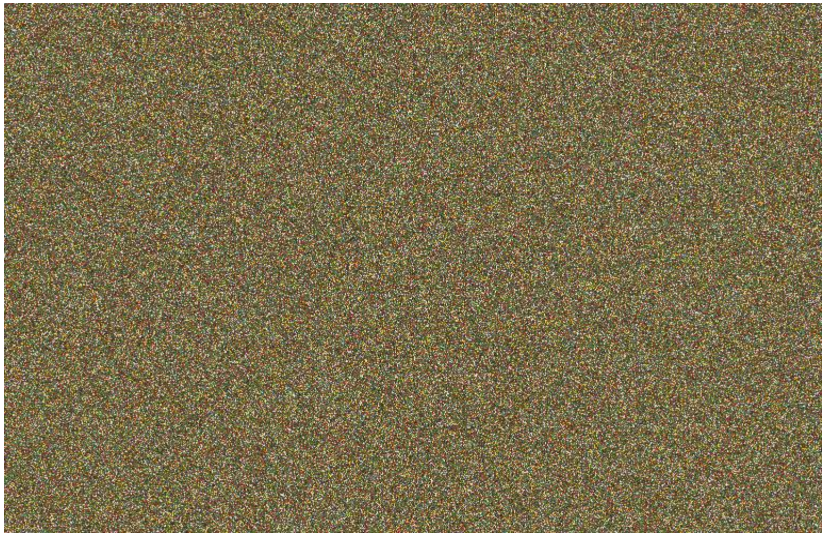
Latin Rectangle Scrambling

720 x 480





512 x 768



Pixel Diffusion

- Update pixel values to avoid differential cryptanalytic attack
- Typically done once per iteration => more iterations needed to achieve better security
- Proposed: perform **Forward** as well as **Backward** diffusions in single iteration => less iterations required

Bi-directional Pixel Diffusion

Plain Image

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
21	22	23	24

LR Scrambled Image

12	6	13	3
22	5	4	19
20	11	14	21
1	24	15	18
23	9	2	8
7	10	16	17

Forward Diffusion

12	6	13	3
22	5	4	19
20	11	14	21
1	55	32	78
167	174	176	136
143	249	233	216

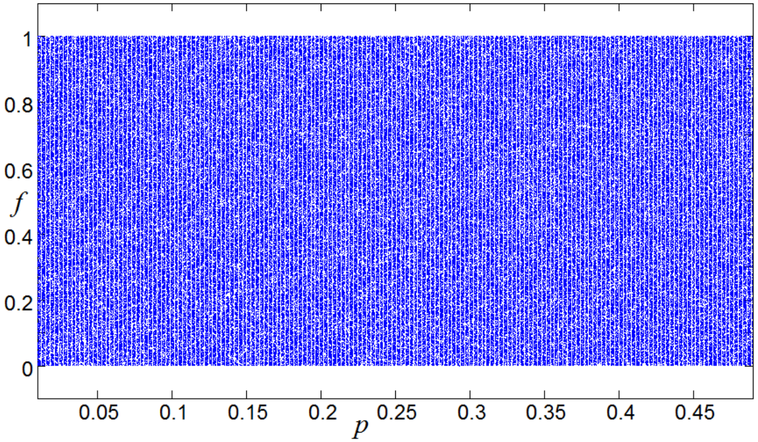
Backward Diffusion

223	216	119	210
203	220	118	98
197	201	9	26
250	37	41	139
249	80	159	146
211	66	100	187

Bi-directional Pixel Diffusion

- Using 1D Piecewise Linear Chaotic Map (PWLCM):

$$f(k) = F\left(f(k-1), p\right) = \begin{cases} f(k-1)/p & \text{if } f(k-1) \in [0, p); \\ \left(f(k-1)-p\right)/\left(0.5-p\right) & \text{if } f(k-1) \in [p, 0.5]; \\ F\left(1-f(k-1), p\right) & \text{if } f(k-1) \in (0.5, 1] \end{cases}$$



Discrete 1D PWLCM:

$$\phi(k) = \mathrm{mod}\left(floor\left(\left(\left(f(k)+1\right)/2\right)\times 10^{8}\right), L\right)$$

Bifurcation Diagram of PWLCM:

Chaotic in the range of control parameter $p \in (0, 0.5)$

Bi-directional Pixel Diffusion

Algorithm III: Bi-directional Pixel Diffusion	
Input:	*SI* – Latin scrambled image
	iD, nD, pD, vD_1, vD_2 – secret keys
Out:	*CI* – Cipher (encrypted image)
1.	$IS = H \times W$
2.	for $k \leftarrow 1$ to IS do $f(k) \leftarrow PWLCM(iD, nD, pD)$
3.	for $k \leftarrow 1$ to IS do $f(k) \leftarrow Digitize(f(k))$ % ***Discrete map***
4.	$SI \leftarrow Vectorize(SI)$ % *1-D vector*
5.	$TI \leftarrow ForwardDiffusion(SI, vD_1)$ % ***Forward Diff.***
6.	$CI \leftarrow BackwardDiffusion(TI, vD_2)$ % ***Backward Diff.***
7.	$CI \leftarrow Reshape(CI, H, W)$

$$TI(1) = \phi(1) \oplus \{[SI(1) + \phi(1)] \bmod N\} \oplus vD_1$$

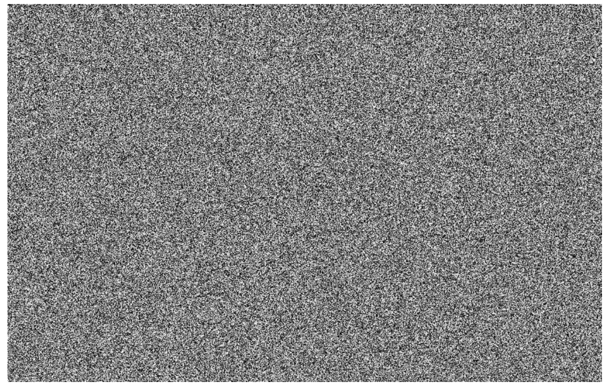
$$TI(k) = \phi(k) \oplus \{[SI(k) + \phi(k)] \bmod N\} \oplus TI(k-1)$$

$$CI(IS) = \phi(IS) \oplus \{[TI(IS) + \phi(IS)] \bmod N\} \oplus vD_2$$

$$CI(k) = \phi(k) \oplus \{[TI(k) + \phi(k)] \bmod N\} \oplus CI(k+1)$$

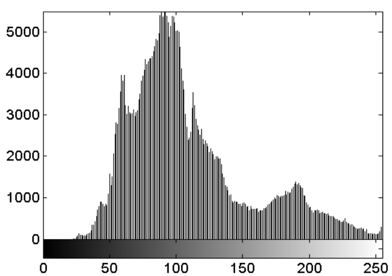
Results: Visual, Entropy, Correlation

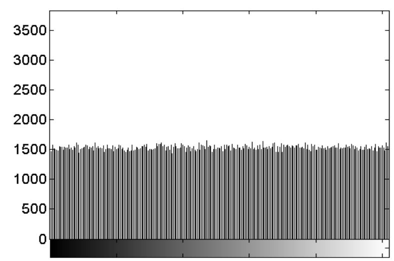
Plain Image (P)



Cipher Image (C)

Decrypted Image (MSE = 0)

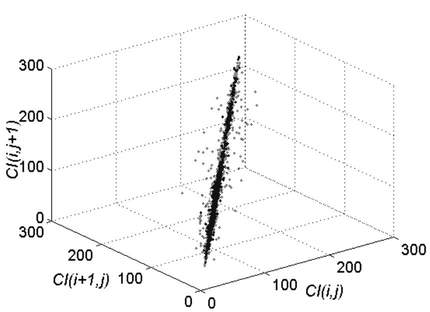


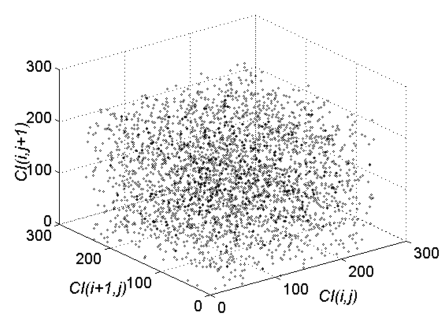


<= Histograms / Entropy

$HG(P)$ = 7.2859, $HG(C)$ = **7.9995**

$HL(P)$ = 5.5016, $HL(C)$ = **7.9109**





<= Correlation Scatter Plots

$\rho(P)$ = 0.9937, $\rho(C)$ = **0.0033**

Results: Diffusion Property Analysis

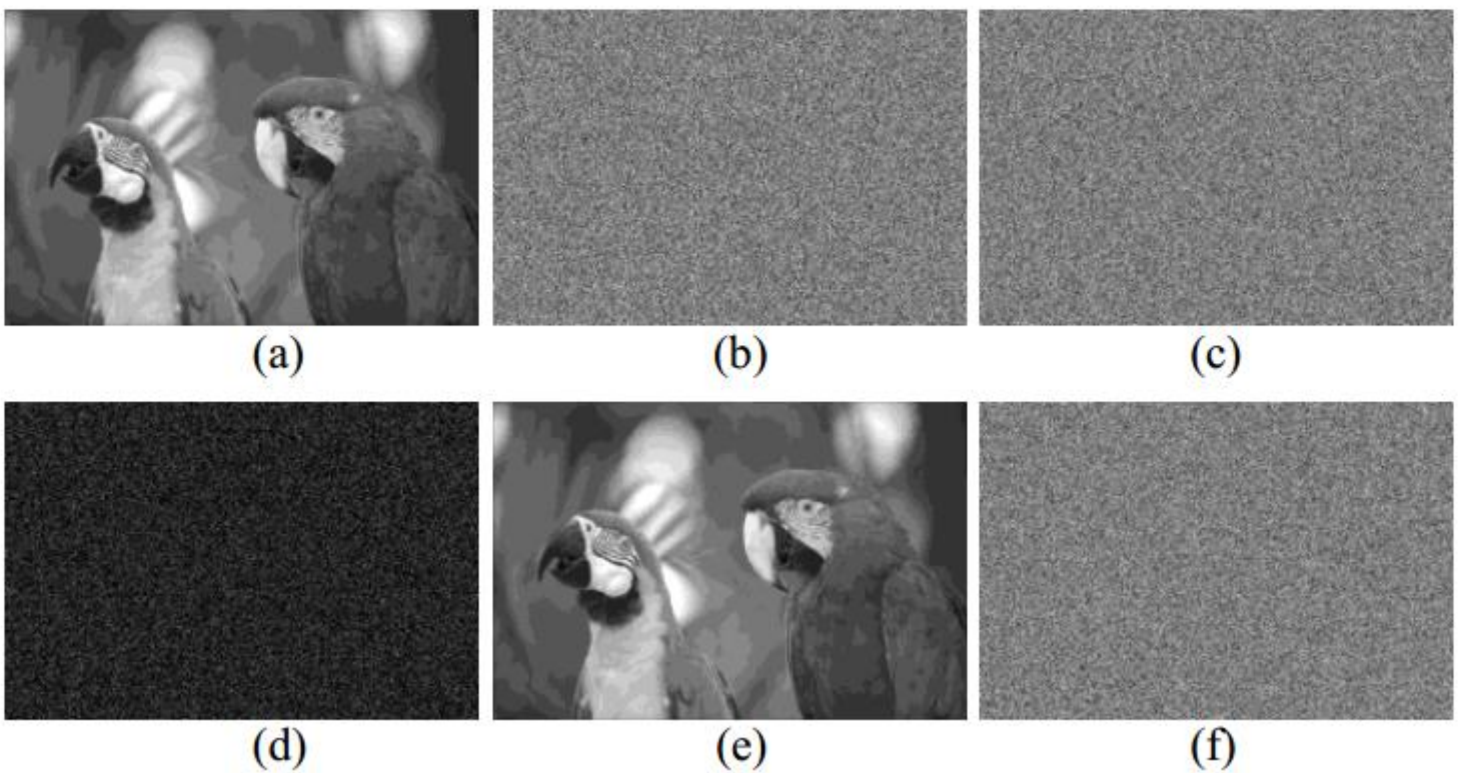
Encryption Method	GDD	NPCR%	UACI%
Ideally	1	> 99.609	> 33.464
Borujeni et al. [7]	0.9173	95.5293	33.4398
Behnia et al. [8]	0.8147	42.8195	32.4671
Proposed	**0.9951**	**99.6442**	**33.5061**

GDD = Gray Difference Degree (to evaluate scrambling)
NPCR = Number of Pixels Change Rate (to evaluate robustness to attacks)
UACI = Unified Average Change Intensity (robustness to attacks)

Results: Key Sensitivity Analysis

$\boldsymbol{K}^1$=79DE299EAF4E68FD4D6047B1318E87C21A165372E293C3D1ECAF635DE481ACC**0**

$\boldsymbol{K}^2$=79DE299EAF4E68FD4D6047B1318E87C21A165372E293C3D1ECAF635DE481ACC**1**



(a) Plain image P

(b) $C^1 = E(P, K^1)$

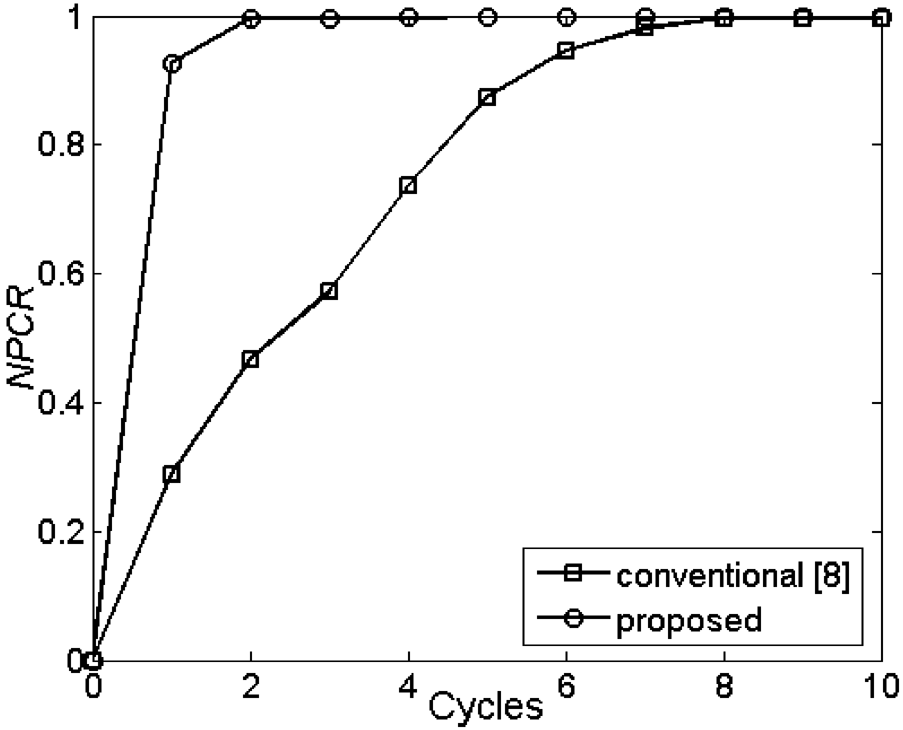
(c) $C^2 = E(P, K^2)$

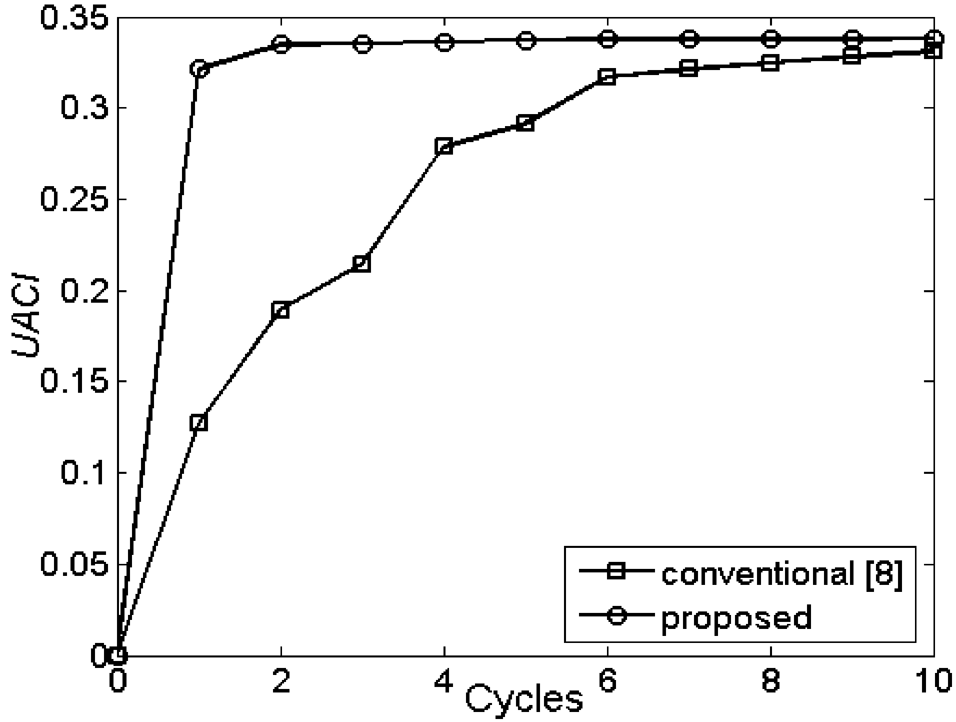
(d) difference $|C^1 - C^2|$

(e) $D^1 = D(C^1, K^1)$

(f) $D^2 = D(C^1, K^2)$

Results: Computation Analysis





Only **3 cycles** needed to achieve enhanced security and resist cryptanalytic attacks with proposed system

Conclusions

- Novel Latin rectangle scrambler for confusion
- Bi-directional pixel diffusion for robustness to differential attacks
- Completely invertible
- Reduced overall number of cycles to achieve enhanced security

References

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8) S. Behnia, A. Akhshani, H. Mahmodi, A. Akhavan, “A novel algorithm for image encryption based on mixture of chaotic maps,” Chaos, Solitons and Fractals, vol. 35, no.2, pp. 408–419, 2008

Appendix

x_0 – double (0.0, 1.0)	nD – integer (>= 25)
y_0 – double (0.0, 1.0)	pD – double (0.0, 0.5)
nC – integer (>= 2)	vD_1 – integer (0, 255)
iD – double (0.0, 1.0)	vD_2 – integer (0, 255)

$$v(b) = \sum_{i=1}^{q} b_{-i} 2^{-i}$$

$$GD_{i,j}^{P} = \frac{1}{4} \sum_{i'j' \in \{-1,+1\}} \left[P(i,j) - P(i',j')\right]^2$$

$$EGD^{P} = \frac{\sum_{i=2}^{H-1} \sum_{j=2}^{W-1} GD^{P}(i,j)}{(H-2)\times(W-2)}$$

$$\boxed{GDD(P,C) = \frac{EGD^{P} - EGD^{C}}{EGD^{P} + EGD^{C}}}$$

$$\boxed{N(C^1, C^2) = \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{Diff(i,j)}{M \times N} \times 100\%}$$

$$Diff(i,j) = \begin{cases} 0, \text{ if } C^1(i,j) = C^2(i,j) \\ 1, \text{ if } C^1(i,j) \neq C^2(i,j) \end{cases}$$

$$\boxed{U(C^1, C^2) = \sum_{i=1}^{M} \sum_{j=1}^{N} \frac{\left|C^1(i,j) - C^2(i,j)\right|}{256 \times M \times N} \times 100\%}$$