Fairest rent division

Santosh Kavhar (22m0787@iitb.ac.in)
Computer Science and Engineering
Indian Institute of Technology, Bombay
Mumbai, India

***Abstract*—How do you collect rent of an apartment which has got rooms with different values from the perspective of the different roommates. One way would be you could fight for it, other way could be there is a lucky draw, but both these are unfair. What if we use some Game Theory to solve this problem of ours? Various approaches have been used for dividing the rent fairly and used properties like envy-freeness, maximal solution and also some are equitable solutions. Many have just implemented the envy-free solution, others have used the combination of above mentioned methods. We have studied one paper titled "Which is the fairest rent division of them all?" This paper has combined the approaches of all the three solutions and given a final solution with welfare maximization. We conclude from our study that the given paper states that the maximin solution which also turns out to be the equitable solution is way better than the normal or any random envy-free solution. It provides an algorithm which uses Linear Programming to solve the problem.**

***Index Terms*—Game Theory, utility, welfare maximization, envy-free solution, maximin solution, equitable solution**

I. Introduction

Several readers may have firsthand knowledge of the rent division issue. When multiple housemates move in together, they must determine who gets which room and at what cost. When the rooms are of different quality, the issue becomes intriguing and, more often than not, a source of aggravation. The next task is to create "rental harmony" by allocating the rooms and fairly sharing the rent. However there are challenges, the cost of rooms will differ in everyone's eye and who is to be allocated which room can be daunting to solve. There is no such thing as "rental harmony", there can only be an approximate solution here and that is what the paper strives to find. There are approaches which talk of an envy-free solution but there is a fault with it. Consider there are 3 players(p1,p2,p3) who want one of the three rooms(r1,r2,r3). If total rent is 3 rs and player i values room i as 3rs and for other 2 rooms the value is 0 rs and then if players get allocated the one with r3 will be indifferent but r1 and r2 allocated people will be more benefited that they are getting their rooms for free! Thus It is not enough to promise excellent outcomes for envy-freeness alone.

It is interesting to enquire whether this problem is still relevant in today's world. As long as people have rooms to share in a single apartment there will be the problem of rental harmony and this problem will rise up again. So, yes this problem is relevant in today's world as well as in the future and this problem can be slightly changed to work out for the problem of splitting goods and other items fairly.

There could be many outcomes for envy free solutions, the difficulty is in selecting among a wide range of feasible envy-free options. The most natural approach to do this, according to some, is to maximise a utility function that satisfies desired social characteristics while adhering to the constraint of envy-freeness. In this study we will only talk about the paper titled "Which is the fairest rent division of them all?", as that is the only paper which talks about the maximal solution that we are going to work on. The best part of this is that the algorithm works in polynomial time.

Fairest rent division will give the best solution possible theoretically yet many a times it's possible that there just isn't an appropriate way to divide the rooms or many are not satisfied with the solution, in those situations you should either hunt for a less expensive apartment or find new roommates. We will however assume that users are concerned about the optimal solution given by this algorithm and that they respect the choice of the algorithm.

There is use of Linear Programming to get the minimum social disparity. The room mapping is also Welfare maximizing.

In section 2 we will talk about the Background of the problem, in section 3 we will talk of the Fairest Rent Division solution. Then we will see the comparison of both these approaches as based on the study. Finally we will see the conclusion and open challenges.

II. Background

Game Theory is a mathematical framework which is used to examine how rational decision-makers interact strategically. It is an area of applied mathematics that focuses on simulating and forecasting how people will act in settings where everyone's decisions may affect the results. However in practice finding rational decision makers is quite difficult. Normal Players tend to just pick up what seems to be best for them without giving the thought about other players moves. If everyone was perfectly rational then this world would have Game Theory as its principal subject, but we know Psychology and other factors still dominate the behaviour rather than Game Theory.

Utility refers to the measure of satisfaction or value that a player receives from a particular outcome in a game. It is a subjective measure of the player's preferences over the available outcomes. Utility can be represented by a numerical value

Under the guidance of Prof. Swaprava Nath

assigned to each outcome, with higher numbers representing outcomes that are more desirable to the player.

The term **welfare maximisation** describes the idea of maximising overall societal welfare, which is the wealth and well-being of people within a society. In economic theory, maximising social welfare—which considers the welfare of all members of society, not just one particular group—is frequently linked with maximising individual welfare.

A scenario where everyone in a group feels they have gotten a fair share of the resources, without feeling jealous of others, is referred to as an **envy-free solution** in game theory. In other words, a resource allocation is envy-free if no member would choose to switch positions with another.

If all players continue to play according to their present strategies, a **maximal solution** is one in which no player can change their strategy and yet enhance their outcome. To put it another way, a maximal solution is when every player is using their best tactics in light of what the other players are doing.

A fair or just solution is one that equally divides the advantages or disadvantages of a game among the participants. An **equitable solution** is one in which each participant receives a reward or result that is thought to be right or fair based on certain standards or principles. Finding each of these 3 solutions takes on very different approaches but the paper we are discussing has brought these all solutions under one umbrella. It gives a maximin solution which is also equitable and envy-free.

III. Fairest Rent Division Solution

All of the solutions, mentioned in the Background section, can be combined together and by their cocktail we get the fairest solution for rent division.

There can exist many envy-free solutions, we need to find the best one of them which is also maximal. The paper [1] proves that every maximal solution is also equitable, but not every equitable solution is maximin. Thus combining all the solutions we get a solution which runs in polynomial time. For maximising the minimum utility of any player subject to envy freeness and for minimising disparity — the maximum difference in utilities, we use Linear Programming based approach which works simultaneously on both these approaches.

For the input we will take the number of rooms that are to be rented by the same number of people, then we will take input each user's valuation for those many rooms. This valuation has to be from each of user's perspectives and it is according to it that rooms are allocated. The allocation is such that each user has maximum utility.

The authors use Linear programming while the objective function is Max U and the constraints are as follows:

1) the utility of each player is greater or equal to U, while the utility is defined as the player's value to his room minus his rent (how much he pays)
2) the envy-freeness constraint – the utility of each player is greater or equal to his value for any other room minus that room's rent
3) the rooms' rent adds up to 1 (or to the total rent if the values are not between 0 and 1)

The first constraint gives the notion of maximizing the utility, the second constraint is about envy-freeness and the third constarint is so that rent adds up to the total.

IV. Experimental Analysis

These findings are based on the questions asked by the the team of Gal et all. to users of application called spliddit.

Solutions	Satisfaction on disparity	Satisfaction on utility of worst-off player
Random Envy-free solution	Low	Medium
Random Equitable solution	Medium	Medium
Proposed Maximal solution	High	High

As the figure shows the highest satisfaction on utility of worst-off player is by the proposed solution of maximal solution which is also equitable. Random equitable solution is medium in terms of satisfaction of disparity as well as satisfaction on utility of worst-off player. Random Envy free solution is low on satisfying the disparity and medium on satisfying the utility of worst-off player.

V. Conclusion and Open Challenges

There have been many approaches to find the fair rent division, most on envy freeness then some based on maximal solution, others on equitable solution. There have also been combined solutions proposed which didn't run in polynomial time. The paper's solution is guaranteed to be envy-free, maximal and equitable and still runs in polynomial time.

Implementation of this code is quite challenging, as far as now we are stuck on the implementation part. The paper's algorithm uses Linear Programming heavily and we haven't figured out how to dynamically create Linear Equations that can be solved. There is also front-end to be created for this application which will be similar to the application of spliddit. But spliddit has closed so we won't get the interface like it had.

References

[1] Ya'akov Gal, Moshe Mash, Ariel D. Procaccia, and Yair Zick. Which is the fairest (rent division) of them all? *Journal of the ACM (JACM)*, 64:1 – 22, 2017.