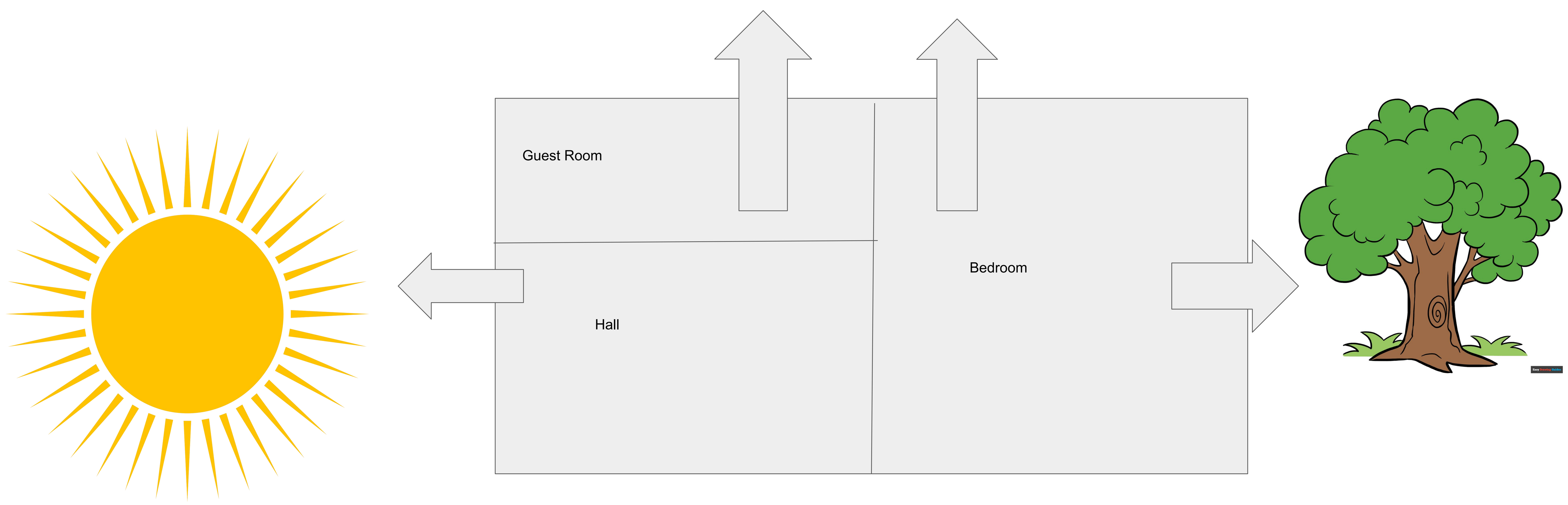
Fairest Room allocation

Problem:

Same rent for every room?



What is LP?

- Linear Objective and Linear inequalities
- A method of optimising operations with some con

$$\text{minimize} \quad \boldsymbol{c}^T \boldsymbol{x}$$
$$\text{subject to} \quad \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$$
$$\boldsymbol{x} \geq \boldsymbol{0}$$
$$\text{where } \mathbf{c} \in \mathbb{R}^{\mathrm{N}}, \mathbf{b} \in \mathbb{R}^{\mathrm{M}}, \text{and } \mathbf{A} \in \mathbb{R}^{\mathrm{M*N}},$$

$$\text{here } \boldsymbol{c}^T = [c_1, c_2, ..., c_n],$$
$$\boldsymbol{x} = [x_1, x_2, ..., x_n]^T \ ,$$
$$\boldsymbol{b} = [b_1, b_2, ..., b_m]^T \ , \text{ and}$$
$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

Example

Maximize: Z = 7x + y

Constraints:

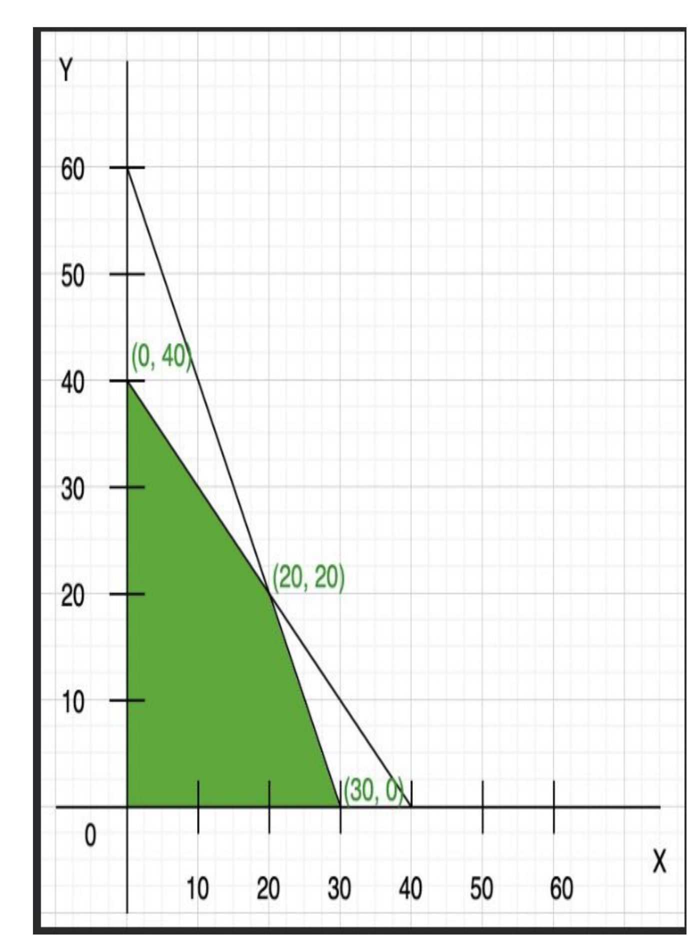
x + y <= 40,

2x + y <= 60,

x >= 0, y >= 0

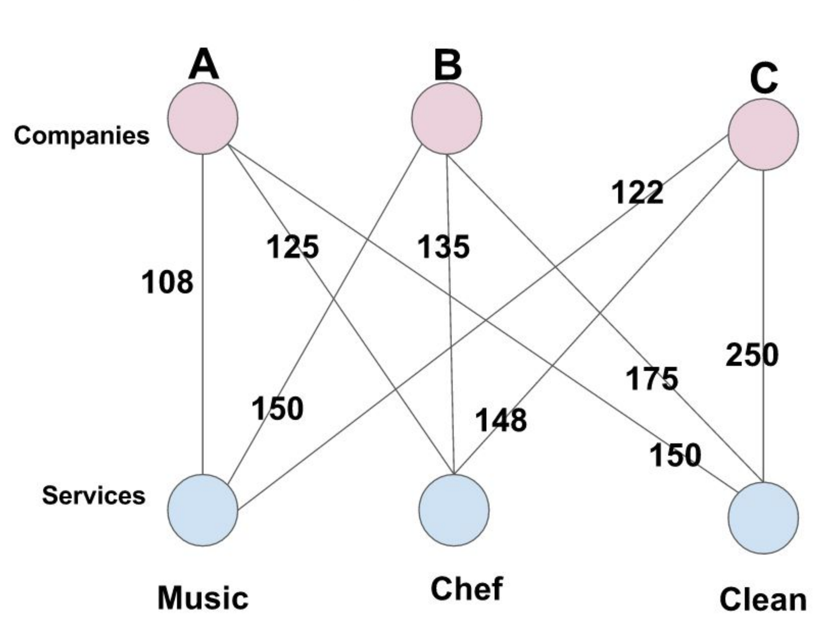
Max at (30, 0) ->

Optimal value 210



Maximum Welfare assignment

-aka Maximum Weight Bipartite Matching



It's Formulation

Maximization Problem

Renter i's value for room j

$\mathbf{x_{ij}}$ is 1 when renter **i** will be assigned room **j**

$$\max \sum_{i \in N} \sum_{j \in M} v_{ij} \cdot x_{ij} \qquad (1)$$

A renter can be allotted at max one room

$$\text{s.t.} \sum_{j \in M} x_{ij} \leq 1, \ \forall i \in N \qquad (2)$$

A room can be allotted to at max one renter

$$\sum_{i \in N} x_{ij} \leq 1, \ \forall j \in M \qquad (3)$$

$$x_{ij} \geq 0, \ \forall i \in N, j \in M. \qquad (4)$$

Non-negativity constraint

Envy-freeness formulation

Minimization Problem:

Room allocated to renter 1

Room allocated to renter n

$$R \leq f_q(v_{1\sigma(1)} - p_{\sigma(1)}, \cdots, v_{n\sigma(n)} - p_{\sigma(n)})$$

$$v_{i\sigma(i)} - p_{\sigma(i)} \geq v_{ij} - p_j \qquad \forall i, j \in [n]$$

Renter i's value for his allocated room

$$\sum_{i=1}^{N} p_i = Rent$$

Final price of Room j

Real allocation

	Room 1	Room 2	Room 3
Renter 1	6	2	1
Renter 2	2	3	4
Renter 3	2	5	2

Table 3.1: Renter's preference for rooms

Renter	Room	Rent
1	1	4
2	3	2
3	2	3

Table 3.3: Final Rent assigned