

XOR of Numbers from 1 to n in Constant Time

To find the XOR of numbers from 1 to n in constant time, you can use a mathematical property based on the pattern of XOR results.

The XOR of numbers from 1 to n can be derived as follows:

1. If $n \bmod 4 = 0$: $\text{XOR} = n$
2. If $n \bmod 4 = 1$: $\text{XOR} = 1$
3. If $n \bmod 4 = 2$: $\text{XOR} = n + 1$
4. If $n \bmod 4 = 3$: $\text{XOR} = 0$

So, the formula to find the XOR from 1 to n in constant time is:

$\text{XOR}(1, n) =$

n if $n \bmod 4 = 0$

1 if $n \bmod 4 = 1$

$n + 1$ if $n \bmod 4 = 2$

0 if $n \bmod 4 = 3$

This method works in $O(1)$ time, as it only involves simple modulus and conditional operations.