Santosh Rajkumar

1. Solving Boundary Value Problem (BVP) with MATLAB ode45

Solve the following ODE for the given boundary conditions using **Shooting Method**,

$$u^{''} = \pi^2 u - 2\pi^2 sin(\pi x) \text{ , boundary condition are: } u(0) = u(1) = 0$$

Solution:
Steps involved:

- Formulate IVP 1 & get the solution u_1.
- Formulate IVP 2 & get the solution u_2.
- The BVP solution is given by $u = u_1 + cu_2$, where $c = \frac{\beta - u_1(1)}{u_2(1)}$

IVP 1:

$$u^{''} = \pi^2 u - 2\pi^2 sin(\pi x) \text{ , } u(0) = 0, u^{'}(0) = 0 \tag{1}$$

*Here we are considering $-2\pi^2 sin(\pi x)$ as the residue function.

IVP 2:

$$u^{''} = \pi^2 u \text{ , } u(0) = 0, u^{'}(0) = 1$$

Solving IVP 1 with MATLAB ode45:
Let, $u_1 = u$, then $u_2 = \dot{u_1}$
Therefore equations we are to use in ode solver are:

$$\dot{u_1} = u_2$$
$$\dot{u_2} = \pi^2 u_1 - 2\pi^2 sin(\pi x)$$

Now, The ODE equation is kept in a MATLAB function file as below:

```
% MATLAB IVP 1 function file

function uPrime = ivp1(x,u)
uPrime = zeros(2,1);
uPrime(1) = u(2);
uPrime(2) = (pi^2)*u(1) - 2*(pi^2)*sin(pi*x);
end
```

Now, the ivp1 function is used in a MATLAB script to solve the ODE using ODE45 as below:

```
%Script to solve  IVP 1 ODE with ode45
tspan = 0:0.25:1; %timespan
u0 = [0 0]; %initial conditions
[x,u] = ode45(@ivp1, tspan, u0); %ode45 solver function
plot(x,u(:,1),'r', 'linewidth', 2.5) %plotting solution u1
xlabel('x')
ylabel('solution (u)')
```

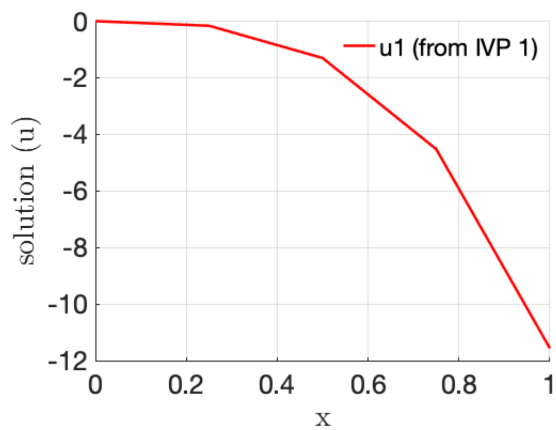

Figure 1: Plot of solution u_1 obtained using MATLAB ode45

Solving IVP 2 using MATLAB:
IVP 2 is also solved in the same way as IVP 1 and the solution u_2 is obtained (Fig 2).

```
% MATLAB IVP 2 function file

function uPrime = ivp1(x,u)
uPrime = zeros(2,1);
uPrime(1) = u(2);
uPrime(2) = (pi^2)*u(1);
end
```

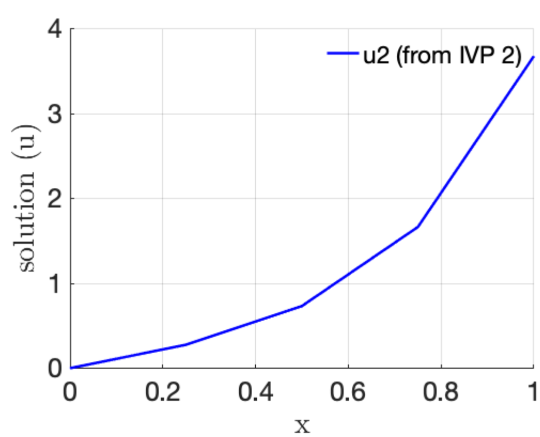

Figure 2: Plot of solution u_2 obtained using MATLAB ode45

The table below shows values of u_1 & u_2 obtained:

x	u1	u2
0	0	0
0.25	-0.1615671	0.27650687
0.5	-1.301302	0.73252705
0.75	-4.5208776	1.66411785
1	-11.548765	3.67608291

Now,

$$c = \frac{\beta - u_1(1)}{u_2(1)} \text{ where } \beta = u(1) = 0, u_1(1) = -11.548756, u_2(1) = 3.67608291$$

Therefore, $c = 3.1416$

Finding the solution of the BVP

The solution of the BVP is

$$u = u_1 + cu_2$$

The table below shows the solution of the BVP obtained using shooting method.

x	u1	u2	u (BVP solution)
0	0	0	0
0.25	- 0.1615671	0.27650687	0.707106836
0.5	-1.301302	0.73252705	1.00000496
0.75	- 4.5208776	1.66411785	0.707115038
1	- 11.548765	3.67608291	0

The solution curves all together are shown below: