Laplace Equation.

$$\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) = 0 \longrightarrow (1)$$

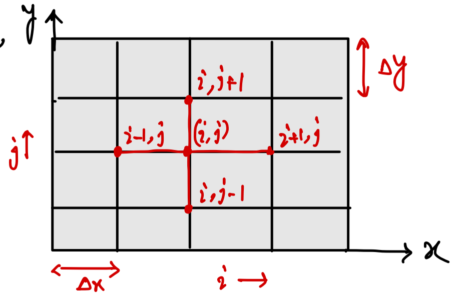
Which satisfies over a domain of [0, 1; 0 1] the following conditions:

$$u(0, y) = u(1, y) = 0 \text{ and } u(x, 0) = \cos(x) \text{ and } u(x, 1) = 2\cos(x)$$

The centreal difference scheme:

In this method, we try to approximate the partial dereivatives with centreal difference scheme.

We can approximate the dereivatives, as centreal finite difference in first ordere as,



$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} = \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2}$$

$$\left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2}$$

$\therefore$ equation (1) $\Rightarrow$

$$\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2} = 0 \longrightarrow (2)$$

if we take, $\Delta x = \Delta y$

$\therefore$ equation (2) $\Rightarrow$

$$u_{i+1,j} - 2u_{i,j} + u_{i-1,j} + u_{i,j+1} - 2u_{i,j} + u_{i,j-1} = 0$$

$$\Rightarrow \quad 4u_{i,j} = u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}$$

$$\Rightarrow \quad \boxed{u_{i,j} = \frac{1}{4}\left[u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1}\right]}$$

Boundary conditions:

Given, $u(0,y) = u(1,y) = 0$

$$u(x,0) = \cos x$$

$$u(x,1) = 2\cos x$$

The boundary conditions are non-homogeneous in nature. Hence, we try to take the approach similar to what we did for separation of variable.