Learning a DFA

Tuesday, October 1, 2019 5:29 PM

maintain Observation Table

- candidate states
  - labeled by strings
  - prefix closed
- query strings
  - suffix closed

	e
s	

$$T(s\cdot e) = \begin{cases} 1 & s\cdot e \in L \\ 0 & \text{otherwise.} \end{cases}$$

Try to get a closed, consistent table.

<u>closed</u> $\forall s: row(s) \in S \Rightarrow row(s.a) \in S.\ \forall a \in \Sigma$

(or $t \in S.\Sigma$, $\exists s \in S$ s.t. $row(t) = row(s)$)

<u>consistent</u> $\forall s_1, s_2 \in S: row(s_1) = row(s_2) \Rightarrow$

$$row(s_1.a) = row(s_2.a) \quad \forall a \in \Sigma$$

$\in S \qquad row(s_1 \cdot a) = row(s_2 \cdot a)$

Algorithm

Start with $S = E = \{\varepsilon\}$ and $T(\varepsilon)$.

$S \cdot \Sigma = \{0, 1\} \quad T(0), T(1).$

If not consistent, i.e. $\exists a \in \Sigma$

s.t. $row(s_1 \cdot a) \neq row(s_2 \cdot a) \quad (row(s_1) = row(s_2))$

i.e. $\exists e \in E$:

$$T(s_1 \cdot e \cdot a) \neq T(s_2 \cdot e \cdot a)$$

then add $e \cdot a$ and all suffixes to E

complete table.

If not closed, i.e. $\exists t \in S \cdot \Sigma$

s.t. $row(t) \neq row(s) \ \forall s \in S$,

add t to S. complete table

If closed & consistent, propose DFA:

$Q = \{ row(s) : s \in S \}$

$Q = \{ row(s) : s \in S \}$

$q_0 = row(\varepsilon)$

$F = \{ row(s) : T(s) = 1 \}$

$\delta(row(s), a) = row(s \cdot a)$.

If counterexample x,
add x and all prefixes to S.
Complete table.

<u>Lemma 1.</u> DFA $M(T)$ is consistent with table T and is the smallest such DFA.

<u>Pf.</u> (1) $\delta(q_0, s) = row(s)$.

(2) $\delta(q_0, s \cdot e) \in F$ iff $T(s \cdot e) = 1$

induction on $|s|$, $|e|$.

Any DFA consistent with T must ... distinct rows of S.

Any DFA consistent ...
have distinct states for distinct rows of S.

<u>Lemma 2</u>. At most $n-1$ conjectures + not closed or not consistent iterations.

<u>Pf.</u> In each such iteration, # states, i.e. distinct rows of S increases by 1.

<u>Thm</u>. Algorithm learns any DFA with n states over alphabet of size k in time poly(n, k, m) where m = size of largest counterexample given.

<u>Another example run.</u>

even #0's
odd 1's

	ε
ε	0

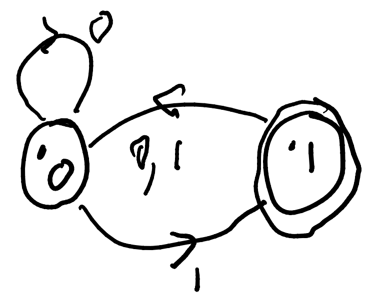
not closed

	ε
ε	0
0	0
1	1

not closed

	ε
ε	0
1	1
0	0
10	0
11	0

closed, consistent

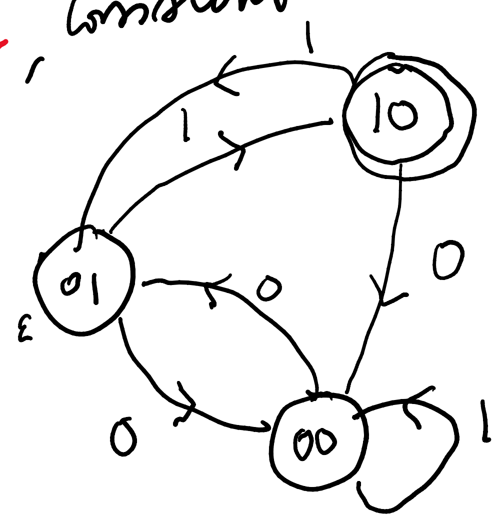


contraexample 001.

	ε	1
ε	0	1
1	1	0
0	0	0
00	0	1
001	1	0
01	0	0
000	0	0
0010	0	0
0011	0	1
10	0	0
11	0	1

closed.
not consistent

closed, consistent



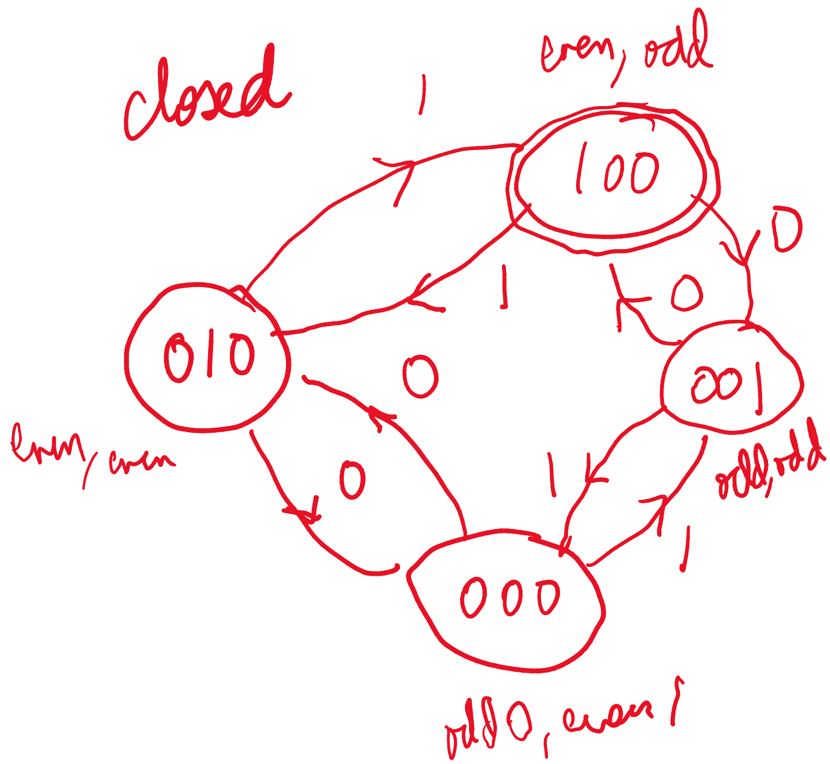
counterexample: 010

	ε	1	0
ε	0	1	0
1	1	0	0
0	0	0	0
00	0	1	0
001	1	0	0
01	0	0	1
010	1	0	0
000	0	0	0
0010	0	0	1
0011	0	1	0
011	0	0	0
0100	0	0	1
0101	0	1	0
10	0	0	1
11	0	1	0

closed,

not consistent!

closed



YES!