Time & Space

Monday, October 28, 2019 6:09 AM

plan
- Password Game

- Turing Machines

"anything computable is computable by a TM"

Nondeterminism does not extend the class of languages accepted by TMs.

Use BFS on computation tree of <u>Configurations</u> = ⟨TAPE CONTENT, STATE, HEAD POS.⟩

<u>Thm</u>. $NTIME(t(n)) \subseteq DTIME(2^{O(t(n))})$.

<u>Pf.</u> $NTIME(t(n)) \Rightarrow \exists$ a computation tree of depth $t(n)$ that accepts L.

What is max size of this tree? $(|\Gamma| + |Q|)^{t(n)}$

since $|\Gamma|, |Q|$ are constants, $= 2^{O(t(n))}$.

BFS.

What about space $s(n)$?

Thm. (EASY) $NSPACE(s(n)) \subseteq DTIME(2^{O(s(n))})$

$\subseteq DSPACE(2^{O(s(n))})$.

Pf. Total number of possible configurations with $s(n)$ space $\leq |\Gamma|^{s(n)} \times |Q| \times s(n)$

$= 2^{O(s(n) + \log(s(n)))}$.

$= 2^{O(s(n))}$

$s(n)$ is a space-constructible function:

$s: \mathbb{N} \to \mathbb{N}$, $s(n) \geq \log_2 n$, $\exists$ TM s.t. on input 1^n outputs $s(n)$ in binary and uses $O(s(n))$ space.

It is possible for a TM to mark off

It is possible for a TM to mark off $s(n)$ squares on its tape.

Thm. $NSPACE(s(n)) \subseteq DSPACE(O(s(n)^2))$.

Pf. with space $s(n)$, # configurations $= 2^{O(s(n))}$.

Imagine a large directed graph $G=(V,E)$

$V=$ configurations

$E=$ transitions of TM.

$\exists$ directed path from starting configuration C_0 to accepting configuration?

DFS?

needs depth # configurations in memory

BFS?

width # configurations

needs width # configurations
in memory

Easy with nondeterminism: guess the next vertex!

space = $s(n)$.

Idea: try all possible "middle" vertices.

PATH(u, v, k): $\exists$ path of length $\leq k$ between u and v?

If $u = v$ OR $((u,v) \in E$ AND $k \geq 1)$
return YES

Else for $w \in V \setminus \{u, v\}$:
if PATH$(u, w, \lfloor \frac{k}{2} \rfloor)$ AND PATH$(w, v, \lceil \frac{k}{2} \rceil)$:
RETURN YES

RETURN NO.

RETURN NO.

Total space used?

$s(n) \times$ Depth of recursion

$$d(k) \leq 1 + d\left(\left\lceil \frac{k}{2} \right\rceil\right)$$

$$\leq \lceil \log_2 k \rceil$$

What is k? $|V|$

$$\log_2 k = O(s(n))$$

$$\Rightarrow \text{total space} = O(s(n)^2).$$

Does more space (time) give TMs more power? i.e. they recognize more L?

<u>Thm</u>. For any space-constructible function $s(n)$, $\exists$ language L that can be decided by a TM

$\exists$ language L that can be ...
using space $O(s(n))$ and cannot be decided
by any TM using space $o(s(n))$.

Pf. Let L be the language accepted by
the following machine D:

On input $\langle M, 1^n \rangle$:

- Mark $s(n)$ space on tape
- Count # steps of D
- Run M on 1^n (if M is not a valid TM description, REJECT)

- if space used exceeds $s(n)$, REJECT
- if time used —— $|\Gamma|^{s(n)}$, REJECT.

- Else, if M accepts REJECT
  if M rejects, ACCEPT.

Claim 1. L can be decided using $O(s(n))$ space.

Pf. D decides L and uses $O(s(n))$ space.
It always terminates.

Claim 2. No TM using space $o(s(n))$ can decide L.

Pf. Suppose there is such a decider M_L.
Run D on $\langle M_L, 1^n \rangle$. Since M_L uses $o(s(n))$ space, D can simulate it.
Then D accepts iff M_L rejects
rejects — M_L accepts.
Contradiction! they decide the same language L.