

# P, NP, and all that

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11:24 AM

julian

- P G

We come to a central definition in Complexity theory - P

$$P = \bigcup_k \text{TIME}(n^k)$$

Class of languages that can be decided in polynomial time.

i.e.  $\exists k \in \mathbb{N}$  s.t.  $L \in \text{TIME}(n^k)$ .

$n = |x|$  length of input.

K is fixed, independent of n.

E.g.  $\{\langle G \rangle : G \text{ is a connected graph}\}$

$\{\langle s_1, s_2, K \rangle : \text{the edit distance between } s_1 \text{ & } s_2 \text{ is at most } K\}$

$A_1 \& A_2$  is at most  $r$  ]

$$\overline{NP = \bigcup_K \text{NTIME}(n^K)}$$

$L \in NP : \exists K, \exists \text{NTM } M \text{ s.t. } \forall x, |x| = n$   
if  $x \in L$   $M$  has an accepting  
path of length  $\leq n^K$ .

**Nondeterministic polynomial time**

$\exists$  "short" proof of membership  
 $x \in L$  has a certificate of length  $\leq n^K$ .

e.g.  $\{\langle G \rangle : G \text{ is Hamiltonian}\}$

$\{\langle G, k \rangle : G \text{ has a } \underline{\text{clique}} \text{ of size } k\}$

$\{\langle a_1, a_2, \dots, a_k \rangle : \exists \text{ partition } A_1, A_2$   
 $\quad \quad \quad \perp \text{ to } \dots \text{ integers s.t.}$

$\{ \langle u_1, u_2, \dots, u_k \rangle$  |  
 integers of these integers s.t.  
 $\sum_{i \in A_1} a_i = \sum_{i \in A_2} a_i \}$

Clearly,  $P \subseteq NP$ .

$Co-NP = \bigcup_K \{ L : \exists \text{TM } M \text{ that accepts } x \in \bar{L} \text{ in } NTIME(n^K) \}$

$L \in NP \Leftrightarrow \bar{L} \in Co-NP$

|                 |                      |
|-----------------|----------------------|
| $x \in L$       | $= x \notin \bar{L}$ |
| has short proof | has short proof.     |

e.g.  $\{ \langle G \rangle : G \text{ is } \underline{\text{not}} \text{ Hamiltonian} \}$

$\{ \langle G, k \rangle : G \text{ does not have a clique of size } k \}$ .

Thm.  $P \subseteq NP \cap Co-NP$ .

Open problem:

$P = NP ?$

E.g. - CLIQUE

- INDEPENDENT SET

$= \{ \langle G, k \rangle : G \text{ has an ind. set. of size } k \}$

- HAM

- VERTEX COVER

$= \{ \langle G, k \rangle : G \text{ has a subset } S \text{ of } k \text{ vertices s.t. every edge has at least one end point in } S \}$

- ILP

$= \{ \langle A, b \rangle : Ax \leq b \text{ has an int. sol. } x \}$

$= \{ \langle A, b \rangle : \begin{matrix} AX = b \\ \text{integer solution } X \end{matrix} \}$

- SAT

$= \{ \langle F \rangle : \begin{matrix} \text{Boolean formula s.t.} \\ \exists x \in \{T/F\}^n \text{ and } F(x) = T \end{matrix} \}$

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