SAT is NP-complete

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SAT = CNF-SAT: $\exists x: (x_1 \vee \bar{x}_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee x_4 \vee x_6) \wedge$ .... ?

$\exists x: F(x) = 1$?

__Thm.__ SAT is NP-complete.

__Pf.__ SAT $\in$ NP.

if $F(x) \in$ SAT, $\exists\ x: F(x) = 1$. NTM can guess x and check $F(x) = 1$ in linear time.

Take some other $L \in NP$.

$\exists$ NTM M_L that takes x as input and if $x \in L$, $\exists$ a sequence of valid transitions of M_L that reaches an accepting configuration in at most n^k steps.

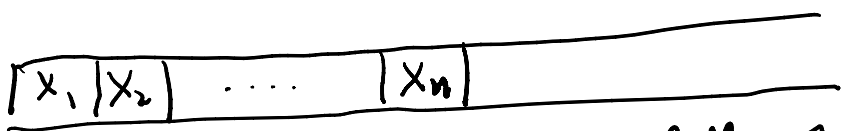
Q. Given M_L, x, does there exist a sequence of configurations that starts with x on the tape and uses valid transitions to reach an accepting state? and # transitions $\leq n^k$?

"SAT-like"

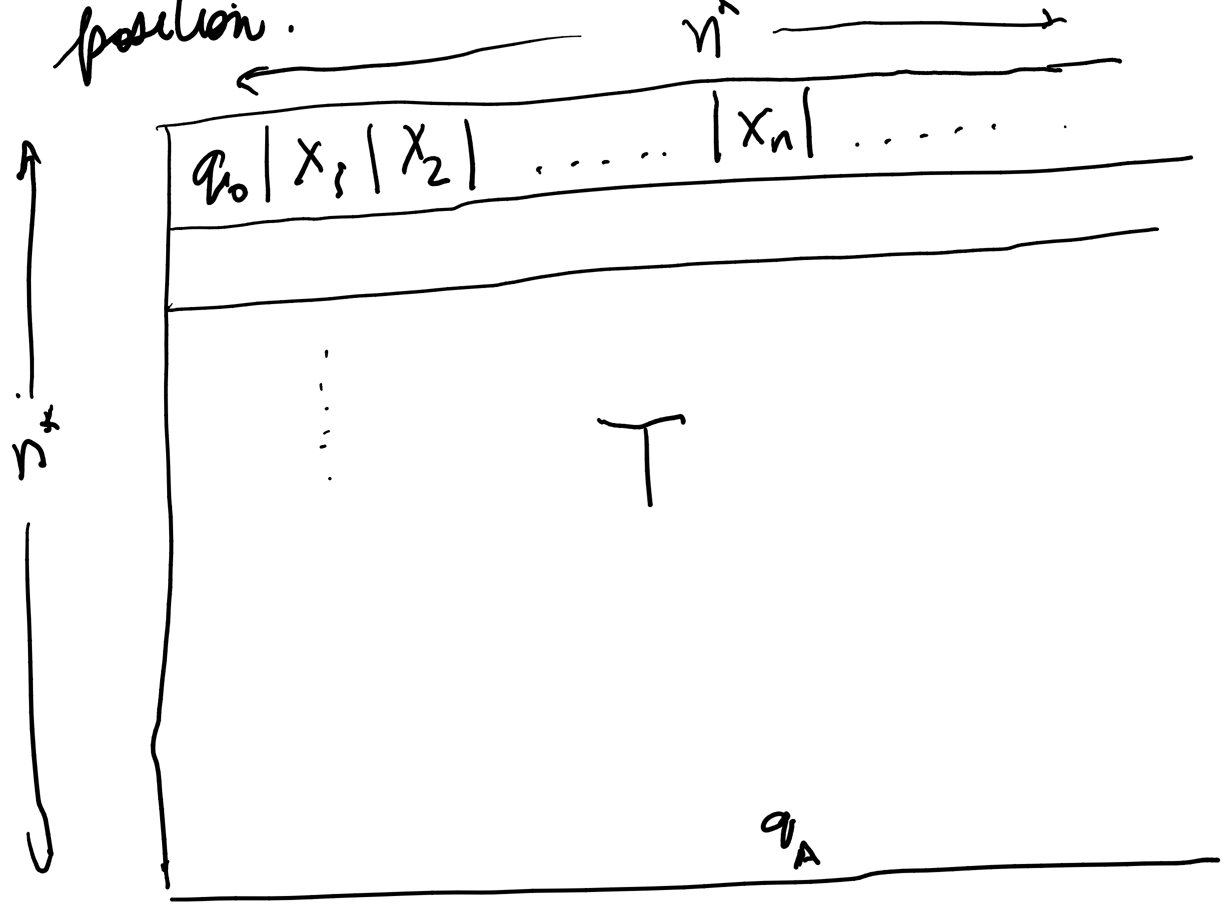
except we need to check tape contents, states etc.

Can we write this as a SAT formula?!

E.g. C_0



We write the current state to the left of the head position.



How to ensure that $T(i,j) = s$?

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Variables $X_{i,j,s}$ Boolean $n^k \times n^k \times (|Q|+|\Gamma|)$.

$$X_{i,j,s} = \begin{cases} 1 & \text{if } T(i,j) = s \\ 0 & \text{o.w.} \end{cases}$$

1. For each i,j exactly one $X_{i,j,s} = 1$.

$$X_{i,j,s_1} \vee X_{i,j,s_2} \vee \cdots \vee X_{i,j,q} \cdots$$

i.e. $\bigvee_{s \in Q \cup \Gamma} X_{i,j,s}$ ← at least one is 1.

But more could be!

X_1, X_2 want exactly one

$$(X_1 \vee X_2) \wedge (\bar{X}_1 \vee \bar{X}_2)$$

at least one is 1. at least one is 0.

$$(X_1 \vee X_2 \vee X_3) \wedge (\bar{X}_1 \vee \bar{X}_2) \wedge (\bar{X}_1 \vee \bar{X}_3) \wedge (\bar{X}_2 \vee \bar{X}_3)$$

exactly one!

exactly one!

① $\bigwedge_{i,j} \left(\bigvee_{s} X_{i,j,s} \right) \wedge \left(\bigwedge_{s_1, s_2} \left(\overline{X_{i,j,s_1}} \vee \overline{X_{i,j,s_2}} \right) \right)$

every table position has exactly one symbol.

② C_0 starting configuration is correct

③ $\exists q_A$ somewhere.

④ Transitions are valid.

② suppose start is

$c_1 c_2 c_3 \ldots\ldots$

want to set $X_{1,j,c_j} = 1$, $X_{1,j,c} = 0$ $\forall c \neq c_j$.

③ $\bigvee_{i,j} X_{i,j,q_A}$

(4) $\delta(q,a) \to q', a', L$

b	q	a
q'	b	a'

$$X_{i,j,b} \wedge X_{i,j+1,q} \wedge X_{i,j+2,a} \Rightarrow X_{i+1,j,q'} \wedge X_{i+1,j+1,b} \wedge X_{i+1,j+2,a'}.$$

($X \Rightarrow Y$ is the same as $\bar{X} \vee Y$)

$$\bigwedge_{i,j} \bigvee_{\substack{\text{valid transitions} \\ (a_1,\dots,a_6)}} \left(X_{i,j,a_1} \wedge X_{i,j+1,a_2} \wedge X_{i,j+2,a_3} \Rightarrow X_{i+1,j,a_4} \wedge X_{i+1,j+1,a_5} \wedge X_{i+1,j+2,a_6} \right.$$

$x \in L$ iff $F(x) = 1$.

$\therefore L \hookrightarrow_p SAT$

total size of formula $= O(n^{2k})$.