Software Correctness: The Construction of Correct Software

Contracts: Test

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Example A: Pure Maximum Function

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- The signature specifies the number of parameters, their type and the return type
- The body contains the implementation code
- Functions that do not return a value can also be defined

```
def fun(x: Z, y: Z) { ... }
```

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@pure def pure_max(x: Z, y: Z): Z = {
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- The `@pure` attribute indicates to Logika that the function is **free of side-effects**

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- Let's have a look at contracts

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- Contracts specify what's **required** *before* the function is executed and what's **ensured** *after* the function has been executed

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- Let's state this informally

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- What's **ensured** is called the **post-condition** of the function

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- What's **required** is called the **pre-condition** of the function
- What's **ensured** is called the **post-condition** of the function
- We have already sketched such contracts using `assume`-`assert`

Example A: Pure Maximum Function Contract

```
@pure def pure_max(x: Z, y: Z): Z = {
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  //   requires x > 0, y > 0
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- It looked similar to this, reading `,` in the contract clauses as conjunctions

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- It looked similar to this, reading `,` in the contract clauses as conjunctions
- What's new is the reference to `Res` in the `ensures` clause
- The special variable `Res` *is needed to deal with* **`return`** *statement*

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- In Slang syntax it looks as follows ...

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- Pure function `pure_max` with its contract can now be used in formulas

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- Remember to read **`return`** statements as assignments to `Res`
- Pure function `pure_max` with its contract can now be used in formulas
- In some context we could write, e.g., `Deduce(|- (z == max(x, y)))`

Example B: Impure Maximum Function

```
var z: Z = randomInt()
def impure_max(x: Z, y: Z) {


  if (x < y) {
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- Function `impure_max` modifies variable `z` outside its scope

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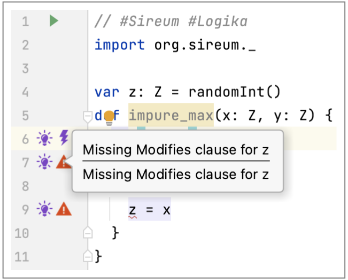

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- Logika rejects the program above

Example B: Impure Maximum Function with Modifies Clause

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var z: Z = randomInt()
def impure_max(x: Z, y: Z) {
  // contract
  //   modifies frame
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- This is done by means of the `modifies` clause that specifies a function's **frame**
- A frame is a (comma-separated) list of variables

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- The frame of function `impure_max` is just the single variable `z`
- We note that for a function with side-effects it is important to know which variables it might modify

Frame of a Function

- Consider a function with a contract

```
var m: Z = randomInt()
var n: Z = randomInt()

def equalize(): Unit = {
  // contract
  //   ensures m == n
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- In general, the result of a function could be obtained by modifying variables that were intended as parameters

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- This could be achieved by assigning `m` to `n`, or `n` to `m`, or a common value to both
- In general, the result of a function could be obtained by modifying variables that were intended as parameters
- The `modifies` clause permits us to describe which variables might change and which do not

Example B: Impure Maximum Function with Modifies Clause

```
var z: Z = randomInt()
def impure_max(x: Z, y: Z) {
  Contract(
    Modifies(z)
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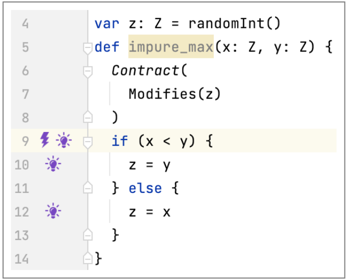

- In the function above the frame is specified in the Slang contract notation
- With the `Modifies` clause added Logika accepts the function

Example B: Impure Maximum Function with Modifies Clause

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var z: Z = randomInt()
def impure_max(x: Z, y: Z) {
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- Now we can specify the complete contract for the function
- Note the use of variable `z` in place of `Res` in the pure version of the function

Specification and Implementation

```
val x = randomInt()
assume(x > 0)
val y = randomInt()
assume(y > 0)

impure_max(x, y)

Deduce(|- (z == pure_max(x, y)))
```

Specification and Implementation

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- We can now program with the impure function and use the pure function in correctness proofs
- (Of course, we can also program with pure functions)
- Pure functions are often clearer but less efficient. They are good specifications
- Impure functions are often more efficient but less clear. They are good implementations

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- The fragment above proves that `impure_max` implements `pure_max` related by `z == pure_max(x, y)`

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- The fragment above proves that `impure_max` implements `pure_max` related by `z == pure_max(x, y)`
- We also say that `impure_max` refines `pure_max`
- Note, the use of `assume` to constrain the values of `x` and `y`
- `assume` is most useful to support proofs of the kind above

Exercise 1

Add contracts to functions `max` and `pure_max` so that the deduction at the end is verified

```
// #Sireum #Logika
import org.sireum._

@pure def pure_max(x: Z, y: Z): Z = {
  if (x < y) {
    return y
  } else {
    return x
  }
}

def max(x: Z, y: Z): Z = {
  if (x < y) {
    return y
  } else {
    return x
  }
}

val x = randomInt()
val y = randomInt()

val z = max(x, y)
Deduce(|- (z == pure_max(x, y)))
```

Invalid conclusion

Example C: Testing the Maximum Function

```
def max(x: Z, y: Z): Z = {
  Contract(
    Requires(x > 0, y > 0),
    Ensures(Res == x | Res == y, x <= Res, y <= Res)
  )
  if (x < y) {
    return y // "Res = y"
  } else {
    return x // "Res = x"
  }
}
```

Example C: Testing the Maximum Function

```
def max(x: Z, y: Z): Z = {
  Contract(
    Requires(x > 0, y > 0),
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  )
  if (x < y) {
    return y // "Res = y"
  } else {
    return x // "Res = x"
  }
}
```

- Suppose the body of function `max` was more complex
  and we would have no proof that the body establishes the post-condition

Example C: Testing the Maximum Function

```
def max(x: Z, y: Z): Z = {
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}
```

- Suppose the body of function `max` was more complex
  and we would have no proof that the body establishes the post-condition
- We would like to have a method that helps us to come up systematically with test cases

Test Cases from Specifications

```
def max(x: Z, y: Z): Z = {
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- We only consider the contract that specifies the behaviour of the function

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- The pre-condition restricts the values to be considered for the **defined** behaviour of the function

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}
```

- We only consider the contract that specifies the behaviour of the function
- The pre-condition restricts the values to be considered
  for the **defined** behaviour of the function
- The post-condition describes possible outcomes depending on the input values
  - If `Res == x` then `y <= x`
  - If `Res == y` then `x <= y`

Equivalence Partitioning

- We can analyse the different variants in which `y <= x` and `x <= y` and their negations are conjoined to predict conditions occurring in an implementation

```
 (y <= x) &  (x <= y)   if and only if x == y
 (y <= x) & !(x <= y)   if and only if x > y
!(y <= x) &  (x <= y)   if and only if x < y
!(y <= x) & !(x <= y)   if and only if false
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- We get three cases to consider: `x < y`, `x == y`, `x > y`

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```

- We get three cases to consider: `x < y`, `x == y`, `x > y`
- These are called equivalence classes

Boundary Value Analysis

- We can analyse the pre-condition `x > 0` and `y > 0`

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- *Aside*. In some cases it is interesting to test the behaviour of a function when the pre-condition is violated, e.g., when security is a concern
- We have collected equivalence classes and boundary values
- These can be used to formulate test cases

Test Case Formulation

- Combining equivalence classes with boundary values we can calculate expected results

Class	Input `x`	Input `y`	Output `Res`
`x < y`	1	2	2
`x == y`	1	1	1
`x > y`	2	1	2

Test Case Formulation

- Combining equivalence classes with boundary values we can calculate expected results

Class	Input `x`	Input `y`	Output `Res`
`x < y`	`1`	`2`	`2`
`x == y`	`1`	`1`	`1`
`x > y`	`2`	`1`	`2`

- The output `Res` must satisfy the condition

```
Res == x | Res == y & x <= Res & y <= Res
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- Inserting the values for `x` and `y` from the table, `Res` can be calculated
- Boundary values have been shown to be good choices for detecting faults in programs
- The derivation of the test cases above is driven by the heuristics of equivalence partitioning and boundary value analysis
  - The test cases are **not** best choices but *heuristically well chosen*
  - The test cases might miss important faults in a program

Test Cases from Implementations

```
def max(x: Z, y: Z): Z = {
  Contract(
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- Taking the body of function `max` we can make sure that all statements and conditions on all branches are tested

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- “all statements and conditions on all branches” is called a **coverage criterion**

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```

- Taking the body of function `max` we can make sure that all statements and conditions on all branches are tested
- “all statements and conditions on all branches” is called a **coverage criterion**
- *Aside*. Other coverage criteria exist (but we will not discuss them)

Test Cases from Implementations

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def max(x: Z, y: Z): Z = {
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- Let's have a look at the fact corresponding to the program

Test Cases from Implementations

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    return y // "Res = y"
  } else {
    return x // "Res = x"
  }
}
```

- Let's have a look at the fact corresponding to the program
- The body has two branches that are followed depending on whether the condition `x < y` is **`true`** or **`false`**

Test Cases from Implementations

- We derive test cases from the fact

  pre-condition &
  body$_{fact}$ & (replacing '`return` e' by '`Res = `e' in body)
  post-condition

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- For function `max` we get

```
x > 0 & y > 0 &                                      // Pre-condition
x < y => Res == y &                                  // Branch 1
!(x < y) => Res == x &                               // Branch 2
(Res == x | Res == y) & x <= Res & y <= Res          // Post-condition
```

Test Cases from Implementations

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x > 0 & y > 0 &                                    // Pre-condition
x < y => Res == y &                                // Branch 1
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(Res == x | Res == y) & x <= Res & y <= Res        // Post-condition
```

- Conjoining either `x < y` or `x >= y`
  we force the choice between branch 1 and branch 2

Test Cases from Implementations

- Two facts result, one for each branch

Test Cases from Implementations

- Two facts result, one for each branch
- Branch 1

```
x > 0 & y > 0 &                                           // Pre-condition
x < y & Res == y &                                        // Branch 1
(Res == x | Res == y) & x <= Res & y <= Res               // Post-condition
```

Test Cases from Implementations

- Two facts result, one for each branch
- Branch 1

```
x > 0 & y > 0 &                                       // Pre-condition
x < y & Res == y &                                    // Branch 1
(Res == x | Res == y) & x <= Res & y <= Res           // Post-condition
```

- Branch 2

```
x > 0 & y > 0 &                                       // Pre-condition
x >= y & Res == x &                                   // Branch 2
(Res == x | Res == y) & x <= Res & y <= Res           // Post-condition
```

Test Cases from Implementations

- Two facts result, one for each branch
- Branch 1

```
x > 0 & y > 0 &                                    // Pre-condition
x < y & Res == y &                                 // Branch 1
(Res == x | Res == y) & x <= Res & y <= Res        // Post-condition
```

- Branch 2

```
x > 0 & y > 0 &                                    // Pre-condition
x >= y & Res == x &                                // Branch 2
(Res == x | Res == y) & x <= Res & y <= Res        // Post-condition
```

- Our prior analysis of the specification suggested that it would be good to split `x >= y` into the cases `x == y` and `x > y` to be tested separately

Test Cases from Implementations

Test Cases from Implementations

- Branch 1

```
x > 0 & y > 0 &                                        // Pre-condition
x < y & Res == y &                                     // Branch 1
(Res == x | Res == y) & x <= Res & y <= Res            // Post-condition
```

Test Cases from Implementations

- Branch 1

```
x > 0 & y > 0 &                                    // Pre-condition
x < y & Res == y &                                 // Branch 1
(Res == x | Res == y) & x <= Res & y <= Res        // Post-condition
```

- Branch 2 (with `x == y`)

```
x > 0 & y > 0 &                                    // Pre-condition
x >= y & Res == x &                                // Branch 2
(Res == x | Res == y) & x <= Res & y <= Res        // Post-condition
```

Test Cases from Implementations

- Branch 1

```
x > 0 & y > 0 &                                          // Pre-condition
x < y & Res == y &                                       // Branch 1
(Res == x | Res == y) & x <= Res & y <= Res             // Post-condition
```

- Branch 2 (with `x == y`)

```
x > 0 & y > 0 &                                          // Pre-condition
x >= y & Res == x &                                      // Branch 2
(Res == x | Res == y) & x <= Res & y <= Res             // Post-condition
```

- Branch 2 (with `x > y`)

```
x > 0 & y > 0 &                                          // Pre-condition
x >= y & Res == x &                                      // Branch 2
(Res == x | Res == y) & x <= Res & y <= Res             // Post-condition
```

Test Cases from Implementations (Using Boundary Values)

- Branch 1: `x == 1`, `y == 2`

```
x > 0 & y > 0 &                                      // Pre-condition
x < y & Res == y &                                   // Branch 1
(Res == x | Res == y) & x <= Res & y <= Res          // Post-condition
```

- Branch 2 (with `x == y`): `x == 1`, `y == 1`

```
x > 0 & y > 0 &                                      // Pre-condition
x >= y & Res == x &                                  // Branch 2
(Res == x | Res == y) & x <= Res & y <= Res          // Post-condition
```

- Branch 2 (with `x > y`): `x == 2`, `y == 1`

```
x > 0 & y > 0 &                                      // Pre-condition
x >= y & Res == x &                                  // Branch 2
(Res == x | Res == y) & x <= Res & y <= Res          // Post-condition
```

Test Cases from Implementations (Using Boundary Values)

- Branch 1: `x == 1`, `y == 2`

```
1 > 0 & 2 > 0 &                                       // Pre-condition
1 < 2 & Res == 2 &                                    // Branch 1
(Res == 1 | Res == 2) & 1 <= Res & 2 <= Res           // Post-condition
```

- Branch 2 (with `x == y`): `x == 1`, `y == 1`

```
1 > 0 & 1 > 0 &                                       // Pre-condition
1 >= 1 & Res == 1 &                                   // Branch 2
(Res == 1 | Res == 1) & 1 <= Res & 1 <= Res           // Post-condition
```

- Branch 2 (with `x > y`): `x == 2`, `y == 1`

```
2 > 0 & 1 > 0 &                                       // Pre-condition
2 >= 1 & Res == 2 &                                   // Branch 2
(Res == 2 | Res == 1) & 2 <= Res & 1 <= Res           // Post-condition
```

Test Cases from Implementations (Using Boundary Values)

- Branch 1: `x == 1`, `y == 2`, `Res == 2`

```
1 > 0 & 2 > 0 &                                    // Pre-condition
1 < 2 & Res == 2 &                                 // Branch 1
(Res == 1 | Res == 2) & 1 <= Res & 2 <= Res        // Post-condition
```

- Branch 2 (with `x == y`): `x == 1`, `y == 1`, `Res == 1`

```
1 > 0 & 1 > 0 &                                    // Pre-condition
1 >= 1 & Res == 1 &                                // Branch 2
(Res == 1 | Res == 1) & 1 <= Res & 1 <= Res        // Post-condition
```

- Branch 2 (with `x > y`): `x == 2`, `y == 1`, `Res == 2`

```
2 > 0 & 1 > 0 &                                    // Pre-condition
2 >= 1 & Res == 2 &                                // Branch 2
(Res == 2 | Res == 1) & 2 <= Res & 1 <= Res        // Post-condition
```

Test Cases from Implementations (Using Boundary Values)

- Branch 1: `x == 1`, `y == 2`, `Res == 2`

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1 > 0 & 2 > 0 &                                 // Pre-condition
1 < 2 & 2 == 2 &                                // Branch 1
(2 == 1 | 2 == 2) & 1 <= 2 & 2 <= 2             // Post-condition
```

- Branch 2 (with `x == y`): `x == 1`, `y == 1`, `Res == 1`

```
1 > 0 & 1 > 0 &                                 // Pre-condition
1 >= 1 & 1 == 1 &                               // Branch 2
(1 == 1 | 1 == 1) & 1 <= 1 & 1 <= 1             // Post-condition
```

- Branch 2 (with `x > y`): `x == 2`, `y == 1`, `Res == 2`

```
2 > 0 & 1 > 0 &                                 // Pre-condition
2 >= 1 & 2 == 2 &                               // Branch 2
(2 == 2 | 2 == 1) & 2 <= 2 & 1 <= 2             // Post-condition
```

Test Cases from Implementations (Using Boundary Values)

- Branch 1

```
satisfied by test case
input: x == 1, y == 2
output: Res == 2
```

- Branch 2

```
with x == y satisfied by test case
input: x == 1, y == 1
output: Res == 1
```

- Branch 2

```
with x > y satisfied by test case
input: x == 2, y == 1
output: Res == 2
```

Symbolic Execution of the Function

```
... Requires(x > 0, y > 0)
... Ensures(Res == x | Res == y, x <= Res, y <= Res)
if (x < y) {
  return y // "Res = y"
} else {
  return x // "Res = x"
}
```

Symbolic Execution of the Function Branch 1

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... Requires(x > 0, y > 0)
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- Entering the function yields (`x: X`, `y: Y`, `Res: RES`), (PC: `X > 0`, `Y > 0`)

Symbolic Execution of the Function Branch 1

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- Entering the function yields (`x: X`, `y: Y`, `Res: RES`), (PC: `X > 0`, `Y > 0`)
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... Requires(x > 0, y > 0)
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- Executing `return y // "Res = y"` yields (`x: X`, `y: Y`, `Res: Y`), (PC: `X > 0`, `Y > 0`, `X < Y`)
- Leaving the function yields (`x: X`, `y: Y`, `Res: Y`), (PC: `X > 0`, `Y > 0`, `X < Y`, `Y == X | Y == Y`, `X <= Y`, `Y <= Y`)

Symbolic Execution of the Function Branch 2

```
... Requires(x > 0, y > 0)
... Ensures(Res == x | Res == y, x <= Res, y <= Res)
if (x < y) {
  return y // "Res = y"
} else {
  return x // "Res = x"
}
```

Symbolic Execution of the Function Branch 2

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... Requires(x > 0, y > 0)
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- Executing `} else {` yields (`x: X`, `x: X`, `Res: RES`), (PC: `X > 0`, `Y > 0`, `X >= Y`)

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- Executing `return x // "Res = x"` yields (`x: X`, `y: Y`, `Res: X`), (PC: `X > 0`, `Y > 0`, `X >= Y`)
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Result of the Symbolic Execution

- Symbolic execution of function `max` produces the following two results
  - (`x: X`, `y: Y`, `Res: Y`),
    (PC: `X > 0`, `Y > 0`, `X < Y`, `Y == X | Y == Y`, `X <= Y`, `Y <= Y`)
  - (`x: X`, `y: Y`, `Res: X`),
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  - (`x: X`, `y: Y`, `Res: Y`),
    (PC: `X > 0`, `Y > 0`, `X < Y`, `Y == X | Y == Y`, `X <= Y`, `Y <= Y`)
  - (`x: X`, `y: Y`, `Res: X`),
    (PC: `X > 0`, `Y > 0`, `X >= Y`, `X == X | X == Y`, `X <= X`, `Y <= X`)
- We can state these as facts
  - `x == X`, `y == Y`, `Res == Y`,
    `X > 0 & Y > 0 & X < Y & (Y == X | Y == Y) & X <= Y & Y <= Y`
  - `x == X`, `y == Y`, `Res == X`,
    `X > 0 & Y > 0 & X >= Y & (X == X | X == Y) & X <= X & Y <= X`

Result of the Symbolic Execution

- Symbolic execution of function `max` produces the following two results
  - (`x: X`, `y: Y`, `Res: Y`),
    (PC: `X > 0`, `Y > 0`, `X < Y`, `Y == X | Y == Y`, `X <= Y`, `Y <= Y`)
  - (`x: X`, `y: Y`, `Res: X`),
    (PC: `X > 0`, `Y > 0`, `X >= Y`, `X == X | X == Y`, `X <= X`, `Y <= X`)
- We can state these as facts
  - `x == X`, `y == Y`, `Res == Y`,
    `X > 0 & Y > 0 & X < Y & (Y == X | Y == Y) & X <= Y & Y <= Y`
  - `x == X`, `y == Y`, `Res == X`,
    `X > 0 & Y > 0 & X >= Y & (X == X | X == Y) & X <= X & Y <= X`
- These facts are very similar to those we have seen when looking at programs as facts

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  - `x == X, y == Y, Res == Y,`
    `X > 0 & Y > 0 & X < Y & (Y == X | Y == Y) & X <= Y & Y <= Y`
  - `x == X, y == Y, Res == X,`
    `X > 0 & Y > 0 & X >= Y & (X == X | X == Y) & X <= X & Y <= X`
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  - We use symbolic values `X`, `Y` and `Res`
  - `Res` has been replaced by `Y` and `X`, respectively

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- These facts are very similar to those we have seen when looking at programs as facts
- The only differences are
  - We use symbolic values `X`, `Y` and `Res`
  - `Res` has been replaced by `Y` and `X`, respectively
- In the second branch we can distinguish `X == Y` and `X > Y` as before

Result of the Symbolic Execution

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  - Test Case 1
    input: `x == X`, `y == Y`, output: `Res == Y`,
    `X > 0 & Y > 0 & X < Y & (Y == X | Y == Y) & X <= Y & Y <= Y`

Result of the Symbolic Execution

- Into the three remaining cases we can insert the boundary values we have determined before
  - Test Case 1
    input: `x == X`, `y == Y`, output: `Res == Y`,
    `X > 0 & Y > 0 & X < Y & (Y == X | Y == Y) & X <= Y & Y <= Y`
  - Test case 2:
    input: `x == X`, `y == Y`, output: `Res == X`,
    `X == Y`,
    `X > 0 & Y > 0 & X >= Y & (X == X | X == Y) & X <= X & Y <= X`

Result of the Symbolic Execution

- Into the three remaining cases we can insert the boundary values we have determined before
  - Test Case 1
    input: `x == X`, `y == Y`, output: `Res == Y`,
    `X > 0 & Y > 0 & X < Y & (Y == X | Y == Y) & X <= Y & Y <= Y`
  - Test case 2:
    input: `x == X`, `y == Y`, output: `Res == X`,
    `X == Y`,
    `X > 0 & Y > 0 & X >= Y & (X == X | X == Y) & X <= X & Y <= X`
  - Test Case 3
    input: `x == X`, `y == Y`, output: `Res == X`,
    `X > Y`,
    `X > 0 & Y > 0 & X >= Y & (X == X | X == Y) & X <= X & Y <= X`

Result of the Symbolic Execution

- Into the three remaining cases we can insert the boundary values we have determined before
  - Test Case 1
    input: `x == 1`, `y == 2`, output: `Res == 2`,
    `1 > 0 & 2 > 0 & 1 < 2 & (2 == 1 | 2 == 2) & 1 <= 2 & 2 <= 2`
  - Test case 2:
    input: `x == 1`, `y == 1`, output: `Res == 1`,
    `1 == 1`,
    `1 > 0 & 1 > 0 & 1 >= 1 & (1 == 1 | 1 == 1) & 1 <= 1 & 1 <= 1`
  - Test Case 3
    input: `x == 2`, `y == 1`, output: `Res == 2`,
    `2 > 1`,
    `2 > 0 & 1 > 0 & 2 >= 1 & (2 == 2 | 2 == 1) & 2 <= 2 & 1 <= 2`

Exercise 2

- Formally determine the test cases for the function below

```
var z: Z = 0
def max(x: Z, y: Z) {
  Contract(
    Requires(x > 0, y > 0),
    Modifies(z),
    Ensures(z == x | z == y, x <= z, y <= z)
  )
  if (x < y) {
    z = y
  } else {
    z = x
  }
}
```

- Remember that this function has a side-effect

Test Cases for Faults in Implementations

```
var x: Z = randomInt()
var y: Z = randomInt()

def tsq() {
  Contract(
    Modifies(x, y),
    Ensures(x >= 0, y >= 0)
  )
  x = x * x
  y = y + y
  if (x < y) {
    y = y - x
  } else {
    x = x - y
  }
}
```

- Is the function above correct?

Test Cases for Faults in Implementations

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var x: Z = randomInt()
var y: Z = randomInt()

def tsq() {
  Contract(
    Modifies(x, y),
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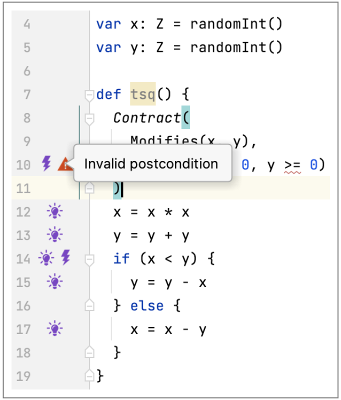
  x = x * x
  y = y + y
  if (x < y) {
    y = y - x
  } else {
    x = x - y
  }
}
```



- Is the function above correct? No.

Test Cases for Faults in Implementations

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var x: Z = randomInt()
var y: Z = randomInt()

def tsq() {
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    Modifies(x, y),
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  )
  x = x * x
  y = y + y
  if (x < y) {
    y = y - x
  } else {
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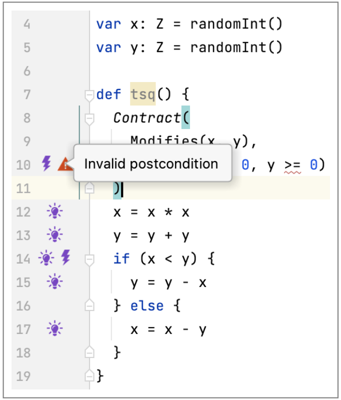
}
```



- Is the function above correct? No.
- Can we determine a test case confirming this?

Counterexamples from Implementations

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$$\mathit{body}_{fact}\ \& \quad \text{(body does not contain a return statement!)}$$
$$!\,\mathit{post\text{-}condition}$$

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- For function `tsq` we get

```
At(x, 1) == At(x, 0) * At(x, 0) &                  // First assignment
At(y, 1) == At(y, 0) + At(y, 0) &                  // Second assignment
(At(x, 1) < At(y, 1)) -> (x == At(x, 1)) &         // Branch 1
(At(x, 1) < At(y, 1)) -> (y == At(y, 1) - x) &
!(At(x, 1) < At(y, 1)) -> (y == At(y, 1)) &        // Branch 2
!(At(x, 1) < At(y, 1)) -> (x == At(x, 1) - y) &
(x < 0 | y < 0)                                    // Negated post-condition
```

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- More formally, this is expressed,

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!(At(x, 1) < At(y, 1)) -> (y == At(y, 1)) &    // Branch 2
!(At(x, 1) < At(y, 1)) -> (x == At(x, 1) - y) &
(x < 0 | y < 0)                                // Negated post-condition
```

- Let's focus on the failing post-condition `y >= 0`, considering branch 1

Test Cases for Faults in Implementations

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At(x, 1) == At(x, 0) * At(x, 0) &                        // First assignment
At(y, 1) == At(y, 0) + At(y, 0) &                        // Second assignment
(At(x, 1) < At(y, 1)) &                                  // Condition to enter branch 1
(At(x, 1) < At(y, 1)) -> (x == At(x, 1)) &               // Branch 1
(At(x, 1) < At(y, 1)) -> (y == At(y, 1) - x) &
y < 0                                                    // Negated failing post-condition
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(At(x, 1) < At(y, 1)) -> (x == At(x, 1)) &          // Branch 1
(At(x, 1) < At(y, 1)) -> (y == At(y, 1) - x) &
y < 0                                               // Negated failing post-condition
```

- This is unsatisfiable
- We have
  - `At(x, 1) >= 0` and `At(y, 1) > At(x, 1)`

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(At(x, 1) < At(y, 1)) &                            // Condition to enter branch 1
(At(x, 1) < At(y, 1)) -> (x == At(x, 1)) &         // Branch 1
(At(x, 1) < At(y, 1)) -> (y == At(y, 1) - x) &
y < 0                                              // Negated failing post-condition
```

- This is unsatisfiable
- We have
  - `At(x, 1) >= 0` and `At(y, 1) > At(x, 1)`
  - Therefore, `At(y, 1) - At(x, 1) > 0` (by algebra)

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At(y, 1) == At(y, 0) + At(y, 0) &                        // Second assignment
(At(x, 1) < At(y, 1)) &                                  // Condition to enter branch 1
(At(x, 1) < At(y, 1)) -> (x == At(x, 1)) &               // Branch 1
(At(x, 1) < At(y, 1)) -> (y == At(y, 1) - x) &
y < 0                                                    // Negated failing post-condition
```

- This is unsatisfiable
- We have
  - `At(x, 1) >= 0` and `At(y, 1) > At(x, 1)`
  - Therefore, `At(y, 1) - At(x, 1) > 0` (by algebra)
  - Therefore, `At(y, 1) - x > 0` (because `x == At(x, 1)`)

Test Cases for Faults in Implementations

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At(x, 1) == At(x, 0) * At(x, 0) &                  // First assignment
At(y, 1) == At(y, 0) + At(y, 0) &                  // Second assignment
 (At(x, 1) < At(y, 1)) &                           // Condition to enter branch 1
 (At(x, 1) < At(y, 1)) -> (x == At(x, 1)) &        // Branch 1
 (At(x, 1) < At(y, 1)) -> (y == At(y, 1) - x) &
y < 0                                              // Negated failing post-condition
```

- This is unsatisfiable
- We have
  - `At(x, 1) >= 0` and `At(y, 1) > At(x, 1)`
  - Therefore, `At(y, 1) - At(x, 1) > 0` (by algebra)
  - Therefore, `At(y, 1) - x > 0` (because `x == At(x, 1)`)
  - Therefore, `y > 0` (by algebra)

Test Cases for Faults in Implementations

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At(x, 1) == At(x, 0) * At(x, 0) &                  // First assignment
At(y, 1) == At(y, 0) + At(y, 0) &                  // Second assignment
(At(x, 1) < At(y, 1)) &                            // Condition to enter branch 1
(At(x, 1) < At(y, 1)) -> (x == At(x, 1)) &         // Branch 1
(At(x, 1) < At(y, 1)) -> (y == At(y, 1) - x) &
y < 0                                              // Negated failing post-condition
```

- This is unsatisfiable
- We have
  - `At(x, 1) >= 0` and `At(y, 1) > At(x, 1)`
  - Therefore, `At(y, 1) - At(x, 1) > 0` (by algebra)
  - Therefore, `At(y, 1) - x > 0` (because `x == At(x, 1)`)
  - Therefore, `y > 0` (by algebra)
  - But also, `y < 0` (the negated post-condition)
- Let's have a look at branch 2

Test Cases for Faults in Implementations

```
At(x, 1) == At(x, 0) * At(x, 0) &                        // First assignment
At(y, 1) == At(y, 0) + At(y, 0) &                        // Second assignment
!(At(x, 1) < At(y, 1)) &                                 // Condition to enter branch 2
!(At(x, 1) < At(y, 1)) -> (y == At(y, 1)) &              // Branch 2
!(At(x, 1) < At(y, 1)) -> (x == At(x, 1) - y) &
y < 0                                                    // Negated failing post-condition
```

Test Cases for Faults in Implementations

```
At(x, 1) == At(x, 0) * At(x, 0) &                    // First assignment
At(y, 1) == At(y, 0) + At(y, 0) &                    // Second assignment
!(At(x, 1) < At(y, 1)) &                             // Condition to enter branch 2
!(At(x, 1) < At(y, 1)) -> (y == At(y, 1)) &          // Branch 2
!(At(x, 1) < At(y, 1)) -> (x == At(x, 1) - y) &
y < 0                                                // Negated failing post-condition
```

- We proceed as if `y < 0` was a post-condition and apply the same methods as before

Test Cases for Faults in Implementations

```
At(x, 1) == At(x, 0) * At(x, 0) &                 // First assignment
At(y, 1) == At(y, 0) + At(y, 0) &                 // Second assignment
!(At(x, 1) < At(y, 1)) &                          // Condition to enter branch 2
!(At(x, 1) < At(y, 1)) -> (y == At(y, 1)) &       // Branch 2
!(At(x, 1) < At(y, 1)) -> (x == At(x, 1) - y) &
y < 0                                             // Negated failing post-condition
```

- We proceed as if `y < 0` was a post-condition and apply the same methods as before
- We find
  - `At(x, 0) == 0`, `At(y, 0) == -1`, `y == -2`

Test Cases for Faults in Implementations

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At(x, 1) == At(x, 0) * At(x, 0) &                // First assignment
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!(At(x, 1) < At(y, 1)) -> (x == At(x, 1) - y) &
y < 0                                            // Negated failing post-condition
```

- We proceed as if `y < 0` was a post-condition and apply the same methods as before
- We find
  - `At(x, 0) == 0`, `At(y, 0) == -1`, `y == -2`
- Failing test case:

```
input: x == 0, y == 0
output: y == ?          // where ? must satisfy ? >= 0
```

Test Cases for Faults in Implementations

```
At(x, 1) == At(x, 0) * At(x, 0) &                    // First assignment
At(y, 1) == At(y, 0) + At(y, 0) &                    // Second assignment
!(At(x, 1) < At(y, 1)) &                             // Condition to enter branch 2
!(At(x, 1) < At(y, 1)) -> (y == At(y, 1)) &          // Branch 2
!(At(x, 1) < At(y, 1)) -> (x == At(x, 1) - y) &
y < 0                                                // Negated failing post-condition
```

- We proceed as if `y < 0` was a post-condition and apply the same methods as before
- We find
  - `At(x, 0) == 0`, `At(y, 0) == -1`, `y == -2`
- Failing test case:

```
input: x == 0, y == 0
output condition: y >= 0
```

Test Cases for Faults in Implementations

```
At(x, 1) == At(x, 0) * At(x, 0) &                      // First assignment
At(y, 1) == At(y, 0) + At(y, 0) &                      // Second assignment
!(At(x, 1) < At(y, 1)) &                               // Condition to enter branch 2
!(At(x, 1) < At(y, 1)) -> (y == At(y, 1)) &            // Branch 2
!(At(x, 1) < At(y, 1)) -> (x == At(x, 1) - y) &
y < 0                                                  // Negated failing post-condition
```

- We proceed as if `y < 0` was a post-condition and apply the same methods as before
- We find
  - `At(x, 0) == 0`, `At(y, 0) == -1`, `y == -2`
- Failing test case:

```
input: x == 0, y == 0
output condition: y >= 0
```

- The test fails until the program has been corrected to satisfy `y >= 0`

Exercise 3

- Use Symbolic execution for the calculation of the counterexample
- Correct the function

Exercise 4

```
var x = randomInt()
var q = randomInt()

def shift(p: Z, y: Z, N: Z) {
  Contract(
    Requires(x * p + y * q == N),
    Modifies(x, q),
    Ensures(x * p + y * q == N)
  )
  x = x - y
  q = q - p
}
```

- Correct and verify the function on the left
- Find a counterexample first by calculation
- Derive test cases for the function
  - What are input and output?
  - Considering the post-condition choose suitable equivalence classes and boundary values (Consider `p == 0` and `y == 0` for the uncorrected function)
  - How many test cases do you get?
- Add deductions that explain that your function is correct

Exercise 5 (Outlook)

```
// linmap yields a * x + b for any a, x, and b
def linmap(a: Z, x: Z, b: Z): Z = {
  ...
}

// given (x - b) % a == 0 revmap yields (x - b) / a for any a, x, and b
def revmap(a: Z, x: Z, b: Z): Z = {
  ...
}

// compose yields x for any a, x, and b
def compose(a: Z, x: Z, b: Z): Z = {
  ...
  var y: Z = linmap(a, x, b)
  y = ... // use function revmap
  return y
}
```

- Add the contracts according to the comments preceding the functions
- Add deductions that explain that your implementation is correct

Summary

- We have discussed pure and impure functions
- We have introduced contracts with
  - Pre-conditions,
  - Post-conditions, and
  - Frames
- We have discussed how to derive test cases from
  - Specifications (resp. function contracts) and
  - Implementation (resp. function bodies – and contracts)
- We have analysed programs that contain faults and derived test cases for those