Problem 1

Consider the following LP:

$$\begin{array}{ll} \text{maximize} & c \cdot x \\ \text{s.t.} & Ax \le b \\ & x \ge 0. \end{array} \qquad (1)$$

where A is an $n \times n$ matrix, c, x and d are vectors of $\mathbb{R}^n$.

1. Prove that if A is invertible then (1) is always feasible once we drop the constraint $x \ge 0$.

2. Let $A = \begin{pmatrix} 2 & 2 \\ -2 & -2 \end{pmatrix}$ and $b = \begin{pmatrix} -2 \\ 9 \end{pmatrix}$.

    (a) Is (1) feasible ? Justify your answer.

    (b) Find the result by using a simplex algorithm.

    (c) Find the rank of A.

3. Now, let $A = \begin{pmatrix} 1 & -2 \\ 1 & 0 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $c = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

    (a) Solve (1) graphically (optimal value and optimal solution).

    (b) Find the result by using a simplex algorithm.

Problem 2

1. Formulate the following problems as LPs:

    (a) 6 batches of pancakes are made using 3 cups of Bisquick, one cup of milk and 2 eggs whereas for 5 batches of waffles you will need 2 cups of Bisquick, 2 cups of milk and 2 eggs 5. You have 24 cups of Bisquick, 18 cups of milk, and 20 eggs. If you want to feed as many people as possible, how many batches of each should you make.

    (b) Kayla works no more than 20 hours per week during the school year. She is paid 10 dollars per hour for tutoring Geometry students and 7 dollars per hour for babysitting. She wants to spend at least 3 hours but no more than 8 hours a week tutoring. Find Kayla's maximum weekly earnings.

2. Solve them using the graphic approach then the simplex algorithm.

Problem 3

Consider the following LP:

$$\begin{aligned} \text{minimize} \quad & x_1 + 2x_2 - x_3 \\ \text{s.t.} \quad & x_1 - x_2 = 1. \\ & x_1 - 3x_2 + x_3 \geq 2. \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \leq 0. \end{aligned} \tag{2}$$

1. (a) Prove that (2) is feasible implies $x_3 \geq 1 + 2x_2$.

   (b) Deduce that (2) is infeasible.

2. (a) Write (2) in the standard form (Hint: consider the change of variable $y = -x_3$).

   (b) Prove that (2) is infeasible using simplex algorithm.