PROBLEM 1:-

Given L.P Max $c \cdot x$

$$\text{st } Ax \le b$$
$$x \ge 0.$$

1) given $b = 0$

(a) To prove:- The above L.P is feasible & deduce a lower bound.

Proof:- W.K.T $Ax \le 0$, so $x = 0$ is a sol. for the given problem

$\Rightarrow$ The problem is feasible.

Sub. $x = 0$ in the problem, we get $z = 0$ which is the lower bound for the problem.

(b) To prove:- The above one is degenarate.

Proof:- Adding slack variables, we get.

$$x_s = b - Ax$$
$$z = cx$$

W.K.T $b = 0$, $x_s = -Ax$
$$z = cx$$

Sub non basic Variables equal to Zero, we get $x_s = 0$.

Since one(or) more basic Variables have the Value Zero, the dictionary is degenarate.

(c) There are two possible sol. for $Ax=0$

<u>Trivial sol</u> :- when $x=0$, this case feasible set is reduced to a single point of R^n.

<u>Non-Trivial sol</u> :- $x \neq 0$, & A is a singular matrix, this case the feasible set is unbounded as x can take any value.

(d) <u>To prove</u> :- The LP is unbounded or, its optimal value is zero.

As we know max Cx & $b=0$.

$$\begin{aligned} \text{s.t } \; & Ax \leq b \\ & x \geq 0 \end{aligned}$$

$\Rightarrow$ $Ax \leq 0$, $x \geq 0$. , so when $x=0$, it satisfies all the conditions and on substituting it in the objective function, we get optimal value $=0$.

$\Rightarrow$ when $x \neq 0$, x can take any values to satisfy the condition $Ax \leq 0$ which implies x can be taken any value without affecting the problem,

$\Rightarrow$ The problem is unbounded

(e) To prove the above using duality theory.

W.K.T dual of the above L.P is

$$\begin{aligned} \text{Min} \quad & b^T y \\ \text{s.t} \quad & A^T y \geq c \\ & y \geq 0 \end{aligned}$$

$\because b=0$; we need to minimize $0 \cdot y$
so whatever is the value of y the optimal value is zero.

2) Given $A = \begin{pmatrix} 0.5 & -5.5 & -2.5 & 9 \\ 0.5 & -1.5 & -0.5 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ $b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ & $c = \begin{pmatrix} 10 \\ -57 \\ -9 \\ -24 \end{pmatrix}$

(a) Initial Dictionary :-

$$\begin{aligned} x_5 &= -0.5x_1 + 5.5x_2 + 2.5x_3 - 9x_4 \\ x_6 &= -0.5x_1 + 1.5x_2 + 0.5x_3 - x_4 \\ x_7 &= 1 - x_1 \\ \hline z &= 10x_1 - 57x_2 - 9x_3 - 24x_4 \end{aligned}$$

Substituting the non-basic variables, $x_1, x_2, x_3, x_4 = 0$ we get $x_5 = 0$; $x_6 = 0$; $x_7 = 1$ & $z = 0$.

$\therefore$ $(0,0,0,0)$ is a solution for the given problem,
$\Rightarrow$ The problem is 'feasible'.

Since the value of more than one basic variables (x_5, x_6) is equal to zero, the given dictionary is 'degenerate'.

(b) Solving the above problem using the following rule
(i) largest coefficient rule for entering variable
(ii) smallest index subscript for leaving variable

To prove :- The simplex method cycles after 6 itera-tions

<u>Proof</u> :- $D_1 \Rightarrow$

$$\begin{aligned} x_5 &= -0.5x_1 + 5.5x_2 + 2.5x_3 - 9x_4 \\ x_6 &= -0.5x_1 + 1.5x_2 + 0.5x_3 - x_4 \\ x_7 &= 1 - x_1 \\ \hline z &= 10x_1 - 57x_2 - 9x_3 - 24x_4 \end{aligned}$$

'x_1' is entering & 'x_5' is leaving Variable.

$$D_2 \Rightarrow \begin{aligned} x_1 &= 11x_2 + 5x_3 - 18x_4 - 2x_5 \\ x_6 &= -4x_2 - 2x_3 + 8x_4 + x_5 \\ x_7 &= 1 - 11x_2 - 5x_3 + 18x_4 + 2x_5 \\ \hline z &= 53x_2 + 41x_3 - 204x_4 - 20x_5 \end{aligned}$$

'x_2' is entering & 'x_6' is leaving variable.

$$D_3 \Rightarrow \begin{aligned} x_2 &= -0.5x_3 + 2x_4 + 0.25x_5 - 0.25x_6 \\ x_1 &= -0.5x_3 + 4x_4 + 0.75x_5 - 2.75x_6 \\ x_7 &= 1 + 0.5x_3 - 4x_4 - 0.75x_5 + 2.75x_6 \\ \hline z &= 14.5x_3 - 98x_4 - 6.75x_5 - 13.25x_6 \end{aligned}$$

'x_3' is entering & 'x_1' is leaving Variable.

$$D_4 \Rightarrow \begin{aligned} x_3 &= -2x_1 + 8x_4 + 1.5x_5 - 5.5x_6 \\ x_2 &= x_1 - 2x_4 - 0.5x_5 + 2.5x_6 \\ x_7 &= 1 - x_1 \\ \hline z &= -29x_1 + 18x_4 + 15x_5 - 93x_6 \end{aligned}$$

'x_4' is entering & 'x_2' is leaving Variable.

$$D_5 \Rightarrow \begin{aligned} x_4 &= 0.5x_1 - 0.5x_2 - 0.25x_5 + 1.25x_6 \\ x_3 &= 2x_1 - 4x_2 - 0.5x_5 + 4.5x_6 \\ x_7 &= 1 - x_1 \\ \hline z &= -20x_1 - 9x_2 + 10.5x_5 - 70.5x_6 \end{aligned}$$

'x_5' is entering & 'x_3' is leaving Variable

$D_6 \Rightarrow x_5 = 4x_1 - 8x_2 - 2x_3 + 9x_6$

$x_4 = -0.5x_1 + 1.5x_2 + 0.5x_3 - x_6$

$x_7 = 1 - x_1$

$z = 22x_1 - 93x_2 - 21x_3 + 24x_6$

'x_6' is entering & 'x_4' is leaving.

$D_7 \Rightarrow x_6 = -0.5x_1 + 1.5x_2 + 0.5x_3 - x_4$

$x_7 = 1 - x_1$

$x_5 = -0.5x_1 + 5.5x_2 + 2.5x_3 - 9x_4$

$z = 10x_1 - 57x_2 - 9x_3 - 24x_4$

$\Rightarrow D_1 = D_7$, so the simplex cycles after 6 iterations.

(c) using Bland's Rule to solve the given problem. until D_6, it is the same as the above solution. After D_6, we choose 'x_1' is entering & 'x_4' is leaving Variable

$D_7 \Rightarrow x_1 = 3x_2 + x_3 - 2x_4 - 2x_6$

$x_7 = 1 - 3x_2 - x_3 + 2x_4 + 2x_6$

$x_5 = 4x_2 + 2x_3 - 8x_4 + x_6$

$z = -27x_2 + x_3 - 44x_4 - 20x_6$.

'x_3' is entering & 'x_7' is leaving.

$D_8 \Rightarrow x_3 = 1 - 3x_2 + 2x_4 + 2x_6 - x_7$

$x_1 = 1 - x_7$

$x_5 = 2 - 2x_2 - 4x_4 + 5x_6 - 2x_7$

$z = 1 - 30x_2 - 42x_4 - 18x_6 - x_7$

FINAL DICTIONARY

Set, $x_2 = x_4 = x_6 = x_7 = 0$, we get $x_3 = 1$; $x_1 = 1$; $x_5 = 2$

$\Rightarrow (x_1, x_2, x_3, x_4) = (1, 0, 1, 0)$ is a sol. with $\boxed{z=1}$

PROBLEM 2:-

Given L.P :- $\max\ 2x_1 + 3x_2 - 5x_3$

$$\text{s.t} \quad x_1 - x_2 \le 3$$
$$-x_1 + x_3 \le 6$$
$$-2x_1 + x_3 \le 2$$
$$-x_1 + x_2 \le -4$$
$$x_1, x_2, x_3 \ge 0$$

1) To check the feasibility of the dual problem without computing it.

Initial dictionary of the above L.P

$$D \Rightarrow \quad \begin{aligned} x_4 &= 3 - x_1 + x_2 \\ x_5 &= 6 + x_1 \quad - x_3 \\ x_6 &= 2 + 2x_1 \quad - x_3 \\ x_7 &= -4 + x_1 - x_2 \\ \hline z &= 2x_1 + 3x_2 - 5x_3 \end{aligned}$$

Since the initial dictionary is infeasible, we need to solve the auxiliary problem first to try to make the Dictionary feasible.

$$D' \Rightarrow \quad \begin{aligned} x_4 &= 3 - x_1 + x_2 \quad + x_0 \\ x_5 &= 6 + x_1 \quad - x_3 + x_0 \\ x_6 &= 2 + 2x_1 \quad - x_3 + x_0 \\ x_7 &= -4 + x_1 - x_2 \quad + x_0 \\ \hline z &= -x_0 \end{aligned}$$

Now, we force x_0 to enter, & x_7 leaves as it has the least coefficient value.

$$D'_1 \Rightarrow \quad \begin{aligned} x_0 &= 4 - x_1 + x_2 - x_7 \\ x_4 &= 7 - 2x_1 + 2x_2 - x_7 \\ x_5 &= 10 + x_2 - x_3 - x_7 \\ x_6 &= 6 + x_1 + x_2 - x_3 - x_7 \\ \hline z &= -4 + x_1 - x_2 + x_7 \end{aligned}$$

'x_1' is the entering & 'x_4' is the leaving variable.

$$D'_2 \Rightarrow \quad \begin{aligned} x_1 &= 3.5 + x_2 - 0.5x_4 - 0.5x_7 \\ x_0 &= 0.5 + 0.5x_4 - 0.5x_7 \\ x_5 &= 10 + x_2 - x_3 - x_7 \\ x_6 &= 9.5 + 2x_2 - x_3 - 0.5x_4 - 1.5x_7 \\ \hline z &= -0.5 - 0.5x_4 + 0.5x_7 \end{aligned}$$

'x_7' is the entering & 'x_0' is the leaving variable.

$$D'_3 \Rightarrow \quad \begin{aligned} x_7 &= 1 + x_4 - 2x_0. \\ x_1 &= 3 + x_2 - x_4 + x_0. \\ x_5 &= 9 + x_2 - x_3 - x_4 + 2x_0. \\ x_6 &= 8 + 2x_2 - x_3 - 2x_4 + 3x_0 \\ \hline z &= -x_0. \end{aligned}$$

Since we have forced x_0 to leave, we can sub. back our original problem.

$$D_1 \Rightarrow \quad \begin{aligned} x_7 &= 1 + x_4 \\ x_1 &= 3 + x_2 - x_4 \\ x_5 &= 9 + x_2 - x_3 - x_4 \\ x_6 &= 8 + 2x_2 - x_3 - 2x_4 \\ \hline z &= 6 + 5x_2 - 5x_3 - 2x_4 \end{aligned}$$

Since, 'x_2' is the entering variable & there is no leaving variable, the given LP is unbounded.

As the primal is unbounded we can now deduce that the Dual is infeasible.

2) Dual of the above L.P is

$$\begin{aligned}
\text{Min} \quad & 3y_1 + 6y_2 + 2y_3 - 4y_4 \\
\text{s.t} \quad & y_1 - y_2 - 2y_3 - y_4 \geq 2 \\
& -y_1 \qquad\qquad + y_4 \geq 3 \\
& \qquad y_2 + y_3 \qquad \geq -5 \\
& \qquad y_2, y_2, y_3, y_4 \geq 0.
\end{aligned}$$

Converting it to std. form.

$$\begin{aligned}
\text{Max} \quad & -3y_1 - 6y_2 - 2y_3 + 4y_4 \\
\text{s.t} \quad & -y_1 + y_2 + 2y_3 + y_4 \leq -2 \\
& y_1 \qquad\qquad - y_4 \leq -3 \\
& \quad - y_2 - y_3 \qquad \leq 5 \\
& \quad y_1, y_2, y_3, y_4 \geq 0.
\end{aligned}$$

Initial dictionary is

$$D \Rightarrow \begin{aligned}
y_5 &= -2 + y_1 - y_2 - 2y_3 - y_4 \\
y_6 &= -3 - y_1 \qquad\qquad + y_4 \\
y_7 &= 5 \qquad + y_2 + y_3 \\
\hline
z &= \quad -3y_1 - 6y_2 - 2y_3 + 4y_4
\end{aligned}$$

Since the Initial dictionary is infeasible, we need to solve the auxiliary problem.

$$D^{a} \Rightarrow \begin{aligned}
y_5 &= -2 + y_1 - y_2 - 2y_3 - y_4 + y_0 \\
y_6 &= -3 - y_1 \qquad\qquad + y_4 + y_0 \\
y_7 &= 5 \qquad + y_2 + y_3 \qquad + y_0 \\
\hline
z &= -y_0
\end{aligned}$$

Now forcing y_0 to enter, we get.

$D_1' \Rightarrow$
$$y_0 = 3 + y_1 \qquad - y_4 + y_6.$$
$$y_5 = 1 + 2y_1 - y_2 - 2y_3 - 2y_4 + y_6.$$
$$y_7 = 8 + y_1 + y_2 + y_3 - y_4 + y_6.$$
$$z = -3 - y_1 + y_4 - y_6.$$

'y_4' is the entering & 'y_5' is the leaving variable.

$D_2' \Rightarrow$
$$y_4 = 0.5 + y_1 - 0.5y_2 - y_3 - 0.5y_5 + 0.5y_6.$$
$$y_0 = 2.5 \qquad + 0.5y_2 + y_3 + 0.5y_5 + 0.5y_6.$$
$$y_7 = 7.5 \qquad + 1.5y_2 + 2y_3 + 0.5y_5 + 0.5y_6.$$
$$z = -2.5 \qquad - 0.5y_2 - y_3 - 0.5y_5 - 0.5y_6.$$

Since the above dictionary is final & set the non basic variables $(y_2, y_3, y_5, y_6) = 0$, we get $z = -2.5$, since the value of the auxilary prob is $-ve$, the given L.P is infeasible.

3) To prove :- If a problem is unbounded, then its dual is infeasible.

Let 'x' be a feasible sol. for P (Primal) & 'y' be a feasible sol. for D (Dual).

Then by Weak Duality theorem.

$$c^T x \leq b^T y$$

Since we are given primal is unbounded, i.e $c^T x = +\infty$ we can see that D (Dual) is infeasible.

4) No, the Inverse isn't true, if Primal is infeasible then Dual can be infeasible (or) unbounded.

Eg:- given L.P max $2x_1 - x_2$

$$\text{s.t} \quad x_1 - x_2 \le 1$$
$$-x_1 + x_2 \le -2$$
$$x_1, x_2 \ge 0.$$

From the constraints, we can conclude that the given L.P is infeasible as the constraints $x_1 - x_2 \le 1$ & $x_1 - x_2 \ge 2$ can't be satisfied.

Dual of the L.P min $y_1 - 2y_2$

$$\text{s.t} \quad y_1 - y_2 \ge 2$$
$$-y_1 + y_2 \ge -1$$
$$y_1, y_2 \ge 0.$$

Converting to Standard form, we get.

Max $-y_1 + 2y_2$

$$\text{st} \quad -y_1 + y_2 \le -2$$
$$y_1 - y_2 \le 1$$
$$y_1 \,\&\, y_2 \ge 0$$

From the Constraints, we can Conclude that the dual is infeasible as the Constraints $y_1 - y_2 \le 1$ & $y_1 - y_2 \ge 2$ can't be satisfied.

$\Rightarrow$ If the primal is infeasible, the dual can be infeasible and not necessarily be unbounded.

PROBLEM 3 :-

Given LP, Max $-x_1 - 2x_2$

s.t $-2x_1 + 7x_2 \le 6$

$-3x_1 + x_2 \le -1$

$9x_1 - 4x_2 \le 6$

$x_1 - x_2 \le 3$

$7x_1 - 3x_2 \le 6$

$-5x_1 + 2x_2 \le -3$

$x_1 \ge 0, x_2 \ge 0$

1) (a) Dual form of above L.P

Min $6y_1 - y_2 + 6y_3 + 3y_4 + 6y_5 - 3y_6$.

s.t $-2y_1 - 3y_2 + 9y_3 + y_4 + 7y_5 - 5y_6 \ge -1$

$7y_1 + y_2 - 4y_3 - y_4 - 3y_5 + 2y_6 \ge -2$

$y_1 \ge 0, y_2 \ge 0, y_3 \ge 0, y_4 \ge 0, y_5 \ge 0, y_6 \ge 0$.

(b) It is easier to solve the Dual as all the primal objective coefficients are all negative, which helps us avoid the Initialisation phase.

→ optimal Sol. & optimal Value of the given L.P -
Since we need to max $-x_1 - 2x_2$ & we know that x_1 & $x_2 \ge 0$.

Let's Start by assuming $x_1 = x_2 = 0$

on sub, the constraints $-3x_1 + x_2 \le -1$ &
$-5x_1 + 2x_2 \le -3$
isn't Satisfied.

From the two constraints we can see that only the value of x_1 could be increased. So on keeping $x_2 = 0$, we get :

$$-3x_1 \le -1 \quad \& \quad -5x_1 \le -3$$
$$x_1 \ge 1/3 \quad \& \quad x_1 \ge 3/5$$

$$\Rightarrow \quad x_1 = 3/5$$

$\therefore$ the optimal Sol. is $(x_1, x_2) = (3/5, 0)$

& optimal value is $z = -3/5$

2) (a) Initial Dictionary of given L.P (i.e the primal)

$$P \Rightarrow \begin{array}{l} x_3 = 6 + 2x_1 - 7x_2 \\ x_4 = -1 + 3x_1 - x_2 \\ x_5 = 6 - 9x_1 + 4x_2 \\ x_6 = 3 - x_1 + x_2 \\ x_7 = 6 - 7x_1 + 3x_2 \\ x_8 = -3 + 5x_1 - 2x_2 \\ \hline z = -x_1 - 2x_2 \\ \hline \end{array} \qquad \begin{array}{l} x_1, x_2, x_3, x_4, x_5 \\ x_6, x_7, x_8 \ge 0 \end{array}$$

Initial Dictionary of its Dual.

$$D \Rightarrow \begin{array}{l} y_7 = 1 - 2y_1 - 3y_2 + 9y_3 + y_4 + 7y_5 - 5y_6 \\ y_8 = 2 + 7y_1 + y_2 - 4y_3 - y_4 - 3y_5 + 2y_6 \\ \hline z = -6y_1 + y_2 - 6y_3 - 3y_4 - 6y_5 + 3y_6 \\ \hline \end{array}$$

$$y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8 \ge 0$$

(b) Dictionary of Primal $\Rightarrow$ Infeasible (as coefficients are -ve)

Dictionary of Dual $\Rightarrow$ feasible

Matrix form of Primal

$$\begin{bmatrix} 0 & -1 & -2 \\ 6 & 2 & -7 \\ -1 & 3 & -1 \\ 6 & -9 & 4 \\ 3 & -1 & 1 \\ 6 & -7 & 3 \\ -3 & 5 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} z \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix}$$

$\Downarrow$

P

Matrix form of Dual.

$$\begin{bmatrix} 0 & -6 & 1 & -6 & -3 & -6 & 3 \\ 1 & -2 & -3 & 9 & 1 & 7 & -5 \\ 2 & 7 & 1 & -4 & -1 & -3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} = \begin{bmatrix} \xi \\ y_7 \\ y_8 \end{bmatrix}$$

$\Downarrow$

D

(c) The relation between the two matrices is,

$$\boxed{P = -D^T}$$

The slack variables of primal becomes the decision variables of dual & decision variables of primal becomes the slack variables of dual.

$x_3 \; x_4 \; x_5 \; x_6 \; x_7 \; x_8 \Rightarrow y_1 \; y_2 \; y_3 \; y_4 \; y_5 \; y_6$

$x_1 \; x_2 \Rightarrow y_7 \; y_8$

3) (a) Initial dictionary of given L.P

$$
D \Rightarrow \begin{aligned}
x_3 &= 6 + 2x_1 - 7x_2 \\
x_4 &= -1 + 3x_1 - x_2 \\
x_5 &= 6 - 9x_1 + 4x_2 \\
x_6 &= 3 - x_1 + x_2 \\
x_7 &= 6 - 7x_1 + 3x_2 \\
x_8 &= -3 + 5x_1 - 2x_2 \\
\hline
z &= \quad - x_1 - 2x_2
\end{aligned}
$$

Since D is infeasible, we must do the Initialisation Phase & solve the Auxiliary problem.

So, now $Z = -x_0$, & new dictionary of the Auxiliary problem is.

$$
D' \Rightarrow \begin{aligned}
x_0 &= 3 - 5x_1 + 2x_2 + x_8 \\
x_3 &= 9 - 3x_1 - 5x_2 + x_8 \\
x_4 &= 2 - 2x_1 + x_2 + x_8 \\
x_5 &= 9 - 14x_1 + 6x_2 + x_8 \\
x_6 &= 6 - 6x_1 + 3x_2 + x_8 \\
x_7 &= 9 - 12x_1 + 5x_2 + x_8 \\
\hline
z &= -3 + 5x_1 - 2x_2 - x_8
\end{aligned}
$$

'x_1' is entering & 'x_0' is leaving Variable.

$$
D'' \Rightarrow \begin{aligned}
x_1 &= 0.6 + 0.4x_2 + 0.2x_8 - 0.2x_0 \\
x_3 &= 7.2 - 6.2x_2 + 0.4x_8 + 0.6x_0 \\
x_4 &= 0.8 + 0.2x_2 + 0.6x_8 + 0.4x_0 \\
x_5 &= 0.6 + 0.4x_2 - 1.8x_8 + 2.8x_0 \\
x_6 &= 2.4 + 0.6x_2 - 0.2x_8 + 1.2x_0 \\
x_7 &= 1.8 + 0.2x_2 - 1.4x_8 + 2.4x_0 \\
\hline
z &= -x_0
\end{aligned}
$$

Sub back the original problem, we get.

$$
D^* \Rightarrow \begin{aligned}
x_1 &= 0.6 + 0.4x_2 + 0.2x_8 \\
x_3 &= 7.2 - 6.2x_2 + 0.4x_8 \\
x_4 &= 0.8 + 0.2x_2 + 0.6x_8 \\
x_5 &= 0.6 + 0.4x_2 - 1.8x_8 \\
x_6 &= 2.4 + 0.6x_2 - 0.2x_8 \\
x_7 &= 1.8 + 0.2x_2 - 1.4x_8 \\
\hline
z &= -0.6 - 0.4x_2 - 0.2x_8
\end{aligned}
$$

Since, no entering variable, D^* is final dictionary
Sub x_2 & $x_8 = 0$, we get.

$x_1 = 0.6$; $x_3 = 7.2$; $x_4 = 0.8$; $x_5 = 0.6$; $x_6 = 2.4$;
$x_7 = 1.8$ & value of $z = -0.6$.

optimal sol. $(x_1, x_2) = (0.6, 0)$
optimal value $= -0.6$.

(b) Initial dictionary of the Dual of the above LP's

$$
D \Rightarrow \begin{aligned}
y_7 &= 1 - 2y_1 - 3y_2 + 9y_3 + y_4 + 7y_5 - 5y_6 \\
y_8 &= 2 + 7y_1 + y_2 - 4y_3 - y_4 - 3y_5 + 2y_6 \\
\hline
z &= -6y_1 + y_2 - 6y_3 - 3y_4 - 6y_5 + 3y_6
\end{aligned}
$$

'y_6' is entering & 'y_7' is leaving variable

$$
D' \Rightarrow \begin{aligned}
y_6 &= 0.2 - 0.4y_1 - 0.6y_2 + 1.8y_3 + 0.2y_4 + 1.4y_5 - 0.2y_7 \\
y_8 &= 2.4 + 6.2y_1 - 0.2y_2 - 0.4y_3 - 0.6y_4 - 0.2y_5 - 0.4y_7 \\
\hline
z &= 0.6 - 7.2y_1 - 0.8y_2 - 0.6y_3 - 2.4y_4 - 1.8y_5 - 0.6y_7
\end{aligned}
$$

Since, no entering variable, D' is final dictionary,
Set y_1, y_2, y_3, y_4, y_5 & $y_7 = 0$, we get.

$y_6 = 0.2$; $y_8 = 2.4$ & $z = 0.6$

Since, we converted the min problem to max. problem; the optimal sol.

$(y_1, y_2, y_3, y_4, y_5, y_6) = (0, 0, 0, 0, 0, 0.2)$
optimal value $= (-z) = -0.6$.

(c) Primal - dual certificate verification.

if x^* feasible solution of primal.
y^* feasible solution of Dual

then $cx^* = dy^*$

From 3(a) we know $x^* = (0.6, 0)$ & $cx^* = -0.6$
III^{ly} 3(b) we know $y^* = (0, 0, 0, 0, 0, 0.2)$ & $dy^* = -0.6$

$\Rightarrow cx^* = dy^*$.

$\therefore$ the primal - dual certificate is verified.