ASSIGNMENT 5

PROBLEM 1

1) $\max \sum_{j=1}^{n} c_j x_j$

$$\text{s.t} \quad \sum_{j=1}^{n} a_{ij} x_j \le b_i \qquad i \in \{1, \dots, m\}$$

$$0 \le x_j \le 1 \qquad x_j \in \mathbb{Z}$$

$x_j = 0$ (or) 1 correspond to not proceeding (or) proceeding with project j.

2) $\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} + \sum_{i=1}^{m} f_i y_i$

$$\text{s.t} \quad \sum_{i=1}^{m} x_{ij} = 1 \qquad i \in \{1, \dots, m\}$$

$$x_{ij} \le y_i \qquad j \in \{1, \dots, n\}$$

$$x_{ij} \ge 0 \; ; \; 0 \le y_i \le 1 \; ; \; y_i \in \mathbb{Z}$$

$x_{ij} \rightarrow$ customer j's requirement supplied from depot i.

$y_i \rightarrow$ takes value 0 (or) 1 corresponding to whether a particular site being not selected (or) selected.

3) $\min \; x_1 + x_2 + x_3 + x_4 + x_5 + x_6.$

$$\text{s.t} \quad x_1 + x_2 \ge 1$$
$$x_1 + x_2 + x_6 \ge 1$$
$$x_3 + x_4 \ge 1$$
$$x_3 + x_4 + x_5 \ge 1$$
$$x_4 + x_5 + x_6 \ge 1$$
$$x_2 + x_5 + x_6 \ge 1$$

$$x_i = \begin{cases} 1 & \text{if fire station built at city } i \\ 0 & \text{else.} \end{cases}$$

$$0 \le x_i \le 1 \quad i \in \{1, \dots, 6\} \quad x_i \in \mathbb{Z}$$

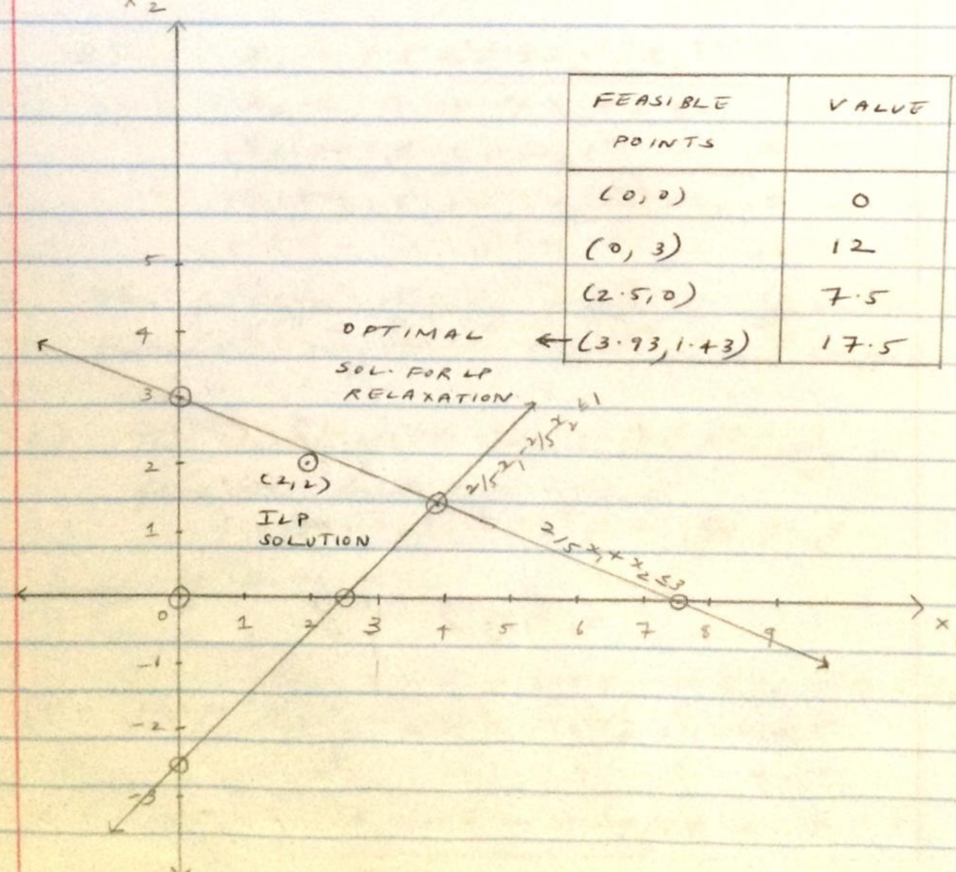
PROBLEM 2

Gn,

$$\text{Maximize } 3x_1 + 4x_2$$
$$\text{s.t. } \tfrac{2}{5}x_1 + x_2 \le 3$$
$$\tfrac{2}{5}x_1 - \tfrac{2}{5}x_2 \le 1$$
$$x \ge 0,\ x \in \mathbb{Z}$$

1) a) LP relaxation of the above problem.

$$\text{Max } 3x_1 + 4x_2$$
$$\text{s.t. } \tfrac{2}{5}x_1 + x_2 \le 3$$
$$\tfrac{2}{5}x_1 - \tfrac{2}{5}x_2 \le 1$$
$$x \ge 0.$$



FEASIBLE POINTS	VALUE
(0, 0)	0
(0, 3)	12
(2.5, 0)	7.5
(3.93, 1.43)	17.5

b) Standard form of the given problem.

$$\max \quad 3x_1 + 4x_2$$
$$\text{st} \quad 2x_1 + 5x_2 \leq 15$$
$$2x_1 - 2x_2 \leq 5$$
$$x \geq 0, \; x \in \mathbb{Z}$$

c) Initial Dictionary,

$$D: \quad \begin{array}{l} x_3 = 15 - 2x_1 - 5x_2 \\ x_4 = 5 - 2x_1 + 2x_2 \\ \hline z = 3x_1 + 4x_2 \\ \hline \end{array}$$

'x_2' is the entering & 'x_3' is the leaving variable.

$$D_1: \quad \begin{array}{l} x_2 = 3 - 0.4x_1 - 0.2x_3 \\ x_4 = 11 - 2.8x_1 - 0.4x_3 \\ \hline z = 12 + 1.4x_1 - 0.8x_3 \\ \hline \end{array}$$

'x_1' is the entering & 'x_4' is the leaving Variable.

$$D_2: \quad \begin{array}{l} x_1 = 11/2.8 - 0.4x_3/2.8 - x_4/2.8 \\ x_2 = 10/7 - x_3/7 + x_4/7 \\ \hline z = 17.5 - x_3 - 0.5x_4 \\ \hline \end{array} \Rightarrow \text{FINAL DICTIONARY}$$

$\therefore$ the optimal sol. is $(11/2.8, 10/7)$ & optimal value is 17.5

2) a) GOMORY CUT ON x_1 ROW.

$$x_1 = 11/2.8 - 0.4x_3/2.8 - x_4/2.8$$

formula,

$$frac(-a_{k_1})x_{I1} + \cdots + frac(-a_{k_n})x_{In} \geq frac(\ldots$$

$$frac(x) = x - \lfloor x \rfloor$$

$$x_1 + x_3/7 + x_4/2.8 = 11/2.8$$

$$\underbrace{x_1}_{A:\ \text{integer.}} + \underbrace{1/7\, x_3 + 1/2.8\, x_4}_{B:\ \text{integer + fraction.}} = 3 + 0.928$$

$$\Rightarrow 1/7\, x_3 + 1/2.8\, x_4 \geq 0.928$$

$\therefore$ the cut to be added to the dictionary is

$$-0.928 + x_3/7 + x_4/2.8$$

b) Final dictionary after adding the cut.

$$x_1 = 11/2.8 - x_3/7 - x_4/2.8$$
$$x_2 = 10/7 - x_3/7 + x_4/7$$
$$x_5 = -0.928 + x_3/7 + x_4/2.8$$
$$z = 17.5 - x_3 - 0.5\, x_4$$

As the dictionary contains negative value for one of the basic variables, the above dictionary is infeasible

c) dual of the above dictionary.

$$y_3 = 1 + 1/7\, y_1 + 1/7\, y_2 - 1/7\, y_5$$
$$y_4 = 0.5 + 1/2.8\, y_1 - 1/7\, y_2 - 1/2.8\, y_5$$
$$\xi = -17.5 - 11/2.8\, y_1 - 10/7\, y_2 + 0.928\, y_5$$

The dual dictionary is feasible, but not final.

d) In the dual dictionary above.

y_5 is the entering and y_4 is the leaving variable

$$y_5 = 1.4 + y_1 - 0.4y_2 - 2.8y_4$$
$$y_3 = 0.8 \qquad + 0.2y_2 + 0.4y_4$$
$$\xi = -16.2 - 3y_1 - 1.8y_2 - 2.6y_4$$

Converting back to primal.

$$x_1 = 3 + 0 \qquad - x_5$$
$$x_2 = 1.8 - 0.2x_3 + 0.4x_5$$
$$x_4 = 2.6 - 0.4x_3 + 2.8x_5$$
$$z = 16.2 - 0.8x_3 - 1.4x_5$$

$\therefore$ the sol. is $(3, 1.8)$, but this doesn't solve the ILP.

3) a) Initialisation phase:

Adding x_0 the dictionary obtained in 2b), we get

$$D: \quad x_1 = 11/2.8 - x_3/7 - x_4/2.8 + x_0$$
$$x_2 = 10/7 - x_3/7 + x_4/7 \qquad + x_0$$
$$x_5 = -2.6/2.8 + x_3/7 + x_4/2.8 + x_0$$
$$z = \qquad - x_0$$

Forcing x_0 to leave, we get

D_1: $x_0 = 2.6/2.8 - x_3/7 - x_4/2.8 + x_5$
$x_1 = 13.6/2.8 - 2x_3/7 - x_4/1.4 + x_5$
$x_2 = 6.6/2.8 - 2x_3/7 - 0.6x_4/2.8 + x_5$
$z = -2.6/2.8 + x_3/7 + x_4/2.8 - x_5$

'x_4' is the entering variable & 'x_0' is leaving.

D_2: $x_4 = 2.6 - 0.4x_3 + 2.8x_5 - 2.8x_0$
$x_1 = 3 - x_5 + 2x_0$
$x_2 = 1.8 - 0.2x_3 + 0.4x_5 + 0.6x_0$
$z = 0$

removing x_0 & sub. back the original z, we get.

D_3: $x_1 = 3 - x_5$
$x_2 = 1.8 - 0.2x_3 + 0.4x_5$ ⇒ FINAL DICTIONARY
$x_4 = 2.6 - 0.4x_3 + 2.8x_5$
$z = 16.2 - 0.8x_3 - 1.4x_5$

∴ The optimal sol. is $(3, 1.8)$ & value is 16.2.

b) Solving the ILP, continuing with the dictionary we got after one cut in 2d), we have:

D: $x_1 = 3 - x_5$
$x_2 = 1.8 - 0.2x_3 + 0.4x_5$
$x_4 = 2.6 - 0.4x_3 + 2.8x_5$
$z = 16.2 - 0.8x_3 - 1.4x_5$

cutting the row, $x_2 + 0.2x_3 - 0.4x_5 = 1.8$, we get:

$$\underbrace{x_2 - x_5}_{A} \underbrace{+ 0.2x_3 + 0.6x_5}_{B} = 1 + 0.8$$

$\Rightarrow \quad 0.2x_3 + 0.6x_5 \geq 0.8$

adding this constraint to the dictionary, we get.

$$
\begin{aligned}
x_1 &= 3 \qquad\qquad -x_5 \\
x_2 &= 1.8 - 0.2x_3 + 0.4x_5 \\
x_4 &= 2.6 - 0.4x_3 + 2.8x_5 \\
x_6 &= -0.8 + 0.2x_3 + 0.6x_5 \\
\hline
z &= 16.2 - 0.8x_3 - 1.4x_5
\end{aligned}
$$

Since the above dictionary is infeasible, but its dual is feasible, we get the dual as:

$$
\begin{aligned}
y_3 &= 0.8 \qquad\qquad + 0.2y_2 + 0.4y_4 - 0.2y_6 \\
y_5 &= 1.4 + y_1 - 0.4y_2 - 2.8y_4 - 0.6y_6 \\
\hline
\xi &= -16.2 - 3y_1 - 1.8y_2 - 2.6y_4 + 0.8y_6.
\end{aligned}
$$

'y_6' is the entering & 'y_5' is the leaving variable.

$$
\begin{aligned}
y_6 &= 1.4/0.6 + 1/0.6\, y_1 - 0.4/0.6\, y_2 - 2.8/0.6\, y_4 - y_5/0.6 \\
y_3 &= 1/3 - 1/3\, y_1 + 1/3\, y_2 + 4/3\, y_4 + y_5/3 \\
\hline
\xi &= -43/3 - 5/3\, y_1 - 7/3\, y_2 - 19/3\, y_4 - 4/3\, y_5
\end{aligned}
$$

Converting back to primal, we get.

$$
\begin{aligned}
x_1 &= 5/3 - 1/0.6\, x_6 + 1/3\, x_3 \\
x_2 &= 7/3 + 0.4/0.6\, x_6 - 1/3\, x_3 \\
x_4 &= 19/3 + 2.8/0.6\, x_6 - 4/3\, x_3 \\
x_5 &= 4/3 + 1/0.6\, x_6 - 1/3\, x_3 \\
\hline
z &= 43/3 - 1.4/0.6\, x_6 - 7/3\, x_3
\end{aligned}
$$

using Gomory cut to cut the x_2 row, we get new row

$x_6 + x_3 \geq 1$

The new Dictionary is,

$$
\begin{aligned}
x_1 &= 5/3 - 1/0.6\, x_6 + 1/3\, x_3 \\
x_2 &= 7/3 + 0.4/0.6\, x_6 + (-1/3\, x_3) \\
x_4 &= 19/3 + 2.8/0.6\, x_6 + (-4/3\, x_3) \\
x_5 &= 4/3 + 1/0.6\, x_6 + (-1/3\, x_3) \\
x_7 &= -1 + x_6 + x_3 \\
\hline
z &= 43/3 - 1.4/0.6\, x_6 - 1/3\, x_3
\end{aligned}
$$

Since this dictionary is infeasible, but its dual is feasible, converting it to dual we get.

$$
\begin{aligned}
y_6 &= 1.4/0.6 + 1/0.6\, y_1 - 2/3\, y_2 - 2.8/0.6\, y_4 - 1/0.6\, y_5 - y_7 \\
y_3 &= 1/3 - 1/3\, y_1 + 1/3\, y_2 + 4/3\, y_4 + 1/3\, y_5 - y_7 \\
\hline
\xi &= -43/3 - 5/3\, y_1 - 7/3\, y_2 - 19/3\, y_4 - 4/3\, y_5 + y_7
\end{aligned}
$$

'y_7' is the entering & 'y_3' is the leaving variable.

$$
\begin{aligned}
y_7 &= 1/3 - 1/3\, y_1 + 1/3\, y_2 + 4/3\, y_4 + 1/3\, y_5 - y_3 \\
y_6 &= 2 - 2y_1 - y_2 - 6y_4 - 2y_5 + y_3 \\
\hline
\xi &= -14 - 2y_1 - 2y_2 - 5y_4 - y_5 - y_3
\end{aligned}
$$

Converting it back to primal, we get.

$$
\begin{aligned}
x_1 &= 2 + 1/3\, x_7 + 2\, x_6 \\
x_2 &= 2 - 1/3\, x_7 + 1\, x_6 \\
x_4 &= 5 - 4/3\, x_7 + 6\, x_6 \\
x_5 &= 1 - 1/3\, x_7 + 2\, x_6 \\
x_3 &= 1 + 1\, x_7 - 1\, x_6 \\
\hline
z &= 14 - 1/3\, x_7 - 2x_6
\end{aligned}
$$

Final dictionary

Substituting x_6 & $x_7 = 0$, we get $x_1 = 2$, $x_2 = 2$, $x_4 = 5$, $x_5 = 1$, $x_3 = 1$

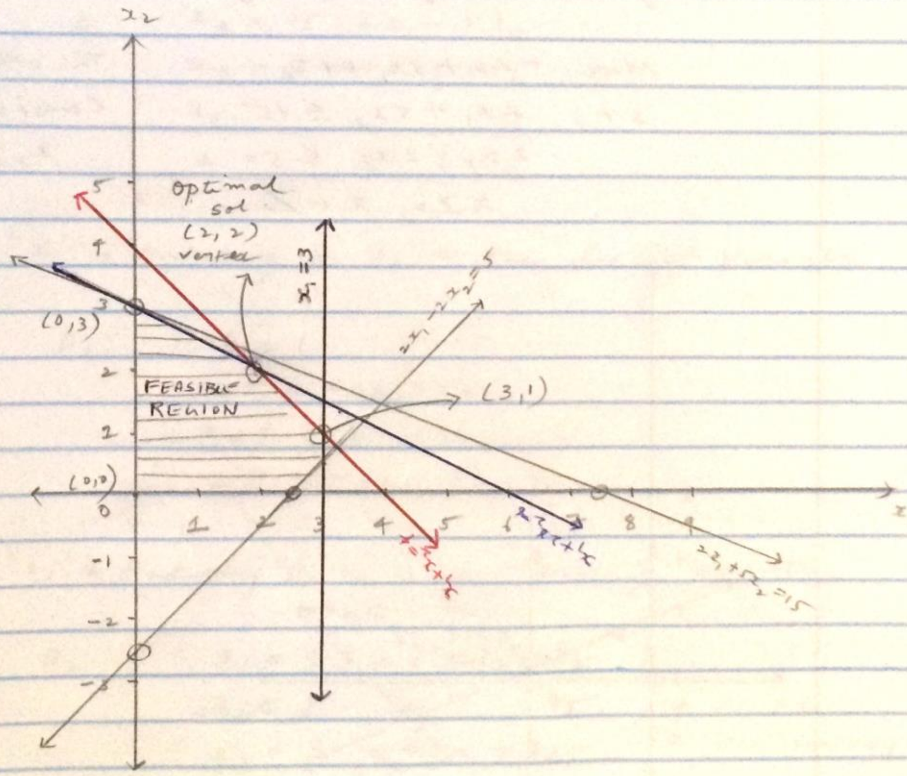
∴ The ILP solution is (2,2) & Value is 14.

c) The cuts that we got are,

$x_1 \le 3$

$x_1 + x_2 \le 4$

$x_1 + 2x_2 \le 6$.



It's clearly evident from the graph, the optimal solution (2,2) is indeed a vertex of the resulted feasible set after different cuts.

PROBLEM 3

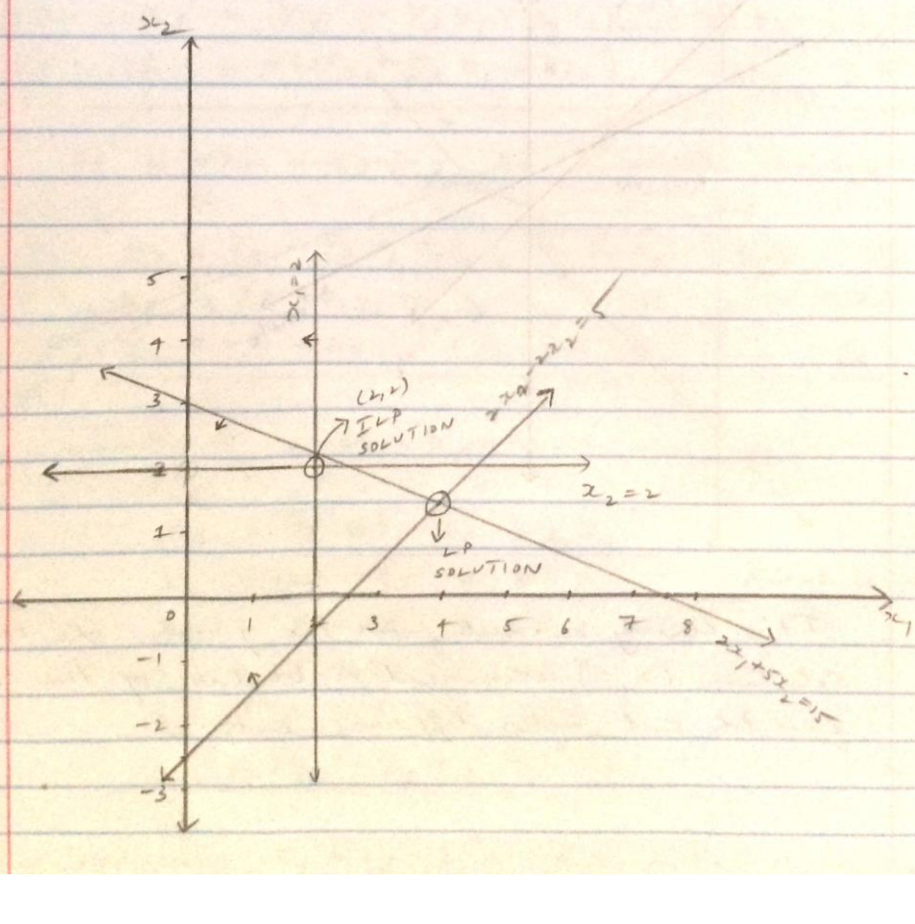
gn, $\quad \min\ 20 - 3x_1 - 4x_2$

$$\begin{aligned} \text{s.t}\quad & \tfrac{2}{5}x_1 + x_2 \le 3 \\ & \tfrac{2}{5}x_1 - \tfrac{2}{5}x_2 \le 1 \\ & x \ge 0,\ x \in \mathbb{Z} \end{aligned}$$

Converting it to std. form, we get.

$$\begin{aligned} \text{Max}\quad & -20 + 3x_1 + 4x_2 \\ \text{s.t}\quad & 2x_1 + 5x_2 \le 15 \\ & 2x_1 - 2x_2 \le 5 \\ & x \ge 0,\ x \in \mathbb{Z} \end{aligned}$$

The other branch constraints are: $x_2 \ge 2$ & $x_1 \le 2$



(x.) Note :- In this problem since Max $-20 + 3x_1 + 4x_2$ can be replaced as Max $3x_1 + 4x_2$ and solved, at last by adding -20 to the answer obtained.

b) The node P_0 is the same problem solved in Problem 2, 1(c), we get the value of x_1 & x_2 as $(3.93, 1.43)$

Now using branch & bound on x_2, we get.

P_{11} :- $x_2 \leq 1$, the initial Dictionary is

$$D: \quad \begin{aligned} x_3 &= 15 - 2x_1 - 5x_2 \\ x_4 &= 5 - 2x_1 + 2x_2 \\ x_5 &= 1 \quad\quad - x_2 \\ \hline z &= \quad 3x_1 + 4x_2 \end{aligned}$$

'x_2' is Entering & 'x_5' is the leaving Variable.

$$D_1: \quad \begin{aligned} x_2 &= 1 \quad\quad - x_5 \\ x_3 &= 10 - 2x_1 + 5x_5 \\ x_4 &= 7 - 2x_1 - 2x_5 \\ \hline z &= 4 + 3x_1 - 4x_5 \end{aligned}$$

'x_1' is Entering & 'x_4' is the leaving Variable.

$$D_2: \quad \begin{aligned} x_1 &= 3.5 - 0.5x_4 - x_5 \\ x_2 &= 1 \quad\quad\quad - x_5 \\ x_3 &= 3 + x_4 + 7x_5 \\ \hline z &= 14.5 - 1.5x_4 - 7x_5 \end{aligned} \quad \Rightarrow \text{FINAL DICTIONARY}$$

∴ the optimal solution is $(3.5, 1)$ & Value is 14.5

P_{12} :- $x_2 \geq 2$, the initial dictionary is infeasible, so initialisation is required.

$$
D :- \quad
\begin{aligned}
x_3 &= 15 - 2x_1 - 5x_2 \\
x_4 &= 5 - 2x_1 + 2x_2 \\
x_5 &= -2 \qquad + x_2 \\
\hline
z &= \quad 3x_1 + 4x_2
\end{aligned}
$$

Adding 'x_0' & proceeding with initialisation phase.

$$
D_1 :- \quad
\begin{aligned}
x_0 &= 2 \qquad - x_2 + x_5 \\
x_3 &= 17 - 2x_1 - 6x_2 + x_5 \\
x_4 &= 7 - 2x_1 + x_2 + x_5 \\
\hline
z &= -2 \qquad + x_2 - x_5
\end{aligned}
$$

'x_2' is entering & 'x_0' is leaving Variable.

$$
D_2 :- \quad
\begin{aligned}
x_2 &= 2 \qquad + x_5 - x_0 \\
x_3 &= 5 - 2x_1 - 5x_5 + 6x_0 \\
x_4 &= 9 - 2x_1 + 2x_5 - x_0 \\
z &= \quad - x_0.
\end{aligned}
$$

Sub-back the original z, we get.

$$
D_3 :- \quad
\begin{aligned}
x_2 &= 2 + x_5 \\
x_3 &= 5 - 5x_5 - 2x_1 \\
x_4 &= 9 + 2x_5 - 2x_1 \\
\hline
z &= 8 + 4x_5 + 3x_1
\end{aligned}
$$

'x_5' is Entering & 'x_3' is leaving Variable.

$$
D_4 :- \quad
\begin{aligned}
x_5 &= 1 - 0.4x_1 - 0.2x_3 \\
x_2 &= 3 - 0.4x_1 - 0.2x_3 \\
x_4 &= 11 - 2.8x_1 - 0.4x_3 \\
\hline
z &= 12 + 1.4x_1 - 0.8x_3
\end{aligned}
$$

'x_1' is entering and 'x_5' is leaving.

D_5 :-

$x_1 = 2.5 - 0.5x_3 - 2.5x_5$

$x_2 = 2 + x_5$ ⇒ FINAL DICTIONARY

$x_4 = 4 - 1.8x_3 - 7x_5$

$z = 15.5 - 1.5x_3 - 3.5x_5$.

∴ optimal Sol. is $(2.5, 2)$ & Value is 15.5

P_{21} :- $x_1 \leq 3$, the initial Dictionary is.

D :-

$x_3 = 15 - 2x_1 - 5x_2$

$x_4 = 5 - 2x_1 + 2x_2$

$x_5 = 1 - x_2$

$x_6 = 3 - x_1$

$z = 3x_1 + 4x_2$

'x_2' is entering & 'x_5' is leaving Variable.

D_1 :-

$x_2 = 1 - x_5$

$x_3 = 10 + 5x_5 - 2x_1$

$x_4 = 7 - 2x_5 - 2x_1$

$x_6 = 3 - x_1$

$z = 4 - 4x_5 + 3x_1$

'x_1' is entering & 'x_6' is leaving Variable.

D_2 :-

$x_1 = 3 - x_6$

$x_2 = 1 - x_5$

$x_3 = 4 + 2x_6 + 5x_5$ ⇒ FINAL DICTIONARY.

$x_4 = 1 + 2x_6 - 2x_5$

$z = 13 - 4x_5 - 3x_6$

$\therefore$ optimal sol is $(3,1)$ & value is 13

P_{22} :- $x_1 \geq 4$, the initial dictionary is infeasible, auxiliary phase needed.

D :-

$$x_3 = 15 - 2x_1 - 5x_2$$
$$x_4 = 5 - 2x_1 + 2x_2$$
$$x_5 = 1 \qquad - x_2$$
$$x_6 = -4 + x_1$$
$$z = \quad 3x_1 + 4x_2$$

Forcing 'x_0' to enter, we get

D_1 :-

$$x_0 = 4 - x_1 + x_6$$
$$x_3 = 19 - 3x_1 + x_6 - 5x_2$$
$$x_4 = 9 - 3x_1 + x_6 - 3x_2$$
$$x_5 = 5 - x_1 + x_6 - x_2$$
$$z = -4 + x_1 - x_6$$

'x_1' is entering and 'x_4' is leaving.

D_2 :-

$$x_0 = 6 + x_2 + x_4/3 + 2x_6/3$$
$$x_1 = 3 - x_2 - x_4/3 + x_6/3$$
$$x_3 = 10 - 2x_2 + x_4$$
$$x_5 = 2 \qquad + x_4/3 + 2x_6/3$$
$$z = -1 - x_2 - x_4/3 - 2x_6/3$$

$\Rightarrow$ FINAL DICTIONARY.

Since the value of z is -ve for the auxiliary problem, the problem is infeasible.

P_{23} :- $x_1 \leq 2$, the initial dictionary is infeasible, initialisation is required.

D :-
$$x_3 = 15 - 2x_1 - 5x_2$$
$$x_4 = 5 - 2x_1 + 2x_2.$$
$$x_5 = -2 + x_2$$
$$x_6 = 2 - x_1$$
$$z = 3x_1 + 4x_2$$

Introducing x_0 we get.

D_1 :-
$$x_0 = 2 - x_2 + x_5$$
$$x_3 = 17 - 6x_2 + x_5 - 2x_1$$
$$x_4 = 7 + x_2 + x_5 - 2x_1$$
$$x_6 = 4 - x_2 + x_5 - x_1$$
$$z = -2 + x_2 - x_5$$

'x_2' is Entering & 'x_0' is leaving.

D_2 :-
$$x_2 = 2 + x_5 - x_0$$
$$x_3 = 5 - 5x_5 - 2x_1 + 6x_0$$
$$x_4 = 9 + 2x_5 - 2x_1 - x_0$$
$$x_6 = 2 - x_1 + x_0$$
$$z = -x_0.$$

removing x_0 & sub. back the original z, we get.

D_3 :-
$$x_2 = 2 + x_5$$
$$x_3 = 5 - 5x_5 - 2x_1$$
$$x_4 = 9 + 2x_5 - 2x_1$$
$$x_6 = 2 - x_1$$
$$z = 8 + 4x_5 + 3x_1$$

'x_5' is entering & 'x_3' is leaving variable.

D_4 :-

$$x_5 = 1 - 0.4x_1 - 0.2x_3$$
$$x_2 = 3 - 0.4x_1 - 0.2x_3$$
$$x_4 = 11 - 2.8x_1 - 0.4x_3$$
$$x_6 = 2 - x_1$$
$$z = 12 + 1.4x_1 - 0.8x_3$$

'x_1' is entering & 'x_6' is leaving Variable.

D_5 :-

$$x_1 = 2 - x_6$$
$$x_2 = 2.2 + 0.4x_6 - 0.2x_3$$
$$x_5 = 0.2 + 0.4x_6 - 0.2x_3$$
$$x_4 = 5.4 + 2.8x_6 - 0.4x_3$$
$$z = 14.8 - 1.4x_6 - 0.8x_3$$

$\Rightarrow$ FINAL DICTIONARY

$\therefore$ optimal sol. is $(2, 2.2)$ & value is 14.8.

P_{24} :- $x_1 \geq 3$, the initial dictionary is infeasible, the initialisation phase is required.

D :-

$$x_3 = 15 - 2x_1 - 5x_2$$
$$x_4 = 5 - 2x_1 + 2x_2$$
$$x_5 = -2 + x_2$$
$$x_6 = -3 + x_1$$
$$z = 3x_1 + 4x_2$$

adding x_0, we get.

D_1 :-

$$x_0 = 3 - x_1 + x_6$$
$$x_3 = 18 - 3x_1 + x_6 - 5x_2$$
$$x_4 = 8 - 3x_1 + x_6 + 2x_2$$
$$x_5 = 1 - x_1 + x_6 + x_2$$
$$z = -3 + x_1 - x_6$$

'x_1' is entering & 'x_5' is leaving variable

$$D_2 :- \quad \begin{aligned} x_1 &= 1 + x_2 - x_5 + x_6. \\ x_0 &= 2 - x_2 + x_5 \\ x_3 &= 15 - 8x_2 + 3x_5 - 2x_6. \\ x_4 &= 5 - x_2 + 3x_5 - 2x_6 \\ \hline z &= -2 + x_2 - x_5 \end{aligned}$$

'x_2' is entering & 'x_0' is leaving variable.

$$D_3 :- \quad \begin{aligned} x_2 &= 2 + x_5 - x_0 \\ x_1 &= 3 + x_6 - x_0 \\ x_3 &= -1 - 5x_5 - 2x_6 + 8x_0. \\ x_4 &= 3 + 2x_5 - 2x_6 + x_0 \\ \hline z &= -x_0 \end{aligned}$$

After removing x_0, & substituting back z, we get $z=0$, but the problem is infeasible as it has a -ve value.

<u>P_3</u> :- $x_2 \leq 2$, the problem's initial dictionary is infeasible & needs initialization.

$$D :- \quad \begin{aligned} x_3 &= 15 - 2x_1 - 5x_2 \\ x_4 &= 5 - 2x_1 + 2x_2 \\ x_5 &= -2 + x_2 \\ x_6 &= 2 - x_1 \\ x_7 &= 2 - x_2 \\ \hline z &= 3x_1 + 4x_2 \end{aligned}$$

Adding x_0, we get.

D_1 :-

$$x_0 = 2 - x_2 + x_5$$
$$x_3 = 17 - 6x_2 + x_5 - 2x_1$$
$$x_4 = 7 + x_2 + x_5 - 2x_1$$
$$x_6 = 4 - x_2 + x_5 - x_1$$
$$x_7 = 4 - 2x_2 + x_5$$
$$z = -2 + x_2 - x_5$$

'x_2' is Entering & 'x_0' is leaving Variable.

D_2 :-

$$x_2 = 2 + x_5 - x_0$$
$$x_3 = 5 - 5x_5 + 6x_0 - 2x_1$$
$$x_4 = 9 + 2x_5 - x_0 - 2x_1$$
$$x_6 = 2 + x_0 - x_1$$
$$x_7 = -x_5 + 2x_0$$
$$z = -x_0.$$

removing x_0 & Sub. back original z, we get.

D_3 :-

$$x_2 = 2 + x_5$$
$$x_3 = 5 - 5x_5 - 2x_1$$
$$x_4 = 9 + 2x_5 - 2x_1$$
$$x_6 = 2 - x_1$$
$$x_7 = -x_5$$
$$z = 8 + 4x_5 + 3x_1$$

'x_1' is entering & 'x_6' is leaving.

D_4 :-

$$x_1 = 2 - x_6.$$
$$x_2 = 2 + x_5$$
$$x_3 = 1 - 5x_5 + 2x_6.$$
$$x_4 = 5 + 2x_5 - 2x_6.$$
$$x_7 = -x_5$$
$$z = 14 + 4x_5 - x_6.$$

'x_5' is entering & 'x_7' is leaving Variable.

$$x_5 = -x_7$$
$$x_1 = 2 - x_6$$
$$x_2 = 2 - x_7$$
$$x_3 = 1 + 5x_7 + 2x_6$$
$$x_4 = 5 - 2x_7 - 2x_6$$
$$z = 14 - 4x_7 - x_6$$

$\Rightarrow$ FINAL DICTIONARY.

$\therefore$ the optimal Value is 14 & sol. is (2,2)

P32 :- The Initial dictionary is infeasible & needs auxiliary phase.

D :-
$$x_3 = 15 - 2x_1 - 5x_2$$
$$x_4 = 5 - 2x_1 + 2x_2$$
$$x_5 = -2 + x_2$$
$$x_6 = 2 - x_1$$
$$x_7 = -3 + x_2$$
$$z = 3x_1 + 4x_2$$

removing the constraint $x_2 \geq 2$, as it is covered by $x_2 \geq 3$ & adding x_0, we get.

D_1 :-
$$x_0 = 3 - x_2 + x_6$$
$$x_3 = 18 - 6x_2 + x_6 - 2x_1$$
$$x_4 = 8 + x_2 + x_6 - 2x_1$$
$$x_5 = 5 - x_2 + x_6 - x_1$$
$$x_0 = -3 + x_2 - x_6$$

'x_2' is entering & 'x_0' is leaving Variable.

D_2 :-
$$\begin{aligned} x_2 &= 3 + x_6 - x_0 \\ x_3 &= \quad -5x_6 - 2x_1 + 6x_0 \\ x_4 &= 11 + 2x_6 - 2x_1 - x_0 \\ x_5 &= 2 \qquad\quad - x_1 + x_0 \\ \hline z &= -x_0. \end{aligned}$$

removing x_0 & substituting back original sol. we get.

D_3 :-
$$\begin{aligned} x_2 &= 3 + x_6 \\ x_3 &= \quad -5x_6 - 2x_1 \\ x_4 &= 11 + 2x_6 - 2x_1 \\ x_5 &= 2 \qquad\quad - x_1 \\ \hline z &= 12 + 4x_6 - 3x_1 \end{aligned}$$

'x_6' is entering & 'x_3' is leaving Variable.

D_4 :-
$$\begin{aligned} x_6 &= -0.2x_3 - 0.4x_1 \\ x_2 &= 3 - 0.2x_3 - 0.4x_1 \\ x_4 &= 11 - 0.4x_3 - 2.8x_1 \\ x_5 &= 2 + 0 \qquad - x_1 \\ \hline z &= 12 - 0.8x_3 - 4.6x_1 \end{aligned}$$
$\Rightarrow$ Final Dictionary.

$\therefore$ the optimal sol. is $(0,3)$ & Value is 12.

Now, the below table gives us the reason for the Node branching (or) Not & the optimal Value & sol. for the Actual problem.

NODE	OPTIMAL SOL.	OPTIMAL VALUE Max For $Z = 3x_1 + 4x_2$	OPTIMAL VALUE Min For $Z = (-20 + 3x_1 + 4x_2) \times -1$	REASON
P_0	$(11/2.8, 10/7)$	17.5	2.5	No Int. Sol
P_{11}	$(3.5, 1)$	14.5	5.5	No Int. Sol.
P_{12}	$(2.5, 2)$	15.5	4.5	No Int Sol
P_{21}	$(3, 1)$	13	<u>7</u>	Pruned
P_{22}	—	—	—	Infeasible
P_{23}	$(2, 2.2)$	14.8	5.2	No Int. Sol.
P_{24}	—	—	—	Infeasible
P_{31}	$(2, 2)$	14	<u>6</u>	FEASIBLE SOL
P_{32}	$(0, 3)$	12	<u>8</u>	Pruned

∴ The ILP Sol. is $(2, 2)$ & Value is 6.