CSCI 5654 Fall 15
Assignment 1 - Solutions

Problem 1

1. A is invertible, so let $x^* = A^{-1}b$, which satisfies $Ax^* \leq b$.

2.

(a) No. $2x_1 + 2x_2 \leq -2$ contradicts with the constraint $x \geq 0$

(b) Initial dictionary is infeasible, so introduce auxiliary problem

$$\begin{array}{ll} \text{maximize} & -x_0 \\ \text{s.t.} & 2x_1 + 2x_2 - x_0 \leq -2 \\ & -2x_1 - 2x_2 - x_0 \leq 9 \\ & x_0, x_1, x_2 \geq 0 \end{array}$$

$$\begin{aligned} \zeta &= -x_0 \\ x_3 &= -2 - 2x_1 - 2x_2 + x_0 \\ x_4 &= 9 + 2x_2 + 2x_2 + x_0 \end{aligned}$$

$$\begin{aligned} \zeta &= -2 - 2x_1 - 2x_2 - x_3 \\ x_0 &= 2 + 2x_1 + 2x_2 + x_3 \\ x_4 &= 11 + 4x_1 + 4x_2 + x_3 \end{aligned}$$

Optimal value of auxiliary problem is $-2 < 0$, so the original problem is infeasible.

(c) $Rank(A) = 1$

3.

(b)

$$\begin{aligned} \xi &= x_1 + x_2 \\ x_3 &= -x_1 + 2x_2 \\ x_4 &= 1 - x_1 \end{aligned}$$

If we choose x_2 as leaving variable, we'll find that there is no constraint on it. The problem is unbounded.

Problem 2

1.

(a)

$$\begin{array}{ll} \text{maximize} & 6x_1 + 5x_2 \\ \text{s.t.} & 3x_1 + 2x_2 \leq 24 \\ & x_1 + 2x_2 \leq 18 \\ & 2x_1 + 2x_2 \leq 20 \\ & x_1, x_2 \geq 0 \end{array}$$

(b)

$$\begin{array}{ll} \text{maximize} & 10x_1 + 7x_2 \\ \text{s.t.} & x_1 + x_2 \leq 20 \\ & -x_1 \leq -3 \\ & x_1 \leq 8 \\ & x_1, x_2 \geq 0 \end{array}$$

2.

(a) First dictionary:

$$\begin{aligned}\xi &= 6x_1 + 5x_2\\ x_3 &= 24 - 3x_1 - 2x_2\\ x_4 &= 18 - x_1 - 2x_2\\ x_5 &= 10 - x_1 - x_2\end{aligned}$$

Last dictionary:

$$\begin{aligned}\xi &= 54 - x_3 - 3x_5\\ x_1 &= 4 - x_3 + 2x_5\\ x_2 &= 6 + x_3 - 3x_5\\ x_4 &= 2 - x_3 + 4x_5\end{aligned}$$

Optimal solution is $x_1 = 4$ (# of 6 pancakes), $x_2 = 6$ (# of 5 waffles); optimal value is 54(# of people)

(b) Initial dictionary is infeasible, so introduce auxiliary problem:

$$\begin{aligned}\zeta &= -x_0\\ x_3 &= 20 - x_1 - x_2 + x_0\\ x_4 &= 8 - x_1 + x_0\\ x_5 &= -3 + x_1 + x_0\end{aligned}$$

Last dictionary of auxiliary problem:

$$\begin{aligned}\zeta &= -x0\\ x_1 &= 3 - x_0 + x_5\\ x_2 &= 17 - 2x_0 - x_2 - x_5\\ x_4 &= 5 - 2x_0 - x_5\end{aligned}$$

Optimal solution of auxiliary problem is 0, remove x_0

$$\begin{aligned}\xi &= 10x_1 + 7x_2 = 30 + 10x_5 + 7x_2\\ x_1 &= 3 + x_5\\ x_2 &= 17 - x_2 - x_5\\ x_4 &= 5 - x_5\end{aligned}$$

Last dictionary:

$$\begin{aligned}\xi &= 164 - 7x_3 - 3x_4\\ x_1 &= 8 - x_4\\ x_2 &= 12 - x_3 + x_4\\ x_5 &= 5 - x_4\end{aligned}$$

Optimal solution is $x_1 = 8$, $x_2 = 12$; optimal value is 164 (maximum weekly earning).

Problem 3

1.

(a) If (2) is feasible, we can get:

$$x_1 = 1 + x_2 \tag{3.1}$$

$$x_1 - 3x_2 + x_3 \geq 2 \tag{3.2}$$

Plug (3.1) into (3.2) we can get the result $x_3 \geq 1 + 2x_2$.

(b) Suppose (2) is feasible, the result of (a) shows $x_3 \geq 1 + 2x_2$, which contradicts with the constraint $x_3 \leq 0$. So (2) is infeasible.

2.

(a) Standard form:

$$\begin{array}{ll}\text{maximize} & -x_1 - 2x_2 - y\\ \text{s.t.} & x_1 - x_2 \leq 1\\ & -x_1 + x_2 \leq -1\\ & -x_1 + 3x_2 + y \leq -2\\ & x_1, x_2, y \geq 0\end{array}$$

(b) Initial dictionary is infeasible, so introduce auxiliary problem:

$$
\begin{aligned}
\zeta &= -x_0 \\
x_3 &= 1 - x_1 + x_2 + x_0 \\
x_4 &= -1 + x_1 - x_2 + x_0 \\
x_5 &= -2 + x_1 - 3x_2 - y + x_0
\end{aligned}
$$

Last dictionary of auxiliary problem:

$$
\begin{aligned}
\zeta &= -\frac{1}{2} - x_2 - \frac{x_3}{2} - \frac{x_5}{2} - \frac{y}{2} \\
x_0 &= \frac{1}{2} + x_2 + \frac{x_3}{2} + \frac{x_5}{2} + \frac{y}{2} \\
x_1 &= \frac{3}{2} + 2x_2 - \frac{x_3}{2} + \frac{x_5}{2} + \frac{y}{2} \\
x_4 &= 1 + 2x_2 + x_5 + y
\end{aligned}
$$

Optimal value of auxiliary problem is $-1/2 < 0$, so the original problem is infeasible.