Total:44/50

PROBLEM 1

①

gn, max. $c \cdot x$

s.t $Ax \leq b$

$x \geq 0$

A is a $n \times n$ matrix.

$c, x$ & $d$ are vectors of $R^n$

2/2 a) given A is invertible,

To prove:- The above L.P is feasible once we drop the constraint $x \geq 0$.

Proof:- For an L.P to be feasible there must be a point st. $Ax \leq b$ is satisfied.

Sub, x in $Ax = b$, with $A^{-1}b$,

we get;

$$Ax = A(A^{-1}b) = (AA^{-1})b = Ib = b.$$

Since $A^{-1}$ & there is a point that satisfies $Ax = b$, the L.P is feasible given the constraint $x \geq 0$ is dropped.

~~7~~/10 $A = \begin{pmatrix} 2 & 2 \\ -2 & -2 \end{pmatrix}$ & $b = \begin{pmatrix} -2 \\ 9 \end{pmatrix}$ ← hint

(2)

don't be misled by question 1, check the problem again....

a) The above L.P isn't feasible as $A^{-1}$ doesn't exist

b) Proof:- L.P of the above problem is infeasible using Simplex.

$$2x_1 + 2x_2 \le -2$$
$$-2x_1 - 2x_2 \le 9$$

adding slack variables,

$$x_3 = -2 - 2x_1 - 2x_2$$
$$x_4 = 9 + 2x_1 + 2x_2$$

Since this dictionary isn't feasible, we must perform Initialization.

$$x_0 = 2 + 2x_1 + 2x_2 + x_3$$
$$x_4 = 11 + 4x_1 + 4x_2 + x_3$$
$$z = -2 - 2x_1 - 2x_2 - x_3$$

Since, no entering Variable can be chosen the above auxiliary problem can't be continued. Hence, the given L.P is Infeasible.

by this step, we have found the optimal value of aux problem, it is -2, which is < 0, so the original LP is infeasible.

c) rank of $A = 1$

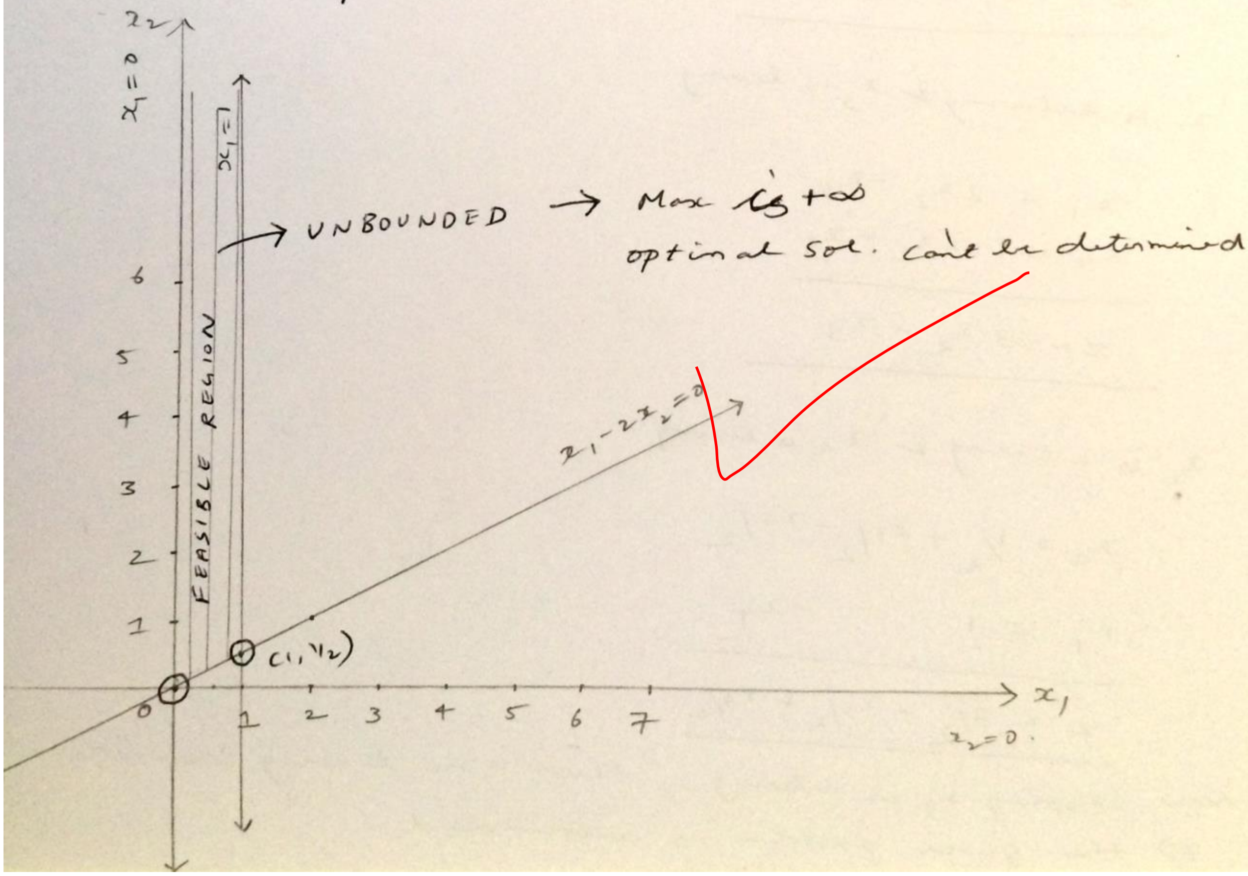
3) 8/8

gn, $A = \begin{pmatrix} 1 & -2 \\ 1 & 0 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ & $c = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

a) $x_1 - 2x_2 \leq 0$

$x_1 \leq 1$ $\quad\quad x_1, x_2 \geq 0$

max $x_1 + x_2$.



b) $x_1 - 2x_2 = 0$

$x_1 = 1$

$z = x_1 + x_2$

(4)

Adding slack variables, ~~--~~ new dictionary is

$x_3 = -x_1 + 2x_2$

$x_4 = 1 - x_1$

$z = x_1 + x_2$

'$x_1$' is entering & '$x_3$' is leaving.

$x_1 = 2x_2 - x_3$

$x_4 = 1 - 2x_2 + x_3$

$z = 3x_2 - x_3$

'$x_2$' is entering & '$x_4$' is leaving.

$x_2 = 1/2 + x_3/2 - x_4/2$

$x_1 = 1 - x_4$

$z = 3/2 + x_3/2 - x_4/2$

Now choosing '$x_3$' as entering, there is no leaving variables,
$\Rightarrow$ the given problem is unbounded.

PROBLEM 20/20



1) a) Let $x_1$ be no. of batches of pancakes.

$x_2$ be no. of batches of Waffles.

objective, $\max Z = 6x_1 + 5x_2$

Constraints
$$3x_1 + 2x_2 \leq 24$$
$$x_1 + 2x_2 \leq 18$$
$$2x_1 + 2x_2 \leq 20$$
$$x_1, x_2 \geq 0.$$

b) Let $x_1$ be no. of hours tutoring Geometry Student

$x_2$ be no. of hours baby sitting.

objective, $\max Z = 10x_1 + 7x_2$

Constraints
$$x_1 + x_2 \leq 20$$
$$x_1 \geq 3$$
$$x_1 \leq 8$$
$$x_1, x_2 \geq 0.$$

## 2) a) (i) Graphical Method

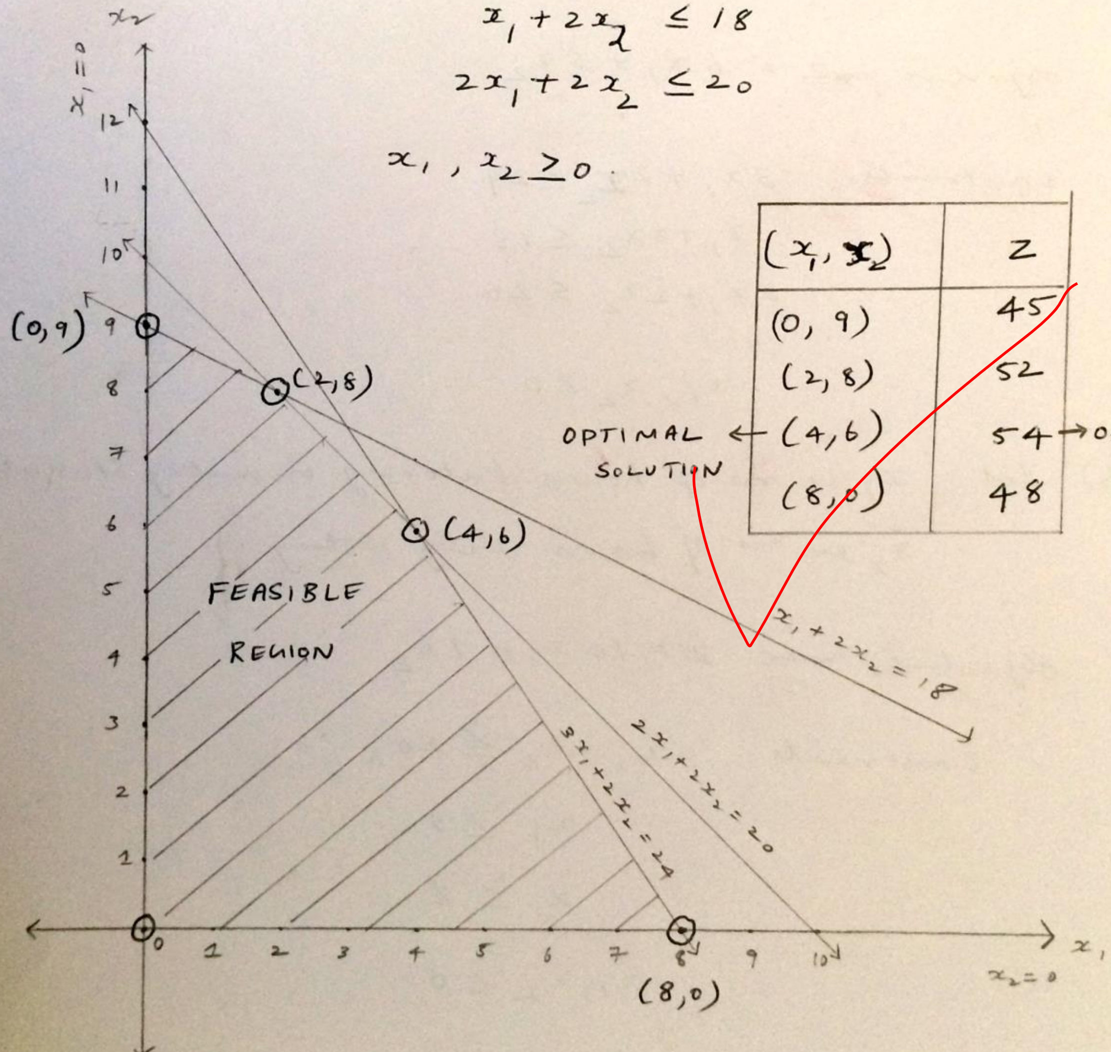
max. $Z = 6x_1 + 5x_2$

such that $3x_1 + 2x_2 \leq 24$

$x_1 + 2x_2 \leq 18$

$2x_1 + 2x_2 \leq 20$

$x_1, x_2 \geq 0.$




| $(x_1, x_2)$ | Z |
|---|---|
| (0, 9) | 45 |
| (2, 8) | 52 |
| (4, 6) | 54 |
| (8, 0) | 48 |

OPTIMAL SOLUTION ← (4, 6)

54 → OPTIMAL VALUE

(ii) Simplex Method

$$\max \quad z = 6x_1 + 5x_2$$

Such that,
$$3x_1 + 2x_2 \le 24$$
$$x_1 + 2x_2 \le 18$$
$$2x_1 + 2x_2 \le 20.$$
$$x_1, x_2 \ge 0.$$

The L.P is in standard form.

→ Adding slack variables, new dictionary is

$$\begin{aligned} x_3 &= 24 - 3x_1 - 2x_2 \\ x_4 &= 18 - x_1 - 2x_2 \\ x_5 &= 20 - 2x_1 - 2x_2 \\ \hline z &= \quad 6x_1 + 5x_2 \end{aligned}$$

'$x_1$' is entering variable & '$x_3$' is leaving variable,

new dictionary is,

$$\begin{aligned} x_1 &= 8 - 2/3\, x_2 - x_3/3 \\ x_4 &= 10 - 4/3\, x_2 + x_3/3 \\ x_5 &= 4 - 2/3\, x_2 + 2x_3/3 \\ \hline z &= 48 + x_2 - 2x_3 \end{aligned}$$

'$x_2$' is entering variable & '$x_5$' is leaving variable. (8)

new dictionary is

$$x_2 = 6 + x_3 - 3/2\, x_5$$

$$x_4 = 2 - 4/3\, x_3 + 2x_5$$

$$x_1 = 4 - x_3 + x_5$$

$$z = 54 - x_3 - 3/2\, x_5$$

Since no entering variable can be chosen, this is our final solution.

Setting nonbasic variables $x_3$ & $x_5 = 0$, we get.

$x_1 = 4$ ; $x_2 = 6$ & $z = 54$.

∴ To feed as many people as possible 24 batches of pancakes & 30 batches of waffles must be made.

optimal solution $(4, 6)$

optimal value $= 54$

b) (i) Graphical Method



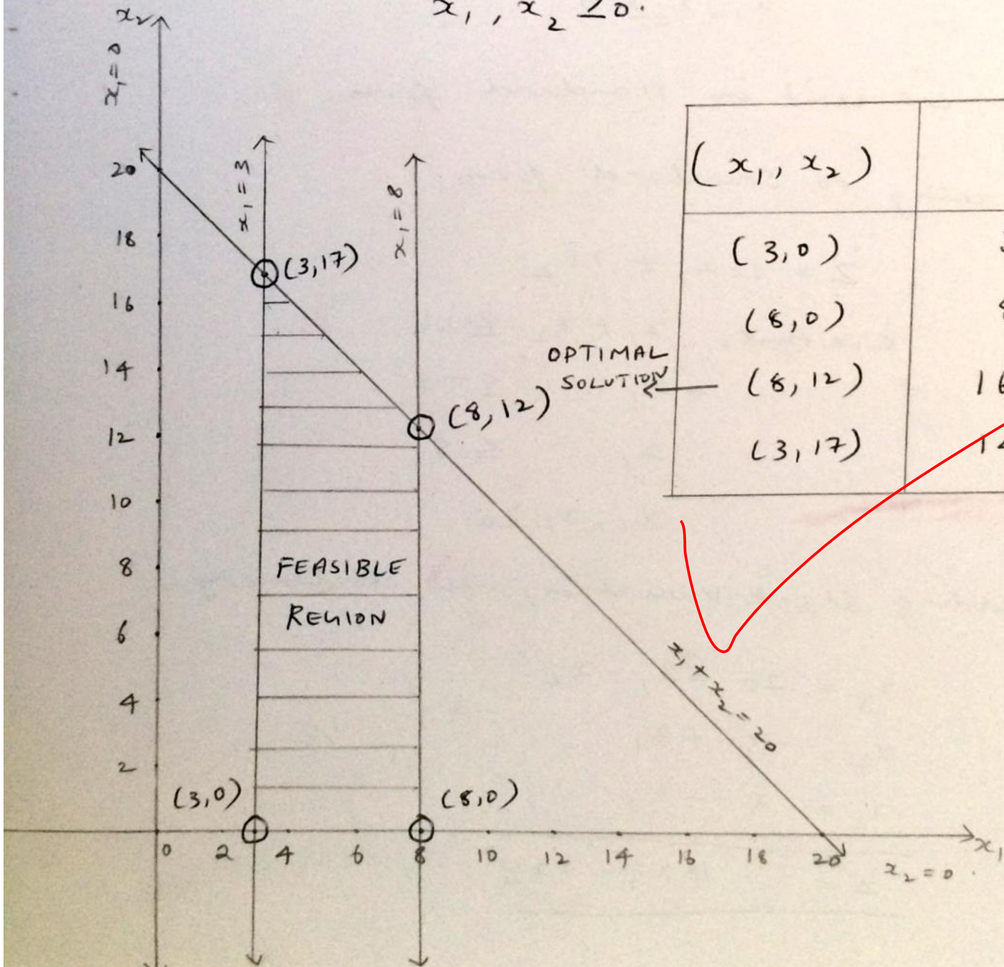
$$\max. \quad Z = 10x_1 + 7x_2$$

Such that $\quad x_1 + x_2 \le 20$

$$x_1 \ge 3$$

$$x_2 \le 8$$

$$x_1, x_2 \ge 0.$$




| $(x_1, x_2)$ | Z |
|---|---|
| (3,0) | 30 |
| (8,0) | 80 |
| (8,12) | 164 |
| (3,17) | 149 |

OPTIMAL SOLUTION ← (8,12)

164 → OPTIMAL VALUE

(ii) <u>Simplex Method</u>



$$\max \quad Z = 10x_1 + 7x_2$$

Such that, $\quad x_1 + x_2 \le 20$

$$x_1 \ge 3$$

$$x_2 \le 8$$

$$x_1, x_2 \ge 0.$$

The L.P isn't in standard form.

→ Converting to standard form,

$$Z = 10x_1 + 7x_2$$

Such that, $\quad x_1 + x_2 \le 20$

$$-x_1 \le -3$$

$$x_1 \le 8$$

$$x_1, x_2 \ge 0.$$

→ Adding slack variables, new dictionary is.

$$
\begin{aligned}
x_3 &= 20 - x_1 - x_2 \\
x_4 &= -3 + x_1 \\
x_5 &= 8 - x_1 \\
\hline
z &= 10x_1 + 7x_2 \\
\hline
\end{aligned}
$$

Since the initial dictionary is infeasible we must first convert to a feasible one by performing initialization & solving the auxiliary problem. 

<u>Initialization</u>

$$\max \; -x_0$$

such that,

$$
\begin{aligned}
x_1 + x_2 + x_3 - x_0 &= 20\\
-x_1 + x_4 - x_0 &= -3\\
x_1 + x_5 - x_0 &= 8\\
\hline
z &= -x_0
\end{aligned}
$$

$x_1, x_2, x_3, x_4, x$

$x_0 \geq 0$

new dictionary is,

$$
\begin{aligned}
x_0 &= 3 - x_1 + x_4\\
x_3 &= 23 - 2x_1 - x_2 + x_4\\
x_5 &= 11 - 2x_1 + x_4\\
\hline
z &= -3 + x_1 - x_4
\end{aligned}
$$

'$x_1$' is the entering variable & '$x_0$' is the leaving variable; the new dictionary is:

$$x_1 = 3 + x_4 - x_0$$
$$x_3 = 17 - x_2 - x_4 + 2x_0$$
$$x_5 = 5 - x_4 + x_0$$

---

$$z = -x_0$$

---

Since the z has reached optimum value, the auxiliary problem is solved. Now, since we have got a feasible dictionary we can start solving, new dictionary is after removing $x_0$,

$$x_1 = 3 + x_4$$
$$x_3 = 17 - x_2 - x_4$$
$$x_5 = 5 - x_4$$

---

$$z = 30 + 7x_2 + 10x_4$$

---

'$x_4$' is the entering variable & '$x_5$' is the leaving variable, new dictionary is .

$$x_4 = 5 - x_5$$
$$x_1 = 8 - x_5$$
$$x_3 = 12 - x_2 + x_5$$

---

$$z = 80 + 7x_2 - 10x_5$$

'$x_2$' is the entering variable & '$x_3$' is the leaving (1
Variable; new dictionary is

$$
\begin{aligned}
x_2 &= 12 - x_3 + x_5 \\
x_4 &= 5 \qquad\quad - x_5 \\
x_1 &= 8 \qquad\quad - x_5 \\
\hline
z &= 164 - 7x_3 - 3x_5
\end{aligned}
$$

Since no entering can be chosen, this is our final solution.

Setting non-basic variables $x_3$ & $x_5 = 0$, we get

$x_1 = 8$; $x_2 = 12$ & $z = 164$.

$\therefore$ Kayla should teach Geometry for 8 hours & babysit for 12 hours.

The maximum weekly earnings of Kayla is $164

optimal solution — (8, 12)

optimal value — 164

## PROBLEM 3 7/10

given, min $x_1 + 2x_2 - x_3$

s.t $x_1 - x_2 = 1 \rightarrow (1)$

$x_1 - 3x_2 + x_3 \geq 2 \rightarrow (2)$

$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \leq 0.$

1) a) To prove; if the above L.P is feasible then $x_3 \geq 1 + 2x_2$.

proof:- In order that above LP is feasible. there must exist atleast one solution solving all the constraints

Sub (1) in (2), we get.

$x_1 = 1 + x_2$ in $x_1 - 3x_2 + x_3 \geq 2$

$\Rightarrow 1 + x_2 - 3x_2 + x_3 \geq 2$

$= x_3 \geq 1 + 2x_2.$

Hence proved.

b) To prove, the above L.P is infeasible ⑮

Proof :-

From the above assumption, inorder that L.P to be feasible

$x_3 \geq 1 + 2x_2 . \rightarrow ①$

but, we know that $x_2 \geq 0$ & $x_3 \leq 0$

$\Rightarrow$ $x_3$ is Zero or negative.

whereas $x_2$ is zero (or) positive.

$\Rightarrow$ assuming $x_3 = 0$ & $x_2 = 0$ & sub in ①

we get $0 \geq 1$, which isn't possible

$\Rightarrow$ That the given L.P is infeasible

2) a) standard form of,

$$\begin{aligned} \min \quad & x_1 + 2x_2 - x_3 \\ \text{s.t} \quad & x_1 - x_2 = 1 \\ & x_1 - 3x_2 + x_3 \geq 2 \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \leq 0 \end{aligned}$$

Standard form

let $y = -x_3$.

So the L.P is

$$\left.\begin{aligned} \max \quad & -x_1 - 2x_2 + (-y) \\ \text{st} \quad & x_1 - x_2 \leq 1 \\ & -x_1 + x_2 \leq -1 \\ & -x_1 + 3x_2 + y \leq 2 \\ & x_1, x_2, y \geq 0. \end{aligned}\right\} \rightarrow \text{Standard Form of given L.P.}$$

$=$ ×

b) To prove, the above L.P is infeasible using Simplex algorithm.

$$\begin{aligned} \max \quad & -x_1 - 2x_2 - y \\ \text{st} \quad & x_1 - x_2 \leq 1 \\ & -x_1 + x_2 \leq -1 \\ & -x_1 + 3x_2 + y \leq 2 \\ & x_1, x_2, y \geq 0 \end{aligned}$$

Adding slack variables,

$$x_3 = 1 - x_1 + x_2$$
$$x_4 = -1 + x_1 - x_2$$
$$x_5 = 2 + x_1 - 3x_2 - y$$
$$z = -x_1 - 2x_2 - y$$

Since, we willn't be able to proceed as the dictionary is infeasible, so we must convert it to a feasible one using initialization;

new dictionary is:

$$x_0 = 1 - x_1 + x_2 + x_4$$
$$x_3 = 2 - 2x_1 + 2x_2 + x_4$$
$$x_5 = 3 \quad - 2x_2 + x_4 - y.$$
$$z = -1 + x_1 - x_2 - x_4$$

'$x_0$' is the leaving variable, & '$x_1$' is the entering vari.

$$x_1 = 1 + x_2 + x_4 - x_0.$$
$$x_3 = \quad - x_4 + 2x_0 \qquad z = -x_0.$$
$$x_5 = 3 - 2x_2 + x_4 \quad - y$$

The new feasible dictionary is 18

$$x_1 = 1 + x_2 + x_4$$
$$x_3 = -x_4$$
$$x_5 = 3 - 2x_2 + x_4 - y.$$

---

$$z = -1 - 3x_2 - x_4 - y.$$

---

Since we aren't able to find an entering variable, the given LP is infeasible.

by this step, we have found the optimal value of aux problem, it is -1, which is < 0, so the original LP is infeasible.