CSCI 5654-Fall15 Assignment 4.
Assigned date: Saturday 10/24/2015,
Due date: Saturday 10/31/2015 (midnight).

Problem 1

Our goal is to solve the following problem using a well known Farkas lemma:

$$\text{Find } \vec{v} \text{ s.t. } (\forall\ \vec{x} \geq 0)\ A\vec{x} \leq \vec{b}\ \Rightarrow\ \vec{v}^{\,t} B\vec{x}\ \geq 0\,. \tag{1}$$

where A and B are given $m \times n$ and $p \times n$ matrices (respectively) and b a given vector.

1. (a) Give the size of the vectors $\vec{v}$, $\vec{x}$ and $\vec{b}$?
   (b) Give a solution to problem (1) .
   (c) Using an optimization problem P, reformulate the problem (1) as follows :
   Find $\vec{v}$ such that the optimal value of P is positive.
   (d) Can you feed P to an LP solver and give a solution for (1) ?

2. For a fixed vector $\vec{v} = \vec{v_0}$, problem (1) becomes:

$$\text{Check that } (\forall\ \vec{x} \geq 0)\ A\vec{x} \leq \vec{b}\ \Rightarrow\ \vec{v_0}^{\,t} B\vec{x}\ \geq 0\,. \tag{2}$$

   (a) Give an expression of a vector c such that problem (2) becomes:

$$\begin{array}{ll} \text{minimize} & \vec{c}^{\,t}\vec{x} \\ \text{s.t.} & A\vec{x} \leq \vec{b} \\ & \vec{x} \geq 0. \end{array} \tag{3}$$

   What will be the size of $\vec{c}$?
   (b) Let $A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 0.5 & -5.5 & -2.5 \\ 0.5 & -1.5 & -0.5 \\ 1 & 0 & 0 \end{pmatrix}$, $b = \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$ and $v_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.
   Check if v_0 satisfies problem (2)

3. (a) Write the dual problem associated to (3).
   (b) Replace $\vec{c}$ using its expression w.r.t $\vec{v}$. Can this problem be solved using an LP solver?
   (c) Deduce that $\vec{v}$ is a solution to the problem (1) if there exist multipliers $\vec{\lambda} \geq 0$ such that

$$A^t\vec{\lambda} + B^t\vec{v} \geq 0 \text{ and } \vec{b}^{\,t}\vec{\lambda}\ \leq 0.$$

   (d) Explain then write a matlab program using 'Linprog' that solve problem (1) while finding the biggest $\vec{v}$ w.r.t to the L1 norm (inputs: matrices A,B, vector b. output: v).

Problem 2

In a grocery store, we measure the waiting time average y (in minutes) in function of the number of available cashiers x.

We obtain the following table :

x	3	4	5	6	8	10	12
y	16	12	9.6	7.9	6	4.7	4

1. Let's consider an affine model $f(x) = ax + b$.
   (a) Can we find f that can exactly fit all the data points .
   (b) Describe an LP that find the best model f w.r.t to $L1$ and L_∞ norm.
   (c) Write a matlab program that solve the LP (using Linprog) and plot a figure with the data points and the two models.
   (d) Add the following data point $(x, y) = (15, 8.5)$ in a new figure and update the two models. Give an interpretation to this update ?
2. Let's consider now the polynomial model $f(x) = a_1 x + a_2 x^2 + b$.
   (a) Describe an LP that find the best model f w.r.t to $L1$ and L_∞ norm.
   (b) Write the corresponding matlab program and plot a new figure (as previously).
   (c) Compare the performance of the affine and the polynomial model for each norm.
3. Write a matlab program that handle polynomial models with up to a degree N that will be given as an input.