CSCI 5654 Fall 15
Assignment 4 - Solutions

Problem 1

1.

(a) $\vec{v} : p \times 1$; $\vec{x} : n \times 1$; $\vec{b} : m \times 1$

(b) $\vec{v} = 0$

(c)

$$\begin{array}{ll} \text{minimize} & \vec{v}^T B \vec{x} \\ \text{s.t.} & A\vec{x} \leq \vec{b} \\ & \vec{x} \geq 0 \end{array}$$

(d) No. The objective function has two variables in a multiplication, which is not linear.

2.

(a) $\vec{c} = B^T \vec{v}$; $\vec{c} : n \times 1$

(b) No

3.

(a)

$$\begin{array}{ll} \text{maximize} & -\vec{b}^T \vec{y} \\ \text{s.t.} & -A^T \vec{y} \leq \vec{c} \\ & \vec{y} \geq 0 \end{array}$$

(b)

$$\begin{array}{ll} \text{maximize} & -\vec{b}^T \vec{y} \\ \text{s.t.} & -A^T \vec{y} \leq B^T \vec{v} \\ & \vec{y} \geq 0 \end{array}$$

Yes.

(c) $A^T \vec{\lambda} + B^T \vec{v} \geq 0$ and $\vec{\lambda} \geq 0$, so $\vec{\lambda}$ is a feasible solution of the dual problem. Because $\vec{b}^T \vec{\lambda} \leq 0$, $-\vec{b}^T \vec{\lambda} \geq 0$, the optimal solution of the dual problem is ≥ 0, so the optimal solution of the primal problem is ≥ 0. Thus $(\forall \vec{x} \geq 0) A\vec{x} \leq \vec{b} \Rightarrow \vec{v}^T B \vec{x} \geq 0$.

(d) The LP is formulated as follows:

$$\begin{array}{ll} \text{maximize} & \sum_{i=1}^{p} t_i \\ \text{s.t.} & -A^T \vec{\lambda} - B^T \vec{v} \leq 0 \\ & \vec{b}^T \vec{\lambda} \leq 0 \\ & \vec{v} - \vec{t} \leq 0 \\ & -\vec{v} - \vec{t} \leq 0 \\ & \vec{\lambda}, \vec{t} \geq 0 \end{array}$$

$$M = \begin{pmatrix} -A^T & -B^T & 0 \\ \vec{b}^T & 0 & 0 \\ 0 & I_p & -I_p \\ 0 & -I_p & -I_p \end{pmatrix}$$

Problem 2

1.

(a) No

(b) L_1:

$$\begin{aligned}
\text{maximize} \quad & \sum_{i=1}^{7} t_i \\
\text{s.t.} \quad & |3a + b - 16| \leq t_1 \\
& |4a + b - 12| \leq t_2 \\
& |5a + b - 9.6| \leq t_3 \\
& |6a + b - 7.9| \leq t_4 \\
& |8a + b - 6| \leq t_5 \\
& |10a + b - 4.7| \leq t_6 \\
& |12a + b - 4| \leq t_7 \\
& t_i \geq 0
\end{aligned}$$

L_∞:

$$\begin{aligned}
\text{maximize} \quad & t \\
\text{s.t.} \quad & |3a + b - 16| \leq t \\
& |4a + b - 12| \leq t \\
& |5a + b - 9.6| \leq t \\
& |6a + b - 7.9| \leq t \\
& |8a + b - 6| \leq t \\
& |10a + b - 4.7| \leq t \\
& |12a + b - 4| \leq t \\
& t \geq 0
\end{aligned}$$

2.

(a) L_1:

$$\begin{aligned}
\text{maximize} \quad & \sum_{i=1}^{7} t_i \\
\text{s.t.} \quad & |3a_1 + 9a_2 + b - 16| \leq t_1 \\
& |4a_1 + 16a_2 + b - 12| \leq t_2 \\
& |5a_1 + 25a_2 + b - 9.6| \leq t_3 \\
& |6a_1 + 36a_2 + b - 7.9| \leq t_4 \\
& |8a_1 + 64a_2 + b - 6| \leq t_5 \\
& |10a_1 + 100a_2 + b - 4.7| \leq t_6 \\
& |12a_1 + 144a_2 + b - 4| \leq t_7 \\
& t_i \geq 0
\end{aligned}$$

L_∞:

$$\begin{aligned}
\text{maximize} \quad & t \\
\text{s.t.} \quad & |3a_1 + 9a_2 + b - 16| \leq t \\
& |4a_1 + 16a_2 + b - 12| \leq t \\
& |5a_1 + 25a_2 + b - 9.6| \leq t \\
& |6a_1 + 36a_2 + b - 7.9| \leq t \\
& |8a_1 + 64a_2 + b - 6| \leq t \\
& |10a_1 + 100a_2 + b - 4.7| \leq t \\
& |12a_1 + 144a_2 + b - 4| \leq t \\
& t \geq 0
\end{aligned}$$