CSCI 5654-Fall15 Assignment 5.
Assigned date: Friday 11/13/2015,
Due date: Friday 11/20/2015 (midnight).

Problem 1

The goal is to able to formulate problems using ILP.

1. A firm has n projects that it would like to undertake but because of budget limitations not all can be selected. In particular project j is expected to produce a revenue of c_j but requires an investment of a_{ij} in the time period i for $i \in \{1, ...m\}$. The capital available in time period i is b_i. The problem is then to maximize the revenue subject to the budget constraints.

2. A company has selected m possible sites for distribution of its products in a certain area. There are n customers in the area and the transport cost of supplying the whole of customer j's requirements over the given planning period from potential site i is c_{ij}. Should site i be developed it will cost f_i to construct a depot there. Which sites should be selected to minimize the total construction plus transport cost?

3. There are six cities in region R. The region must determine where to build fire stations. The region wants to build the minimum number of fire stations and ensure that at least one fire station is near 15 minutes of each city. The times (in minutes) required to drive between cities are:

	1	2	3	4	5	6
1	0	10	20	30	30	20
2	10	0	25	35	20	10
3	20	25	0	15	30	20
4	30	35	15	0	15	25
5	20	20	30	15	0	14
6	20	10	20	25	14	0

Problem 2

Consider the following ILP:

$$\begin{array}{ll} \text{maximize} & 3x_1 + 4x_2 \\ \text{s.t.} & \frac{2}{5}x_1 + x_2 \leq 3 \\ & \frac{3}{5}x_1 - \frac{2}{5}x_2 \leq 1 \\ & x \geq 0 \ , \ x \in \mathbb{Z}. \end{array} \tag{1}$$

The goal is to solve the previous ILP using the cutting plane method.

1. (a) Solve graphically the LP relaxation of (1) and the ILP.
   (b) Write the ILP (1) in a standard form.
   (c) Use the simplex method with the biggest coefficient rule for the entering variable to solve the LP relaxation of (1).

2. (a) Using Gomory cuts, show how a cut can be found based on x_1 row.

(b) Discuss the feasibility of the final dictionary after adding this cut.

(c) Discuss the feasibility of the associated dual.

(d) Proceed using the dual dictionary. Does the final dictionary solves the ILP (1).

3. (a) Find the previous result while using the initialization phase.

(b) Solve the ILP (1) (Hints: two more cuts using the row x_2 are needed).

(c) Plot the different cuts and check that the optimal value of (1) is indeed a vertex of the resulted feasible set.

Problem 3

Consider the following ILP:

$$\begin{array}{ll} \text{minimize} & 20 - 3x_1 - 4x_2 \\ \text{s.t.} & \frac{2}{5}x_1 + x_2 \leq 3 \\ & \frac{2}{5}x_1 - \frac{2}{5}x_2 \leq 1 \\ & x \geq 0 \ , \ x \in \mathbb{Z}. \end{array} \tag{2}$$

The goal is to solve the previous ILP using the Branch and Bound method.

(a) Solve graphically the LP relaxation of (2) and the ILP.

(b) Using the simplex method and the following tree, solve the problem while giving for each node the reason why it should or should not be expanded.