CSCI 5654-Fall15 Assignment 2.
Assigned date: Friday 09/18/2015,
Due date: Friday 09/25/2015 (midnight).

Problem 1

Consider the following LP:

$$\begin{array}{ll} \text{maximize} & c \cdot x \\ \text{s.t.} & Ax \leq b \\ & x \geq 0. \end{array} \tag{1}$$

1. Let $b = 0$ i.e all the components are equal to zero.

   (a) Prove that (1) is feasible and deduce a lower bound.

   (b) Prove that (1) is degenerate.

   (c) Show that the feasible set has to be unbounded or reduced to a single point of $\mathbb{R}^n$.

   (d) Deduce that (1) is unbounded or its optimal value is zero.

   (e) Find the previous result using the duality theory.

2. Let $A = \begin{pmatrix} 0.5 & -5.5 & -2.5 & +9 \\ 0.5 & -1.5 & -0.5 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$, $b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ and $c = \begin{pmatrix} 10 \\ -57 \\ -9 \\ -24 \end{pmatrix}$

   (a) Write the initial dictionary and prove that it is feasible and degenerate.

   (b) Using the largest coefficient rule for the entering variable and the smallest index subscript for the leaving variable, prove that simplex method cycles after 6 iterations.

   (c) Solve the problem using the Brandt (Lexicographic) rule.

Problem 2

Let's consider the following LP:

$$\begin{array}{ll} \text{maximize} & 2x_1 + 3x_2 - 5x_3 \\ \text{s.t.} & x_1 - x_2 \leq 5 \\ & -x_1 + x_3 \leq 6 \\ & -2x_1 + x_3 \leq 2 \\ & -x_1 + x_2 \leq 4 \\ & x_1, x_2, x_3 \geq 0. \end{array} \tag{2}$$

1. Check the feasibility of the dual problem without computing it.

2. Compute the dual of (2) and find the previous result.

3. Prove the result in the general case (1) .

4. Is the inverse true? Hint: use the following LP:

$$\begin{array}{ll} \text{maximize} & 2x_1 - x_2 \\ \text{s.t.} & x_1 - x_2 \leq 1 \\ & -x_1 + x_2 \leq -2 \\ & x_1, x_2 \geq 0. \end{array}$$

Problem 3

Consider the following LP:

$$\begin{array}{ll} \text{maximize} & -x_1 - 2x_2 \\ \text{s.t.} & -2x_1 + 7x_2 \leq 6 \\ & -3x_1 + x_2 \leq -1 \\ & 9x_1 - 4x_2 \leq 6 \\ & x_1 - x_2 \leq 3 \\ & 7x_1 - 3x_2 \leq 6 \\ & -5x_1 + 2x_2 \leq -3 \\ & x_1 \geq 0,\ x_2 \geq 0. \end{array} \tag{3}$$

1. (a) Compute the dual form of (3).
   (b) Is it easier to solve the primal or the dual ? Give the optimal solution and the optimal value of (3) without using simplex.
2. (a) Compute the initial dictionaries associated to (3) its dual.
   (b) Discuss the feasibility of the two initial dictionaries and give their associated matrices.
   (c) What is the relation between the two matrices.
3. (a) Solve (3) using the simplex method.
   (b) Deduce the optimal solution for the dual problem.
   (c) Check the primal-dual certificates in order to verify your result.