Network Analysis and Modelling - CSCI 5352

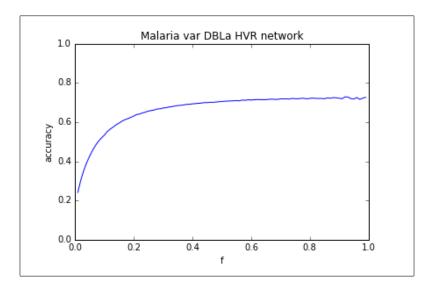
Problem Set 5

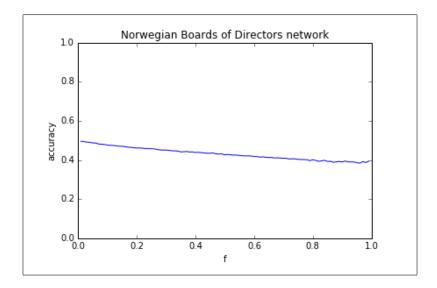
Santhanakrishnan Ramani

Tuesday 1st November, 2016

Problem 1

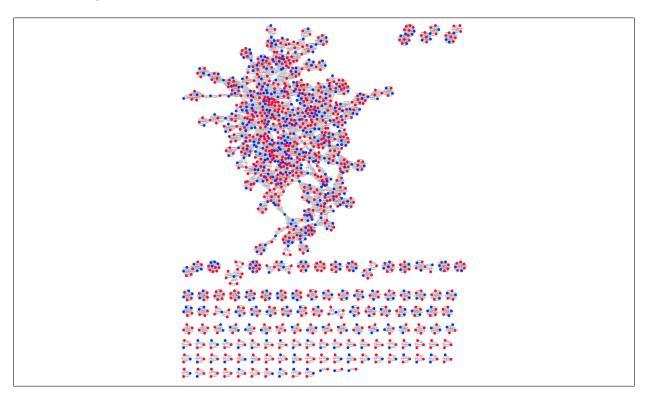
(a) The plots below shows us the accuracy of the GbA heuristic as a function of the fraction $f \in (0,1)$ of the empirical labels observed, chosen uniformly at random for both the Norwegian Boards of Directors and Malaria var DBLa HVR networks. From the plots below we can see that GbA heuristic to predict the missing labels, performs well in Malaria network but not in the Norwegian network.

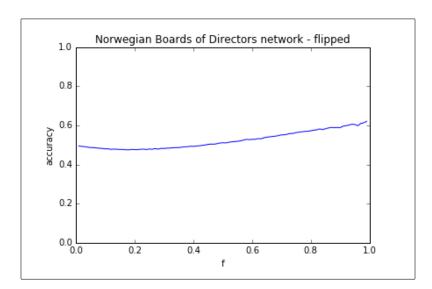




In the Malaria network we can see that as the fraction of observed label increases the accuracy in predicting missing labels increases which implies that there is assortative mixing in the network, because that's when the GbA heuristic works the best. But, still the max accuracy achieved is only about 0.7 which could be attributed to the fact that the node is assigned a label randomly when there is a tie.

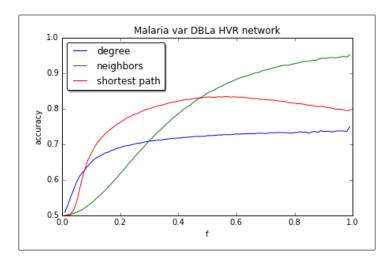
In the case of Norwegian network, we can see that there is a decrease in the accuracy as the fraction of observed labels increases, which hints us that there isn't much assortative mixing in the network as expected. Which became more conclusive when I visualized the network. The below image is the visualization of the network where the red represents male and blue represents female. We could see that there exists many small individual isolated cliques and can see that females are more central in the network. The number of male-female connections or vice versa is more compared to that of male-male or female-female connections. The above two reasons can be attributed to why the prediction goes wrong when using the GbA heuristic.

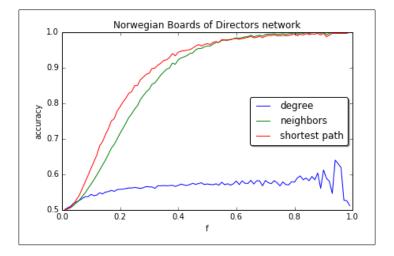




In order to verify the disassortative mixing pattern in the network, I assigned the nodes with missing label a label that is the lowest in the distribution of non-missing labels observed among it's nearest neighbors instead of the mode in the GbA heuristic. The results of it can be seen in the flipped plot above, and we can see that there is an increase in the accuracy as fraction of observed labels increases.

(b) The plots below shows the accuracy defined as AUC of the three heuristics namely degree product, normalized common neighbours, shortest path as a function of the fraction $f \in (0,1)$ of the edges observed for both the Norwegian Boards of Directors and Malaria var DBLa HVR networks averaged over 100 and 10 iterations for each value of f respectively. We can clearly see that the neighbour and shortest path heuristic perform better than the degree heuristic in predicting the missing edges in both the networks.





While using the normalized common neighbours heuristic, we can clearly see that the accuracy keeps increasing for both the networks as the fraction of edges considered in the train set increases. This can be attributed to the fact that the malaria network is a single connected component, and the Norwegian network contains many small cliques and a large connected component, so more the number of edges observed more the probability of having common neighbours, and the prediction accuracy increases.

While using the shortest path heuristic, we can see that the accuracy increases faster initially and starts stabilizing with further increase in the value of f in the malaria network, whereas the accuracy keeps increasing with the increase in the value of f in the Norwegian network. The behaviour in the Malaria network, can be attributed to the fact that the network is connected, so the accuracy increases rapidly the until emergence of a Giant component in the network and stops increasing after that. But since the Norwegian network contains many small isolated cliques, so more the number of observed edges better the accuracy in predicting the missing edge.

While using the degree product heuristic, we can see that the accuracy increases with the increase in the value of f for both the networks, but the accuracy achieved isn't that great. This heuristic assumes that nodes with higher degree connect with each other. The low accuracy in the Norwegian network is due to the presence of many isolated cliques, as the heuristics assumes that there exists an edge between nodes having higher degree in different cliques but in reality it doesn't exist. The low accuracy in the Malaria dataset is because it becomes difficult to differentiate between the degree product of different pairs after the emergence of giant component.

Problem 2

The tables below lists the top 10 nodes of the moderately sized Caltech36 and Reed98 networks from the facebook100 network, ordered by their spreading centrality along with their degree.

Table 1: Caltech36 Network - Transmission Probability 0.023

$node_id$	spreading centrality	degree
709	269.98	248
90	256.521	203
623	255.168	160
664	255.086	184
257	253.063	172
223	252.137	194
278	251.56	171
687	251.026	160
626	246.128	156
377	245.254	144

Table 2: Reed98 Network - Transmission Probability 0.025

node_id	spreading centrality	degree
679	342.011	313
873	340.101	213
889	334.342	211
646	330.438	183
147	329.819	182
328	323.153	154
562	321.744	175
230	320.066	155
293	318.884	158
236	318.744	154

From the tables above, we can clearly see that the spreading centrality of a node in a network has a direct dependence on the degree of the node where the epidemic starts. Which implies that, higher the degree of the node which gets infected higher the chances that epidemics spreads and vice versa. It is very much evident in the two networks used, even though the transmission probability is very low for the two networks, still we could see a max of around 40% of the network being affected when the node that gets infected first is the node with the highest degree.

Code for Problem 1a

```
from __future__ import division
import random
import csv
import networks as nx
from collections import defaultdict
from collections import Counter
dataPath = "/home/santa/Dropbox/NAM/Problem_Set_5/"
files = [("net1m_2011 - 08 - 01.txt", "data_people.txt"), ("HVR_5.txt", "data_people.txt", "data_peo
        metadata_CysPoLV.txt")]
for (filename1, filename2) in files:
         lines = [line.rstrip('\n') for line in open(dataPath+"data/"+filename1)]
         G = nx.Graph()
         for line in lines:
                   vertexes = line.split(" \ t")
                  G. add_edge(int(vertexes[0]), int(vertexes[1]))
                  G. add_node(int(vertexes[0]))
                  G. add_node(int(vertexes[1]))
         lines = [line.rstrip('\n') for line in open(dataPath+"data/"+filename2)]
         labels = defaultdict(int)
         if filename1.startswith("net1m"):
                   for line in lines [1:]:
                            node_id = int(line.split("_\"")[0])
                            label = int(line.split("\""")[1])
                            if node_id in G.nodes():
                                     labels[node\_id] = label
         else:
                  cnt = 1
                   for line in lines:
                            if cnt in G.nodes():
                                     labels [cnt] = int(line.strip())
                            cnt += 1
         labels_set = set(labels.values())
         no\_of\_iter = 200
         no\_of\_nodes = G.number\_of\_nodes()
         fp = open(dataPath+"Code/prob1a_"+filename1.split("_")[0]+".csv", 'a')
         writer = csv.writer(fp)
         writer.writerow(("f", "acc"))
         for f in range(1, 100):
                  print f
                   f /= 100
                   train_size = int(f*no_of_nodes)
                   avg\_accuracy = 0
                   for i in range(no_of_iter):
                            train_set = defaultdict(int)
                            test_set = defaultdict(int)
                            while len(train_set) <= train_size:
                                     key = int(random.choice(labels.keys()))
                                     train_set [key] = int(labels [key])
                            for node in G. nodes():
                                     if not int(node) in train_set:
                                               edges = G. edges (node)
                                               lst = []
                                               for edge in edges:
                                                        if edge [1] in train_set:
                                                                 lst.append(train_set[edge[1]])
```

```
data = Counter(lst)
                     max\_count = 0 if len(data) == 0 else max(data.values())
                     mode = [k for k, v in data.items() if v == max_count]
                     if len(mode) == 1:
                         test\_set[node] = int(mode[0])
                     else:
                         if len(mode) != 0:
                             test_set [node] = int(random.choice(mode))
                             test_set [node] = int(random.sample(labels_set, 1)
            cnt = 0
            for key in test_set.keys():
                 if test_set[key] == labels[key]:
                     cnt += 1
            accuracy = cnt/len(test_set)
            avg_accuracy += accuracy
        writer.writerow((f, (avg_accuracy/no_of_iter)))
    fp.close()
import matplotlib.pyplot as plt
import pandas as pd
files = ['net1m', 'HVR', 'flip_netm']
title = \{\}
title ['net1m'] = 'Norwegian_Boards_of_Directors_network'
title ['HVR'] = 'Malaria_var_DBLa_HVR_network'
title ['flip_net1m'] = 'Norwegian_Boards_of_Directors_network_-_flipped'
for filename in files:
    csv_file = "/home/santa/Dropbox/NAM/Problem_Set_5/Code/prob1a_"+filename+".
       csv"
    df = pd.read_csv(csv_file)
    x = df['f'].tolist()
    y = df['acc'].tolist()
    plt.plot(x, y)
plt.xlabel('f')
    plt.ylim([0, 1])
    plt.ylabel('accuracy')
    plt.title(title[filename])
    plt.savefig("/home/santa/Dropbox/NAM/Problem_Set_5/Latex/images/1a_"+
       filename+".png")
    plt.clf()
```

Code for Problem 1b

```
from __future__ import division
import csv
import random
import networkx as nx
import numpy as np
from sklearn import metrics

def calc_degree_score(graph, u, v, degree_score_set):
    degree = graph.degree(u) * graph.degree(v)
    if degree in degree_score_set:
        degree += random.uniform(0, 1)/no_of_nodes
    return degree

def calc_neighbor_score(graph, u, v, neighbor_score_set):
```

```
i_neighbor = set (graph.neighbors (u))
        j_n = ighbor = set(graph.neighbors(v))
        union = len(i_neighbor | j_neighbor)
        intersection = len(i_neighbor & j_neighbor)
        neighbor = intersection/union if union != 0 else 0
        if neighbor in neighbor_score_set:
                 neighbor += random.uniform(0, 1) / no_of_nodes
        return neighbor
def calc_path_score(graph, u, v, path_score_set):
                 path = 1/nx.shortest_path_length(graph, source=u, target=v)
        except:
                 path = 0
        if path in path_score_set:
                 path += random.uniform (0, 1) / no_of_nodes
        return path
dataPath = "/home/santa/Dropbox/NAM/Problem_Set_5/"
files = [("net1m_2011 - 08 - 01.txt", "data_people.txt"), ("HVR_5.txt", "data_people.txt", "data_people.txt"), ("HVR_5.txt", "data_people.txt", "data_peo
       metadata_CysPoLV.txt")]
for (filename1, filename2) in files:
        lines = [line.rstrip('\n') for line in open(dataPath+"data/"+filename2)]
        tot_no_of_nodes = len(lines)
        if filename1.startswith("data"):
                 tot_no_of_nodes = 1
        A_Full = np.zeros(shape=(tot_no_of_nodes, tot_no_of_nodes))
        edges = []
        nodes = set()
        lines = [line.rstrip('\n') for line in open(dataPath+"data/"+filename1)]
        for line in lines:
                 vertexes = line.split("\t")
                 edges.append((int(vertexes[0]), int(vertexes[1])))
                 nodes.add(int(vertexes[0]))
                 nodes.add(int(vertexes[1]))
                 A_Full[int(vertexes[0]) - 1][int(vertexes[1]) - 1] = 1
                 A_Full[int(vertexes[1]) - 1][int(vertexes[0]) - 1] = 1
        no\_of\_edges = len(edges)
        node_list = list (nodes)
        no\_of\_nodes = len(node\_list)
        fp = open(dataPath+"Code/prob1b_"+filename1.split("_")[0]+".csv", "wt")
        writer = csv.writer(fp)
        writer.writerow(("f", "d", "n", "p"))
        fp.close()
        no\_of\_iter = 10
        for f in range (1, 100):
                 print f
                 f /= 100
                 train\_size = int(f*no\_of\_edges)
                 degree\_accuracy = 0
                 neighbor_accuracy = 0
                 path\_accuracy = 0
                 for itr in range(no_of_iter):
                         train_set = random.sample(edges, train_size)
                         G = nx.Graph()
                         G. add_edges_from (train_set)
                         G. add_nodes_from (node_list)
```

```
A = np.zeros(shape=(tot_no_of_nodes, tot_no_of_nodes))
            for (x, y) in train_set:
                A[x-1][y-1] = 1
                A[y-1][x-1] = 1
            y_true = []
            y_degree_score = []
            y_degree_score_set = set()
            y_neighbor_score = []
            y_neighbor_score_set = set()
            y_path_score = []
            y_path_score_set = set()
            for x in range(0, no_of_nodes):
                i = node_list[x]
                for y in range(x+1, no_of_nodes):
                    j = node_list[y]
                    if A[i-1][j-1] == 0:
                        if A_Full[i-1][j-1] == 1:
                             y_true.append(1)
                        else:
                             y_{true}. append (0)
                        degree_score = calc_degree_score(G, i, j,
                            y_degree_score_set)
                        y_degree_score.append(degree_score)
                        y_degree_score_set.add(degree_score)
                         neighbor_score = calc_neighbor_score(G, i, j,
                            y_neighbor_score_set)
                        y_neighbor_score.append(neighbor_score)
                        y_neighbor_score_set.add(neighbor_score)
                        path_score = calc_path_score(G, i, j, y_path_score_set)
                        y_path_score.append(path_score)
                        y_path_score_set.add(path_score)
            y_degree_score = np.asarray(y_degree_score)
            y_degree_score /= sum(y_degree_score)
            y_neighbor_score = np.asarray(y_neighbor_score)
            y_neighbor_score /= sum(y_neighbor_score)
            y_path_score = np.asarray(y_path_score)
            y_path_score /= sum(y_path_score)
            degree_accuracy += metrics.roc_auc_score(y_true, y_degree_score)
            neighbor_accuracy += metrics.roc_auc_score(y_true, y_neighbor_score)
            path_accuracy += metrics.roc_auc_score(y_true, y_path_score)
        fp = open(dataPath+"Code/prob1b_"+filename1.split("_")[0]+".csv", "a")
        writer = csv.writer(fp)
        writer.writerow((f, (degree_accuracy/no_of_iter), (neighbor_accuracy/
            no_of_iter), (path_accuracy/no_of_iter)))
        fp.close()
import matplotlib.pyplot as plt
import pandas as pd
files = ['net1m', 'HVR']
title = \{\}
title ['net1m'] = 'Norwegian_Boards_of_Directors_network'
title ['HVR'] = 'Malaria_var_DBLa_HVR_network'
for filename in files:
    csv_file = "/home/santa/Dropbox/NAM/Problem_Set_5/Code/prob1b_"+filename+".
```

Code for Problem 2 - EC

```
from __future__ import division
import random
import networks as nx
import csv
from collections import defaultdict
dataPath = "/home/santa/Dropbox/NAM/Problem_Set_5/"
files = ["Caltech36", "Reed98"]
for filename in files:
    lines = [line.rstrip('\n') for line in open(dataPath+"data/"+filename+".txt"
       ) ]
    G = nx.Graph()
    for line in lines:
        vertexes = line.split("\t")
        G. add_edge(vertexes[0], vertexes[1])
    avg_deg = sum(list(G.degree(G.nodes()).values()))/G.number_of_nodes()
    p = 1/avg_deg
    no\_of\_iter = 1000
    f = open(dataPath+"Code/"+filename+".csv", 'a')
    writer = csv.writer(f)
    writer.writerow(("node_id", "spreading_centrality", "degree"))
    for i in G. nodes():
        print i
        epidemic_size = 0
        for j in range(no_of_iter):
            infected_list = defaultdict(int)
            infected_list[i] = 1
            lst = [i]
            while len(lst):
                newList = []
                for infected_node in lst:
                     edges = G. edges (infected_node)
                     for edge in edges:
                         if random.uniform(0, 1) \le p:
                             if infected_list[edge[1]] != 1:
                                 newList.append(edge[1])
                                 infected_list[edge[1]] = 1
                lst = newList
            epidemic_size += len(infected_list)
        writer.writerow((i, (epidemic_size/no_of_iter), G.degree(i)))
    f.close()
```