Probabilistic Models of Human and Machine Learning
CSCI 7222
Assignment 7

Assigned 11/3/2013
Due 11/12/2013

Goal

The goal of this assignment is to give you a first-hand appreciation of the Dirichlet process mixture model. You will use the Chinese restaurant process for computing probabilities and sampling.

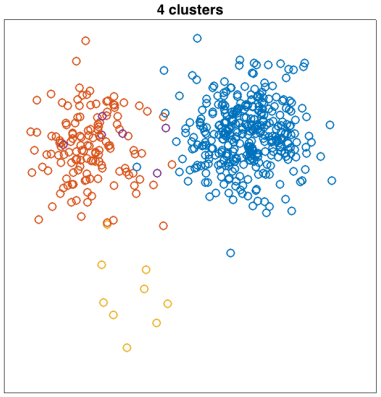
Task 1

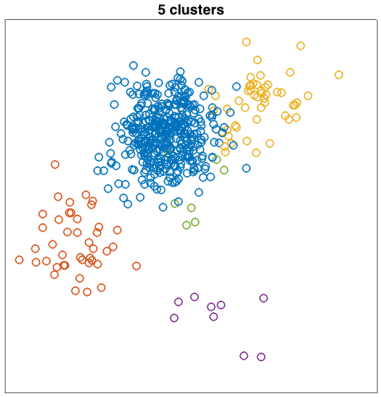
Make a graph showing the probability that a new customer will be assigned to an empty table in the CRP, as a function of the number of customers already in the restaurant. For example, when there are 0 customers in the restaurant, the next customer -- customer 1 -- will be assigned to a new table with probability 1. Use $\alpha = 0.5$, and plot your graph for up to 500 customers.

Hint: If you're thinking about running a sampler, you should think again. You can compute this probability analytically.

Task 2

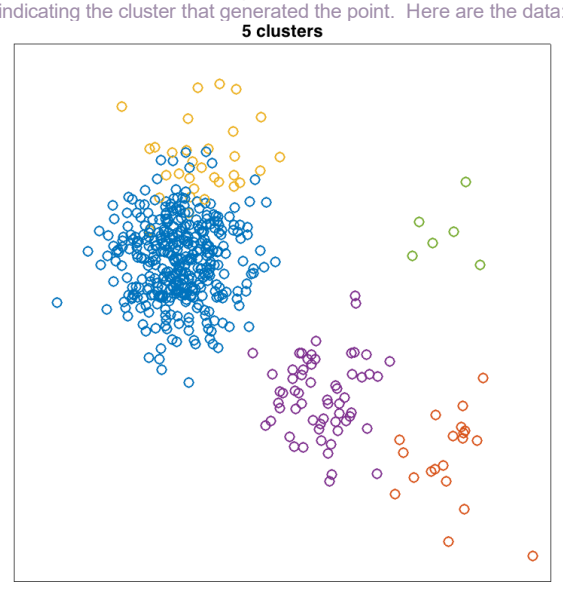
Using the CRP, draw samples from a Dirichlet process mixture model with Gaussian components in a 2 dimensional observation space. For this draw, use $\alpha = 0.5$ and G0 = (X,Y) with X ~ Uniform(0,1) and Y ~ Uniform(0,1) specifying the Gaussian means. Use $\sigma = 0.1$ for all components. Plot some sample draws from the DPMM with 500 points. Below are 3 examples that I generated. I color coded the points to indicate the compnent that generated the observation..





Task 3 [OPTIONAL]

If you are ambitious, write a Gibbs sampler to infer the underlying clustering of a set of data points assumed to have been generated by a Dirichlet process mixture model. I have created a set of data points either in matlab format or in raw text format. Along with the (x,y) coordinates of each point is a label indicating the cluster that generated the point. Here are the data:



Running the sampler involves first assigning each customer (data point) to a table (cluster). Then you'll pick random customers, remove them from the restaurant, and draw from the CRP posterior to pick a new table for them. For both the initial assignment and the re-assignment, the posterior probability of assigning a customer to a table is proportional to product of (1) the CRP prior, which is based on table occupancies and α, and (2) the likelihood of the observed (x,y) value given estimated parameters for each table.
In my implementation, I used a maximum likelihood estimate of the table parameters, obtained by computing the (x,y) mean of all customers at a table.

The implementation requires a lot of bookkeeping. Each time a customer is removed from the restaurant for reassignment, you'll want to recompute the parameter values associated with that customer's former table, and similarly each time a customer is added to a table. You'll also have to deal with the fact that as customers shift around, occupied tables can become unoccupied. Unoccupied tables must be deleted. The CRP should allow only one unoccupied table at a time.

In playing with this simulation, I discovered that even though the data were generated with $\alpha = 0.5$, I needed to use $\alpha = 0.05$ to get reasonable results. The model had too strong of a bias to assign customers to new tables because even though the prior for choosing a new table was small, the likelihood associated with a new table was very high. Because the table was empty, the maximum likelihood estimate of the Gaussian (x,y) center is simply the location of the sole customer being added to the table, which yields a high likelihood. A proper Bayesian implementation, with a prior over the (x,y) centers would probably be a better approach.

Here are two examples of inferred clusters which I obtained by running the sampler for 100 updates of each customer's table assignment. The example on the left is pretty good; the example on the right is not great.