HW2 R Markdown

Santanu Mukherjee

10/10/2021

R Markdown

Problem 1

Answer 1a

Craps is a dice game in which players bet on the outcomes of a pair of dice.

Simplest way to play and rules of the game

A **‘Natural’** means that the result of rolling the pair of dice (sum of the two dice) is a 7 or a 11. When this happens, the person wins and get to roll the pair of dice again.

When the result of rolling the pair of dice (sum of the two dice) is 2, 3, or 12, it is called **“craps”** or **“crapping out”**, and anyone betting the Pass line loses.

The other possible outcomes are the point numbers: 4, 5, 6, 8, 9, and 10. If the shooter rolls one of these numbers on the come-out roll, this establishes the **“point”** – to “pass” or “win”, the point number must be rolled again before a seven to win.

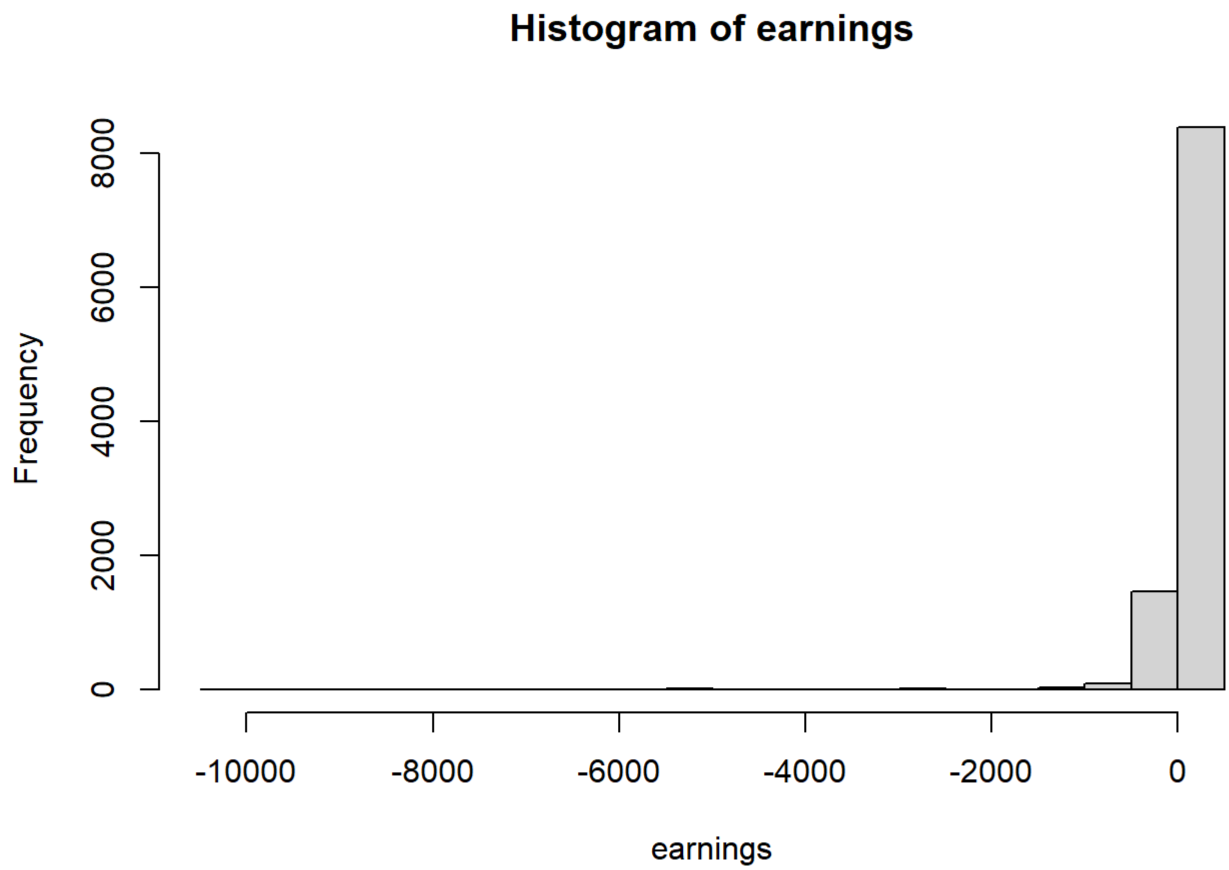
Answer 1b and 1c (Monte Carlo Simulation)

```
playcraps = function(){
    num = append(1:6, 5:1)
    first.roll = sample(2:12, 1, prob = num /36)
    if (first.roll %in% c(2,3,12)){
        return(0)
    } else if (first.roll %in% c(7,11)){
        return(1)
    } else  {
            point = first.roll
            next.roll = sample(2:12, 1, prob = num /36)
            while (next.roll != point  && next.roll !=7) {
              next.roll = sample(2:12, 1, prob = num /36)
            }
            if (next.roll== point){
              return(1)
            }
            return(0)

    }
}
```

```
set.seed(1)
sims  = 10000     # Number of times the game is simulated
n     = 10        # Number of games played
i.bet = 10        # Initial bet of $ 10

earnings = rep (0,sims)

for (t in 1:sims){
    bet = i.bet
    tmp = 0
    for (i in 1:n) {
      res = playcraps()
      if (res==0){
        tmp = tmp - bet
        bet = 2*bet

      }
      if (res==1){
        tmp = tmp + bet
        bet = i.bet
      }
    }
    #print(t)
    earnings[t] = tmp

}


print(paste('The expected amount ( in dollars) after playing for 10 games is :', mean(earning
s)))
```

```
## [1] "The expected amount ( in dollars) after playing for 10 games is : 0.741"
```

```
hist(earnings)
```




```
n = 10000
res = rep(-99,n)
for (i in 1:n) {
  res[i] = playcraps()

}

mean(res)
```

```
## [1] 0.4925
```

```
print(paste('The probability of expected winning is :', mean(res)))
```

```
## [1] "The probability of expected winning is : 0.4925"
```

I do not think this is a good betting strategy for winning.

Problem 2

Life in years for Microwave Magnetron tube has exponential distribution with rate parameter β

Answer 2a, 2b & 2c

HW2

Problem 2

Magnetron tube has an exponential distribution probability distribution.

(a) Likelihood function of rate parameter β is

$$L(\beta, x_1 \dots x_n) = \beta^n \exp\left(-\beta \sum_{j=1}^{n} x_j\right)$$

(b) MLE of β; $\hat{\beta} = \dfrac{n}{\sum_{j=1}^{n} x_j} = \dfrac{100}{691.74} = 0.14456$

(c) So the uniform prior: $P(\beta) = 1$.

Posterior distribution of β.

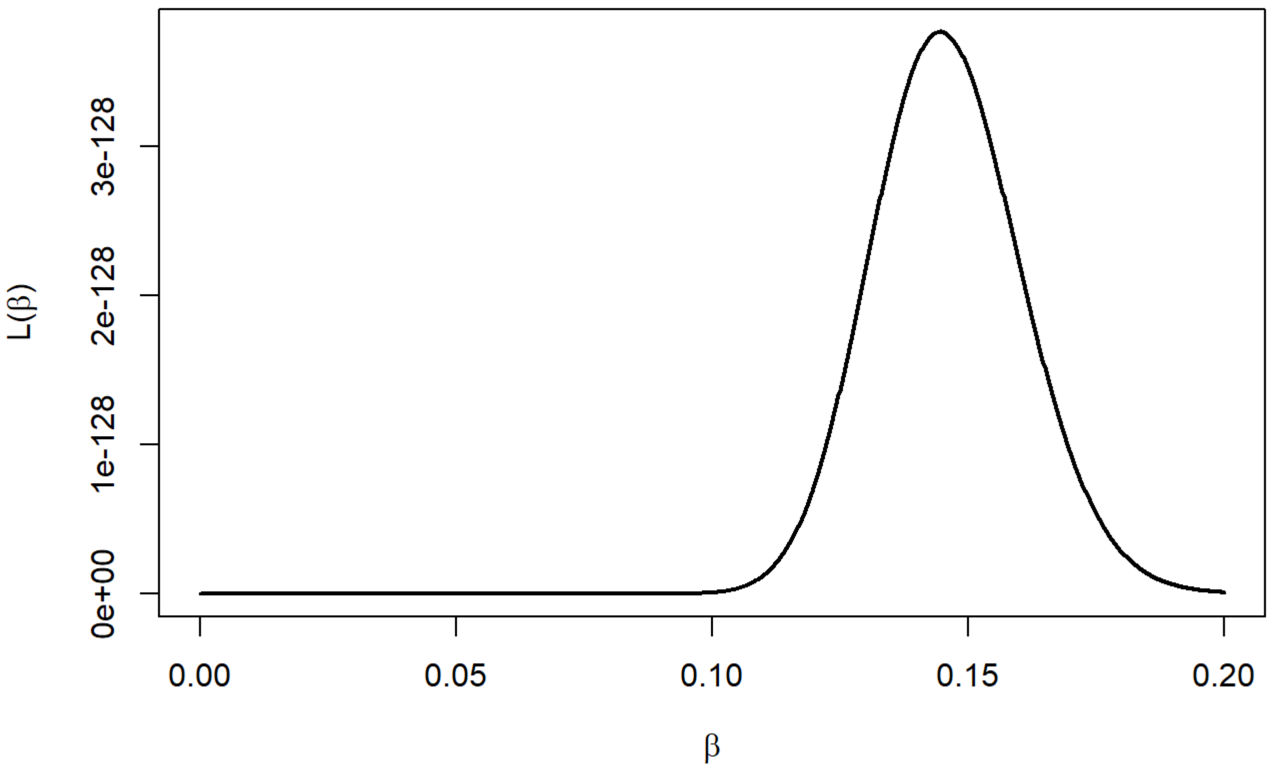
$$P(\beta|x) \propto \underbrace{\beta^n \exp\left(-\beta \sum_{j=1}^{n} x_j\right)}_{\text{Likelihood}} \underbrace{1}_{\text{Uniform Prior}}$$

$$P(\beta|x) \propto \beta^n \exp\left(-\beta \sum_{j=1}^{n} x_j\right), \quad \beta > 0$$

which is the kernel of Gamma distribution $\text{Gamma}(n+1, b = \sum_{j=1}^{n} x_j)$

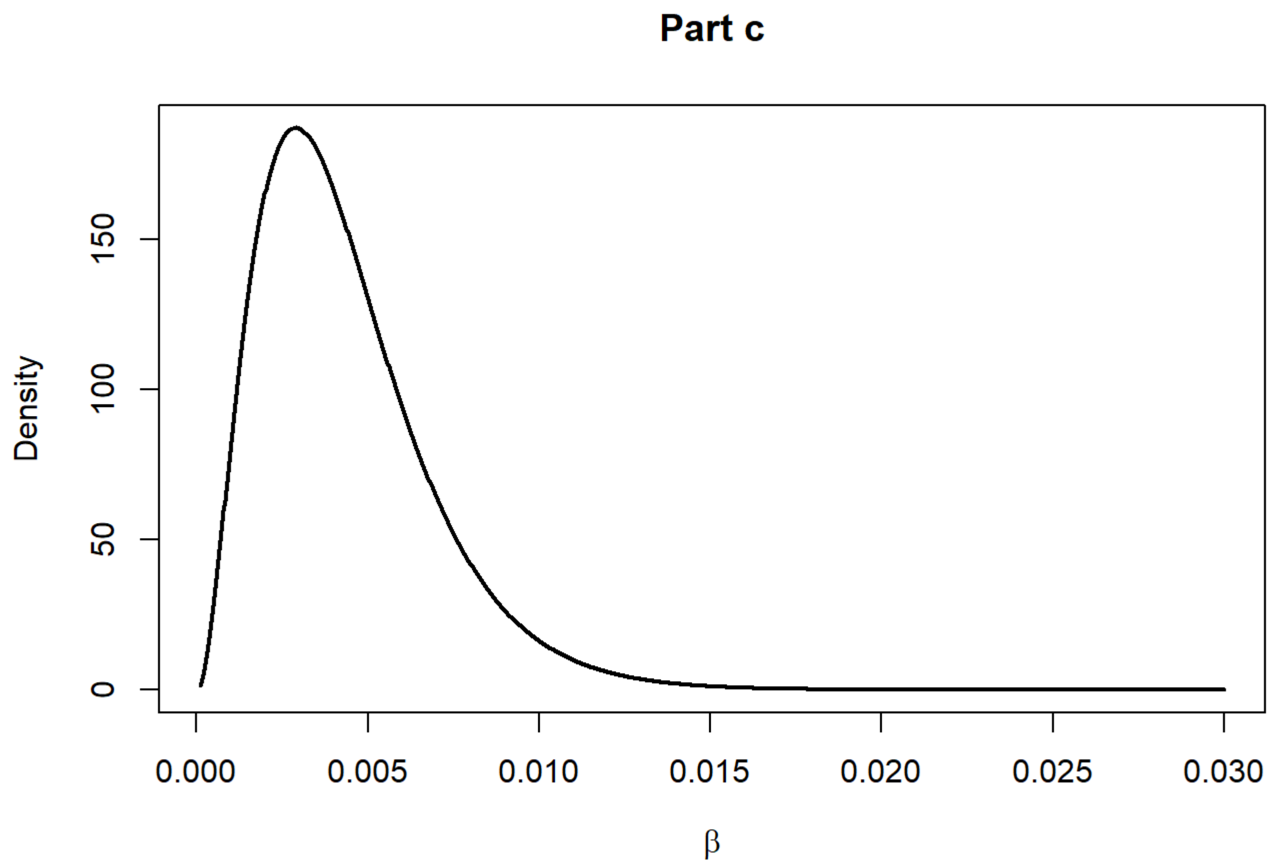
Posterior Mean $= \dfrac{n+1}{\sum_{j=1}^{n} x_j} = \dfrac{101}{691.74} = 0.146$

Problem 2-a-b-c



Graph for Answer 2c

```
x = seq(.0001, .03, length=1000)

plot(x, dgamma(x, length(df2) + 1 ,rate=sum(df2$x)), main='Part c',
type='l', lwd=2, xlab=expression(beta), ylab='Density')
```


Part c
Density
0
50
100
150
0.000
0.005
0.010
0.015
0.020
0.025
0.030
β

Answer 2d

Problem 2

(d) So, when the prior is Gamma distribution

$$p(\beta) \propto \beta^{a-1} \exp(-\beta b),$$

Posterior distribution of β is

$$p(\beta \mid x) \propto \underbrace{\beta^{n} \exp\left(-\beta \sum_{i=1}^{n} x_i\right)}_{\text{Likelihood}} \underbrace{\beta^{a-1} \exp(-b\beta)}_{\text{Gamma prior}}$$

$$\propto \beta^{n+a-1} \exp\left(-\beta \left(b + \sum_{i=1}^{n} x_i\right)\right), \beta > 0$$

which is kernel of Gamma $(n+a, b+\sum_{i=1}^{n} x_i)$

So, posterior distribution of β under Gamma prior is

$$\beta / x \sim \text{Gamma}\left(n+a, b+\sum_{i=1}^{n} x_i\right)$$

$$\text{Posterior Mean} = \frac{n+a}{b+\sum x_i} = \frac{100+a}{b+691.74}$$

Answer 2e and 2g

```
## [1] 0.1189264 0.1758273
```

```
## [1] "Equal_Tailed Credible Interval for rate : 0.1189264 0.1758273"
```

```
## [1] "Equal_Tailed Credible Interval for life of magnetron tube: 5.6874 8.4086"
```

So, the 95% Bayesian credible interval for the average length of life of a magnetron tube under the uniform prior is between **5.69 and 8.41 years**

Answer 2f

The posterior probability that the average length of life of a magnetron tube is less than 10 years

p(1/β < 10 |x) = p(β > 0.10 |x) = 1 - pgamma(0.10,100,691.74)

```
# (1-alpha) 100% equal tailed (Quantile) credible interval
# Quantile Function for equal credible interval

w <- round(pgamma(0.10,NROW(df2),sum(df2final)),4)
1 - w
```

```
## [1] 0.9997
```

p(1/β < 10 |x) = p(β > 0.10 |x) = 1 - pgamma(0.10,100,691.74) = **0.9997**

Answer 2g

So, the 95% Bayesian credible interval for β is : **(0.1189264, 0.1758273)** AND
the 95% Bayesian credible interval for $1/\beta$ is : **(5.68784, 8.4086)**.

Equal tailed credit interval is INVARIANT UNDER ONE-TO-ONE TRANSFORMATION.