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Problem 1

(a) $X \sim N(\mu_1, \sigma_1^2)$, $\sigma_1 = 0.00003$ $\Sigma X_i = 15.19991$

Prior of μ_1 follows $N(1.52000, 0.0001^2)$

↳ μ_K ↳ σ_K^2

Posterior distribution:

$$P(\mu_1/x) = P(x/\mu) \cdot P(\mu)$$

$$P(\mu_1/x) \propto exp\left\{ -\frac{1}{2\sigma_1^2} \Sigma(x_i - \mu_1)^2 \right\} exp\left\{ -\frac{1}{2\sigma_K^2} (\mu_1 - \mu_K)^2 \right\}$$

$$\propto exp\left[-\frac{1}{2}\left(\frac{1}{\sigma_1^2} n\mu_1^2 - \frac{2}{\sigma_1^2}\Sigma x_i \mu_1 + \frac{1}{\sigma_K^2}\mu_1^2 - \frac{2}{\sigma_K^2}\mu_K \cdot \mu_1 \right) \right]$$

Putting values for $\mu_K = 1.52000$, $\sigma_K^2 = 0.0001^2$, $\sigma_1 = 0.00003$, $n = 10$

$$\propto exp\left\{ -\frac{1}{2}\left[\left(\frac{10}{0.00003^2} + \frac{1}{0.0001^2} \right)\mu_1^2 - \left(\frac{2}{0.00003^2} \times 15.19991 \right. \right. \right.$$
$$\left. \left. \left. + \frac{2}{0.0001^2} \times 1.52000 \right)\mu_1 \right] \right\}$$

$$\propto exp\left\{ -\frac{1}{2} \frac{1}{8.92e-11} \left[\mu_1 - \frac{17040788889}{11211111111} \right]^2 \right\}$$

↓   ↓

$8.92e-11$ 1.51999108

∴ $\mu_1/x \sim N(1.51999108,\ 8.92e-11)$

means $\mu_1/x \sim N(1.51999108,\ 0.0000000000891972)$

                        μ σ^2