1. El sistema tiene dos grados de libertad: el ángulo θ que forma la varilla de longitud l con la vertical y el ángulo ϕ que forma la varilla de longitud a con la horizontal.

Para la masa 1.

$$x_1 = l\,\text{sen}\,\theta - \frac{a}{2}\cos(\phi) \quad y_1 = -l\cos(\theta) - \frac{a}{2}\,\text{sen}(\phi)$$

Para la masa 2.

$$x_2 = l\,\text{sen}(\theta) + \frac{a}{2}\cos(\phi) \quad y_2 = -l\cos(\theta) + \frac{a}{2}\,\text{sen}(\phi)$$

La energía cinética (T) será igual a:

$$T = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2)$$

$$T = \frac{1}{2}m_1\left[\left(l\dot{\theta}\cos(\theta) + \frac{a}{2}\,\text{sen}(\phi)\dot{\phi}\right)^2 + \left(l\dot{\theta}\,\text{sen}(\theta) - \frac{a}{2}\cos(\phi)\dot{\phi}\right)^2\right]$$

$$+ \frac{1}{2}m_2\left[\left(l\dot{\theta}\cos(\theta) - \frac{a}{2}\,\text{sen}(\phi)\dot{\phi}\right)^2 + \left(l\dot{\theta}\,\text{sen}(\theta) + \frac{a}{2}\cos(\phi)\dot{\phi}\right)^2\right]$$

$$T = \frac{1}{2}m_1\left[l^2\dot{\theta}^2\cos^2(\theta) + l\dot{\theta}\cos(\theta)\,a\,\text{sen}(\phi)\dot{\phi} + \frac{a^2}{4}\text{sen}^2(\phi)\dot{\phi}^2 + l^2\dot{\theta}^2\text{sen}^2(\theta) - l\dot{\theta}\,\text{sen}(\theta)\,a\cos(\phi)\dot{\phi} + \frac{a^2}{4}\cos^2(\phi)\dot{\phi}^2\right]$$

$$+ \frac{1}{2}m_2\left[\;\right]$$

$$T = \frac{1}{2}(m_1+m_2)\ell^2\dot{\theta}^2 + \frac{1}{8}(m_1+m_2)a^2\dot{\varphi}^2 + \frac{1}{2}\ell a\dot{\theta}\dot{\varphi}[(\rightarrow$$

$$\rightarrow m_2-m_1)\cos\theta\,\text{sen}\varphi + (m_1-m_2)\,\text{sen}\theta\cos\varphi]$$

La Energía Potencial será

$$U = m_1 g y_1 + m_2 g y_2 = -g\ell(m_1+m_2)\cos\theta + \frac{ga}{2}(m_1-m_2)\,\text{sen}\varphi$$

$$L = T-U$$

Para θ

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\theta}}\right) - \frac{\partial L}{\partial\theta} = 0$$

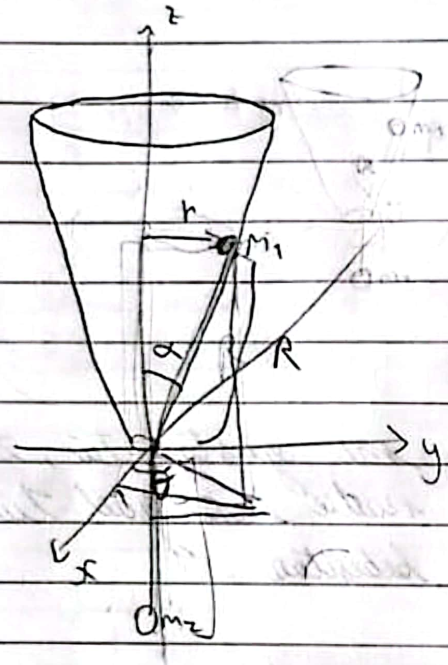
Resolviendo queda:

$$1)\ (m_1+m_2)\ell^2\ddot{\theta} + \frac{1}{2}\ell a[\ddot{\varphi}((m_2-m_1)\cos\theta\,\text{sen}\varphi + (m_1-m_2)\,\text{sen}\theta\cos\varphi)$$
$$+\dot{\varphi}(-\dot{\theta}(m_2-m_1)\,\text{sen}\theta\,\text{sen}\varphi + \dot{\theta}(m_1-m_2)\cos\theta\cos\varphi)] + g\ell(m_1+m_2)\,\text{sen}\theta = 0$$

Para φ

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{\varphi}}\right) - \frac{\partial L}{\partial\varphi} = 0$$

$$2)\ \frac{1}{4}(m_1+m_2)a^2\ddot{\varphi} + \frac{1}{2}\ell a[\ddot{\theta}((m_2-m_1)\cos\theta\,\text{sen}\varphi + (m_1-m_2)\,\text{sen}\theta\cos\varphi +$$
$$\dot{\theta}(-\dot{\theta}(m_2-m_1)\,\text{sen}\theta\,\text{sen}\varphi + \dot{\varphi}(m_1-m_2)\cos\theta\cos\varphi)] - \frac{ga}{2}(m_1-m_2)\cos\varphi = 0$$



$q_1 = \theta \quad q_2 = r$

$q_3 = z_2$

m_1	m_2
$x_1 = r\cos\theta$	$x_2 = 0$
$y_1 = r\,\text{sen}\,\theta$	$y_2 = 0$
$z_1 = r\cot\alpha$	$z_2 = l - r\sec\alpha$

$$T_1 = \frac{1}{2} m_1 (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = \frac{1}{2} m_1 (\dot{r}^2 \csc^2\alpha + r\dot{\theta}^2)$$

$$T_2 = \frac{1}{2} m_2 \dot{z}_2^2$$

$$V_1 = mgr\cot\alpha$$

$$V_2 = mgz_2$$

$\tan^2 x + 1 = \sec^2 \alpha$ $\qquad$ $\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = 0.$

$\cot^2 x + 1 = \csc^2 x$

$$\mathcal{L} = \frac{1}{2} m_1 \dot{r}^2 \csc^2\alpha + \frac{1}{2} m_1 r^2 \dot{\theta}^2 + \frac{1}{2} m_2 \dot{z}_2^2 - m_1 g r \cot\alpha + m_2 g z_2$$

$m_2 \dot{z}_2$

$$\frac{\partial \mathcal{L}}{\partial \theta} = 0$$

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{m_1 \dot{\theta}^2 r}{2} - m_1 g \cot\alpha = m_1 \left(r\dot{\theta}^2 - g \cot\alpha \right)$$

$$\frac{\partial \mathcal{L}}{\partial z_2} = m_2 g$$

$m_1 \ddot{r} \csc^2\alpha - m_1 r \dot{\theta}^2 + m_1 g \cot\alpha = 0$ $\qquad$ $m_1 r^2 \dot{\theta} = c$

$\ddot{r} = -r\dot{\theta}^2 \sin^2\alpha - g \cos\alpha \sin\alpha =$

$m_2 \ddot{z}_2 - m_2 g = 0.$

$\ddot{z}_2 = g$ $\qquad$ $m_1 r^2 \dot{\theta} = c_1$

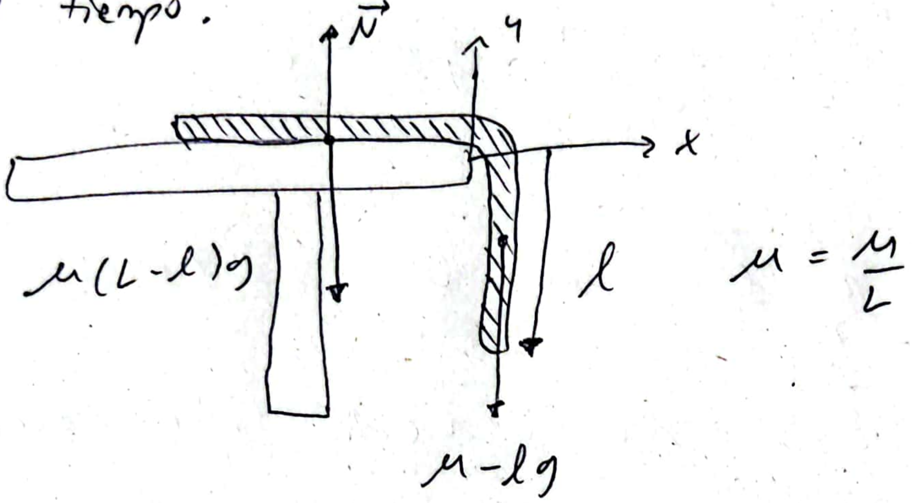
$\int r(t)\,dt$

$\ddot{r}$ $\qquad$ $\frac{c_1^2}{m_1 r^3}$ $\qquad$ $\dot{\theta} = \frac{c_1}{m_1 r^2}$

$$\theta = \frac{c_1 t}{m_1 r^2} + c_2$$

$\frac{1}{\cos^2 x \sin}$

4. Una cuerda uniforme de masa M y longitud L se encuentra sobre una mesa sin fricción. La cuerda se suelta desde el reposo cuando una sección de longitud l está colgando. Encuentre la trayectoria de la cuerda en función del tiempo.



$$\begin{aligned} x&=0 \\ z&=0 \end{aligned} \longrightarrow s=1 \longrightarrow \begin{aligned} y &= -l(t) \\ \dot{y} &= -\dot{l} \end{aligned}$$

$$dT = \frac{1}{2}\frac{M}{L}\,dl\,\dot{l}^2 \longrightarrow T = \int_0^{l} \frac{1}{2}\frac{M}{L}\dot{l}^2\,dl$$

$$T = \frac{1}{2}\frac{M}{L}\,l\,\dot{l}^2$$

$$dU = \frac{M}{L}\,l g\,dl \longrightarrow V = \frac{M}{L}\,g\int_0^{-l} l\,dl = \frac{1}{2}\frac{M}{L}\,g l^2$$

$$L = T - U = \frac{1}{2}\frac{M}{L}\,l\dot{l}^2 - \frac{1}{2}\frac{M}{L}\,g l^2$$

<u>Ecuaciones Euler - Lagrange:</u>

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{l}}\right) - \frac{\partial L}{\partial l} = 0$$

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$$\frac{d}{dt}\left(\frac{m}{L}l\dot{l}\right)-\frac{1}{2}\frac{m}{L}\dot{l}^2-\frac{m}{L}gl=0$$

$$\frac{m}{L}\dot{l}^2+\frac{m}{L}l\ddot{l}-\frac{1}{2}\frac{m}{L}\dot{l}^2-\frac{m}{L}gl=0$$

$$l\ddot{l}+\dot{l}^2-\frac{1}{2}\dot{l}^2-gl=0$$

Entonces:

$$\ddot{l}+\frac{1}{2}\frac{\dot{l}^2}{l}-g=0 \quad , \quad l\neq 0$$

Es una E.D. NO LINEAL por tener un exponente mayor a 1, por series de taylor:

sea $l(0)=l$, $\dot{l}(0)=0$,

$$\ddot{l}=g-\frac{1}{2}\frac{\dot{l}^2}{l} \longrightarrow \ddot{l}(0)=g$$

$$\dddot{l}=-\frac{1}{2}\left(\frac{2\dot{l}\ddot{l}l-\dot{l}^2\dot{l}}{l^2}\right) \rightarrow \dddot{l}(0)=-\frac{1}{2}\left(\frac{0-0}{l^2}\right)=$$

$$l^{(4)}=-\frac{1}{2}\left[\left(\frac{2\ddot{l}^2l^3+2\dot{l}\dddot{l}l^3+2\dot{l}^2\ddot{l}l^2-4\dot{l}^2\ddot{l}\ldots}{l^4}\right)\right.$$

$$\left(\frac{2\dot{l}\ddot{l}l^3+\dot{l}^3l^2-2\dot{l}^3l^2}{l^4}\right)$$

$$l^{(4)}(0) = -\frac{1}{2}\left[\left(\frac{2g^2 l^3 + 2(0) + 2(0) - 4(0)}{l^4}\right) - \left(\frac{2(0) + 0 - 0}{l^4}\right)\right] = -g^2\frac{l^3}{l^4} = -\frac{g^2}{l}$$

Por último:

$$l(t) = l + t\dot{l}(0) + \frac{t^2}{2}\ddot{l}(0) + \dots + \frac{t^n}{n!}l^{(n)}(0)$$

$$= l + \frac{g}{2}t^2 - \frac{g^2}{6l}t^4 + \dots + \frac{t^n}{n!}l^{n}(0)$$

Que es la expansión de Taylor de $l(t)$ al rededor de $t=0$.

Considerando solo $t=0$ y $t=2$

$$l(t) = l + \frac{g}{2}t^2$$