$X \in \mathbb{R}^{d} \leftarrow$ rango

$\underbrace{X_1, \dots, X_n}_{\text{obs}}$ ind muestras X

$$X = \begin{pmatrix} X_1 \\ \vdots \\ X_d \end{pmatrix} \longrightarrow \mathbb{X} = \begin{pmatrix} - X_1^T - \\ \vdots \\ - X_n^T - \end{pmatrix} = \begin{pmatrix} X_{11} & \cdots & X_{1d} \\ \vdots & X_{ij} & \vdots \\ X_{n1} & \cdots & X_{nd} \end{pmatrix}_{n \times d}$$

$$E[X] = \begin{bmatrix} E[X_{n1}] \\ \vdots \\ E[X_{nd}] \end{bmatrix} = \begin{bmatrix} E[X^1] \\ \vdots \\ E[X^d] \end{bmatrix}$$ $\leftarrow$ centralidad

$$\Sigma = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{1d} \\ \vdots & & \vdots \\ \sigma_{1d} & \cdots & \sigma_{dd} \end{bmatrix}_{d \times d} \qquad \sigma_{jk} = Cov(X^j, X^k)$$

$$Cov(X^j, X^k) = E[X^j X^k] - E[X^j]E[X^k] \checkmark$$

$$\Sigma = \underbrace{E[XX^T]}_{j} - \underbrace{E[X]E[X^T]}_{(1)}$$

$$E[XX^T]_{jk} = E[(XX^T)_{jk}] = E[X^j X^k]$$

$$(1)\ (E[X]E[X^T])_{jk} = E[X]_j E[X^T]_k = E[X^j]E[X^k]$$

$$\Sigma = E((X - E(X))(X - E(X))^T)$$

$$= E(XX^T - XE(X)^T - E(X)X^T + E(X)E(X)^T)$$

$$= E[XX^T] - E[X]E[X^T] - E(X)E[X^T] + E[X]E(X^T)$$

$$= E[XX^T] - E[X]E[X^T]$$

$$E \sim \frac{1}{n}\sum_{i=1}^{n} \qquad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad \checkmark \qquad \mathbb{X}^T = \begin{bmatrix} \vdots & & \vdots \\ x_1 & \cdots & x_n \\ \vdots & & \vdots \end{bmatrix}_{d\times n} \qquad \mathbb{1}_{n\times 1} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}_{n\times 1} \in \mathbb{R}^n$$

$$\Sigma \sim S = \frac{1}{n}\sum_{i=1}^{n} x_i x_i^T - \bar{x}\bar{x}^T$$

$$= \frac{1}{n}\sum_{i=1}^{n} (x_i - \bar{x})(x_i - \bar{x})^T$$

$$\frac{1}{n}\mathbb{X}^T \mathbb{1} = \left(\sum_{i=1}^{n} x_i\right)\frac{1}{n} = \bar{x} = \begin{bmatrix} \bar{x}_1 \\ \vdots \\ \bar{x}_d \end{bmatrix}$$

$$\mathbb{X}^T\mathbb{X}$$

$$\mathbb{X}^T = \underbrace{\begin{bmatrix} x_1 & 0 \end{bmatrix}}_{M_1} + \underbrace{\begin{bmatrix} 0 & x_2 & 0 \end{bmatrix}}_{M_2} + \cdots \underbrace{\begin{bmatrix} 0 & x_n \end{bmatrix}}_{M_n}$$

$$AB = \begin{bmatrix} -a_1- \\ -a_n- \end{bmatrix}\begin{bmatrix} | & & | \\ b & & b \\ | & & | \end{bmatrix}$$

$$\mathbb{X}^T\mathbb{X} = \left(\sum_{i=1}^{n} M_i\right)\left(\sum_{i=1}^{n} M_i^T\right)$$

$$= \sum_{i=1}^{n} M_i M_j^T = \sum_{i=1}^{n} M_i M_i^T = \sum_{i=1}^{n} x_i x_i^T$$

$i \neq j$ $\quad \begin{bmatrix} x_1 & 0 \end{bmatrix}\begin{bmatrix} 0 & x_2 & 0 \end{bmatrix}^T = 0$

$$S = \frac{1}{n}\mathbb{X}^T\mathbb{X} - \frac{1}{n^2}(\mathbb{X}^T\mathbb{1})(\mathbb{X}^T\mathbb{1})^T \qquad (A^TB)^T = B^TA$$

$$= \frac{1}{n}\mathbb{X}^T\mathbb{X} - \frac{1}{n^2}\mathbb{X}^T\mathbb{1}\mathbb{1}^T\mathbb{X}$$

$$= \mathbb{X}^T\left[\frac{1}{n}I_n - \frac{1}{n^2}\mathbb{1}\mathbb{1}^T\right]\mathbb{X} = \frac{1}{n}\mathbb{X}^T\underbrace{\left[I_n - \frac{1}{n}\mathbb{1}\mathbb{1}^T\right]}_{\substack{\text{projecta} \\ H}}\mathbb{X}$$

$$H = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{n} & \cdots & \frac{1}{n} \\ \vdots & \frac{1}{n} & \vdots \\ \frac{1}{n} & \cdots & \frac{1}{n} \end{bmatrix} = \begin{bmatrix} 1-\frac{1}{n} & -\frac{1}{n} & \cdots & -\frac{1}{n} \\ & 1-\frac{1}{n} & & \\ & & \ddots & \\ -\frac{1}{n} & & & -\frac{1}{n} \end{bmatrix} \qquad H^2 = H$$

$v \in \mathbb{R}^d$

$$Hv = v - \overbrace{\frac{1}{n}(v^T\mathbb{1})}^{}\mathbb{1}$$

$$\mathbb{1}^T v$$

$$= v - \bar{v}\mathbb{1} \quad \leftarrow \text{estandarización}$$

$u \in \mathbb{R}^d \qquad X \in \mathbb{R}^d$

$$u^T \Sigma u = u^T \left[E[XX^T] - E[X]E[X]^T \right] u$$

$$= E[u^T X X^T u] - E[u^T X]\, E[X^T u]$$

$$= E[(u^T X)^2] - E[u^T X]\, E[u^T X]^T$$

$$= Var(u^T X)$$

$$\boxed{\text{Encontrar } u \text{ que } \max_{\substack{u \in \mathbb{R}^d \\ \|u\|_2 = 1}} u^T \Sigma u}$$

$$u = \begin{pmatrix} u_1 \\ u_k \end{pmatrix}$$

$$\|u\|_2 = u_1^2 + \dots + u_k^2$$

$$A_{n\times m} B_{m\times j} = C_{n\times j}$$

Descomposición de vectores Singulares

matriz es cuadrada = Teorema Espectral

cuadrados

$$S = P \Delta P^T$$

$$X_{n\times d} = U \Delta V^T = U_{n\times e}\, \Delta_{e\times e}\, V^T_{e\times d}$$

obs

varianza

columnas

Matriz Ortogonal

$$PP^T = P^T P = I$$

$$P_i P_i = 1$$

$$P_i P_j = 0$$

$$\Delta \to diag(\Delta) = \lambda_j$$

$$\Sigma v_1 = P \Delta P^T v_1 = \lambda_1 v_1$$

$$P D \begin{bmatrix} - & v_1^T & - \\ & \vdots & \\ - & v_n & - \end{bmatrix} \begin{bmatrix} | \\ v_1 \\ | \end{bmatrix} = P \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = P \begin{bmatrix} \lambda_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \lambda_1 v_1$$

autovalores

$$[\quad]_{n\times d}$$

$$= \begin{bmatrix} | & & | \\ v_1 & \dots & v_n \\ | & & | \end{bmatrix} \begin{bmatrix} \lambda_1 \\ 0 \\ 0 \end{bmatrix}$$

autovectores