CSE 575: Statistical Machine Learning Self-Evaluation Exam

Instructor: Prof. Jingrui He January 11, 2019

First Name:			
Last Name:			
Email:			
ASU ID:			
Q	Topic	Max Score	Score
1	Probability	24	
2	Iterative Algorithms	21	
3	MLE	20	
4	Quadratic Optimization	17	
5	Distance Metric	18	
Total:		100	

- This exam book has 6 pages, including this cover page and a blank page at the end.
- Good luck!

1 Probability (24 points)

(7 points.) If A and B are **DISJOINT** events, and P(B) > 0, what is the value of P(A|B)?

(7 points.) Suppose that the PDF of a random variable X is as follows:

$$f(x) = \begin{cases} \frac{4}{3}(1 - x^3), & \text{for } 0 \le x \le 1\\ 0, & \text{otherwise} \end{cases}$$

Then what is the value of P(X < 0)?

(10 points.) Suppose that X is a random variable for which $E(X) = \mu$ and $Var(X) = \sigma^2$, and let c be an arbitrary constant. What is the value of $E[(X-c)^2]$? Hint: What is the definition of variance?

2 Iterative Algorithms and Big-O (21 points)

[7 points.] Write a program/function (Not Pseudocode) to calculate the following sum (c++ or any of your favorite programming language). Your program takes an integer n $(n \ge 1)$ as the input; and returns r defined as $r = \sum_{i=1}^n \sum_{j=1}^i j$.

(7 points.) What is the value r returned by your program? Express your answer as a function of n and use closed-form solution.

(7 points.) Using O() notation, give the worst-case running time of your program.

3 Maximum Likelihood Estimation (20 points)

[10 points.] Suppose we flip a coin, and observe either a head or a tail. The probability of observing a head in each trial is p ($0 \le p \le 1$). If we flip the coin five times, and observe (head, head, tail, tail, head), what is the maximum likelihood estimate of p? Justify your answer.

[10 points.] Suppose we flip a coin, and observe either a head or a tail. The probability of observing a head in the first trial is p ($0 \le p \le 1$). The probability of observing a head in the second trial is 2p. The probability of observing a head in the third and fourth trials is 3p, respectively. The probability of observing a head in the fifth trial is 5p. If we flip the coin five times, and observe (head, head, tail, tail, head), what is the maximum likelihood estimate of p? Justify your answer.

4 Quadratic Optimization (17 points)

[7 points.] Solve the following optimization problem

$$\operatorname{argmin}_x 3x^2 - 12x + 8$$

[10 points.] Solve the following optimization problem

$$\begin{aligned} & \operatorname{argmin}_x 3x^2 - 12x + 8 \\ & \operatorname{subject to} \ 6 < x < 10 \end{aligned}$$

5 Distance Metric (18 points)

[18 points]. Given two data points $x_1 = (0,0)'$ and $x_2 = (1,2)'$ in two-dimensional space, what is the L_2 distance between them? What is the L_1 distance between them? What is the L_∞ distance between them? Justify your answer.