Test Problems for Unconstrained Optimization

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Abstract:

This report contains a description of subroutines which can be used for testing unconstrained optimization codes. These subroutines can easily be obtained either by using the anonymous ftp address ftp://ftp.cs.cas.cz/pub/msdos/opt (file TEST28.FOR) or from the web homepage http://www.cs.cas.cz/luksan/test.html. Furthermore, all test problems contained in these subroutines are presented in the analytic form.

Keywords:
unconstrained optimization, test problems

1 Introduction

This report describes subroutines `TIUD28`, `TFFU28`, `TFGU28`, `TFBU28`, which contain 92 general problems for testing unconstrained optimization codes. Some of these problems were chosen from [20], but additional problems with dense Hessian matrices are included. All subroutines are written in the standard Fortran 77 language. Their names are derived from the following rule:

- The first letter is `T` - test subroutines.

- The second letter is either `I` - initiation, or `F` - objective function.

- The third letter is either `U` - initiation of unconstrained problem, or `F` - computation of the function value, or `G` - computation of the gradient vector, or `B` - computation of both the function value and the gradient vector simultaneously.

- The fourth letter is either `U` - universal subroutine, or `D` - subroutine for dense problems.

The last two digits determine a given collection (the numbering corresponds to the UFO system [21], which contains similar collections).

Initiation subroutines use the following parameters (array dimensions are given in parentheses):

`N`	input	number of variables,
`X(N)`	output	vector of variables,
`FMIN`	output	lower bound of the objective function value,
`XMAX`	output	maximum stepsize,
`NEXT`	input	number of the problem selected,
`IERR`	output	error indicator (0 - correct data, 1 - `N` is too small).

Although `N` is an input parameter, it can be changed by the initiation subroutine when its value does not satisfy the required conditions. For example, most of the problems require `N` to be even or a multiple of a positive integer.

Evaluation subroutines use the following parameters (array dimensions are given in parentheses):

`N`	input	number of variables,
`X(N)`	input	vector of variables,
`F`	output	value of the objective function,
`G(N)`	output	gradient of the objective function,
`NEXT`	input	number of the problem selected.

2 Test problems for general unconstrained optimization

Calling statements have the form

```
CALL TIUD28(N,X,FMIN,XMAX,NEXT,IERR)
CALL TFFU28(N,X,F,NEXT)
CALL TFGU28(N,X,G,NEXT)
CALL TFBU28(N,X,F,G,NEXT)
```

with the following significance:

TIUD28 – initiation of vector of variables X, which has dimension N.

TFFU28 – evaluation of the general objective function value F at the point X.

TFGU28 – evaluation of the general objective function gradient G at the point X.

TFBU28 – evaluation of the general objective function value F and gradient G at the point X.

We seek a minimum of a general objective function $F(x)$ from the starting point $\bar{x}$. For positive integers k and l, we use the notation $\text{div}(k, l)$ for integer division, i.e., maximum integer not greater than k/l, and $\text{mod}(k, l)$ for the remainder after integer division, i.e., $\text{mod}(k, l) = l(k/l - \text{div}(k, l))$. The description of individual problems follows.

Problem 1. Chained Rosenbrock function [7].

$$F(x) = \sum_{i=2}^{n}[100(x_{i-1}^2 - x_i)^2 + (x_{i-1} - 1)^2],$$
$$\bar{x}_i = -1.2, \ \text{mod}(i, 2) = 1, \quad \bar{x}_i = 1.0, \ \text{mod}(i, 2) = 0.$$

Problem 2. Chained Wood function [7].

$$F(x) = \sum_{j=1}^{k}[100(x_{i-1}^2 - x_i)^2 + (x_{i-1} - 1)^2 + 90(x_{i+1}^2 - x_{i+2})^2$$
$$+ \ (x_{i+1} - 1)^2 + 10(x_i + x_{i+2} - 2)^2 + (x_i - x_{i+2})^2/10],$$
$$i = 2j, \ k = (n-2)/2,$$
$$\bar{x}_i = -3, \ \text{mod}(i, 2) = 1, \ i \leq 4, \quad \bar{x}_i = -2, \ \text{mod}(i, 2) = 1, \ i > 4,$$
$$\bar{x}_i = -1, \ \text{mod}(i, 2) = 0, \ i \leq 4, \quad \bar{x}_i = 0, \ \text{mod}(i, 2) = 0, \ i > 4.$$

Problem 3. Chained Powel singular function [7].

$$F(x) = \sum_{j=1}^{k}\left[(x_{i-1} + 10x_i)^2 + 5(x_{i+1} - x_{i+2})^2 + (x_i - 2x_{i+1})^4 + 10(x_{i-1} - x_{i+2})^4\right],$$
$$i = 2j, \ k = (n-2)/2,$$
$$\bar{x}_i = 3, \ \text{mod}(i, 4) = 1, \quad \bar{x}_i = -1, \ \text{mod}(i, 4) = 2,$$
$$\bar{x}_i = 0, \ \text{mod}(i, 4) = 3, \quad \bar{x}_i = 1, \ \text{mod}(i, 4) = 0.$$

Problem 4. Chained Cragg and Levy function [7].

$$F(x) = \sum_{j=1}^{k} \left[(\exp(x_{i-1}) - x_i)^4 + 100(x_i - x_{i+1})^6 + \tan^4(x_{i+1} - x_{i+2}) + x_{i-1}^8 + (x_{i+2} - 1)^2 \right],$$
$$i = 2j, \quad k = (n-2)/2,$$
$$\overline{x}_i = 1, \quad i = 1, \qquad \overline{x}_i = 2, \quad i > 1.$$

Problem 5. Generalized Broyden tridiagonal function [23].

$$F(x) = \sum_{i=1}^{n} |(3 - 2x_i)x_i - x_{i-1} - x_{i+1} + 1|^p,$$
$$p = 7/3, \quad x_0 = x_{n+1} = 0,$$
$$\overline{x}_i = -1, \quad i \geq 1.$$

Problem 6. Generalized Broyden banded function [23].

$$F(x) = \sum_{i=1}^{n} \left| (2 + 5x_i^2)x_i + 1 + \sum_{j \in J_i} x_j(1 + x_j) \right|^p,$$
$$p = 7/3, \quad J_i = \{j : j \neq i, \ \max(1, i-5) \leq j \leq \min(n, i+1)\},$$
$$\overline{x}_i = -1, \quad i \geq 1.$$

Problem 7. Seven-diagonal generalization of the Broyden tridiagonal function [7].

$$F(x) = \sum_{i=1}^{n} |(3 - 2x_i)x_i - x_{i-1} - x_{i+1} + 1|^p + \sum_{i=1}^{n/2} \left| x_i + x_{i+n/2} \right|^p,$$
$$p = 7/3, \quad x_0 = x_{n+1} = 0,$$
$$\overline{x}_i = -1, \quad i \geq 1.$$

Problem 8. Sparse modification of the Nazareth trigonometric function.

$$F(x) = \frac{1}{n} \sum_{i=1}^{n} \left(n + i - \sum_{j \in J_i} (a_{ij} \sin x_j + b_{ij} \cos x_j) \right)^2,$$
$$a_{ij} = 5[1 + \text{mod}(i, 5) + \text{mod}(j, 5)], \quad b_{ij} = (i + j)/10,$$
$$J_i = \{j : \max(1, i-2) \leq j \leq \min(n, i+2)\} \cup \{j : |j - i| = n/2\},$$
$$\overline{x}_i = 1/n, \quad i \geq 1.$$

Problem 9. Another trigonometric function.

$$F(x) = \frac{1}{n} \sum_{i=1}^{n} \left(i(1 - \cos x_i) + \sum_{j \in J_i} (a_{ij} \sin x_j + b_{ij} \cos x_j) \right),$$
$$a_{ij} = 5[1 + \text{mod}(i, 5) + \text{mod}(j, 5)], \quad b_{ij} = (i + j)/10,$$
$$J_i = \{j : \max(1, i-2) \leq j \leq \min(n, i+2)\} \cup \{j : |j - i| = n/2\},$$
$$\overline{x}_i = 1/n, \quad i \geq 1.$$

Problem 10. Toint trigonometric function [28].

$$F(x) = \frac{1}{n}\sum_{i=1}^{n}\sum_{j\in J_i} a_{ij}\sin(b_{ij} + c_i x_i + c_j x_j),$$

$$a_{ij} = 5[1 + \mathrm{mod}(i,5) + \mathrm{mod}(j,5)], \quad b_{ij} = (i+j)/10,$$

$$c_i = 1 + i/10, \quad c_j = 1 + j/10,$$

$$J_i = \{j : \max(1, i-2) \le j \le \min(n, i+2)\} \cup \{j : |j - i| = n/2\},$$

$$\overline{x}_i = 1, \ i \ge 1.$$

Problem 11. Augmented Lagrangian function [7].

$$F(x) = \sum_{i\in J}\left\{\exp\left(\prod_{j=1}^{5} x_{i+1-j}\right) + 10\left[\left(\sum_{j=1}^{5} x_{i+1-j}^2 - 10 - \lambda_1\right)^2\right.\right.$$

$$\left.\left. + \ (x_{i-3}x_{i-2} - 5x_{i-1}x_i - \lambda_2)^2 + (x_{i-4}^3 + x_{i-3}^3 + 1 - \lambda_3)^2\right]\right\},$$

$$\lambda_1 = -0.002008, \quad \lambda_2 = -0.001900, \quad \lambda_3 = -0.000261,$$

$$J = \{i, \ \mathrm{mod}(i,5) = 0\},$$

$$\overline{x}_i = -2, \ \mathrm{mod}(i,5) = 1, \ i \le 2, \quad \overline{x}_i = -1, \ \mathrm{mod}(i,5) = 1, \ i > 2,$$

$$\overline{x}_i = 2, \ \mathrm{mod}(i,5) = 2, \ i \le 2, \quad \overline{x}_i = -1, \ \mathrm{mod}(i,5) = 2, \ i > 2,$$

$$\overline{x}_i = 2, \ \mathrm{mod}(i,5) = 3, \quad \overline{x}_i = -1, \ \mathrm{mod}(i,5) = 4,$$

$$\overline{x}_i = -1, \ \mathrm{mod}(i,5) = 0.$$

Problem 12. Generalization of the Brown function 1 [7].

$$F(x) = \sum_{j=1}^{k}\left[(x_{i-1} - 3)^2/1000 - (x_{i-1} - x_i) + \exp(20(x_{i-1} - x_i))\right]$$

$$+ \left(\sum_{j=1}^{k}(x_{i-1} - 3)\right)^2,$$

$$i = 2j, \ k = n/2,$$

$$\overline{x}_i = 0, \ \mathrm{mod}(i,2) = 1, \quad \overline{x}_i = -1, \ \mathrm{mod}(i,2) = 0.$$

Problem 13. Generalization of the Brown function 2 [7].

$$F(x) = \sum_{j=1}^{k}\left[(x_{i-1}^2)^{(x_i^2+1)} + (x_i^2)^{(x_{i-1}^2+1)}\right],$$

$$i = 2j, \ k = n/2,$$

$$\overline{x}_i = -1, \ \mathrm{mod}(i,2) = 1, \quad \overline{x}_i = 1, \ \mathrm{mod}(i,2) = 0.$$

Problem 14. Discrete boundary value problem [23].

$$F(x) = \sum_{i=1}^{n}\left[2x_i - x_{i-1} - x_{i+1} + h^2(x_i + ih + 1)^3/2\right]^2,$$

$$h = 1/(n+1), \quad x_0 = x_{n+1} = 0,$$

$$\overline{x}_i = ih(1 - ih), \ i \ge 1.$$

Problem 15. Discretization of a variational problem [28].

$$
\begin{aligned}
F(x) &= 2\sum_{i=1}^{n}\left[(x_i(x_i - x_{i+1}))/h + 2h\sum_{i=0}^{n}[(\exp(x_{i+1}) - \exp(x_i))/(x_{i+1} - x_i)]\right], \\
h &= 1/(n+1), \quad x_0 = x_{n+1} = 0, \\
\overline{x}_i &= ih(1 - ih), \quad i \geq 1.
\end{aligned}
$$

Problem 16. Banded trigonometric problem.

$$
\begin{aligned}
F(x) &= \sum_{i=1}^{n} i\left[(1 - \cos x_i) + \sin x_{i-1} - \sin x_{i+1}\right], \\
x_0 &= x_{n+1} = 0, \\
\overline{x}_i &= 1, \quad i \geq 1.
\end{aligned}
$$

Problem 17. Variational problem 1 [10].

This problem is a finite difference analogue of a variational problem defined as a minimization of the functional

$$
F(x) = \int_0^1 \left[\frac{1}{2}\dot{x}^2(t) + \exp(x(t)) - 1\right] dt,
$$

where $x(0) = 0$ and $x(1) = 0$. We use the trapeziodal rule together with 3-point finite differences on a uniform grid having $n + 1$ internal nodes. The starting point is given by the formula $\overline{x}_i = x(t_i) = ih(1 - ih)$, where $h = 1/(n+1)$.

Problem 18. Variational problem 2 [10].

This problem is a finite difference analogue of a variational problem defined as a minimization of the functional

$$
F(x) = \int_0^1 \left[\dot{x}^2(t) - x^2(t) - 2tx(t)\right] dt,
$$

where $x(0) = 0$ and $x(1) = 0$. We use the trapeziodal rule together with 3-point finite differences on a uniform grid having $n + 1$ internal nodes. The starting point is given by the formula $\overline{x}_i = x(t_i) = ih(1 - ih)$, where $h = 1/(n+1)$.

Problem 19. Variational problem 3 [10].

This problem is a finite difference analogue of a variational problem defined as a minimization of the functional

$$
F(x) = \int_0^1 \left[\dot{x}^2(t) + x^2(t) + 2x(t)\exp(2t)\right] dt,
$$

where $x(0) = 1/3$ and $x(1) = \exp(2/3)$. We use the trapeziodal rule together with 3-point finite differences on a uniform grid having $n + 1$ internal nodes. The starting point is given by the formula $\overline{x}_i = x(t_i) = (ih\exp(2) + 1)/3$, where $h = 1/(n+1)$.

Problem 20. Variational problem 4 [10] (Calvar 3 [11]).

This problem is a finite difference analogue of a variational problem defined as a minimization of the functional

$$F(x) = \int_0^1 \left[\exp(-2x^2(t))(\dot{x}^2(t) - 1) \right] dt,$$

where $x(0) = 1$ and $x(1) = 0$. We use the trapeziodal rule together with 3-point finite differences on a uniform grid having $n + 1$ internal nodes. The starting point is given by the formula $\bar{x}_i = x(t_i) = 1 - ih$, where $h = 1/(n + 1)$.

Problem 21. Variational problem 5 [10] (Calvar 1 [11]).

This problem is a finite difference analogue of a variational problem defined as a minimization of the functional

$$F(x) = \int_0^1 \left[x^2(t) + \dot{x}(t) \arctan \dot{x}(t) - \log \sqrt{1 + \dot{x}^2(t)} \right] dt,$$

where $x(0) = 1$ and $x(1) = 2$. We use the trapeziodal rule together with 3-point finite differences on a uniform grid having $n + 1$ internal nodes. The starting point is given by the formula $\bar{x}_i = x(t_i) = ih + 1$, where $h = 1/(n + 1)$.

Problem 22. Variational problem Calvar 2 [11].

This problem is a finite difference analogue of a variational problem defined as a minimization of the functional

$$F(x) = \int_0^1 \left[100(x(t) - \dot{x}^2(t))^2 + (1 - \dot{x}(t))^2 \right] dt,$$

where $x(0) = 0$ and $x(1) = 0$. We use the trapeziodal rule together with 3-point finite differences on a uniform grid having $n + 1$ internal nodes. The starting point is given by the formula $\bar{x}_i = x(t_i) = ih(1 - ih)$, where $h = 1/(n + 1)$.

Problem 23. Penalty function 2 [23]

$$
\begin{aligned}
F(x) &= \left(x_1 - \frac{1}{5} \right)^2 + 10^{-5} \sum_{i=2}^{n} \left(\exp(10^{-2} x_i) + \exp(10^{-2} x_{i-1}) - y_i \right)^2 \\
&\quad + 10^{-5} \sum_{i=2}^{n} \left(\exp(10^{-2} x_i) - \exp(10^{-2}) \right)^2 + \left(\sum_{j=1}^{n} (n - j + 1) x_j^2 - 1 \right)^2, \\
y_i &= \exp(10^{-2} i) + \exp(10^{-2}(i - 1)), \quad 1 \le i \le n, \\
\bar{x}_0 &= 1/2, \quad l \ge 1.
\end{aligned}
$$

Problem 24. Penalty function 3 [11]

$$
\begin{aligned}
F(x) &= 1 + \exp(x_n) \sum_{i=1}^{n-2} (x_i + 2x_{i+1} + 10x_{i+2} - 1)^2 + \\
&\quad + \left(\sum_{i=1}^{n-2} (x_i + 2x_{i+2} + 10x_{i+2} - 1)^2 \right) \left(\sum_{i=1}^{n-2} (2x_i + x_{i+1} - 3)^2 \right) + \\
&\quad + \exp(x_{n-1}) \sum_{i=1}^{n-2} (2x_i + x_{i+1} - 3)^2 + \left(\sum_{i=1}^{n} (x_i^2 - n) \right)^2 + \sum_{i=1}^{n/2} (x_i - 1)^2, \\
\bar{x}_l &= l/(n + 1), \quad l \ge 1.
\end{aligned}
$$

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Problem 25. Extended Rosenbrock function [23].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n}f_k^2(x),$$

$$f_k(x) = 10(x_k^2 - x_{k+1}) \quad , \quad \mathrm{mod}(k,2) = 1$$

$$f_k(x) = x_{k-1} - 1 \qquad , \quad \mathrm{mod}(k,2) = 0$$

$$\bar{x}_l = -1.2, \quad \mathrm{mod}(l,2) = 1, \qquad \bar{x}_l = 1.0, \quad \mathrm{mod}(l,2) = 0$$

Problem 26. Extended Powell singular function [23].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n}f_k^2(x),$$

$$f_k(x) = x_k + 10x_{k+1} \qquad\qquad , \quad \mathrm{mod}(k,4) = 1,$$

$$f_k(x) = \sqrt{5}\,(x_{k+2} - x_{k+3}) \quad , \quad \mathrm{mod}(k,4) = 2,$$

$$f_k(x) = (x_{k+1} - 2x_{k+2})^2 \qquad , \quad \mathrm{mod}(k,4) = 3,$$

$$f_k(x) = \sqrt{10}\,(x_k - x_{k+3})^2 \quad , \quad \mathrm{mod}(k,4) = 0,$$

$$\bar{x}_l = 3, \quad \mathrm{mod}(l,4) = 1, \qquad \bar{x}_l = -1, \quad \mathrm{mod}(l,4) = 2,$$

$$\bar{x}_l = 0, \quad \mathrm{mod}(l,4) = 3, \qquad \bar{x}_l = 1, \quad \mathrm{mod}(l,4) = 0.$$

Problem 27. Penalty function 1 [23]

$$F(x) = \frac{1}{2}\sum_{k=1}^{m}f_k^2(x)\,,$$

$$f_k(x) = \frac{1}{\sqrt{100000}}\,(x_k - 1), \quad 1 \le k \le n,$$

$$f_k(x) = \sum_{i=1}^{n}x_i^2 - \frac{1}{4}, \quad k = n+1\,,$$

$$m = n+1\,,$$

$$\bar{x}_l = l, \quad l \ge 1.$$

Problem 28. Variably dimensioned function [23]

$$F(x) = \frac{1}{2}\sum_{k=1}^{m}f_k^2(x)\,,$$

$$f_k(x) = x_k - 1, \quad 1 \le k \le n\,,$$

$$f_k(x) = \sum_{i=1}^{n}i(x_i - 1), \quad k = n+1,$$

$$f_k(x) = \left(\sum_{i=1}^{n}i(x_i - 1)\right)^2, \quad k = n+2,$$

$$m = n+2,$$

$$\bar{x}_l = 1 - l/n, \quad l \ge 1.$$

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Problem 29. Brown almost linear function [23]

$$F(x) = \frac{1}{2}\sum_{k=1}^{m} f_k^2(x),$$

$$f_k(x) = x_k + \sum_{i=1}^{n} x_i - (n+1), \quad 1 \le k \le n,$$

$$f_k(x) = \left(\prod_{i=1}^{n} x_i\right) - 1, \quad k = n,$$

$$m = n,$$

$$\overline{x}_l = 1/2, \quad l \ge 1.$$

Problem 30. Discrete boundary value function [23]

$$F(x) = \frac{1}{2}\sum_{k=1}^{m} f_k^2(x),$$

$$f_k(x) = x_k + \frac{h}{2}\left[(1-kh)\sum_{i=1}^{k} ih(x_i + ih + 1)^3 + kh \sum_{i=k+1}^{n} (1-ih)(1+ih+1)^3\right],$$

$$m = n, \quad h = 1/(n+1),$$

$$\overline{x}_l = lh(lh-1), \quad l \ge 0.$$

Problem 31. Broyden tridiagonal function [23].

$$F(x) = \frac{1}{2}\sum_{k=1}^{m} f_k^2(x),$$

$$f_k(x) = (3-2x_k)x_k - x_{k-1} - 2x_{k+1} + 1,$$

$$m = n, \quad x_0 = x_{n+1} = 0,$$

$$\overline{x}_l = -1, \quad l \ge 1.$$

Problem 32. Generalized Broyden tridiagonal function [17].

$$F(x) = \frac{1}{2}\sum_{k=1}^{m} f_k^2(x),$$

$$f_k(x) = (3-2x_k)x_k + 1 - x_{k-1} - x_{k+1},$$

$$m = n, \quad x_0 = x_{n+1} = 0,$$

$$\overline{x}_l = -1, \quad l \ge 1.$$

Problem 33. Generalized Broyden banded function [17].

$$F(x) = \frac{1}{2}\sum_{k=1}^{m} f_k^2(x),$$

$$f_k(x) = (2+5x_k^2)x_k + 1 + \sum_{j=k_1}^{k_2} x_j(1+x_j),$$

$$m = n, \quad J_k = \{j : j \ne k, \ \max(1, k-5) \le j \le \min(n, k+1)\},$$

$$\overline{x}_l = -1, \quad l \ge 1.$$

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Problem 34. Chained Freudenstein and Roth function [30].

$$
\begin{aligned}
F(x) &= \frac{1}{2} \sum_{k=1}^{m} f_k^2(x), \\
f_k(x) &= x_i + x_{i+1}((5 - x_{i+1})x_{i+1} - 2) - 13 \quad, \quad \mathrm{mod}(k, 2) = 1, \\
f_k(x) &= x_i + x_{i+1}((1 + x_{i+1})x_{i+1} - 14) - 29, \quad \mathrm{mod}(k, 2) = 0, \\
m &= 2(n - 1), \quad i = \mathrm{div}(k + 1, 2), \\
\bar{x}_l &= 0.5, \quad l < n, \qquad \bar{x}_l = -2, \quad l = n.
\end{aligned}
$$

Problem 35. Wright and Holt zero residual problem [31].

$$
\begin{aligned}
F(x) &= \frac{1}{2} \sum_{k=1}^{m} f_k^2(x), \\
f_k(x) &= (x_i^a - x_j^b)^c, \\
a &= 1, \quad k \le m/2, \quad a = 2, \; k > m/2, \\
b &= 5 - \mathrm{div}(k, m/4), \quad c = \mathrm{mod}(k, 5) + 1, \\
m &= 5n, \quad i = \mathrm{mod}(k, n/2) + 1, \quad j = i + n/2, \\
\bar{x}_l &= \sin^2(l), \quad l \ge 1.
\end{aligned}
$$

Problem 36. Toint quadratic merging problem [30].

$$
\begin{aligned}
F(x) &= \frac{1}{2} \sum_{k=1}^{m} f_k^2(x), \\
f_k(x) &= x_i + 3x_{i+1}(x_{i+2} - 1) + x_{i+3}^2 - 1 \quad &, \quad \mathrm{mod}(k, 6) = 1, \\
f_k(x) &= (x_i + x_{i+1})^2 + (x_{i+2} - 1)^2 - x_{i+3} - 3 \quad &, \quad \mathrm{mod}(k, 6) = 2, \\
f_k(x) &= x_i x_{i+1} - x_{i+2} x_{i+3} \quad &, \quad \mathrm{mod}(k, 6) = 3, \\
f_k(x) &= 2x_i x_{i+2} + x_{i+1} x_{i+3} - 3 \quad &, \quad \mathrm{mod}(k, 6) = 4, \\
f_k(x) &= (x_i + x_{i+1} + x_{i+2} + x_{i+3})^2 + (x_i - 1)^2 \quad &, \quad \mathrm{mod}(k, 6) = 5, \\
f_k(x) &= x_i x_{i+1} x_{i+2} x_{i+3} + (x_{i+3} - 1)^2 - 1 \quad &, \quad \mathrm{mod}(k, 6) = 0, \\
m &= 3(n - 2), \quad i = 2 \, \mathrm{div}(k + 5, 6) - 1, \\
\bar{x}_l &= 5, \quad l \ge 1.
\end{aligned}
$$

Problem 37. Chained exponential problem [17].

$$
\begin{aligned}
F(x) &= \frac{1}{2} \sum_{k=1}^{m} f_k^2(x), \\
f_k(x) &= 4 - \exp(x_i) - \exp(x_{i+1}) \quad &, \quad \mathrm{mod}(k, 2) = 1, \; i = 1, \\
f_k(x) &= 8 - \exp(3x_{i-1}) - \exp(3x_i) \\
&\quad + 4 - \exp(x_i) - \exp(x_{i+1}) \quad &, \quad \mathrm{mod}(k, 2) = 1, \; 1 < i < n, \\
f_k(x) &= 8 - \exp(3x_{i-1}) - \exp(3x_i) \quad &, \quad \mathrm{mod}(k, 2) = 1, \; i = n, \\
f_k(x) &= 6 - \exp(2x_i) - \exp(2x_{i+1}) \quad &, \quad \mathrm{mod}(k, 2) = 0, \\
m &= 2n - 1, \quad i = \mathrm{div}(k + 1, 2), \\
\bar{x}_l &= 0.2, \quad l \ge 1.
\end{aligned}
$$

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Problem 38. Chained serpentine function [18].

$$F(x) = \frac{1}{2}\sum_{k=1}^{m} f_k^2(x),$$

$$
\begin{aligned}
f_k(x) &= 10(2x_i/(1+x_i^2) - x_{i+1}) &, \quad \mathrm{mod}(k,2) = 1, \\
f_k(x) &= x_i - 1 &, \quad \mathrm{mod}(k,2) = 0, \\
m &= 2(n-1), \quad i = \mathrm{div}(k+1,2), \\
\bar{x}_l &= -0.8, \quad l \geq 1.
\end{aligned}
$$

Problem 39. Chained and modified problem HS47 [18].

$$F(x) = \frac{1}{2}\sum_{k=1}^{m} f_k^2(x),$$

$$
\begin{aligned}
f_k(x) &= 10(x_i^2 - x_{i+1}) &, \quad \mathrm{mod}(k,6) = 1, \\
f_k(x) &= x_{i+2} - 1 &, \quad \mathrm{mod}(k,6) = 2, \\
f_k(x) &= (x_{i+3} - 1)^2 &, \quad \mathrm{mod}(k,6) = 3, \\
f_k(x) &= (x_{i+4} - 1)^3 &, \quad \mathrm{mod}(k,6) = 4, \\
f_k(x) &= x_i^2 x_{i+3} + \sin(x_{i+3} - x_{i+4}) - 10 &, \quad \mathrm{mod}(k,6) = 5, \\
f_k(x) &= x_{i+1} + x_{i+2}^4 x_{i+3}^2 - 20 &, \quad \mathrm{mod}(k,6) = 0, \\
m &= 6(\mathrm{div}(n-5,3) + 1), \quad i = 3\,\mathrm{div}(k+5,6) - 2, \\
\bar{x}_l &= -1, \quad l \geq 1.
\end{aligned}
$$

Problem 40. Chained and modified problem HS48 [18].

$$F(x) = \frac{1}{2}\sum_{k=1}^{m} f_k^2(x),$$

$$
\begin{aligned}
f_k(x) &= 10(x_i^2 - x_{i+1}) &, \quad \mathrm{mod}(k,7) = 1, \\
f_k(x) &= 10(x_{i+1}^2 - x_{i+2}) &, \quad \mathrm{mod}(k,7) = 2, \\
f_k(x) &= (x_{i+2} - x_{i+3})^2 &, \quad \mathrm{mod}(k,7) = 3, \\
f_k(x) &= (x_{i+3} - x_{i+4})^2 &, \quad \mathrm{mod}(k,7) = 4, \\
f_k(x) &= x_i + x_{i+1}^2 + x_{i+2} - 30 &, \quad \mathrm{mod}(k,7) = 5, \\
f_k(x) &= x_{i+1} - x_{i+2}^2 + x_{i+3} - 10 &, \quad \mathrm{mod}(k,7) = 6, \\
f_k(x) &= x_i x_{i+4} - 10 &, \quad \mathrm{mod}(k,7) = 0, \\
m &= 7(\mathrm{div}(n-5,3) + 1), \quad i = 3\,\mathrm{div}(k+6,7) - 2, \\
\bar{x}_l &= -1, \quad l \geq 1.
\end{aligned}
$$

Problem 41. Sparse signomial function [18].

$$F(x) = \frac{1}{2}\sum_{k=1}^{m} f_k^2(x),$$

$$f_k(x) = y_j - \sum_{p=1}^{3}(p^2/j)\prod_{q=1}^{4}\mathrm{sign}(x_{i+q})|x_{i+q}|^{q/(pj)},$$

$$
\begin{aligned}
m &= 4(\operatorname{div}(n-4,2)+1), \quad i = 2\operatorname{div}(k+3,4)-2, \quad j = \operatorname{mod}(k-1,4)+1, \\
\bar{x}_l &= -0.8 \quad , \quad \operatorname{mod}(l,4)=1, \quad y_1 = 14.4, \\
\bar{x}_l &= 1.2 \quad , \quad \operatorname{mod}(l,4)=2, \quad y_2 = 6.8, \\
\bar{x}_l &= -1.2 \quad , \quad \operatorname{mod}(l,4)=3, \quad y_3 = 4.2, \\
\bar{x}_l &= 0.8 \quad , \quad \operatorname{mod}(l,4)=0, \quad y_4 = 3.2.
\end{aligned}
$$

Problem 42. Sparse exponential function [18].

$$
F(x) = \frac{1}{2}\sum_{k=1}^{m} f_k^2(x),
$$

$$
f_k(x) = y_j - \sum_{p=1}^{3}(p^2/j)\exp\left(\sum_{q=1}^{4} x_{i+q}q/(pj)\right),
$$

$$
\begin{aligned}
m &= 4(\operatorname{div}(n-4,2)+1), \quad i = 2\operatorname{div}(k+3,4)-2, \quad j = \operatorname{mod}(k-1,4)+1, \\
\bar{x}_l &= -0.8 \quad , \quad \operatorname{mod}(l,4)=1, \quad y_1 = 35.8, \\
\bar{x}_l &= 1.2 \quad , \quad \operatorname{mod}(l,4)=2, \quad y_2 = 11.2, \\
\bar{x}_l &= -1.2 \quad , \quad \operatorname{mod}(l,4)=3, \quad y_3 = 6.2, \\
\bar{x}_l &= 0.8 \quad , \quad \operatorname{mod}(l,4)=0, \quad y_4 = 4.4.
\end{aligned}
$$

Problem 43. Sparse trigonometric function [18].

$$
F(x) = \frac{1}{2}\sum_{k=1}^{m} f_k^2(x),
$$

$$
f_k(x) = y_j - \sum_{q=1}^{4}[(-1)^q jq^2 \sin(x_{i+q}) + j^2 q\cos(x_{i+q})],
$$

$$
\begin{aligned}
m &= 4(\operatorname{div}(n-4,2)+1), \quad i = 2\operatorname{div}(k+3,4)-2, \quad j = \operatorname{mod}(k-1,4)+1, \\
\bar{x}_i &= -0.8 \quad , \quad \operatorname{mod}(l,4)=1, \quad y_1 = 30.6, \\
\bar{x}_i &= 1.2 \quad , \quad \operatorname{mod}(l,4)=2, \quad y_2 = 72.2, \\
\bar{x}_i &= -1.2 \quad , \quad \operatorname{mod}(l,4)=3, \quad y_3 = 124.4, \\
\bar{x}_i &= 0.8 \quad , \quad \operatorname{mod}(l,4)=0, \quad y_4 = 187.4.
\end{aligned}
$$

Problem 44. Countercurrent reactors problem 1 [6] (modified).

$$
\begin{aligned}
F(x) &= \frac{1}{2}\sum_{k=1}^{n} f_k^2(x), \\
f_k(x) &= \alpha - (1-\alpha)x_{k+2} - x_k(1+4x_{k+1}) && , \ k = 1, \\
f_k(x) &= -(2-\alpha)x_{k+2} - x_k(1+4x_{k-1}) && , \ k = 2, \\
f_k(x) &= \alpha x_{k-2} - (1-\alpha)x_{k+2} - x_k(1+4x_{k+1}) && , \ \operatorname{mod}(k,2)=1, \ 2 < k < n-1, \\
f_k(x) &= \alpha x_{k-2} - (2-\alpha)x_{k+2} - x_k(1+4x_{k-1}) && , \ \operatorname{mod}(k,2)=0, \ 2 < k < n-1, \\
f_k(x) &= \alpha x_{k-2} - x_k(1+4x_{k+1}) && , \ k = n-1, \\
f_k(x) &= \alpha x_{k-2} - (2-\alpha) - x_k(1+4x_{k-1}) && , \ k = n, \\
\alpha &= 1/2,
\end{aligned}
$$

$$\begin{aligned}
\overline{x}_l &= 0.1 \quad, \bmod(l,8)=1, & \overline{x}_l &= 0.2 \quad, \bmod(l,8)=2, \\
\overline{x}_l &= 0.3 \quad, \bmod(l,8)=3, & \overline{x}_l &= 0.4 \quad, \bmod(l,8)=4, \\
\overline{x}_l &= 0.5 \quad, \bmod(l,8)=5, & \overline{x}_l &= 0.4 \quad, \bmod(l,8)=6, \\
\overline{x}_l &= 0.3 \quad, \bmod(l,8)=7, & \overline{x}_l &= 0.2 \quad, \bmod(l,8)=0.
\end{aligned}$$

Problem 45. Tridiagonal system [16].

$$\begin{aligned}
F(x) &= \frac{1}{2}\sum_{k=1}^{n} f_k^2(x), \\
f_k(x) &= 4(x_k - x_{k+1}^2) & &, \ k=1, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) + 4(x_k - x_{k+1}^2) & &, \ 1<k<n, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1-x_k) & &, \ k=n, \\
\overline{x}_l &= 12, \quad l \geq 1.
\end{aligned}$$

Problem 46. Structured Jacobian problem [12].

$$\begin{aligned}
F(x) &= \frac{1}{2}\sum_{k=1}^{n} f_k^2(x), \\
f_k(x) &= -2x_k^2 + 3x_k - 2x_{k+1} + 3x_{n-4} - x_{n-3} \\
&\quad - x_{n-2} + 0.5x_{n-1} - x_n + 1 & &, \ k=1, \\
f_k(x) &= -2x_k^2 + 3x_k - x_{k-1} - 2x_{k+1} + 3x_{n-4} - x_{n-3} \\
&\quad - x_{n-2} + 0.5x_{n-1} - x_n + 1 & &, \ 1<k<n, \\
f_k(x) &= -2x_k^2 + 3x_k - x_{k-1} + 3x_{n-4} - x_{n-3} \\
&\quad - x_{n-2} + 0.5x_{n-1} - x_n + 1 & &, \ k=n, \\
\overline{x}_l &= -1, \quad l \geq 1.
\end{aligned}$$

Problem 47. Modified discrete boundary value problem [18].

$$\begin{aligned}
F(x) &= \frac{1}{2}\sum_{k=1}^{n} f_k^2(x), \\
f_k(x) &= 2x_k + (1/2)h^2(x_k + hk + 1)^3 - x_{k-1} - x_{k+1} + 1, \\
h &= 1/(n+1), \quad x_0 = x_{n+1} = 0, \\
\overline{x}_l &= lh(lh-1), \quad l \geq 1.
\end{aligned}$$

Problem 48. Chained and modified problem HS48 [18].

$$\begin{aligned}
F(x) &= \frac{1}{2}\sum_{k=1}^{m} f_k^2(x), \\
f_k(x) &= 10(x_i^2 - x_{i+1}) & &, \quad \bmod(k,7)=1, \\
f_k(x) &= x_{i+1} + x_{i+2} - 2 & &, \quad \bmod(k,7)=2, \\
f_k(x) &= x_{i+3} - 1 & &, \quad \bmod(k,7)=3,
\end{aligned}$$

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$$
\begin{aligned}
f_k(x) &= x_{i+4} - 1 & , & \quad \mathrm{mod}(k,7) = 4, \\
f_k(x) &= x_i + 3x_{i+1} & , & \quad \mathrm{mod}(k,7) = 5, \\
f_k(x) &= x_{i+2} + x_{i+3} - 2x_{i+4} & , & \quad \mathrm{mod}(k,7) = 6, \\
f_k(x) &= 10(x_{i+1}^2 - x_{i+4}) & , & \quad \mathrm{mod}(k,7) = 0, \\
m &= 7(\mathrm{div}(n-5,3)+1) & , & \quad i = 3\,\mathrm{div}(k+6,7) - 2, \\
\bar{x}_l &= -1, \quad l \geq 1.
\end{aligned}
$$

Problem 49. Attracting-Repelling problem [18].

$$
\begin{aligned}
F(x) &= \frac{1}{2}\sum_{k=1}^{m} f_k^2(x), \\
f_k(x) &= x_1 - 1 & , & \quad k = 1, \\
f_k(x) &= 10(x_i^2 - x_{i+1}), & , & \quad k > 1, \quad \mathrm{mod}(k,2) = 0, \\
f_k(x) &= 2\exp(-(x_i - x_{i+1})^2) + \exp(-2(x_{i+1} - x_{i+2})^2) , & \quad k > 1, \quad \mathrm{mod}(k,2) = 1, \\
m &= 2(n-1), \quad i = \mathrm{div}(k,2), \\
\bar{x}_l &= -1.2 \;,\; \mathrm{mod}(l,2) = 1, \qquad \bar{x}_l = 1.0 \;,\; \mathrm{mod}(l,2) = 0.
\end{aligned}
$$

Problem 50. Toint exponential-trigonometric merging problem [30].

$$
\begin{aligned}
F(x) &= \frac{1}{2}\sum_{k=1}^{m} f_k^2(x), \\
f_k(x) &= (1 + \cos(c))(x_j - \sin(x_i) - 1 + \sin(1))^2 \\
&\quad + 5(x_i - 1)\exp(\sin(c)x_j) + \frac{1}{2}\sum_{\substack{l=r \\ l \neq i,j}}^{r+6} \sin(x_l) - \sin(1), \\
s &= \min(\max(\mathrm{mod}(k,13)-2,1),7), \quad r = \mathrm{div}(k+12,13), \quad c = 3s/10, \\
m &= 13(n-6), \quad i = r+s-1, \quad j = r+s, \quad s < 7, \quad j = r, \quad s = 7, \\
\bar{x}_l &= 5, \quad l \geq 1.
\end{aligned}
$$

Problem 51. Countercurrent reactors problem 2 [6] (modified).

$$
\begin{aligned}
F(x) &= \frac{1}{2}\sum_{k=1}^{n} f_k^2(x), \\
f_k(x) &= x_1 - (1 - x_1)x_{k+2} - \alpha(1 + 4x_{k+1}) & , & \quad k = 1, \\
f_k(x) &= -(1 - x_1)x_{k+2} - \alpha(1 + 4x_k) & , & \quad k = 2, \\
f_k(x) &= \alpha x_1 - (1 - x_1)x_{k+2} - x_k(1 + 4x_{k-1}) & , & \quad k = 3, \\
f_k(x) &= x_1 x_{k-2} - (1 - x_1)x_{k+2} - x_k(1 + 4x_{k-1}) & , & \quad 3 < k < n-1, \\
f_k(x) &= x_1 x_{k-2} - x_k(1 + 4x_{k-1}) & , & \quad k = n-1, \\
f_k(x) &= x_1 x_{k-2} - (1 - x_1) - x_k(1 + 4x_{k-1}) & , & \quad k = n, \\
\alpha &= 0.414214, \\
\overline{x}_i &= 0.1 \;,\; \mathrm{mod}\,(i,8) = 1, \qquad \overline{x}_i = 0.2 \;,\; \mathrm{mod}\,(i,8) = 2, \\
\overline{x}_i &= 0.3 \;,\; \mathrm{mod}\,(i,8) = 3, \qquad \overline{x}_i = 0.4 \;,\; \mathrm{mod}\,(i,8) = 4,
\end{aligned}
$$

$$\overline{x}_i = 0.5 \ , \ \mathrm{mod}\,(i,8) = 5, \qquad \overline{x}_i = 0.4 \ , \ \mathrm{mod}\,(i,8) = 6,$$
$$\overline{x}_i = 0.3 \ , \ \mathrm{mod}\,(i,8) = 7, \qquad \overline{x}_i = 0.2 \ , \ \mathrm{mod}\,(i,8) = 0.$$

Problem 52. Trigonometric - exponential system (trigexp 1) [29].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$
$$f_k(x) = 3x_k^3 + 2x_{k+1} - 5 + \sin(x_k - x_{k+1})\sin(x_k + x_{k+1}) , \quad k = 1,$$
$$f_k(x) = 3x_k^3 + 2x_{k+1} - 5 + \sin(x_k - x_{k+1})\sin(x_k + x_{k+1})$$
$$+ \ 4x_k - x_{k-1}\exp(x_{k-1} - x_k) - 3 \qquad\qquad , \quad 1 < k < n,$$
$$f_k(x) = 4x_k - x_{k-1}\exp(x_{k-1} - x_k) - 3 \qquad\qquad , \quad k = n,$$
$$\overline{x}_i = 0, \quad i \geq 1.$$

Problem 53. Trigonometric - exponential system (trigexp 2) [29].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$
$$f_k(x) = 3(x_k - x_{k+2})^3 - 5 + 2x_{k+1},$$
$$+ \ \sin(x_k - x_{k+1} - x_{k+2})\sin(x_k + x_{k+1} - x_{k+2}) \qquad , \ \mathrm{mod}\,(k,2) = 1, \ k = 1,$$
$$f_k(x) = -6(x_{k-2} - x_k)^3 + 10 - 4x_{k-1}$$
$$- \ 2\sin(x_{k-2} - x_{k-1} - x_k)\sin(x_{k-2} + x_{k-1} - x_k)$$
$$+ \ 3(x_k - x_{k+2})^3 - 5 + 2x_{k+1}$$
$$+ \ \sin(x_k - x_{k+1} - x_{k+2})\sin(x_k + x_{k+1} - x_{k+2}) \qquad , \ \mathrm{mod}\,(k,2) = 1, \ 1 < k < n,$$
$$f_k(x) = -6(x_{k-2} - x_k)^3 + 10 - 4x_{k-1}$$
$$- \ 2\sin(x_{k-2} - x_{k-1} - x_k)\sin(x_{k-2} + x_{k-1} - x_k) \quad , \ \mathrm{mod}\,(k,2) = 1, \ k = n,$$
$$f_k(x) = 4x_k - (x_{k-1} - x_{k+1})\exp(x_{k-1} - x_k - x_{k+1}) - 3 \ , \ \mathrm{mod}\,(k,2) = 0,$$
$$\overline{x}_i = 1, \quad i \geq 1.$$

Problem 54. Singular Broyden problem [12].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$
$$f_k(x) = ((3 - 2x_k)x_k - 2x_{k+1} + 1)^2 \qquad\qquad , \ k = 1,$$
$$f_k(x) = ((3 - 2x_k)x_k - x_{k-1} - 2x_{k+1} + 1)^2 \qquad , \ 1 < k < n,$$
$$f_k(x) = ((3 - 2x_k)x_k - x_{k-1} + 1)^2 \qquad\qquad , \ k = n,$$
$$\overline{x}_i = -1, \quad i \geq 1.$$

Problem 55. Five-diagonal system [16].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$
$$f_k(x) = 4(x_k - x_{k+1}^2) + x_{k+1} - x_{k+2}^2 \qquad\qquad , \ k = 1,$$

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$$
\begin{aligned}
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1 - x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k+1} - x_{k+2}^2 &, \ k = 2, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1 - x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} - x_{k+2}^2 &, \ 2 < k < n - 1, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1 - x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} &, \ k = n - 1, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1 - x_k) + x_{k-1}^2 - x_{k-2} &, \ k = n, \\
\overline{x}_i &= -2, \quad i \geq 1.
\end{aligned}
$$

Problem 56. Seven-diagonal system [16].

$$
\begin{aligned}
F(x) &= \frac{1}{2} \sum_{k=1}^{n} f_k^2(x), \\
f_k(x) &= 4(x_k - x_{k+1}^2) + x_{k+1} - x_{k+2}^2 + x_{k+2} - x_{k+3}^2 &, \ k = 1, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1 - x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 + x_{k+1} - x_{k+2}^2 + x_{k+2} - x_{k+3}^2 &, \ k = 2, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1 - x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} - x_{k+2}^2 \\
&+ x_{k-2}^2 + x_{k+2} - x_{k+3}^2 &, \ k = 3, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1 - x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} - x_{k+2}^2 \\
&+ x_{k-2}^2 + x_{k+2} - x_{k-3} - x_{k+3}^2 &, \ 3 < k < n - 2, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1 - x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} - x_{k+2}^2 \\
&+ x_{k-2}^2 + x_{k+2} - x_{k-3} &, \ k = n - 2, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1 - x_k) \\
&+ 4(x_k - x_{k+1}^2) + x_{k-1}^2 - x_{k-2} + x_{k+1} \\
&+ x_{k-2}^2 - x_{k-3} &, \ k = n - 1, \\
f_k(x) &= 8x_k(x_k^2 - x_{k-1}) - 2(1 - x_k) + x_{k-1}^2 - x_{k-2} \\
&+ x_{k-2}^2 - x_{k-3} &, \ k = n, \\
\overline{x}_i &= -3, \quad i \geq 1.
\end{aligned}
$$

Problem 57. Extended Freudenstein and Roth function [5].

$$
\begin{aligned}
F(x) &= \frac{1}{2} \sum_{k=1}^{n} f_k^2(x), \\
f_k &= x_k + ((5 - x_{k+1})x_{k+1} - 2)x_{k+1} - 13 \ , \quad \mathrm{mod}(k, 2) = 1, \\
f_k &= x_{k-1} + ((x_k + 1)x_k - 14)x_k - 29 \quad , \quad \mathrm{mod}(k, 2) = 0, \\
\overline{x}_i &= 90 \ , \quad \mathrm{mod}(i, 2) = 1, \quad \overline{x}_i = 60 \ , \quad \mathrm{mod}(i, 2) = 0.
\end{aligned}
$$

Problem 58. Extended Cragg and Levy problem [23].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = (\exp(x_k) - x_{k+1})^2 \quad, \quad \mathrm{mod}(k,4) = 1,$$
$$f_k(x) = 10(x_k - x_{k+1})^3 \quad, \quad \mathrm{mod}(k,4) = 2,$$
$$f_k(x) = \tan^2(x_k - x_{k+1}) \quad, \quad \mathrm{mod}(k,4) = 3,$$
$$f_k(x) = x_k - 1 \quad, \quad \mathrm{mod}(k,4) = 0,$$
$$\overline{x}_i = 1 \ , \ \mathrm{mod}(i,4) = 1, \quad \overline{x}_i = 2 \ , \ \mathrm{mod}(i,4) \neq 1.$$

Problem 59. Broyden tridiagonal problem [23].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = x_k(0.5x_k - 3) + 2x_{k+1} - 1 \quad, \quad k = 1,$$
$$f_k(x) = x_k(0.5x_k - 3) + x_{k-1} + 2x_{k+1} - 1 \quad, \quad 1 < k < n,$$
$$f_k(x) = x_k(0.5x_k - 3) - 1 + x_{k-1} \quad, \quad k = n,$$
$$\overline{x}_i = -1, \quad i \geq 1.$$

Problem 60. Extended Powell badly scaled function [23].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = 10000 \, x_k \, x_{k+1} - 1 \quad, \quad \mathrm{mod}(k,2) = 1,$$
$$f_k(x) = \exp(-x_{k-1}) + \exp(-x_k) - 1.0001 \quad, \quad \mathrm{mod}(k,2) = 0,$$
$$\overline{x}_i = 0 \ , \ \mathrm{mod}(i,2) = 1, \quad \overline{x}_i = 1 \ , \ \mathrm{mod}(i,2) = 0.$$

Problem 61. Extended Wood problem [13].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = -200x_k(x_{k+1} - x_k^2) - (1 - x_k) \quad, \quad \mathrm{mod}(k,4) = 1,$$
$$f_k(x) = 200(x_k - x_{k-1}^2) + 20.2(x_k - 1) + 19.8(x_{k+2} - 1) \quad, \quad \mathrm{mod}(k,4) = 2,$$
$$f_k(x) = -180x_k(x_{k+1} - x_k^2) - (1 - x_k) \quad, \quad \mathrm{mod}(k,4) = 3,$$
$$f_k(x) = 180(x_k - x_{k-1}^2) + 20.2(x_k - 1) + 19.8(x_{k-2} - 1) \quad, \quad \mathrm{mod}(k,4) = 0,$$
$$\overline{x}_i = -3, \ \mathrm{mod}(i,2) = 1, \quad \overline{x}_i = -1, \ \mathrm{mod}(i,2) = 0.$$

Problem 62. Tridiagonal exponential problem [5].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = x_k - \exp(\cos(k(x_k + x_{k+1}))) \quad, \quad k = 1,$$
$$f_k(x) = x_k - \exp(\cos(k(x_{k-1} + x_k + x_{k+1}))) \quad, \quad 1 < k < n,$$
$$f_k(x) = x_k - \exp(\cos(k(x_{k-1} + x_k))) \quad, \quad k = n,$$
$$\overline{x}_i = 1.5, \quad i \geq 1.$$

Problem 63. Brent problem [4].

$$F(x) = \frac{1}{2} \sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = 3x_k(x_{k+1} - 2x_k) + x_{k+1}^2/4 \qquad\qquad , \; k = 1,$$

$$f_k(x) = 3x_k(x_{k+1} - 2x_k + x_{k-1}) + (x_{k+1} - x_{k-1})^2/4 \;\;, \; 1 < k < n,$$

$$f_k(x) = 3x_k(20 - 2x_k + x_{k-1}) + (20 - x_{k-1})^2/4 \qquad , \; k = n,$$

$$\overline{x}_i = 10, \quad i \geq 1.$$

Problem 64. Troesch problem [26].

$$F(x) = \frac{1}{2} \sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = 2x_k + \rho h^2 \sinh(\rho x_k) - x_{k+1} \qquad\qquad , \; k = 1,$$

$$f_k(x) = 2x_k + \rho h^2 \sinh(\rho x_k) - x_{k-1} - x_{k+1} \;\;, \; 1 < k < n,$$

$$f_k(x) = 2x_k + \rho h^2 \sinh(\rho x_k) - x_{k-1} - 1 \qquad , \; k = n,$$

$$\rho = 10, \quad h = 1/(n+1),$$

$$\overline{x}_i = 1, \quad i \geq 1.$$

Problem 65. Flow in a channel [3].

$$F(x) = \frac{1}{2} f^T(x) f(x),$$

where equation $f(x) = 0$ is a finite difference analogue of the following nonlinear ordinary differential equation

$$u'''' = R(u' u'' - u u'''), \quad R = 500$$

over unit interval Ω with boundary conditions $u(0) = 0$, $u'(0) = 0$, $u(1) = 1$, $u'(1) = 0$. We use standard 5-point finite differences on an uniform grid having 5000 internal nodes. The initial approximate solution is a discretization of $u_0(x) = (x - 1/2)^2$.

Problem 66. Swirling flow [3].

$$F(x) = \frac{1}{2} f^T(x) f(x),$$

where equation $f(x) = 0$ is a finite difference analogue of the following system of two nonlinear ordinary differential equations

$$u'''' + R(u u''' + v v') = 0$$
$$v'' + R(u v' + u' v) = 0, \quad R = 500$$

over unit interval Ω with boundary conditions $u(0) = u'(0) = u(1) = u'(1) = 0$, $v(0) = -1$, $v(1) = 1$. We use standard 5-point finite differences on an uniform grid having 2500 internal nodes. The initial approximate solution is a discretization of $u_0(x) = (x - 1/2)^2$ and $v_0(x) = x - 1/2$.

Problem 67. Bratu problem [14].

$$F(x) = \frac{1}{2}f^T(x)f(x),$$

where equation $f(x) = 0$ is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u + R\exp(u) = 0, \quad R = 6.8$$

over unit square Ω with Dirichlet boundary conditions $u = 0$ on $\partial\Omega$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x, y) = 0$.

Problem 68. Poisson problem 1 [12].

$$F(x) = \frac{1}{2}f^T(x)f(x),$$

where equation $f(x) = 0$ is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u = \frac{u^3}{1 + x^2 + y^2}$$

over unit square Ω with Dirichlet boundary conditions $u(0, y) = 1$, $u(1, y) = 2 - \exp(y)$, $u(x, 0) = 1$, $u(x, 1) = 2 - \exp(x)$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x, y) = -1$.

Problem 69. Poisson problem 2 [22].

$$F(x) = \frac{1}{2}f^T(x)f(x),$$

where equation $f(x) = 0$ is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u + \sin(2\pi u) + \sin\left(2\pi\frac{\partial u}{\partial x}\right) + \sin\left(2\pi\frac{\partial u}{\partial y}\right) + f(x, y) = 0,$$

where $f(x, y) = 1000((x-1/4)^2 + (y-3/4)^2)$, over unit square Ω with Dirichlet boundary conditions $u = 0$ on $\partial\Omega$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x, y) = 0$.

Problem 70. Porous medium problem [8].

$$F(x) = \frac{1}{2}f^T(x)f(x),$$

where equation $f(x) = 0$ is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u^2 + R\left(\frac{\partial u^3}{\partial x} + f(x, y),\right) = 0, \quad R = 50,$$

18

where $f(1/71, 1/71) = 1$ and $f(x, y) = 0$ for $(x, y) \neq (1/71, 1/71)$, over unit square Ω with Dirichlet boundary conditions $u(0, y) = 1$, $u(1, y) = 0$, $u(x, 0) = 1$, $u(x, 1) = 0$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x, y) = 1 - xy$.

Problem 71. Convection-difussion problem [15].

$$F(x) = \frac{1}{2} f^T(x) f(x),$$

where equation $f(x) = 0$ is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta u - Ru \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) + f(x, y) = 0, \quad R = 20,$$

where $f(x, y) = 2000x(1 - x)y(1 - y)$, over unit square Ω with Dirichlet boundary conditions $u = 0$ on $\partial\Omega$. We use standard 5-point finite differences on a uniform grid having 70×70 internal nodes. The initial approximate solution is a discretization of $u_0(x, y) = 0$.

Problem 72. Nonlinear biharmonic problem [19].

$$F(x) = \frac{1}{2} f^T(x) f(x),$$

where equation $f(x) = 0$ is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta\Delta u + R \left(\max(0, u) + \text{sign}(x - \frac{1}{2}) \right) = 0, \quad R = 500$$

over unit square Ω with the boundary conditions $u = 0$ on $\partial\Omega$ and $\partial u(0, y)/\partial x = 0$, $\partial u(1, y)/\partial x = 0$, $\partial u(x, 0)/\partial y = 0$, $\partial u(x, 1)/\partial y = 0$. We use standard 13-point finite differences on a shifted uniform grid having 50×50 internal nodes [14]. The initial approximate solution is a discretization of $u_0(x, y) = 0$.

Problem 73. Driven cavity problem [14].

$$F(x) = \frac{1}{2} f^T(x) f(x),$$

where equation $f(x) = 0$ is a finite difference analogue of the following nonlinear partial differential equation

$$\Delta\Delta u + R \left(\frac{\partial u}{\partial y} \frac{\partial \Delta u}{\partial x} - \frac{\partial u}{\partial x} \frac{\partial \Delta u}{\partial y} \right) = 0, \quad R = 500$$

over unit square Ω with the boundary conditions $u = 0$ on $\partial\Omega$ and $\partial u(0, y)/\partial x = 0$, $\partial u(1, y)/\partial x = 0$, $\partial u(x, 0)/\partial y = 0$, $\partial u(x, 1)/\partial y = 1$. We use standard 13-point finite differences on a shifted uniform grid having 50×50 internal nodes [14]. The initial approximate solution is a discretization of $u_0(x, y) = 0$.

Problem 74.

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = x_k - 1 - \frac{1}{5n}x_k\left(1 + \frac{k}{k+n}x_n + 2\sum_{i=1}^{n-1}\frac{k}{k+i}x_i\right),$$

$$\overline{x}_l = 1, \quad \mathrm{mod}(l, 2) = 1, \quad \overline{x}_l = 3, \quad \mathrm{mod}(l, 2) = 0.$$

Problem 75. Problem 201 in [27].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = x_k - 1 \qquad\qquad, \quad k = 1,$$

$$f_k(x) = 10(k-1)(x_k - x_{k-1})^2, \quad 1 < k \le n,$$

$$\overline{x}_l = -1.2, \quad 1 \le l < n, \quad x_l = -1, \quad l = n.$$

Problem 76. Problem 202 in [27].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = x_k - \frac{x_{k+1}^2}{10}, \quad 1 \le k < n,$$

$$f_k(x) = x_k - \frac{x_1^2}{10}, \quad k = n,$$

$$\overline{x}_l = 2, \quad l \ge 1.$$

Problem 77. Problem 205 in [27].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = x_k - \frac{\displaystyle\sum_{i=1}^{n} x_i^3 + k}{2n}, \quad 1 \le k \le n,$$

$$\overline{x}_l = 3/2, \quad l \ge 1.$$

Problem 78. Problem 206 in [27].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = x_{k-1} - 2x_k + x_{k+1} - h^2\exp(x_k), \quad 1 \le k \le n,$$

$$h = 1/(n+1), \quad x_0 = x_{n+1} = 0,$$

$$\overline{x}_l = 1, \quad l \ge 1.$$

Problem 79. Problem 207 in [27].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$

$$f_k(x) = (3 - x_k/10)x_k + 1 - x_{k-1} - 2x_{k+1}, \quad 1 \le k \le n,$$
$$x_0 = x_{n+1} = 0,$$
$$\overline{x}_l = -1, \quad l \ge 1.$$

Problem 80. Problem 208 in [27].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$
$$f_k(x) = (1 + x_k^2)x_k + 1 - \sum_{i \in I_k}(x_i + x_i^2), \quad 1 \le k \le n,$$
$$I_k = \{i : i \ne k, \max(1, k-3) \le i \le \min(n, k+3)\},$$
$$\overline{x}_l = -1, \quad l \ge 1.$$

Problem 81. Problem 209 in [27].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$
$$f_k(x) = x_k^2 - 1 \quad\quad, \quad k = 1,$$
$$f_k(x) = x_{k-1}^2 + \log x_k - 1, \quad 1 < k \le n,$$
$$\overline{x}_l = 1/2, \quad l \ge 1.$$

Problem 82. Problem 212 in [27].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$
$$f_k(x) = x_k \quad\quad\quad, \quad k = 1,$$
$$f_k(x) = \cos(x_{k-1}) + x_k - 1), \quad 1 < k \le n,$$
$$\overline{x}_l = 1/2, \quad l \ge 1.$$

Problem 83. Problem 213 in [27].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k^2(x),$$
$$f_k(x) = 2x_k + h^2(x_k + \sin(x_k)) - x_{k-1} - x_{k+1}, \quad 1 \le k \le n,$$
$$h = 1/(n+1), \quad x_0 = 0, \quad x_{n+1} = 1,$$
$$\overline{x}_l = 1, \quad l \ge 1.$$

Problem 84. Problem 214 in [27].

$$F(x) = \frac{1}{2}\sum_{k=1}^{n} f_k(x),$$
$$f_k(x) = x_k(2 + 5x_k^2) + 1 - \sum_{i \in I_k}x_i(1 + x_i), \quad 1 \le k \le n,$$
$$I_k = \{i : i \ne k, \max(1, k-5) \le i \le \min(n, k+1)\},$$
$$\overline{x}_l = -1, \quad l \ge 0.$$

Problem 85. Gheri and Mancino problem [9].

$$F(x) = \frac{1}{2} \sum_{k=1}^{n} f_k(x),$$

$$f_k(x) = 14nx_k + \left(k - \frac{n}{2}\right)^3 +$$

$$+ \sum_{i=1,i\neq k}^{n} z_{ki} \left[\sin^5 \log(z_{ki}) + \cos^5 \log(z_{ki})\right], \quad 1 \leq k \leq n,$$

$$z_{ki} = \sqrt{x_i^2 + k/i}, \quad 1 \leq k \leq n, \quad 1 \leq i \leq n,$$

$$\overline{x}_l = -f_l(0)\frac{a+b}{2ab}, \quad 1 \leq i \leq l.$$

$$a = 14n - 6(n-1), \quad b = 14n + 6(n-1).$$

Problem 86. Ortega and Rheinboldt problem [24].

$$F(x) = \frac{1}{2} \sum_{k=1}^{n} f_k(x),$$

$$f_k(x) = \left(1 - \frac{1}{8n}\right)x_k - 1 - \sum_{i=1}^{n} a_{ki}x_kx_i, \quad 1 \leq k \leq n,$$

$$a_{ki} = \frac{k}{2n}\frac{1}{2(k+i)}, \quad 1 \leq i < n,$$

$$a_{ki} = \frac{k}{2n}\frac{1}{4(k+i)}, \quad i = n,$$

$$\overline{x}_l = 1, \quad l \geq 1.$$

Problem 87. Ascher and Russel boundary value problem [2].

$$F(x) = \frac{1}{2} \sum_{k=1}^{n} f_k(x),$$

$$f_k(x) = 2x_k - 2h^2\left(x_k^2 + \frac{x_{k+1} - x_{k-1}}{2h}\right) - x_{k-1} - x_{k+1}, \quad 1 \leq k \leq n,$$

$$h = 1/(n+1), \quad x_0 = 0, \quad x_{n+1} = 1/2,$$

$$\overline{x}_l = 1, \quad l \geq 1.$$

Problem 88. Ascher and Russel boundary value problem [2].

$$F(x) = \frac{1}{2} \sum_{k=1}^{n} f_k(x),$$

$$f_k(x) = x_k - \frac{kh}{2} - 2h^2\sum_{i=1}^{k} i(1-kh)\left(x_i^2 + \frac{x_{i+1} - x_{i-1}}{2h}\right) -$$

$$-2h^2\sum_{i=k+1}^{n} k(1-ih)\left(x_i^2 + \frac{x_{i+1} - x_{i-1}}{2h}\right), \quad 1 \leq k \leq n,$$

$$h = 1/(n+1), \quad x_0 = 0, \quad x_{n+1} = 1/2,$$

$$\overline{x}_l = 1, \quad l \geq 1.$$

Problem 89. Allgower and Georg boundary value problem [1].

$$
\begin{aligned}
F(x) &= \frac{1}{2}\sum_{k=1}^{n} f_k(x)\,, \\
f_k(x) &= 2x_k + 0.3h^2\left[\exp(20(x_k + 25(kh - 1))) - \exp(-20(x_k + 25kh)) - t_k\right] \\
&\quad -x_{k-1} - x_{k+1}\,, \\
t_k &= \mathrm{sign}(kh - 0.009)\,, \quad k \geq 1\,, \\
h &= 0.01/(n+1)\,, \quad x_0 = 0\,, \quad x_{n+1} = 25\,, \\
\overline{x}_l &= 1\,, \quad l \geq 1\,.
\end{aligned}
$$

Problem 90. Potra and Rheinboldt boundary value problem [25].

$$
\begin{aligned}
F(x) &= \frac{1}{2}\sum_{k=1}^{n} f_k(x)\,, \\
f_k(x) &= 2x_k - x_{k-1} - x_{k+1} + h^2(x_k^2 + x_k + 0.1x_{k+n/2} - 1.2)\,, \quad 1 \leq k \leq n/2\,, \\
f_k(x) &= 2x_k - x_{k-1} - x_{k+1} + h^2(0.2x_{k-n/2}^2 + x_k^2 + 2x_k - 0.6)\,, \quad n/2 < k \leq n\,, \\
h &= 1/(n/2 + 1)\,, \quad x_0 = x_{n+1} = 0\,, \\
\overline{x}_l &= lh(1 - lh)\,, \quad \overline{x}_{l+n/2} = \overline{x}_l\,, \quad 1 \leq l \leq n/2\,.
\end{aligned}
$$

Problem 91.

$$
\begin{aligned}
F(x) &= \frac{1}{2}\sum_{k=1}^{n} f_k(x)\,, \\
f_k(x) &= 4x_k - x_{k-1} - x_{k+1} - x_{k-\sqrt{n}} - x_{k+\sqrt{n}} + h^2\exp(x_k)\,, \quad 1 \leq k \leq n\,, \\
h &= 1/(\sqrt{n} + 1)\,, \quad x_l = 0 \quad \text{for} \quad l < 1 \quad \text{or} \quad l > n, \\
\overline{x}_l &= 1\,, \quad 1 \leq l \leq n\,.
\end{aligned}
$$

Problem 92.

$$
\begin{aligned}
F(x) &= \frac{1}{2}\sum_{k=1}^{n} f_k(x)\,, \\
f_k(x) &= 4x_k - x_{k-1} - x_{k+1} - x_{k-\sqrt{n}} - x_{k+\sqrt{n}} + h^2 x_k^2 - y_k\,, \quad 1 \leq k \leq n\,, \\
h &= 1/(\sqrt{n} + 1)\,, \quad x_l = 0 \quad \text{for} \quad l < 1 \quad \text{or} \quad l > n, \\
\overline{x}_l &= 1\,, \quad 1 \leq l \leq n\,.
\end{aligned}
$$

References

[1] Allgover, E.L., and Georg, K., *Computational Solution of Nonlinear Systems of Equations,* Lectures in Applied Mathematics 26, American Mathematical Society, 1990.

[2] Ascher, U.M., and Russel, R.D., *Numerical Boundary Value ODEs,* Birkhauser, Boston 1985.

[3] Averick, B.M., and Carter, R.G., and Moré, J.J., *The Minpack-2 Test Problem Collection,* Research Report No. ANL/MCS-TM-150, Mathematics and Computer Science Division, Argonne National Laboratory, Argonne 1991.

[4] Alefeld, G., and Gienger, A., and Potra, F., *Efficient Validation of Solutions of Nonlinear Systems,* SIAM Journal on Numerical Analysis, Vol 31, pp. 252-260, 1994.

[5] Bing, Y., Lin, G., *An Efficient Implementation of Merrill's Method for Sparse or Partially Separable Systems of Nonlinear Equations,* SIAM Journal on Optimization, Vol. 2, pp. 206-221, 1991.

[6] Bogle, I.D.L., Perkins, J.D., *A New Sparsity Preserving Quasi-Newton Update for Solving Nonlinear Equations,* SIAM Journal on Scientific and Statistical Computations, Vol. 11, pp. 621-630, 1990.

[7] Conn, A.R., Gould, N.I.M, Toint, P., *Testing a Class of Methods for Solving Minimization Problems with Simple Bounds on the Variables,* Mathematics of Computation, Vol. 50, pp. 399-430, 1988.

[8] Eisenstat, S.C., Walker, H.F., *Choosing the Forcing Terms in an Inexact Newton Method,* SIAM Journal on Scientific Computation, Vol. 17, pp. 16-32, 1996.

[9] Gheri, G., and Mancino, O.G., *A Significant example to test solving systems of nonlinear equations,* Calcolo Vol. 8, pp. 107-114, 1971.

[10] Gill, P.E., Murray, W., *The Numerical Solution of a Problem in the Calculus of Variations,* Technical Report NAC 28, National Physical Laboratory, Teddington, Middlesex, England 1972.

[11] Gill, P.E., Murray, W., *Conjugate-Gradient Methods for Large-Scale Nonlinear Optimization,* Technical Report SOL 79-15, System Optimization Laboratory, Department of Operations Research, Stanford University, Stanford, California, 1979.

[12] Gomez-Ruggiero, M.A., Martinez, J.M., and Moretti, A.C., *Comparing Algorithms for Solving Sparse Nonlinear Systems of Equations,* SIAM Journal on Scientific and Statistical Computations, Vol. 13, pp. 459-483, 1992.

[13] Incerti, S., Zirilli, F., and Parisi, V., *Algorithm 111. A Fortran Subroutine for Solving Systems of Nonlinear Simultaneous Equations,* Computer Journal, Vol 24, pp. 87-91, 1981.

[14] Kaporin, I.E., Axelsson, O., *On a Class of Nonlinear Equation Solvers Based on the Residual Norm Reduction Over a Sequence of Affine Subspaces,* SIAM Journal on Scientific and Statistical Computations, Vol 16, pp. 228-249, 1995.

[15] Kelley, C.T., *Iterative Methods for Linear and Nonlinear Equations,* SIAM, Philadelphia, Pennsylvania, 1995.

[16] Li, G., *Successive Column Correction Algorithms for Solving Sparse Nonlinear Systems of Equations,* Mathematical Programming, Vol. 43, pp. 187-207, 1989.

[17] Lukšan, L., *Inexact Trust Region Method for Large Sparse Nonlinear Least Squares,* Kybernetika, Vol. 29, pp. 305-324, 1993.

[18] Lukšan L., *Hybrid Methods for Large Sparse Nonlinear Least Squares,* Journal of Optimization Theory and Applications, Vol. 89, pp. 575-595, 1996.

[19] Lukšan, L., Vlček, J., *Computational Experience with Globally Convergent Descent Methods for Large Sparse Systems of Nonlinear Equations,* Optimization Methods and Software, Vol. 8, pp. 201-223, 1998.

[20] Lukšan, L., Vlček, J., *Sparse and Partially Separable Test Problems for Unconstrained and Equality Constrained Optimization,* Research Report No. V-767, Institute of Computer Science, Academy of Sciences of the Czech Republic, Prague, Czech Republic, 1998.

[21] Lukšan, L., Šiška, M., Tůma, M., Vlček, J., Ramešová, N., *Interactive System for Universal Functional Optimization (UFO), Version 2002,* Research Report No. V-883, Institute of Computer Science, Academy of Sciences of the Czech Republic, Prague, Czech Republic, 2002.

[22] Martinez, J.M., *A Quasi-Newton Method with Modification of One Column per Iteration,* Computing, Vol. 33, pp. 353-362, 1984.

[23] Moré, J.J., Garbow, B.S., Hillström, K.E., *Testing Unconstrained Optimization Software,* ACM Transactions on Mathematical Software, Vol. 7, pp. 17-41, 1981.

[24] Ortega, J.M., and Rheinboldt, W.C., *Iterative Solution of Nonlinear Equitions in Several Variables,* Academic Press, New York, 1970.

[25] Potra, F.A., and Rheinboldt, W.C., *On the Monotone Convergence of Newton's Method, Computing, Vol 36, pp. 81-90, 1986.*

[26] Roberts, S.M., and Shipman, J.S., On the Closed Form Solution of Troesch's Problem, Journal of Computational Physics, Vol. 21, pp. 291-304, 1976.

[27] Roose, A., Kulla, V., Lomp, M., Meressoo, T., *Test Examples for systems of nonlinear equations,* Estonian Software and Computer Service Company, Tallin 1990.

[28] Toint, P.L., *Some Numerical Results Using a Sparse Matrix Updating Formula in Unconstrained Optimization,* Mathematics of Computation, Vol. 143, pp. 839-851, 1978.

[29] Toint, P.L., *Numerical Solution of Large Sets of Algebraic Equations,* Mathematics of Computation, Vol. 46, pp. 175-189, 1986.

[30] Toint, P.L., *On Large Scale Nonlinear Least Squares Calculations,* SIAM Journal on Scientific and Statistical Computations, Vol. 8, pp. 416-435, 1987.

[31] Wright, S.J., Holt, J.N., *An Inexact Levenberg-Marquardt Method for Large Sparse Nonlinear Least Squares,* Journal of the Australian Mathematical Society, Vol. B26, pp. 387-403, 1985.