

Numerical optimization for large scale problems

UNCONSTRAINED OPTIMIZATION

a.y. 2023/24

PLEASE, CAREFULLY READ ALL THE INSTRUCTIONS BEFORE
PREPARING AND SUBMITTING YOUR REPORT

Recall that the homework allows you to score up to 12 points, according both to the report contents and defense. Suitably addressing all the points reported, allows you to score the corresponding points.

1 Assignment

- i. Implement **exactly two** out of the following numerical methods for unconstrained optimization:

- (a) [2 points] Nelder-Mead
- (b) [1 points] Steepest descent method
- (c) [1 points] Nonlinear conjugate gradient method (either Fletcher and Reeves or Polak-Ribière)
- (d) [2 points] Modified Newton method
- (e) [2 points] Modified Newton method with finite differences
- (f) [1 points] Inexact Newton method (*)
- (g) [2 points] Truncated Newton method

In all the cases (except for Nelder-Mead), complement the method with a backtracking strategy for the line search, imposing sufficient decrease condition with parameters $\rho = 0.5$ and $c = 10^{-4}$. Try to tune the parameters, if it is not working well.

- ii. [2 points] Test your implementations on the Rosenbrock function

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2$$

with starting point $x_0 = (1.2, 1.2)$ and $x_0 = (-1.2, 1)$, reporting the behaviour.

- iii. [6 points] Apply the codes to **exactly three** test problems taken from

https://www.researchgate.net/publication/325314497_Test_Problems_for_Unconstrained_Optimization

using several starting points, and using **large** values of the dimension n (i.e., $n = 10^d$, d at least 3 or 4).

Realize a throughout comparison among the methods, reporting data in tables and/or figures. Comment your results also in view of the expected theoretical behaviour. The comparison should be performed comparing, for each test problem:

- (a) number of failures/successfull runs;
- (b) number of iterations to satisfy a fixed stopping criterion;
- (c) expertimental rate of convergence;
- (d) execution time;
- (e) whatever else you think it may be useful to understand the behavior of the methods.

Recommendations:

- (a) in case of inexact Newton method: use at least two different forcing terms which are expected to yield different convergence rate, and report number of both inner and outer iterations;
- (b) whenever you use iterative solver for solving linear systems, report results both without preconditioning and preconditioning.

2 Guidelines for writing the report

- i. The document is expected to report:
 - (a) an introductory analysis of the problem
 - (b) a brief description of the methods
 - (c) tables and/or figures summarizing your results
 - (d) comments on your results.
- ii. Please use **captions** in order to explain what every table and/or figure is reporting, and quote it also in the text (e.g., "In Figure xx we report the plot of...", "In Table yy we compare ...").
- iii. In general you are expected to test your solvers on some common problems, with different values of some parameters and possibly different starting points. In all the cases you should compare the results obtained, for example in terms of number of iterations and computing time, commenting your results also in view of the values of the parameters used and of the theory.
- iv. As an **appendix** of the report, please add the commented scripts/functions you implemented in your favorite programming language. Please make sure to use sensible names for the variables and functions, and to provide enough comments and explanations to render the code readable to a non expert of the specific language.

3 Submission guidelines

- i. You are expected to submit **a single pdf file** per group (and **not** per person). **Avoid** compressed folders. If you upload something which is not a single pdf file, I'll ask you to resubmit.
- ii. The file should be submitted through the **Consegna elaborati** tab on the course page.
- iii. If you work in a group (max 3 people) please upload the file **only once** but clearly state in the file name the family names of all team-mates.
- iv. The **deadline** for submission is **one week before the date of the official call** at which you aim at taking the exam.
- v. Despite it is recommended that people in the same group takes the exam in the same call, you are allowed to take the exam in different calls, by submitting the report only once.