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Assignment 10

$x'' = x \sin t + x' \cos t - \exp(t)$ ——(1)   for $t \in (0,1)$

Replacing, $x'' = \dfrac{1}{h^2}\left[x_{i+1} - 2x_i + x_{i-1}\right]$

$x' = \dfrac{1}{2h}\left[x_{i+1} - x_{i-1}\right]$

$[at\ x_i, t_i]$

Putting these in the given equation,

$\dfrac{1}{h^2}\left[x_{i+1} - 2x_i + x_{i-1}\right] = x_i \sin t_i + \dfrac{1}{2h}\left[x_{i+1} - x_{i-1}\right]\cos t_i - \exp(t_i)$

$\Rightarrow x_{i+1} - 2x_i + x_{i-1} = h^2 \times x_i \sin t_i + \dfrac{h}{2}\left[x_{i+1} - x_{i-1}\right]\cos t_i - h^2 \exp(t_i)$

$\Rightarrow x_{i+1}\left[1 - \dfrac{h}{2}\cos t_i\right] + x_i\left[-2 - h^2(\sin(t_i))\right]$

$+ x_{i-1}\left[1 + \dfrac{h}{2}\cos t_i\right] = -h^2 \exp(t_i)$ ——(2)

Now, let. $C_i = 1 - \dfrac{h}{2}\cos t_i$ $\bigg|$ $x_0 = 0$ &

$d_i = -2 - h^2 \sin t_i$ $\bigg|$ $x_n = 1$.

$a_i = 1 + \dfrac{h}{2}\cos t_i$

$\&\ b_i = -h^2 \exp(t_i)$

$\underline{i=1}$ $\Rightarrow C_1 x_2 + d_1 x_1 + a_1 x_0 = b_1$

$\Rightarrow d_1 x_1 + C_1 x_2 = b_1 - a_1 x_0$

$\underline{i=2}$ $\Rightarrow$ $a_2 x_1 + d_2 x_2 + C_2 x_3 = b_2$

$i=n-1 \Rightarrow a_{n-1}\, x_{n-2} + d_{n-1}\, x_{n-1} + c_{n-1}\, x_n = b_{n-1}$

$\Rightarrow a_{n-1}\, x_{n-2} + d_{n-1}\, x_{n-1} = b_{n-1} - c_{n-1}\, x_n$

Forming The $\underline{An = B}$ form

$$\begin{bmatrix} d_1 & c_1 & 0 & 0 & \cdots\cdots & 0 & 0 \\ a_2 & d_2 & c_2 & 0 & \cdots\cdots & 0 & 0 \\ 0 & a_3 & d_3 & c_3 & \cdots\cdots 0 & 0 \\ & & & & \vdots & \\ & & & & \vdots & \\ 0 & 0 & 0 & 0 & & a_{n-1} & d_{n-1} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ \vdots \\ x_{n-1} \end{Bmatrix} = \begin{Bmatrix} b_1 - a_1 x_0 \\ b_2 \\ \cdot \\ \cdot \\ \cdot \\ b_{n-1} - c_{n-1} x_n \end{Bmatrix}$$

Solve this eq^n with any of the matrix solver and get the solution $x_1, x_2 \cdots, x_{n-1}$ at $t_1, t_2 \cdots, t_{n-1}$.