

Stability Prediction in Nonlinear Turning Processes Using the Muller Algorithm

Anurag Singh
Department of Mechanical Engineering
Indian Institute of Technology Kanpur
Kanpur, India
anuragsingh2021@iitk.ac.in

Mohit Law
Department of Mechanical Engineering
Indian Institute of Technology Kanpur
Kanpur, India
mlaw@iitk.ac.in*

Abstract— This paper reports the application of a root-finding method, the Muller method, for predicting machining stability in turning processes exhibiting process damping related nonlinearities. We use the method to evaluate the process stability of a flexible tool modelled as a single degree of freedom system that interacts with a rotating workpiece. The system experiences the regeneration phenomena responsible for chatter. Prediction accuracies of the proposed method are found to agree with standard frequency and time domain methods of solving the resulting nonlinear delay differential equation. Though the method is computationally more demanding than the others, its advantages lie in being easy to implement and in being robust in handling process related nonlinearities.

Keywords—Muller algorithm, Stability, Turning, Chatter, Process damping, Nonlinear.

I. INTRODUCTION

Instabilities in turning processes occur due to the closed-loop interactions of the vibrating cutting tool with a rotating workpiece. A phase difference between vibrations imprinted on the surface already machined and being machined in the present period/rotation results in a regenerative phenomenon that causes large amplitude unstable chatter vibrations. These vibrations, if not mitigated, will damage the tool, the workpiece, and even elements of the machine tool system. There has hence been sustained research in developing models that can predict such instabilities [1, 2]. All models predict a stability boundary. Any combination of speed and depth of cut above the boundary results in unstable cutting. And any combination below results in stable cutting. Prediction of the boundary requires solving the nonlinear delay differential equations of motion that characterize the closed-loop regenerative interactions between the flexible tool and the rotating workpiece.

Early, classical models solved for the stability boundaries by linearizing the nonlinear delay differential equations by a transformation to the Laplace or to the frequency domain [3-6]. Though useful, the resulting linearization could not account for multiple regenerative effects which happen when the current vibration levels depend on vibrations that happened several time periods before. Such phenomena result in the tool out-of-cut effect that results in limit cycle oscillations that the frequency domain methods failed to characterize [7, 8]. Moreover, linearizing meant that nonlinear effects such the process damping phenomena could not be accounted for. In process damping, the flank face rubs with the vibrations on the machined surface, and results in a local low-speed increase in the stable depth of cut that could not be accounted for in the classical frequency domain models [9].

To predict stability while accounting for the tool out-of-cut effect and process damping related nonlinearities, time-

domain methods of numerical integration were preferred [7, 8]. Since numerical integration methods required solving the resulting nonlinear delay differential equations for every combination of speed and depth of cut and then interrogating the response to check for stability, the methods were computationally inefficient. To address this, Stépán presented a time-domain semi-discretization method of stability prediction in machining operations [10] that was computationally a lot more efficient. Since prediction of stability remains an important problem, there have been several other high-fidelity time- and frequency-domain methods of solution [11].

As an alternative to all existing methods for prediction of machining stability, this paper presents the use of the Muller method [12]. The Muller method is a root-finding technique. To solve for machining stability with this method, the equations of motion are transformed to the frequency domain to obtain a characteristic equation – much like in the classical frequency domain methods [3-5]. The only difference is that no linearization is necessary. The method works by checking for which of the roots lie in the left half plane that describe stability. Since the method can handle nonlinearities well, it has already been used for prediction in machining stability of circular sawing processes [13-15]. Though the method has shown to be effective, no comparisons with classical time- and frequency domain methods were presented to establish the efficacy of the Muller methods in being able to handle nonlinearities.

The main aim and modest new technical contribution of this paper is to demonstrate the workings of the Muller method and to contrast it with other classical methods to predict stability in a turning process exhibiting process damping related nonlinearity. We illustrate the method by evaluating the process stability of a flexible tool modelled as a single degree of freedom (SDOF) system that interacts with a rotating workpiece. This system and its equation of motion is described in Section II of the paper. Section III discusses methods of solution that include the classical frequency domain solution method, the time-domain solution method, and the Muller method. Section IV presents and contrasts results with all three methods. Main conclusions are presented in Section V.

II. MODEL AND EQUATIONS OF THE SYSTEM

A simplistic model of a flexible turning tool is shown in Fig. 1(a), which depicts a tool cutting the workpiece. In this study, the tool holder is considered as a lumped spring, mass and damper attached to tool post as shown in Fig. 1(b) and these are denoted by k , m and c , respectively. The equivalent cutting force ($F(x, t)$) is oriented by an angle β to the axis of the tool as is the case in most turning operations.

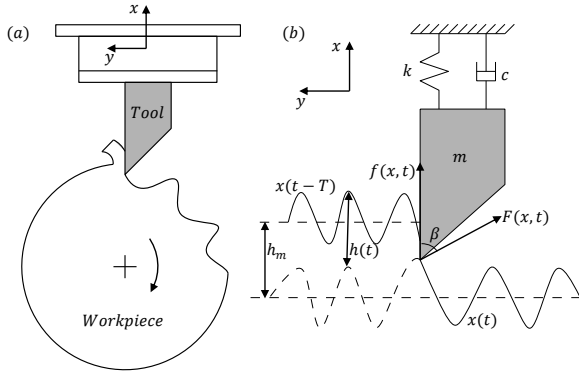


Fig. 1: (a) Schematic of a turning tool cutting the workpiece, and (b) Mechanical model showing SDOF system of turning tool with equivalent force at an orientation.

$x(t)$ in Fig. 1(b) represents the present vibrations being left by the tool on the surface, and $x(t - T)$ represents the vibrations left behind in the previous rotation, i.e., during one prior spindle period, T . The mean static chip thickness is denoted by h_m .

For the system shown, the governing equation of motion can be shown to be:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(x, t) \quad (1)$$

wherein $f(x, t)$ is expressed as follows:

$$f(x, t) = K_s b h(t) \cos(\beta) - \frac{C_s b}{v} \dot{x}(t) \quad (2)$$

wherein $h(t)$ represents the dynamic chip thickness, and it equals $h_m + x(t - T) - x(t)$, K_s is the tool-workpiece material combination specific cutting force coefficient, b is chip width or depth/width of cut, C_s is coefficient of process damping, and v is linear speed of the workpiece at the contact point with tool that is represented as: $v = \pi d \Omega / 60$, with, d being diameter of workpiece and Ω as the spindle speed in rpm.

The equation of motion is of the form of a nonlinear delay differential equation in which the response at the present time instant is also governed by the response in the previous time instants. And, though it is also possible to account for multiple regenerations, i.e., when the dynamic chip thickness is additionally a function of vibrations left behind by two or more spindle periods, i.e., $x(t - 2T)$, $x(t - 3T)$..., for analysis herein we limit our investigations to one spindle period. Methods to solve this equation are discussed next.

III. METHODS TO SOLVE THE EQUATION OF MOTION

We discuss herein three approaches to solve the delay differential equation.

A. Frequency domain method

The classical frequency domain approach to solve for stability involved a linearizing Laplace transform to Eq. (1). The limiting stable depth of cut was shown to be [16, 17]:

$$b_{lim} = -\frac{1}{2K_s \cos(\beta) \text{Re}[G]} \quad (3)$$

wherein $\text{Re}[G]$ represents the real part of the structure's transfer function, $G(\omega)$ that in turn is represented by:

$$G(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{k - m\omega^2 + jc\omega} \quad (4)$$

wherein ω represents the frequency in rad/s.

Since Eq. (3) only accounts for the limiting depth of cut, to factor how that changes with spindle speed, the following equation too has to be solved:

$$\Omega = \frac{f_c}{l + \frac{\epsilon}{2\pi}} \quad (5)$$

wherein f_c represents the chatter frequency in Hz, l represents the the complete number of vibration waves imprinted on the surface, and ϵ represents the phase shift between waves.

Details of obtaining Eqs. (3-5) are available elsewhere [16, 17], as are the algorithmic steps of obtaining the stability boundary. Those are hence not presented here. What is however important to note is that the classical frequency domain method does not account for the process damping related nonlinear effect.

B. Time domain simulation method

The time domain method of solving for stability provides a local view about which combination of depth of cut and speed results in stable or unstable cutting. For a given depth of cut, speed, feed, and tool-workpiece material combination, the simulation procedure begins with determining the instantaneous chip thickness, i.e., solving for $h(t)$. However, since $h(t)$ depends on present and previous vibration levels, those must be first evaluated by reordering the equation of motion such that, at first the response is obtained as:

$$\ddot{x}_n = \frac{f(x, t) - c\dot{x}_n - kx_n}{m} \quad (6)$$

wherein \dot{x} and x are from the previous time step, and are set to be zero to start with. The velocities and positions for the current time step are then determined by numerical integration. Using the Euler method (other higher order schemes too can be used), these can be shown to be:

$$\dot{x}_{n+1} = \dot{x}_n + \ddot{x}_n dt \quad (7)$$

and

$$x_{n+1} = x_n + \dot{x}_{n+1} dt \quad (8)$$

wherein dt , the time step must be chosen to be sufficiently small to ensure numerical stability.

Once responses are obtained as above, forces can be calculated and the above procedure is to be repeated for a sufficiently long period which allows the solution to evolve into being stable or not. Since the form of force used in Eq. (6) makes no linearizing assumptions, the process damping term can be easily integrated and its influence checked.

Since above procedures are also routine, details of these can be found in [17].

C. Muller Method

Muller's algorithm is a root-finding method especially helpful in finding the complex roots of equations [12]. This method utilizes a quadratic (parabolic) interpolation through three points to find the next probable root, and that makes this better at finding complex roots than other classical methods such as the Newton-Raphson method. Muller's method can also be considered an extension of the Secant method as it increases the degree of the interpolation polynomial from one to two, and this increases the rate of convergence.

To implement this method in our case of turning stability problem, the response is assumed in the form of $x(t) = Xe^{\lambda t}$ and thus the characteristic equation becomes:

$$\{m\lambda^2 + c\lambda + k\}Xe^{\lambda t} = f(x, t) \quad (9)$$

and the forcing term becomes:

$$f(x, t) = K_s b \{h_m + Xe^{\lambda t} e^{-\lambda T} - Xe^{\lambda t}\} \cos(\beta) - \lambda \frac{C_s b}{v} Xe^{\lambda t} \quad (10)$$

replacing the forcing term (Eq. (10)) into Eq. (9) and segregating the terms associated with $Xe^{\lambda t}$, we get:

$$\left[m\lambda^2 + \left(c + \frac{C_s b}{v} \right) \lambda + \{k - K_s b(e^{-\lambda T} - 1) \cos(\beta)\} \right] Xe^{\lambda t} = K_s b h_m \quad (11)$$

The homogeneous response of the Eq. (11) defines the stability of the system. If the real part of the values of the complex root λ are negative then system's response will decay over time and thus will be stable. If it is zero, the system will have a harmonic response, and positive values of the real part of the roots will signify a growing response, and the system will be unstable.

For the case of without process damping, the roots of the following equation needs to be obtained:

$$m\lambda^2 + c\lambda + \{k - K_s b(e^{-\lambda T} - 1) \cos(\beta)\} = 0 \quad (12)$$

and for the process damping inclusion case the damping term in Eq. (12) becomes $(c + C_s b/v)$.

As Eq. (12) is non-linear in λ , the Muller algorithm can be applied directly to find the roots. The caution to observe herein is in supplying the initial guess to the Muller algorithm. It should be a smart guess to make sure that the root converges to the desired value. A best initial guess in the present case will be to supply the roots of the simple quadratic equation $m\lambda^2 + c\lambda + k = 0$, which are known. This initial guess will make sure that Muller algorithm searches the nearby zeros and yields a complex conjugate values as the roots.

The overall procedure to solve for stability using the Muller's algorithm can be summarized as:

- i. Initialize material, dynamic, and cutting parameters
- ii. Loop over spindle speed and chip width
- iii. Define the characteristic equation in homogeneous form
- iv. Define the initial guess of the roots
- v. Solve for the roots
- vi. Check the real part of the roots for stability and store the data
- vii. Update the initial guess with the new roots for the next iteration

We use the above procedure to predict for stability and contrast our predictions with those obtained with the frequency domain and time domain methods.

IV. RESULTS AND DISCUSSION

This section presents results obtained with the Muller's method and benchmarks those with results obtained from the time- and frequency-domain methods. We first present results for a linear model, in which we ignore the influence of process damping, and subsequently present comparisons with the process damping nonlinearity being included in the model.

To generate results, we take the system model parameters to be: $k = 6.48 \times 10^6$ N/m, $m = 0.561$ kg, $c = 145$ N s/

m. For assumed cutting of steel, we take, $K_s = 2927 \times 10^6$ N/m², $\beta = 61.79^\circ$, $d = 0.035$ m and $C_s = 6.11 \times 10^5$ N/m. These parameters are typical and are similar to those reported in [17].

Since the time-domain method and the Muller's method of solution require checks for many combinations of depths of cut and spindle speeds, we define a grid and interrogate the response at every grid point to check for stability. Since we simulate cutting of steel, we limit our spindle speed range to between 1500 rpm and 3000 rpm, discretized in steps of 1 rpm. At every rpm, we evaluate response at an initial depth of cut of 0.30 mm, and then increase that depth of cut by a step of 0.01 mm until we find the response to be unstable. That point defines the stability boundary. Computational time for each simulation run was documented. All simulations were run on MATLAB running on a laptop with the following specifications: i7-1255U, 1700 Mhz, 16.0 GB RAM.

A. Stability predictions for a linear model

First, we present the results of the comparison of the stability lobes obtained from the three methods without including process damping into the analysis. These results are shown in Fig. 2. The absolute speed-independent limiting depth of cut is 0.37 mm, and this remains the same for all three methods of solution. Moreover, the nature of the boundaries is also the same. This comparison suggests that the Muller method is able to consistently predict the boundary and that predictions agree with well-established methods, and as such the Muller method may deemed to be validated.

Though the Muller method is accurate, it is computationally less efficient. To generate the results shown in Fig. 2, the frequency domain method takes a run time of around 1 seconds, the time-domain method takes a run time of approximately 1,048 seconds, whereas the Muller method takes a significantly longer time, taking nearly 2,329 seconds.

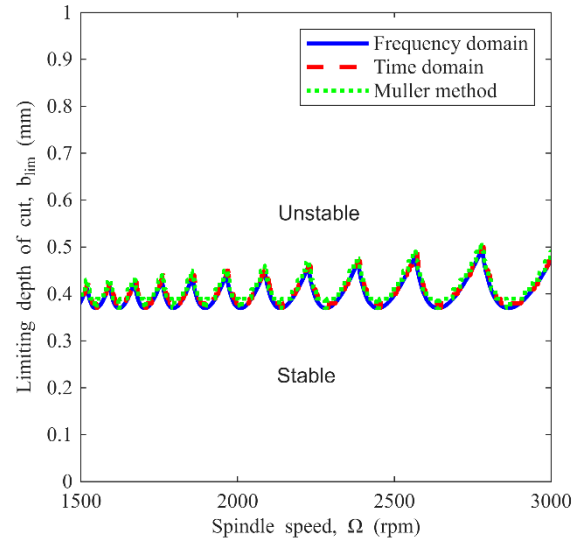


Fig. 2: Stability predictions with three different methods for a linear model.

B. Stability predictions with process damping

Results with the inclusion of process damping are shown in Fig. 3. As is evident, the stability boundary increases at lower speed on account of process damping. This effect is captured well by the Muller algorithm and its results agree

with those obtained using the time domain method, suggesting that the Muller method is well capable of predicting stability in systems with nonlinearities. Since the frequency domain method ignores the process damping term, it naturally cannot predict the correct boundaries as predicted by the time domain method and the Muller's method.

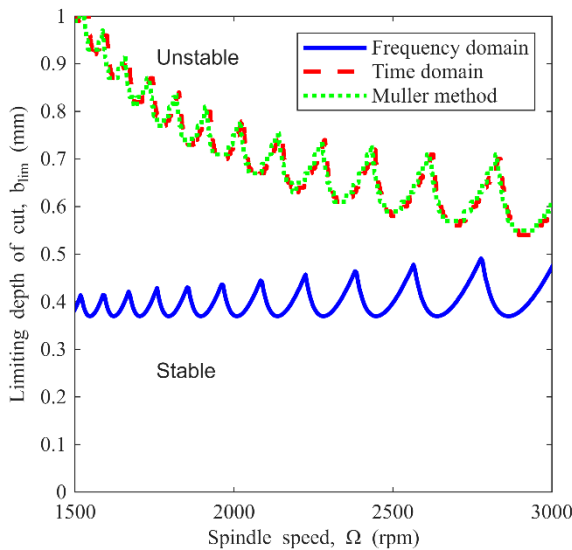


Fig. 3: Stability predictions with the three different methods for when process damping nonlinearities are included in the model.

Like for the case of linear stability, in this case too the Muller method, though accurate, is computationally less efficient than the time domain method. To obtain the results shown in Fig. 3, the time domain method took roughly 5,162 seconds, whereas the Muller method took almost 5,332 seconds.

V. CONCLUSION

Predicting stability of turning processes with nonlinearities remains important to guide selection of cutting parameters that will not result in unstable chatter vibrations. We showed that the Muller method offers a robust alternative to the classical time-and frequency domain methods to solve for stability in systems exhibiting process damping related nonlinearities. Though the method is computationally less efficient than the others, its advantage lies in its simple implementation. Future follow-on research can focus on

testing the Muller method's efficacy in predicting stability of other machining processes with other types of nonlinearities.

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