Date:

TASK 2: Pauli Matrices and Eigenvalues/Eigenvectors

Aim:

To analyze Pauli matrices through application on qubit states and eigenvalue decomposition.

1. **Mathematical Model**

The **Pauli matrices** are a set of three 2×2 complex Hermitian and unitary matrices that are widely used in **quantum mechanics**, particularly in spin systems (spin-1/2 particles), quantum computing, and quantum information theory. They are denoted as σ_x, σ_y, and σ_z, and are defined as follows:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

1.1. Eigenvalues and Eigenvectors of the Pauli Matrices

Each Pauli matrix has eigenvalues $\lambda = \pm 1$ and corresponding eigenvectors:

i. **σ_x (Pauli-X Matrix)**
- **Eigenvalues**: $\lambda = +1, -1$
- **Eigenvectors**:
  - For $\lambda = +1$

$$|+\rangle_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

  - For $\lambda = -1$

$$|-\rangle_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

ii. **σ_y (Pauli-Y Matrix)**
- **Eigenvalues**: $\lambda = +1, -1$
- **Eigenvectors**:
  - For $\lambda = +1$

$$|+\rangle_y = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix}$$

- For $\lambda = -1$

$$|-\rangle_y = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix}$$

iii. **σ_z (Pauli-Z Matrix)**

- **Eigenvalues**: $\lambda = +1, -1$
- **Eigenvectors**:
  - For $\lambda = +1$

$$|+\rangle_z = \begin{pmatrix}1\\ 0\end{pmatrix}$$

  - For $\lambda = -1$

$$|-\rangle_z = \begin{pmatrix}0\\ 1\end{pmatrix}$$

2. **Observations**
   - All three Pauli matrices have eigenvalues ± 1.
   - Their eigenvectors are orthonormal, i.e., $\langle +|-\rangle = 0$ and $\langle \pm|\pm\rangle = 1$.
   - The eigenvectors of σ_x and σ_y are superpositions of the eigenvectors of σ_z, reflecting the non-commutativity of the Pauli matrices ($[\sigma_i,\sigma_j]=2i\epsilon_{ijk}\sigma_k$).

3. **Physical Interpretation**

In quantum mechanics, the Pauli matrices represent spin measurements along the x, y, and z axes for a spin-1/2 particle (like an electron). The eigenvalues ± 1 correspond to the possible outcomes of a spin measurement (spin-up or spin-down), and the eigenvectors represent the spin states along the respective axes.

4. **Algorithm**
   - Define Pauli-X, Y, and Z matrices.
   - Apply these matrices to $|0\rangle$ and $|1\rangle$ states.
   - Use linear algebra to compute eigenvalues and eigenvectors.

- Print matrix properties.

5. **Program**

```
print("\n" + "="*50)
print("TASK 2: PAULI MATRICES AND EIGEN-ANALYSIS")
print("="*50)
# Define Pauli matrices
pauli_x = np.array([[0, 1], [1, 0]])
pauli_y = np.array([[0, -1j], [1j, 0]])
pauli_z = np.array([[1, 0], [0, -1]])
print("Pauli-X matrix:")
print(pauli_x)
print("\nPauli-Y matrix:")
print(pauli_y)
print("\nPauli-Z matrix:")
print(pauli_z)
# Apply to qubit states
qubit_0 = np.array([1, 0]) # |0⟩
qubit_1 = np.array([0, 1]) # |1⟩
print("\nApplying Pauli-X to |0⟩:", pauli_x @ qubit_0)
print("Applying Pauli-X to |1⟩:", pauli_x @ qubit_1)
# Compute eigenvalues and eigenvectors
def analyze_operator(matrix, name):
    eigenvals, eigenvecs = eig(matrix)
    print(f"\n{name} Eigenvalues:", eigenvals)
    print(f"{name} Eigenvectors:")
    for i, vec in enumerate(eigenvecs.T):
    print(f" λ={eigenvals[i]:.1f}: {vec}")

analyze_operator(pauli_x, "Pauli-X")
analyze_operator(pauli_y, "Pauli-Y")
analyze_operator(pauli_z, "Pauli-Z")
```

Output:

```
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TASK 2: PAULI MATRICES AND EIGEN-ANALYSIS
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Pauli-X matrix:
[[0 1]
 [1 0]]

Pauli-Y matrix:
[[ 0.+0.j -0.-1.j]
 [ 0.+1.j  0.+0.j]]

Pauli-Z matrix:
[[ 1  0]
 [ 0 -1]]

Applying Pauli-X to |0⟩: [0 1]
Applying Pauli-X to |1⟩: [1 0]

Pauli-X Eigenvalues: [ 1. -1.]
Pauli-X Eigenvectors:
 λ=1.0: [0.70710678 0.70710678]
 λ=-1.0: [-0.70710678  0.70710678]

Pauli-Y Eigenvalues: [ 1.+0.j -1.+0.j]
Pauli-Y Eigenvectors:
 λ=1.0+0.0j: [-0.        -0.70710678j  0.70710678+0.j        ]
 λ=-1.0+0.0j: [0.70710678+0.j         0.        -0.70710678j]

Pauli-Z Eigenvalues: [ 1. -1.]
Pauli-Z Eigenvectors:
 λ=1.0: [1. 0.]
 λ=-1.0: [0. 1.]
```

Result

Pauli matrices were applied, and their eigenvalues and eigenvectors were correctly determined.