

SKILL #21

CODE: FT.3

Completing the Square

**Core Concept**

A method that rewrites any quadratic expression $(ax^2 + bx + c)$ into the form $(x + k)^2 + \text{constant}$, so solving, graphing, or analyzing becomes easy.

Example

Complete the square: $x^2 + 6x + 2$

STEP 1: $\rightarrow a = 1, b = 6, c = 2$ $a = 1$

STEP 2: $\frac{b}{2} = \frac{6}{2} = 3 \rightarrow 3^2 = 9$

STEP 3: $\rightarrow x^2 + 6x + 2 + 9 - 9$

STEP 4: $x^2 + 6x + 9 + 2 - 9$

STEP 5: $(x^2 + 6x + 9) - 7 \rightarrow (x + 3)^2 - 7$

Example 2: When $a \neq 1$

Complete the square: $2x^2 + 12x + 1$

STEP 1: $\rightarrow a = 2, b = 12, c = 1$ factor 2 out

$$2(x^2 + 6x) + 1$$

STEP 2: $\frac{b}{2} = \frac{6}{2} = 3 \rightarrow 3^2 = 9$

STEP 3: $\rightarrow 2(x^2 + 6x + 9 - 9) + 1$

STEP 4: $2(x^2 + 6x + 9) - 18 + 1$ (take -9 out)

STEP 5: $2(x + 3)^2 - 17$

When to use it?

- When solving quadratics by factoring fails.
- To find the vertex (highest / lowest point) of a parabola.

GULDEN RULE

STEP 1: Make sure the x^2 coefficient is 1 (divide if needed).

STEP 2: Take half of the x -coefficient (b), then square it.

STEP 3: Add & subtract that square inside the expression.

STEP 4: Rearrange the constants.

STEP 5: Factor the perfect square trinomial.

Must have $a = 1$ before taking half of b .

If $a \neq 1$, factor it out first!

**Common Mistakes to Avoid**

✗ Forgetting to balance

Whatever you add inside the square, also subtract

✗ Leaving x^2 coefficient $\neq 1$

Always factor it out first.

✗ Wrong Half-and-Square:

For $x^2 + 8x$, it's $\left(\frac{8}{2}\right)^2 = 16$, not $8^2 = 64$.

Always halve first, then square.

**Additional Resources**