Solving Equations By Factoring

Example 1: Basic Quadratic Equation

Solve: $x^2 + 7x + 12 = 0$

Step 1: The equation is already set equal to zero ✅

Step 2: Factor the left side

- We need two numbers that multiply to 12 and add to 7
- Those numbers are 3 and 4
- So: $x^2 + 7x + 12 = (x + 3)(x + 4)$

Step 3: Set each factor equal to zero

- $x + 3 = 0 \;\; or \;\; x + 4 = 0$

Step 4: Solve each equation

- $x = -3 \;\; or \;\; x = -4$

Step 5: Check both solutions

- For $x = -3$: $(-3)^2 + 7(-3) + 12 = 9 - 21 + 12 = 0$ ✅
- For $x = -4$: $(-4)^2 + 7(-4) + 12 = 16 - 28 + 12 = 0$ ✅

Example 2: $x^2 + bx + c$ (Negative c)

Solve: $x^2 + 5x - 6 = 0$

Step 1: The equation is already set equal to zero ✅

Step 2: Factor

- We need two numbers that multiply to -6 and add to 5
- Those numbers are 6 and -1
- So: $x^2 + 5x - 6 = (x + 6)(x - 1)$

Step 3: Set each factor equal to zero

- $x + 6 = 0 \quad or \quad x - 1 = 0$

Step 4: Solve

- $x = -6 \quad or \quad x = 1$

Example 3: $ax^2 + bx + c$ ($a \neq 1$, Factorable)

Solve: $2x^2 + 7x + 3 = 0$

Step 1: The equation is already set equal to zero ✅

Step 2: Factor using trial and error or grouping

- We need factors of $2 \times 3 = 6$ that add to 7
- Those are 6 and 1
- Rewrite: $2x^2 + 6x + x + 3 = 0$
- Group: $2x(x + 3) + 1(x + 3) = 0$
- Factor out common factor: $(2x + 1)(x + 3) = 0$

Step 3: Set each factor equal to zero

- $2x + 1 = 0 \quad or \quad x + 3 = 0$

Step 4: Solve

- $x = -\frac{1}{2} \quad or \quad x = -3$

Example 4: $ax^2 + bx + c$ (Larger coefficients)

Solve: $6x^2 - 11x + 3 = 0$

Step 1: Factor using grouping method

- We need factors of $6 \times 3 = 18$ that add to -11
- Those are -9 and -2
- Rewrite: $6x^2 - 9x - 2x + 3 = 0$
- Group: $3x(2x - 3) - 1(2x - 3) = 0$
- Factor: $(3x - 1)(2x - 3) = 0$

Step 2: Set each factor equal to zero

- $3x - 1 = 0 \quad or \quad 2x - 3 = 0$

Step 3: Solve

- $x = \frac{1}{3} \quad or \quad x = \frac{3}{2}$

Example 5: Difference of Squares $x^2 - b^2$

Solve: $x^2 - 25 = 0$

Step 1: Recognize the pattern $(a^2 - b^2)$

- $x^2 - 25 = x^2 - 5^2$

Step 2: Factor using difference of squares formula: $a^2 - b^2 = (a + b)(a - b)$

- $x^2 - 25 = (x + 5)(x - 5)$

Step 3: Set each factor equal to zero

- $x + 5 = 0 \quad or \quad x - 5 = 0$

Step 4: Solve

- $x = -5 \quad or \quad x = 5$

Example 6: Difference of Squares (Larger numbers)

Solve: $4x^2 - 49 = 0$

Step 1: Recognize both terms are perfect squares

- $4x^2 = (2x)^2$ and $49 = 7^2$

- So: $(2x)^2 - 7^2 = 0$

Step 2: Apply difference of squares

- $(2x + 7)(2x - 7) = 0$

Step 3: Set each factor equal to zero

- $2x + 7 = 0 \quad or \quad 2x - 7 = 0$

Step 4: Solve

- $x = -\frac{7}{2} \quad or \quad x = \frac{7}{2}$

Example 7: Common Factor First ($abx^2 + acx$)

Solve: $3x^2 + 12x = 0$

Step 1: Factor out the greatest common factor

- Both terms have 3x in common
- $3x^2 + 12x = 3x(x + 4) = 0$

Step 2: Set each factor equal to zero

- $3x = 0 \quad or \quad x + 4 = 0$

Step 3: Solve

- $x = 0 \quad or \quad x = -4$

Note: Don't forget that $x = 0$ is often a solution when there's no constant term!

Example 8: Common Factor with Trinomial

Solve: $2x^2 + 8x + 6 = 0$

Step 1: Factor out common factor first

- $2x^2 + 8x + 6 = 2(x^2 + 4x + 3)$

Step 2: Factor the trinomial inside

- $x^2 + 4x + 3 = (x + 1)(x + 3)$
- So: $2(x + 1)(x + 3) = 0$

Step 3: Set factors equal to zero

- Note: 2 ≠ 0, so we only need: $(x + 1) = 0 \quad or \quad (x + 3) = 0$

Step 4: Solve

- $x = -1 \quad or \quad x = -3$

Example 9: Perfect Square Trinomial

Solve: $x^2 + 10x + 25 = 0$

Step 1: Recognize the perfect square pattern $a^2 + 2ab + b^2$

- $x^2 + 10x + 25 = x^2 + 2(5)x + 5^2$

Step 2: Factor as a perfect square

- $(x + 5)^2 = 0$

Step 3: Set the factor equal to zero

- $x + 5 = 0$

Step 4: Solve

- $x = -5$ (This is called a repeated root-same solution twice!)

Example 10: Perfect Square Trinomial (Subtraction)

Solve: $x^2 - 12x + 36 = 0$

Step 1: Check if it's a perfect square: $a^2 - 2ab + b^2$

- $x^2 - 12x + 36 = x^2 - 2(6)x + 6^2$

Step 2: Factor as perfect square

- $(x - 6)^2 = 0$

Step 3: Solve

- $x - 6 = 0 \to x = 6$

Example 11: Difference of Cubes

Solve: $x^3 - 8 = 0$

Step 1: Recognize as difference of cubes: $a^3 - b^3$

- $x^3 - 8 = x^3 - 2^3$

Step 2: Use difference of cubes formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

- $x^3 - 8 = (x - 2)(x^2 + 2x + 4) = 0$

Step 3: Set each factor equal to zero

- $x - 2 = 0 \quad or \quad x^2 + 2x + 4 = 0$

Step 4: Solve the first equation

- $x = 2$

Step 5: Try to solve $x^2 + 2x + 4 = 0$

- We need two numbers that multiply to 4 and add to 2
- There are no real numbers that work! (We'll learn a formula for this later)

Answer: $x = 2$ is the only real solution

Example 12: Sum of Cubes

Solve: $x^3 + 27 = 0$

Step 1: Recognize as sum of cubes: $a^3 + b^3$

- $x^3 + 27 = x^3 + 3^3$

Step 2: Use sum of cubes formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

- $x^3 + 27 = (x + 3)(x^2 - 3x + 9) = 0$

Step 3: Set each factor equal to zero

- $x + 3 = 0 \quad or \quad x^2 - 3x + 9 = 0$

Step 4: Solve the first equation

- $x = -3$

Step 5: Try to solve $x^2 - 3x + 9 = 0$

- We need two numbers that multiply to 9 and add to -3

- There are no real numbers that work!

Answer: $x = -3$ is the only real solution

Example 13: Not Factorable Equation

Solve: $x^2 + 3x + 1 = 0$

Step 1: Try to factor $x^2 + 3x + 1$

- We need two numbers that multiply to 1 and add to 3
- Let's try: 1 and 1 → multiply to 1 ✅, but add to 2 ✗
- No integer factors work!

Step 2: Check if we made an error

- The equation is already in standard form
- This trinomial cannot be factored with integers

Conclusion: This equation is not factorable using integer factors.

Good news! There's still a way to solve it using the Quadratic Formula, which will be our next skill. The quadratic formula works for ALL quadratic equations, even ones that don't factor nicely!

Example 14: Another Non-Factorable Example

Solve: $2x^2 + 3x - 4 = 0$

Step 1: Try to factor $2x^2 + 3x - 4$

- For ax²+bx+c, we need factors of $ac = 2(-4) = -8$ that add to $b = 3$
- Factors of -8: $\pm 1, \pm 8$ or $\pm 2, \pm 4$
- Check: $1 + (-8) = -7$ ✗, $8 + (-1) = 7$ ✗, $2 + (-4) = -2$ ✗, $4 + (-2) = 2$ ✗
- None work!

Conclusion: This equation cannot be factored with integers but can be solved using the Quadratic Formula.