Sum and difference of cubes

Example 1: Basic Sum of Cubes

Factor: $x^3 + 8$

Solution:

- Step 1: Identify $\rightarrow x^3 + 8$ (both perfect cubes)
- Step 2: Cube roots $\rightarrow \sqrt[3]{x^3} = x, \sqrt[3]{8} = 2$
- Step 3: Sum of cubes $\rightarrow (x + 2)(x^2 - 2x + 4)$
- Step 4: Check $\rightarrow (x + 2)(x^2 - 2x + 4) = x^3 + 8$ ✅

Check:

$$(x + 2)(x^2 - 2x + 4) = x^3 - 2x^2 + 4x + 2x^2 - 4x + 8$$
$$= x^3 + 8$$

Example 2: Basic Difference of Cubes

Factor: $y^3 - 27$

Solution:

- Step 1: Identify $\rightarrow y^3 - 27$ (both perfect cubes)
- Step 2: Cube roots $\rightarrow \sqrt[3]{y^3} = y, \sqrt[3]{27} = 3$
- Step 3: Difference of cubes $\rightarrow (y - 3)(y^2 + 3y + 9)$
- Step 4: Check $\rightarrow (y - 3)(y^2 + 3y + 9) = y^3 - 27$ ✅

Check:

$$(y - 3)(y^2 + 3y + 9) = y^3 + 3y^2 + 9y - 3y^2 - 9y - 27$$
$$= x^3 - 27$$

Example 3: Sum of Cubes with Coefficients

Factor: $8a^3 + 125$

Solution:

- Step 1: Identify $\rightarrow 8a^3 + 125 (8a^3 = (2a)^3, 125 = 5^3)$
- Step 2: Cube roots $\rightarrow \sqrt[3]{8a^3} = 2a, \sqrt[3]{125} = 5$
- Step 3: Sum of cubes $\rightarrow (2a + 5)(4a^2 - 10a + 25)$
- Step 4: Check $\rightarrow (2a + 5)(4a^2 - 10a + 25) = 8a^3 + 125$ ✅

Check:

$$(2a + 5)(4a^2 - 10a + 25) = 8a^3 - 20x^2 + 50x + 20x^2 - 50x + 125$$
$$= x^3 + 8$$

Example 4: Difference of Cubes with Variables

Factor: $27x^3 - 64y^3$

Solution:

- Step 1: Identify $\rightarrow 27x^3 - 64y^3 \rightarrow (27x^3 = (3x)^3, 64y^3 = (4y)^3)$
- Step 2: Cube roots $\rightarrow \sqrt[3]{27x^3} = 3x, \sqrt[3]{64y^3} = 4y$
- Step 3: Difference of cubes $\rightarrow (3x - 4y)(9x^2 + 12xy + 16y^2)$
- Step 4: Check $\rightarrow (3x - 4y)(9x^2 + 12xy + 16y^2) = 27x^3 - 64y^3$ ✅

Check:

$$(3x - 4y)(9x^2 + 12xy + 16y^2) = 27x^3 + 36x^2y + 48xy^2 - 36x^2y - 48xy^2 + 64y^3$$
$$= 27x^3 + 64y^3$$

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Example 5: Large Perfect Cubes

Factor: $x^3 + 216$

Solution:

- Step 1: Identify → $x^3 + 216$ (is 216 a perfect cube? $6^3 = 216$) ✅
- Step 2: Cube roots → $\sqrt[3]{x^3} = x$, $\sqrt[3]{216} = 6$
- Step 3: Sum of cubes → $(x + 6)(x^2 - 6x + 36)$
- Step 4: Check → $(x + 6)(x^2 - 6x + 36) = x^3 + 216$ ✅

Example 6: Testing if it's NOT Sum/Difference of Cubes

Factor: $x^3 + 16$

Solution:

- Step 1: Check → $x^3 + 16$ (x^3 is perfect cube, but is 16 a perfect cube?)
- Analysis: $\sqrt[3]{16} \approx 2.52$, not a whole number
- Conclusion: This is NOT a sum of cubes pattern
- Result: Cannot be factored using sum/difference of cubes formulas