

Completing the Square



LEVEL 1: The Basics

Complete the square for each expression. Leading coefficient is 1.

Basic Completing the Square

❖ $x^2 + 4x =$ Answer --> $x^2 + 4x + 4 - 4 \rightarrow (x + 2)^2 - 4$

❖ $x^2 + 6x =$

❖ $x^2 + 10x =$

❖ $x^2 - 8x =$

❖ $x^2 - 12x =$

❖ $x^2 + 2x =$

❖ $x^2 - 6x =$

❖ $x^2 + 14x =$

❖ $x^2 - 4x =$

❖ $x^2 + 16x =$

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LEVEL 2: Dive Deeper

Complete the square:

❖ $x^2 + 8x + 7 =$ Answer --> $x^2 + 8x + 7 + 16 - 16 \rightarrow x^2 + 8x + 16 + 7 - 16 \rightarrow (x + 4)^2 - 9$

❖ $x^2 - 6x + 5 =$

❖ $x^2 + 4x + 1 =$

❖ $x^2 - 10x + 21 =$

❖ $x^2 + 12x + 20 =$

❖ $x^2 - 2x + 3 =$

❖ $x^2 + 6x - 2 =$

❖ $x^2 - 8x + 10 =$

❖ $x^2 + 10x + 9 =$

❖ $x^2 - 4x - 5 =$

❖ $2x^2 + 8x + 3 =$

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🎯 LEVEL 3: Mastering the Concept

❖ $3x^2 - 12x + 7 = \text{Answer} \rightarrow 3(x^2 - 4x) + 7 \rightarrow 3(x^2 - 4x + 4 - 4) + 7 \rightarrow 3(x^2 - 4x + 4) - 12 + 7$
 $\rightarrow 3(x - 2)^2 - 5$

❖ $4x^2 + 16x - 5 =$

❖ $-x^2 + 6x - 2 =$

❖ $5x^2 - 20x + 8 =$

❖ $-2x^2 + 12x - 10 =$

❖ $3x^2 + 18x + 15 =$

❖ $-4x^2 - 8x + 1 =$

❖ $6x^2 - 24x + 18 =$

❖ $-3x^2 + 18x - 12 =$

❖ $2x^2 + 12x - 5 =$

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Challenge Problems

Q1: Complete the square: $4x^2 - 12x + 9$. What do you notice about this expression?

Q2: For what value of k does $x^2 + 6x + k$ have exactly one solution?

Q3: Complete the square: $x^2 + bx + c = 0$, solving for x in terms of b and c.

Q4: Pattern Recognition: Complete the square for $x^2 + 2x, x^2 + 4x, x^2 + 6x, x^2 + 8x$. What pattern do you see in the constant terms of the completed squares?

Q5: Complex Coefficients: Complete the square for $3x^2+9x+5$. Express your answer with exact fractions.