

01. Number Systems and Codes

- ⇒ Review of number systems: Decimal systems, Binary, Octal & Hexa decimal Numbering systems,
- ⇒ Number base conversions, binary arithmetic, complements
- ⇒ Signed binary numbers, Binary codes: weighted, Non-weighted error detecting and error correcting codes - hamming codes and other codes.
- ⇒ Boolean Algebra and logic Gates: Basic definition, Basic logic operations, Basic theorems and properties, Truth tables, Boolean functions, Representations of Boolean functions: Canonical and standard forms, Other logic operations
- Digital logic gates, universal gates.

02. Gate Level Minimization

- ⇒ Algebraic simplification of Boolean expression, the map method, four variable K-map, five variable K-map, Prime implicants, PD's & SOP simplification,
- ⇒ Don't care conditions, Tabular Method - Simplification of Boolean function using tabulation Method, two level and multi-level NAND & NOR Implementation, Other two level implementation circuits, Ex-OR function and properties.

03. Combinational Circuits

- ⇒ Combinational circuits, analysis and synthesis (Design)
- ⇒ procedure, Design using conventional logic gates:
- ⇒ Binary Adder - Subtractor - Decimal Adder - Binary multiplier
- ⇒ Magnitude comparator - Parity generators and checkers - Code converters, Hazards, Decoders, Encoders, Multiplexers, Demultiplexers, logic Design using MSI Components.

04. Synchronous Sequential Circuits

- ⇒ Sequential circuits, latches, flip-flops - Level and Edge triggering, Master-slave flip-flops & Edge triggered flip-flops,
- ⇒ Characteristic table and equation, Excitation table, flip flop conversions,
- ⇒ Analysis of synchronous sequential circuits, state reduction and assignment, Design procedure of synchronous sequential circuits, Design examples: Serial binary adder, sequence detector.

05. Shift Registers and Counters

- ⇒ Registers, Shift Registers: SISO, SIPO, PIPO, PIPD and universal shift registers, applications, Counters: Ripple Counters, Synchronous counters, other counters and Application of counters
- ⇒ finite state Machines: Introduction - General Model of FSM, classifications of FSM (Mealy & Moore Models), Design of FSM, Design examples, Capabilities, and Limitations of FSM.

06. Memory and Programmable Logic

- ⇒ Introduction, Random Access Memory, Memory Decoding
- ⇒ Read only memory, Programmable Logic Array
- ⇒ Programmable Array Logic: Sequential Programmable devices
- ⇒ Complex Programmable Logic devices,
- ⇒ field Programmable Gate Arrays.

Introduction to Digital Electronics

- The term, 'digital,' is derived from the way in which computers perform operations by counting.
- ⇒ Digital electronics involves circuits and systems in which there are only two possible states which are typically represented by amplitude levels.
- ⇒ Digital is the way in which the computer performs the arithmetical operations.
- ⇒ Some digital signals are keyboard signals.
- ⇒ Sine wave is known as the fundamental or universal wave.
- ⇒ In digital systems, two states are used to represent numbers, symbols, characters etc i.e 1 & 0.
- * Difference between analog signal and digital signal:
- (i) Analog signal is defined as amplitude variable whose size is proportional to the quantity it represents.
 - (ii) Analog signal is continuous in nature.
 - (iii) Analog signal has infinite set of possible values.
 - (iv) Examples:- Sound, Velocity, Wave, Pressure, Temperature
- (i) A Digital signal is one which changes states between two discrete amplitude levels.
 - (ii) Digital signals are discrete in nature.
 - (iii) There are 2 possible set of values 1 & 0.
 - (iv) Examples are:- Telegraph signal, Keyboard signal

* Advantages of digital systems:-

- 01) Data accuracy and precision are greater in digital systems. & can be fabricated easily.
- 02) Noise is less in case of digital systems.
- 03) Digital systems are less time consuming.
- 04) Easier to understand, easy to design.
- 05) Complexity is less than analog systems.
information can be stored easily, reusable & versatile

* Disadvantages of digital systems:-

⇒ Most physical quantities are analog in nature. To take advantage of digital technique, the analog inputs are first to be converted to digital signal by the help of analog to digital converter (ADC). Then after processing converted back to analog form.

* Application of Digital Signals:-

- 01) Consumer electronics ex:- Television, Computers
- 02) Communication systems
- 03) Signal Processing, image & speech processing

Number systems and codes

Bit: Binary digit '0' or 1.

1 Nibble \rightarrow 4 bits;

1 byte \rightarrow 8 bits;

1 word \rightarrow 2 bytes; 16 bits.

1 double word - 4 bytes; 32 bits.

1 quad word - 8 bytes; 64 bits.

1 Kilo byte $\rightarrow 2^{10}$ bytes; 1024 bytes.

1 Mega byte $\rightarrow 2^{10} \cdot 2^{10}$ bytes; 1024 Kilo bytes.

1 Giga byte $\rightarrow 2^{10} \cdot 2^{10} \cdot 2^{10}$ bytes; 1024 MB.

1 Tera byte $\rightarrow 2^{10} \cdot 2^{10} \cdot 2^{10} \cdot 2^{10}$ bytes; 1024 GB.

Number Systems: There are four number systems

01. Binary Number system: $b_1=2$, '0' & '1'

02. Octal Number System: $b_2=8$, 0, 1, 2, 3, 4, 5, 6, 7

03. Decimal Number System: $b_3=10$, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

04. Hexadecimal number system: $b_4=16$ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 & ABCDEF

Number System conversion:

Binary to decimal:

$$(i) (110110)_2 = 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ = 32 + 16 + 0 + 4 + 2 = (54)_{10}$$

$$(ii) (1101101110)_2 = 1 \times 2^9 + 1 \times 2^8 + 0 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 \\ + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 \\ = 512 + 256 + 64 + 32 + 8 + 4 + 2 = (878)_{10}$$

Decimal	Hexadecimal	Binary	Octal
0	0	00000000	0
01	1	00000001	1
02	2	00000010	2
03	3	00000011	3
04	4	00000100	4
05	5	00000101	5
06	6	00000110	6
07	7	00000111	7
08	8	00001000	10
09	9	00001001	11
10	A	00001010	12
11	B	00001011	13
12	C	00001100	14
13	D	00001101	15
14	E	00001110	16
15	F	00001111	17
16	10	00010000	20
17	11	00010001	21
18	12	00010010	22
19	13	00010011	23
20	14	00010100	24
21	15	00010101	25
22	16	00010110	26
23	17	00010111	27
24	18	00011000	30
25	19	00011001	31
26	1A	00011010	32
27	1B	00011011	33
28	1C	00011100	34
29	1D	00011101	35
30	1E	00011110	36

$$\begin{aligned}
 (11011.101)_2 &= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\
 &\quad + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} \\
 &= 16 + 8 + 0 + 2 + 1 + 0.5 + 0 + 0.125 \\
 &= (27.625)_{10}
 \end{aligned}$$

$$(iv) (1010.1100)_2 = (10.75)_{10}$$

(02) Octal to Decimal :

$$(i) (36)_8 = \frac{3}{8^1} \times 8^1 + \frac{6}{8^0} \times 8^0 = 24 + 6 = (30)_{10}$$

$$(ii) (563)_8 = 5 \times 8^2 + 6 \times 8^1 + 3 \times 8^0 = 320 + 48 + 3 = (371)_{10}$$

$$\begin{aligned}
 (iii) (125.32)_8 &= 1 \times 8^2 + 2 \times 8^1 + 5 \times 8^0 + \frac{3}{8^1} \times 8^{-1} + \frac{2}{8^0} \times 8^{-2} \\
 &= 64 + 16 + 5 + 0.375 + 0.03125 \\
 &= (85.40625)_{10}
 \end{aligned}$$

(03) Hexadecimal to decimal :

$$\begin{aligned}
 (i) (5AE)_{16} &= 5 \times 16^2 + A \times 16^1 + E \times 16^0 \\
 &= 256 \times 5 + (10 \times 16) + (14 \times 1) \\
 &= 1280 + 160 + 14 = (1454)_{10}
 \end{aligned}$$

$$\begin{aligned}
 (ii) (35D.4F)_{16} &= (\dots)_{10} \\
 &= 3 \times 16^2 + 5 \times 16^1 + 0 \times 16^0 + 4 \times 16^{-1} + 15 \times 16^{-2} \\
 &= 3 \times 256 + 80 + 13 \times 1 + \frac{4}{16} + \frac{15}{256} \\
 &= (861.3085938)_{10}
 \end{aligned}$$

$$\begin{aligned}
 (iii) (675.AC)_{16} &= 6 \times 16^2 + 7 \times 16^1 + 5 \times 16^0 + \frac{10}{16} + \frac{12}{256} \\
 &= 1920 + 112 + 5 + 1.6 + 0.046875 \\
 &= (1653.671875)_{10}
 \end{aligned}$$

04) Decimal to Binary:

$$(i) (565)_{10} = (1000110101)_2$$

$$\begin{array}{r} 2 \mid 565 \\ 2 \mid 284 - 1 \\ 2 \mid 142 - 0 \\ 2 \mid 71 - 0 \\ 2 \mid 35 - 0 \\ 2 \mid 17 - 1 \\ 2 \mid 85 - 1 \\ 2 \mid 42 - 1 \\ 2 \mid 21 - 0 \\ 2 \mid 101 - 1 \\ 2 \mid 5 - 0 \\ \hline \end{array}$$

$$(ii) (342.625)_{10} = (101010110.101)_2$$

$$0.625 \times 2 = 1.250$$

$$0.250 \times 2 = 0.500$$

$$0.5 \times 2 = 1.000$$

$$0.1000 \times 2 = 0.2000$$

$$0.2000 \times 2 = 0.4000$$

$$0.4000 \times 2 = 0.8000$$

$$0.8000 \times 2 = 1.6000$$

$$0.6000 \times 2 = 1.2000$$

$$0.2000 \times 2 = 0.4000$$

$$0.4000 \times 2 = 0.8000$$

$$0.8000 \times 2 = 1.6000$$

$$0.6000 \times 2 = 1.2000$$

$$(iii) (125.123)_{10} = (1111101.00011)_2$$

$$0.123 \times 2 = 0.246$$

$$0.246 \times 2 = 0.492$$

$$0.492 \times 2 = 0.984$$

$$0.984 \times 2 = 1.968$$

$$0.968 \times 2 = 1.936$$

$$0.936 \times 2 = 1.872$$

$$0.872 \times 2 = 1.744$$

$$0.744 \times 2 = 1.488$$

$$0.488 \times 2 = 0.976$$

$$0.976 \times 2 = 1.952$$

$$0.952 \times 2 = 1.904$$

$$0.904 \times 2 = 1.808$$

$$0.808 \times 2 = 1.616$$

$$0.616 \times 2 = 1.232$$

$$0.232 \times 2 = 0.464$$

$$0.464 \times 2 = 0.928$$

$$0.928 \times 2 = 1.856$$

$$0.856 \times 2 = 1.712$$

$$0.712 \times 2 = 1.424$$

$$0.424 \times 2 = 0.848$$

$$0.848 \times 2 = 1.696$$

$$0.696 \times 2 = 1.392$$

$$0.392 \times 2 = 0.784$$

$$0.784 \times 2 = 1.568$$

$$0.568 \times 2 = 1.136$$

$$0.136 \times 2 = 0.272$$

$$0.272 \times 2 = 0.544$$

05) Decimal to Octal:

$$(i) (365)_{10} = (555)_8$$

$$\begin{array}{r} 8 \mid 365 \\ 8 \mid 45 \text{ } 5 \uparrow \\ 8 \mid 5 \text{ } 1 \uparrow \\ 8 \mid 0 \text{ } 5 \uparrow \\ \hline \end{array}$$

$$(ii) (286.052)_{10} = (436.032)_8$$

$$0.052 \times 8 = 0.416$$

$$0.416 \times 8 = 3.328$$

$$0.328 \times 8 = 2.624$$

$$\begin{array}{r} 8 \mid 286 \\ 8 \mid 35 \text{ } 6 \uparrow \\ 8 \mid 4 \text{ } 3 \uparrow \\ 8 \mid 0 \text{ } 0 \uparrow \\ \hline \end{array}$$

$$(iii) (482.58)_{10} = (742.450)_8$$

$$0.58 \times 8 = 4.64$$

$$0.64 \times 8 = 5.12$$

$$0.12 \times 8 = 0.96$$

$$\begin{array}{r} 8 \mid 982 \\ 8 \mid 6 \text{ } 2 \uparrow \\ 8 \mid 0 \text{ } 6 \uparrow \\ 8 \mid 0 \text{ } 0 \uparrow \\ \hline \end{array}$$

06) Decimal to Hexadecimal:

$$(i) (1532)_{10} = (5FC)_{16}$$

$$\begin{array}{r} 16 \mid 1532 \\ 16 \mid 95 \text{ } . \text{ } C \uparrow \\ 16 \mid 5 \text{ } . \text{ } 7 \uparrow \\ 16 \mid 0 \text{ } . \text{ } 7 \uparrow \\ \hline \end{array}$$

$$(ii) (356.49)_{10} = (164.7D7)_{16}$$

$$0.49 \times 16 = 7.84$$

$$0.84 \times 16 = 13.44$$

$$\begin{array}{r} 16 \mid 356 \\ 16 \mid 22 \text{ } . \text{ } 6 \uparrow \\ 16 \mid 16 \text{ } . \text{ } 1 \uparrow \\ 16 \mid 10 \text{ } . \text{ } 1 \uparrow \\ 16 \mid 6 \text{ } . \text{ } 1 \uparrow \\ 16 \mid 4 \text{ } . \text{ } 1 \uparrow \\ 16 \mid 4 \text{ } . \text{ } 1 \uparrow \\ \hline \end{array}$$

$$(iii) (25B.2.86)_{10} = (9E4.DC28)_{16}$$

16 | 2532 11

$$0.86 \times 16 = 13.76$$

$$0.76 \times 16 = 12.16$$

$$0.16 \times 16 = 2.56$$

$$0.56 \times 16 = 8.96$$

07. Octal to Binary :- radix = 8 \Rightarrow 8 = 2^3

each number will be represented in 3 digits.

$$(i). (563)_8 = (101110011)_2$$

$5 \rightarrow 3$
4 21
4 11

$$(ii). (445.31)_8 = (100100101.011001)_2$$

$$(iii). (1234.56)_8 = (001010011100.101110)_2$$

08) Binary to Octal :-

$$(i) (1101110010)_2 = (15627)_8$$

001|101|110|010
1 5 6 2

$$(ii) (011110000111.111100)_2 = (3607.74)_8$$

$$(iii) (110011010101.00110)_2 = (6325.14)_8$$

09) Hexadecimal to Binary :-

radix = 16, $16 = 2^4$, 4-bit

$$(i) (A(F))_{16} = (101011001111)_2$$

$$(ii) (1AD.EC)_{16} = (000110101101.11101100)_2$$

$$(iii) (25B.CD)_{16} = (001001011011.11001101)_2$$

10) Binary to Hexadecimal :-

$$(i) (11011100101)_2 = (6E5)_{16}$$

$$(ii) (111000110011.110110)_2 = (E33.0B)_{16}$$

$$(iii) (1010101011.110011)_2 = (2AB.CC)_{16}$$

(11) Octal to Hexadecimal :- [Octal \rightarrow Binary \rightarrow Hex]

(i) $(357)_8 = (\text{ E F })_{16}$

(ii) $(536.53)_8 = (15E. AC)_{16}$

(iii) $(135.67)_8 = (5D. DC)_{16}$

(12) Hexadecimal to Octal :-

(i) $(3AF.DE)_{16} = (1657.674)_8$

(ii) $(4BC.5A)_{16} = (2274.264)_8$

(iii) $(125.46)_{16} = (445.214)_8$

(iv) $(ABC.DB)_{16} = (5274.674)_8$

Problems :-

01. $(253.56)_8 = (\dots)_{10}$

02. $(123.4F8)_{16} = (\dots)_{10}$

03. $(536.32)_{10} = (\dots)_8$

04. $(1010101.010)_{2} = (\dots)_{10}$

05. $(4326.95)_{10} = (\dots)_8$

06. $(1534.516)_{10} = (\dots)_{16}$

07. $(5316.72)_8 = (\dots)_2$

08. $(101011.110)_2 = (\dots)_{16}$

09. $(1101101101111.110101)_{2} = (\dots)_8$

10. $(5FD.CA)_{16} = (\dots)_{16}$

11. $(3561.52)_8 = (\dots)_2$

12. $(4FA.(B))_{16} = (\dots)_{10}$

13. $(4AC.AB)_{16} = (\dots)_{10}$

14. $(952.58)_{10} = (\dots)_{16}$

15. $(101010.110)_2 = (\dots)_{10}$

$$01) (253.56)_8 = (\quad)_{10}$$

Solution: $\therefore 2 \times 8^2 + 5 \times 8^1 + 3 \times 8^0 + \frac{5}{8} + \frac{6}{8^2}$
 $= 2 \times 64 + 40 + 3 + 0.625 + 0.09375$
 $= (171.71875)_{10}$

$$02) (123.4F8)_{16} = (\quad)_{10}$$

Solution: $\therefore 1 \times 16^2 + 2 \times 16^1 + 3 \times 1 + \frac{4}{16} + \frac{15}{256} + \frac{8}{16^3}$
 $= 256 + 32 + 3 + 0.25 + 0.05859375 + 0.00195$
 $= (291.3105469)_{10}$

$$03) (536.32)_{10} = (1030.2436)_8$$

Solution:

$$0.32 \times 8 = 2.56$$

$$0.56 \times 8 = 4.48$$

$$0.48 \times 8 = 3.84$$

$$0.84 \times 8 = 6.72$$

$$04) (1010101.010)_2 = (\quad)_{10}$$

Solution: $\therefore 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$
 $+ 0 \times 2^{-1} + \frac{1}{2^2} + 0 \times 2^{-3}$
 $= 2^6 + 2^4 + 2^2 + 1 + \frac{1}{4} = 64 + 16 + 4 + 0.25 = (85.25)_{10}$

$$05) (4326.95)_{10} = (\quad)_{16}$$

Solution: $(10E6.F33)_{16}$

$$0.95 \times 16 = 15.2$$

$$0.2 \times 16 = 3.2$$

$$0.2 \times 16 = 3.2$$

$$\begin{array}{r} 1614326 \\ -16 \\ \hline 14326 \\ -16 \\ \hline 1376 \\ -16 \\ \hline 120 \\ -16 \\ \hline 8 \\ -8 \\ \hline 0 \end{array}$$

$$(06) (1534.516)_{10} = (101111110.1000)_2$$

$$0.516 \times 2 = 1.032$$

$$0.032 \times 2 = 0.064$$

$$0.064 \times 2 = 0.128$$

$$0.128 \times 2 = 0.256$$

2 | 1534
 2 | 767 - 0
 2 | 383 - 1
 2 | 191 - 1
 2 | 95 - 1
 2 | 47 - 1
 2 | 23 - 1
 2 | 11 - 1
 1 | 5 - 1
 2 | 2 - 1
 1 | 0

$$(07) (5316.72)_8 = (\text{ACE.EB})_{16}$$

$$5316 \Rightarrow 1010|1100|110.1110|0001$$

A C E . B B

$$(08) (101011.110)_2 = (53.6)_8$$

$$101|011|.110|$$

5 3 6

$$(09) (1101101101111.110101)_2 = (1B6F.D4)_{16}$$

$$1|101|10110|1111.1|1101|0100$$

1 B 6 F D 4

$$(10) (5FD.(A))_{16} = (2775.624)_8$$

$$\begin{array}{r} 8421 & 8421 & 8421 & . & 8421 & 8421 \\ 0101 & 1111 & 1101 & . & 1100 & 1010 \\ \hline 010111110111001010 \\ \hline 2 \quad 7 \quad 7 \quad 5 : 6 \quad 2 \quad 4 \end{array}$$

$$(11) (3561.72)_8 = (\text{ })_2$$

$$\begin{array}{r} 421 & 421 & 421 & 421 & 421 & 421 \\ 011 & 101 & 110 & 001 & 111 & 010 \end{array}$$

$$(11101110001.11101)_2$$

$$(12) (4FA \cdot (B))_{16} = (\quad)_{10}$$

$$= 4 \times 16^2 + 15 \times 16 + 10 \times 1 + \frac{12}{16} + \frac{13}{256}$$

$$= (1274, 792969)_{10}$$

$$(4FA \cdot (B))_{16} = (\quad)_2$$

$$\begin{array}{cccccc} 8421 & 8421 & 8421 & 8421 & 8421 \\ 0100 & 1111 & 1000 & 1100 & 1011 \end{array}$$

$$(1001111101011001011)_2$$

$$(13) (4AC \cdot AB)_{16} = (\quad)_{10}$$

Solution :- $4 \times 16^2 + 10 \times 16 + 12 \times 1 + \frac{10}{16} + \frac{11}{256}$

$$= (1196, 669969)_{10}$$

$$(14) (45.258)_{10} = (1(4.997)_{16})$$

$$0.58 \times 16 = 9.28$$

$$0.28 \times 16 = 4.48$$

$$0.48 \times 16 = 7.68$$

$$(15) (101010.110)_2 = (\quad)_{10}$$

$$1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 + \frac{1}{2} + \frac{1}{4} +$$

$$= 32 + 8 + 2 \cdot \frac{1}{2} + \frac{1}{4} = 42.4075 = 42.75$$

Conversion from any number system to any other number system :-

$$(i) (5643, 12)_{10} = (\quad)_5$$

Solution :-

$$5 \times 7^3 + 6 \times 7^2 + 4 \times 7 + 3 + \frac{1}{7} + \frac{2}{49}$$

$$5 \times 343 + 6 \times 49 + 28 + 3 + \frac{1}{7} + \frac{2}{49}$$

$$= (2040, 184)$$

$$(2040.184)_{10} = (31130.043)_5$$

$$0.184 \times 5 = 0.92$$

$$0.92 \times 5 = 4.6$$

$$0.6 \times 5 = 3.$$

$$\begin{array}{r} 12090 \\ 403 \times 0 \\ 315 \times 1 \\ 316 \times 1 \\ 23 \times 1 \\ 0 - 3 \end{array}$$

$$(ii) (143.2)_5 = (\quad)_9$$

$$1 \times 5^3 + 4 \times 5^2 + 3 \times 5 + 2 \times 5^0 = 125 + 4 \times 25 + 15 + 2$$

$$= (242)_{10} = (288)_9$$

$$\begin{array}{r} 1242 \\ 9 \times 2 \times 0 \\ 2 - 8 \\ 0 - 1 \\ 1 \end{array}$$

$$(iii) (1453.52)_6 = (\quad)_8$$

$$= 1 \times 6^3 + 4 \times 6^2 + 5 \times 6^1 + 3 \times 6^0 + \frac{5}{36} + \frac{2}{36}$$

$$= 216 + 144 + 30 + 3 + 0.833 + 0.556$$

$$= (393.888889)_{10}$$

$$0.8889 \times 8 = 7.1112$$

$$0.1112 \times 8 = 0.8896$$

$$0.8896 \times 8 = 7.1168$$

$$0.1168 \times 8 = 0.9344$$

$$\begin{array}{r} 393 \\ 8 \times 4 \times 0 \\ 49 - 4 \\ 8 \times 6 - 1 \\ 6 - 6 \\ 0 - 6 \\ 6 \end{array}$$

$$= (393.888889)_{10} = (611.7070)$$

find the value of 'x' for the following

$$(1 \times 5.32)_8 = (85.40625)_{10}$$

$$(85.40625)_{10} = (125.32)_8$$

$$\begin{array}{r} 8 | 85 \\ 8 | 10 - 5 \\ 1 - 2 \end{array}$$

$$x = 2, 1$$

Compliments:

⇒ To simplify the subtraction operation

⇒ Logical calculations

they are of 2 types:

01. Radix complement (r's complement)

02. Diminished Radix complement ($(r-1)$'s complement)

Decimal Number System:

10's Compliment

9's Compliment

→ $(r-1)$'s compliment can be calculated $[r^D - N]$
where $r \rightarrow$ radix; $n \rightarrow$ digits no ; $N \rightarrow$ Given number

→ To calculate r_6 's compliment $[r^D - N]$

Calculate the 9's & 10's compliment of the following.

$$(i) (263.35)_{10} \Rightarrow (736.64)_{10} = (738.65)_{10}$$

9's compliment 10's compliment,

$$(ii) (345.67)_{10} \Rightarrow (654.32)_{10} = (654.33)_{10}$$

Binary Number System:

Calculate the 1's & 2's complement of the following

$$(i) (1011011)_2 \Rightarrow (0100100)_2 \quad 1^{\text{st}} \text{ Compliment} \quad 2^{\text{nd}} \text{ Compliment}$$

$$(ii) (11100101)_2 \Rightarrow (00011010)_2 \quad 1^{\text{st}} \text{ Compliment} \quad 2^{\text{nd}} \text{ Compliment}$$

$$(iii) (1010110)_2 \Rightarrow (0101001)_2 \quad 1^{\text{st}} \text{ Compliment} \quad 2^{\text{nd}} \text{ Compliment}$$

Calculate the 7's & 8's compliment of the following

$$(i) (4536)_8 \Rightarrow (3241)_8 \quad 7^{\text{th}} \text{ Compliment} \quad 8^{\text{th}} \text{ Compliment}$$

$$(2) (1354)_8 \Rightarrow (6423)_8 \quad 7^{\text{th}} \text{ Compliment} \quad 8^{\text{th}} \text{ Compliment}$$

Calculate the 15^{1's} & 16^{1's} complement of the following

$$(i) (ABF)_{16} = \begin{pmatrix} 5 & 4 & 0 \end{pmatrix}_{16} = \begin{pmatrix} 5 & 4 & 1 \end{pmatrix}_{16} \quad \begin{matrix} 15^{\text{1's Compliment}} \\ 16 \end{matrix} \quad \begin{matrix} 16^{\text{1's Compliment}} \\ 16 \end{matrix} \quad \begin{matrix} 567 \\ 1 \end{matrix}$$

$$(ii) (354C)_{16} = \begin{pmatrix} C & A & B & 3 \end{pmatrix}_{16} = \begin{pmatrix} C & A & B & 4 \end{pmatrix}_{16} \quad \begin{matrix} 1 \\ 570 \end{matrix}$$

$$\begin{pmatrix} A & 3 & D & F \end{pmatrix}_{16} = \begin{pmatrix} A & 3 & E & 0 \end{pmatrix}_{16} \quad \begin{matrix} A3DF \\ +1 \\ 16 \\ \hline A3E0 \end{matrix}$$

Q1. Write the 1st 10 numbers in Radix 4,

Decimal Radix 4 Careful imp

0	0
1	1
2	2
3	3
4	10
5	11
6	12
7	13
8	20
9	21

Subtraction using complements.

Q1) Decimal Number System

Subtraction using 9's complements.

Step 01:- Calculate the 9's compliment of the subtrahend

Step 02:- Add it to the minuend.

Step 03:- If carry is present, the result is +ve.

Step 04:- If carry is not present, the result is -ve &

again calculate 9's compliment for the answer.

Step 05:- And end around the carry

Perform the following subtraction using 9's complement

(i) $534 - 236 = +298$ (ii) $325 - 616 = -291$

$$\begin{array}{r} 534 \\ - 236 \\ \hline 298 \end{array}$$

$\xrightarrow{-616} \begin{array}{r} 325 \\ 383 \\ \hline 708 \end{array}$ No carry
M.S.B \Rightarrow Most Significant Bit $\Rightarrow -291$
M.S.D \Rightarrow Most significant digit
L.S.B \Rightarrow Least significant Bit
L.S.D \Rightarrow Least significant digit.

Perform the following using 9's complement method

(i) $536.4 - 329.6$

$$\begin{array}{r} 536.4 \rightarrow \text{Minuend} \\ + 670.3 \rightarrow 9\text{'s compliment of } 329.6 \\ \hline 1206.7 \end{array}$$

$\xrightarrow{\text{Ignore carry}}$ front around carry.

$$\begin{array}{r} 1206.8 \end{array}$$

(ii) $315.23 - 654.15$

Solution:

$$\begin{array}{r} 315.23 \\ - 345.84 \\ \hline 661.07 \end{array}$$

As no carry is present,
result is negative

\therefore again calculating 9's compliment $\Rightarrow -338.92$ [9's complement of 661.07]

Perform the following using 10's complement method

(i) $536.4 - 329.6$

Solution: $536.4 \rightarrow \text{Minuend}$

$$\begin{array}{r} 536.4 \\ + 670.4 \rightarrow 10\text{'s compliment of } 329.6 \\ \hline 1206.8 \end{array}$$

$\xrightarrow{\text{Ignore the carry}}$ 206.8

$$(ii) 315.23 - \underline{654.15}$$

Solution: $315.23 \rightarrow$ Minuend
 $345.85 \rightarrow$ 10's complement of 654.15
 $\underline{661.08} \rightarrow$ As no carry is present, result is -ve.

∴ Solution is -338.92 (10's complement of 661.08)
a's $\Rightarrow 338.91$

$$(iii) 123.61 - 429.54$$

Solution:
$$\begin{array}{r} 123.61 \\ 570.46 \\ \hline 694.07 \end{array} \rightarrow$$
 no carry therefore calculating 10's complement of 694.07 $\Rightarrow [305.93]$
 $= -305.93$

Binary addition:

$$(i) 0+0=0$$

$$(ii) 0+1=1$$

$$(iii) 1+0=1$$

$$(iv) 1+1=0 \text{ with carry is } 1$$

$$(v) 1+1+1=1 \text{ with carry is } 1$$

$$(a) \begin{array}{r} 1000 \\ + 0101 \\ \hline 1101 \end{array}$$

$$(b) 1010111 \rightarrow 87$$

$$+ 0110$$

$$\underline{\underline{1111}} \rightarrow 55$$

$$+ 1000$$

$$\underline{\underline{1000}} \rightarrow 142$$

$$+ 1101$$

$$\underline{\underline{0001}} \rightarrow 142_1$$

$$+ 1010$$

$$+ 1110$$

$$+ 1100$$

$$(c) 10101010 \rightarrow 140$$

$$+ 11101101 \rightarrow 237$$

$$\underline{\underline{11001011}} \rightarrow 407$$

$$(d) 101011.10$$

$$+ 011111.010$$

$$\underline{\underline{1001010.111}}$$

Binary Subtraction:

$$0 - 0 = 0$$

$$1 - 0 = 1$$

$$1 - 1 = 0$$

$0 - 1 = 1$ with borrow with 1

$$(a) \begin{array}{r} 10 \\ - 0101 \\ \hline 0101 \end{array}$$

$$(b) \begin{array}{r} 100 \\ - 0110 \\ \hline 0010 \end{array}$$

$$(c) \begin{array}{r} 10101101 \\ - 01010100 \\ \hline 01011001 \end{array} \rightarrow 173 \rightarrow 84 \rightarrow 89$$

$$(d) \begin{array}{r} 10100 \\ - 011101010 \\ \hline 000101010 \end{array} \rightarrow 277 \rightarrow 234 \rightarrow 43$$

M.S.B $\rightarrow 0 \rightarrow +ve$ number
 M.S.B $\rightarrow 1 \rightarrow -ve$ number.

Subtraction using 1's complement method:

Step 01: Calculate the given 1's complement of subtrahend

Step 02: Add it to the minuend.

Step 03: If carry is present, the result is +ve and end around the carry

Step 04: If carry is not present, the result is -ve, again calculate 9's complement for the answer.

$$(a) X = 1000 \rightarrow 8$$

$$Y = 0101 \rightarrow 5$$

$$(i) X - Y \rightarrow 1000 \rightarrow \text{Minuend}$$

$$+ 1010 \rightarrow 1's \text{ complement of } 0101$$

$$\begin{array}{r} 10010 \\ \oplus 1010 \\ \hline 0011 \end{array} \rightarrow \text{end around carry}$$

$$\begin{array}{r} 0011 \\ \hline \end{array} \rightarrow \text{final answer}$$

(ii) $y-x \Rightarrow 0101 \rightarrow \text{Minuend}$
 $\underline{0111} \rightarrow 1's \text{ complement of } 1000$
 $\underline{1000} \rightarrow \text{carry not present \& result is negative}$

$\therefore \text{solution is } -0011 \Rightarrow 1's \text{ complement of } 1100$

$$(b) x = 10101011$$

$$y = 01110111$$

$$\begin{array}{r} x-y = 10101011 \\ - 01110111 \\ \hline 10001000 \\ \boxed{00110011} \\ \hline 00110100 \end{array}$$

$$y-x \Rightarrow 01110111$$

$$\begin{array}{r} 01010100 \\ \underline{1111} \\ \boxed{10010111} \end{array}$$

$\Rightarrow \text{carry is absent}$

$\therefore \text{solution is } -\underline{00110100}$

$$(c) x = 11100110 \rightarrow 462$$

$$y = 101110110 \rightarrow 374$$

$$\begin{array}{r} x-y = 111001110 \\ - 010001000 \\ \hline 100101011 \\ \hline 001011000 \end{array}$$

$$y-x \Rightarrow 101110110$$

$$\begin{array}{r} 000110001 \\ \underline{111} \\ \hline 110100111 \end{array}$$

$\Rightarrow -\underline{001011000}$

344
88
462

Subtraction using 2's complement:

Step 01: Calculate 2's complement of subtrahend

i.e add 1 to 1's complement

Step 02: Add it to minuend

Step 03: If carry is present, the result is +ve and ignore the carry

Step 04: If carry is absent, the result is -ve and again calculate 2's complement for answer.

$$(a) X = 1000$$

$$Y = 0101$$

$$(i) X-Y \Rightarrow 1000 \rightarrow \text{minuend}$$

$$\begin{array}{r} 1011 \\ \hline \text{ignoring } \boxed{1} \\ \text{carry } \quad \underline{0011} \\ \hline \end{array} \rightarrow \text{2's complement of } Y$$

$\rightarrow \text{final answer}$

$$(ii) Y-X \Rightarrow 0101 \rightarrow \text{Minuend}$$

$$\text{no carry } \quad \underline{1000} \rightarrow \text{2's complement}$$

M.S.B $\leftarrow \boxed{1}01 \rightarrow$ for answer, calculate 2's complement

$$\text{i.e } (0010+1)-\underline{0011} \rightarrow \text{final answer.}$$

$$(b) X = 10101011$$

$$Y = 01110111$$

$$(i) X-Y \Rightarrow 10101011 \rightarrow \text{Minuend}$$

$$\begin{array}{r} 10001000 \\ \hline \text{ignore } \boxed{1} \\ \underline{00110100} \\ \hline \end{array} \rightarrow \text{2's complement}$$

$\rightarrow \text{final answer}$

$$(ii) Y-X \Rightarrow 01110111 \rightarrow \text{Minuend}$$

$$\text{no carry } \& \quad \underline{01010101} \rightarrow \text{2's complement}$$

M.S.B is $\leftarrow \boxed{1}1001100 \rightarrow$

Indicates a -ve number

$$\begin{array}{r} 01010100 \\ \hline 01010101 \\ \hline 111 \\ \hline 00110011 \\ \hline 00110011 \\ \hline \end{array} \quad \left. \begin{array}{l} \text{2's complement} \\ \text{of } (11001100) \end{array} \right\}$$

$$(C) X = 111001110$$

$$Y = 101110110$$

$$(i) X - Y \Rightarrow 111001110$$

$$\begin{array}{r} 010001001 \\ - 101110110 \\ \hline 001011000 \end{array}$$

ignore $\underline{\underline{1}}$

$$\begin{array}{r} 010001001 \\ - 101110110 \\ \hline 001011000 \end{array}$$

$$(ii) Y - X \Rightarrow 101110110$$

no carry
and M.S.B
is 1

$$\begin{array}{r} 000110001 \\ - 11111 \\ \hline 110101000 \end{array}$$

$$\begin{array}{r} 000110001 \\ - 110011001 \\ \hline 000001000 \end{array}$$

calculating 2's complement i.e $[001010111 + 1]$

$$\begin{array}{r} 001010111 \\ + 1 \\ \hline 001011000 \end{array}$$

22nd January 2018

Signed Binary Numbers: M.S.B determines the sign of number.

Decimal	Signed magnitude form	Signed 1's complement	Signed 2's complement
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
0	0000	0000	0000
-0	1000	1111	1111
-1	1001	1110	1111
-2	1010	1101	1110
-3	1011	1100	1110
-4	1100	1011	1100
-5	1101	1010	1100
-6	1110	1001	1100
-7	1111	1000	1100

1's complement subtraction for signed numbers

$$+6 \rightarrow 00000110$$

$$-6 \rightarrow 10000110$$

$$\begin{matrix} 1's \\ comp \end{matrix} \rightarrow 11111001$$

$$+13 \rightarrow 00001101$$

$$-13 \rightarrow 10001101$$

$$\begin{matrix} 1's \\ comp \end{matrix} \rightarrow 11110010$$

$$+6 \rightarrow 00000110$$

$$\begin{array}{r} +13 \\ -6 \\ \hline 19 \end{array}$$

$$\begin{array}{r} 00001101 \\ \hline 00010011 \end{array}$$

$$+13 \rightarrow 00001101$$

$$-6 \rightarrow 11111001$$

$$\begin{array}{r} 11111001 \\ -6 \\ \hline 00000110 \end{array}$$

$$00000111$$

$$-13 \rightarrow 11110010$$

$$+6 \rightarrow 00000110$$

$$\begin{array}{r} +13 \\ -6 \\ \hline -19 \end{array}$$

calculating 1's complement

$$10000111$$

$$-13 \rightarrow 11110010$$

$$-6 \rightarrow 11111001$$

$$\begin{array}{r} 11111001 \\ -6 \\ \hline 11110101 \end{array}$$

$$\begin{array}{r} 11110101 \\ -19 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 11110101 \\ -19 \\ \hline 11 \end{array}$$

$$\begin{array}{r} 11110101 \\ -19 \\ \hline 11 \end{array}$$

$$+38 \rightarrow 00100110$$

$$-38 \rightarrow 10100110$$

1's complement

$$-38 \rightarrow 11011001$$

$$+38 \rightarrow 00100110$$

$$+25 \rightarrow 00011001$$

$$-25 \rightarrow 10011001$$

$$\begin{matrix} 1's \\ comp \end{matrix} \rightarrow 11100110$$

$$+25 \rightarrow 00011001$$

$$\begin{array}{r} 00011001 \\ -25 \\ \hline 63 \end{array}$$

$$00111111$$

$$+38 \rightarrow 00100110$$

$$-25 \rightarrow 11100110$$

$$\begin{array}{r} 11100110 \\ +13 \\ \hline 00001100 \end{array}$$

$$-38 \rightarrow 11011001$$

$$+25 \rightarrow 00011001$$

$$\begin{array}{r} 00011001 \\ -25 \\ \hline 63 \end{array}$$

$$11111111$$

$$00001100 \rightarrow +13$$

$$-10001101 \rightarrow -13$$

-38 → 10100110 11011001

-25 →

$$\begin{array}{r} 11100110 \\ + 1 \\ \hline 10111111 \end{array}$$

M.S.B is 0 → $\begin{array}{r} \text{ve} \\ \leftarrow \end{array} \boxed{1} 0 0 0 0 0 0 0$

calculating 1's complement for result obtained

10111111

LectureNotes.in

2's complement subtraction for signed numbers

+6 → 0000 0110

+13 → 0000 1101

-6 → 1000 0110

-13 → 1000 1101

1's com

-6 → 1111 0010

1's com

2's com

-6 → 1111 0110

2's com

-13 → 1111 0010

+13 → 0000 1101

-6 → 1111 1010

$\frac{-}{+}$ 1111 0011 Neglecting carry.

+ve

-13 → 1111 0011

+6 → 0000 0110

$\frac{-}{+}$ 1111 1001

-ve number

∴ calculating 2's complement

10000110

no carry

$\frac{-}{+}$ 10000110

$\Rightarrow -7$

-13 → 1111 0011

-6 → 1111 1010

$\frac{-}{+}$ 1111 1010

-ve number

$\Rightarrow 10010010$

$\Rightarrow -10010010$

$+38 \rightarrow 00100110$

$-38 \rightarrow 10100110$

1^{st} comp

$-38 \rightarrow 11011001$

2^{nd} com

$-38 \rightarrow 11011010$

$+25 \rightarrow 00011001$

$-25 \rightarrow 10011001$

1^{st} comp

$-25 \rightarrow 11100110$

2^{nd} comp

$-25 \rightarrow 11100111$

$+38 \rightarrow 00100110$

LectureNotes.in

$+25 \rightarrow 00011001$

$\begin{array}{r} 1 \\ + 63 \\ \hline + 63 \end{array}$

$\begin{array}{r} 00111111 \\ \hline 00111111 \end{array}$

$+38 \rightarrow 00100110$

$-25 \rightarrow 11100111$

$\begin{array}{r} 1 \\ + 13 \\ \hline + 13 \\ 00001101 \end{array} \rightarrow +13,$

$-38 \rightarrow 11011010$

$+25 \rightarrow 00011001$

$\begin{array}{r} 1 \\ -13 \\ \hline 11100011 \end{array} \text{ no carry.}$

-ve
number

calculating 1^{st} complement of $11110000 \oplus 10001100$

$-38 \rightarrow 11011010$

$\begin{array}{r} 1 \\ 10001100 \\ \hline 10001100 \end{array} = -13$

$-25 \rightarrow 11100111$

$\begin{array}{r} 1 \\ -63 \\ \hline 10000001 \end{array}$

neglect

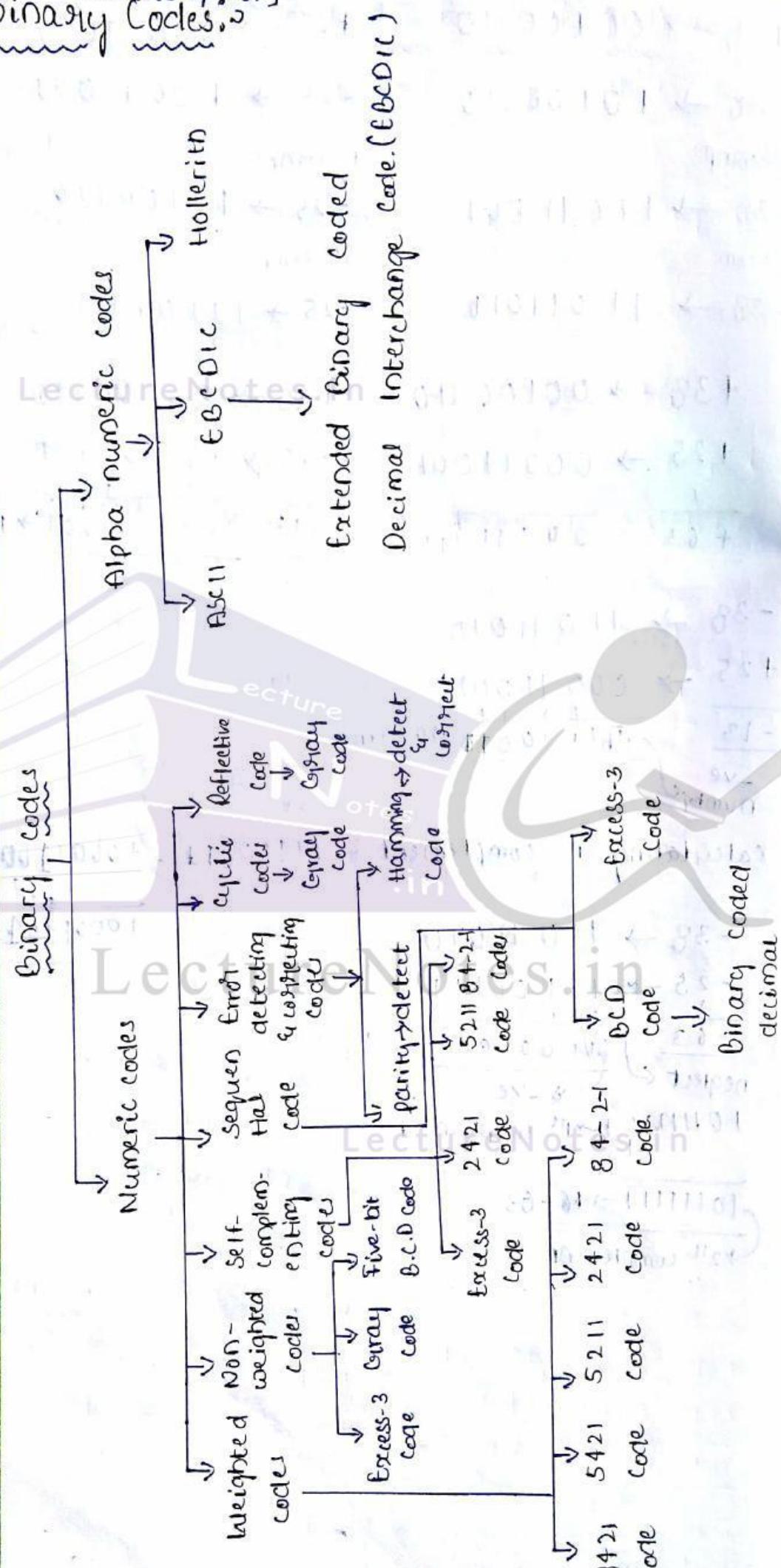
-ve

$10111110 \rightarrow 1^{\text{st}}$ complement

$\begin{array}{r} 10111111 \\ \hline 10111111 \end{array} \Rightarrow +6 - 63$

$\rightarrow 2^{\text{nd}}$ complement

23rd January, 2019
 Binary Codes



Decimal number	8421 code	5421 code	2421 code	5211 code	B4-2-1 code	Excess-3 code. (non-weighted code)
0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 1 1 - 4
1	0 0 0 1	0 0 0 1	0 0 0 1	0 0 0 1	0 1 1 1	0 1 0 0 - 5
2	0 0 1 0	0 0 1 0	0 0 1 0	0 0 1 1	0 1 1 0	0 1 0 1 - 6
3	0 0 1 1	0 0 1 1	0 0 1 1	0 0 1 0	0 1 0 1	0 1 1 0 - 7
4	0 1 0 0	0 1 0 0	0 1 0 0	0 1 1 1	0 0 1 0 0 0	0 1 1 1 - 8
5	0 1 0 1	1 0 0 0	1 0 1 1	1 0 0 0	1 0 1 1	1 0 0 0 - 9
6	0 1 1 0	1 0 0 1	1 1 0 0	1 0 1 0	1 0 1 0	1 0 0 1 - 10
7	0 1 1 1	1 0 0 0	1 1 0 1	1 1 0 0	1 0 0 1	1 0 1 1 - 11
8	1 0 0 0	1 0 1 1	1 1 1 0	1 1 1 0	1 0 0 0	1 0 1 1 - 12
9	1 0 0 1	1 1 0 0	1 1 1 1	1 1 1 1	1 1 1 1	1 1 0 0 - 13

Self complementing codes:

$$\Rightarrow 9-6=3 \rightarrow 0110 ; 9-7=2 \rightarrow 0101$$

\downarrow \uparrow \downarrow \uparrow

1001 1's complement 1010

⇒ Self complementing codes are the codes in which $9-N=x$ where x is the 1's complement of N .

ex: (i) Excess-3 code

(ii) 2421 code

(iii) 5211 code

(iv) 84-2-1 code.

Sequential Codes:

1111-0100-1001

0111-0100-0101

0110-0100-0100

0001-0100-0100

0000-0100-0100

Binary coded Decimal (B.C.D code) (8421)

→ In B.C.D code, each and every digit is represented with 4 bits.

→ In B.C.D 1010, 1011, 1100, 1101, 1110, 1111 are invalid codes.

* B.C.D addition:

→ The correction factor in B.C.D is +6.

→ The correction factor in Excess-3 code is +3,

→ Add 6 (0110) to the number if we get invalid code.

$$\begin{array}{r}
 25 \rightarrow 0010\ 0101 \\
 + 42 \rightarrow 0100\ 0010 \\
 \hline
 67 \quad \quad \quad \underbrace{0110\ 0111}_{6+7}
 \end{array}$$

$$\begin{array}{r}
 54 \rightarrow 0101\ 0100 \\
 + 49 \rightarrow 0100\ 1001 \\
 \hline
 103 \quad \quad \quad \underbrace{1001\ 1101}_{+10} \quad \quad \quad +0110
 \end{array}$$

$$\begin{array}{r}
 653 \rightarrow 0110\ 0101\ 0011 \\
 + 169 \rightarrow 0001\ 0110\ 1001 \\
 \hline
 822 \quad \quad \quad \underbrace{0111\ 1011\ 1100}_{7+11+12}
 \end{array}$$

0111 1011 1100

$$\begin{array}{r}
 \quad \quad \quad 0110\ 0110 \\
 \underline{1111\ 1111} \\
 \underline{1000\ 0000} \quad \quad \quad \text{Illegal code}
 \end{array}$$

8 2 0

* B.C.D subtraction:

$$\begin{array}{r}
 85 \rightarrow 1000\ 0101 \\
 - 34 \rightarrow 0011\ 0100 \\
 \hline
 51 \quad \quad \quad \underbrace{0101\ 0001}_{5-4}
 \end{array}$$

$$\begin{array}{r}
 95 \rightarrow 1001\ 0101 \\
 - 67 \rightarrow 0110\ 0111 \\
 \hline
 28 \quad \quad \quad \underbrace{0010\ 1110}_{2-14 \rightarrow \text{Invalid}}
 \end{array}$$

-0110
0010 1000

$$(c) 845 \rightarrow 01000\ 0100\ 0101$$

$$- 257 \rightarrow 00100\ 0101\ 0111$$

$$\underline{588} \quad \underline{\begin{array}{r} 010111101110 \\ 01100110 \end{array}}$$

$$\underline{\begin{array}{r} 010110001000 \\ 5 \quad 8 \quad 8 \end{array}}$$

(D) $756.4 \rightarrow 01110\ 0101\ 0100$

$$- 378.5 \rightarrow 00110\ 01111\ 0000.0101$$

$$\underline{378.9} \quad \underline{\begin{array}{r} 001111011101.1111 \\ 3 \quad 13 \quad 13 \quad 15 \end{array}}$$

$$\therefore 00110\ 0101.0101.1111$$

$$\underline{\begin{array}{r} 011001100110 \\ 0011024909999001 \\ 3 \quad 7 \quad 7 \quad . \quad 9 \end{array}}$$

Perform the following subtraction in 8421 BCD code using 9's complement method

(i) $56 \rightarrow 56 \rightarrow 01010110 \rightarrow 56$ in B.C.B

$$- 32 \rightarrow 67 \rightarrow 01100111 \rightarrow 9^{\text{'}}\text{s complement of } 32$$

$$\underline{\begin{array}{r} 24 \\ 23 \\ \boxed{2} \\ 24 \end{array}} \quad \underline{\begin{array}{r} 01100111 \\ 10111101 \\ 10111101 \\ \hline 00100100 \end{array}}$$

$+ 0110 + 0110$ (Adding 0110 for illegal code)

0	0	1	0	0	1
---	---	---	---	---	---

(End around the carry)

(Answer in B.C.D)

(ii) $635.4 \rightarrow 0111001011\ 01011.01010 \rightarrow 635.4$

$$- 427.7 \rightarrow 01101010110\ 01111.01010 \rightarrow 572.2$$

$$\underline{\begin{array}{r} 207.7 \\ 1207.6 \\ \hline 207.7 \end{array}}$$

$$635.4 \rightarrow 0110\ 0011\ 0101.0100$$

$$572.2 \rightarrow 0101\ 0100\ 0010.0010$$

$$\begin{array}{r} 11 \\ \hline 1011\ 00100111.0110 \\ 11\ 10\ 7\ 6 \end{array}$$

$$\begin{array}{r} 0110\ 0110 \\ \hline 0111\ 11 \\ \hline 0010\ 000000111.0110 \end{array}$$

$$\begin{array}{r} 0010\ 0000\ 0111.0111 \\ 2\ 0\ 7\ 7 \end{array}$$

$$(c) 256.5 \rightarrow 256.5 \rightarrow 0010\ 0101\ 0110.010 \rightarrow 1$$

$$\begin{array}{r} -429.7 \\ \hline -173.2 \\ \hline 826.7 \end{array} \quad \begin{array}{r} \rightarrow 570.2 \\ \hline 0101\ 0111\ 0000.0010 \rightarrow 1 \\ \hline 0111\ 1100\ 0110.0111 \end{array}$$

$$\begin{array}{r} -173.2 + 9's \\ \text{complement of } 826.7 \\ \hline 10110 \end{array} \quad \begin{array}{r} \rightarrow 0110 \\ \hline 1000\ 0010\ 0110.0111 \end{array} \quad \begin{array}{l} \rightarrow \text{adding } 0110 \\ \text{no carry} \end{array}$$

$$(d) 123.6 \rightarrow 123.6 \rightarrow 0001\ 0010\ 0011.0110$$

$$\begin{array}{r} -578.4 \\ \hline -454.8 \\ \hline 545.1 \end{array} \quad \begin{array}{r} \rightarrow 421.5 \\ \hline 0100\ 0010\ 0001.0101 \end{array}$$

$$123.6 \rightarrow 0001\ 0010\ 0011.0110 \rightarrow 123.6 \text{ in B.C.D}$$

$$421.5 \rightarrow 0100\ 0010\ 0001.0101 \rightarrow 9's \text{ complement of } 578$$

$$\begin{array}{r} \rightarrow 0110 \\ \hline 0101\ 0100\ 0101.0001 \\ 5\ 4\ 5\ 1 \end{array} \quad \begin{array}{l} \rightarrow \text{adding } 0110 \\ \text{no carry} \end{array}$$

Perform the following subtraction in 8421 in B.C.D by using 9's complement.

$$(a) 95 \rightarrow 95$$

$$\begin{array}{r} -64 \rightarrow 36 \rightarrow 10^{'s} \text{ complement of } 64 \\ \hline 31\ 31 \end{array}$$

95 → 1001 0101 → 95 in B.C.D

36 → 0011 0110 → a's complement of 64 in B.C.D

31 → 1100 1011

0110 0110 → adding 0110 to illegal codes

0010 0001 → carry propagated and end carry ignored

0011 0001 → Answer is '31' in B.C.D

(b) 625.3 → 625.3 → 0110 0010 0101. 0011

-456.4 → 543.6 → 0101 0100 0011. 0110
1689 → 1011 0110 1000. 1001
14 6 8 9

1011 0110 1000. 1001

0110
0001 0110 1000. 1001
1 6 8 . 9
→ ignore carry

(c) 456.5 → 456.5 → 0100 0101 0110. 0101 → 456.5 in B.C.D

-648.7 → 351.3 → 0011 0101 0001. 0011 → a's complement
807.8 → 0111 1010 0111. 1000
-192.2 → 0110 → adding 0110
0111 0000 0111. 1000
1000 0000 0111. 1000
8 0 7 . 8 . →
- 1 9 2 . 2 → 10's complement

(d) 256.4 → 256.4

-625.8 → 374.2 → 10's complement of 625.8

630.6 → no carry ∵ result is -ve

-369.4 → 10's complement of 630.6

$$256.4 \rightarrow 0010\ 0101\ 0110.0100 \rightarrow 256.4 \text{ in B.C.D}$$

$$374.2 \rightarrow 0011\ 0111\ 0100.0010 \rightarrow 10^3 \text{ complement of } 625.$$

$$\begin{array}{r} 630.6 \\ -369.4 \\ \hline \end{array} \quad \begin{array}{r} 0110\ 0110\ 0110.0110 \\ 0101\ 0010\ 0000.0110 \\ \hline 0100\ 0011\ 0000.0110 \end{array} \quad \begin{array}{l} \rightarrow \text{adding } 0110 \text{ to illegal code,} \\ \rightarrow \text{carry propagated} \end{array}$$

6 3 0 6 → calculating 10^3 complement as no carry is present
- 369.4

Excess-3 (X_5-3) Code:

⇒ Excess-3 is non-weighted BCD code

⇒ It is sequential and self complementing.

⇒ Invalid codes are 0000(0), 0001(1), 0010(2), 1101(13), 1110(14), 1111(15)

⇒ The correction factor for Excess-3 code is +3

⇒ (0), (1), (2), (13), (14), (15) are invalid or illegal codes in X_5-3 code because, X_5-3 code starts with 3 and ends at 12,

Excess-3 Addition:

→ If carry is generated, add 3(+0011)

→ If carry is not generated, subtract 3(-0011)

(i) $35 \rightarrow 0110\ 1000 \rightarrow 0110\ 1000$ (35 in X_5-3)

$+46 \rightarrow 0111\ 1001 \rightarrow 0111\ 1001$ (46 in X_5-3)

$$\begin{array}{r} 81 \\ -35 \\ \hline 46 \end{array} \quad \begin{array}{r} 0110\ 1000 \\ +0011 \\ \hline 0111\ 1001 \end{array} \quad \begin{array}{r} 0001 \\ | \\ 1101 \\ | \\ 1101\ 0001 \end{array} \quad \begin{array}{l} \text{(carry propagated)} \\ \text{1100 0001 (add 0011 if carry)} \end{array}$$

$-0011\ 0011$ generated subtract
 $\begin{array}{r} 1101\ 0100 \\ -0011\ 0011 \\ \hline 1101\ 0100 \end{array}$ 0011 if carry not generated

11-3 4-3

8 → Answer is 81 in XS-3

(ii) $546 \cdot 32 \rightarrow 1000\ 0111\ 1001 \cdot 0110\ 0101$

$$\begin{array}{r} + 354 \cdot 16 \rightarrow 0110\ 1000\ 0111 \cdot 0100\ 1001 \\ \hline 900 \cdot 48 \end{array}$$

$$\begin{array}{r} 1110\ 1111\ 0000 \cdot 1010\ 1110 \\ \hline \text{(carry propag} \\ \text{ated)} \end{array}$$

$$\begin{array}{r} 1110\ 0000\ 0000 \cdot 1010\ 1110 \\ \hline 1111\ 0001\ -0011 \end{array}$$

$$\begin{array}{r} 1100\ 0011\ 0011 \cdot 0111\ 1011 \\ \hline 12 \quad 3 \quad 3 \quad 7 \quad 11 \end{array} \text{(Answer } 900.48 \text{ in XS-3)}$$

900.48(iii) $236.54 \rightarrow 0101\ 0110\ 1001 \cdot 1000\ 0111 \rightarrow 236.54 \text{ in XS-3}$ $7.23.13 \rightarrow 1010\ 0101\ 0110 \cdot 0100\ 0110 \rightarrow 7.23.13 \text{ in XS-3}$

$$\begin{array}{r} + 959.67 \\ \hline 1111\ 1011\ 1111\ 1100\ 1001 \end{array} \text{(No carry propagation)}$$

$$\begin{array}{r} -0011\ -0011\ -0011 \cdot 0011\ -0011 \end{array} \text{(Add } 1 \text{ to } 0011 \text{ if)}$$

$$\begin{array}{r} 1100\ 1000\ 1100 \cdot 1001\ 1010 \\ \hline 12 \quad 8 \quad 12 \quad 9 \quad 10 \end{array} \text{ carry generated}$$

Subtract 0011
generalization

959.67Excess-3 subtraction:

→ if borrow is taken, subtract 3(0011)

→ if borrow is not taken add 3(0011)

(i) $75 \rightarrow 1010\ 1000 \text{ (75 in XS-3)}$

$$\begin{array}{r} - 39 \rightarrow 0110\ 1100 \text{ (39 in XS-3)} \\ \hline 36 \end{array}$$

$\begin{array}{r} 0011\ 1100 \\ + 0011\ -0011 \\ \hline 11 \end{array}$ (if borrow taken subtract 3
Otherwise add 3)

$\begin{array}{r} 0110\ 1001 \\ 6 \quad 9 \end{array}$ (Answer 36 in XS-3)

11-3 4-3

8 → Answer is 81 in XS-3

(ii) $546 \cdot 32 \rightarrow 1000\ 0111\ 1001 \cdot 0110\ 0101$

$$\begin{array}{r}
 + 354 \cdot 16 \rightarrow 0110\ 1000\ 0111 \cdot 0100\ 1001 \\
 \hline
 900 \cdot 48 \quad 1110\ 1111\ 0000 \cdot 1010\ 1110
 \end{array}$$

(carry propag
ated)

$$\begin{array}{r}
 1110\ 0000\ 0000 \cdot 1010\ 1110 \\
 -0011\ 0011\ -0011 \quad -0011\ -0011 \\
 \hline
 1100\ 0011\ 0011 \cdot 0111\ 1011
 \end{array}$$

(Answer
900.48
in XS-3)

900.48

(iii) $236.54 \rightarrow 0101\ 0110\ 1001 \cdot 1000\ 0111 \rightarrow 236.54$ in XS-3

$723.13 \rightarrow 1010\ 0101\ 0110 \cdot 0100\ 0110 \rightarrow 723.13$ in XS-3

$$\begin{array}{r}
 959.67 \quad 1111\ 1011\ 1111 \cdot 1100\ 1001 \\
 \hline
 -0011\ -0011\ -0011 \cdot 0011\ -0011
 \end{array}$$

(No carry propagation)
(Add if 0011 if
carry generated)
(Subtract 0011
generally)

959.67

Excess-3 subtraction?

→ if borrow is taken, subtract 3(0011)

→ if borrow is not taken add 3(0011)

(i) $75 \rightarrow 1000\ 1000$ (75 in XS-3)

$- 39 \rightarrow 0110\ 1100$ (39 in XS-3)

$$\begin{array}{r}
 36 \quad 0011\ 1100 \\
 \hline
 -0011\ -0011
 \end{array}$$

(if borrow taken subtract 3)

$+ 0011 \quad 0011$ Otherwise add 3)

$\hline 0110\ 1001$ (Answer 36 in XS-3)

6 9

$$\begin{array}{r}
 \text{(ii)} \quad 845.32 \rightarrow 1011\ 0111\ 1000\ .0110\ 0101 \\
 -634.58 \rightarrow 1001\ 0111\ 0111\ .1000\ 1011 \\
 \hline
 210.74 \qquad \qquad \qquad \qquad \qquad
 \end{array}$$

$$\begin{array}{r}
 \text{0010} & \text{0001} & \text{0000} & \text{+ 1101} & \text{1010} \\
 \hline
 \text{0011} & \text{0011} & \text{0011} & \cdot \text{0011} & \text{-0011} \\
 \hline
 \text{0101} & \text{0100} & \text{0011} & \text{: 1001} & \text{0111} \text{ Answer in } X_5
 \end{array}$$

$$\begin{array}{r}
 \text{(iii)} \quad 736.21 \rightarrow 0101\ 0110\ 1010\ .1000 \\
 -498.35 \rightarrow 0111\ 1100\ 1011\ .0110\ 1000 \\
 \hline
 237.86 \qquad \qquad \qquad \qquad \qquad
 \end{array}$$

$$\begin{array}{r}
 \text{0010} & \text{1001} & \text{1001} & \text{: 1110} & \text{1100} \\
 \hline
 \text{0011} & \text{-0011} & \text{-0011} & \text{: 0011} & \text{-0011} \\
 \hline
 \text{0101} & \text{0110} & \text{1010} & \text{: 1011} & \text{1001} \\
 \hline
 \text{237.86} & & \text{5} & \text{6} & \text{10} \quad \text{11} \quad \text{9}
 \end{array}$$

Q1. Perform the following subtraction in X_5-3 using 9's complement method.

$$\begin{array}{r}
 \text{(i) } 63.54 \rightarrow 63.54 \rightarrow 1001\ 0110\ 1000\ 0111 \text{ (63.54 in } X_5-3) \\
 39.42 \rightarrow 60.57 \rightarrow 1001\ 0011\ 1000\ 1010 \text{ (9's complement)} \\
 \hline
 24.12 \qquad \qquad \qquad \qquad \qquad
 \end{array}$$

$$\begin{array}{r}
 \text{0010} & \text{0001} & \text{0000} & \text{0000} & \text{0000} \\
 \hline
 \text{0011} & \text{-0011} & \text{-0011} & \text{-0011} & \text{-0011} \\
 \hline
 \text{0101} & \text{0111} & \text{0100} & \text{0101} & \text{0101} \\
 \hline
 \text{24.12} & & \text{5} & \text{6} & \text{10} \quad \text{11} \quad \text{9}
 \end{array}$$

Carry propagation
 0010 1010 * 0001 0010 end around carry.
 +0011 -0011 = 10011 + 0011 add 0011 if carry
 0101 0111 0100 0101 generated subtract
 5 6 10 11 9 add 0011 if carry
 (not generated)

$$\begin{array}{r}
 \text{(ii) } 845.32 \rightarrow 845.32 \rightarrow 1011\ 0111\ 1000\ .0110\ 0101 \\
 634.58 \rightarrow 365.41 \rightarrow 0110\ 1001\ 1000\ .0111\ 0100 \\
 \hline
 210.74 \qquad \qquad \qquad \qquad \qquad
 \end{array}$$

$$\begin{array}{r}
 \text{0001} & \text{0000} & \text{0000} & \text{+ 1101} & \text{1001} \\
 \hline
 \text{1001} & \text{-1001} & \text{-1001} & \text{: 0011} & \text{-0011} \\
 \hline
 \text{0101} & \text{0100} & \text{0111} & \text{: 1010} & \text{0111} \\
 \hline
 \text{210.74} & & \text{5} & \text{4} & \text{3} \quad \text{10} \quad \text{7}
 \end{array}$$

$$\begin{array}{r}
 \text{(iii)} \quad 328.56 \rightarrow 328.56 \text{ (g's complement of } 653.75) \\
 -653.75 \rightarrow 346.24 \text{ (as no carry present again} \\
 \hline
 674.80 \text{ calculate g's complement)} \\
 325.19
 \end{array}$$

$$\begin{array}{r}
 328.56 \rightarrow 0110\ 0101\ 1011\ 1000\ 1001 \\
 346.24 \rightarrow 0110\ 0111\ 1001\ 0101\ 0111 \\
 \hline
 1900\ 1100\ 0100\ 1101\ 0000 \\
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 1101\ 1000\ 0111\ 1011\ 0011 \\
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 6 \quad 7 \quad 4 \quad 8 \quad 0
 \end{array}$$

Lecture Notes.in

325.19 [g's complement of 674.80]

$$\begin{array}{r}
 \text{(iv)} \quad 251.78 \rightarrow 251.78 \\
 -648.43 \rightarrow 351.56 \\
 \hline
 603.34 \rightarrow \text{As no carry present again} \\
 \text{i.e. } 396.65 \text{ calculate g's complement}
 \end{array}$$

$$\begin{array}{r}
 251.78 \rightarrow 0101\ 1000\ 0100\ 1010\ 1011 \\
 351.56 \rightarrow 0110\ 1000\ 0100\ 1000\ 1001 \\
 \hline
 1011\ 1000\ 1000\ 0010\ 0100 \\
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 1100\ 0000\ 0001\ 0011\ 0100 \\
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 -0011\ 1001\ 1001\ 0011\ 1001 \\
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 1010\ 0011\ 0100\ 0110\ 0111 \\
 \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\
 9 \quad 3 \quad 6 \quad 6 \quad 7
 \end{array}$$

Lecture Notes.in

$28/01/19$ $603.34 \Rightarrow 396.65$ [g's complement of 603.34]

Perform the following subtraction in Excess-3 using

10th complement method

$$\begin{array}{r}
 \text{(i)} \quad 936 \\
 -521 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(ii)} \quad 745 \\
 -368 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \text{(iii)} \quad 745 \\
 -368 \\
 \hline
 \end{array}$$

(i) $936 \rightarrow 936 \rightarrow 1100\ 0110\ 1001 \rightarrow 936$ in $\bar{x}_3's_3$

$$\begin{array}{r} -521 \rightarrow 479 \rightarrow 0111\ 1010\ 1100 \rightarrow 10^3 \text{ complement of } 521 \\ \hline 415 \end{array}$$

$\frac{\overline{0011}}{0000} \quad \frac{\overline{11}}{0101}$

+ carry propagate

$$\begin{array}{r} 478 \\ +1 \\ \hline 479 \end{array}$$

$\frac{0100}{0011} \quad \frac{0001}{0011} \quad \frac{0101}{1111}$

Adding to all if carry generated

LectureNotes.In 7 4 1 8 Answer in $\bar{x}_3's_3$

(ii) $745 \rightarrow 745 \rightarrow 1010\ 0111\ 1000 \rightarrow 745$ in $\bar{x}_3's_3$

$$\begin{array}{r} -368 \rightarrow 632 \rightarrow 1001\ 0110\ 0101 \rightarrow 10^3 \text{ complement of } 368 \\ \hline 377 \end{array}$$

$+0011 - 0011 - 0011 \rightarrow$ subtracting -0011

$$\begin{array}{r} 634 \\ +1 \\ \hline 635 \end{array}$$

$\frac{11}{0110\ 1010\ 1010}$

when no carry present

6 10 10 Answer in $\bar{x}_3's_3$

3 res 7 7 // answer

(iii) $542 \rightarrow 542 \rightarrow 1000\ 0111\ 0101 \rightarrow 542$ in $\bar{x}_3's_3$

$$\begin{array}{r} -736 \rightarrow 264 \rightarrow 0101\ 1010\ 0111 \rightarrow 10^3 \text{ complement of } 736 \\ \hline -194 \end{array}$$

$-0011 + 0011 - 0011 \rightarrow$ carry propagate

$$\begin{array}{r} 263 \quad 1.93 \\ +1 \quad +1 \\ \hline -194 \end{array}$$

$\frac{11}{1001\ 0100\ 1001}$

for presence of carry

$$\begin{array}{r} 264 \\ -11 \quad 8 \quad 9 \\ \hline \end{array}$$

8 0 6 \rightarrow answer

1 9 3 \rightarrow 9³ complement of 806

$$\begin{array}{r} +1 \\ \hline -1194 \end{array}$$

4 \rightarrow final answer.

(01) $346 \rightarrow 346 \rightarrow 0110\ 0100\ 100 \rightarrow 346 \text{ in } X^3$

$$\begin{array}{r} -531 \rightarrow 469 \rightarrow 0111\ 1001\ 1100 \xrightarrow{\text{10's complement of 531}} \\ \hline -185 \quad 815 \quad 1101\ 0000\ 0101 \xrightarrow{\text{carry propagated}} \end{array}$$

$\begin{array}{r} 468 \quad 184 \\ +1 \quad +1 \\ \hline 469 \quad -185 \end{array}$ $\begin{array}{r} -0011\ 0001\ 0011 \xrightarrow{\text{adding +0011}} \\ \hline 1011\ 0100\ 1000 \end{array}$ $\begin{array}{r} 11 \quad 4 \quad 8 \\ 8 \quad 1 \quad 5 \\ \hline 11 \quad 8 \quad 4 \end{array}$ $\xrightarrow{\text{calculating 10's comp}}$

$\begin{array}{r} -11 \quad 8 \quad 5 \xrightarrow{\text{final answer}} \\ \hline \end{array}$

Error detecting and correcting codes.

→ Error detecting code → Parity

→ Error correction code → Hamming code.

→ Parity $\begin{cases} \text{Even Parity} \rightarrow \text{Even no. of 1's} \\ \text{Odd Parity} \rightarrow \text{Odd no. of 1's} \end{cases}$

Decimal	Binary Code	Even Parity	Odd parity
0	0000	0	1
1	0001	1	0
2	0010	1	0
3	0011	0	1
4	0100	1	0
5	0101	0	1
6	0110	0	1
7	0111	1	0
8	1000	1	0
9	1001	0	1

In an even parity scheme, which of the following words contain an error

- (a) 10110110 (b) 11011011 (c) 10101000

Answer is a & c as they contain odd number of ones.

In an odd parity scheme which of the following words contain an error

- (a) 11011101 (b) 11011010 (c) 111011000

Answer is a, b, c as they contain even number of ones.

Hamming Code: it is a corrective and detection code. It detects only one error.

These are of 3 types

01) 7-bit hamming code

02) 12-bit hamming code

03) 15-bit hamming code

7-bit hamming code

→ encode the message bit 1010 in an even parity hamming code

1	2	3	4	5	6	7
P_1	P_2	D_3	P_4	D_5	D_6	D_7
1			0	1	0	

P_1 is set to 0 if 1 to have even parity scheme.

$$P_1 \rightarrow (P_1 D_3 D_5 D_7) = (P_1 1 0 0) = 1$$

$$P_2 \rightarrow (P_2 D_3 D_5 D_6) = (P_2 1 0 1) = 0$$

$$P_4 \rightarrow (P_4 D_5 D_6 D_7) = (P_4 0 1 0) = 1$$

Transmitted code $\Rightarrow X = P_1 \ P_2 \ D_3 \ P_4 \ D_5 \ D_6 \ D_7$

Received code $\Rightarrow Y = 1 \ 0 \ 1 \ 0 \ 1 \ 0 \ 1 \ 0$

check sum:

$$c_1 = P_1 \oplus D_3 \oplus D_5 \oplus D_7 = 1 \oplus 0 \oplus 0 \oplus 0 \Rightarrow 1$$

$$c_2 = P_2 \oplus D_3 \oplus D_6 \oplus D_7 = 0 \oplus 0 \oplus 1 \oplus 0 \Rightarrow 1$$

$$c_3 = P_4 \oplus D_5 \oplus D_6 \oplus D_7 = 1 \oplus 0 \oplus 1 \oplus 0 \Rightarrow 0$$

+ 3

3rd bit has

an error.

for 1011, 1110, 0101 generate a 7-bit hamming code using even parity.

$P_1 \ P_2 \ D_3 \ P_4 \ D_5 \ D_6 \ D_7$

$$P_1 \rightarrow (P_1 \oplus D_3 \oplus D_5 \oplus D_7) \rightarrow (P_1 + 01) = 0$$

$$P_2 \rightarrow (P_2 \oplus D_3 \oplus D_6 \oplus D_7) \rightarrow (P_2 + 11) = 1$$

$$P_4 \rightarrow (P_4 \oplus D_5 \oplus D_6 \oplus D_7) \rightarrow (P_4 + 01) = 0$$

7-bit hamming code is $P_1 \ P_2 \ D_3 \ P_4 \ D_5 \ D_6 \ D_7$

(b2) 1110 $\Rightarrow P_1 \ P_2 \ D_3 \ P_4 \ D_5 \ D_6 \ D_7$

$$P_1 \rightarrow (P_1 \oplus D_3 \oplus D_5 \oplus D_7) \rightarrow (P_1 + 110) \Rightarrow 0$$

$$P_2 \rightarrow (P_2 \oplus D_3 \oplus D_6 \oplus D_7) \rightarrow (P_2 + 110) \Rightarrow 0$$

$$P_4 \rightarrow (P_4 \oplus D_5 \oplus D_6 \oplus D_7) \rightarrow (P_4 + 110) \Rightarrow 0$$

7-bit hamming code is $P_1 \ P_2 \ D_3 \ P_4 \ D_5 \ D_6 \ D_7$

10 0 1 0 1 1 0

(iii) $D(0) \rightarrow P_1, P_2, D_3, P_4, D_5, D_6, P_7$

 $P_1 \rightarrow (P_1, D_3, D_5, D_7) \rightarrow (P_1, 011) \rightarrow 0$
 $P_2 \rightarrow (P_2, D_3, D_6, D_7) \rightarrow (P_2, 001) \rightarrow 1$
 $P_4 \rightarrow (P_4, D_5, D_6, D_7) \rightarrow (P_4, 01) \rightarrow 0$

7-bit hamming code is $P_1' P_2' D_3' P_4' D_5' D_6' D_7'$
 $0, 1, 1, 0, 0, 1, 0, 1$

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In a communication system the received code words are 1011011, 1010101, 1110110 using 7-bit hamming code. Find transmitted code words.

$$P_1 \quad P_2 \quad D_3 \quad P_4 \quad D_5 \quad D_6 \quad P_7$$
 $1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1$
 $e_1 = (P_1, D_3, D_5, D_7) \Rightarrow 1 \oplus 1 \oplus 0 \oplus 1 \Rightarrow 1$
 $e_2 = P_2 \oplus D_3 \oplus D_6 \oplus D_7 \Rightarrow 0 \oplus 1 \oplus 1 \oplus 1 \Rightarrow 1$
 $e_3 = P_4 \oplus D_5 \oplus D_6 \oplus D_7 \Rightarrow 1 \oplus 0 \oplus 1 \oplus 1 \Rightarrow 1$

The transmitted code 7-bit hamming code is 1011010

(ii) 1010101

$$P_1' \quad P_2' \quad D_3' \quad P_4' \quad D_5' \quad D_6' \quad D_7'$$
 $1 \quad 0 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1$
 $e_1 = P_1 \oplus P_3 \oplus D_5 \oplus D_7 \Rightarrow 1 \oplus 1 \oplus 1 \oplus 1 \Rightarrow 0$
 $e_2 = P_2 \oplus D_3 \oplus D_6 \oplus D_7 \Rightarrow 0 \oplus 1 \oplus 0 \oplus 1 \Rightarrow 0$
 $e_3 = P_4 \oplus D_5 \oplus D_6 \oplus D_7 \Rightarrow 0 \oplus 1 \oplus 0 \oplus 1 \Rightarrow 0$

Transmitted code is 1010101

(iii) 1110110

P_1	P_2	D_3	P_4	D_5	D_6	D_7
1	1	0	1	1	0	

$$e_1 = P_1 \oplus D_3 \oplus D_5 \oplus D_7 \Rightarrow 1 \oplus 1 \oplus 1 \oplus 0 \Rightarrow 1$$

$$e_2 = P_2 \oplus D_3 \oplus D_6 \oplus D_7 \Rightarrow 1 \oplus 1 \oplus 1 \oplus 0 \Rightarrow 1$$

$$e_3 = P_4 \oplus D_5 \oplus D_6 \oplus D_7 \Rightarrow 0 \oplus 1 \oplus 1 \oplus 0 \Rightarrow 0$$

error is in 3rd bit

transmitted code is 1100110

generate a 12 bit hamming code, using even parity scheme if the message bits are 10110110

Solution:

P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	P_{10}	D_{11}	P_{12}
1	0	1	1	0	1	1	0	1	1	0	

P_1 is set to 0 or 1 using even parity.

$$P_1 \rightarrow (P_1, D_3, D_5, D_7, D_9, D_{11}) \rightarrow (P_1, 10, 1, 1) = 1$$

P_2 is set to 0 or 1 using even parity

$$P_2 \rightarrow (P_2, D_3, D_6, D_7, D_{10}, D_{11}) \rightarrow (P_2, 11, 1) = 1$$

P_4 is set to 0 or 1 using even parity

$$P_4 \rightarrow (P_4, D_5, D_6, D_7, D_{12}) \rightarrow (P_4, 0, 1, 1, 0) = 0$$

P_8 is set to 0 or 1 using even parity.

$$P_8 \rightarrow (P_8, D_9, D_{10}, D_{11}, D_{12}) = (P_8, 0, 1, 1, 0) = 0$$

12-bit hamming code is 111001100110

In a 12-bit hamming code, if the received hamming code is 1,11001101110, when transmitted through a noisy channel, find the transmitted code word.

$$Y = P_1 \oplus P_2 \oplus D_3 \oplus P_4 \oplus D_5 \oplus D_6 \oplus P_7 \oplus P_8 \oplus D_9 \oplus P_{10} \oplus D_{11} \oplus P_{12}$$

$$1 \quad 1 \quad 1 \quad , 0 \quad 0 \quad 1 \quad , 1 \quad 0 \quad 1 \quad , 1 \quad 1 \quad 0$$

$$C_1 = P_1 \oplus D_3 \oplus D_5 \oplus D_7 \oplus D_9 \oplus D_{11} = 1 \oplus 1 \oplus 0 \oplus 1 \oplus 1 \oplus 1 = 1$$

$$C_2 = P_2 \oplus D_3 \oplus D_6 \oplus D_7 \oplus D_{10} \oplus D_{11} = 1 \oplus 1 \oplus 1 \oplus 1 \oplus 1 \oplus 1 = 0$$

$$C_4 = P_4 \oplus D_5 \oplus D_6 \oplus D_7 \oplus D_{12} = 0 \oplus 0 \oplus 1 \oplus 1 \oplus 0 = 0$$

$$C_8 = P_8 \oplus D_9 \oplus D_{10} \oplus D_{11} \oplus D_{12} = 0 \oplus 1 \oplus 1 \oplus 1 \oplus 0 = 1$$

The error is at 9th bit

∴ The transmitted code is 111001100110

03. Generate a 15-bit hamming code for the following message bits

(a) 10110110101 (b) 11101011110 (c) 11001100111

(a) 10110110101

$$Y = P_1 \oplus P_2 \oplus D_3 \oplus P_4 \oplus D_5 \oplus D_6 \oplus P_7 \oplus P_8 \oplus P_9 \oplus P_{10} \oplus D_{11} \oplus D_{12} \oplus P_{13} \oplus P_{14} \oplus P_{15}$$

$$1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1$$

P_1 is set to zero or one using even parity.

$$P_1 \rightarrow (P_1 \oplus D_3 \oplus D_5 \oplus D_7 \oplus D_9 \oplus D_{11} \oplus D_{13} \oplus D_{15})$$

$$= P_1 \oplus 1 \oplus 0 \oplus 1 \oplus 0 \oplus 1 \oplus 1 \oplus 1 = (P_1, 1010111)_2$$

P_2 is set to zero or one using even parity.

$$P_2 \rightarrow (P_2 D_3 D_6 D_7 D_{10} D_{11} D_{14} D_{15}) = (P_2 1 1 1 1 1 0) = 0$$

P_2 is set to zero or one using even parity.

$$P_4 \rightarrow (P_4 D_5 D_6 D_7 D_{12} D_{13} D_{14} D_{15}) = (P_4 0 1 1 0 1 0 1) = 0$$

$$P_8 \rightarrow (P_8 D_9 D_{10} D_{11} D_{12} D_{13} D_{14} D_{15}) = (P_8 0 1 1 0 1 0 1) = 0$$

The 15-bit Hamming code is 101001100110101

LectureNotes.in
(b) 11101011110

$$Y = P_1 P_2 D_3 P_4 D_5 D_6 D_7 P_8 D_9 D_{10} D_{11} D_{12} D_{13} D_{14} D_{15}$$

1	1	1	0	1	0	1	1	1	0
---	---	---	---	---	---	---	---	---	---

P_1 is set to zero or one using even parity.

$$P_1 \rightarrow (P_1 D_3 D_5 D_7 D_9 D_{11} D_{13} D_{15}) \rightarrow (P_1 1 1 0 1 1 1 0) = 1$$

P_2 is set to zero or one using even parity.

$$P_2 \rightarrow (P_2 D_3 D_6 D_7 D_{10} D_{11} D_{14} D_{15}) \rightarrow (P_2 1 1 0 0 1 1 0) = 0$$

P_4 is set to zero or one using even parity.

$$P_4 \rightarrow (P_4 D_5 D_6 D_7 D_{12} D_{13} D_{14} D_{15}) \rightarrow (P_4 1 1 0 1 1 1 0) = 1$$

P_8 is set to zero or one using even parity

$$P_8 \rightarrow (P_8 D_9 D_{10} D_{11} D_{12} D_{13} D_{14} D_{15}) \rightarrow (P_8 1 0 1 1 1 0) = 1$$

The transmitted code is 101111011011110

LectureNotes.in
(c) 11001100111

$$Y = P_1 P_2 D_3 P_4 D_5 D_6 D_7 P_8 D_9 D_{10} D_{11} D_{12} D_{13} P_{14} D_{15} D_{16}$$

1	1	0	0	1	1	0	0	1	1
---	---	---	---	---	---	---	---	---	---

$$P_1 \rightarrow (P_1 D_3 D_5 D_7 D_9 D_{11} D_{13} D_{15}) = (P_1 1 1 0 1 0 1 1) = 1$$

$$P_2 \rightarrow (P_2 D_3 D_6 D_7 D_{10} D_{11} D_{14} D_{15}) = (P_2 1 0 0 1 0 1 1) = 0$$

$$P_4 \rightarrow (P_4 D_5 D_6 D_7 D_{12} D_{13} D_{14} D_{15}) = (P_4 1 0 0 0 1 1 1) = 0$$

$$P_8 \rightarrow (P_8 D_9 D_{10} D_{11} D_{12} D_{13} D_{14} D_{15}) = (P_8 1 1 0 0 1 1 1) = 1$$

The transmitted code is 101010011100111

Gray Code

- It is a non-weighted code
- It is cyclic and Reflective code
- Successive codewords differ by 1 bit position.
- It is also called as unit-distance code.

<u>1 bit</u>	<u>2 bit</u>	<u>3 bit</u>
0	00	000
	01	001
	11	011
	10	010
		110
		111
		101
		100

Calculate the 4-bit Gray code for the binary decimal numbers zero to 15.

Solution

<u>Decimal Numbers</u>	<u>Binary Code</u>	<u>Gray code</u>
0	0000	0000 - 0
1	0001	0001 - 1
2	0010	0011 - 3
3	0011	0101 - 2
4	0100	0110 - 6
5	0101	0111 - 7
6	0110	0101 - 5
7	0111	01100 - 4
8	1000	1100 - 12
9	1001	1101 - 13
10	1010	1111 - 15
11	1011	1110 - 14
12	1100	1010 - 10
13	1101	1011 - 11
14	1110	1001 - 9
15	1111	1000 - 8

calculate the Gray code for the following
binary codes

- (a) 10110110 (b) 11100010 (c) 11001010 (d) 11110000

Solution

$$\begin{array}{lll} \text{(a)} & 10110110 & \text{(b)} & 11100010 & \text{(c)} & 11001010 \\ & \downarrow & & \downarrow & & \downarrow \\ & 11011011 & & 10010010 & & 10101111 \\ & \text{(d)} & & \text{(a)} & & \end{array}$$

LectureNotes.in

for the following Gray code, calculate the
numbers, in decimal

$$\text{(a)} 11101101 \rightarrow 1001110110 = 128 + 32 + 16 + 4 + 2 = 182$$

$$\text{(b)} 10010010 \rightarrow 11100010 = 226$$

$$\text{(c)} 10101111 \rightarrow 11001010 = 202$$

$$\text{(d)} 10001000 \rightarrow 11110000 = 240$$

$$\begin{array}{ll} \text{(a)} 10001000 & \text{(b)} 10101111 \\ \begin{array}{ccccccccc} \uparrow & \uparrow \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array} & \begin{array}{ccccccccc} \uparrow & \uparrow \\ 1 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \end{array} \\ \begin{array}{ccccccccc} \downarrow & \downarrow \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \end{array} & \begin{array}{ccccccccc} \downarrow & \downarrow \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \end{array} \end{array}$$

$$\Rightarrow 10110110 \Rightarrow 128 + 32 + 16 + 4 + 2 = 182,$$

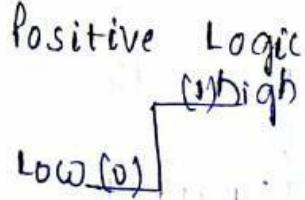
$$\Rightarrow 11100010 = 128 + 64 + 32 + 2 = 226,$$

$$\Rightarrow 11001010 = 128 + 64 + 0 + 2 = 202,$$

$$\Rightarrow 11110000 = 128 + 64 + 32 + 16 = 240,$$

Logic Gates:

- These are the basic building blocks of digital systems.
- Logic Gate has the ability to make the decision.



Logic design:

- The interconnection of gates, to perform a logical operation is known as 'Logic design'.

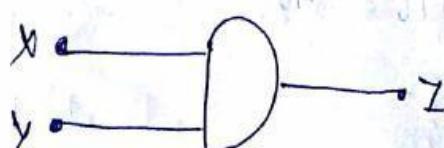
Truth Table:

* The list of all possible combinations with inputs and outputs is known as truth table.

There are 7 logic gates.

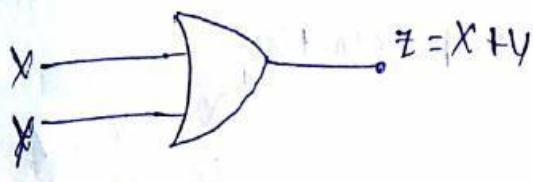
- | | |
|--------------|-------------|
| 01. AND Gate | Basic Gates |
| 02. OR Gate | |
| 03. NOT Gate | |
-
- | | |
|----------------|-----------------|
| 04. NAND Gate | Universal Gates |
| 05. NOR Gate | |
| 07. Ex-OR Gate | |
-
- | | |
|-----------------|--|
| 08. Ex-NOR Gate | |
|-----------------|--|

01) AND Gate $\Rightarrow (X \cdot Y)$



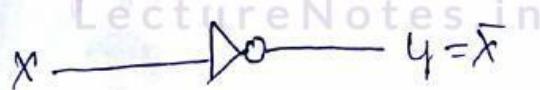
X	Y	Z
0	0	0
1	0	0
0	1	0
1	1	1

OR Gate : $Z = X + Y$



X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

03. NOT Gate : $Z = \bar{X}$



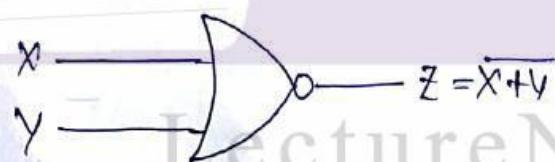
X	Y
0	1
1	0

04. NAND Gate :



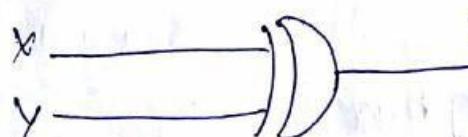
X	Y	Z
0	0	1
0	1	1
1	0	1
1	1	0

05. NOR Gate :



X	Y	Z
0	0	1
0	1	0
1	0	0
1	1	0

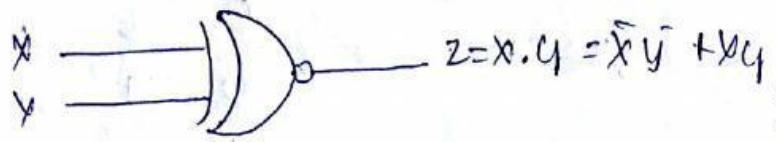
06. Ex-OR Gate :



$$Z = X \oplus Y = \bar{X} \cdot Y + X \cdot \bar{Y}$$

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	0

07) Ex- NOR Gate :-



X	Y	Z
0	0	1
0	1	0
1	0	0
1	1	0

30th January 2019 :-

Boolean Algebra:-

* Boolean Algebra is a system of mathematical logic, any complex function can be represented as logic design

Logic Operation:-

01) AND Operation

02) OR Operation

03) NOT Operation

04) NAND & NOR Operation

05) X-OR & X-NOR Operation

06) Axioms of Boolean Algebra

07) Axioms \Rightarrow No mathematical equation to prove.

01) $0 \cdot 0 = 0$

02) $0 \cdot 1 = 0$

03) $1 \cdot 0 = 0$

04) $1 \cdot 1 = 1$

05) $0 + 0 = 0$

06). $0+1=1$

07). $1+0=1$

08). $1+1=1$

09). $\bar{0}=1$

10). $\bar{1}=0$

* Laws of Boolean Algebra:

(01) Complementation Law:

(i) $\bar{0}=1$

(ii) $\bar{1}=0$

(iii) If $A=0$ then $\bar{A}=1$

(iv) If $A=1$ then $\bar{A}=0$

(v) $\bar{\bar{A}}=A$,

(02) AND Law:

(i) $A \cdot 0 = 0$

(ii) $A \cdot 1 = A$

(iii) $A \cdot A = A$

(iv) $A \cdot \bar{A} = 0$

(03) OR Law:

(i) $A + 0 = A$

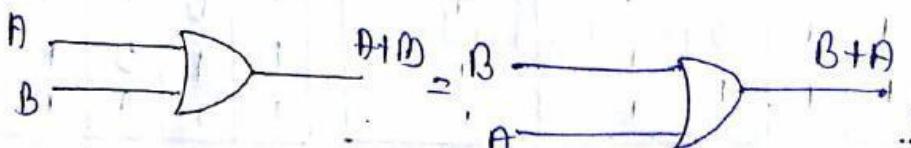
(ii) $A + 1 = 1$

(iii) $A + A = A$

(iv) $A + \bar{A} = 1$,

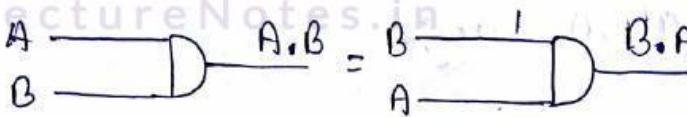
(04) Commutative Law:

(i), $A+B = B+A$, \Rightarrow law (i)



A	B	A	B	A	B+A
0	0	0	0	0	0
0	1	1	0	1	1
1	0	1	1	0	1
1	1	1	1	1	1

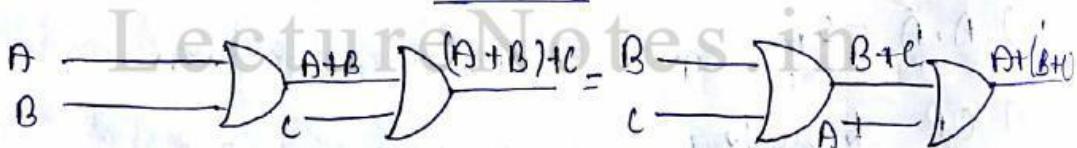
law (ii) : $\underline{A \cdot B} = \underline{B \cdot A}$



A	B	A · B	B	A	B · A
0	0	0	0	0	0
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	1	1	1

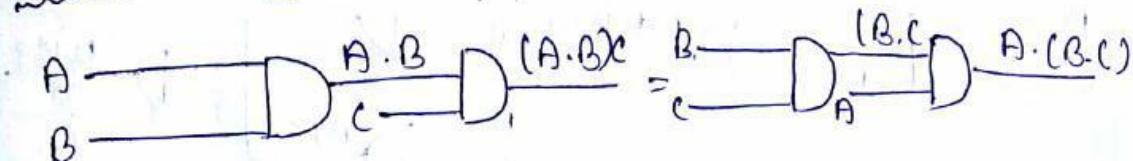
(05) Associative law:

law (i) : $\underline{(A+B)+C} = \underline{A+(B+C)}$



A	B	C	A+B	(A+B)+C	B+C	A+(B+C)
0	0	0	0	0	0	0
0	0	1	0	1	1	1
0	1	0	1	1	1	1
1	0	0	1	1	0	1
0	1	1	1	1	1	1
1	0	1	1	1	0	1
1	1	0	1	1	1	1
1	1	1	1	1	1	1

law (ii) : $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

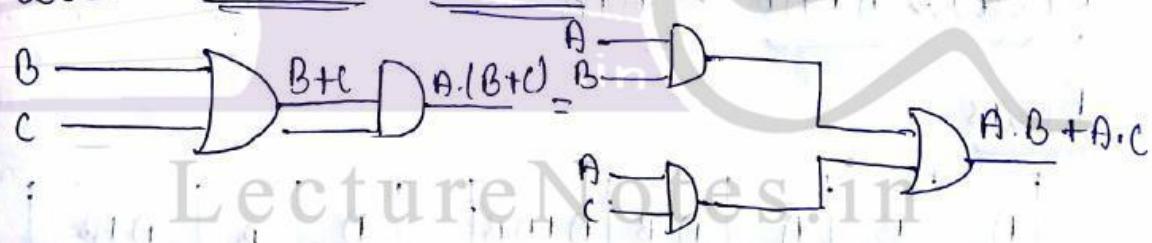


A	B	C	$A \cdot B$	$(A \cdot B) \cdot C$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	0	0
1	0	1	0	0
1	1	0	1	0
1	1	1	1	1

A	B	C	$(B \cdot C)$	$A \cdot (B \cdot C)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	0
1	0	0	0	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

(06) Distributive Law

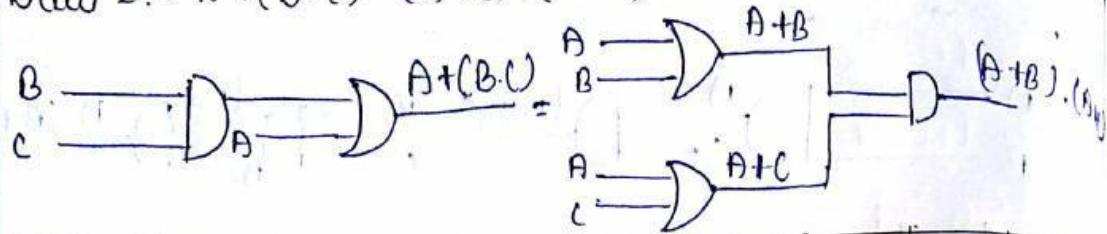
law (i) : $A \cdot (B+C) = AB + AC$



A	B	C	$B+C$	$A \cdot (B+C)$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	0
1	1	0	1	1
1	1	1	1	1

A	B	C	$A \cdot B$	$A \cdot C$	$A \cdot B + A \cdot C$
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

$$\text{Law 2: } A + (B \cdot C) = (A + B) \cdot (A + C)$$



A	B	C	$B \cdot C$	$A + C$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	1
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

A	B	C	$A + B$	$A + C$	$(A + B) \cdot (A + C)$
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	0	0
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

07. Redundant Literal Rule [RLR]

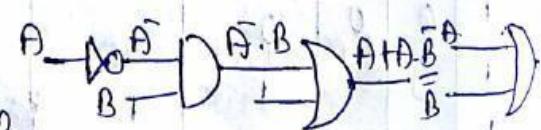
$$\text{Law(i)}: A + \bar{A}B = A + B$$

Proofs

A	B	\bar{A}	$\bar{A} \cdot B$	$A + \bar{A}B$
0	0	1	0	0
0	1	1	0	1
1	0	0	0	1
1	1	0	0	1

A	\bar{B}	$A + B$
0	0	0
0	1	0
1	0	1
1	1	1

$$\begin{aligned}
 \text{Proof! - L.H.S} &= A + \bar{A}B \\
 &= (A + \bar{A})(A + B) \\
 &= 1 \cdot (A + B) \\
 &= \underline{\underline{A + B}}
 \end{aligned}$$



law (ii): $\underline{\underline{A \cdot (\bar{A} + B) = A \cdot B}}$

A	B	\bar{A}	$\bar{A} + B$	$A \cdot (\bar{A} + B)$
0	0	1	1	0
0	1	1	1	0
1	0	0	0	0
1	1	0	1	1

A	B	$A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

$$L.H.S = A \cdot (\bar{A} + B)$$

$$= A \cdot \bar{A} + A \cdot B \Rightarrow 0 + A \cdot B$$

$$= A \cdot B$$

(8) Idempotente Law:

law (i): $\underline{\underline{A + A = A}}$

Proof: If $A=0$, then $A+A=0+0=0=A$

If $A=1$ then $A+A=1+1=1=A$

law (ii): $\underline{\underline{A \cdot A = A}}$

Proof: If $A=0$ then $A \cdot A=0 \cdot 0=0=A$

if $A=1$, then $A \cdot A=1 \cdot 1=1=A$

(9) Absorption Law:

law 1: $\underline{\underline{A + AB = A}}$

Proof: $A(1+B) = A(1) = A$

A	B	AB	$A+AB$
1	0	0	1
1	1	1	1
0	0	0	0

law(ii): $\underline{\underline{A \cdot (\bar{A} + B) = A \cdot B}}$

A	B	\bar{A}	$\bar{A} + B$	$A \cdot (\bar{A} + B)$
0	0	1	1	0
0	1	1	1	0
1	0	0	0	0
1	1	0	1	1

A	B	$A \cdot B$
0	0	0
0	1	0
1	0	0
1	1	1

$$L.H.S = A \cdot (\bar{A} + B)$$

$$= A \cdot \bar{A} + A \cdot B \Rightarrow 0 + A \cdot B$$

$$= A \cdot B$$

(8) Idempotence Law: $\underline{\underline{A+A=A}}$

law(i): $\underline{\underline{A+A=A}}$

Proof: If $A=0$, then $A+A=0+0=0=A$

If $A=1$ then $A+A=1+1=1=A$

law(ii): $\underline{\underline{A \cdot A=A}}$

Proof: If $A=0$ then $A \cdot A=0 \cdot 0=0=A$

if $A=1$ then $A \cdot A=1 \cdot 1=1=A$

(9) Absorption Law:

law 1: $\underline{\underline{A+A \cdot B=A}}$

Proof: $A(1+B) = A(1) = A$

A	B	$A \cdot B$	$A+A \cdot B$
1	0	0	1
1	1	1	1
0	0	0	0
0	1	0	0

law (ii) $A \cdot (A+B) = A$

Proof: $A \cdot (A+B) = A \cdot A + A \cdot B$
= $A + AB = A(1+B)$
= A

A	B	$A+B$	$A \cdot (A+B)$
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

(b) Consensus Theorem: [Included factor Theorem]

law (ii) $AB + \bar{A}C + BC = AB + \bar{A}C$

Proof: $AB + \bar{A}C + BC = AB + \bar{A}C + BC(A + \bar{A})$

$$= AB + \bar{A}C + ABC + \bar{A}BC$$

$$= AB(1+C) + \bar{A}C(1+B)$$

$$= AB + \bar{A}C$$

law (ii) $\therefore (A+B)(\bar{A}+C)(B+C) = (A+B)(\bar{A}+C)$

Proof: $A\bar{A} + A(\bar{A}+C)B + B(\bar{A}+C) \cdot (B+C) = A\bar{A} + A(\bar{A}+C) + BC + \bar{A}C + \bar{A}B + BC \cdot (B+C) =$

L.H.S = $AC + \bar{A}B + BC(B+C) =$

$$= ABC + \bar{A}BC + BC + \bar{A}C + \bar{A}B + BC$$

$$= ABC + BC + \bar{A}B + \bar{A}BC + \bar{A}C$$

$$= AC(1+B) + \bar{A}B(1+C) + BC$$

$$= AC + \bar{A}B + BC = AC + \bar{A}B$$

ii) Transposition Theorem :-

$$\text{law(i)} = AB + \bar{A}C = (\bar{A} + B)(A + C)$$

Proof :- R.H.S = $(\bar{A} + B)(A + C)$

$$= A \cdot \bar{A} + \bar{A}C + AB + BC$$

$$= AB + \bar{A}C + BC$$

$$= AB + \bar{A} [\text{using Consensus theorem}]$$

(12) DeMorgan's Theorem :-

$$\text{law(ii)} = \overline{A+B} = \bar{A} \cdot \bar{B}$$

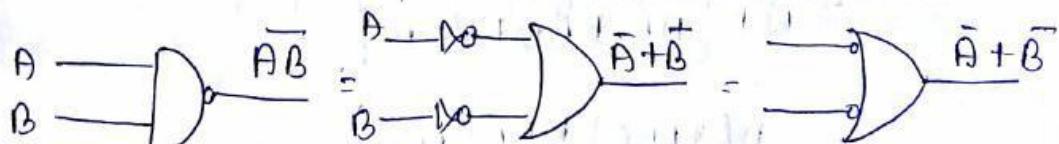


Bubbled AND Gate = NOR Gate

A	B	$A+B$	$\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

A	B	\bar{A}	\bar{B}	$\bar{A} \cdot \bar{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

$$\text{law(iii)} : \overline{A \cdot B} = \bar{A} + \bar{B}$$



Bubbled OR Gate = NAND Gate

A	B	$A \cdot B$	$\overline{A \cdot B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

A	B	\bar{A}	\bar{B}	$\bar{A} + \bar{B}$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

Reduce the following expressions using laws
of Boolean Algebra:-

(a) $A[B+C(\overline{AB}+\overline{AC})]$

Solution: $\sim A[B+C(\overline{A}\overline{B}, \overline{A}\overline{C})]$

$$= A[B+C((\overline{A}+\overline{B})(\overline{A}+\overline{C}))]$$

$$= A[B+C(\overline{A}+\overline{A}(\overline{A}\overline{B}+\overline{B}C))]$$

$$= A[B+\overline{A}\overline{C}+\overline{A}C\overline{C}+\overline{A}\overline{B}\overline{C}+\overline{B}C\overline{C}]$$

$$= A[B+\overline{A}\overline{C}+0+0+\overline{A}\overline{B}\overline{C}+0]$$

$$= A[B+\overline{A}\overline{C}+\overline{A}\overline{B}\overline{C}]$$

$$= AB+A\overline{A}\overline{C}+A\overline{A}\overline{B}\overline{C}$$

$$\therefore = AB+0+0$$

$$= \underline{\underline{AB}}$$

(b) $A+B[A(C+(B+C)D)]$

Solution: $\sim A+B[A(C+B(D+C)D)]$

$$= A+[ABC+BD+BCD]$$

$$= A(1+BC)+BD(1+C)$$

$$= \underline{\underline{A+BD}}$$

$$(c) (\overline{A+B\bar{C}})(AB + A\bar{B}C)$$

$$\underline{\text{Solution:}} \quad (\bar{A} \cdot \bar{B}\bar{C}) (AB + A\bar{B}C)$$

$$= (\bar{A} \cdot \bar{B}C) (A\bar{B} + A\bar{B}C)$$

$$= \underset{0}{A\bar{A}\bar{B}} + \underset{0}{A\bar{A}BC} + \underset{0}{B\bar{B}AC} + \underset{0}{ABC}$$

$$= 0$$

$$(a) (B+BC) (B+\bar{B}C) (B+D)$$

$$\underline{\text{Solution:}} \quad (B+BC) (B+\bar{B}C) (B+D)$$

$$(B+BC)(B+D) = B + B\bar{B}C + \underset{0}{BC} + \bar{B}C B (B+D)$$

$$B(1+C)(B+D) = B + BC [B+D]$$

$$B(B+D) = B + BC + \bar{B}C D$$

$$B+B\bar{D} = B[1+\bar{D}] = B+BC[1+\bar{D}] = B+BC = B[1+C] = B$$

Show that $A\bar{B} + A\bar{B}C + B\bar{C} = AC + B\bar{C}$

$$\underline{\text{Solution:}}$$

$$\begin{aligned} & A(\bar{B} + \bar{B}C) + B\bar{C} \\ & \xrightarrow{\text{Redundant Literal Rule}} A(\bar{B} + C) + B\bar{C} \end{aligned}$$

$$A\bar{B} + AC + B\bar{C}$$

$$\begin{aligned} & = A\cancel{\bar{B} + C} + AB + B\bar{C} \\ & \quad \text{By Consensus Theorem} \\ & = AC + B\bar{C} \end{aligned}$$

Show that $A\bar{B}C + B + B\bar{D} + A\bar{B}\bar{D} + \bar{A}C = B+C$,

$$\underline{\text{Solution:}}$$

$$A\bar{B}C + B + B\bar{D} + A\bar{B}\bar{D} + \bar{A}C$$

$$A\bar{B}C + B(1 + \bar{D} + A\bar{B}) + B\bar{D} + \bar{A}C$$

$$A\bar{B}C + B + A\bar{B}\bar{D} + \bar{A}C$$

$$A(\bar{B}C + B\bar{D}) + B + \bar{A}C$$

$$\begin{aligned}
 &= A\bar{B}C + B[\bar{A} + \bar{A}\bar{D}] + \bar{A}C \\
 &= A\bar{B}C + B + \bar{A}C \\
 &= C[A\bar{B} + \bar{A}] + B \\
 &= C[\bar{A} + \bar{B}] + B \\
 &= AC + B + B \\
 &= AC + (B + \bar{B}) + (B + C) \\
 &= AC + B + C = B + C[\bar{A} + 1] = B + 1, \text{ hence proved.}
 \end{aligned}$$

Duality and Complement:

Find the duality of the following expressions

$$(a) A \cdot (\bar{A} \cdot B) = A \cdot B$$

$$A + (\bar{A} + B) = A + B$$

$$(b) \overline{\bar{A}B + \bar{A} + AB} = 0$$

$$(\bar{A} + B) * \bar{A} * (\bar{A} + B) = 1$$

$$(c) AB + \bar{A}C + A\bar{B}C [AB + C] = 1$$

$$(A + B) * (\bar{A} + C) * (A + \bar{B} + C) + [A + B] * C = 0$$

$$(d) A + B(\overline{C + DE}) = A + B\bar{C}DE$$

$$A \cdot B + (C \cdot \overline{DE}) = A \cdot B + \bar{C} + D + E$$

$$(e) \overline{A\bar{B} + ABC} + A(B + A\bar{B}) = 0$$

$$\overline{A + \bar{B}} * (A + B + C) * A + (B \cdot A + \bar{B}) = 1$$

1st february 2019.

find the complement of the following expressions,

(a) $\bar{A}B + \bar{A}\bar{B}\bar{C} + \bar{A}BCD + \bar{A}\bar{B}\bar{C}\bar{D}\bar{E}$

$$\bar{A}B + \bar{A}\bar{B}\bar{C} + \bar{A}BCD + \bar{A}\bar{B}\bar{C}\bar{D}\bar{E} = \overline{\bar{A}B} \cdot \overline{\bar{A}BCD} \cdot \overline{\bar{A}\bar{B}\bar{C}\bar{D}\bar{E}}$$

$$= (\bar{A} + \bar{B})(\bar{A} + \bar{B} + \bar{C} + \bar{D})(\bar{A} + \bar{B} + \bar{C} + \bar{D} + \bar{E})$$

(b) $\bar{B}\bar{C}D + (\bar{B} + C + D) + \bar{B}\bar{C}\bar{D}\bar{E} = \overline{\bar{B}\bar{C}D} + \overline{(\bar{B} + C + D)} + \overline{\bar{B}\bar{C}\bar{D}\bar{E}}$

$$= \bar{B} + \bar{C} + \bar{D} \cdot (\bar{B} + C + D) \cdot (\bar{B} + C + D + \bar{E})$$

LectureNotes.in

(c) $(A\bar{B} + A\bar{C})(B\bar{C} + B\bar{C})(ABC)$

$$= (\bar{A}\bar{B} + A\bar{C})(B\bar{C} + B\bar{C})(ABC) = \overline{(\bar{A}\bar{B} + A\bar{C})} + \overline{(B\bar{C} + B\bar{C})} + \overline{(ABC)}$$

$$= (\bar{A} + B \cdot \bar{A} + C) + (\bar{B} + \bar{C} \cdot \bar{B} + C) + (\bar{A} + \bar{B} + C)$$

(d) $A + \bar{B}C(A + B + \bar{C}) = \overline{A + \bar{B}C(A + B + \bar{C})}$

$$= \bar{A} + \bar{B}C + \overline{(A + B + C)} \Rightarrow \bar{A} \cdot (B + \bar{C}) + (\bar{A} \cdot \bar{B} \cdot \bar{C})$$

(e) $(\bar{ABC})(\bar{A} + B + \bar{C})$

Solution: apply whole complement

$$(\bar{ABC})(\bar{A} + B + \bar{C})$$

Replace 'and' with 'or'

$$(\bar{ABC}) + (\bar{A} + B + \bar{C})$$

Sum of products [S.O.P] and Product of sums [P.O.S]

Ex:- $\bar{A}B + \bar{A}\bar{B}\bar{C} + ABC + A$ forms:
Product Product Product

\Rightarrow S.O.P form (or) disjunctive normal form [DNF]

$\Rightarrow A\bar{B}\bar{C}, \bar{A}\bar{B}\bar{C}, \bar{A}B\bar{C} \Rightarrow$ Standard SOP (or) Expanded SOP,
disjunctive canonical form (or) Canonical SOP

Ex:- $(\bar{A} + B)(\bar{A} + B + \bar{C}) \cdot (B + \bar{C})$

↓ ↓ ↓
Sum Sum Sum

The above form is P.O.S form (or) conjunctive normal form

↳ converts it into
standard POS (d) Expanded POS, conjunctive canonical
form (e) Canonical POS

Minterm:-

Each product term is standard, SOP form is known as Minterm.

Maxterm:- Each sum term in standard POS form is known as Maxterm.

X Y Z	Minterm term Designation	Maxterm term Designation
0 0 0	$\bar{x}\bar{y}\bar{z}$	$x+y+z$
0 0 1	$\bar{x}\bar{y}z$	$x+y+\bar{z}$
0 1 0	$\bar{x}y\bar{z}$	$x+\bar{y}+z$
0 1 1	$\bar{x}yz$	$x+\bar{y}+\bar{z}$
1 0 0	$\bar{x}yz$	$\bar{x}+y+z$
1 0 1	$x\bar{y}\bar{z}$	$\bar{x}+y+\bar{z}$
1 1 0	$x\bar{y}z$	$\bar{x}+\bar{y}+z$
1 1 1	xyz	$\bar{x}+\bar{y}+\bar{z}$

P2) Convert the following expressions into standard SOP form.

$$(i) F(A, B) = \bar{A}B + B$$

Solution $\Rightarrow \bar{A}B + B(A+B)$

$$= \bar{A}B + BA + \bar{A}B$$

$$= \bar{A}B + BA \Rightarrow 01+11 \Rightarrow m_1 + m_3 \Rightarrow \sum m(1, 3)$$

$$(ii) F(A, B, C) = ABC + A\bar{B}C + AB\bar{C} + \bar{A}BC$$

Solution $= ABC + A\bar{B}C + AB\bar{C} + \bar{A}BC + (A+B+C)(A+\bar{B}+\bar{C})$

$$= ABC + A\bar{B}C + AB\bar{C} + \bar{A}BC + ABC + \bar{A}B\bar{C}$$

$$= ABC + A\bar{B}C + AB\bar{C} + \bar{A}BC$$

$$\sum m(0,1,0+1,1+1,0+1,0,1) \Rightarrow \Sigma m(3,5,6,7)$$

$$(iii) f(A, B, C) = A + AB + BCA$$

$$\underline{\text{Solution:}} \quad A(B+\bar{B})(C+C) + AB(C+C) + BCA$$

$$= AB + A\bar{B}(C+C) + ABC + A\bar{B}C + BCA$$

$$= ABC + ABC\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + ABC + A\bar{B}C + BCA$$

$$= ABC + A\bar{B}C + A\bar{B}\bar{C} + BCA$$

$$= 111 + 110 + 101 + 100$$

$$= M_7 + M_6 + M_5 + M_4$$

$$= \Sigma m(4, 5, 6, 7)$$

Expand the following expressions, into standard POS form

$$(i) f(A, B, C) = A + AB + BCA$$

$$(ii) f(A, B) = (\bar{A} + B)A$$

$$= (\bar{A} + B)(A + B \cdot \bar{B})$$

$$L = \begin{matrix} (\bar{A} + B) & (A + B) & (A + \bar{B}) \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{matrix}$$

$$= M_2, M_3, M_4$$

$$(iii) f(A, B, C) = \prod m(0, 1, 2, 1)$$

$$(ii) f(A, B, C) = A(\bar{A} + B)(\bar{A} + B + C)$$

$$\underline{\text{Solution:}} \quad = (\bar{A} + B \cdot \bar{B}) \cdot (C \cdot \bar{C}) (\bar{A} + B + C \cdot \bar{C}) (\bar{A} + \bar{B} + \bar{C})$$

$$= [(A + B)(A + \bar{B}) + (C \cdot \bar{C})] [(\bar{A} + B)C + \bar{C}(A + B)] (\bar{A} + \bar{B} + \bar{C})$$

$$= [\bar{A} + B + C \cdot \bar{C}] (\bar{A} + \bar{B} + C \cdot \bar{C}) (\bar{A} + B + C) (\bar{A} + B + \bar{C}) (\bar{A} + B + C)$$

$$= (\bar{A} + B + C) (\bar{A} + B + \bar{C}) (\bar{A} + \bar{B} + C) (\bar{A} + \bar{B} + \bar{C}) (\bar{A} + B + \bar{C}) (\bar{A} + B + C)$$

$$= \begin{matrix} (\bar{A} + B + C) & (\bar{A} + B + \bar{C}) & (\bar{A} + \bar{B} + C) & (\bar{A} + \bar{B} + \bar{C}) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{matrix}$$

$$= \Sigma m(0,1,2,3,4,5)$$

Expand the following into minterms and
maxterms

$$(a) f(A,B) = \bar{A} + \bar{B}$$

Solution: $\bar{A}(\bar{B} + \bar{B}) + \bar{B}(A + \bar{A})$

$$= \bar{A}\bar{B} + \bar{A}\bar{B} + \bar{B}A + \bar{A}\bar{B}$$

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$$\begin{array}{ccc} 0 & 1 & \\ 0 & 0 & 1 \\ 1 & & \end{array}$$

$$= 1 \quad 0 \quad 2$$

$\Rightarrow \Sigma m(0,1,2) \rightarrow \text{Minterm}; \Sigma M(3) \rightarrow \text{Maxterm}$

$$(b) f = A + BC\bar{C} + AB\bar{D} + ABCD$$

Solution: $A(\bar{B} + \bar{B})(C + \bar{C})(D + \bar{D}) + BC\bar{C}(A + \bar{A})(D + \bar{D})$
 $+ AB\bar{D}(C + \bar{C}) + ABCD$

$$= AB + A\bar{B}((C + \bar{C})(D + \bar{D})) + ABC\bar{C} + \bar{A}BC\bar{C}(D + \bar{D}) + ABC\bar{D} + ABC\bar{C}\bar{D}$$

 $+ ABCD$

$$= AB(C + AB\bar{C}) + A\bar{B}C + A\bar{B}\bar{C}\bar{C}(D + \bar{D}) + A\bar{B}CD + A\bar{B}\bar{C}\bar{D} + \bar{A}BC\bar{D}$$

 $+ A\bar{B}\bar{C}\bar{D} + ABC\bar{C} + ABC\bar{D} + ABCD$

$$= ABCD + ABC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D}$$

 $+ A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D}$
 $+ A\bar{B}\bar{C}\bar{D} + ABCD$

$$= ABCD + ABC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D}$$

 $+ A\bar{B}\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D}$

$$\begin{aligned}
 & A(B+\bar{B})(C+\bar{C})(D+\bar{D}) + B\bar{C}(A+\bar{A})(D+\bar{D}) + AB\bar{D}(C+\bar{C}) \\
 & AB + A\bar{B} \cdot [(D+C\bar{D}+\bar{C}\bar{D}) + B\bar{C}A + \bar{A}B\bar{C}(D+\bar{D}) + ABC\bar{D} + A\bar{B}\bar{C}\bar{D} \\
 & = ABCD + ABC\bar{D} + A\bar{B}\bar{C}D + ABC\bar{D} + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} \\
 & + A\bar{B}\bar{C}\bar{D} + B\bar{C}AD + B\bar{C}\bar{A}\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D} + A\bar{B}C\bar{D} + A\bar{B}\bar{C}\bar{D} \\
 & = ABCD + ABC\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}\bar{C}\bar{D}
 \end{aligned}$$

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Weekly Assessment Test - 2

- (01) Each product sum term in standard product of sums is _____.
 - (02) Illegal codes in $X_5 \cdot 3$ are _____.
 - (03) _____ Binary code is Reflective, Cyclic and Non-weighted code.
- (04) Add the octal numbers 273.56 + 425.07

$$\begin{array}{r}
 273.56 \rightarrow 010111011 \cdot 101110 \\
 425.07 \rightarrow 100010101 \cdot 000111 \\
 \hline
 1111110000011101
 \end{array}$$

$$\begin{array}{r}
 273.56 \rightarrow \\
 425.07 \rightarrow \\
 \hline
 65
 \end{array}$$

(05) Subtract the hexadecimal $67F2.6B$
- $4A0E.A9$

(06) Complement of, $(\overline{ABC}) (\overline{A} + \overline{B} + \overline{C})$ is

(07) $f(A, B, C) = A(\overline{A} + B)(\overline{A} + B + \overline{C})$ is

(08) find the error in 1010100

(09) $463.12 - 875.34$ in excess - 3 using 10¹³ complement method

(10) State & prove De Morgan's laws.

5th February 2019

(c) $f = A(\bar{B}+A)B$

Solution: $f = A(\bar{B}+A)B$

$$= (A+B\bar{B})(\bar{B}+A)(B+A\cdot\bar{A})$$

$$= (A+B)(A+\bar{B})(A+B)$$

0 0 0 1 1 0

$$= \Pi M(0,1,2)$$

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$$= m(3)$$

(d) $f = A(\bar{A}+B)(\bar{A}+B+\bar{C})$

Solution: $(A+B\bar{B}+C\bar{C})(\bar{A}+B+C\bar{C})(\bar{A}+B+C)$

$$= (A+B+C\bar{C})(A+\bar{B}+C\bar{C})(\bar{A}+B+C)(\bar{A}+B+\bar{C})$$

$$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+\bar{C})(A+\bar{B}+C)(A+B+C)(\bar{A}+B+\bar{C})$$

0 0 0 0 0 1 0 1 0 0 1 1 1 0 0 1 0 1

$$= \Pi M(0,1,2,3,4,5)$$

$$= \Sigma m(6,7)$$

(e) Write the algebraic terms of a variable expression having the following minterms.

(a) m_0

(b) m_5

(c) m_9

(d) m_{14}

Given minterms $m_0 \quad m_5 \quad m_9 \quad m_{14}$

Binary form $0000 \quad 0101 \quad 1001 \quad 1110$

Algebraic form $\bar{A}\bar{B}\bar{C}\bar{D} \quad \bar{A}B\bar{C}D \quad A\bar{B}\bar{C}D \quad AB\bar{C}\bar{D}$

(f) Write the algebraic term of a four variable expression having the following minterms.

(a) M_3

(b) M_9

(c) M_{11}

(d) M_{14} .

Given minterm	m_3	m_9	m_{11}	m_{14}
Binary form	0011	1001	1011	1110
Algebraic form	$A+B+\bar{C}+\bar{D}$	$\bar{A}+B+C+\bar{D}$	$\bar{A}+B+\bar{C}+\bar{D}$	$\bar{A}+\bar{B}+C+D$

02. Boolean Algebra

K maps

→ 'K' stands for Karnaugh Maps

→ A K-map is a chart or graph composed of arrangement of adjacent cells, each representing a sum term (or) product term

→ A n -variable K-map will have 2^n possible combinations (or) cells (or) squares.

→ 2-variable K-map → 4 squares

→ 3-variable K-map → 8 squares

→ 4-variable K-map → 16 squares

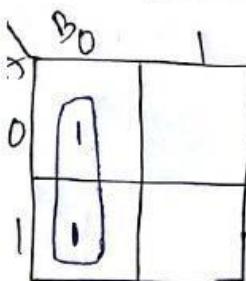
→ 5-variable K-map → 32 squares

→ 6-variable K-map → 64 squares

2-variable K-map:

$$m_0 = \bar{A}\bar{B} \quad m_1 = \bar{A}B \quad m_2 = A\bar{B} \quad m_3 = AB$$

A	B	0	1
0	$\bar{A}\bar{B}$	0	$\bar{A}B$
1	$A\bar{B}$	2	3



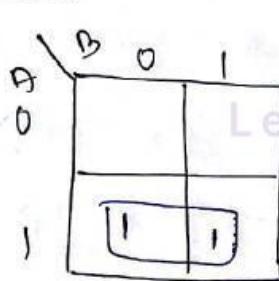
$$f = B$$

$$\bar{A}\bar{B} + A\bar{B}$$

$$B(A + \bar{A})$$

$$= B$$

$$= \bar{B}$$



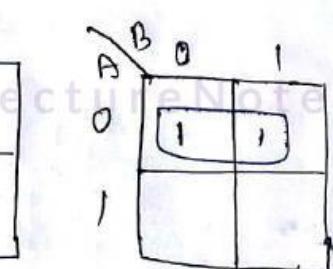
$$f = A$$

$$A$$

$$0$$

$$1$$

$$f = B$$



$$f = \bar{A}$$

$$A$$

$$0$$

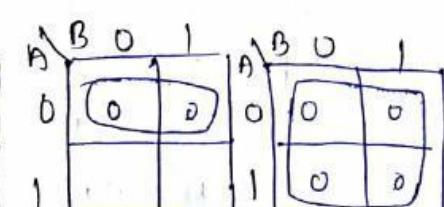
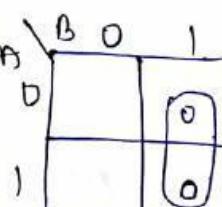
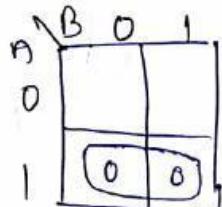
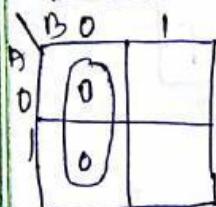
$$1$$

$$f = 1$$

mapping of P.O.S Expressions:

$$M_0 = A + B \quad M_1 = A + \bar{B} \quad M_2 = \bar{A} + B \quad M_3 = \bar{A} + \bar{B}$$

A	B
A+B	A+B
$\bar{A}+B$	$\bar{A}+B$



$$F = B$$

$$F = \bar{A}$$

$$F = \bar{B}$$

$$F = A$$

$$F = 0$$

$$(A+B)(\bar{A}+B)$$

$$A\bar{A} + AB + \bar{A}B + B$$

$$0 + AB + \bar{A}B + B$$

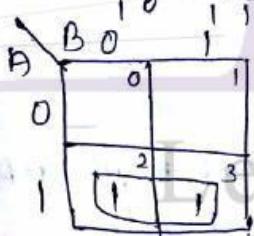
$$B(A\bar{A}) + B$$

$$B + B$$

$$= B$$

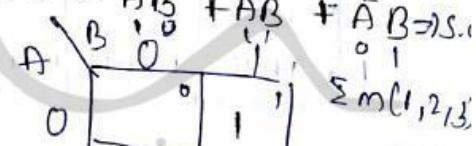
Reduce the following expressions by using K-map.

$$\text{(i)} F = A\bar{B} + A\bar{B} \Rightarrow \text{S.O.P}$$



$$\text{(ii)} f = \bar{A}\bar{B} + \bar{A}B + A\bar{B} \Rightarrow \text{S.O.P}$$

$$\Sigma m(2,3)$$



$$f = A\bar{B} + AB + \bar{A}B$$

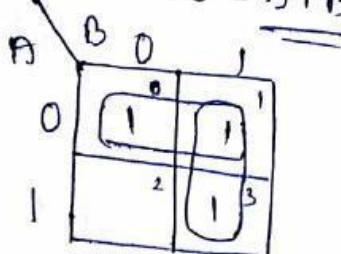
$$= A(B + \bar{B}) + \bar{A}B$$

$$= A + \bar{A}B$$

$$= (A + \bar{A})(A + B)$$

$$= \underline{\underline{A + B}}$$

$$= 1(\bar{A} + B) = \bar{A} + B$$



Reduce the following expression using K-map.

$$(i) (A+B)(\bar{A}+B)(\bar{A}+\bar{B})$$

	B	0	1
A	0	$\begin{matrix} 0 \\ 1 \end{matrix}$	$\begin{matrix} 1 \\ 1 \end{matrix}$
	0	0	1
	1	0	0

$\Pi M(0,1,2,3)$

$$f = B \cdot A' = \bar{A}B$$

$$(ii) f = (\bar{A}+B)(A+\bar{B})(\bar{A}+\bar{B})$$

	B	0	1
A	0	$\begin{matrix} 0 \\ 0 \end{matrix}$	$\begin{matrix} 0 \\ 1 \end{matrix}$
	0	2	3
	1	0	0

$\Sigma M(0,1,3)$

$$f = A\bar{B}$$

3- Variable K-map:

$$\begin{array}{c} BC \\ \diagdown \quad \diagup \\ A \end{array}$$

	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	$\bar{A}B\bar{C}$
1	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$A\bar{B}\bar{C}$

$$\begin{array}{c} BC \\ \diagdown \quad \diagup \\ A \end{array}$$

	00	01	11	10
0	$A+B+C$	$A+B+\bar{C}$	$A+\bar{B}+\bar{C}$	$A+\bar{B}+C$
1	$\bar{A}+B+C$	$\bar{A}+B+\bar{C}$	$\bar{A}+\bar{B}+\bar{C}$	$\bar{A}+\bar{B}+C$

$000 \rightarrow 000 \rightarrow$, Gray code
 $001 \rightarrow 001 \rightarrow 2$
 $010 \rightarrow 011 \rightarrow 3$
 $011 \rightarrow 010 \rightarrow 2$
 $100 \rightarrow 110 \rightarrow 6$
 $101 \rightarrow 111 \rightarrow 7$
 $110 \rightarrow 101 \rightarrow 5$
 $111 \rightarrow 100 \rightarrow 4$

Reduce the expression $f = \bar{A}BC + A\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$ using K-map.

$$f = \bar{A}B\bar{C} + A\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$$

	00	01	10	11	11
0	0	1	3	1	2
1	4	5	7	6	6

$= \Sigma m(1,2,5,6,7)$

$$f = BC + B\bar{C} + AB$$

$$f = (A+B+C)(\bar{A}+\bar{B}+\bar{C})(\bar{A}+\bar{B}+\bar{C})(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+C)$$

Solution: ~ K-M(0,5,7,3,6)

	BC	00	01	11	10
A	0	0	1	0	2
C	0	4	5	7	6

$$f = (\bar{A}+\bar{C})(\bar{B}+\bar{C})(\bar{A}+\bar{B})(A+B+C)$$

If we combine only one term \rightarrow we get 3 literals.

If we combine only 2 terms \rightarrow we get 2 literals.

If we combine only 4 terms \rightarrow we get only 1 literal.

Implement the following functions in NAND gate

$$(i) f(A,B,C) = \Sigma(1,2) \quad (ii) f(A,B,C) = \Sigma(0,6)$$

$$(iii) f(A,B) = \Sigma(1,2)$$

$$(i) f(A,B,C) = \Sigma(1,2) \Rightarrow \text{taking 3-variable K-map.}$$

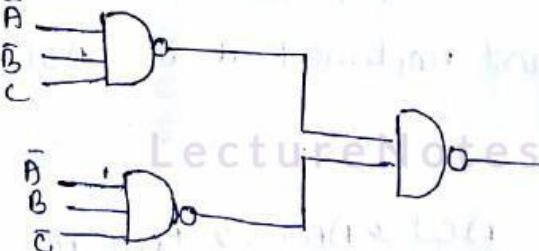
	BC	00	01	11	10
A	0	0	1	3	2
C	1	4	5	7	6

$$f = \bar{A}\bar{B}C + \bar{A}B\bar{C}$$

$$f = A\bar{B}C + \bar{A}BC$$

$$= \bar{A}\bar{B}C + \bar{A}BC$$

$$= \bar{A}\bar{B}C \cdot \bar{A}BC$$

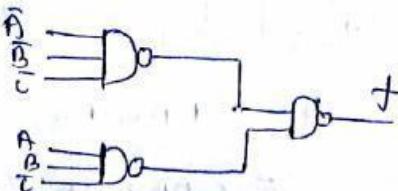


$$(ii) f(A,B,C) = \Sigma(0,6) \Rightarrow \text{considering 3-variable K-map}$$

	BC	00	01	11	10
A	0	0	1	3	2
C	1	4	5	7	6

$$f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$$

$$f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} \Rightarrow \bar{A}\bar{B}\bar{C} \cdot A\bar{B}\bar{C}$$



(iii) $f(A, B) = \Sigma(1, 2)$ \Rightarrow Considering 2 variable K-map

	B	0	1
A	0	0	1
1	1	2	3

$$f = \bar{A}\bar{B} + A\bar{B}$$

$$= \overline{\bar{A}\bar{B} + A\bar{B}} = \overline{\bar{A}\bar{B}} \cdot \overline{A\bar{B}}$$



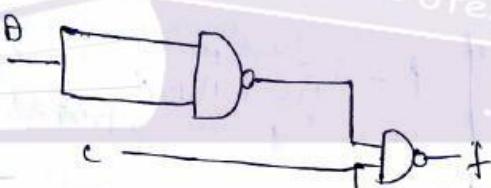
Simplify using K-map and implement the same using NAND gates

$$(ii) f(A, B, C) = \Sigma(0, 2, 4, 5, 6, 7)$$

	B	C	00	01	11	10
A	0	0	1	0	0	1
1	1	1	1	0	1	0

Group 4, 5, 6, 7 so that we will get 1 variable Group 0, 4, 2, 6

$$f = A + \bar{C} \Rightarrow \overline{A + \bar{C}} = \overline{A} \cdot \bar{C}$$



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Reduce the expression $f = \Sigma m(0, 2, 3, 4, 5, 6)$ using mapping and implement it in AOI logic and NAND logic.

Solution: AOI \Rightarrow And or Inverter

And and OR

Inverter

$$f = \Sigma m(0, 2, 3, 4, 5, 6)$$

	B	00	01	11	10
A	0	1	0	1	1
1	1	1	0	1	0

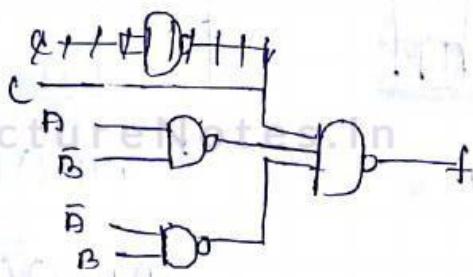
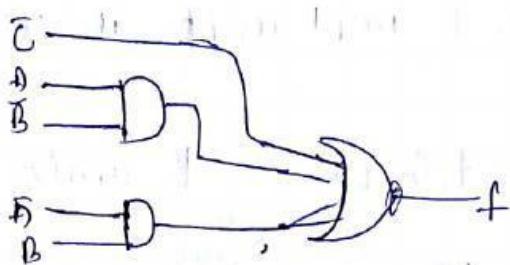
Grouping 0, 4, 2, 6 gives 1 variable

3, 5 are grouped to 7th ans sub

$$f = \bar{C} + A\bar{B} + \bar{A}B$$

$$= \overline{\bar{C} \cdot \bar{A}B} \cdot \overline{\bar{A}B} = C \cdot \bar{A}B \cdot \bar{A}B$$

OA NOR logic



Reduce the expression $f = \prod M(0,1,2,3,4,7)$ using mapping and implement it in OA NOR logic and NOR logic

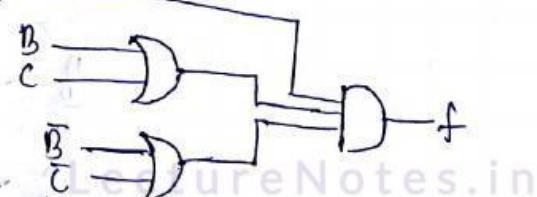
Logic

Solution:

		B\ C	00	01	11	10
		A	0	0	0	0
		1	0	1	0	1

OA NOR And
Invertor

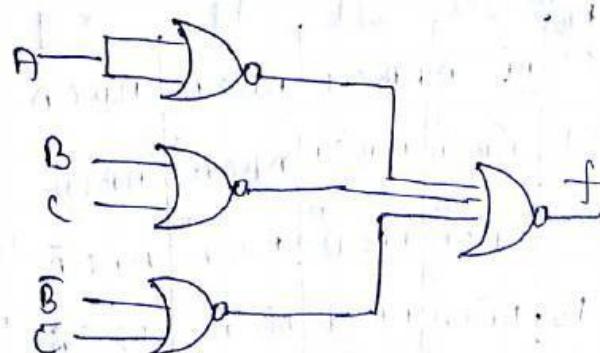
$$f = A(B+C)(\bar{B}+\bar{C})$$



$$f = A(B+C)(\bar{B}+\bar{C})$$

$$= \overline{A(B+C)(\bar{B}+\bar{C})}$$

$$= \overline{\overline{A} + (\overline{B+C}) + (\overline{\bar{B}+\bar{C}})}$$



Obtain the real minimal expression for
 $f = \Sigma m(1, 2, 4, 6, 7)$ and implement it using Universal gates.

Solution $f = \Sigma m(1, 2, 4, 6, 7)$

	BC	00	01	11	10
A	0	0	1	1	1
C	1	1	0	0	0
f	0	1	0	1	1

$$f = AB + A\bar{C} + \bar{A}\bar{B}C + B\bar{C}$$

$f = \Sigma M(0, 3, 5)$

	BC	00	01	11	10
B	0	0	0	0	2
C	1	0	5	7	6
f	0	1	0	1	1

$$f = (A+B+C)(\bar{A}+B+\bar{C})(A+\bar{B})$$

$$= \overline{(A+B+C)}(\bar{A}+B+\bar{C})(A+\bar{B}+\bar{C})$$

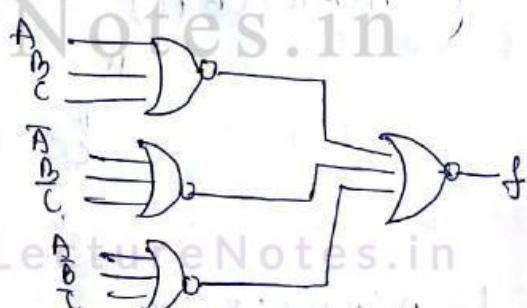
$$= (\bar{A}+\bar{B}+\bar{C}) + (\bar{A}+B+\bar{C}) + (A+\bar{B}+\bar{C})$$

As real minimal expression is considered we need to take any one of the above through which we can minimize the no. of gates.

$f = \Sigma m(1, 2, 4, 6, 7)$ needs 5 gates and $f = \Sigma M(0, 3, 5)$ needs only 4 gates.

∴ And if we get same no. of gates we should go for NAND.

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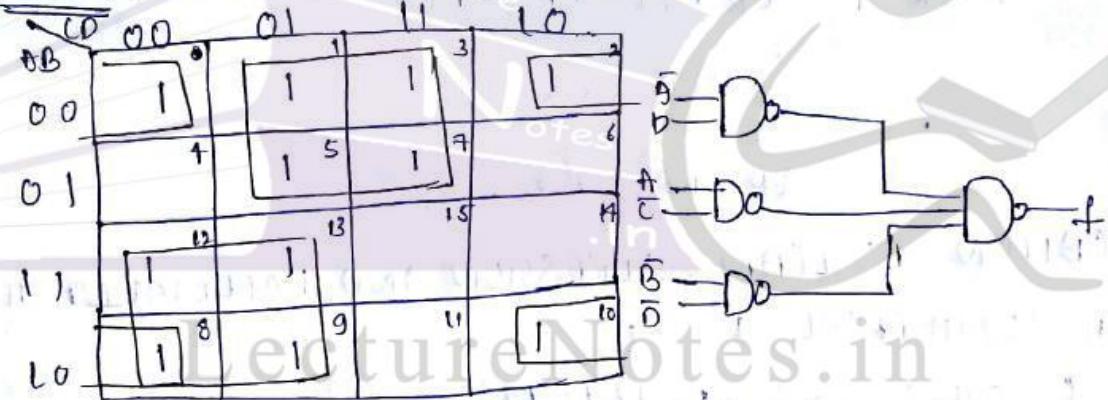
4-variable K-maps

	BC	00	01	11	10
AB	0	0	1	3	2
CD	4	5	7	6	
AB	8	9	13	11	10
CD	12	14	15	16	17

AB	00	01	11	10
CD	00	01	11	10
00	$A + B + C + D$	$A + B + C + \bar{D}$	$A + \bar{B} + C + \bar{D}$	$A + \bar{B} + C + D$
01	$A + \bar{B} + C + D$	$A + \bar{B} + C + \bar{D}$	$A + \bar{B} + \bar{C} + \bar{D}$	$A + \bar{B} + \bar{C} + D$
11	$\bar{A} + \bar{B} + C + D$	$\bar{A} + \bar{B} + C + \bar{D}$	$\bar{A} + \bar{B} + \bar{C} + \bar{D}$	$\bar{A} + \bar{B} + \bar{C} + D$
10	$\bar{A} \bar{B} C D$	$\bar{A} \bar{B} C \bar{D}$	$\bar{A} B \bar{C} \bar{D}$	$A B \bar{C} D$

Reduce the expression $f = \Sigma m(0, 1, 2, 3, 5, 7, 8; 9, 10, 12, 13)$ and implement the expression by using universal logic

Solution : Draw a 4-variable K-map.



$$f = \bar{A}D + A\bar{C} + \bar{B}\bar{D},$$

$$= \overline{\bar{A}D + A\bar{C} + \bar{B}\bar{D}}$$

$$= \overline{\bar{A}B}, \overline{A\bar{C}}, \overline{\bar{B}\bar{D}}$$

Reduce the expression $\Sigma m(2, 8, 9, 10; 11, 12, 14)$ and implement the expression by using universal logic

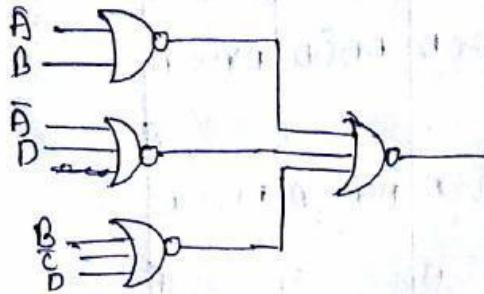
AB	00	01	11	10
CD	00	01	11	10
00				0
01	1	1	1	1
11	0	1	1	1
10	1	0	0	1

$$f = (\bar{A} + D)(B + \bar{C} + D)(\bar{A} + B)$$

$$= \overline{(\bar{A} + D)(B + \bar{C} + D)(\bar{A} + B)}$$

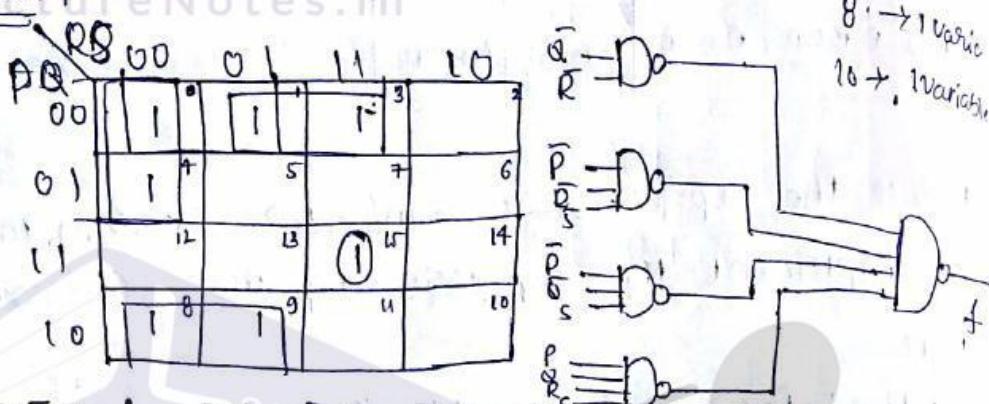
$$= \overline{(\bar{A} + D)} + \overline{(B + \bar{C} + D)} + \overline{(\bar{A} + B)}$$

Using NOR Gate



$$f(P, Q, R, S) = \sum m(0, 1, 3, 4, 8, 9, 15)$$

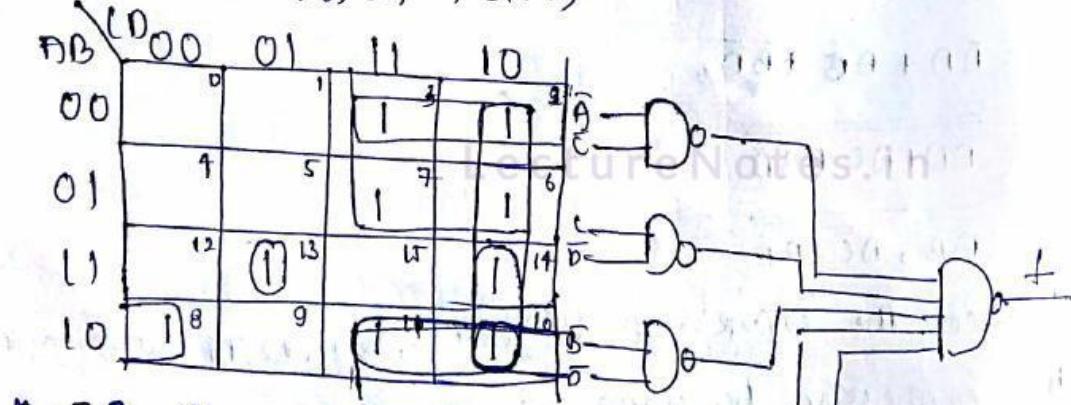
Solution:



$$\begin{aligned} f &= P\bar{Q}\bar{R}\bar{S} \\ f &= \bar{Q}\bar{R} + \bar{P}\bar{R}\bar{S} + \bar{P}\bar{Q}\bar{S} + P\bar{Q}\bar{R}\bar{S} \\ &= \overline{\bar{Q}\bar{R} + \bar{P}\bar{R}\bar{S} + \bar{P}\bar{Q}\bar{S} + P\bar{Q}\bar{R}\bar{S}} \\ &= \overline{\bar{Q}\bar{R} \cdot \bar{R}\bar{R}\bar{S} \cdot \bar{P}\bar{Q}\bar{S} \cdot \bar{P}\bar{Q}\bar{R}\bar{S}} \end{aligned}$$

Obtain Boolean Expression and Implement it in Universal Logic

$$f = \sum m(2, 3, 6, 7, 8, 10, 11, 13, 14)$$



$$f = \bar{A}\bar{C} + \bar{A}\bar{C}$$

$$f = \bar{A}C + C\bar{D} + \bar{B}\bar{D} + A\bar{B}\bar{D} + A\bar{B}\bar{C}\bar{D}$$

$$= \overline{\bar{A}C + C\bar{D} + \bar{B}\bar{D} + A\bar{B}\bar{D} + A\bar{B}\bar{C}\bar{D}}$$

$$= \overline{\bar{A}C \cdot \bar{C}\bar{D} \cdot \bar{B}\bar{D} \cdot A\bar{B}\bar{D} \cdot A\bar{B}\bar{C}\bar{D}}$$

K-MAP SIMPLIFICATION USING DON'T CARES:

Reduce the following expression using K-Maps

$$f = \sum m(1, 5, 6, 12, 13, 14) + d(2, 4)$$

		CD	00	01	11	10	
		AB	00	1		3	X
		00	X	1		7	6
		01	X	1		7	6
		11	1	1		15	14
		10	8	9	11	10	

$$f = B\bar{C} + B\bar{D} + \bar{A}\bar{C}D$$

$$f = \sum m(9, 10, 12) + d(3, 5, 6, 7, 11, 13, 14, 15)$$

		CD	00	01	11	10	
		AB	00		X	3	2
		00		X	5	7	6
		01	X		X		X
		11	1	X	X	X	X
		10	8	9	X	11	10

$$f = AB + AD + AC$$

Reduce the following expression using K-map
and implement it with universal logic.

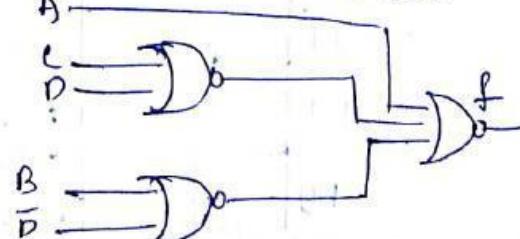
$$f = \sum m(4, 5, 7, 8) + d(10, 11)$$

		CD	00	01	11	10	
		AB	00			3	2
		00					
		01	1	1	1		
		11	12	13	15		14
		10	8	9	X	X	10

$$f = \sum m(1, 3, 4, 8, 9, 11, 15) + d(0, 10, 12, 13, 14)$$

		CD	00	01	11	10	
		AB	00	X	0	0	2
		00	0				
		01	0				
		11	X	X	0	X	
		10	0	0	0	X	

$$f = (\bar{A} + D)(B + \bar{D})(\bar{A})$$

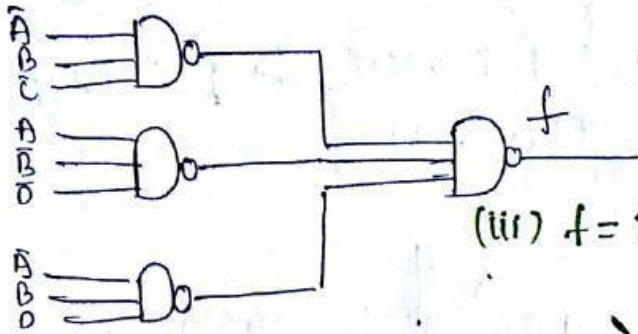


$$f = \bar{A}B\bar{C} + A\bar{B}\bar{D} + \bar{A}B\bar{D}$$

$$= \cancel{\bar{A}B\bar{C}} + A\bar{B}\bar{D} + \cancel{\bar{A}B\bar{D}}$$

$$= \bar{A}B\bar{C}, \bar{A}\bar{B}\bar{D}, \bar{A}B\bar{D}$$

Q1)



$$(iii) f = \sum m(1, 6, 10, 11, 12, 13, 15) + \bar{m}(4, 5, 7, 8, 14)$$

$$(iv) f = \sum m(1, 3, 7, 11, 15) + \bar{m}(0, 2, 5)$$

AB	CD	00	01	11	10
00	X	1	1	1	X
01		X	1	1	
11			1	1	
10			1	1	

AB	CD	00	01	11	10
00		1			
01		X	1	X	
11		1	1	1	X
10		X		1	1

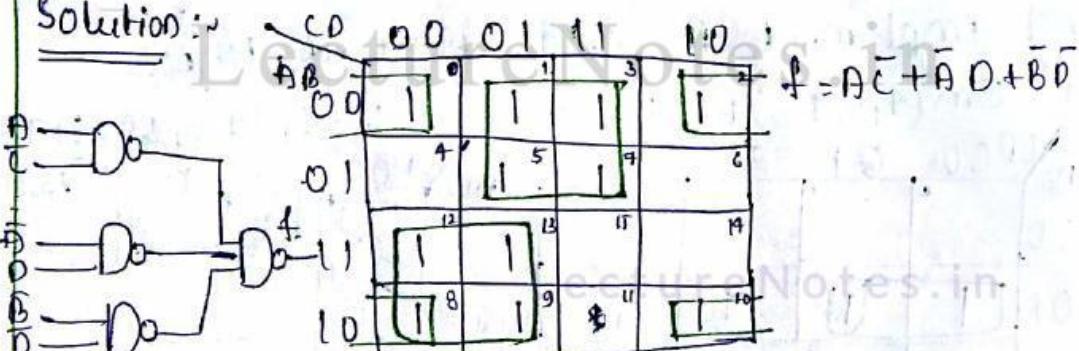
$$f = B + AC + \bar{A}\bar{C}D$$

$$f = CD + \bar{A}\bar{B}$$

Reduce using mapping and implement the minimal expression in universal logic

$$f = \sum m(0, 1, 2, 3, 5, 7, 8, 9, 10, 12, 13)$$

Solution:



$$f = \prod M(4, 6, 11, 14, 15)$$

Compared to NOR, NAND has less power consumption

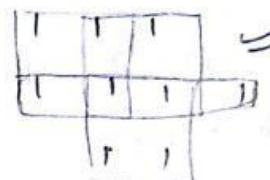
AB	CD	00	01	11	10
00					
01		0			0
11				0	0
10				0	0

$$f = \bar{A}B\bar{D}((A+B+D)(\bar{A}+\bar{C}+\bar{D}) \\ (A+B+C))$$

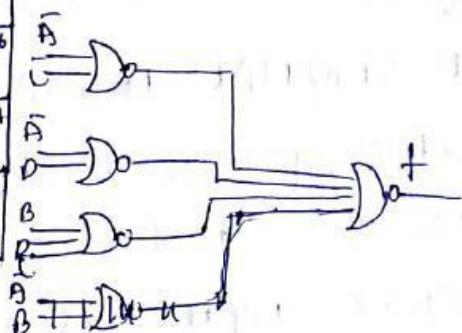
$$(ii) f = \Sigma m(2, 8, 9, 10, 11, 12, 14)$$

$$f = \Sigma m'(0, 1, 3, 4, 5, 6, 7, 13, 15)$$

	CD	00	01	11	10
AB	00	1	1	1	1
00	1	1	1	1	1
01	1	1	1	1	1
11	1	1	1	1	1
10	1	1	1	1	1



$$f = \bar{A}\bar{C} + \bar{A}\bar{D} + BD + \bar{A}B$$



$$(i) f = \Sigma m(0, 2, 4, 6, 7, 8, 10, 12, 13, 15)$$

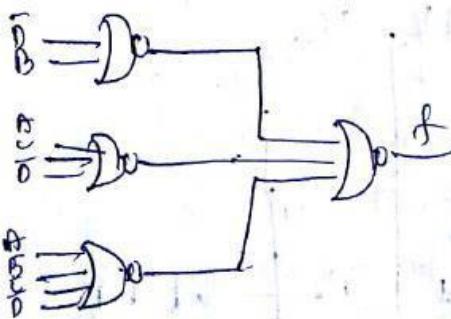
	CD	00	01	11	10
AB	00	1	1	1	1
00	1	1	1	1	1
01	1	1	1	1	1
11	1	1	1	1	1
10	1	1	1	1	1

$$f = \bar{C}\bar{D} + \bar{B}\bar{D}$$

$$f = \Sigma m(1, 3, 5, 9, 11, 14)$$

	CD	00	01	11	10
AB	00	1	1	0	1
00	1	1	0	1	1
01	1	0	1	1	1
11	1	0	1	1	1
10	1	0	1	1	1

$$f = (\bar{D} + B)(A + \bar{B}\bar{C}\bar{D})(A + C + \bar{D})$$



- * PRIME IMPlicant: ~ Each square or rectangle is made up of adjacent minterms called sub-cube.
 - Each sub cube is known as a prime implicant
 -
- * ESSENTIAL PRIME IMPlicant: ~ (EPI)
 - The prime implicant which contains atleast one '1' which can't be covered by other prime implicants is known as Essential Prime Implicant

* REDUNDANT PRIME IMPlicant: - (RPI)

- The prime implicant whose each '1' is covered by other E.P.I is known as Redundant Prime Implicant

		00	01	11	10	
	AB	00	01	11	10	
	00	1	1	1		
E.P.I	01					
	11			1	1	
E.P.I	10			1	1	
	ABC					

* SELECTIVE PRIME IMPlicant:

The prime implicant which is neither E.P.I nor R.P.I is known as Selective prime implicant

		00	01	11	10	
	AB	00	01	11	10	
	00	1	1			
E.P.I	01	1		1		
SPI	11		1	1		
SPI	10			1	1	
	ABC					

* FALSE PRIME IMPlicants:

- ⇒ The max terms are called False minterms.
- ⇒ The prime-implicants obtained by using max terms are known as False-Prime Implicants

* FALSE ESSENTIAL, PRIME IMPlicants:

- ⇒ The False prime Implicant which contains atleast one zero which can't be covered by other F.P.I is known as F.E.P.I

* FALSE REDUNDANT PRIME IMPlicants:

- ⇒ The False Prime Implicant whose each zero is covered by other F.E.P.I is known as False Redundant Prime Implicant

* FALSE SELECTIVE PRIME IMPlicants:

- ⇒ The False Prime Implicant which is neither F.E.P.I nor F.R.P.I is known F.S.P.I

AB\CD	00	01	11	10
00	0	1	3	2
01	4	5	6	7
11	8	9	10	11
10	12	13	14	15

AB\CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	8	12	13	11
10	10	9	14	15

Calculate the F.P.I., F.E.P.I., F.R.P.I., F.S.P.I.
 $f(A, B, C, D) = \Sigma m(0, 4, 5, 10, 11, 13, 15)$

				$= \prod M(1, 2, 3, 6, 7, 8, 9, 12, 14)$
AB	CD	00	01	F.S.P.I.
00	00	0	0	F.E.P.I.
01	01	0	0	F.S.P.I.
11	11	0	0	F.S.P.I.
10	10	0	0	F.S.P.I.

13/2/19
 find the number of implicants, P.I., E.P.I., R.P.I., S.P.I. for the following functions

$$(i) f = \Sigma (1, 5, 6, 7, 11, 12, 13, 15)$$

				Implicant $\rightarrow 08$	
AB	CD	00	01	11	10
00	00	0	1	1	0
01	01	1	0	1	1
11	11	1	1	0	1
10	10	0	1	1	0

Prime Implicant $\rightarrow 05$,
 Essential Prime Implicant $\rightarrow 04$,
 Selective prime Implicant $\rightarrow 0$

$$(ii) f = \Sigma (0, 1, 5, 8, 12, 13)$$

				Implicant $\rightarrow 6$	
AB	CD	00	01	11	10
00	00	1	0	1	0
01	01	0	1	0	1
11	11	1	1	0	1
10	10	1	0	0	1

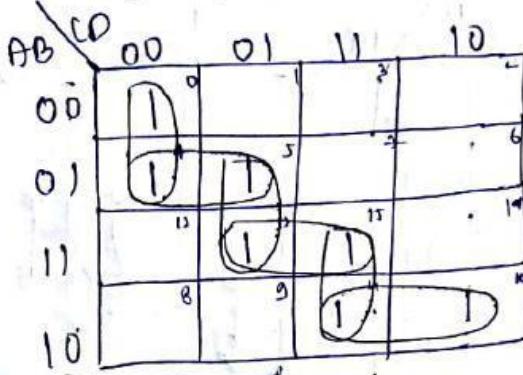
Prime Implicant $\rightarrow 6$,
 E.P.I. $\rightarrow 0$,
 R.P.I. $\rightarrow 0$,
 S.P.I. $\rightarrow 6$.

$$(iii) f = \Sigma (0, 1, 5, 7, 10, 13, 15)$$

				Implicant $\rightarrow 7$	
AB	CD	00	01	11	10
00	00	1	0	1	0
01	01	0	1	0	1
11	11	1	1	0	1
10	10	0	0	1	1

P.I. $\rightarrow 6$,
 E.P.I. $\rightarrow 2$,
 R.P.I. $\rightarrow 0$,
 S.P.I. $\rightarrow 4$

$$04) f = \Sigma(0, 4, 5, 10, 11, 13, 15)$$



Implicant $\rightarrow 07$

P.I. $\rightarrow 6$

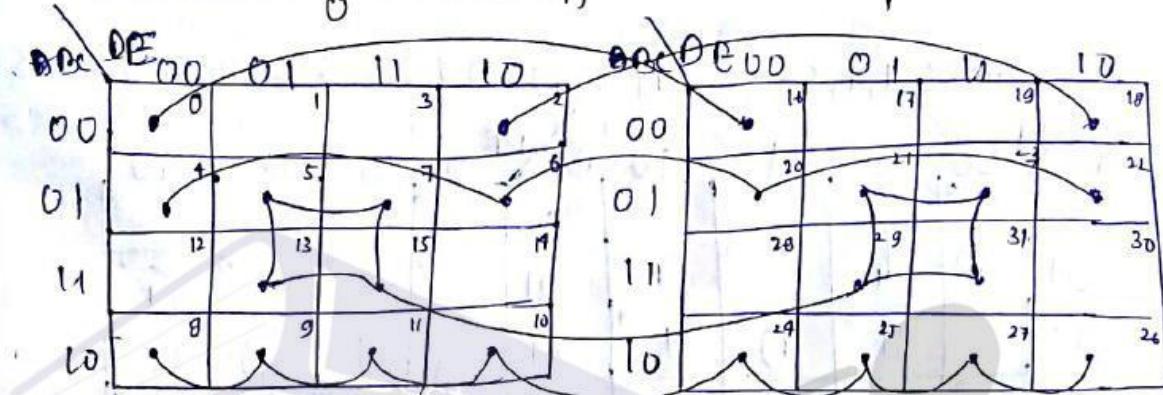
E.P.I. $\rightarrow 2$

R.P.I. $\rightarrow 0$

S.P.I. $\rightarrow 4$

5- Variable K-map:

lectureNotes.in



2 terms \rightarrow 4 literals

4 \rightarrow 3 literals.

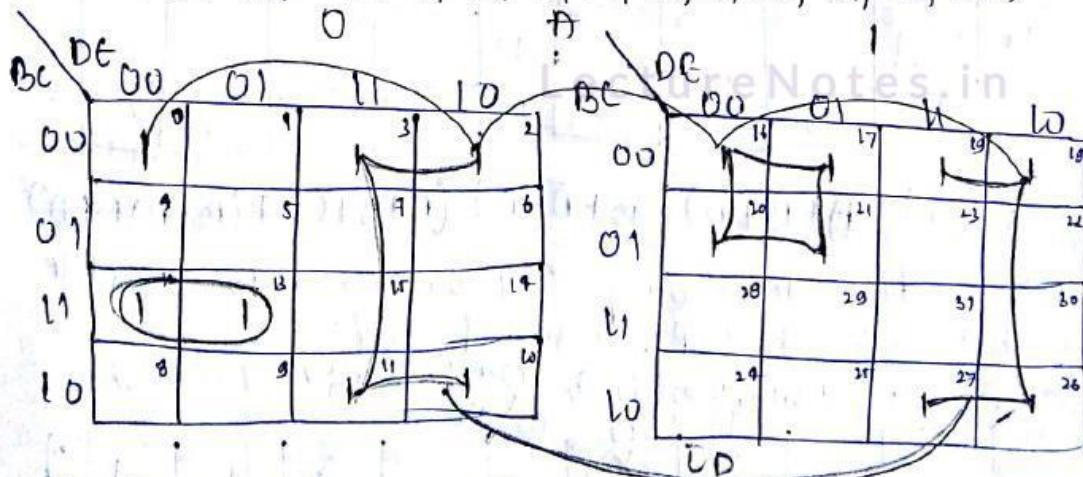
8 terms \rightarrow 2 literals

16 terms \rightarrow 1 literal.

32 terms \rightarrow 0 literals & 1 will be the answer.

Reduce the following expressions in S.O.P and P.O.S forms using mapping

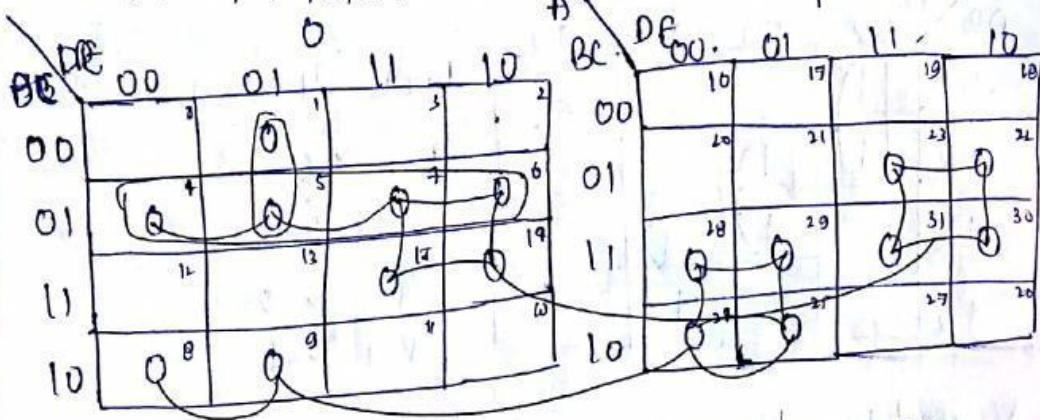
$$f = \Sigma m(0, 2, 3, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 26, 27)$$



$$f = \bar{C}D + A\bar{B}\bar{D} + \bar{A}BC\bar{D} + \bar{B}CD\bar{E}$$

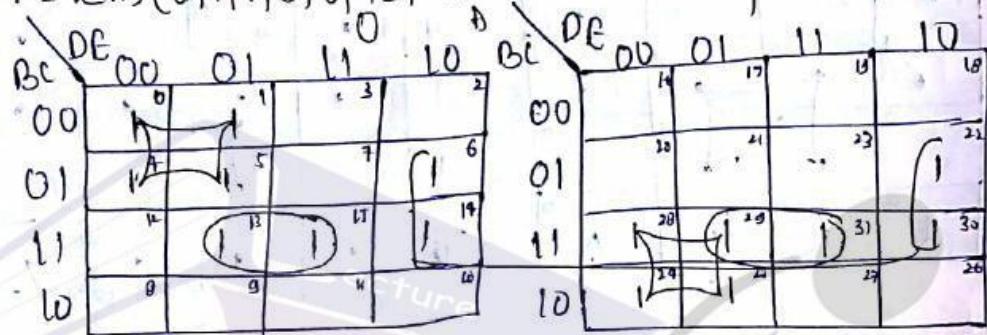
K-map \rightarrow reduce product terms

$$f = \sum m(1, 4, 5, 6, 7, 8, 9, 14, 15, 22, 23, 24, 25, 28, 29, 30, 31)$$



$$f = (\bar{C} + \bar{D}) (\bar{A} + \bar{B} + D) (\bar{B} + C + D) (A + B + \bar{C}) (A + B + D + \bar{E})$$

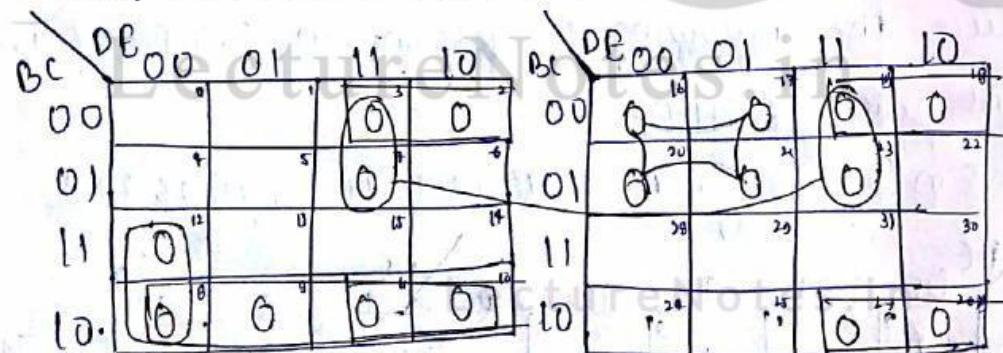
$$f = \sum m(0, 1, 4, 5, 6, 13, 14, 15, 22, 24, 25, 28, 29, 30, 31)$$



$$f = \bar{A}\bar{B}D + C\bar{D}\bar{E} + BCE + AB\bar{D}$$

P.O.S

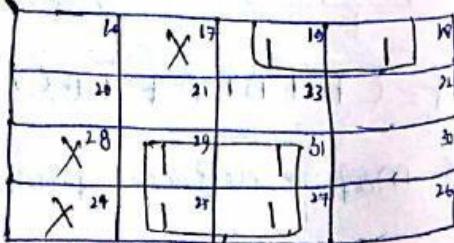
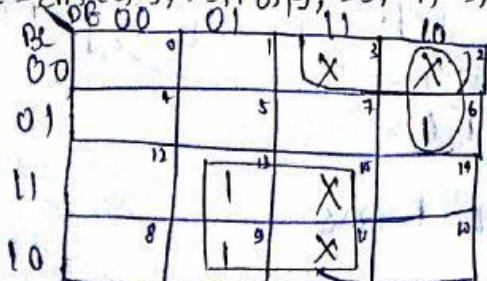
$$f = \sum m(2, 3, 7, 8, 9, 10, 11, 12, 16, 17, 18, 19, 20, 21, 23, 26, 27)$$



$$f = (C + \bar{D})(\bar{A} + B + D)(B + \bar{D} + \bar{E})(A + B + C) + (\bar{B} + D + E)A$$

Reduce the following expressions into simplest possible P.O.S & S.O.P forms $f = \sum m($

$$f = \sum m(6, 9, 13, 18, 19, 25, 27, 29, 31) + \sum m(2, 3, 11, 15, 17, 24, 28)$$



$$f = BE + BC'D + ABE'$$

$$P.O.S = \overline{f} M(0, 1, 4, 5, 7, 8, 10, 12, 14, 16, 20, 21, 22, 23, 26, 30) + \\ d(2, 3, 11, 15, 17, 24, 28)$$

BC \ DE	00	01	11	10
00	0	0	X	X
01	0	0	0	
11	0		X	0
10	0		X	0

BC \ DE	00	01	11	10
00	0	X		
01	0	0	0	0
11	X			
10	X			

$$f = (D+E)(\bar{A}+B+\bar{C})(\bar{B}+E)(A+B+\bar{E})$$

Tabular Method: ~ Quine-McClusky Method: ~

P1) Obtain the minimal expression for $f = \sum m(1, 2, 3, 5, 6, 7, 8, 9, 12, 13, 15)$

Step 01:

$$\begin{aligned} \text{Index 1: } & 1 \rightarrow 0001 \\ & 2 \rightarrow 0010 \\ & 8 \rightarrow 1000 \end{aligned}$$

$$\begin{aligned} \text{Index 2: } & 3 \rightarrow 0011 \\ & 5 \rightarrow 0101 \\ & 6 \rightarrow 0110 \\ & 9 \rightarrow 1001 \\ & 12 \rightarrow 1100 \end{aligned}$$

$$\begin{aligned} \text{Index 3: } & 7 \rightarrow 0111 \\ & 13 \rightarrow 1101 \end{aligned}$$

$$\text{Index 4: } 15 \rightarrow 1111$$

Step 02:

$$(1, 3, 5, 7)(2, 4) \Rightarrow 0 - - 1$$

$$(1, 5, 9, 13)(4, 8) \Rightarrow - - 01$$

$$(2, 3, 6, 7)(1, 9) \Rightarrow 0 - 1 -$$

$$(3, 9, 12, 13)(1, 4) \Rightarrow 1 - 0 -$$

$$(5, 7, 13, 15)(2, 8) \Rightarrow - 1 - 1$$

Step 02:

$$(1, 3)(2) \Rightarrow 00 - 1$$

$$(1, 5)(4) \Rightarrow 0 - 01$$

$$(1, 9)(8) \Rightarrow - 001$$

$$(2, 3)(1) \Rightarrow 001 -$$

$$(2, 6)(4) \Rightarrow 0 - 10$$

$$(8, 9)(1) \Rightarrow 100 -$$

$$(8, 12)(4) \Rightarrow 1 - 00$$

$$(3, 7)(4) \Rightarrow 0 - 11$$

$$(5, 7)(2) \Rightarrow 01 - 1$$

$$(5, 13)(8) \Rightarrow - 101$$

$$(6, 7)(1) \Rightarrow 011 -$$

$$(9, 13)(4) \Rightarrow 1 - 01$$

$$(12, 13)(1) \Rightarrow 110 -$$

$$(7, 15)(8) \Rightarrow - 111$$

$$(13, 15)(2) \Rightarrow 11 - 1$$

Step 04: Prime Implicant Chart

PI/minterm	1	2	3	5	6	7	8	9	12	13	15
P(1,3,5,7)	X		X	X		X					
Q(1,5,9,13)	X			X		.	X			X	
R(2,3,6,7)		(X)	X		(X)	X					
S(8,9,12,13)						(X)	X	(X)	X		
T(5,7,13,15)					X	X			X	X	(X)

$$\begin{aligned}
 f &= R + S + T + P(0) \\
 f &= R + S + T + Q \\
 f &= R + S + T + Q \\
 f &= A - 1 - + 1 - 0 - + - 1 - 1 + 0 - - 1 \\
 f &= A\bar{C} + A\bar{C} + B\bar{D} + \bar{B}\bar{D} \\
 f &= A\bar{C} + A\bar{C} + B\bar{D} + \bar{B}\bar{D}
 \end{aligned}$$

Checking :-

AB	00	01	11	10
00	0	1	1	1
01	4	1	1	1
11	1	1	1	1
10	9	9	0	0

$$f = (w, x, y, z) = \sum m(0, 1, 5, 7, 8, 10, 14, 15)$$

Step 01:-

Step 02:-

Step 03:-

$$\text{Index } 0: (1, 0)(1) \Rightarrow 000-$$

$$0000-0 \quad (0, 8)(8) \Rightarrow -000$$

$$\text{Index } 1: (1, 5)(4) \Rightarrow 0-01$$

$$0001-1 \quad (8, 10)(2) \Rightarrow 10-0$$

$$1000-8$$

$$\text{Index } 2: (5, 7)(2) \Rightarrow 01-1$$

$$0101-5 \quad (10, 14)(4) \Rightarrow 1-10$$

$$1010-10$$

$$(7, 15)(8) \Rightarrow -111$$

$$\text{Index } 3:$$

$$0111-7$$

$$1110-14$$

$$\text{Index } 4:$$

$$1111-15$$

Step 03: Prime Implicant Chart

P.I / minterm	0	1	5	7	8	10	14	15
A(0,1)*	X	X						
B(0,8)	X				X			
C(1,5)		X	X		X	X		
D(8,10)*			X	X				
E(5,7)*		X					X	
F(10,14)				X		X		
G(7,15)				X				X
H(14,15)*						X	X	

$$f = A + D + C + H$$

$$= \bar{0}00 - + 10 - 0 + D1 - 1 + 111 -$$

$$= \bar{w}\bar{x}\bar{y} + w\bar{x}\bar{z} + \bar{w}x\bar{z} + wxy$$

$$f = \Sigma m(6,7,8,9) + \Sigma (10,11,12,13,14,15)$$

Step 01

Index 1

8 → 1000

Index 2

6 → 0110

9 → 1001

10 → 1010

12 → 1100

Index 3

7 → 011

11 → 1011

13 → 1101

14 → 1100

Index 4

15 → 111

Step 2

(8,9)(1) 1000-

(8,10)(2) 10-0

(8,12)(4) 1-00

(6,7)(1) 0,11-

(6,14)(8) -110

(9,11)(2) 10-1

(9,13)(4) 1-01

(10,11)(1) 101-

(10,14)(4) 1-10

(12,13)(1) 110-

(12,14)(2) 11-0

(7,15)(0) -111

(11,15)(4) -1-11

(13,15)(2) 11-1

(14,15)(1) 111-

Step 03

P' (8,9,10,11)(1,2) 10-

R' (8,9,12,13)(1,4) 1-0-

(8,10,12,14) (2,4) 1--0 ✓

S (6,7,14,15) (1,8) -11-

{ x (10,11,14,15) (1,4) 1-1-

{ x (12,13,14,15) (1,2) 11--

(9,11,13,15) (2,4) 1--1 ✓

Step 04

P (8,9,10,11,12,13,14,15)

(1,2,4) 1---

PI/minterm	6	7	8	9
P(8,9,10,11, 12,13,14,15)			X	X
Q(8,9,10,11)	.		X	X
R(8,9,12,13)			X	X
* S(6,7,14,15)	(X)	(X)		

$$f = P + Q$$

$$I - - + - U - = A + BC$$

Check K-map:

AB \ CD		00	01	11	10
00	0	1	3	2	
01	4	5	7	6	
11	X	X	X	X	
10	1	1	X	X	

$$f = A + BC$$

find the minimal expression for $f = \prod M(2,3,8,12,13)$

a (6,14) using tabular method

Step 01

Step 02: $\sum m(2,3)(1)001 - 4$

$0010 - 2$ $\sum m(8,10,12,14)(2,4)1 - - 0$

$1000 - 8$ $\sum m(2,10)(8) - 010 - B$

$1001 - 1$ $\sum m(8,10)(4)10 - 0$

$0001 - 3$ $\sum m(8,12)(4)1 - 00 -$

$1010 - 1$ $\sum m(10,14)(4)1 - 10 -$

$12 + 1000 - 1$ $\sum m(10,14)(4)1 - 10 -$

$1100 - 1$ $\sum m(12,13)(1)110 - C$

$13 - 1101 - 1$ $\sum m(12,14)(2)11 - 0 -$

$14 - 1110 - 1$

Step 03: $\sum m(1)110 - C$

$0010 - 2$ $\sum m(8,10,12,14)(2,4)1 - - 0$

$1000 - 8$ $\sum m(2,10)(8) - 010 - B$

$1001 - 1$ $\sum m(8,10)(4)10 - 0$

$0001 - 3$ $\sum m(8,12)(4)1 - 00 -$

$1010 - 1$ $\sum m(10,14)(4)1 - 10 -$

$12 + 1000 - 1$ $\sum m(10,14)(4)1 - 10 -$

$1100 - 1$ $\sum m(12,13)(1)110 - C$

$13 - 1101 - 1$ $\sum m(12,14)(2)11 - 0 -$

$14 - 1110 - 1$

Step 04: $\sum m(1)110 - C$

$f = A, C, D$

$$= (001 -).(110 -)(1 - 0)$$

$$= (A + B + \bar{C}) . (\bar{A} + \bar{B} + C) (A + D)$$

Prime cant chart	2	3	8	10	12	13	14
A(2,3)	X	(X)					
B(2,10)	X			X			
C(12,13)					X	(X)	
D(8,10,12,14)			(X)	X	X		(X)

		00	01	11	10
		00	01	01	00
		01	00	00	01
A	B	0	0	0	1
00	00	0	0	0	0
01	01	0	0	0	1
11	00	0	0	1	1
10	01	0	1	1	0
00	11	1	1	1	0
01	10	1	1	0	0
11	11	1	0	0	0
10	10	1	0	0	1

$$f = (\bar{A} + D) \cdot (\bar{A} + \bar{B} + C) \cdot (A + B + \bar{C})$$

NAND-NAND IMPLEMENTATION:

* Bubbled AND gate is NOR gate

* Bubbled OR gate is NAND gate

Step 01:

→ for AND gate keep the bubble at the end.

→ for OR gate keep the bubble at the starting point of the gate.

Step 02:

→ If ever bubble was kept, keep a NOT gate.

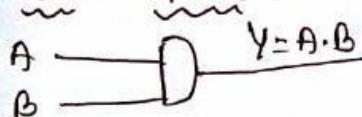
Step 03:

→ Replace NOT gate with NAND gate.

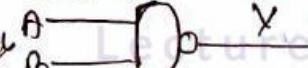
01. NOT GATE:



02. AND GATE:



Step 01

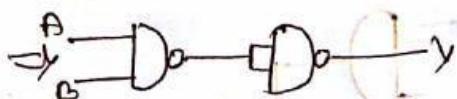


2 → NAND are required.

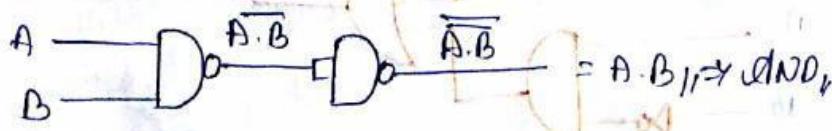
Step 02



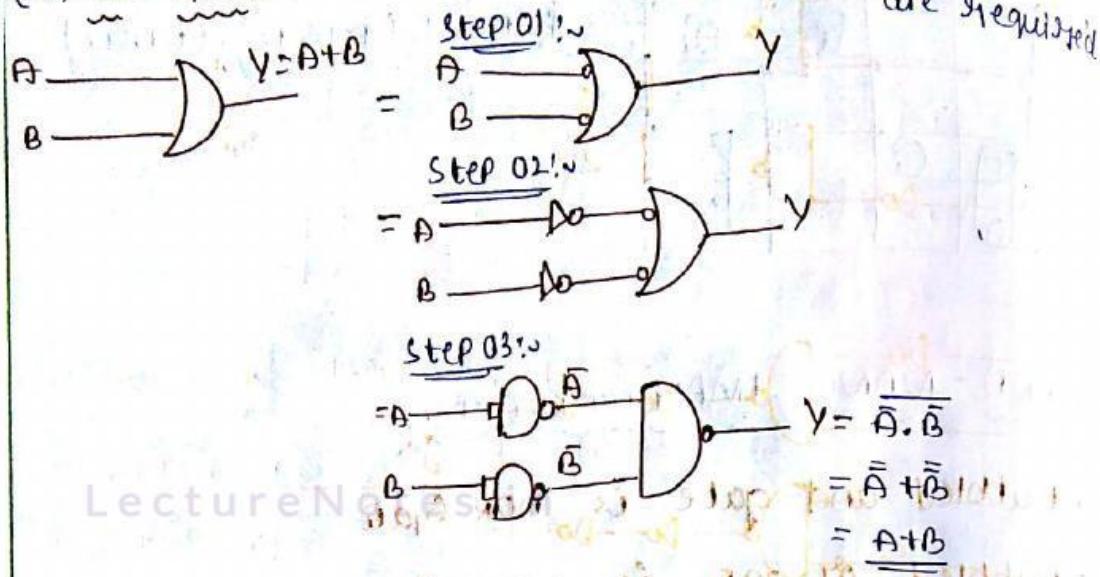
Step 03



CHECK:-



(03) OR Gate

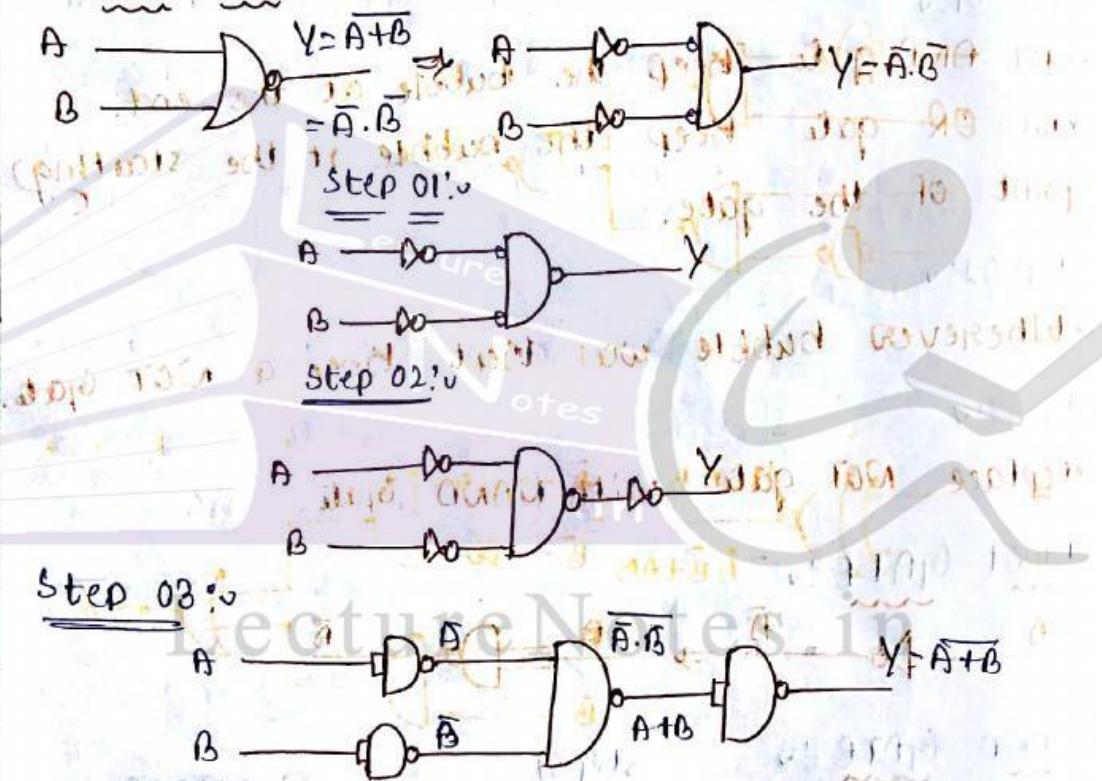


3-NAND Gates

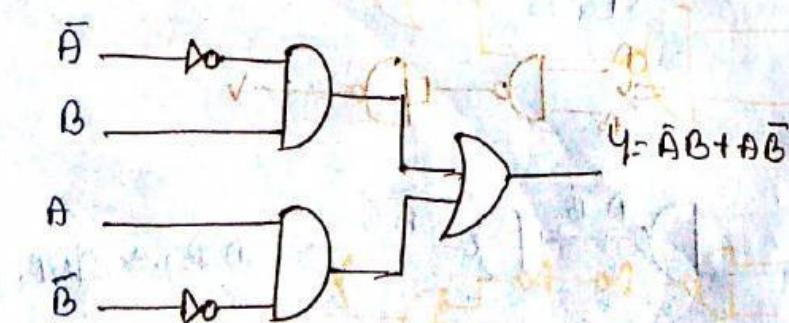
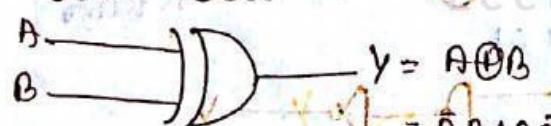
are required

(04) NOR Gate

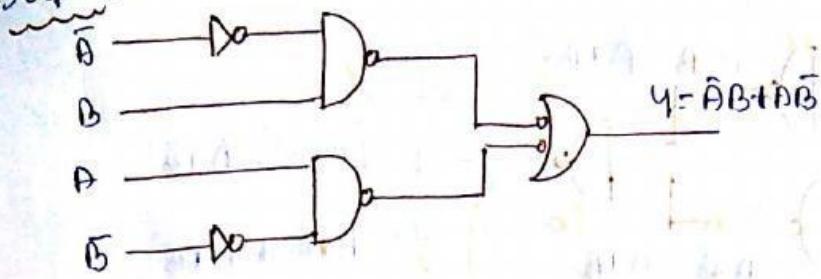
4-NAND Gates are required



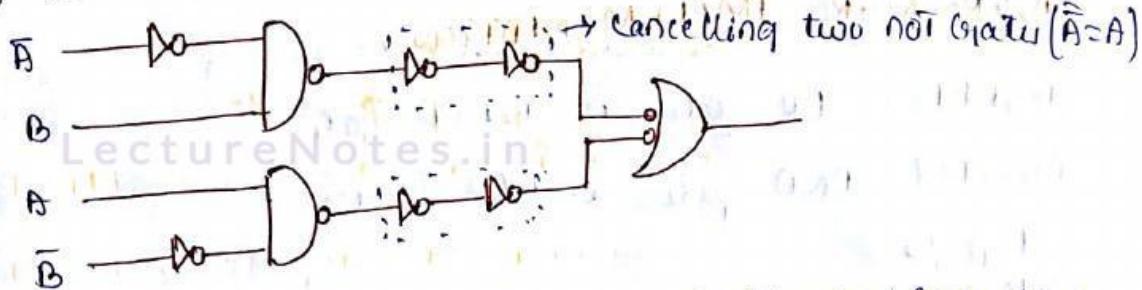
(05) EX-OR Gate



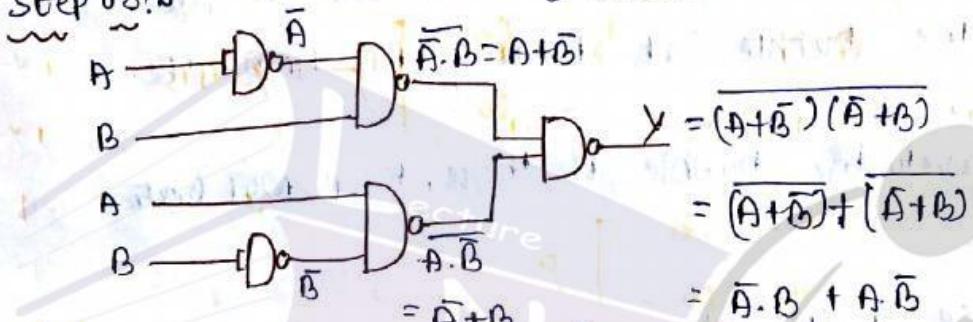
Step 01:



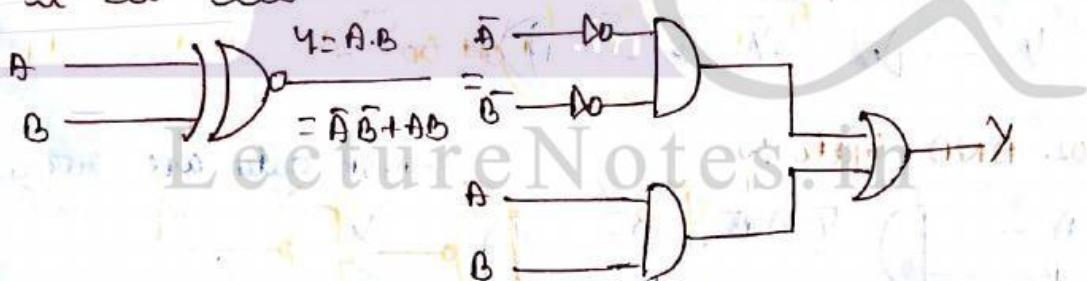
Step 02:



Step 03:

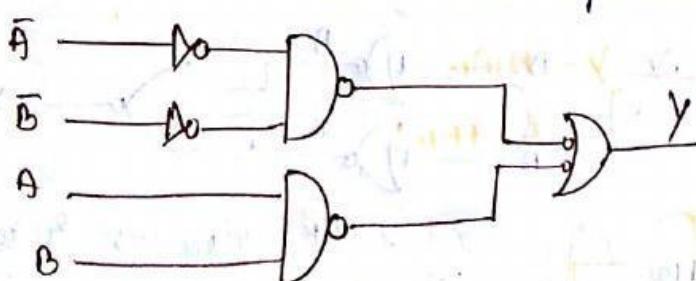


(06) Ex-NOR Gate :-

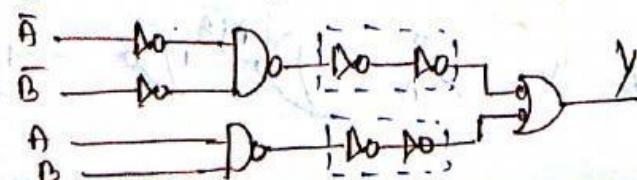


Step 01: AND Gate → End bubble

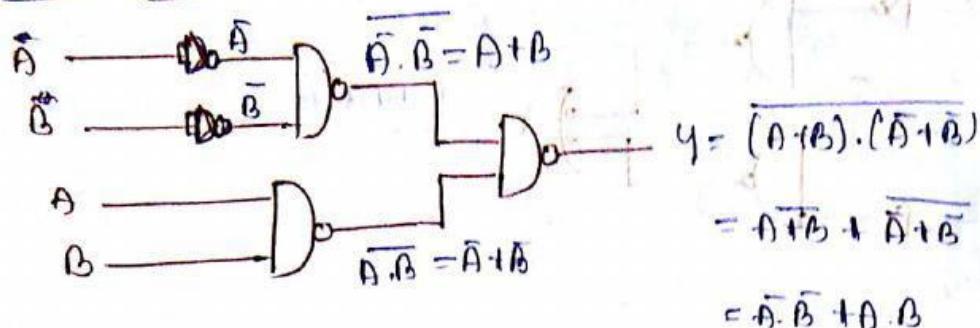
OR Gate → Starting point bubble.



Step 02: Keeping NOT Gate near bubble.



Step 03:-



NOR-NOR IMPLEMENTATION :-

Bubbled OR gate is NAND Gate

Bubbled AND gate is NOR Gate.

Step 01:-

→ Keep the bubble at the end for OR Gate

→ Keep the bubble at starting for AND Gate

Step 02:-

→ Whenever the bubble is there, keep NOT Gate

Step 03:-

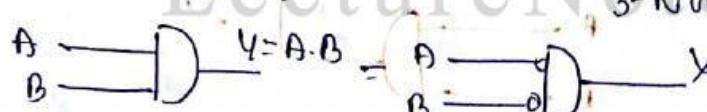
→ Replace NOT gate with NOR Gate

01. NOT Gate :-

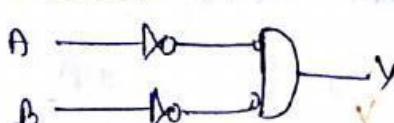


02. AND Gate :-

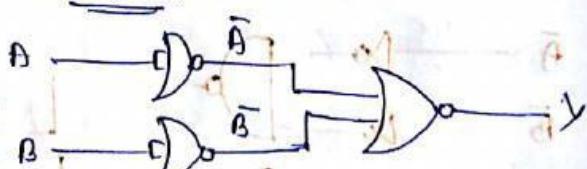
3-NOR gate are required.



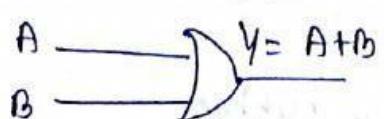
Step 02:-



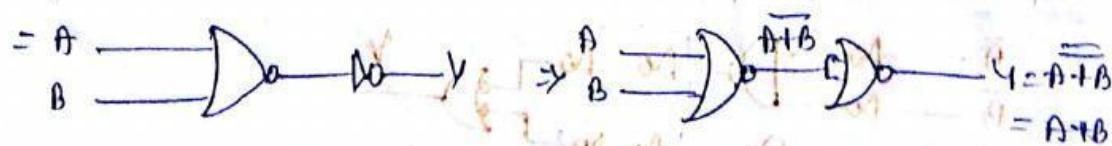
Step 03:-



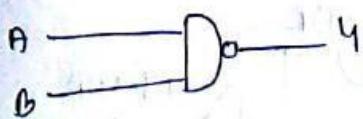
03. OR Gate :-



2-NOR Gate are required.



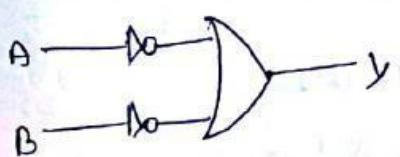
04. NAND Gate :-



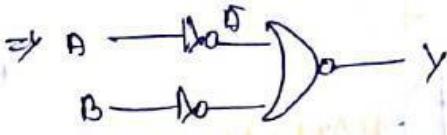
$$Y = \overline{A \cdot B} = \bar{A} + \bar{B}$$

4-NOR gates are required

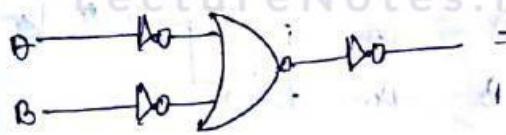
Step 01:-



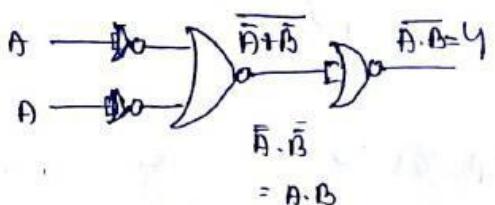
Step 02:-



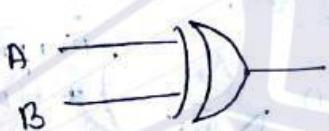
Step 02:-



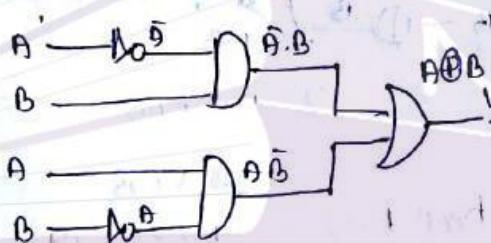
Step 03:-



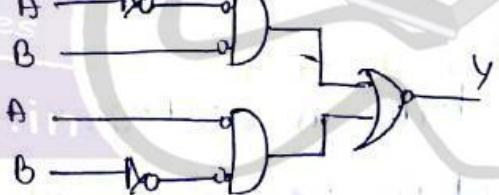
05. EX-OR Gate :-



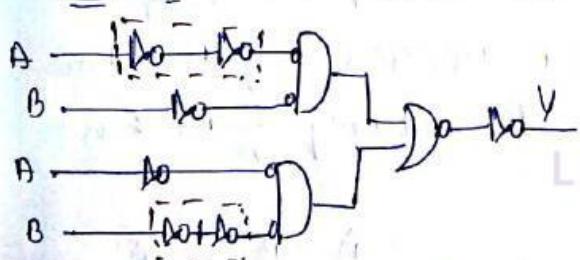
$$Y = A \oplus B \\ = \bar{A}B + A\bar{B}$$



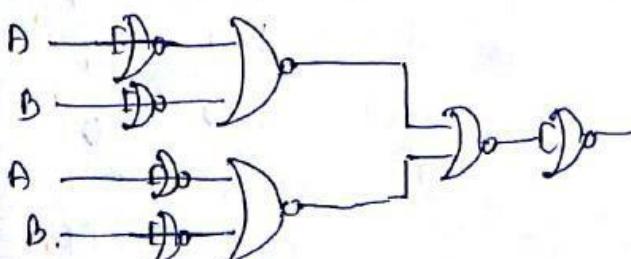
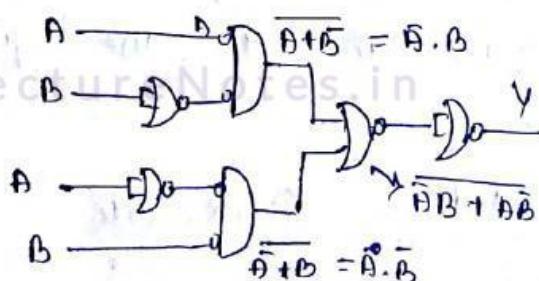
Step 01



Step 02:-



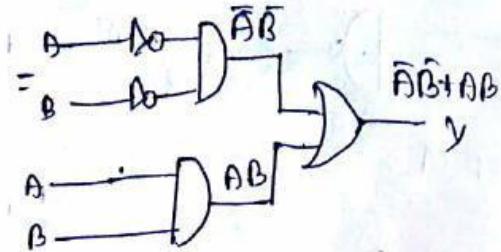
Step 03:-



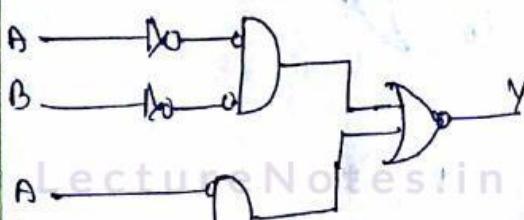
$$Y = \overline{\overline{A}\bar{B} + \overline{A}\bar{B}} \\ = \overline{AB} + \overline{AB}$$

Ex-NOR Gate :-

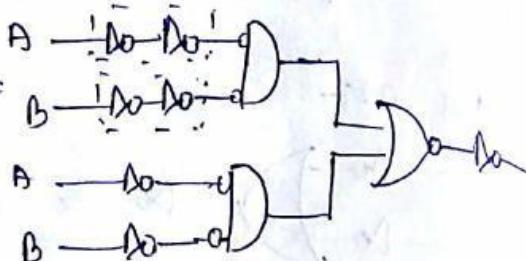
$$\text{Y} = \bar{A}\bar{B} + A\bar{B}$$



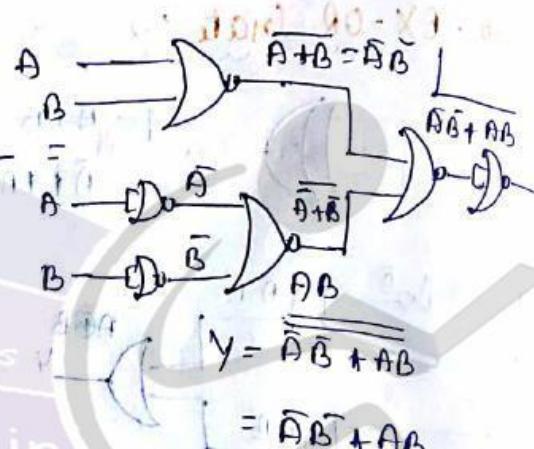
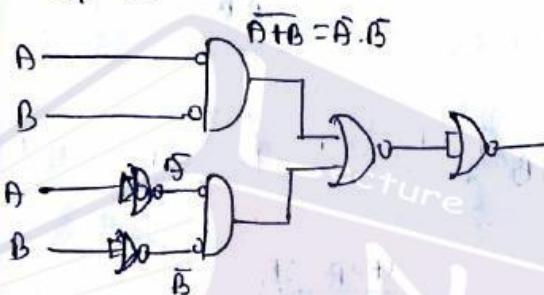
Step - 01 :-



Step 02 :-



Step - 03 :-



A bulb in a stair-case has two switches. One switch being at ground floor and other on first floor. The bulb can be turned on and off by any one of the switches irrespective of the state of the other switch. The logic of switching of the bulb resembles

Solution:- X-OR Gate

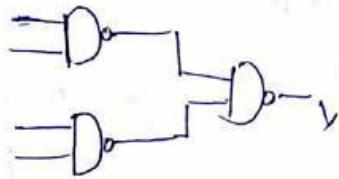
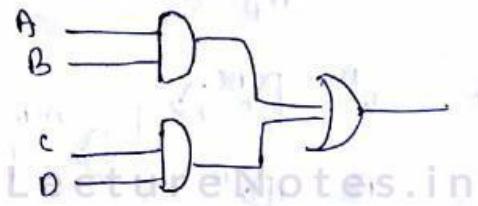
A	B	XOR
0	0	0
0	1	1
1	1	1
1	0	0

The Boolean function $Y = AB + CD$ is to be realised using 2 input NAND gates. The minimum number of gates required is

Solution: Draw the logic gate for the given expression

$$Y = AB + CD \Rightarrow$$

$$\Rightarrow Y = \overline{AB} \cdot \overline{CD} = \overline{\overline{AB} \cdot \overline{CD}}$$



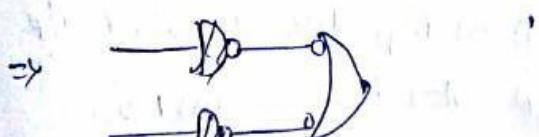
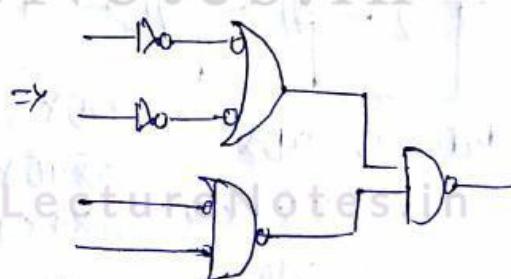
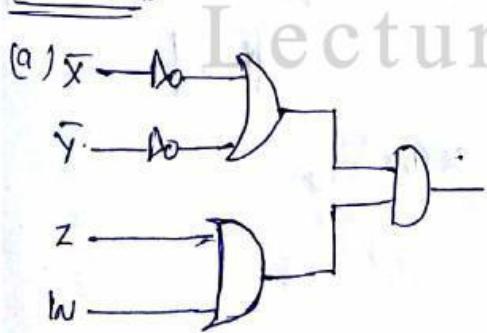
What is the minimum number of 2-input NAND gates required to implement the following Boolean functions

$$(a) f = (\bar{x} + \bar{y})(z + w)$$

$$(b) z = A\bar{B}C$$

$$(c) f = A + A\bar{B} + A\bar{B}C$$

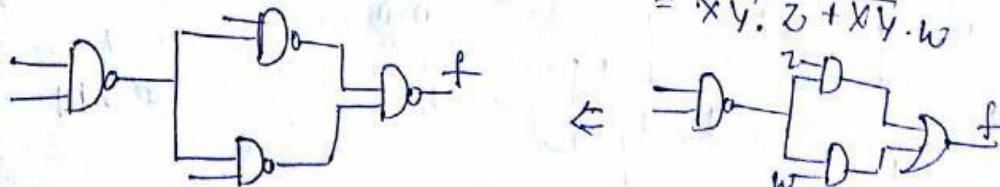
Solution



$$(1) f = (\bar{x} + \bar{y})(z + w)$$

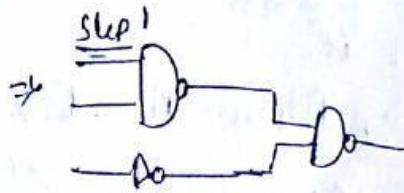
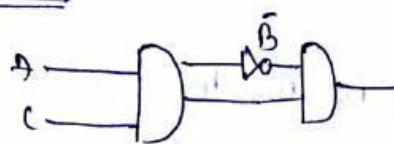
$$= \bar{x}\bar{y}(z + w)$$

$$= \bar{x}\bar{y} \cdot z + \bar{x}\bar{y} \cdot w$$

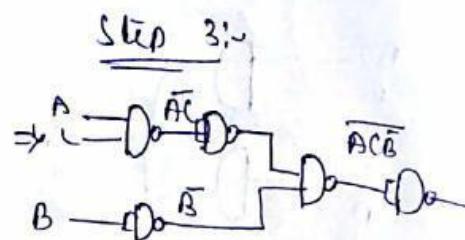
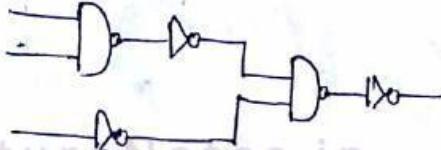


$$(b) Z = A \bar{B} C$$

Solution:



Step 2:



LectureNotes.in

$$(1) f = A + A\bar{B} + A\bar{B}C$$

$$= A(1 + \bar{B} + \bar{B}C) = A,$$

Zero NAND gates are required to realise.

- (b) The o/p of the logic gate in the fig is



$$\begin{array}{l} \text{A} \\ \text{---} \\ \text{A} \cdot \text{G} \end{array} \quad A \cdot B = \bar{A}\bar{B} + AB$$

$$f = A \cdot 0 + \bar{A} \cdot 1 \Rightarrow 0 + \bar{A} = \bar{A}$$

(a) 0

(b) 1

(c) A

(d) A'

- (c) for the circuit shown below, the o/p 'f' is given by



$$f = 0$$

Solution: $X \oplus X$

$$0 \oplus X$$

$$X \oplus X = 0,$$

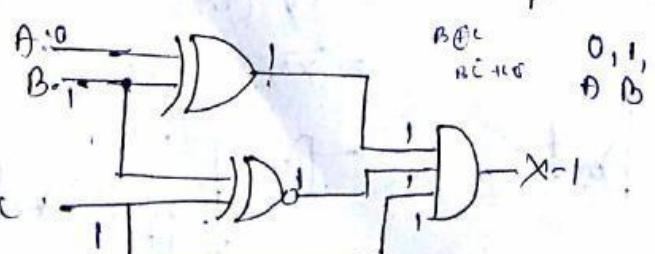
$$0 \bar{X} + \bar{X} \cdot X$$

$$\bar{X} + X \cdot 1$$

$$= 0$$

$$= X$$

- (d) for the logic circuit shown in fig, the required i/p conditions to make the o/p $Z=1$ is



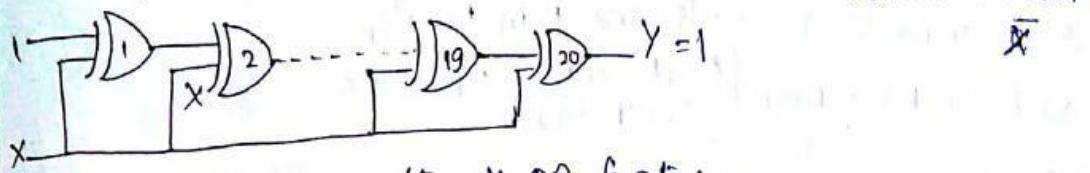
(a) 1, 0, 1

(b) 0, 0, 1

(c) 1, 1, 1

for $Z=1$

If the i/p to a digital circuit shown consisting of cascaded 20 X-OR Gates is X , the o/p Y is equal to



$$1 \oplus X \Rightarrow 1 \bar{x} + x \bar{1} \Rightarrow \bar{x}$$

Solution:- O/p at 1st X-OR Gate:-

$$1 \oplus X \Rightarrow 1 \bar{x} + x \cdot 0 = \bar{x}$$

O/p at 2nd X-OR Gate:-

$$\bar{x} \oplus x \Rightarrow \bar{x} \cdot \bar{x} + x \cdot x = \bar{x} + x = 1.$$

O/p at 19th X-OR $\Rightarrow \bar{x}$

O/p at 20th X-OR $\Rightarrow 1$

Which of the following boolean expression correctly represent the relation b/w $PQR \& M_1$,

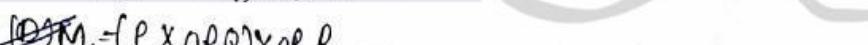
Solution :- (a) $M_1 = (P \text{OR } Q) \text{XOR } R$



(b) $M_1 = (P \text{AND } Q) \text{XOR } R$



(c) $M_1 = (P \text{NOR } Q) \text{XOR } R$



~~(d) $M_1 = (P \text{XOR } Q) \text{XOR } R$~~

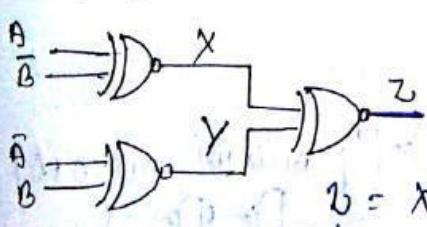
$$X = \overline{P \cdot Q}; \quad Y = P + Q$$

$$Z = X \cdot Y = \overline{P \cdot Q} \cdot (P + Q) \Rightarrow (\overline{P} + \overline{Q})(P + Q) = \underbrace{(\overline{P} + \overline{Q})P}_0 (\overline{P}Q + \overline{Q}P)$$

$$= (P \text{XOR } Q)$$

$$M_1 = Z \text{XOR } R \Rightarrow (P \text{XOR } Q) \text{XOR } R$$

The o/p of the circuit show is equal to



$$A \oplus \bar{B} \Rightarrow \bar{A} \cdot \bar{B} + \bar{B} \cdot \bar{A} \Rightarrow A \bar{B} + \bar{A} B \Rightarrow A \oplus B$$

$$\bar{A} \oplus B \Rightarrow \bar{A} \cdot B + \bar{B} \cdot \bar{A} \Rightarrow A \oplus B$$

$$Z = X \oplus Y \Rightarrow (A \oplus B) \oplus (A \oplus B)$$

$$\Rightarrow (A \oplus B) / (A \oplus B) + \bar{(A \oplus B)} (A \oplus B)$$

$$\Rightarrow (A \oplus B) + (\bar{A} \oplus \bar{B}) = 1,$$

Implement the following functions using NAND gates

(a) $f = \overline{xy + xz}$

(b) $f(A, B, C, D) = \Sigma m(0, 3, 4, 8, 11, 12, 15)$

(c) $f = (\bar{D} + \bar{B})(AB)$ } assume both true &
(d) $f = \bar{C}\bar{D} + \bar{B}(C\bar{D} + A\bar{C})$ } complemented i/p are
available

(e) $\Rightarrow \overline{xy + xz} \Rightarrow \overline{xy} \cdot \overline{xz} \Rightarrow (\bar{x} + \bar{y}) \cdot (\bar{x} + \bar{z})$
4-NAND Gates required.

LectureNotes.in

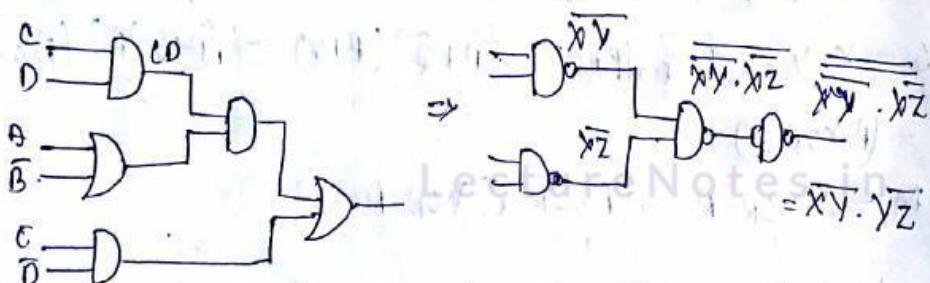


(b) $f(A, B, C, D) = \Sigma m(0, 3, 4, 8, 11, 12, 15)$

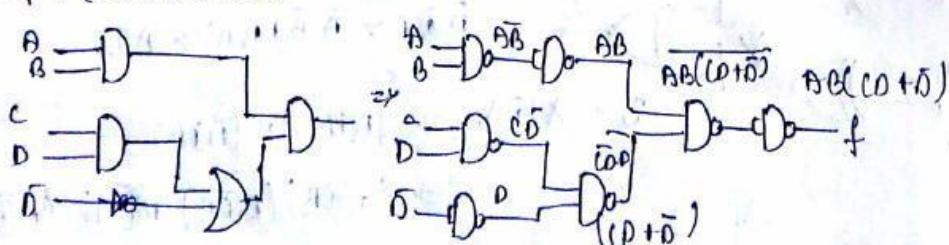
A	B	C	D	00	01	11	10
00	1			1		1	1
01	1			1	1	0	0
11	1			1	0	1	0
10	1	0	0	0	1	0	1

$$f = \bar{C}\bar{D} + \bar{A}CD + \bar{B}CD$$

(d) $f = \bar{C}\bar{D} + \bar{B}(C\bar{D} + A\bar{C}) \Rightarrow \bar{C}\bar{D} + (\bar{D}(A\bar{B}))$

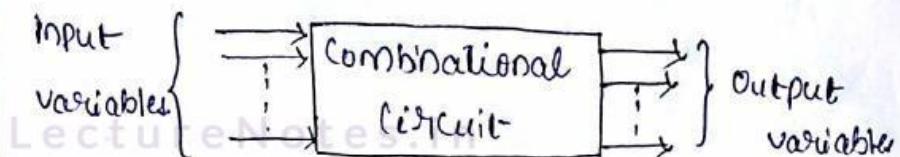


(e) $f = (\bar{C}\bar{D} + \bar{B})(AB)$



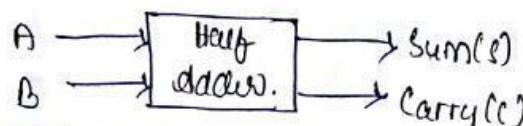
03. Combinational Circuits.

- ⇒ There are 2 types of circuits ① Sequential circuit ② Combinational
- ⇒ In a digital circuit if the o/p at any instant of time depends only on the present i/p, then the circuit is said to be a combinational circuit.



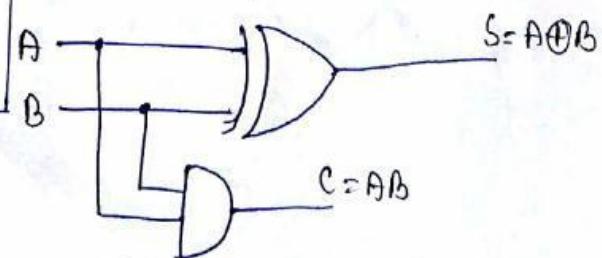
Ex:- Adders, Subtractors, Multipliers, Multiplexers, Encoders, Decoders.

- * Analysis and design of Combinational Circuits
 - ⇒ Identify the number of input and output variables
 - ⇒ Express the output function in terms of input variables
 - ⇒ Truth table and logic diagram.
 - ⇒ An adder is a combinational circuit, which performs the addition of 2 bits [half adder] and addition of 3 bits [full adder]
- * Half Adder



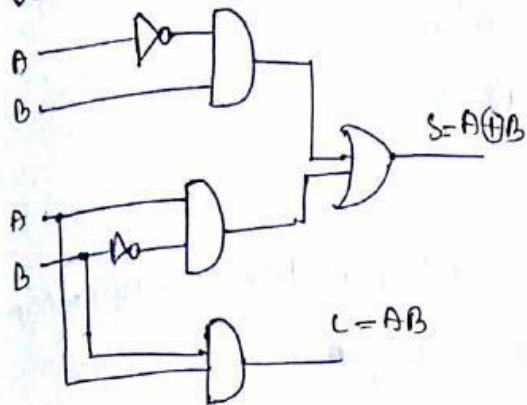
Inputs		Outputs	
A	B	C	D
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

(a) Truth Table.



(b) Logic Diagram.

AOI logic



$$S = A \oplus B$$

$$= \bar{A}B + A\bar{B}$$

$$= \bar{A}B + A\bar{B} + A\bar{A} + B\bar{B}$$

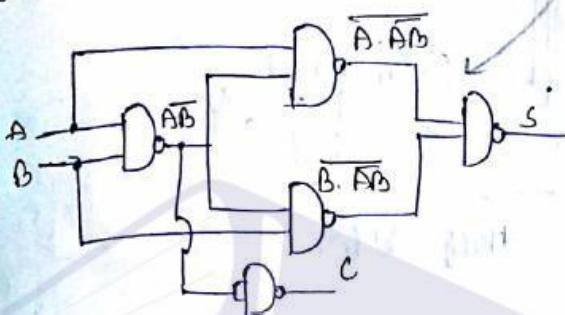
$$= A(\bar{A} + \bar{B}) + B(\bar{A} + \bar{B})$$

$$= A \cdot \bar{A}B + B \cdot \bar{A}B$$

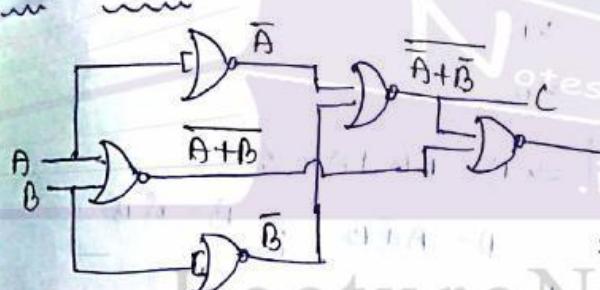
$$= \overline{A \cdot \bar{A}B} + \overline{B \cdot \bar{A}B}$$

$$= \overline{\overline{A \cdot \bar{A}B} \cdot \overline{B \cdot \bar{A}B}}$$

* NAND logic



NOR logic



$$S = \bar{A}B + A\bar{B}$$

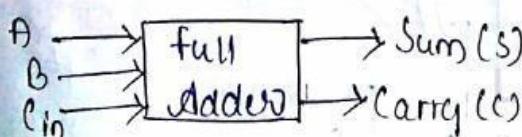
$$= \bar{A}B + A\bar{B} + A \cdot \bar{A} + B \cdot \bar{B}$$

$$= A(\bar{A} + \bar{B}) + B(\bar{A} + \bar{B})$$

$$= (A + B)(\bar{A} + \bar{B})$$

$$= \overline{(A + B)(\bar{A} + \bar{B})} = \overline{\overline{A + B} + \overline{\bar{A} + \bar{B}}}$$

full adder



Inputs			Outputs	
A	B	C _{in}	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = \bar{A}\bar{B}C_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}\bar{C}_{in} + AB\bar{C}_{in}$$

$$= \bar{A}[\bar{B}C_{in} + BC_{in}] + A[\bar{B}\bar{C}_{in} + B\bar{C}_{in}]$$

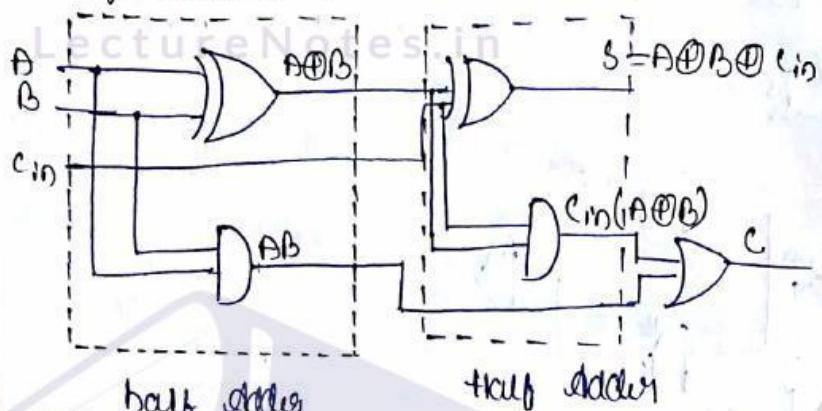
$$= \bar{A}(B \oplus C_{in}) + A(\bar{B} \oplus C_{in})$$

$$S = A \oplus B \oplus C_{in} \Rightarrow \text{Expression for sum}$$

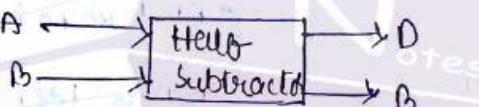
Expression for complementary :-

$$\begin{aligned}
 C &= ABc_{in} + A\bar{B}c_{in} + AB\bar{C}_{in} + A\bar{B}\bar{C}_{in} \\
 &= c_{in}(\bar{A}B + A\bar{B}) + AB(c_{in} + \bar{c}_{in}) \\
 &= \boxed{C = c_{in}(A \oplus B) + AB}
 \end{aligned}$$

* Show that one full adder is the combination
2 half adder plus one OR gate



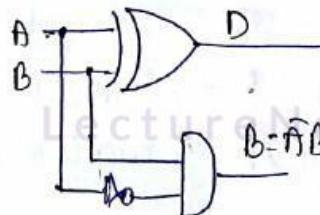
20/2/19 → Wednesday.
Half Subtractor :-



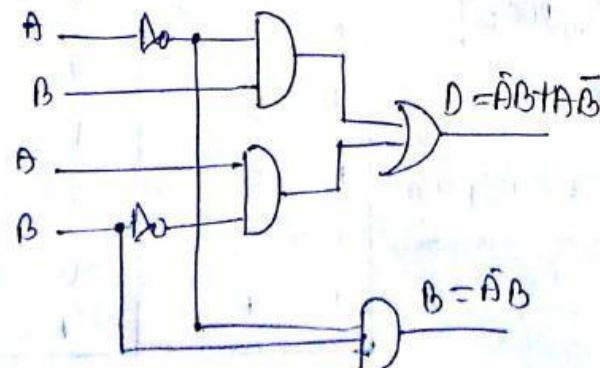
Inputs		Output	
A	B	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$\begin{aligned}
 D &= A\bar{B} + \bar{A}B ; \quad B = \bar{A}B \\
 D &= A \oplus B ; \quad B = \bar{A}B
 \end{aligned}$$

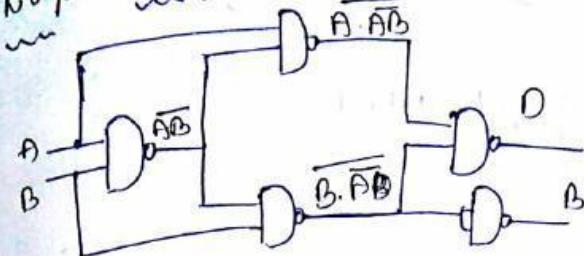
Logic Diagram :-



Logic Diagram in AOI Logic :-



logic diagram in NAND Logic



$$D = \bar{A}B + A\bar{B} + A \cdot \bar{A} + B\bar{B}$$

$$= B(\bar{A} + \bar{B}) + A(\bar{A} + \bar{B})$$

$$= B(\bar{A}\bar{B}) + A(\bar{A}\bar{B})$$

$$= \overline{B(\bar{A}\bar{B})} \cdot \overline{A(\bar{A}\bar{B})}$$

Logic:

$$D = A\bar{B} + \bar{A}B + A \cdot \bar{A} + B\bar{B}$$

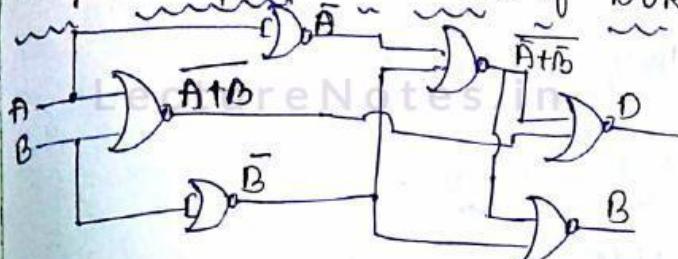
$$= \bar{A}B + A\bar{B} + A(\bar{A} + \bar{B})$$

$$= (\bar{A} + \bar{B}) \cdot (A + B)$$

$$= \overline{(\bar{A} + \bar{B})} \cdot \overline{(A + B)}$$

$$D = \overline{(\bar{A} + \bar{B})} + \overline{(A + B)}$$

logic Diagram in terms of NOR



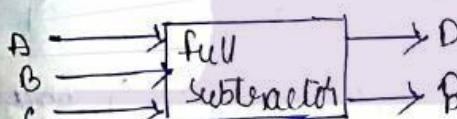
$$B = \bar{A}B + A \cdot \bar{A}$$

$$= \bar{A}(A + B) = \overline{\bar{A}(A + B)}$$

$$= \overline{A + (\bar{A} + B)}$$

$$B = \bar{A}B + B \cdot \bar{B} \Rightarrow \overline{B(\bar{A} + \bar{B})} = \overline{B + (\bar{A} + \bar{B})}$$

full subtractor :



$$D = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

$$= \bar{A}(\bar{B}C + BC) + A(\bar{B}\bar{C} + BC)$$

$$= \bar{A}(B \oplus C) + A(C \oplus C)$$

$$= \bar{A}(B \oplus C) + A(\bar{B} \oplus \bar{C})$$

$$D = A \oplus B \oplus C$$

$$B = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}C + A\bar{B}C$$

$$= \bar{A}((\bar{B}C + BC) + AC) + ABC$$

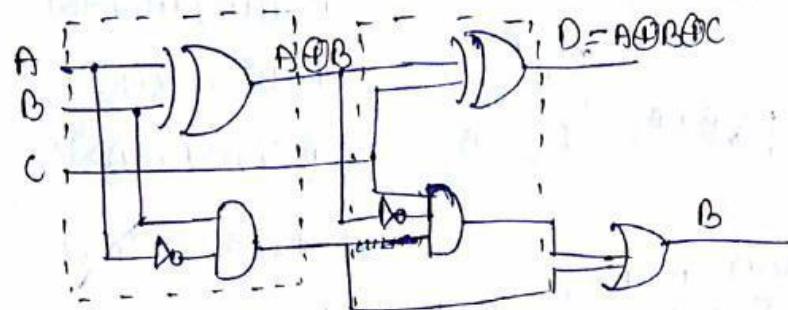
$$= ((\bar{A}\bar{B} + AB) + \bar{A}B)(C + \bar{C})$$

$$= ((A \oplus B) + AB)$$

$$B = ((A \oplus B) + AB)$$

Inputs	Outputs
A B C	D B
0 0 0	0 0
0 0 1	1 1
0 1 0	1 1
0 1 1	0 1
1 0 0	1 0
1 0 1	0 0
1 1 0	0 0
1 1 1	1 1

Logic Diagrams for full Subtractor:

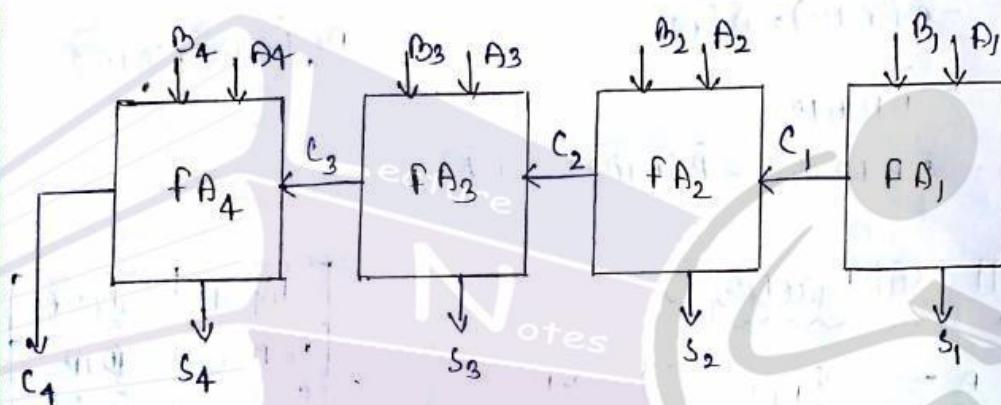


Half Subtractor

Half Subtractor

Applications of full adder:

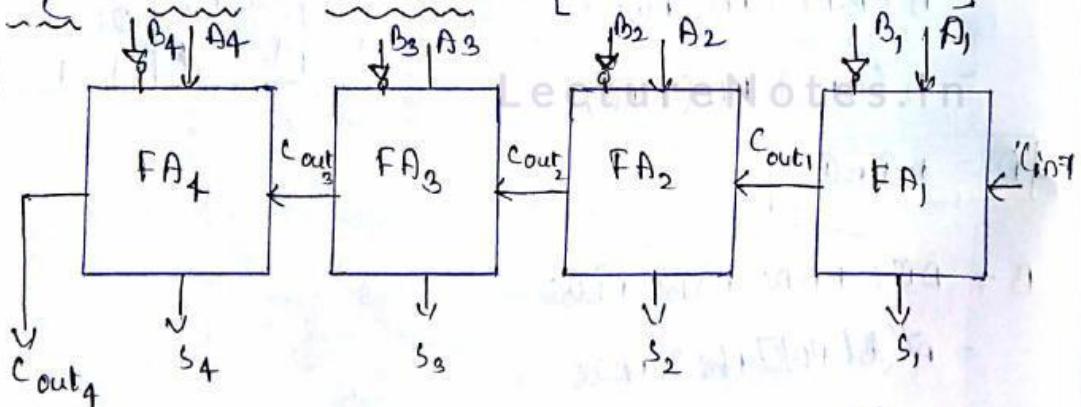
* Binary Parallel Adder:



Disadvantage of the circuit is S_3 & S_4 depend on C_1 , which results in the delaying of o/p.

Binary Parallel

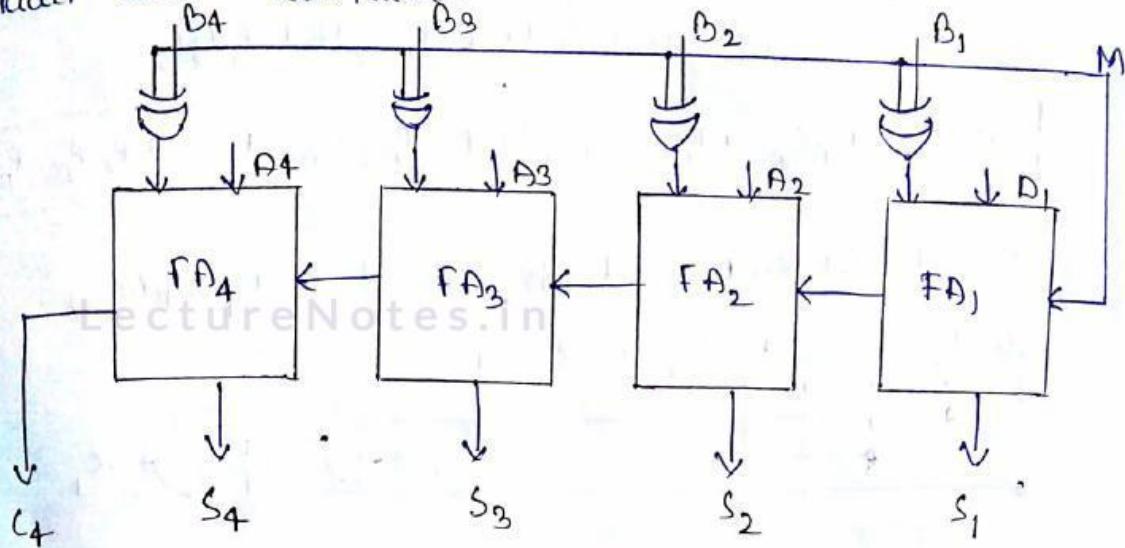
Subtractor: [2's complement method]



Nor gates are used to calculate complement as we are doing 2nd complement subtraction when we add $C_{in}=1$ we will find 2nd complement

Binary parallel Adder and Subtractor:

Out of the 7-Logic gates X-OR acts as both adder and Subtractor



when $M=0 \Rightarrow$ acts as adder

when $M=1 \Rightarrow$ acts as Subtractor

\Rightarrow X-OR has more applications.

when compared to X-NOR gate.

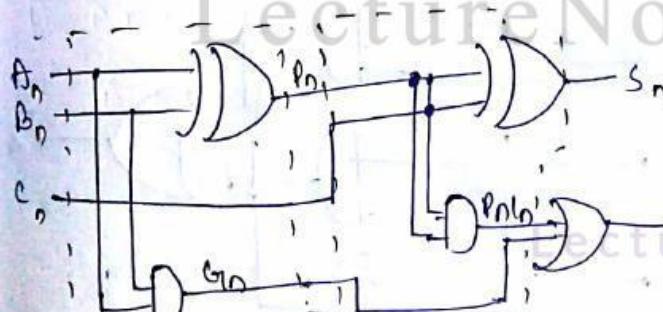
20/2/19

book ahead Carry Adder:-

XOR gate output

$$M=0 \Rightarrow 0 \oplus B = 0 \bar{B} + 1 \cdot B$$

$$\begin{aligned} M=1 \Rightarrow 1 \oplus B &= 0 \cdot \bar{B} + 1 \cdot \bar{B} \\ &= \bar{B} \end{aligned}$$



half adder half adder

$$P_n = A_n \oplus B_n$$

$$S_n = P_n \oplus C_n$$

$$C_{n+1} = B_n + P_n C_n ; \text{ when } n=0 \Rightarrow C_1 = G_{10} + P_0 C_0$$

$$\text{when } n=1 \Rightarrow C_2 = G_1 + P_1 C_1 \Rightarrow G_1 + P_1 (G_{10} + P_0 C_0) = G_1 + P_1 G_{10} + P_1 P_0 C_0$$

$$\text{when } n=2 \Rightarrow C_3 = G_2 + P_2 C_2 = G_2 + P_2 (G_1 + P_1 G_{10} + P_1 P_0 C_0) = G_2 + P_2 G_1 + P_2 P_1 G_{10}$$

$$N=2 \Rightarrow C_3 = G_{12} + P_2 C_2 = G_{12} + P_2 (G_1 + P_1 G_{10} + P_1 P_0 C_0)$$

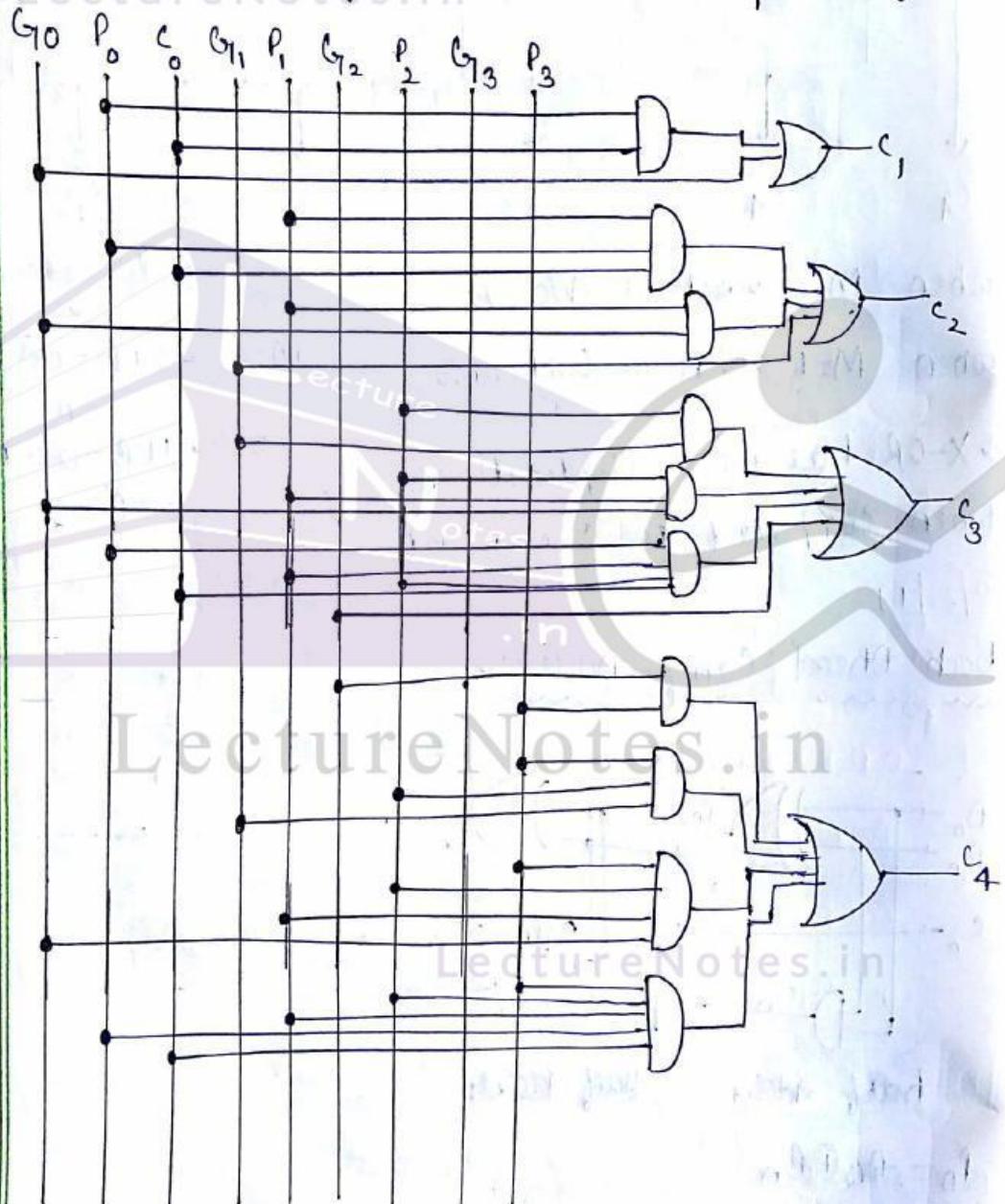
$$= G_{12} + P_2 G_{11} + P_2 P_1 G_{10} + P_2 P_1 P_0 C_0$$

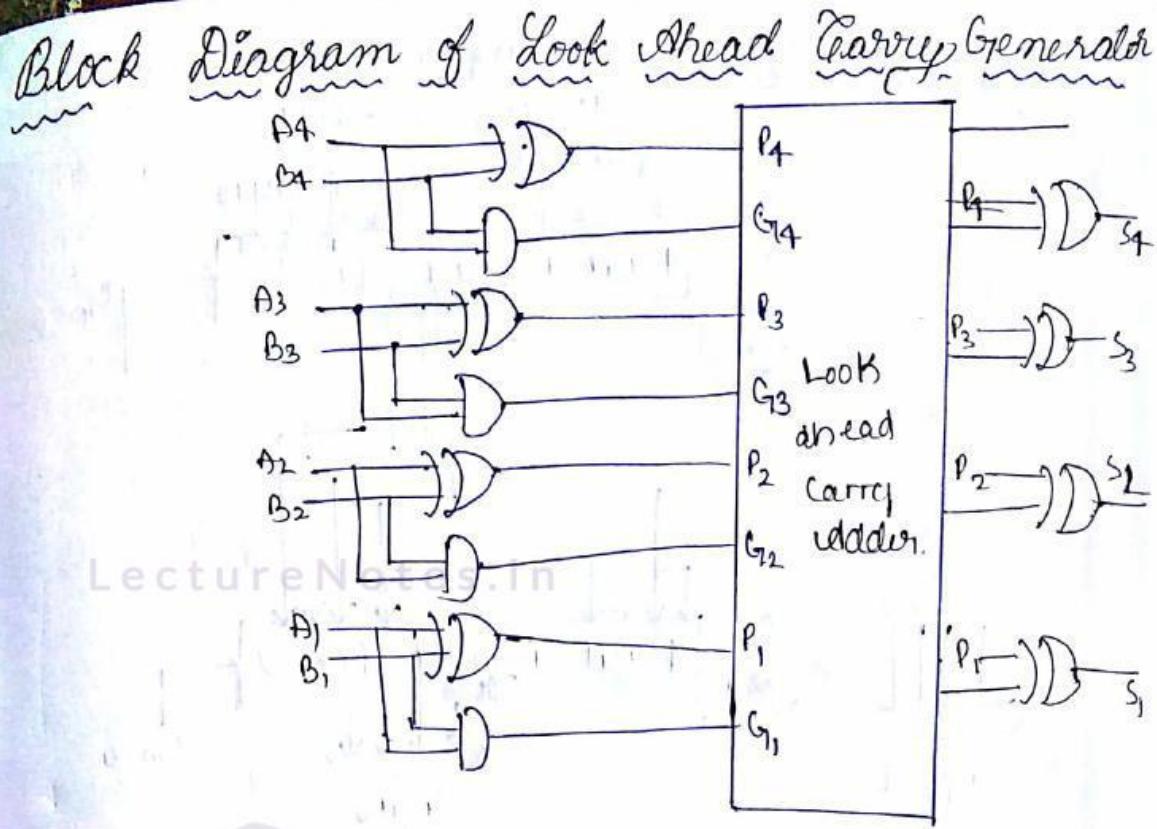
$$N=3 \Rightarrow C_4 = G_{13} + P_3 C_3$$

$$= G_{13} + P_3 G_{12} + P_3 P_2 G_{11} + P_3 P_2 P_1 G_{10} + P_3 P_2 P_1 P_0 C_0$$

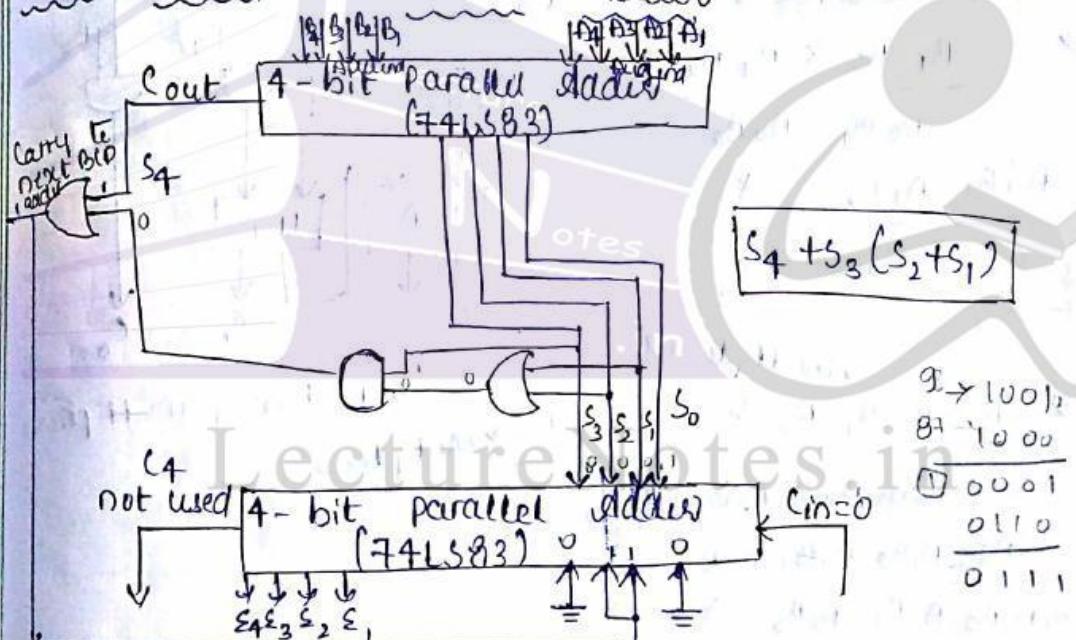
$$C_{n+1} = G_{1n} + P_n G_{1n-1} + P_n P_{n-1} G_{1n-2} + P_n P_{n-1} P_{n-2} G_{1n-3} + \dots + P_n P_{n-1} P_{n-2} \dots P_1 C_0$$

Internal circuit of Look ahead adder.





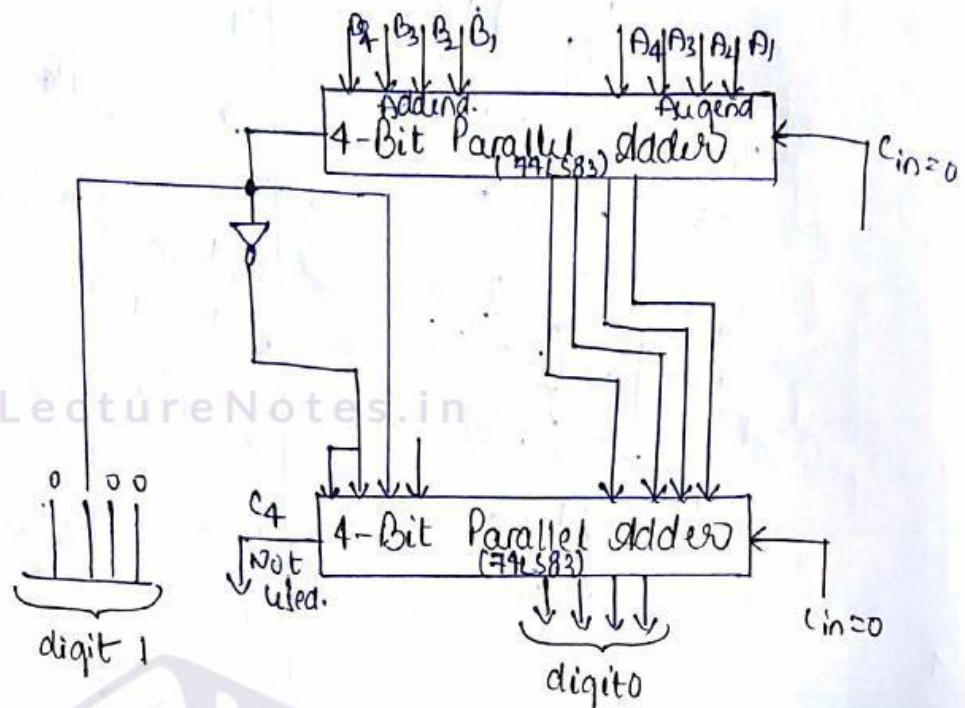
B.C.D Adder (or) Decimal adder



Excess-3 Adder:

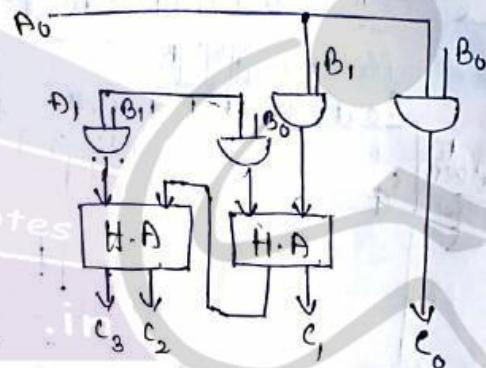
If carry = 1 \rightarrow add 3

If carry = 0 \rightarrow add 13



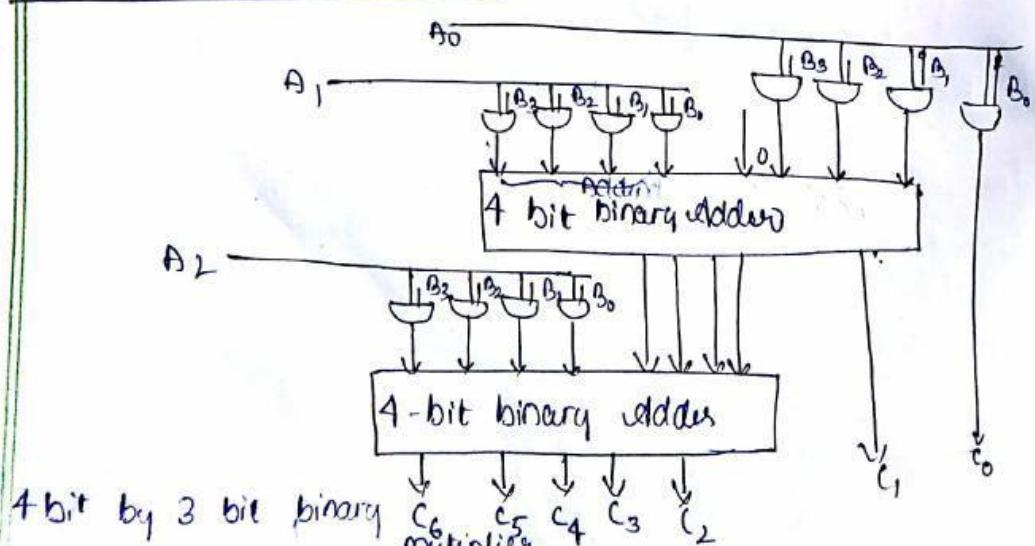
Binary Multiplier:

$$\begin{array}{r} \overline{B_1 \ B_0 \times A_1 \ A_0} \\ \hline A_0 \ B_1 \ A_0 \ B_0 \\ A_1 \ B_1 \ A_1 \ B_0 \quad X \\ \hline c_3 \ c_2 \quad c_1 \quad c_0 \end{array}$$



\Rightarrow Multiplicand.

$$\begin{array}{r} \overline{B_3 \ B_2 \ B_1 \ B_0 \times A_3 \ A_2 \ A_1 \ A_0} \rightarrow \text{2-bit binary multiplier} \\ \hline A_0 B_3 \ A_0 B_2 \ A_0 B_1 \ A_0 B_0 \\ A_1 B_3 \ A_1 B_2 \ A_1 B_1 \ A_1 B_0 \quad X \\ A_2 B_3 \ A_2 B_2 \ A_2 B_1 \ A_2 B_0 \quad X \\ \hline c_6 \ c_5 \ c_4 \ c_3 \ c_2 \ c_1 \ c_0 \end{array}$$



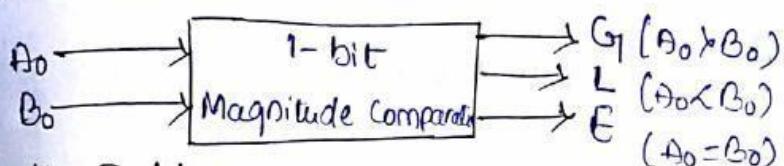
$J \times K$ AND gates are required : $k \rightarrow$ Multiplicands

$(J-1)K$ bit address ; $J \rightarrow$ Multiplier

product from $J+K$ bits

Magnitude Comparator :-

→ A magnitude comparator is a one combinational circuit which is used to compare the magnitude of numbers represented in binary.



Truth Table:-

A ₀	B ₀	G ₁	L	E
0	0	0	0	1
0	1	0	1	0
1	0	1	0	0
1	1	0	0	1

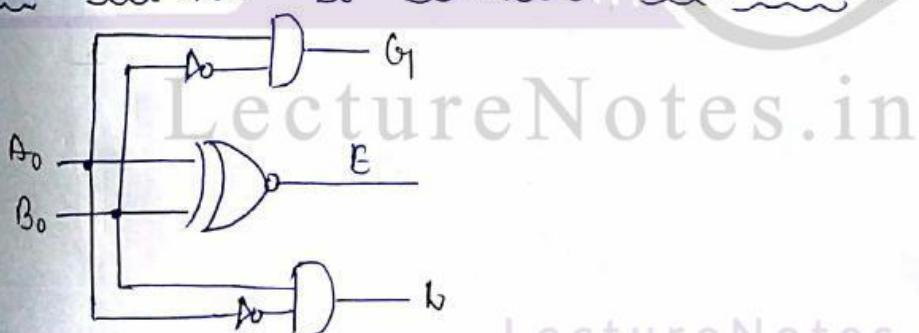
$$G_1 \Rightarrow A_0 \bar{B}_0$$

$$L \Rightarrow \bar{A}_0 B_0$$

$$E \Rightarrow A_0 \oplus B_0 \Rightarrow A_0 \oplus B_0$$

$$E \Rightarrow A_0 \oplus B_0$$

Logic Diagram of Magnitude Comparator :-



* 2-bit Magnitude Comparator:-

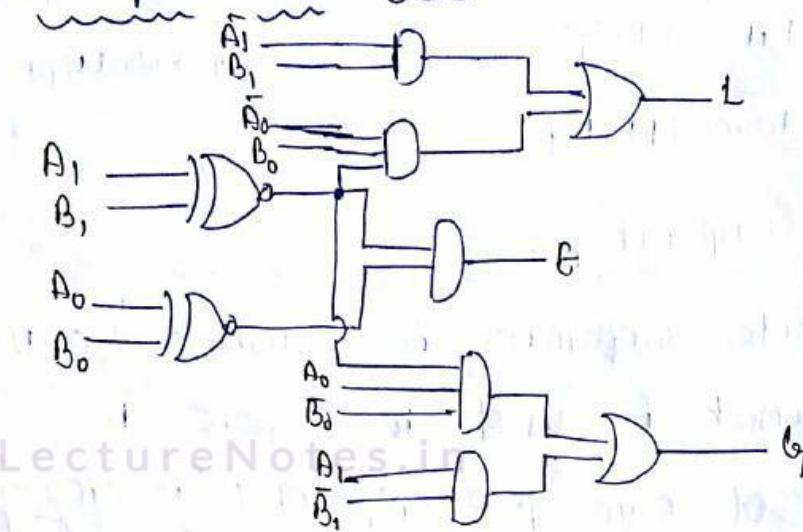
Equations are as follows:-

$$G_1 = A_1 \bar{B}_1 + (A_1 \oplus B_1) A_0 \bar{B}_0$$

$$L = \bar{A}_1 B_1 + (A_1 \oplus B_1) \bar{A}_0 B_0$$

$$E = (A_1 \oplus B_1) \cdot (A_0 \oplus B_0)$$

Logic Diagrams for 2-bit Magnitude Comparator



4-bit Magnitude Comparator

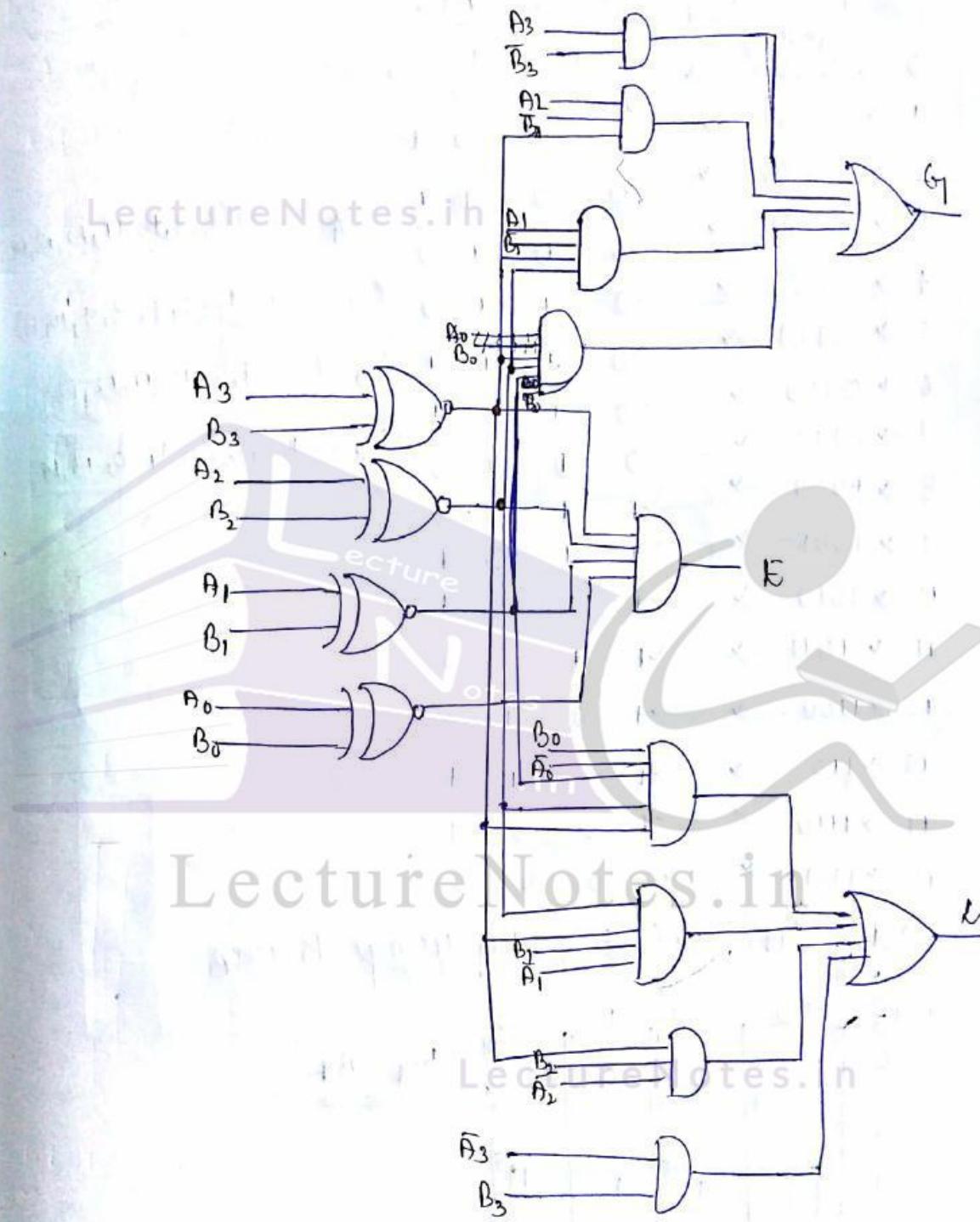
Equations:

$$G = A_3 \bar{B}_3 + (A_3 \oplus B_3) A_2 \bar{B}_2 + (A_3 \oplus B_3) (A_2 \oplus B_2) A_1 \bar{B}_1 + (A_3 \oplus B_3) (A_2 \oplus B_2) (A_1 \oplus B_1) A_0 \bar{B}_0$$

$$L = \bar{A}_3 B_3 + (A_3 \oplus B_3) \bar{A}_2 B_2 + (A_3 \oplus B_3) (A_2 \oplus B_2) \bar{A}_1 B_1 + (A_3 \oplus B_3) (A_2 \oplus B_2) (A_1 \oplus B_1) \bar{A}_0 B_0$$

$$E = \underline{\underline{(A_3 \oplus B_3) (A_2 \oplus B_2) (A_1 \oplus B_1) (A_0 \oplus B_0)}}$$

Logic Diagram for 4-bit Magnitude Comparator



Binary to Gray Code Conversion

	$B_4 B_3 B_2 B_1$	$G_4 + G_3 \quad G_2 \quad G_1$
0	0 0 0 0	0 0 0 0
1	0 0 0 1	0 0 1
2	0 0 1 0	0 1 1
3	0 0 1 1	0 1 0
4	0 1 0 0	1 1 0
5	0 1 0 1	1 1 1
6	0 1 1 0	1 0 1
7	0 1 1 1	1 0 0
8	1 0 0 0	1 0 0
9	1 0 0 1	0 0 1
10	1 0 1 0	0 1 1
11	1 0 1 1	0 1 0
12	1 1 0 0	1 0 1 0
13	1 1 0 1	1 0 1 1
14	1 1 1 0	1 0 0 1
15	1 1 1 1	1 0 0 0

Solve G_4 , G_3 , G_2 & G_1 by using K-map.

$B_4 B_3$	00	01	11	10
00				
01	4	5	7	6
11	1	1	1	1
10	8	9	11	10

$$G_4 = B_4$$

$B_4 B_3$	00	01	11	10
00				
01	4	5	7	6
11	1	1	1	1
10	8	9	11	10

$$G_3 = \bar{B}_4 B_3 + \bar{B}_3 B_4 = \underline{\underline{B_3 \oplus B_4}}$$

$B_2 B_3$	00	01	11	10
00	0	1	1	1
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

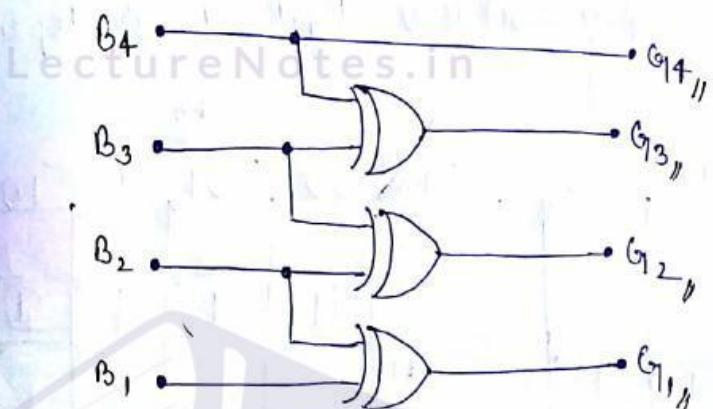
$$G_2 = B_3 \bar{B}_2 + B_2 \bar{B}_3$$

$$= \underline{\underline{B_3 \oplus B_2}}$$

$B_2 B_3$	00	01	11	10
00	0	1	1	1
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$$G_1 = \bar{B}_2 B_1 + B_2 \bar{B}_1$$

$$= \underline{\underline{B_2 \oplus B_1}}$$



Gray Code to Binary Code Converter :-

Input \rightarrow Gray Code ; Output \rightarrow Binary Code.

$$\begin{array}{cccc} G_4 & G_3 & G_2 & G_1 \\ 0 \leftarrow 0 & 0 & 0 & 0 \\ \xrightarrow{0 \leftarrow 0} & \xrightarrow{0 \leftarrow 0} & \xrightarrow{0 \leftarrow 0} & \xrightarrow{0 \leftarrow 0} \\ 1 \leftarrow 0 & 0 & 0 & 1 \\ \xrightarrow{1 \leftarrow 0} & \xrightarrow{0 \leftarrow 0} & \xrightarrow{0 \leftarrow 0} & \xrightarrow{1 \leftarrow 0} \\ 3 \leftarrow 0 & 0 & 1 & 1 \\ \xrightarrow{3 \leftarrow 0} & \xrightarrow{0 \leftarrow 1} & \xrightarrow{1 \leftarrow 1} & \xrightarrow{1 \leftarrow 1} \\ 2 \leftarrow 0 & 0 & 1 & 0 \\ \xrightarrow{2 \leftarrow 0} & \xrightarrow{0 \leftarrow 1} & \xrightarrow{1 \leftarrow 0} & \xrightarrow{0 \leftarrow 0} \\ 1 \leftarrow 0 & 1 & 1 & 0 \\ \xrightarrow{1 \leftarrow 0} & \xrightarrow{1 \leftarrow 1} & \xrightarrow{0 \leftarrow 0} & \xrightarrow{0 \leftarrow 1} \\ 7 \leftarrow 0 & 1 & 1 & 1 \\ \xrightarrow{7 \leftarrow 0} & \xrightarrow{1 \leftarrow 1} & \xrightarrow{1 \leftarrow 1} & \xrightarrow{1 \leftarrow 1} \\ 5 \leftarrow 0 & 1 & 0 & 1 \\ \xrightarrow{5 \leftarrow 0} & \xrightarrow{1 \leftarrow 0} & \xrightarrow{0 \leftarrow 1} & \xrightarrow{1 \leftarrow 0} \\ 4 \leftarrow 0 & 1 & 0 & 0 \\ \xrightarrow{4 \leftarrow 0} & \xrightarrow{1 \leftarrow 0} & \xrightarrow{0 \leftarrow 0} & \xrightarrow{0 \leftarrow 0} \\ 12 \leftarrow 1 & 1 & 0 & 0 \\ \xrightarrow{12 \leftarrow 1} & \xrightarrow{1 \leftarrow 1} & \xrightarrow{0 \leftarrow 0} & \xrightarrow{0 \leftarrow 0} \\ 13 \leftarrow 1 & 1 & 0 & 1 \\ \xrightarrow{13 \leftarrow 1} & \xrightarrow{1 \leftarrow 1} & \xrightarrow{0 \leftarrow 0} & \xrightarrow{1 \leftarrow 1} \\ 15 \leftarrow 1 & 1 & 1 & 1 \\ \xrightarrow{15 \leftarrow 1} & \xrightarrow{1 \leftarrow 1} & \xrightarrow{1 \leftarrow 1} & \xrightarrow{1 \leftarrow 1} \\ 14 \leftarrow 1 & 1 & 1 & 0 \\ \xrightarrow{14 \leftarrow 1} & \xrightarrow{1 \leftarrow 1} & \xrightarrow{1 \leftarrow 0} & \xrightarrow{0 \leftarrow 1} \\ 10 \leftarrow 1 & 0 & 1 & 0 \\ \xrightarrow{10 \leftarrow 1} & \xrightarrow{0 \leftarrow 1} & \xrightarrow{1 \leftarrow 0} & \xrightarrow{0 \leftarrow 0} \\ 4 \leftarrow 1 & 0 & 1 & 1 \\ \xrightarrow{4 \leftarrow 1} & \xrightarrow{0 \leftarrow 1} & \xrightarrow{1 \leftarrow 1} & \xrightarrow{1 \leftarrow 1} \end{array}$$

$$g_{41} \ 0 \ 01 \rightarrow 1 \ 1 \ 1 \ 0$$

$$g_{41} \ 0 \ 00 \rightarrow 1 \ 1 \ 1 \ 1$$

$$B_4 = \Sigma m(8, 9, 10, 11, 12, 13, 14, 15)$$

$$B_3 = \Sigma m(4, 5, 6, 7, 12, 13, 14, 15) \Rightarrow B_3 = \Sigma m(4, 5, 6, 7, 8, 9, 10, 11)$$

$$\underline{B_2 = \Sigma m(2, 3, 6, 7, 10, 11, 14, 15)} \Rightarrow B_2 = \Sigma m(2, 3, 4, 5, 8, 9, 14, 15)$$

$$B_1 = \Sigma m(1, 3, 5, 7, 9, 11, 13, 14) \Rightarrow B_1 = \Sigma m(1, 2, 4, 7, 8, 11, 13, 14)$$

Solving the expression for B_4, B_3, B_2, B_1 by using K-map

		<u>B_4</u>				
		00	01	11	10	
<u>$G_1 G_2$</u>		00	0	1	3	4
00		0	0	0	0	
01		1	1	1	1	
11		1	1	1	1	
10		1	1	1	1	

		<u>B_3</u>				
		00	01	11	10	
<u>$G_1 G_2 G_3$</u>		00	0	1	3	4
00		0	0	0	0	
01		1	1	1	1	
11		1	1	1	1	
10		1	1	1	1	

$$\underline{B_4 = G_{14}}$$

$$B_3 = \bar{G}_{14} G_{13} + G_{14} \bar{G}_{13} = G_{14} \oplus G_{13}$$

		<u>B_2</u>				
		00	01	11	10	
<u>$G_1 G_2 G_3 G_4$</u>		00	0	1	3	4
00		0	0	0	0	
01		1	1	1	1	
11		1	1	1	1	
10		1	1	1	1	

		<u>B_2</u>				
		00	01	11	10	
<u>$G_1 G_2 G_3 G_4$</u>		00	0	1	3	4
00		0	0	0	0	
01		1	1	1	1	
11		1	1	1	1	
10		1	1	1	1	

$$B_2 = \bar{G}_{14} \bar{G}_{13} G_{12} + \bar{G}_{14} G_{13} \bar{G}_{12} + G_{14} \bar{G}_{13} \bar{G}_{12} + G_{14} G_{13} G_{12}$$

$$= \bar{G}_{14} (\bar{G}_{13} G_{12} + G_{13} \bar{G}_{12}) + G_{14} (\bar{G}_{13} \bar{G}_{12} + G_{13} G_{12})$$

$$= \bar{G}_{14} (G_{13} \oplus G_{12}) + G_{14} (\bar{G}_{13} \oplus G_{12})$$

$$= G_{14} \oplus G_{13} \oplus G_{12}$$

$$\underline{B_2 = B_2 \oplus G_{12}}$$

$$B_1 = \bar{G}_{14} \bar{G}_{13} \bar{G}_{12} G_{11} + \bar{G}_{14} \bar{G}_{13} G_{12} \bar{G}_{11} + \bar{G}_{14} G_{13} \bar{G}_{12} \bar{G}_{11} + \bar{G}_{14} G_{13} G_{12} G_{11}$$

$$+ G_{14} \bar{G}_{13} \bar{G}_{12} G_{11} + G_{14} G_{13} G_{12} \bar{G}_{11} + G_{14} \bar{G}_{13} G_{12} \bar{G}_{11} + G_{14} \bar{G}_{13} G_{12} G_{11}$$

$$\Rightarrow \bar{G}_{14} \bar{G}_{13} (\bar{G}_{12} G_{11} + G_{12} \bar{G}_{11}) + \bar{G}_{14} G_{13} (\bar{G}_{12} \bar{G}_{11} + G_{12} G_{11}) + G_{14} \bar{G}_{13} (\bar{G}_{12} G_{11} + G_{12} \bar{G}_{11}) \\ + G_{14} G_{13} (\bar{G}_{12} \bar{G}_{11} + G_{12} G_{11})$$

$$\Rightarrow \underbrace{\bar{G}_{14} \bar{G}_{13} (\bar{G}_{12} \oplus G_{11})}_{(G_2 \oplus G_1)} + \underbrace{\bar{G}_{14} G_{13} (\bar{G}_{12} \oplus \bar{G}_{11})}_{(G_2 \oplus \bar{G}_1)} + \underbrace{G_{14} \bar{G}_{13} (G_{12} \oplus G_{11})}_{(G_2 \oplus G_1)} + \underbrace{G_{14} G_{13} (G_{12} \oplus \bar{G}_{11})}_{(G_2 \oplus \bar{G}_1)}$$

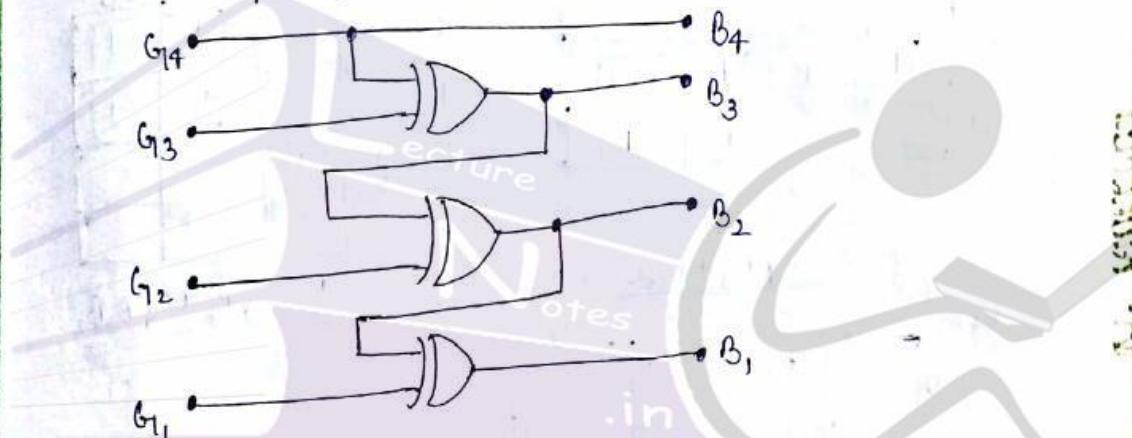
$$\Rightarrow (G_2 \oplus G_1) (\bar{G}_{14} \oplus G_{13}) + (G_2 \oplus \bar{G}_1) (G_{14} \oplus G_{13}) = G_{14} \oplus G_{13} \oplus G_2 \oplus G_1$$

LectureNotes.in

$$= B_3 \oplus B_2 \oplus G_1$$

$$B_1 = B_2 \oplus G_1$$

LOGIC DIAGRAM FOR GRAY TO BINARY CONVERTOR.



4-Bit Binary to B-C-D Code Converter.

Binary Code B-C-D Code

B₄ B₃ B₂ B₁ A B C D E

0 → 0 0 0 → 0 0 0 0 0

1 → 0 0 1 → 0 0 0 0 1

2 → 0 1 0 → 0 0 0 1 0

3 → 0 1 1 → 0 0 0 0 1

4 → 1 0 0 → 0 0 1 0 0

5 → 1 0 1 → 0 0 1 0 1

6 → 1 1 0 → 0 0 1 1 0

7 → 1 1 1 → 0 0 0 1 1

8 → 1 0 0 → 0 1 0 0 0

9 → 1 0 1 → 0 1 0 0 1

10 → 1 1 0 → 1 0 0 0 0

11 → 1 0 1 1 → 1 0 0 0 1

12 → 1 1 0 0 → 1 0 0 1 0

13 → 1 1 0 1 → 1 0 0 1 1

14 → 1 1 1 0 → 1 0 1 0 0

15 → 1 1 1 1 → 1 0 1 0 1

$$A = \Sigma m(10, 11, 12, 13, 14, 15)$$

$$B = \Sigma m(8, 9)$$

$$C = \Sigma m(4, 5, 6, 7, 14, 15)$$

$$D = \Sigma m(2, 3, 6, 7, 12, 13)$$

$$E = \Sigma m(1, 3, 5, 7, 9, 11, 13, 15)$$

0	0	0	0	0
1	0	0	0	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0
8	1	0	0	0
9	1	0	0	1
10	1	1	0	0
11	0	1	1	0
12	1	0	1	1
13	1	1	0	1
14	1	1	1	1
15	1	1	1	0

$$A = B_4 \bar{B}_3 + B_4 B_2 ; \quad B = B_4 \bar{B}_3 \bar{B}_2 ; \quad C = \bar{B}_4 B_3 + B_3 B_2$$

$$D = \bar{B}_4 B_2 + B_4 B_3 \bar{B}_2 ; \quad E = B_1$$

$B_2 B_3$	00	01	11	10
$B_4 B_1$	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	0	1	1	1

$$A = B_4 B_3 + B_4 B_2$$

$B_2 B_3$	00	01	11	10
$B_4 B_1$	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

$$B = B_4 \bar{B}_3 \bar{B}_2$$

$B_2 B_3$	00	01	11	10
$B_4 B_1$	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

$$C = \bar{B}_4 B_3 + B_3 B_2$$

$B_2 B_3$	00	01	11	10
$B_4 B_1$	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	1	1	1	1
10	1	1	1	1

$$D = \bar{B}_4 B_2 + B_4 B_3 \bar{B}_2$$

$B_2 B_3$	00	01	11	10
$B_4 B_1$	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$E = B_1$$

25/2/19

4-Bit B.C.D to Excess-3 Code Conversion

4-Bit B.C.D

$B_4 B_3 B_2 B_1$

0 0 0 0

0 0 0 1

0 0 1 0

0 0 1 1

0 0 0 0

0 1 0 1

0 1 1 0

1 0 1 0

0 1 1 0

$X_4 X_3 X_2 X_1$

0 0 1 1

0 1 0 0

0 1 0 1

0 1 1 0

0 1 1 1

0 0 0 0

1 0 0 1

1 0 1 0

0 1 0 0

0 1 1 0

1000 → 1011

1001 → 1100

$$x_4 = \Sigma m(5, 6, 7, 8, 9) + \bar{m}(10, 11, 12, 13, 14, 15)$$

$$x_3 = \Sigma m(1, 2, 3, 4, 9) + \bar{m}(10, 11, 12, 13, 14, 15)$$

$$x_2 = \Sigma m(0, 3, 4, 7, 8) + \bar{m}(10, 11, 12, 13, 14, 15)$$

$$x_1 = \Sigma m(0, 2, 4, 6, 8) + \bar{m}(10, 11, 12, 13, 14, 15)$$

	B_4	B_3	B_2	B_1	00	01	11	10
B_4	00	00	00	00	0	1	1	1
01	01	01	01	01	1	1	1	1
11	11	X	X	X	X	X	X	X
10	10	1	1	X	X	X	X	X

	B_4	B_3	B_2	B_1	00	01	11	10
B_4	00	00	00	00	0	1	1	1
01	01	01	01	01	1	1	1	1
11	11	X	X	X	X	X	X	X
10	10	1	1	X	X	X	X	X

$$X = B_4 + B_3 B_1 + B_3 B_2$$

	B_4	B_3	B_2	B_1	00	01	11	10
B_4	00	00	00	00	0	1	1	1
10	10	1	1	1	1	5	7	6
11	11	X	X	X	X	X	X	X
10	10	1	1	X	X	X	X	X

$$X_2 = \bar{B}_2 \bar{B}_1 + B_2 B_1$$

$$= B_2 \oplus B_1$$

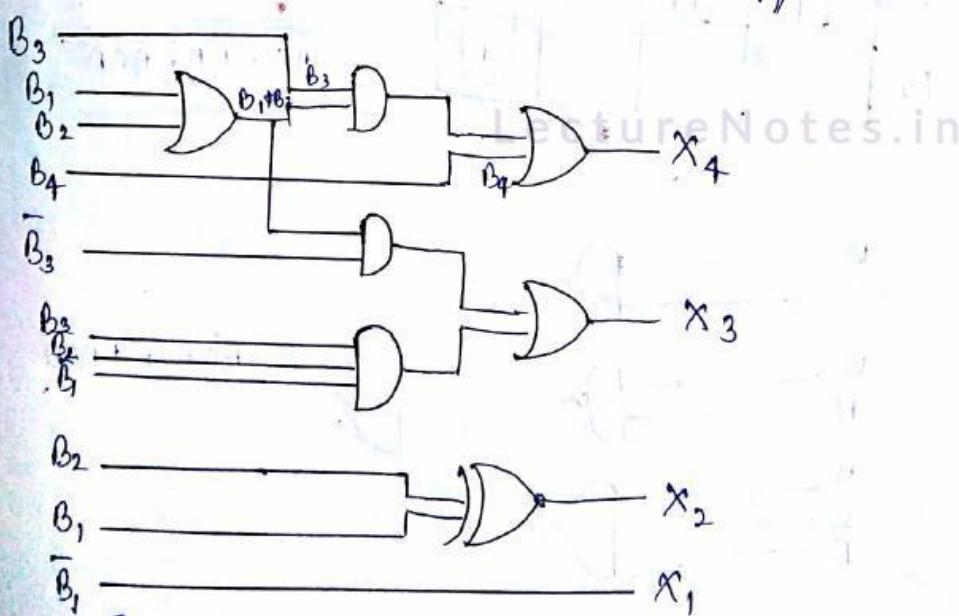
$$X_3 = \bar{B}_3 B_1 + \bar{B}_3 B_2 + B_3 \bar{B}_2 \bar{B}_1$$

	B_4	B_3	B_2	B_1	00	01	11	10
B_4	00	00	00	00	0	1	1	1
10	10	1	1	1	1	5	7	6
11	11	X	X	X	X	X	X	X
10	10	1	1	X	X	X	X	X

$$X_1 = \bar{B}_2 \bar{B}_1 + B_2 \bar{B}_1$$

$$= \bar{B}_1 (\bar{B}_2 + B_2)$$

$$= \bar{B}_1 Y$$



Logic Diagram of 4-bit B-C-D to XS-3

Design an S.O.P. circuit to detect the decimal numbers 5 through 12 in a 4-bit Gray code

Input

Solution:

Decimal	Binary	Gray Code	Output 'f'
0	0 0 0 0	0 0 0 0	0
1	0 0 0 1	0 0 0 1	0
2	0 0 1 0	0 0 1 1	0
3	0 0 1 1	0 0 1 0	0
4	0 1 0 0	0 1 1 0	0
5	0 1 0 1	0 1 1 1	1
6	0 1 1 0	0 1 0 1	1
7	0 1 1 1	0 1 0 0	1
8	1 0 0 0	1 1 0 0	1
9	1 0 0 1	1 1 0 1	1
10	1 0 1 0	1 1 1 1	1
11	1 0 1 1	1 1 1 0	1
12	1 1 0 0	1 0 1 0	1
13	1 1 0 1	1 0 1 1	0
14	1 1 1 0	1 0 0 1	0
15	1 1 1 1	1 0 0 0	0

$$f = \Sigma m(5, 6, 7, 8, 9, 10, 11, 12)$$

$$f = \Sigma m(7, 5, 4, 12, 13, 15, 14, 10) = \Sigma m(5, 4, 7, 10, 12, 13, 14, 15)$$

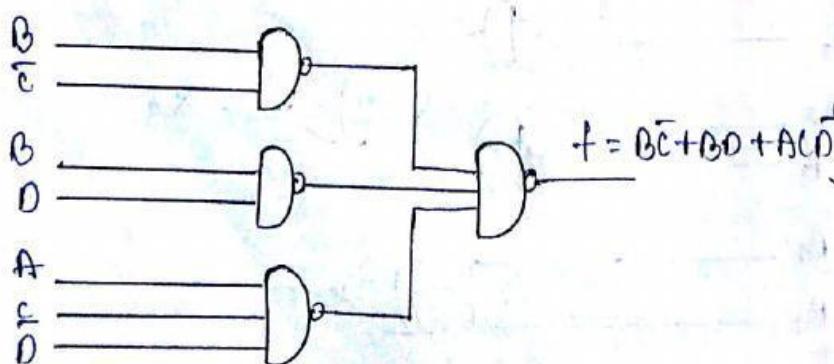
CD	00	01	11	10
AB	00			
	00			
00				
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

$$f = \underline{B\bar{C} + B\bar{D} + A\bar{C}\bar{D}}$$

$$f = \underline{\bar{B}\bar{C} + B\bar{D} + A\bar{C}\bar{D}}$$

$$= \underline{\bar{B}\bar{C} \cdot \bar{B}\bar{D} \cdot A\bar{C}\bar{D}}$$

$$= \underline{\bar{B}\bar{C} \cdot \bar{B}\bar{D} \cdot A\bar{C}\bar{D}}$$



4-Bit B.C.D to Gray Code Conversion

4-Bit B.C.D

$B_4 \ B_3 \ B_2 \ B_1$

0 0 0 0

0 0 0 1

0 0 1 0

0 0 1 1

0 1 0 0

0 1 0 1

0 1 1 0

0 1 1 1

1 0 0 0

1 0 0 1

Gray Code

$G_{14} \ G_{13} \ G_{12} \ G_{11}$

0 0 0 0

0 0 0 1

0 0 1 1

0 0 1 0

0 1 1 0

0 1 1 1

0 1 0 1

0 1 0 0

1 1 1 0

1 1 0 1

$$G_{14} = \Sigma m(8, 9) + d(10, 11, 12, 13, 14, 15)$$

$$G_{13} = \Sigma m(4, 5, 6, 7, 8, 9) + d(10, 11, 12, 13, 14, 15)$$

$$G_{12} = \Sigma m(2, 3, 4, 5) + d(10, 11, 12, 13, 14, 15)$$

$$G_{11} = \Sigma m(1, 2, 5, 6, 9)$$

$$+ d(10, 11, 12, 13, 14, 15)$$

	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	X	X	X	X
10	1	1	X	X

	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	X	X	X	X
10	1	1	X	X

	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	X	X	X	X
10	1	1	X	X

	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	X	X	X	X
10	1	1	X	X

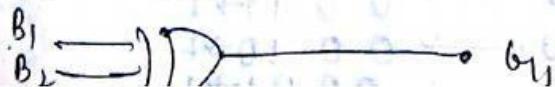
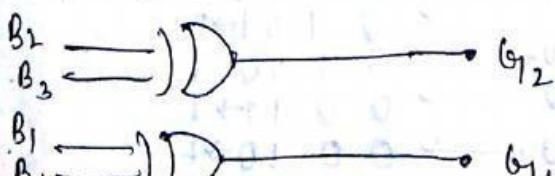
$$G_{14} = B_4$$

$$G_{12} = B_3 \oplus B_2$$

$$G_{13} = B_3 + B_4$$

$$G_{11} = \bar{B}_2 B_1 + B_2 \bar{B}_1$$

$$G_{10} = B_1 \oplus B_2$$

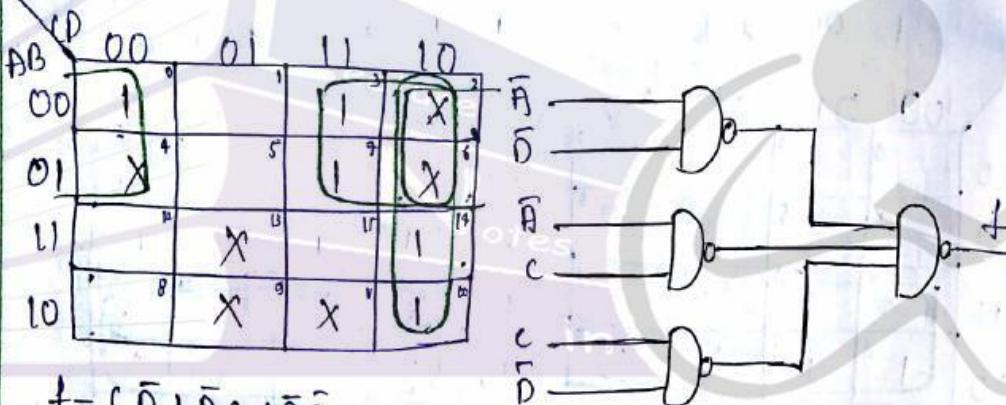


Design an S.O.P circuit to detect the decimal numbers 0, 2, 4, 6, 8 in a 5211 B.C.D code input.

Solution:

Decimal	Input	Output
0	00 00	→ 1
1	00 01	→ 0
2	00 11	→ 1
3	01 01	→ 0
4	01 11	→ 1
5	10 00	→ 0
6	10 10	→ 1
7	11 00	→ 0
8	11 10	→ 1
9	11 11	→ 0

$$f = \sum m(0, 3, 7, 10, 14) + d(2, 4, 6, 9, 11, 13)$$



Design a combinational circuit to produce 2's complement of 4-bit Binary number.

Solution:

Decimal	Input	Output [2's complement]
0	0 0 0 0	0 0 0 0 → 0
1	0 0 0 1	1 1 1 1 → 15
2	0 0 1 0	1 1 1 0 → 14
3	0 0 1 1	1 1 0 1 → 13
4	0 1 0 0	1 1 0 0 → 12
5	0 1 0 1	1 0 1 1 → 11
6	0 1 1 0	1 0 1 0 → 10
7	0 1 1 1	1 0 0 1 → 9
8	1 0 0 0	1 0 0 0 → 8
9	1 0 0 1	0 1 1 1 → 7
10	1 0 1 0	0 1 1 0 → 6
11	1 0 1 1	0 1 0 1 → 5
12	1 1 0 0	0 1 0 0 → 4
13	1 0 1 0	0 0 1 1 → 3
14	1 1 1 0	0 0 1 0 → 2
15	1 1 1 1	0 0 0 1 → 1

$$T_4 = \sum m(1, 2, 3, 4, 5, 6, 7, 8)$$

$$T_3 = \sum m(1, 2, 3, 4, 9, 10, 11, 12)$$

$$T_2 = \sum m(1, 2, 5, 6, 9, 10, 13, 14)$$

$$T_1 = \sum m(1, 3, 5, 7, 9, 11, 13, 15)$$

$B_4\bar{B}_3$	$\bar{B}_2\bar{B}_1$	00	01	11	10
B_4B_3		00	01	11	10
$\bar{B}_4\bar{B}_1$		00	01	11	10
00	1	1	1	1	1
01	1	1	1	1	1
11	1	1	1	1	1
10	1	1	1	1	1

$B_4\bar{B}_3$	$\bar{B}_2\bar{B}_1$	00	01	11	10
B_4B_3		00	01	11	10
$\bar{B}_4\bar{B}_1$		00	01	11	10
00	1	1	1	1	1
01	1	1	1	1	1
11	1	1	1	1	1
10	1	1	1	1	1

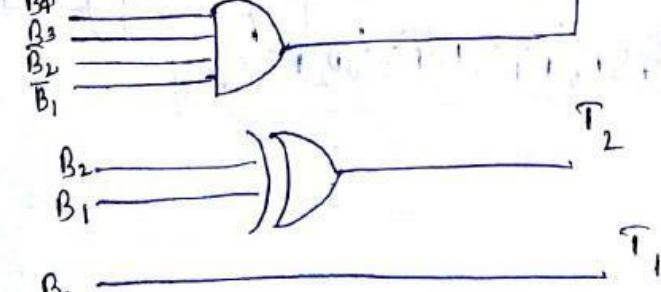
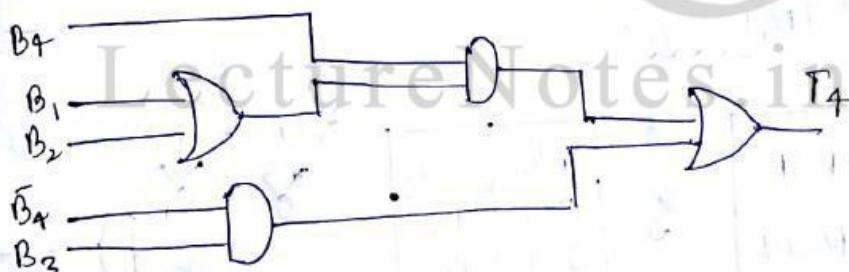
$$T_4 = B_4\bar{B}_3\bar{B}_2\bar{B}_1 + \bar{B}_4B_3 + \bar{B}_4\bar{B}_1 + \bar{B}_4B_2$$

$$= B_4(\bar{B}_2 + B_1) + \bar{B}_4B_3$$

$B_4\bar{B}_3$	$\bar{B}_2\bar{B}_1$	00	01	11	10
B_4B_3		00	01	11	10
$\bar{B}_4\bar{B}_1$		00	01	11	10
00	1	1	1	1	1
01	1	1	1	1	1
11	1	1	1	1	1
10	1	1	1	1	1

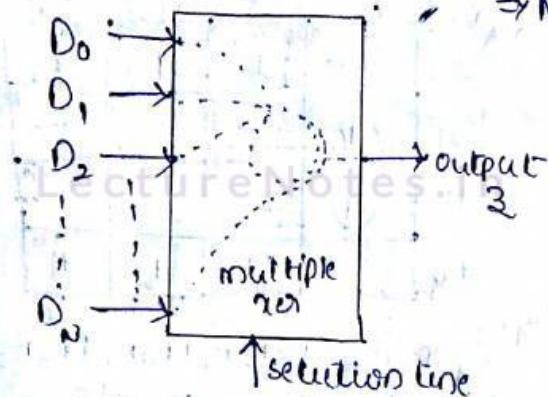
$B_4\bar{B}_3$	$\bar{B}_2\bar{B}_1$	00	01	11	10
B_4B_3		00	01	11	10
$\bar{B}_4\bar{B}_1$		00	01	11	10
00	1	1	1	1	1
01	1	1	1	1	1
11	1	1	1	1	1
10	1	1	1	1	1

$$T_2 = \bar{B}_2\bar{B}_1 + B_2\bar{B}_1 \\ = B_2 \oplus B_1$$



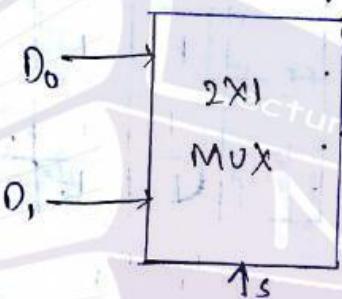
Multiplexers:

→ A multiplexer is a combinational circuit that accepts several data inputs and select only one of them as the output depending upon the selection lines
 ⇒ 2^D i/p lines; n selection lines, 1 o/p line.



⇒ Multiplexer is data selector

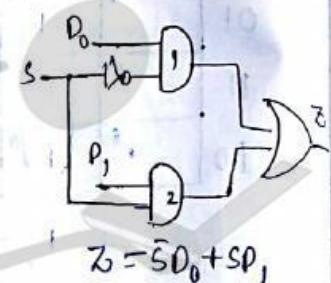
2X1 Multiplexer:



Fig(a) Block Diagram

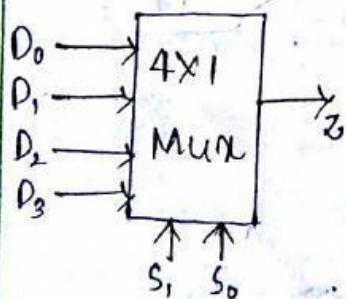
S	Z
0	D_0
1	D_1

Fig(b) Truth Table



Logic Diagram

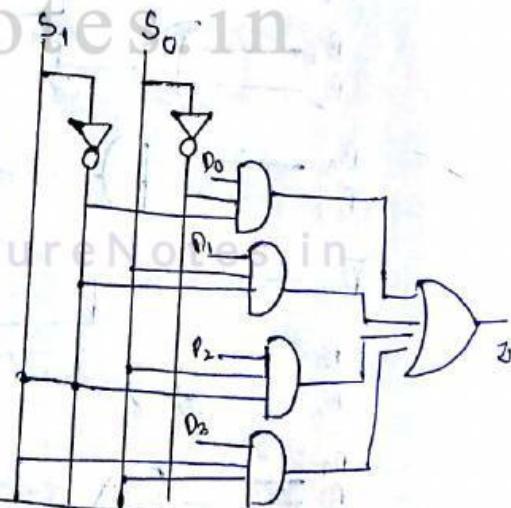
4X1 Multiplexer:



Fig(c) Block Diagram

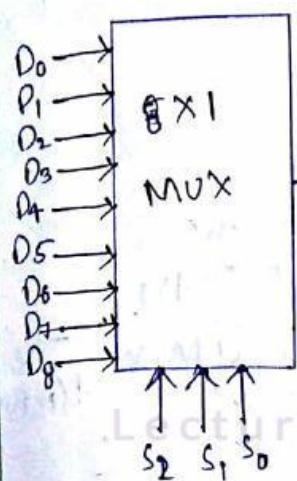
$S_1\ S_0$	Z
0 0	D_0
0 1	D_1
1 0	D_2
1 1	D_3

Fig(d) Truth Table

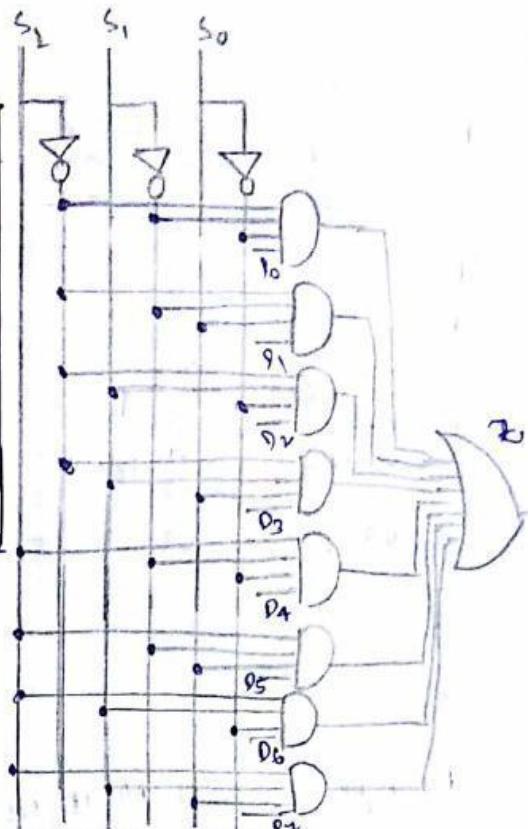


$$Z = \bar{S}_1 \bar{S}_0 D_0 + S_1 \bar{S}_0 D_1 + S_1 S_0 D_2 + S_1 S_0 D_3$$

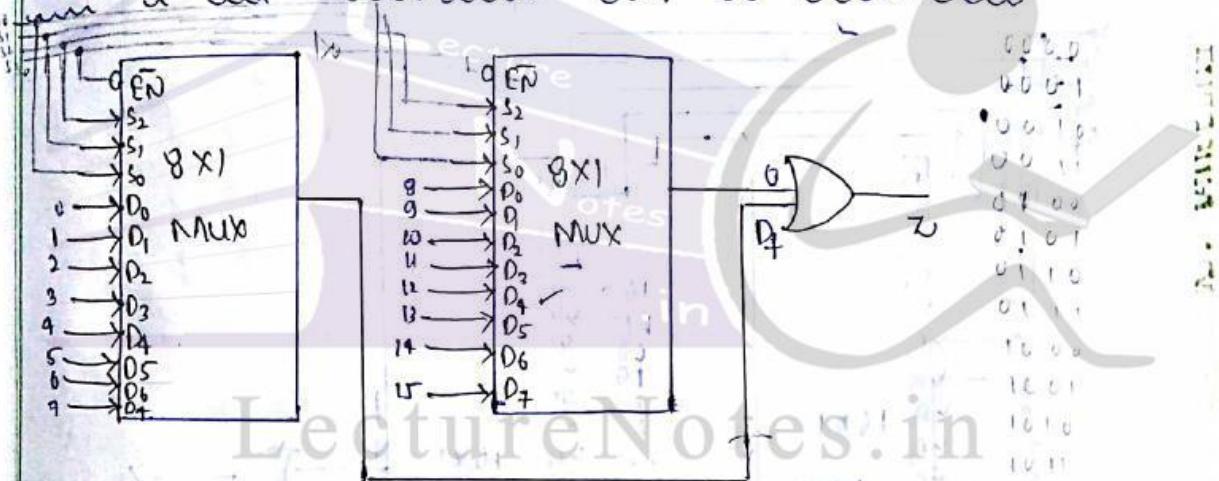
8x1 Multiplexer



S_2, S_1, S_0	Z
0 0 0	D_0
0 0 1	D_1
0 1 0	D_2
0 1 1	D_3
1 0 0	D_4
1 0 1	D_5
0 1 0	D_6
1 1 1	D_7



Design of 16x1 Multiplexer using 8x1 Multiplexers



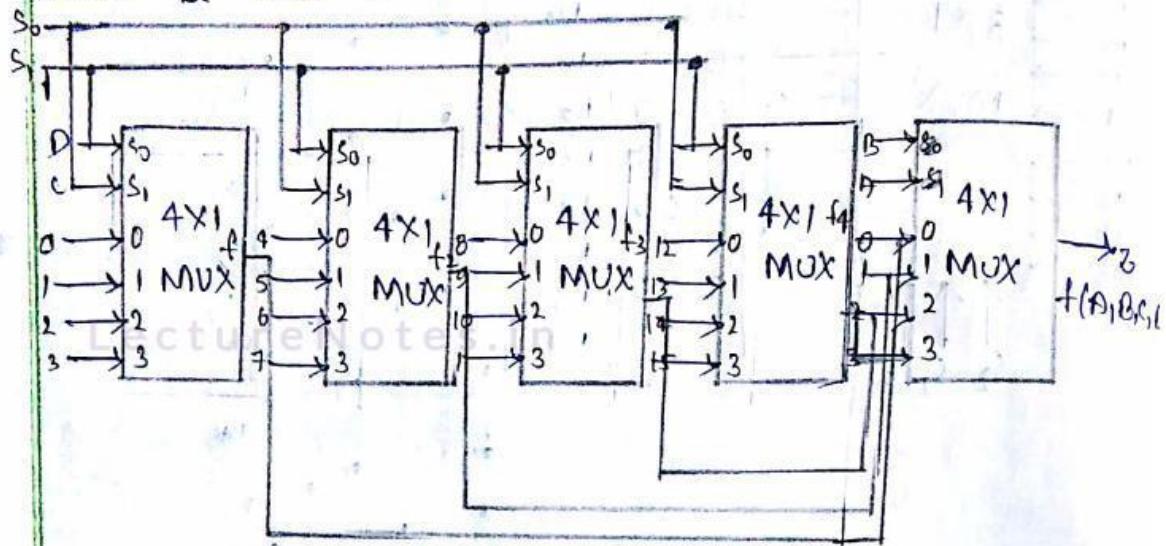
Design of 32x1 Multiplexer using 16x1 multiplexer

① 00 ② 10

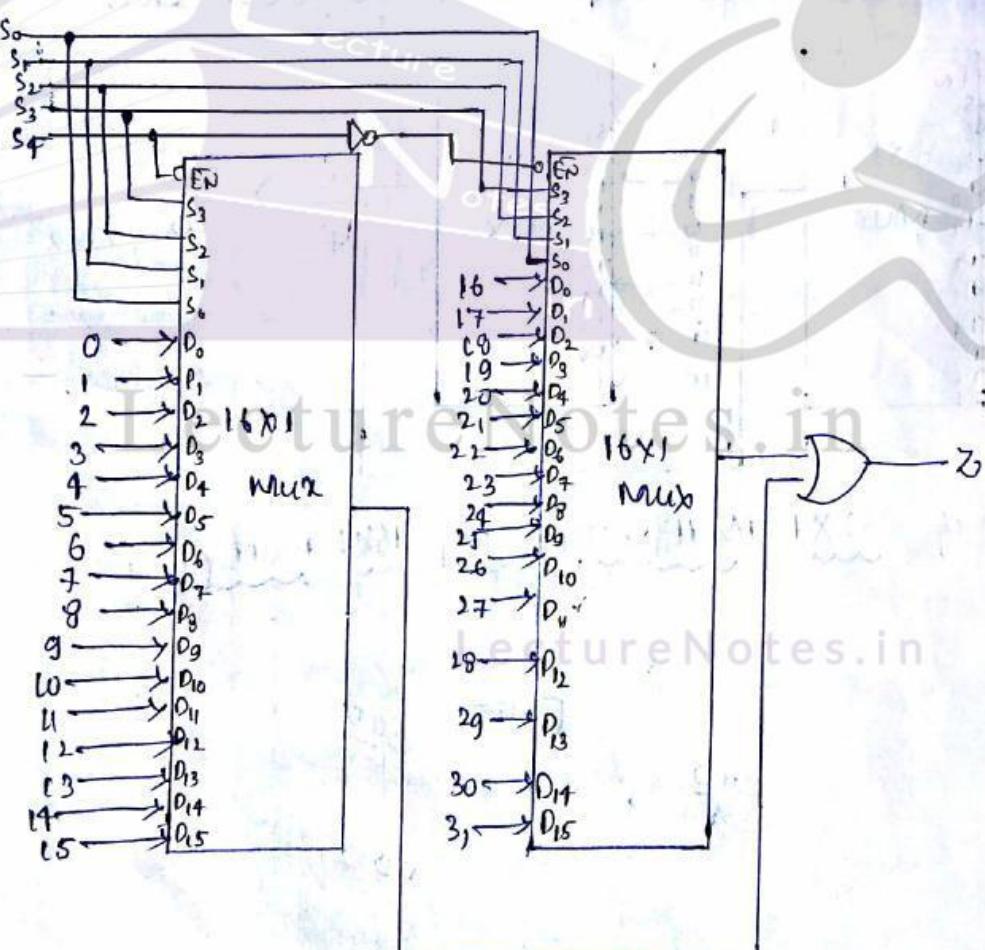
③ ④ 00
S₃ S₂ S₁

100

Design of 16×1 MUX using 4×1 MUX's

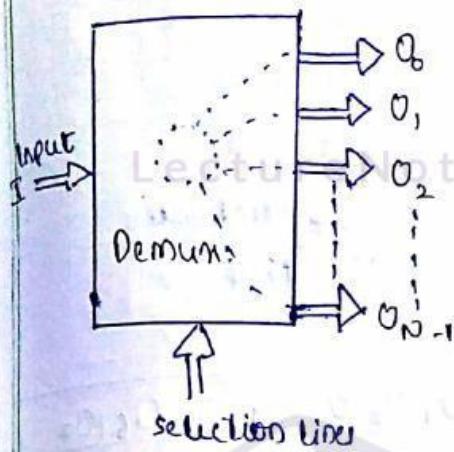


Design of 32×1 Multiplier using 16×1 Multipliers

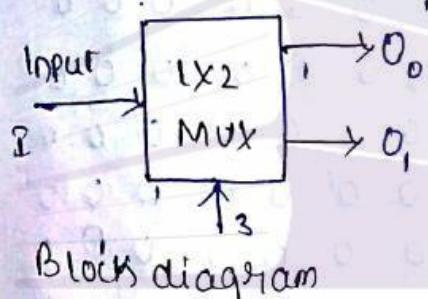


Demultiplexer (Data distributor):

- ⇒ Demultiplexer is also known as Data Distributor.
- ⇒ It is a reverse operation of multiplexer.
- ⇒ 1 input line and 2^P output lines.
- ⇒ P selection lines.

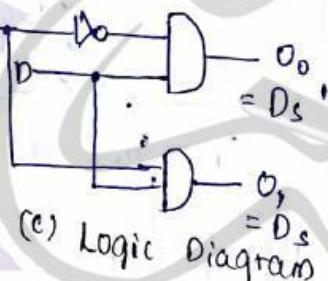


1 line to 2-line Demultiplexer:

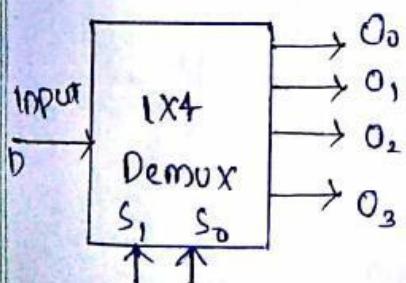


S	O
0	O ₀
1	O ₁

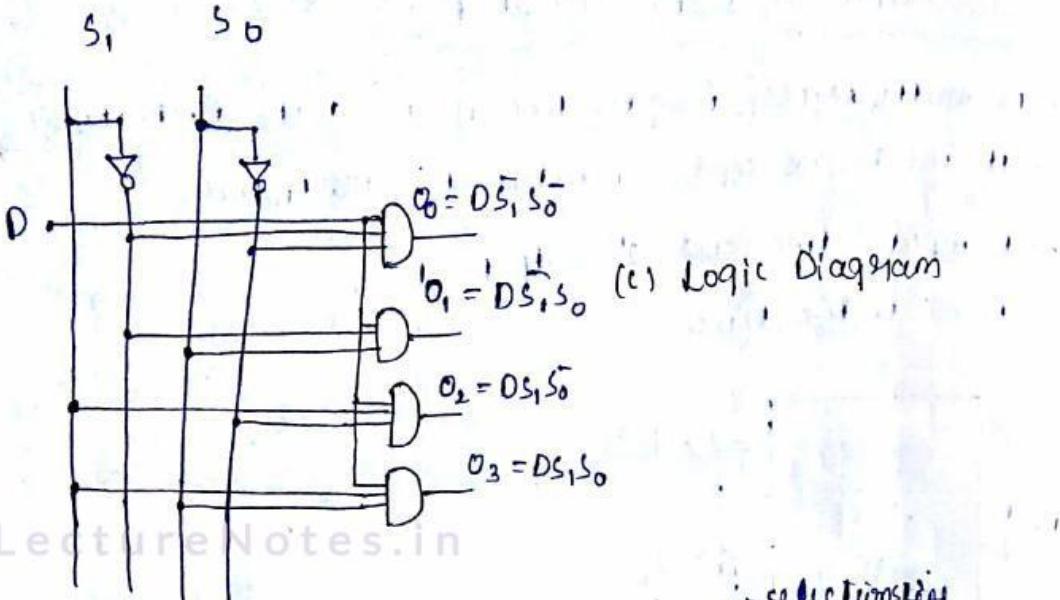
(b) Truth Table



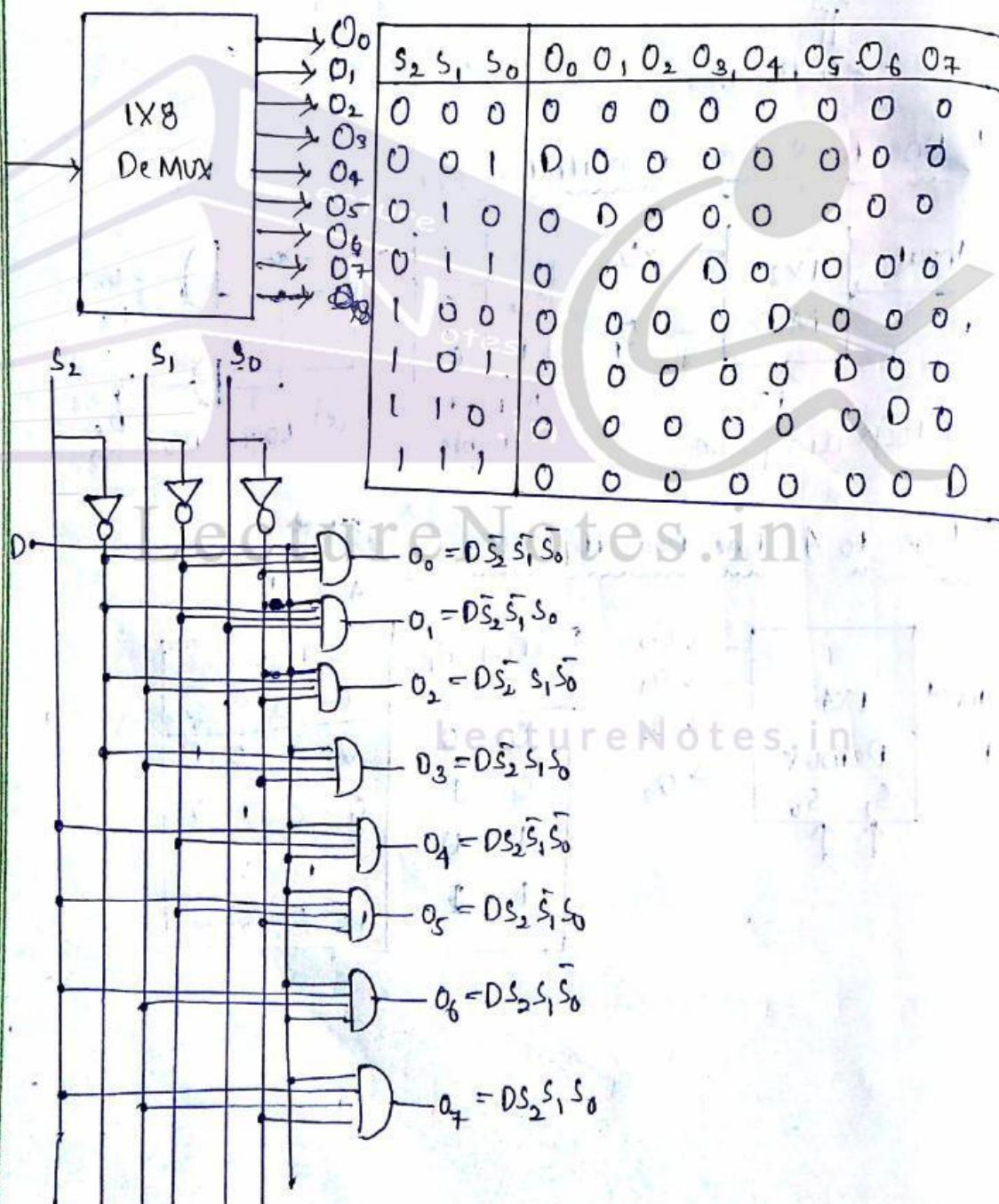
1 line to 4-line Demultiplexer:

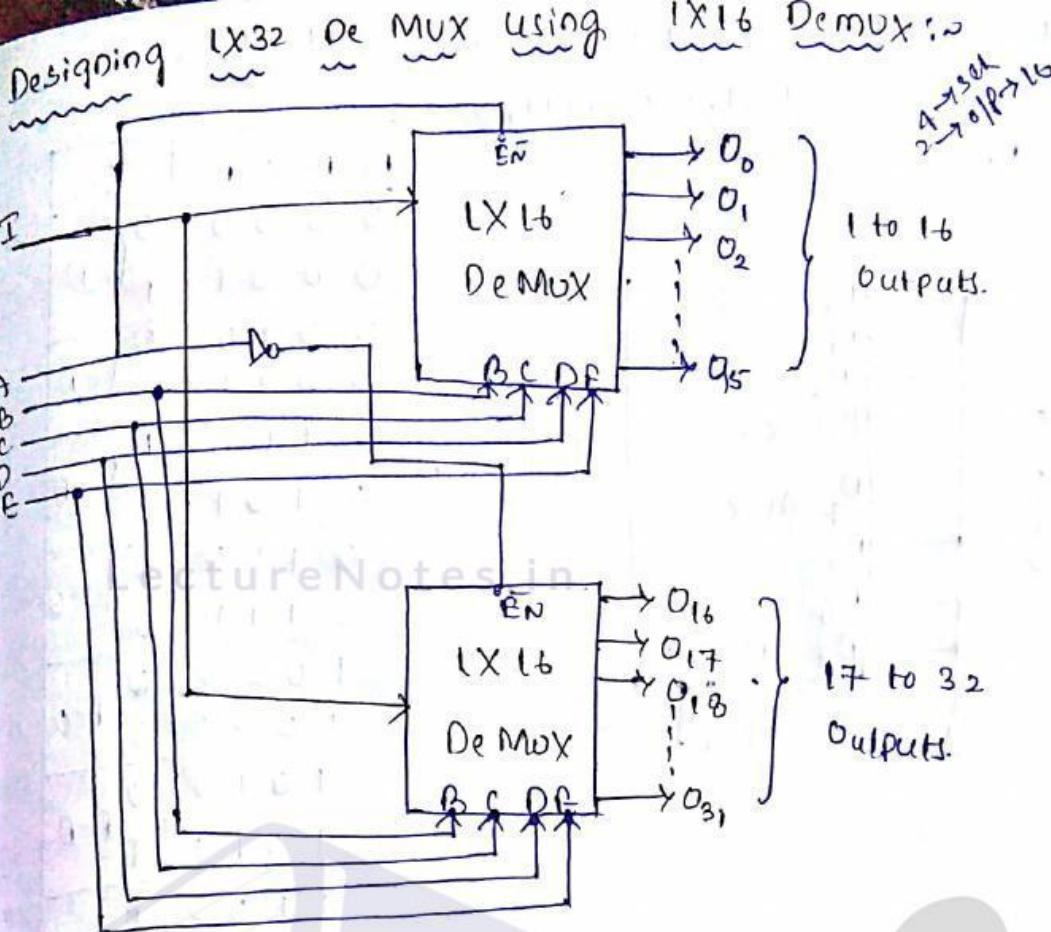


select i/p		Outputs
S ₁	S ₀	O ₃ O ₂ O ₁ O ₀
0	0	0 0 0 D
0	1	0 0 D 0
1	0	0 D 0 0
1	1	D 0 0 0



1- Line to 8 Line De-Multiplexer: $8 \Rightarrow 2^3$ \rightarrow Selection Lines

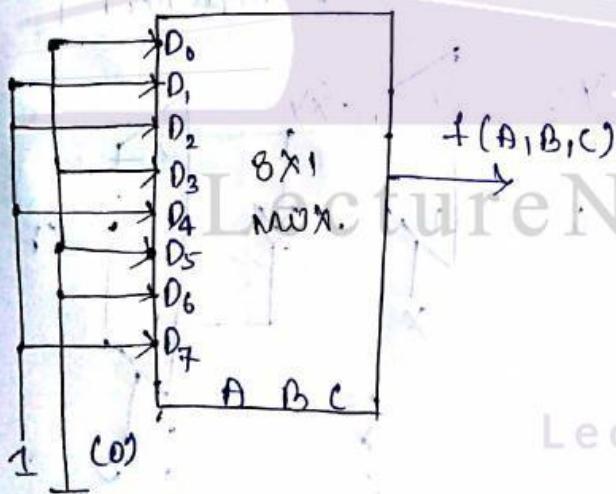




a) Use a Multiplexer to implement a Logic function

$$f = A \oplus B \oplus C$$

Solution :-

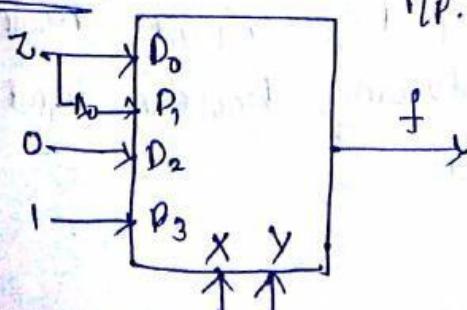


A B C	$f = A \oplus B \oplus C$
0 0 0	0 D_0
0 0 1	1 D_1
0 1 0	1 D_2
0 1 1	0 D_3
1 0 0	1 D_4
1 0 1	0 D_5
1 1 0	0 D_6
1 1 1	1 D_7

b) Implement the boolean function $f(x, y, z) = \Sigma(1, 2, 6, 7)$

using 4x1 MUX

Solution :-



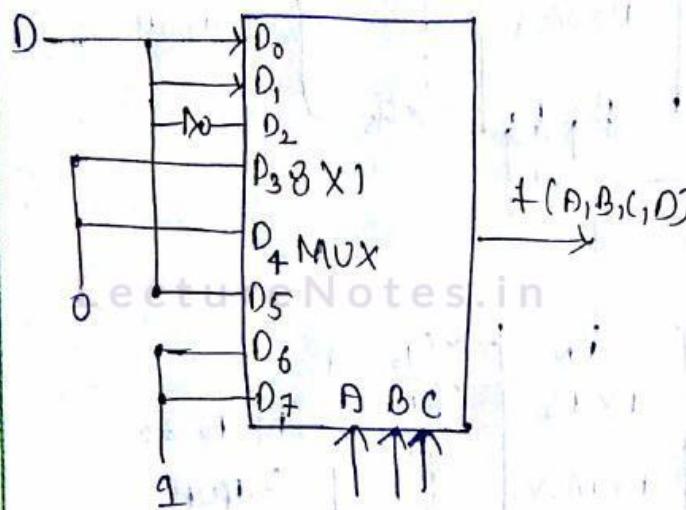
Z is external i/p.

x y z	f
0 0 0	0
0 0 1	1, $f=z$
0 1 0	1
0 1 1	0, $f=\bar{z}$
1 0 0	0
1 0 1	0, $f=0$
1 1 0	1
1 1 1	1, $f=1$

Implement the Boolean function using 8x1 MUX

$$f(A, B, C) = \sum (1, 3, 4, 11, 12, 13, 14, 15)$$

Solution:



A	B	C	D	f
0	0	0	0	0
0	0	0	1	$f=0$
0	0	1	0	0
0	0	1	1	$f=0$
0	1	0	0	1
0	1	0	1	$f=0$
0	1	1	0	0
0	1	1	1	$f=0$
1	0	0	0	0
1	0	0	1	$f=0$
1	0	1	0	0
1	0	1	1	$f=0$
1	1	0	0	0
1	1	0	1	$f=1$
1	1	1	0	1
1	1	1	1	$f=1$

Consider the circuit shown in the fig, The Boolean expression f implemented by the circuit is

Solution:

$$A = \bar{x}y + x \cdot 0$$

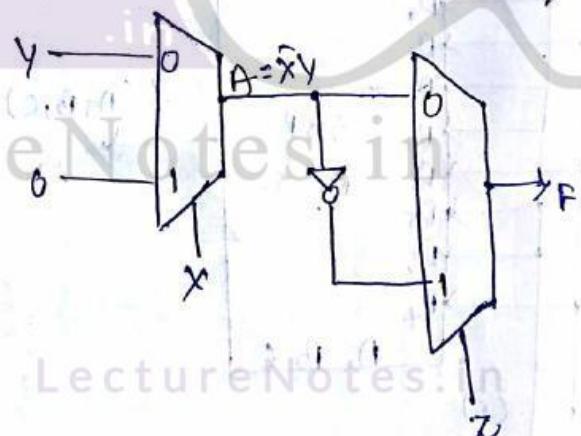
$$= \bar{x}y$$

$$f = \bar{z} \cdot A + z \cdot \bar{A}$$

$$= \bar{z}(\bar{x}y) + z(\bar{x}y)$$

$$= \bar{z}(\bar{x}y) + z(x+y)$$

$$= \bar{x}yz + xy\bar{z}$$



Consider a multiplexer based logic circuit shown in the fig which of the following boolean functions is realised by the circuit

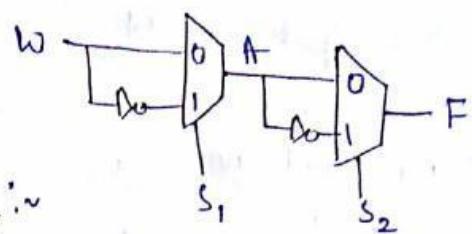
$$(a) f = w \overline{s_1 s_2}$$

$$(b) F = \bar{w}s_1 + ws_2 + s_1 s_2$$

$$(c) \bar{w}s_1 + ws_2 + \bar{s}_1 s_2$$

~~$$(d) w \oplus s_1 \oplus s_2$$~~

Solution:



$$A = \bar{s}_1 \bar{w} + s_1 \bar{w} = s_1 \oplus w$$

$$f = \bar{s}_1 (s_1 \oplus w) + s_2 (\bar{s}_1 \oplus w)$$

$$= w \oplus s_1 \oplus s_2$$

P₆) A 4x1 MUX is to be used for generating the output carry of a full adder. A & B are the bits to be added while C_{in} is the i/p carry and C_{out} is the output carry. A & B are used as the select bits. A being the Most significant select bit. Which one of the following statements correctly describes the choice of the signals to be connected to the i/p I₀, I₁, I₂, I₃, so that the o/p is C_o.

~~(a) I₀ = 0, I₁ = C_{in}, I₂ = C_{in} & I₃ = 1~~

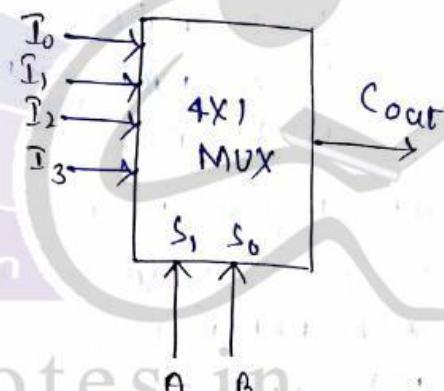
~~(b) I₀ = 1, I₁ = C_{in}, I₂ = C_{in} & I₃ = 1~~

~~(c) I₀ = C_{in}, I₁ = 0, I₂ = 1 & I₃ = C_{in}~~

~~(d) I₀ = 0, I₁ = C_{in}, I₂ = 1 & I₃ = C_{in}~~

Solution:

$$I_0 = 0; I_1 = C_{in}; I_2 = C_{in} \& I_3 = 1,$$



A	B	C _{in}	S	C _{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

1/3/19 Some QM problems on Multiplexers and Combinational Circuits

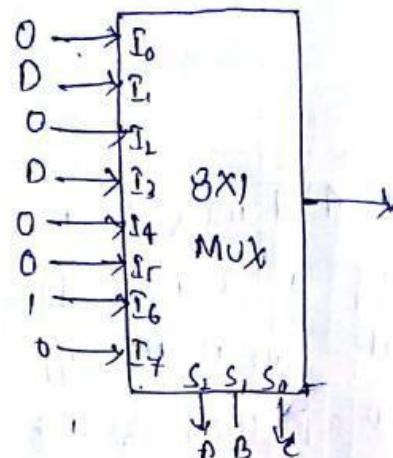
Q1) An 8x1 MUX is used to implement a logical function Y as shown in the fig. The o/p Y is given by

(a) $Y = A\bar{B}C + A\bar{C}D$

(b) $Y = \bar{A}\bar{B}C + A\bar{B}D$

(c) $Y = AB\bar{C} + \bar{A}CD$

(d) $Y = \bar{A}\bar{B}D + A\bar{B}C$.



$$000 \quad \bar{A}\bar{B}\bar{C}$$

$$001 \quad \bar{A}\bar{B}C \rightarrow I_1 = \bar{A}\bar{B}CD + \bar{A}\bar{B}CD + A\bar{B}\bar{C}$$

$$010 \quad \bar{A}\bar{B}\bar{C}$$

$$011 \quad \bar{A}\bar{B}\bar{C} \rightarrow I_3 = \bar{A}CD(\bar{A}\bar{B} + \bar{A}B) + A\bar{B}\bar{C}$$

$$100 \quad A\bar{B}\bar{C} = A$$

$$101 \quad A\bar{B}C$$

$$110 \quad A\bar{B}\bar{C} \rightarrow I_6$$

$$111 \quad ABC$$

Q2) The logical function implemented by the circuit below is [ground implies a logic '0']

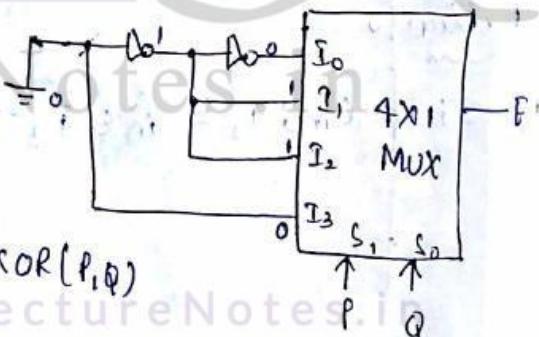
(a) $F = \bar{P}ND(P, Q)$

(b) $F = DR(P, Q)$

(c) $F = \bar{X}NOR(P, Q)$

~~(d) $F = X \oplus P$~~ $\Rightarrow F = X \oplus P$

$$\bar{P}Q + P\bar{Q} = X \oplus P$$



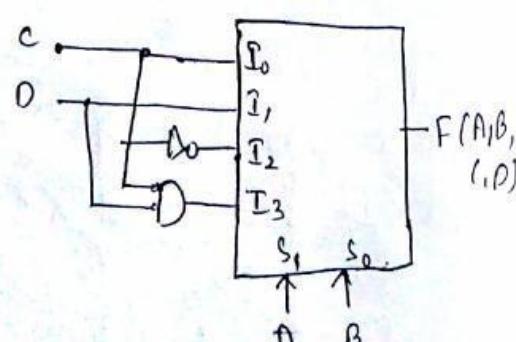
Q3) The boolean function realized by the logic circuit shown is

(a) $F = \sum m(0, 1, 3, 5, 9, 10, 14)$

(b) $F = \sum m(2, 3, 5, 7, 8, 12, 13)$

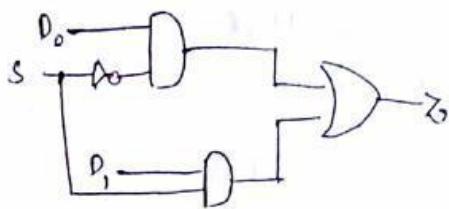
(c) $F = \sum m(1, 2, 4, 5, 11, 14, 15)$

~~(d) $F = \sum m(2, 3, 5, 7, 8, 9, 12)$~~



Q4) What are the min. no. of 2x1 multiplexers required to generate a 2-i/p AND gate & a 2 i/p EX-OR gate?

- (a) 1 \times 2
- (b) 1 \times 3
- (c) 1 \times 1
- (d) 2 \times 2

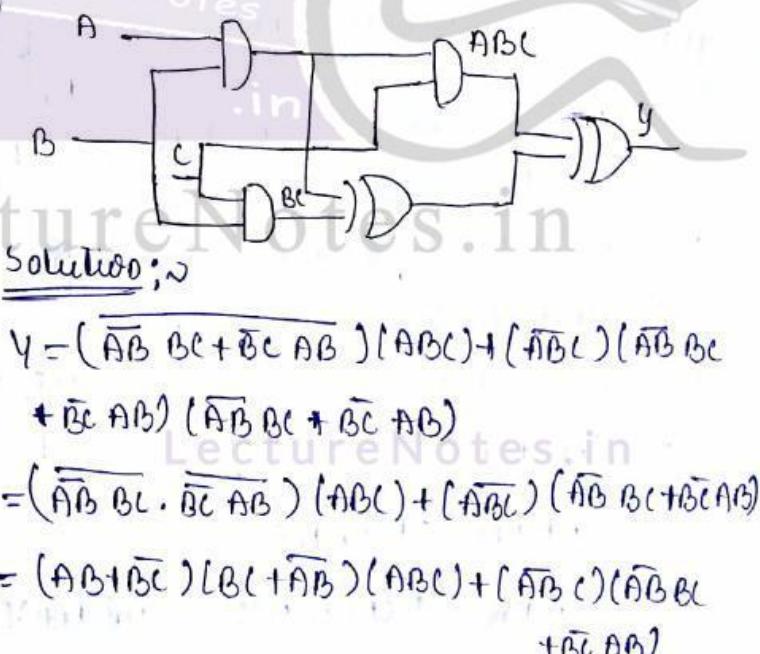


Q5) The o/p Y of a 2-bit comparator is logic 1 whenever the 2-bit i/p A is greater than the 2-bit i/p B. The no. of combination for which the o/p is logic 1 is

- (a) 4 ~~(b) 6~~ (c) 8 (d) 10.

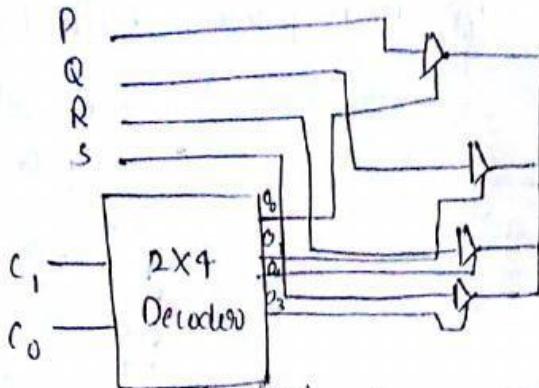
A ₁	A ₀	B ₁	B ₀	Y
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	1	0

Q6) The o/p of the combinational circuit given below is (a) A+B+C
 (b) A(B+C) ~~(c) B(A+A)~~ (d) ((A+B))



07) The functionality implemented by the circuit below is:

- (a) 2 to 1 multiplexer
- ~~(b)~~ 4 to 1 multiplexer
- (c) 7 to 1 multiplexer
- (d) 6 to 1 multiplexer



08) For the circuit shown in the following figure $\bar{I}_0 - \bar{I}_3$ are i/p to the 4×1 multiplexer. If R (MSB), S are control bits the o/p Z can be represented by

- (a) $PQ + P\bar{Q}S + \bar{P}\bar{Q}\bar{S}$
- (b) $P\bar{Q}\bar{R} + \bar{P}QR + \bar{P}\bar{Q}\bar{R}S + \bar{P}Q\bar{R}S$
- (c) $PQR + PQR\bar{S} + P\bar{Q}\bar{R}S + \bar{Q}\bar{R}\bar{S}$
- (d) $P\bar{Q} + P\bar{Q}\bar{R} + \bar{P}\bar{Q}\bar{S}$

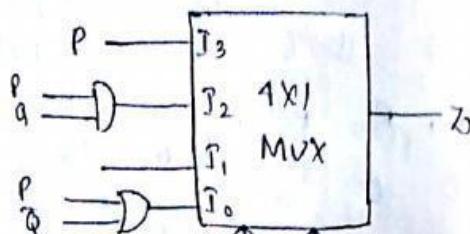
$$PRS + PQR\bar{S} + (P+\bar{Q})\bar{R}\bar{S} + P\bar{R}\bar{S}\bar{Q}$$

$$(D) I_1 \quad 1110 \quad 1 \quad 0 \quad 00 \quad 1001$$

$$PRS + PQR\bar{S} + \bar{P}\bar{R}\bar{S} + \bar{Q}\bar{R}\bar{S} + PR\bar{S}$$

$$I_0 \quad 1110 \quad 1000 \quad 0000 \quad 1001$$

$$I_1 \quad 1111 \quad 1100 \quad 1000 \quad 1101$$



		I_0	I_1	I_2	I_3	
PQ	RS	00	01	11	10	
00	00	1				
00	01	1	1	1	1	
01	00	1	1	1	1	
01	01	1	1	1	1	
11	00	1	1	1	1	
11	01	1	1	1	1	
11	11	1	1	1	1	
11	10	1	1	1	1	

09) In the following circuit, X is given by

$$(a) X = A\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}C + ABC$$

$$(b) X = \bar{A}BC + A\bar{B}\bar{C} + ABC + \bar{A}\bar{B}\bar{C}$$

$$(c) X = AB + BC + AC$$

$$(d) X = \bar{A}B + \bar{B}\bar{C} + \bar{A}\bar{C}$$

$$Y = (\bar{A}B + A\bar{B})$$

$$X = (\bar{A}B + A\bar{B})C + (\bar{A}B + A\bar{B})\bar{C}$$

$$X = (\bar{A}B + A\bar{B})C + (\bar{A}B + A\bar{B})\bar{C} \Rightarrow (\bar{A}B + A\bar{B})(1 + (\bar{A}B + A\bar{B}))C$$

$$\bar{A}B(1 + A\bar{B}) + A\bar{B}(1 + \bar{A}B) + ABC + ABC$$

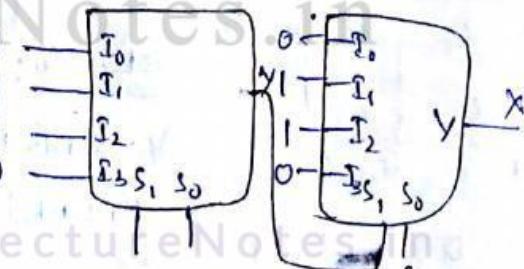
10) In the circuit shown, W, Y are MSBs of the control i/p. The o/p F is given by 4:1 Mux.

$$(a) F = W\bar{x} + W\bar{x} + \bar{Y}Z$$

$$(b) F = W\bar{x} + \bar{W}x + \bar{Y}Z$$

$$(c) F = W\bar{x}\bar{y} + \bar{W}x\bar{y}$$

$$(d) F = W\bar{x}\bar{y} + \bar{W}x\bar{y}$$



Solution:

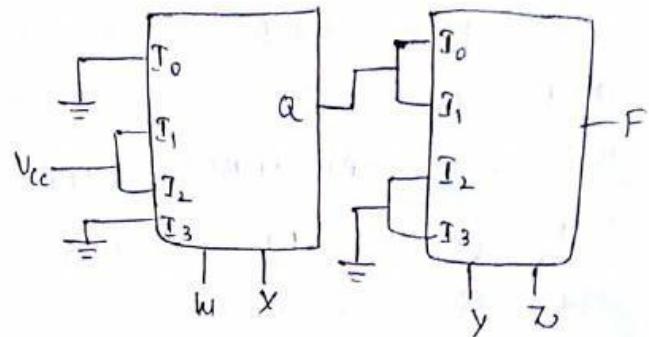
$$Q = (W \oplus X)$$

$$F = \bar{Q} Y \bar{Z} + Y Z \bar{Q}$$

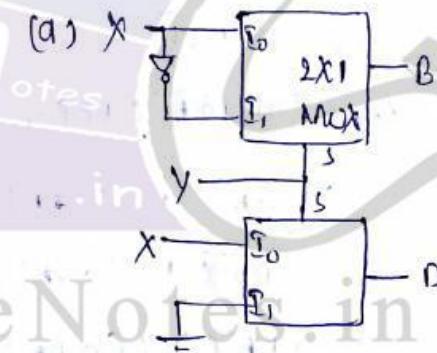
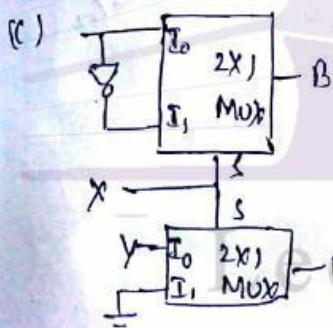
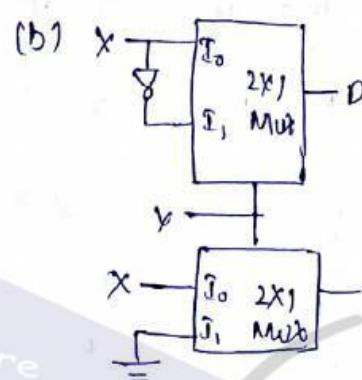
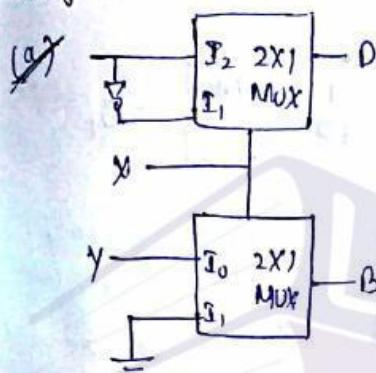
$$= Q (\bar{Y} \bar{Z} + Y Z)$$

$$= Q (\bar{Y} (Z + \bar{Z}))$$

$$= (W \bar{X} + \bar{W} X) (\bar{Y}) = W \bar{X} \bar{Y} + \bar{W} X \bar{Y}$$



11. If X & Y are i/p to the difference ($D = X - Y$) & the borrow (B) are the o/p, which one of the following diagrams implement a half-subtractor?



- 12) The minimum no. of 2 to 1 multiplexers required to realize a 4 to 1 multiplexer is

- (a) 1 (b) 2 (c) 3 (d) 4

- 13) The boolean function of implemented in the figure using two i/p multiplexers is

(a) $A\bar{B}C + A\bar{B}\bar{C}$

(b) $A\bar{B}C + A\bar{B}\bar{C}$

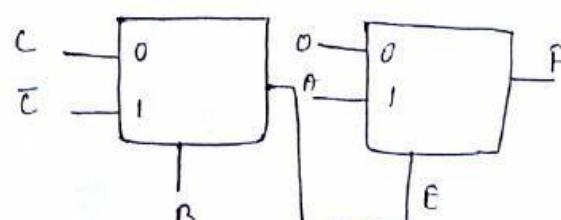
(c) $\bar{A}BC + \bar{A}\bar{B}\bar{C}$

(d) $\bar{A}\bar{B}C + A\bar{B}\bar{C}$

$$F = \bar{B}C + B\bar{C}$$

$$= A(\bar{B}C + B\bar{C})$$

$$= A\bar{B}C + A\bar{B}\bar{C}$$



(14) The logic realized by the circuit shown in the fig is

(a) $F = A \cdot C$

(b) $F = A \oplus C$

(c) $F = B \cdot C$

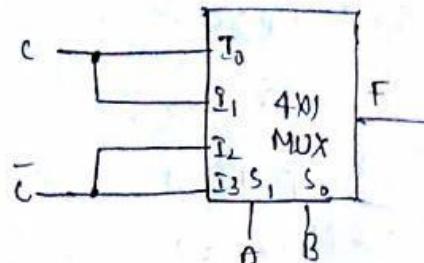
(d) $F = B + C$

$$\bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + ABC$$

$$A(C(\bar{B}\bar{A}) + \bar{C}(B+A))$$

$$= AC + A\bar{C}$$

$$= A \oplus C$$



(15) A MUX network is shown in figure (11). $Z_1 = ?$

(a) $a+b+c$ (b) $a+b+c$ (c) $a+b+c$ (d) $a \oplus b \oplus c$

(ii) $Z_2 = ?$

(a) $\bar{a}b+b\bar{c}+ca$ (b) $a+b+c$ (c) $a+b+c$ (d) $a+b+c$

(iii) This circuit acts as a

(a) Full adder

(b) Half adder

(c) Full subtractor

(d) Half subtractor

$$Z_0 = ab + \bar{a}\bar{b}$$

$$Z_1 = (\bar{a}b + \bar{a}\bar{b})\bar{c} + c(ab + \bar{a}\bar{b})$$

$$= (\overline{a \oplus b})\bar{c} + c(a \oplus b)$$

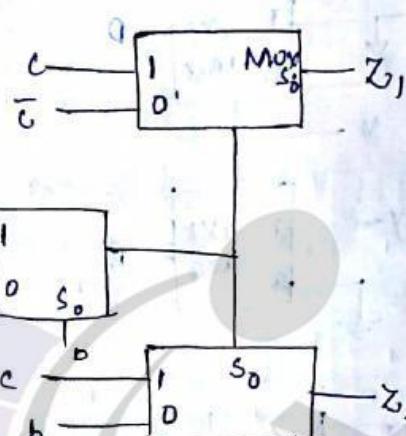
$$= (a \oplus b)\bar{c} + c(\overline{a \oplus b})$$

$$= (a \oplus b)\bar{c} + ((\overline{a \oplus b})) = a \oplus b \oplus c$$

$$Z_2 = (ab + \bar{a}\bar{b})c + b(ab + \bar{a}\bar{b})$$

$$= abc + \bar{a}\bar{b}c + b(ab + \bar{a}\bar{b})$$

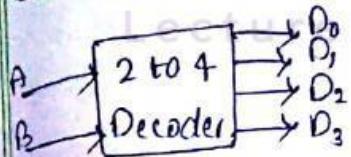
$$= abc + \bar{a}\bar{b}c + b\bar{a} + b\bar{a}$$



5/03/19
Decoder & S/in

→ A decoder is a combinational circuit that converts n -bit binary data into 2^n output lines. Only one line is activated at a time for each possible combination of the input data.

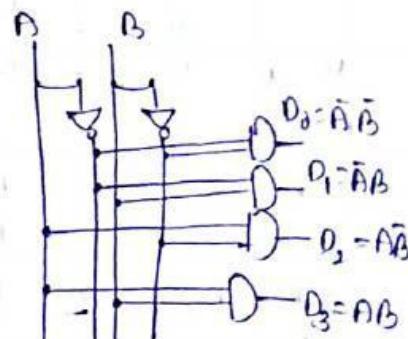
2-to-4 Decoder:



Block Diagram

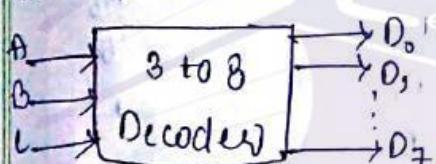
i/p	o/p			
A B	D ₀	D ₁	D ₂	D ₃
0 0	1	0	0	0
0 1	0	1	0	0
1 0	0	0	1	0
1 1	0	0	0	1

Truth Table

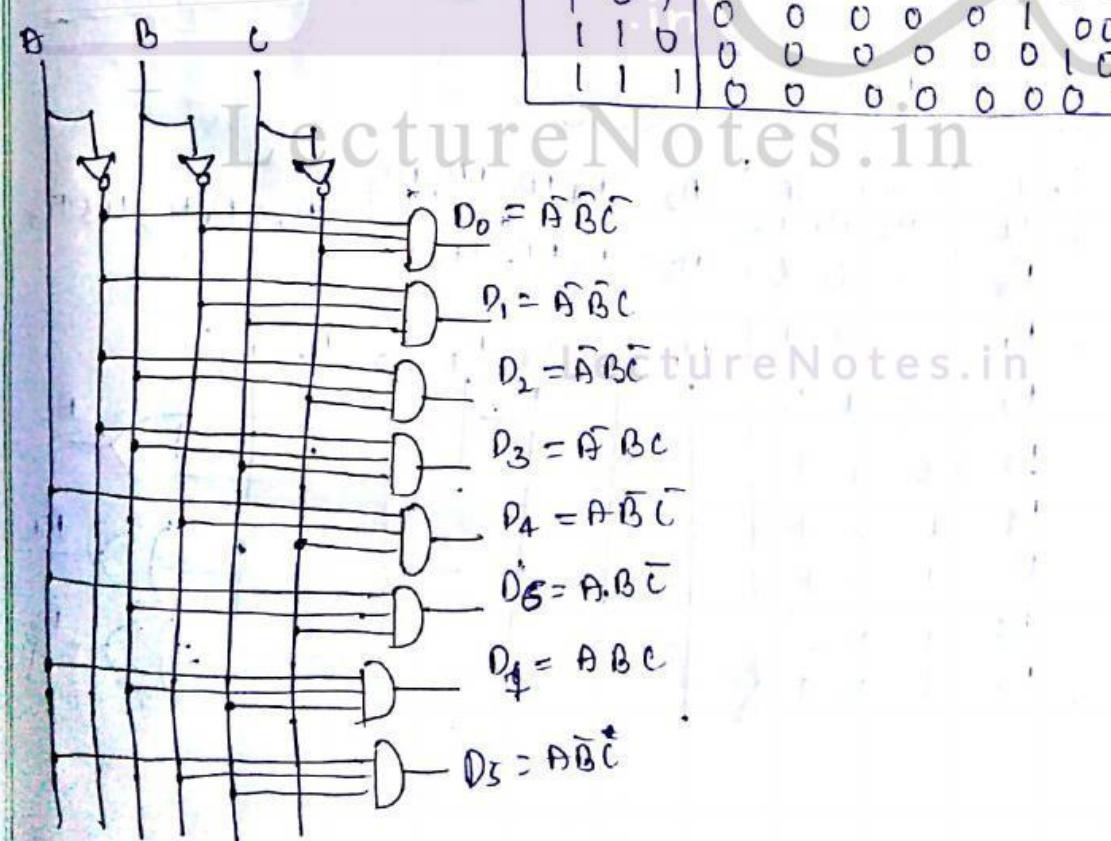


Logic Diagram

3-to-8 Decoder:

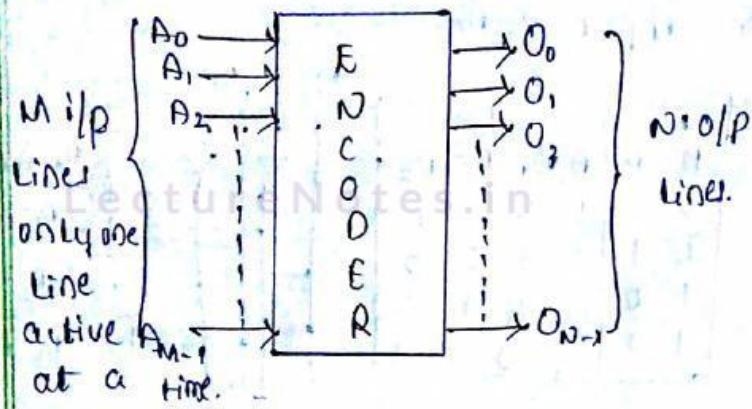


i/p	o/p							
A B C	D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇
0 0 0	1	0	0	0	0	0	0	0
0 0 1	0	1	0	0	0	0	0	0
0 1 0	0	0	1	0	0	0	0	0
0 1 1	0	0	0	1	0	0	0	0
1 0 0	0	0	0	0	1	0	0	0
1 0 1	0	0	0	0	0	1	0	0
1 1 0	0	0	0	0	0	0	1	0
1 1 1	0	0	0	0	0	0	0	1

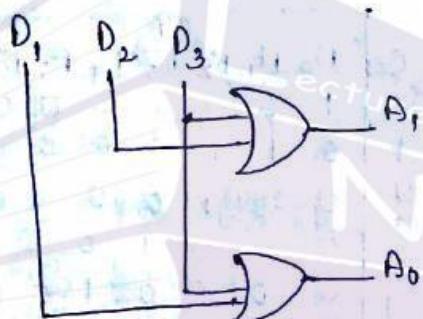


Encoder:

→ An encoder is a combinational circuit whose inputs are decimal digits and/or alphabetical characters, and whose outputs are coded representations of those inputs.



4 line to 2 line Encoder:

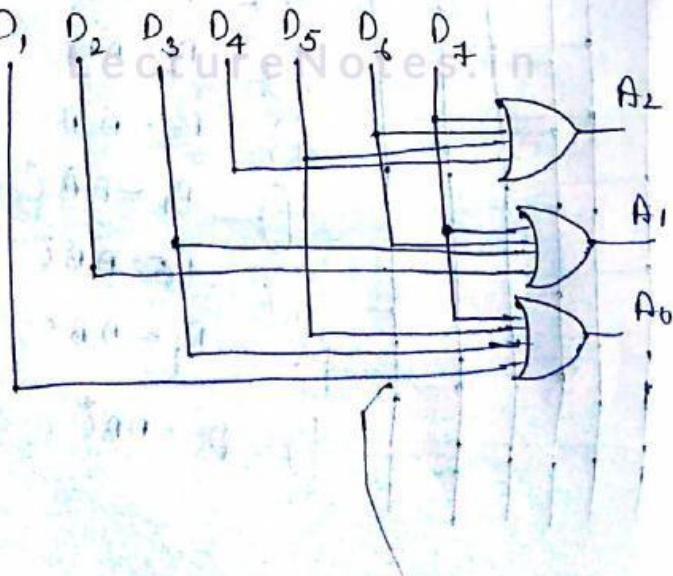


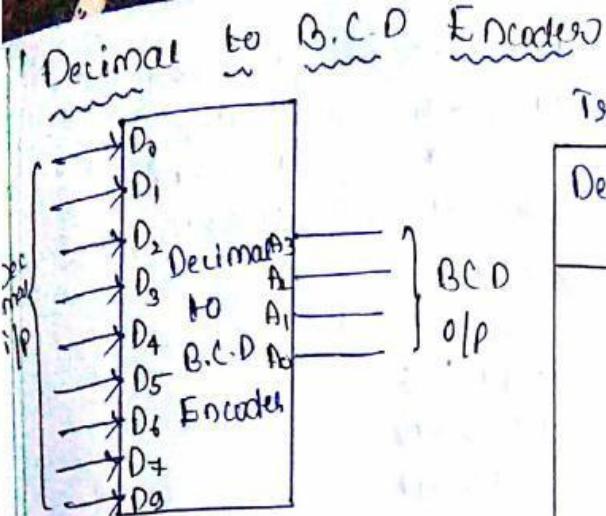
Inputs	Coded	Output
D ₀	0	A ₁ A ₀
D ₁	0	0 1
D ₂	1	1 0
D ₃	1	1 1

Octal to Binary Encoder:

Output i/p	Coded o/p		
	A ₂	A ₁	A ₀
D ₀	0	0	0
D ₁	0	0	1
D ₂	0	1	0
D ₃	0	1	1
D ₄	1	0	0
D ₅	1	0	1
D ₆	1	1	0
D ₇	1	1	1

$$\begin{aligned}
 A_2 &= D_4 + D_5 + D_6 + D_7 & A_0 &= D_1 + D_3 + D_5 + D_7 \\
 A_1 &= D_2 + D_3 + D_6 + D_7
 \end{aligned}$$





Truth Table

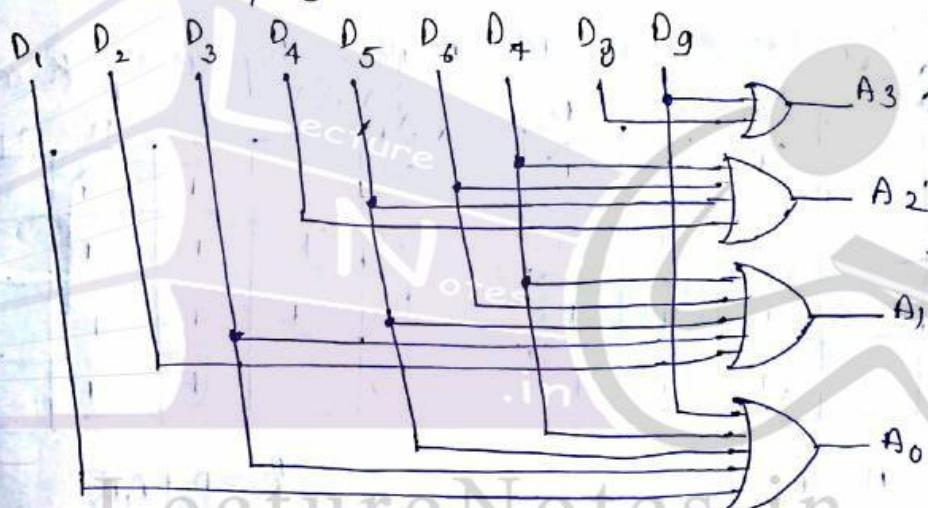
Decimal i/p	coded o/p A ₃ A ₂ A ₁ A ₀
D ₀	0 0 0 0
D ₁	0 0 0 1
D ₂	0 0 1 0
D ₃	0 0 1 1
D ₄	0 1 0 0
D ₅	0 1 0 1
D ₆	0 1 1 0
D ₇	0 1 1 1
D ₈	1 0 0 0

$$A_3 = D_8 + D_9$$

$$A_2 = D_9 + D_5 + D_6 + D_7$$

$$A_1 = D_2 + D_3 + D_6 + D_7$$

$$A_0 = D_1 + D_3 + D_5 + D_7 + D_9$$



4 bit priority Encoder

D ₀	D ₁	D ₂	D ₃	A	B	V
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

$$A = \text{Em} (1, 2, 3, 5, 6, 7, 9_{10}, 11, 13, 17, 15)$$

A \rightarrow	X X 1 0
	0 0 1 0 \rightarrow 2
	0 1 1 0 \rightarrow 6
	1 0 1 0 \rightarrow 10
	1 1 1 0 \rightarrow 14
	X X X 1
	0 0 0 1 \rightarrow 1
	0 0 1 1 \rightarrow 3
	0 1 0 1 \rightarrow 5
	0 1 1 1 \rightarrow 7
	1 0 0 1 \rightarrow 9
	1 0 1 0 \rightarrow 10
	1 0 1 1 \rightarrow 11
	1 1 0 1 \rightarrow 13
	1 1 1 1 \rightarrow 15

$B \rightarrow X_1 00$
 $01 00 \rightarrow 4$
 $11 00 \rightarrow 12$
 $\times \times X_1 \rightarrow$
 $0001 \rightarrow 1$
 $0011 \rightarrow 5$
 $0101 \rightarrow 5$
 $0111 \rightarrow 7$
 $1001 \rightarrow 9$
 $1011 \rightarrow 11$
 $1101 \rightarrow 13$
 $1111 \rightarrow 15$

$B \rightarrow 1000 \rightarrow 8$
 $X_1 00 \rightarrow 6$
 $01 00 \rightarrow 4$
 $11 00 \rightarrow 12$
 $V \rightarrow XX10$
 $0010 \rightarrow 2$
 $0010 \rightarrow 6$
 $1010 \rightarrow 10$
 $1110 \rightarrow 14$
 $V \rightarrow XXX1$
 $0001 \rightarrow 1$
 $0101 \rightarrow 5$
 $0011 \rightarrow 3$
 $0111 \rightarrow 7$

$$B = \Sigma(1, 3, 4, 5, 7, 9, 11, 13, 15)$$

$$V = \Sigma m(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$$

Solving expression for A, B, V by using K-map

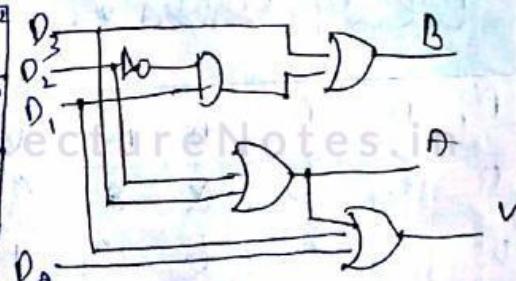
	$D_2 D_3$	$D_2 D_3'$	$D_2' D_3$	$D_2' D_3'$
$D_0 D_1$	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

	$D_2 D_3$	$D_2 D_3'$	$D_2' D_3$	$D_2' D_3'$
$D_0 D_1$	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

$$A = D_2 + D_3$$

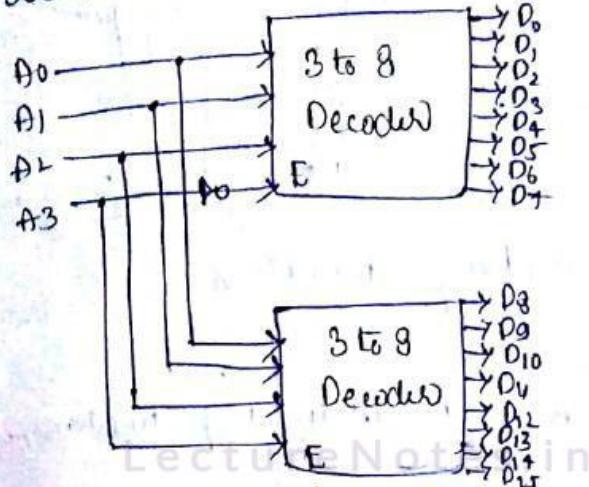
$$B = D_3 + D_1 D_2'$$

	$D_2 D_3$	$D_2 D_3'$	$D_2' D_3$	$D_2' D_3'$
$D_0 D_1$	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1



$$V = D_0 + D_1 + D_2 + D_3$$

5/03/19
4 to 16 Decoder using 3 to 8 Decoder:



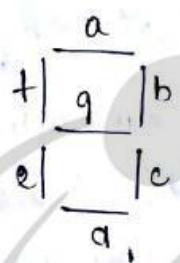
→ No 'not' gate is used to keep the decoder in high to low modes.

If we give 'not' gate to 2nd decoder, 1st decoder will be disabled.

If we give 'not' gate to 1st decoder, 2nd decoder will be disabled.

B.C.D to 7-Segment Display:

B.C.D Data	Output
A ₃ A ₂ A ₁ A ₀	a b c d e f g
0 0 0 0	1 1 1 1 1 1 0
0 0 0 1	0 1 1 0 0 0 0
0 0 1 0	1 1 0 1 1 0 1
0 0 1 1	1 1 1 1 0 0 1
0 1 0 0	0 1 1 0 0 1 1
0 1 0 1	1 0 1 1 0 1 1
0 1 1 0	1 0 1 1 1 1 1
0 1 1 1	1 1 0 0 0 0 1
1 0 0 0	1 1 1 1 1 1 1
1 0 0 1	1 1 1 1 0 1 1



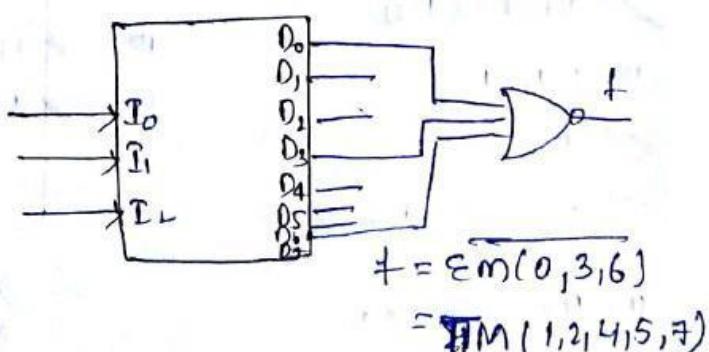
Problems on Decoders:

(a) $f(x_2, x_1, x_0) = ?$

(b) $f(x_2, x_1, x_0) = ?$

(c) $f(x_2, x_1, x_0) = ?$

(d) None



Q2) find f_1, f_2 & $f_1 \cdot f_2 = ?$

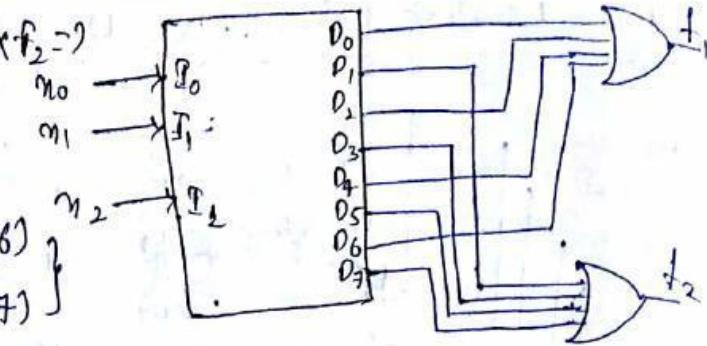
(a) $\pi_0 \oplus \pi_2$

(b) $\pi_0 \oplus \pi_1 \oplus \pi_2$

(c) 1

~~(d)~~ 0

$$\begin{aligned} f_1 &= \{0, 2, 4, 6\} \\ f_2 &= \{1, 3, 5, 7\} \end{aligned}$$



Product of complementary functions
are zero.

Q3) The logic circuit which is shown in fig implements

(a) $D[B \odot C + \bar{A} \bar{C}]$

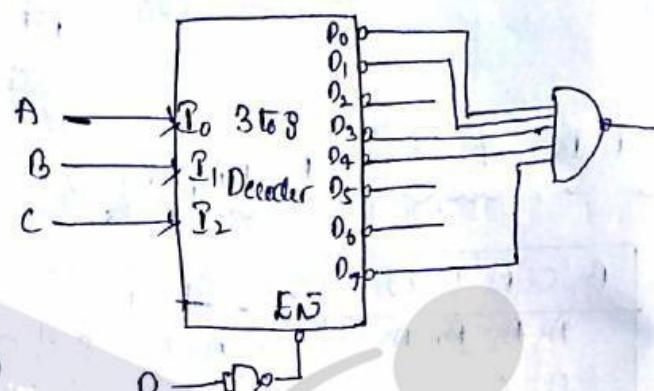
(b) $D[B \oplus C + \bar{A} \bar{C}]$

(c) $D[B \oplus C + \bar{A} \bar{B}]$

~~(d)~~ $D[B \odot C + \bar{A} \bar{B}]$

$$\Sigma m(0, 1, 3, 4, 7)$$

$$= \prod M(2, 5, 6, 8)$$



Solving by using 3-variable K-map

A	BC	00	01	11	10
0	1	1	1	1	1
1	1	1	1	1	1

$f = \bar{B}\bar{C} + \bar{A}\bar{B} + BC$
 $D(B \odot C) + \bar{A}\bar{B}$

The building block shown in fig is a active high off decoder.

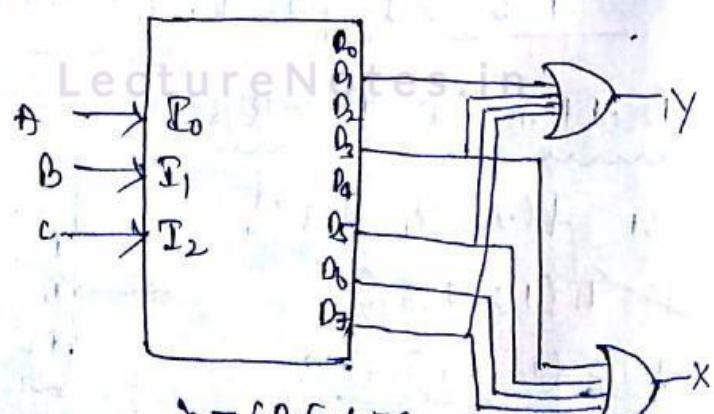
(i) The o/p x is

~~(a)~~ $\bar{A}B + B(C + \bar{A})$

(b) $A + B + C$

(c) $A\bar{B}C$

(d) None of the above



$$X = \{3, 5, 6, 7\}$$

$$Y = \{1, 2, 4, 7\}$$

(ii) The o/p Y is

(a) $A + B$ (b) $B + C$

(c) $C + A$ ~~(d)~~ None of the above

Solving for X

	00	01	11	10
0	0	1	1	0
1	1	0	0	1

$$X = AC + BC + AB$$

Solving for Y

	00	01	11	10
0	0	1	1	0
1	1	1	1	1

$$Y = C_{11}$$

The logic circuit consists of 2 2 to 4 decoders as shown
The output of the decoder are

(i) $D_0 = 1$ when $A_0 = 0, A_1 = 0$

(ii) $D_1 = 1$ when $A_0 = 1, A_1 = 0$

(iii) $D_2 = 1$ when $A_0 = 0, A_1 = 1$

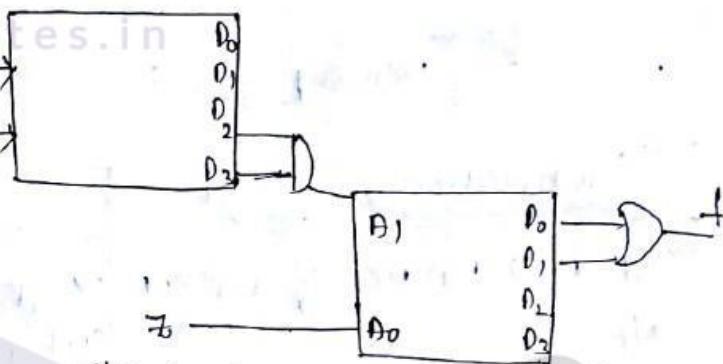
(iv) $D_3 = 1$ when $A_0 = 1, A_1 = 1$

$$D_0 = \bar{A}_0 \bar{A}_1 \Rightarrow 1, \bar{Z},$$

$$D_1 = A_0 \bar{A}_1;$$

$$D_2 = \bar{A}_0 A_1;$$

$$D_3 = A_0 A_1$$



The values of $f(X, Y, Z)$ are

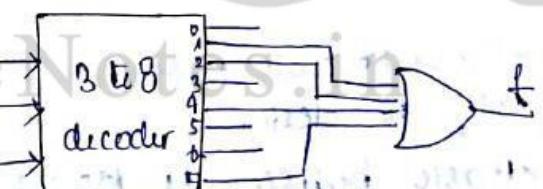
- (a) 0 (b) 2 (c) \bar{Z} (d) Z

The circuit shown below is

- (a) Full adder (b) 3 i/p X-OR gate (c) Half adder (d) 3 sink

Solution

	00	01	11	10
0	0	1	1	0
1	1	0	0	1



$$\begin{aligned} f &= \bar{A} \bar{B} C + \bar{A} B \bar{C} + A \bar{B} \bar{C} + A B C \\ &= \bar{A} (\bar{B} C + B \bar{C}) + A (\bar{B} \bar{C} + B C) \end{aligned}$$

We have no information about carry, so we can't justify the circuit as full adder. So our answer is 3 i/p X-OR.

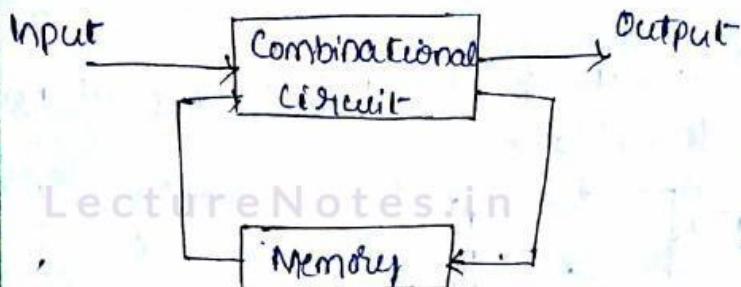
$$= A \oplus B \oplus C \rightarrow \text{sum of full adder}$$

06/3/19

Synchronous Sequential Circuits.

Sequential Circuit:

- Sequential circuits are the circuits where the o/p is a function of present input and past i/p and o/p values.



Combinational Circuit	Sequential circuit
→ o/p is a function of present i/p	→ o/p is a function of present and past i/p & o/p.
→ i/p → combination circuit → o/p	→ combination circuit → o/p ↓ Memory
→ Memory element is absent	→ Memory element is present
→ feedback does not exist	→ feedback exists.
→ Combinational circuits are faster	→ Sequential circuits are slower
→ Combinational circuits are easy to design	→ Difficult to design
→ Basic building blocks are logic gates	→ Basic building blocks are flip flop.
→ Combinational circuits are time independent	→ Sequential circuits are time dependent
→ Ex:- adders, Multiplexers, Demultiplexers, Encoders	→ Ex:- flip flops, Counter, Register

Sequential Circuits are classified as:

01. Synchronous Sequential circuit → controlled by clock
02. Asynchronous Sequential circuit → does not use any clock.

* Latch and flip flop:

⇒ flip flop is a basic memory element which is used to save the binary data.

⇒ Latch saves the data in asynchronous manner

⇒ flip flop saves the data in synchronous manner.

⇒ flip flop has clock.

⇒ Latch doesn't have clock.

⇒ Latch is level triggered

⇒ flip-flop is edge triggered

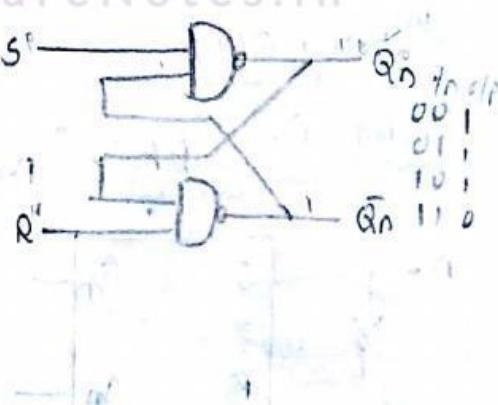
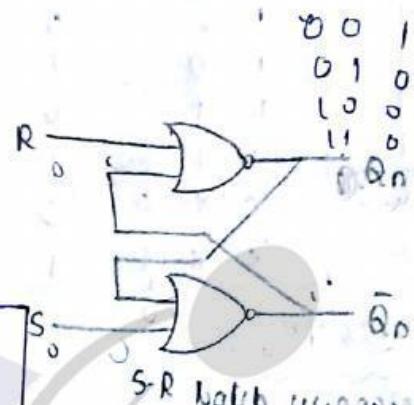
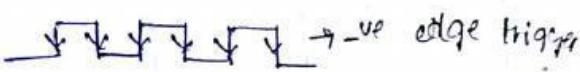
* S-R Latch:

⇒ Set(1) R → Reset(0)

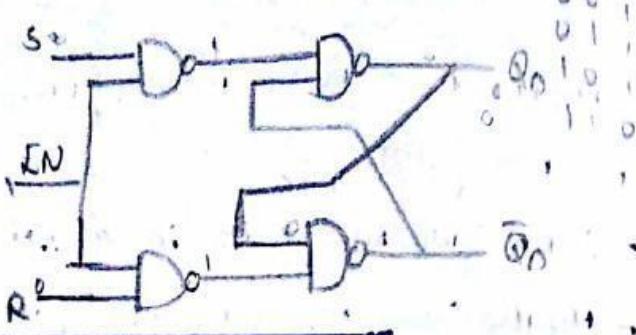
Inputs S R	Present state Q_n	Next state Q_{n+1}	State
0 0	0	0	No change
0 0	1	1	change
0 1	0	0	Reset
0 1	1	0	
1 0	0	1	Set
1 0	1	1	
1 1	0	X	Invalid
1 1	1	X	Invalid

S-R Latch using NOR Gates:

Inputs S R	Present state Q_n	Next state Q_{n+1}	State
0 0	0	X	Invalid
0 0	1	X	Invalid
0 1	0	1	Set
0 1	1	1	
1 0	0	0	Reset
1 0	1	0	
1 1	0	0	No change
1 1	1	1	

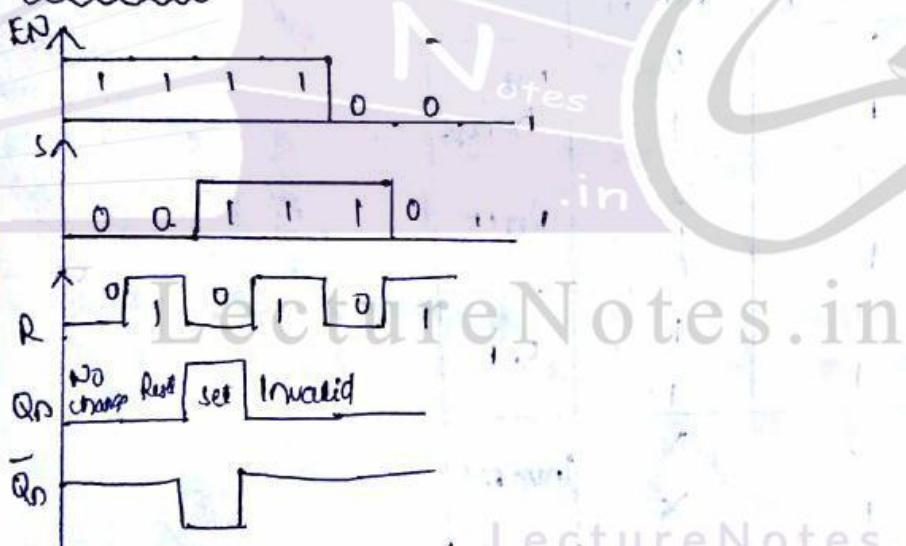


* Gated S-R Latch:



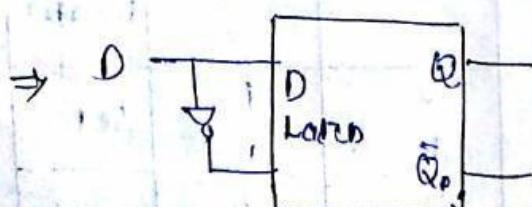
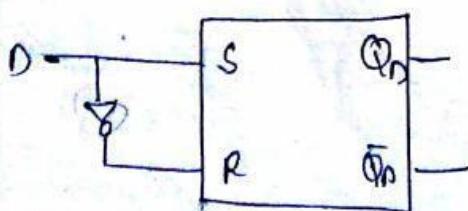
Enable EN	Inputs S R	Present state Q0	Next state Q0+1	State
1	0 0	0	0	No change
1	0 0	1	1	No change
1	0 1	0	0	Reset
1	0 1	1	0	Set
1	1 0	0	1	
1	1 0	1	1	
1	1 1	0	X	Invalid
1	1 1	1	X	Invalid
0	XX	0	0	No change
0	XX	1	1	No change

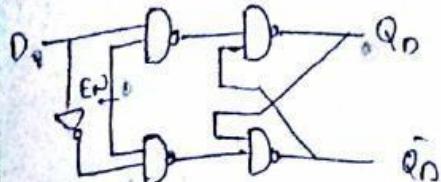
Waveforms



8th March 2019

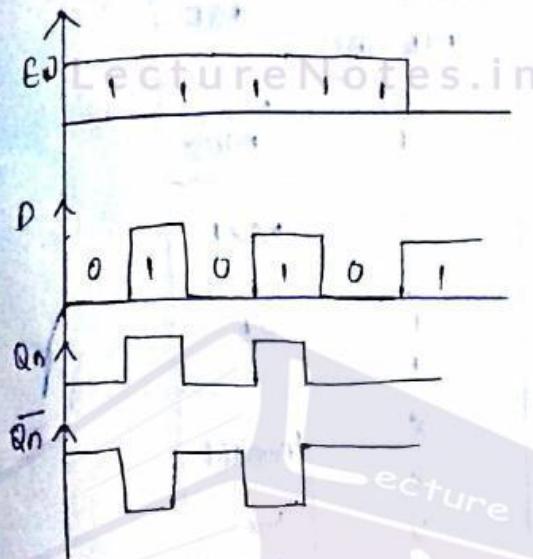
* Gated D-Latch: [D \rightarrow Delay, D \rightarrow Data]





Enable EN	Input D	Present state Qn	Next state Qn+1	State
1	0	0	0	
1	0	1	0	Reset
1	1	0	1	Sat
0	X	0	0	No change
0	X	1	1	

Wavesforms:-



flip flop: [JK flip flop \rightarrow Jack Kilby]

There are 4 types of flip flops

01) SR flip flop

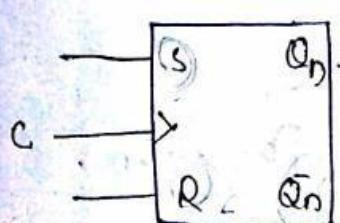
02) D flip flop

03) JK flip flop

04) T flip flop. [$T \rightarrow$ Toggle (inversion of bit)]

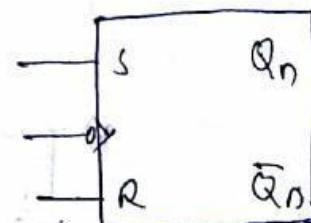
01) SR flip flop:-

 m m m



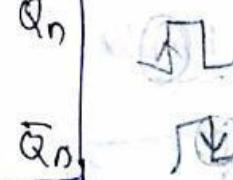
Positive edge triggered.

SR flip flop.



Negative edge triggered.

SR flip flop.

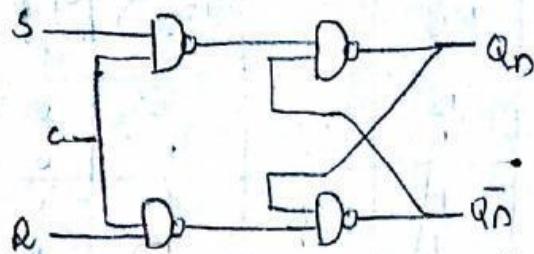


01. Truth Table

02. characteristic Equation

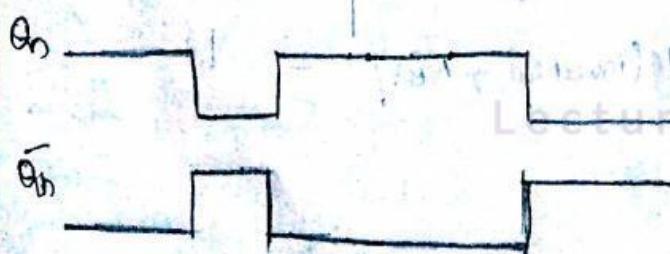
03. characteristic value

04. Excitation table

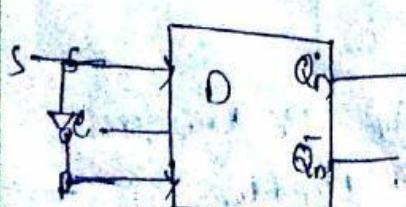


Circuit Diagram.

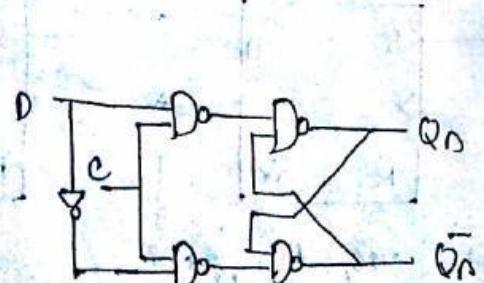
Clock C	Inputs S R	Present state Qn	Next state Qn+1	state
↑↑L	0 0	0	0	No change
↑↑L	0 0	1	1	No change
↑↑L	0 1	0	0	
↑↑L	0 1	1	0	Reset
↑↑L	1 0	0	1	Set
↑↑L	1 0	1	1	
↑↑L	1 1	0	X	Invalid
↑↓L	XX	0	0	No change
↑↓L	XX	1	1	No change



D- flip Flop:



Block Diagram

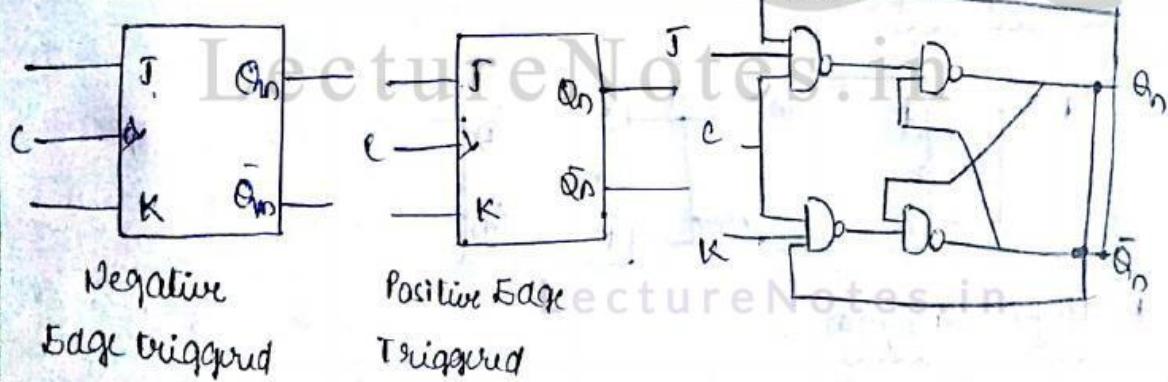


Logic Diagram.

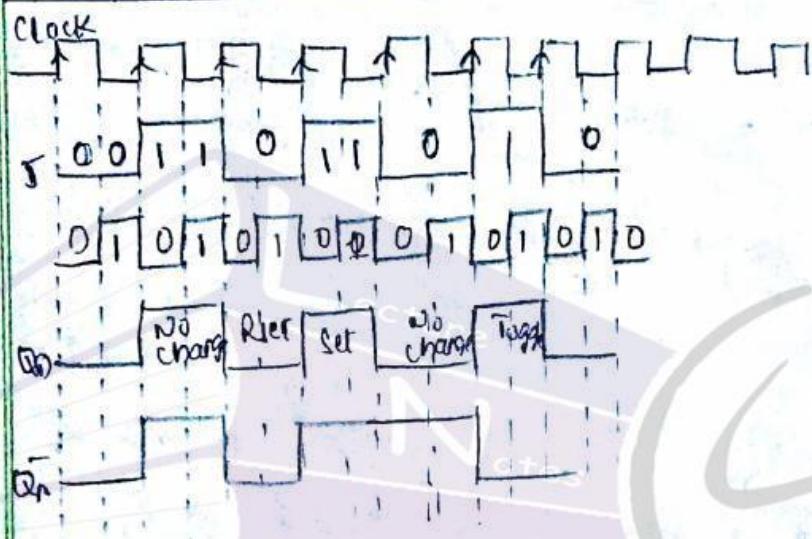
Clock c	Inputs S R	Present state Qn	Next state Qn+1	state
↑	0	0	0	Reset
↑	0	1	0	
↑	1	0	1	Set
↑	1	1	1	
↓	X	0	0	No change
↓	X	1	1	

13/03/19

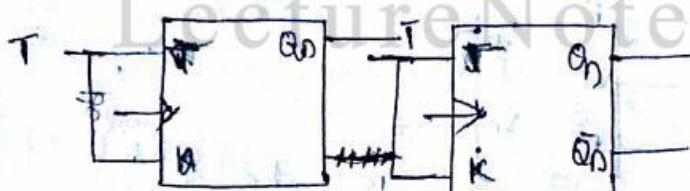
J K flipflop:



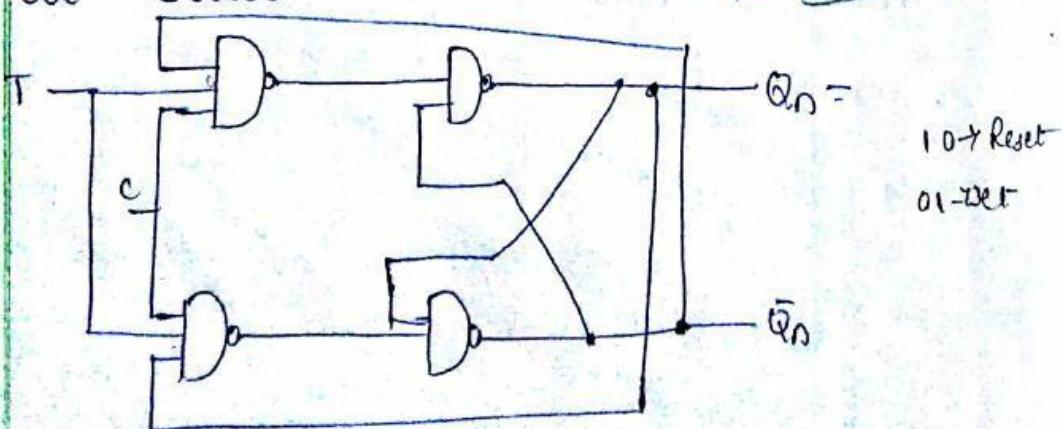
Clock C	Inputs J K	Present state Q_n	Next state Q_{n+1}	state
↑	0 0	0	0	No change
↑	0 0	1	1	change
↑	0 1	0	0	
↑	0 1	1	0	Reset
↑	1 0	0	1	Set
↑	1 0	1	1	
↑	1 1	0	1	Toggle
↑	1 1	1	0	
↓	XX	0	0	No change
↓	XX	1	1	



* T-Flip Flop:



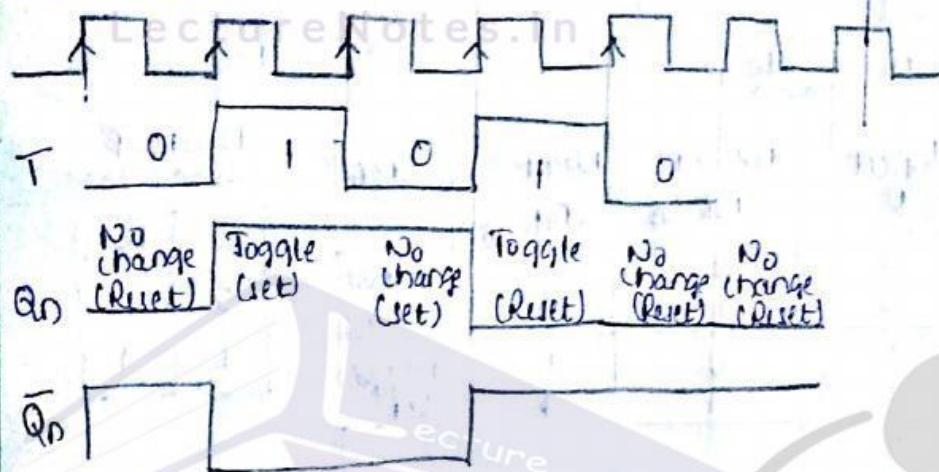
* Logic Diagram:



10 → Reset
01 → Set

Clock	Input	Present State Q _n	Next State Q _{n+1}	State
↑L	0	0	0	No change
↑L	0	1	1	No change
↑L	1	0	1	Set
↑L	1	1	0	Reset
↑K	X	0	0	No change
↑K	X	1	1	No change

Timing Diagrams



SR-Flipflop:

Characteristic Table:

Clock C.	Inputs S R	Present State Q _n	Next State Q _{n+1}	State
↑L	0 0	0	0	No change
↑L	0 0	1	1	No change
↑L	0 1	0	0	
↑L	0 1	1	0	Reset
↑L	1 0	0	1	Set
↑L	1 0	1	1	
↑L	1 1	0	X	
↑L	1 1	1	X	Invalid

Truth Table:

S R	Q _{n+1}
0 0	Q _n
0 1	0
1 0	1
1 1	X

Characteristic Equation:

S	00	01	11	10
R	0	0	1	1
Q _n	0	1	0	X
Q _{n+1}	0	0	1	X

$$Q_{n+1} = S + \bar{R}Q_n$$

Excitation Table:-

Q_n	Q_{n+1}	S, R
0	0	0 X
0	1	1 0
1	0	0 1
1	1	X 0

Characteristic equation for JK flip flop

Q_n	00	01	11	10
0	0	1	1	0
1	1	0	0	1

$$Q_{n+1} = S\bar{Q}_n + RQ_n$$

D-Flip flop:-

Lecture Notes.in

Characteristic Table:-

Clock	Input D	Present state Q_n	Next state Q_{n+1}	state
ATL	0	0	0	No Change
ATL	0	1	0	No Change
ATL	1	0	1	Toggle
ATL	1	1	0	Set
ATL	X			
ATL	X			

Truth Table:-

D	Q_{n+1}
0	0
1	1

Characteristic Equation:-

D	Q_n	1
0	0	0
1	1	1

$$Q_{n+1} = D$$

Excitation Table:-

Q_n	Q_{n+1}	D
0	0	0
0	1	1
1	0	0
1	1	1

JK - Flip Flop:-

Q1. Characteristic Table

Clock	Inputs J K	Present state Q_n	Next state Q_{n+1}	state
ATL	0 0	0	0	No change
ATL	0 0	1	1	No change
ATL	0 1	0	0	Reset
ATL	0 1	1	0	Set
ATL	1 0	0	1	Set
ATL	1 0	1	1	Set
ATL	1 1	0	0	Toggle.
ATL	1 1	1	0	Toggle.

Q_n	Q_{n+1}	J K
0	0	0 X
0	1	1 X
1	0	X 1
1	1	X 1

Truth Table:-

J K	Q_{n+1}
0 0	Q_n
0 1	0
1 0	1
1 1	\bar{Q}_n