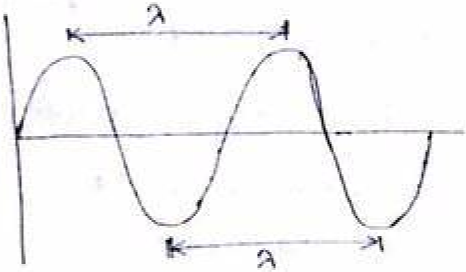
01/11/01/2016 ①

Refer : D.M. Pozar.



$\lambda = \frac{c}{f}$.

f = 300 MHz – 300 GHz (IEEE standard)

f = 1 GHz – 300 GHz (US standard)

- Microwave refers to alternating current signals with frequencies 300 MHz - 300 GHz (A/c to IEEE standard), with corresponding electrical wavelength between 1m to 1mm.
- A/c to US standard, wavelength is 30cm - 1mm.

Region	Frequency Range	Wavelength Range
Microwave	300 MHz - 300 GHz	1m - 1mm
Millimeter wave	30 GHz - 300 GHz	1cm - 1mm
Submillimeter	> 300 GHz	
Infrared	1000 GHz - 10000 GHz	0.3 mm - 30 μm
Visible	430000 GHz - 750000 GHz	700 nm - 400 nm

Approximate Band Designation:

L-band ⇒ 1-2 GHz
S-band ⇒ 2-4 GHz
C-band ⇒ 4-8 GHz
X-band ⇒ 8-12 GHz
Ku-band ⇒ 12-18 GHz
K-band ⇒ 18-26 GHz
Ka-band ⇒ 26-40 GHz
U-band ⇒ 40-60 GHz
Millimeter ⇒ 40-300 GHz

Q) What is 'different" about high frequency?

High frequency denote, the small wavelength i.e. wavelength is small compared to size of object.

In Broad sense, the high frequency is generally considered $f > 100MHz$ ($\lambda = 3m$ in air), because we live in a world of characteristic sizes of this wavelength.

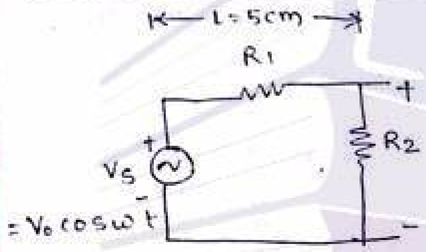
For example:

Height of people (5-6 feet) - (1.66 - 2) mtr

~~weight~~ length of car (8-12 feet) - (2.66 - 4) mtr

Height of building (10-30 feet) - (3.33 - 10) mtr

Lumped & Distributed Circuit:



$$V_{out} = \frac{R_2}{R_1+R_2} V_s$$

Consider the discrete ckt as shown in above fig:

Case 1 : $f = 20KHz$

$$\lambda = \frac{c}{f} = \frac{3\times10^{10}}{20\times10^3} = 15\,000m = 15\times10^6 cm$$

$$\frac{L}{\lambda} = \frac{5cm}{15\times10^6 cm} = 3.3\times10^{-6} << 1$$

The circuit length is very short compared to wavelength. Low frequency approximations are applicable. Hence,

$$V_{out} = \frac{R_2}{R_1+R_2} V_s.$$

Case 2: $f = 6GHz$

$$\lambda = \frac{c}{f} = \frac{3\times10^{10}}{6\times10^9} = 5cm$$

$$\frac{L}{\lambda} = \frac{5cm}{5cm} = 1$$

The ckt length is equal to wavelength. Hence, low frequency approximation is not applicable.

Case 3:

$f = 300\,GHz$

$\lambda = \frac{3\times10^{10}}{300\times10^{9}} = 0.1\,cm$

$\frac{L}{\lambda} = \frac{5cm}{0.1cm} = 50 >> 1$

The circuit length is greater than wavelength. So low frequency approximation is not applicable.

- From case(2 & 3)

- The voltage of line will be function of position along line.
- Practically the circuit will not work since it will radiate energy to space.
- Dispersion will degrade the signal before it reachs the output.
- Since, the circuit is sensitive to length dependent impedance, which must be carefully matched for efficient power transfer.

Case 4:

$f = 6\,GHz$, $L = 1.666\times10^{-6}\,cm$

$\lambda = 5\,cm$

$\frac{L}{\lambda} = 3.3\times10^{-6} << 1$.

- Because of the high frequencies (short wavelengths), standard ckt theory generally can't be used directly to solve microwave network problems (valid only Maxwell's eq.).

- Microwave components are often distributed elements where the phase of the voltage or current changes significantly over the physical extent of the device. Because, the device dimensions are of the order of microwave length.

- A field theory solution generally provides a complete description of the electromagnetic field, At every point in the space, which is usually much more informative i.e. power, impedance, voltage & current which can be expressed interms of circuit theory concepts.

Advantages of Microwave:

1) Antenna gain $\propto$ frequency.

$$\text{gain } (G) = \frac{4\pi A_e}{\lambda_e}$$

$$f \propto \frac{1}{\lambda}$$

2) Narrow beam width for higher frequency or

Beam width $\propto \frac{1}{f}$ i.e. more directivity (it is the measure of power density in the intended direction of radiation.).

3) More bandwidth (information carrying capacity) can be realized at high frequencies.

Example: Let us consider a number of 4kHz wide voice signal has to transmitte with the wireless Line. There are two wireless system, one is operating at 500MHz and another is 4GHz each with a 10% bandwidth around its centre frequency.

$$\text{No. of channels} = \left[\frac{\text{operating frequency of the system} \times \text{\% of bandwidth around centre freq}}{\text{Bandwidth per channel}}\right]$$

For 500MHz,

$$\text{No. of channels} = \frac{500\times 10^6\times 0.1}{4\times 10^3} = \frac{5}{4}\times 10^4 = 12500$$

For 4GHz,

$$\text{No. of channels} = \frac{4\times 10^9\times 0.1}{4\times 10^3} = 10^5$$

4) The effective reflection area (RADAR crossection) of a RADAR target is usually proportional to the target of electrical size.

5) Microwave signals travell by line of sight and are not bend by ionosphere, as are lower frequency signals. Hence, low ~~dispersion~~ distortion and attenuation.

Disadvantages:

1) LOS affected by obstracles such as hillocks, mountains, tall buildings etc.

2) Attenuation due to atmospheric disturbances rain, fog, snow etc.

3) Easily reflected from flat and metal surfaces.

4) It gets diffracted from solid objects.

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Applications: (Read from D.M. Pozar or sec-B Notes)

Various molecular, atomic and nuclear resonance ~~of~~ occur at microwave frequencies creating a variety of unique applications in the area of basic science, medical diagnosis, remote sensing and also heating method. RADAR systems are used for detecting and locating air, ground or sea going targets and for air traffic control system missions, tracking RADAR, automobile collision avoidance system whenever prediction, motion detector & wide varity of remote sensing. International and other long-haul telephone data and TV transmission. Wireless communication systems PCSs, WLANs, CV, GPS.

Maxwell's Equation :

$$\nabla\cdot\vec{D} = \rho \quad \oint_S \vec{D}\cdot ds = \int_V \rho\, dv$$

$$\nabla\cdot\vec{B} = 0 \quad \oint_S \vec{B}\cdot ds = 0$$

$\left.\begin{matrix}\nabla\cdot\vec{D} = \rho \\ \nabla\cdot\vec{B} = 0\end{matrix}\right\}$ static eq's

$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad \oint_C \vec{E}\cdot d\ell = -\int \frac{\partial B}{\partial t}\, d\vec{a}$$

$$\nabla\times\vec{H} = J + \frac{\partial D}{\partial t} \quad \oint_C \vec{H}\cdot d\vec{l} = \int_S \left(\vec{J} + \frac{\partial\vec{D}}{\partial t}\right) d\vec{a}$$

$\left.\begin{matrix}\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{H} = J + \frac{\partial D}{\partial t}\end{matrix}\right\}$ dynamic eq

(Differential form)

where $\rho \rightarrow$ charge density

$D \rightarrow$ Electric flux

$D = \epsilon E$.

$B \rightarrow$ Mag. flux density

Ampere Circuital Law : $H \times \ell = NI$

i.e. number of turns multiplied by current is the amount of magnetic field at distance 'ℓ'.

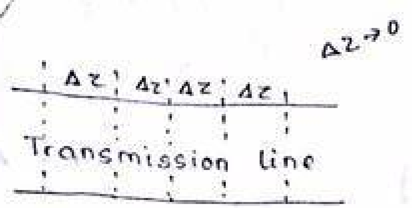
TRANSMISSION LINE :

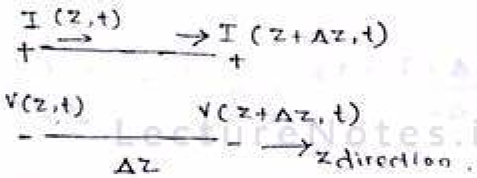
1) A transmission line is a two wire cable used to convey RF energy bet. two different pieces of communication equipment or bet. an antenna and a receiver or transmitter

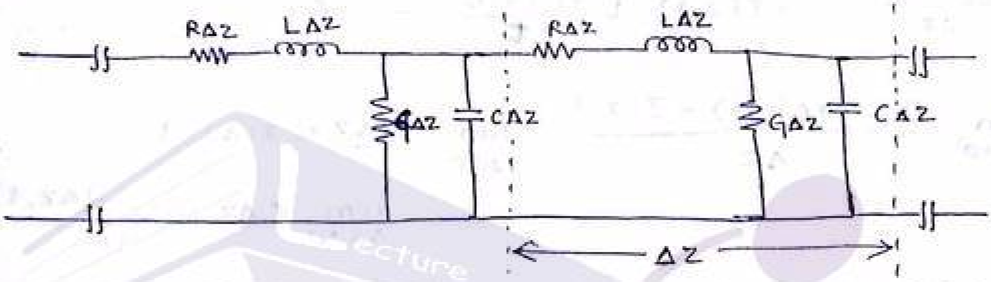
2) A transmission Line is a distributed parameter network where voltages and current can vary in magnitude and phase over its length. The physical dimensions of the transmission line may be considerable fraction of wavelength or many wavelength in size.

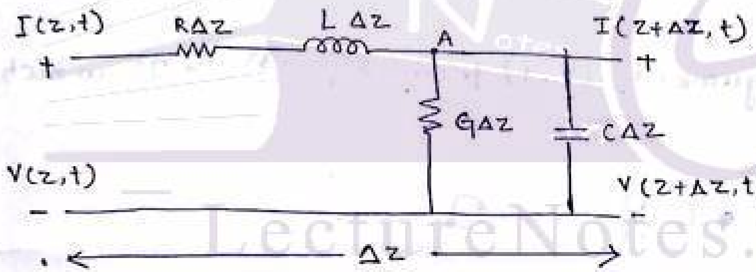
e.g.- parallel wires, microstripe lines, co-axial lines, rectangular waveguides, cylindrical waveguides.

The Lumped-element model for transmission line : ⑦









$R \rightarrow \Omega/m$

$L \rightarrow H/m$

$G \rightarrow$ mho/m, S/m

$C \rightarrow F/m$.

The series inductance L represents the total self inductance of the two conductors and shunt capacitance C is due to the close proximity of the two conductors.

The series resistance R represents the resistance due to finite conductivity of the conductors and shunt conductance G is due to dielectric loss in the material between the conductors. R & G are loss elements of transmission line.

Applying KVL to the small ckt, we have

$$V(z,t) - R\Delta z\, I(z,t) - L\Delta z \cdot \frac{dI(z,t)}{dt} - V(z+\Delta z, t) = 0 \quad \text{— ①}$$

Applying KCL to node A,

$$I(z,t) - G\Delta z\, V(z+\Delta z,t) - C\frac{dV(z+\Delta z),t}{dt} - I(z+\Delta z,t) = 0 \quad \text{--- (2)}$$

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Dividing eq. (1) & (2) by Δz individually, and taking limit $\Delta z \to 0$, we have,

$$\lim_{\Delta z\to 0}\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = \lim_{\Delta z\to 0}\frac{-R\Delta z\, I(z,t)}{\Delta z} - \lim_{\Delta z\to 0}\frac{L\Delta z\,\partial I(z,t)}{\Delta z\,\partial t}$$

$$\Rightarrow \frac{\partial V(z,t)}{\partial z} = -RI(z,t) - L\frac{\partial I(z,t)}{\partial t} \quad \text{--- (3)}$$

$$\lim_{\Delta z\to 0}\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = \lim_{\Delta z\to 0} -G\Delta z V(z+\Delta z,t) - \lim_{\Delta z\to 0} C\Delta z\frac{\partial V(z+\Delta z,t)}{\partial t}$$

$$\Rightarrow \frac{\partial I(z,t)}{\partial t} = -GV(z,t) - C\frac{\partial V(z,t)}{\partial t} \quad \text{--- (4)}$$

Omitting the argument (z,t) from eq. (3) & (4) which is understood,

$$\boxed{\begin{aligned}\frac{\partial V}{\partial z} &= -RI - L\frac{\partial I}{\partial t} \quad \text{--- (5)}\\ \frac{\partial I}{\partial z} &= -GV - C\frac{\partial V}{\partial t} \quad \text{--- (6)}\end{aligned}}$$

Eq. (5) & (6) are time domain form of Transmission Line known as Telegrapher eq.

For sinusoidal steady state condition with cosine based phasors, we can derive eq. (5) & (6) as

$$\frac{dV}{dz} = -RI - j\omega LI = -(R+j\omega L)I = ZI \quad \text{--- (7)} \qquad \left[\because \frac{\partial}{\partial t} = j\omega\right]$$

$$\frac{dI}{dz} = -(G+j\omega C)V = YV \quad \text{--- (8)}$$

Wave Propagation on a transmission line:

Differentiating eq (7) & (8) w.r.to z, it gives wave eq. for V & I (making voltage dependent voltage & current dependent current).

$$\frac{d^2V}{dz^2} = -(R+j\omega L)\frac{dI}{dz} = -(R+j\omega L)\left[-(G+j\omega C)V\right]$$

$$\frac{d^2V}{dz^2} = (R+j\omega L)(G+j\omega C)V = \gamma^2 V \quad \text{---} \ (9)$$

Similarly

$$\frac{d^2I}{dz^2} = (R+j\omega L)(G+j\omega C)I = \gamma^2 I \quad \text{---} \ (10)$$

where γ = Propagation constant dependent on freq.

$$= \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= \alpha + j\beta$$

where α - attenuation constant (neper/m)

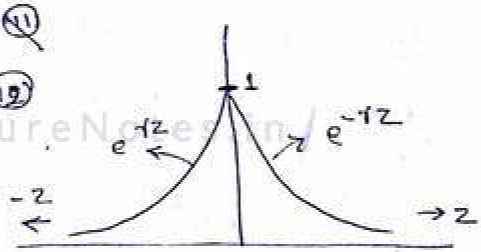
β - Phase constant (rad/m).

Solution to the transmission line eq:

Solution to the eq (9) & (10), can be written as

$$\cancel{V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}} \quad \text{---} \ (11)$$

$$\cancel{I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}} \quad \text{---} \ (12)$$

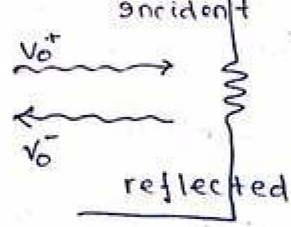


$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad \text{---} \ (11)$$

$$I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad \text{---} \ (12)$$

V(z) in time domain,

$$V(z,t) = |V_0^+| \cos(\omega t - \beta z + \phi^+) e^{-\alpha z}$$
$$+ |V_0^-| \cos(\omega t + \beta z + \phi^-) e^{\alpha z}$$



Q) The current in Transmission Line is given as $i(t) = 1.2\cos(1.51\times10^{10}t - 80.3z)$. Determine the wavelength, dielectric constant, phasor representation of this current.

Wavelength:

$\omega = 2\pi f = 1.5\times10^{10}$

$f = 2.4 GHz$

$\beta = \frac{2\pi}{\lambda}$

$\Rightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{80.3} = 0.07782 m$.

Dielectric constant:

$v_p = \frac{\omega}{\beta} = \frac{1.5\times10^{10}}{80.3} = 1.88\times10^{8} m/s$

$\varepsilon_r = \sqrt{\frac{c}{v_p}} = 1.263$

Phasor Representation:

$I = 1.2\angle -80.3z$.

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Differentiate eq. (11) & (13) w.r.to z,

$$\frac{dV(z)}{dz} = -\gamma(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}) \quad \text{— (13)}$$

$$\frac{dI(z)}{dz} = -\gamma(I_0^+ e^{-\gamma z} - I_0^- e^{\gamma z}) \quad \text{— (14)}$$

On comparing eq. (13) - (7) & (14) - (8)

we have,

$$-(R+j\omega L)\, I(z) = -\gamma(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z})$$

$$\Rightarrow I(z) = \frac{\gamma}{R+j\omega L}(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}) = \frac{1}{Z_0}(V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z})$$

where $\frac{\gamma}{R+j\omega L}$ = conductance & Z_0 = characteristic impedance.

$$I(z) = \left(\frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}\right)$$

$$Z_0 = \frac{R+j\omega L}{\gamma} = \frac{R+j\omega L}{\sqrt{(R+j\omega L)(G+j\omega C)}}$$

VVI

$$\Rightarrow \boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}}$$

$\lambda = \frac{2\pi}{\beta}$ = wavelength of line

$V_p = \frac{\omega}{\beta} = \lambda f$ = phase velocity

Characteristics impedance, propagation constant, general soln of voltage, current, wavelength, phase velocity:

Medium of Transmission line:

- Lossless
- Distortionless
- Low-loss line
- Lossy.

Lossless Transmission Line ($R = G = 0$):

1. Characteristic Impedance:

$$\boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}}}$$

2. Propagation constant:

$$\boxed{\begin{aligned}\gamma &= \sqrt{(R+j\omega L)(G+j\omega C)} \\ &= j\omega\sqrt{LC}\end{aligned}}$$

$\gamma = \alpha + j\beta$

β = propagation costant

$$\Rightarrow \boxed{\beta = \omega\sqrt{LC}}$$

$\alpha = 0$

3) Voltage eq^n :

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$\Rightarrow \boxed{V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}}$$

4) Current eq^n :

$$\boxed{I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}}$$

5) Wavelength :

$$\boxed{\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}}$$

6) Phase Velocity :

$$\boxed{V_P = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}}$$

Never asked ✗

Distortionless $\left(\frac{R}{L} = \frac{G}{C}\right)$:

1) Characteristic Impedance :

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{R\left(1 + j\omega L/R\right)}{G\left(1 + j\omega C/G\right)}}$$

$$\Rightarrow Z_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

2) Propagation Constant :

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$= \sqrt{RG\left(1 + \frac{j\omega L}{R}\right)\left(1 + j\omega\frac{C}{G}\right)}$$

$$= \left(1 + j\omega\frac{L}{R}\right)\sqrt{RG} = \sqrt{RG} + j\omega\sqrt{\frac{G}{R}}\cdot L = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\cancel{\boxed{\left(1 + j\omega\frac{C}{G}\right)\sqrt{RG}}}$$

Here $\alpha = \sqrt{RG}$

$\beta = \omega\sqrt{LC}$

3) Voltage eq^n:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

4) Current eq^n:

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} + \frac{V_0^-}{Z_0} e^{\gamma z} = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

5) Wave length:

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}$$

6) Phase velocity:

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

VVI

Low-Less Line ($R \ll \omega L$ & $G \ll \omega C$):

1) Characteristic Impedance:

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{j\omega L\left(1+\frac{R}{j\omega L}\right)}{j\omega C\left(1+\frac{G}{j\omega C}\right)}} = \sqrt{\frac{L}{C}}\left(1+\frac{R}{j\omega L}\right)^{1/2}\left(1+\frac{G}{j\omega C}\right)^{-1/2}$$

$$\Rightarrow Z_0 = \sqrt{\frac{L}{C}}\left(1+\underbrace{\frac{R}{2j\omega L}}_{\ll 1}\cdots\right)\left(1-\underbrace{\frac{G}{2j\omega C}}_{\ll 1}+\cdots\right)$$ [Applying binomial theorm]

$$\Rightarrow \boxed{Z_0 = \sqrt{\frac{L}{C}}}$$

2) Propagation constant:

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

$$= j\omega\sqrt{LC}\sqrt{\left(1+\frac{R}{j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)}$$

$$= j\omega\sqrt{LC}\left(1+\frac{R}{2j\omega L}\right)\left(1+\frac{G}{j\omega C}\right)$$

$$= j\omega\sqrt{LC}\left[1+\frac{1}{2}\left(\frac{R}{j\omega L}+\frac{G}{j\omega C}\right) - \cancel{\frac{RG}{4\omega^2 LC}}^{\,0}\right]$$

$$= j\omega\sqrt{LC} + \frac{1}{2}\sqrt{LC}\left(\frac{R}{L}+\frac{G}{C}\right)$$

$= j\omega\sqrt{LC} + \frac{1}{2}\left[\sqrt{\frac{C}{L}}R + \sqrt{\frac{L}{C}}G\right]$

$= j\beta + \alpha$

Here $\beta = \omega\sqrt{LC}$

$$\alpha = \frac{1}{2}\left[\underbrace{\sqrt{\frac{C}{L}}\cdot R}_{\substack{\alpha_c \\ \text{attenuation} \\ \text{due to} \\ \text{conductor}}} + \underbrace{\sqrt{\frac{L}{C}}\cdot G}_{\substack{\alpha_d \\ \text{attenuation} \\ \text{due to} \\ \text{dielectric}}}\right]$$

3) Voltage eq^n:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

4) Current eq^n:

$$I(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} + \frac{V_0^-}{Z_0} e^{\gamma z}$$

$$= I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

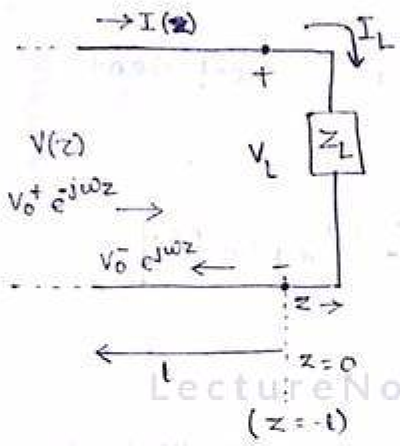
5) Wavelength:

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}}$$

6) Phase Velocity:

$$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

The Terminated Lossless Transmission Line:



Assume that incident wave of the form $V_0^+ e^{-j\omega z}$ is generated from a source $z<0$.

When the line is terminated in an arbitary load i.e. $Z_L \neq Z_0$, the ratio of voltage to the current at the load must be Z_L.

$$V(z) = V_0^+ e^{-j\omega z} + V_0^- e^{j\omega z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\omega z} - \frac{V_0^-}{Z_0} e^{j\omega z}$$

$$Z_L = \frac{V(z=0)}{I(z=0)} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

$$\Rightarrow \frac{Z_L}{Z_0} = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}$$

By applying componendo & dividendo,

$$\boxed{\frac{Z_L - Z_0}{Z_L + Z_0} = \frac{V_0^-}{V_0^+}}$$

a) Reflection coefficient (Γ):

The ratio betn reflected voltage to the incident voltage wave is known as voltage reflection coefficient.

$$\boxed{\Gamma = \Gamma(0) = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

$Z_L = 0 \Rightarrow$ short circuit

$Z_L = \infty \Rightarrow$ open circuit

For $Z_L = 0$, $\Gamma = -1$, range $-1 < \Gamma < 1$

For $Z_L = \infty$, $\Gamma = \dfrac{1 - Z_0/Z_L}{1 + Z_0/Z_L} = 1$, range $0 < \Gamma < 1$

For $Z_L = Z_0 \Rightarrow$ impedance matching

$\Gamma = 0$.

Voltage & current interms of reflection coefficient can be written as:

$$V(z) = V_0^+\left[e^{-j\beta z} + \frac{V_0^-}{V_0^+}e^{j\beta z}\right] = V_0^+\left[e^{j\beta z} + \Gamma e^{j\beta z}\right]$$

$$I(z) = \frac{V_0^+}{Z_0}\left[e^{-j\beta z} - \Gamma e^{j\beta z}\right]$$

The time average power flow along the line of the point 'z' can be

$$P_{avg} = \frac{1}{2}\text{Re}\left[V(z)I^*(z)\right]$$

$$= \frac{1}{2}\frac{|V_0^+|^2}{Z_0}\text{Re}\left[\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)\left(e^{j\beta z} - \Gamma^* e^{-j\beta z}\right)\right]$$

$$= \frac{1}{2}\frac{|V_0^+|^2}{Z_0}\text{Re}\left[1 - \Gamma^* e^{-2j\beta z} + \Gamma e^{2j\beta z} - |\Gamma|^2\right]$$

$A - A^* = 2j\,\text{im}(A)$ = Purely imaginary.

$$P_{avg} = \frac{1}{2}\frac{|V_0^+|^2}{Z_0}\left[1 - |\Gamma|^2\right]$$

$$= \underbrace{\frac{1}{2}\frac{|V_0^+|^2}{Z_0}}_{\text{Incident power}} - \underbrace{\frac{1}{2}\frac{|V_0^+|^2}{Z_0}|\Gamma|^2}_{\text{Reflected power}}$$

Therefore total power delivered to load is equal to incident power.

b) <u>Return loss:</u>

The return loss of a device is defined as the ratio of the reflected power to the incident power at its input.

$$\text{Return loss} = \frac{\text{Related Power}}{\text{Incident Power}} = \frac{\frac{|V_0^+|^2}{2Z_0}|\Gamma|^2}{\frac{|V_0^+|^2}{2Z_0}} = |\Gamma|^2$$

$$\Rightarrow 10\log|\Gamma|^2 = 20\log\Gamma \text{ dB}.$$

c) Insertion loss (Transmission Loss):

Insertion loss of a device is defined as ratio of the ~~reflected power to the~~ incident power at its ~~input~~ power transmitted (i.e. power avialable at the output port) to that of power incident at its input.

$$P_{tr} = P_{inc} - P_{ref} = \frac{|V_0^+|^2}{2Z_0}\left[1-|\Gamma|^2\right]$$

$$I.L \text{ or } T.L = \frac{P_{tr}}{P_{inc}} = 1-|\Gamma|^2$$

$$\Rightarrow 10\log\left[1-|\Gamma|^2\right] \approx -20\log|\Gamma|^2$$

Transmission coefficent $T = 1+\Gamma$

$$\boxed{T = 1+\frac{Z_L - Z_0}{Z_L+Z_0} = \frac{2Z_L}{Z_L+Z_0}}$$

d) Voltage standing wave ratio (VSWR):

When load mismatched, i.e. $Z_L \neq Z_0$, the presence of reflected wave leads to standing waves where magnitude of voltage on the line is constant.

$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$= V_0^+ e^{-j\beta z}\left[1+\frac{V_0^-}{V_0^+}e^{2j\beta z}\right]$$

$$|V(z)| = |V_0^+|\left[1+\Gamma e^{2j\beta z}\right]$$

$$|V(l)| = |V_0^+|\left[1+\Gamma e^{j\beta l}\right]$$

For $z = -l$,

$$V(l) = |V_0^+|\left[1+|\Gamma| e^{-j\theta} e^{-j2\beta l}\right] \quad \because \Gamma = |\Gamma| e^{j\theta}$$

$$\Rightarrow |V(l)| = V_0^+|\left[1+|\Gamma| e^{j(\theta-2\beta l)}\right]$$

Maximum voltage occur when,

$$e^{j(\theta-2\beta l)} = e^0 = 1$$

$$\therefore V_{max} = |V_0^+|\left[1+|\Gamma|\right]$$

Minimum voltage occurs when

$$e^{j(\theta - 2\beta l)} = -1$$

$$\therefore \; V_{min} = |V_0^+| \left[1 - |\Gamma|\right]$$

The ratio of max^m voltage to min^n voltage is known as voltage standing wave ratio (VSWR).

$$VSWR = \frac{V_{max}}{V_{min}} = \frac{[1+|\Gamma|]V_0^+}{[1-|\Gamma|]V_0^+}$$

$$\Rightarrow \boxed{S = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

for $0 < |\Gamma| < 1$, $1 < S < \infty$.

The distance between two successive voltage maxima or minima can be given by:

For ~~maxima~~ l_{max}, $e^{j(\theta - 2\beta l)} = e^0 = 1$

$$\Rightarrow \theta - 2\beta l = 0$$

$$\Rightarrow l = \frac{\theta}{2\beta}$$

$$\theta = 2\pi, \; \beta = \frac{2\pi}{\lambda}$$

$$\therefore \; l_{max} = \frac{2\pi}{2 \times \frac{2\pi}{\lambda}}$$

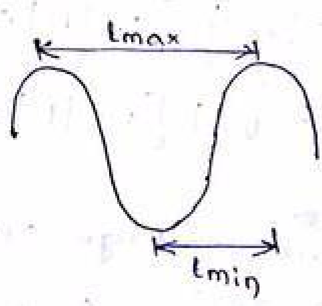
$$\Rightarrow \boxed{l_{max} = \frac{\lambda}{2}}$$

The distance between a minima & maxima can be given by:

$$\theta = \pi, \; \beta = \frac{2\pi}{\lambda}$$

$$\therefore \; l_{min} = \frac{\pi}{2 \times \frac{2\pi}{\lambda}}$$

$$\Rightarrow \boxed{l_{min} = \frac{\lambda}{4}}$$



Reflection Coefficient:

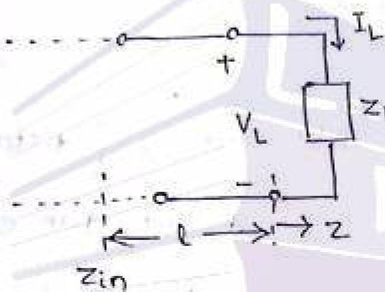
The reflection coefficient at any point 'l' on the line i.e. $z=-l$, is the ratio of the reflected component to the incident component.

$V_0^+ e^{-j\beta z} \longrightarrow$ incident $\qquad V_0^- e^{j\beta z} \longleftarrow$ reflected

$$\Gamma' = \frac{V_0^- e^{-j\beta l}}{V_0^+ e^{j\beta l}} \qquad \text{Put } z=-l$$

$$= \Gamma(0)\, e^{-2j\beta l}$$

e) Input Impedance (Z_{in}):



$(l=-z)$

The input impedance at any point looking towards the load is given by,

$$Z_{in} = \frac{V(z)}{I(z)} = \frac{V(z=l)}{I(z=-l)} = \frac{V_0^+ [e^{j\beta l} + \Gamma e^{-j\beta l}]}{\frac{V_0^+}{Z_0}[e^{j\beta l} - \Gamma e^{-j\beta l}]}$$

$$\left[\begin{array}{l} \because V(z) = V_0^+ [e^{-j\beta z} + \Gamma e^{j\beta z}] \\ I(z) = \frac{V_0^+}{Z_0}[e^{-j\beta z} - \Gamma e^{j\beta z}] \end{array}\right]$$

$$Z_{in} = \frac{Z_0 \left[e^{j\beta l} + \frac{Z_L - Z_0}{Z_L + Z_0} e^{-j\beta l}\right]}{e^{j\beta l} - \frac{Z_L - Z_0}{Z_L + Z_0} e^{-j\beta l}}$$

$$= \frac{Z_0[(Z_L + Z_0)e^{j\beta l} + (Z_L - Z_0)e^{-j\beta l}]}{(Z_L + Z_0)e^{j\beta l} - (Z_L - Z_0)e^{-j\beta l}}$$

$$= Z_0 \left[\frac{Z_L(e^{j\beta l} + e^{-j\beta l}) + Z_0(e^{j\beta l} - e^{-j\beta l})}{Z_L(e^{j\beta l} - e^{-j\beta l}) + Z_0(e^{j\beta l} + e^{-j\beta l})}\right]$$

$$= \frac{Z_0 [Z_L \cos\beta l + jZ_0 \sin\beta l]}{jZ_L \sin\beta l + Z_0 \cos\beta l}$$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] \rightarrow \text{for lossless}$$

$$Z_{in} = \frac{Z_L + Z_0 \tanh\gamma l}{Z_0 + Z_L \tanh\gamma l} \longrightarrow \text{lossy medium.}$$

Properties:

1) Line characteristics repeats after every λ value.

$$Z\left(l + \frac{\lambda}{2}\right) = Z(l)$$

$$Z_{in}\left(l + \frac{\lambda}{2}\right) = Z_0 \frac{Z_L + jZ_0 \tan\beta\left(l + \frac{\lambda}{2}\right)}{Z_0 + jZ_L \tan\beta\left(l + \frac{\lambda}{2}\right)}$$

$$\tan\left(\beta l + \beta\frac{\lambda}{2}\right) = \tan\beta l$$

2) Normalized impedance inverse after every $\frac{\lambda}{4}$ distance.

$$\bar{Z}\left(l + \frac{\lambda}{4}\right) = \frac{1}{\bar{Z}(l)} \qquad \frac{Z_{in}}{Z_0} = \text{normalized impedance.}$$

3) For load impedance, $Z_{in} = Z_0$, the impedance at any point on the line is:

$$Z(l) = Z_0$$

Not imp

= Transmission line as a circuit element:

Transmission lines don't only behave as communication channel betn. transmitter & receiver but section of transmission line acts as a circuit element also.

1. Short circuit line (T.L. terminated on s.c. load):

A(a)

$$Z_L = 0, \quad \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \infty$$

$$V(z) = V_0^+ \left[e^{-j\beta z} - e^{j\beta z}\right] = -2jV_0^+ \sin\beta z$$

$$I(z) = \frac{V_0^+}{Z_0}\left[e^{-j\beta z} + e^{j\beta z}\right] = \frac{2V_0^+}{Z_0} \cos\beta z$$

$$Z_{in} = \frac{V(-l)}{I(-l)} = \frac{2jV_0^+ \sin\beta l}{\frac{2V_0^+}{Z_0}\cos\beta l}$$

$$\Rightarrow \quad Z_{in} = jZ_0 \tan\beta l \quad \text{——— ①}$$

2. Open circuit line (T.L. terminated in o.c. load):

$$Z_L = \infty, \quad \Gamma = 1, \quad VSWR = \infty$$

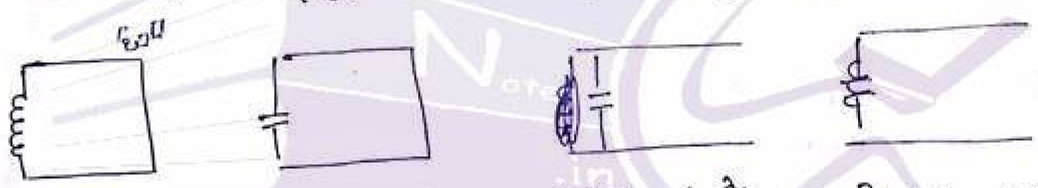
$$V(z) = V_0^+\left[e^{-j\beta z} + e^{j\beta z}\right] = 2V_0^+\cos\beta z$$

$$I(z) = \frac{V_0^+}{Z_0}\left[e^{-j\beta z} - e^{j\beta z}\right] = -2j\frac{V_0^+}{Z_0}\sin\beta z$$

$$Z_{in} = \frac{V(-l)}{I(-l)} = \frac{2V_0^+\cos\beta l}{\frac{2jV_0^+}{Z_0}\sin\beta l} = -jZ_0\cot\beta l \quad \text{— ②}$$

$$Z_0 = \sqrt{(Z_{in})_{s.c}\,(Z_{in})_{o.c}}$$

$$l_{s.c.} = \frac{1}{\beta}\tan^{-1}\left(\frac{X}{Z_0}\right) \qquad l_{o.c} = \frac{1}{\beta}\cot^{-1}\left(\frac{-X}{Z_0}\right)$$



$0 < l_{sc} < \lambda/4$ $\qquad \frac{\lambda}{4} < l_{sc} < \frac{\lambda}{2}$ $\qquad 0 < l_{oc} < \lambda/4$ $\qquad \frac{\lambda}{4} < l_{oc} < \frac{\lambda}{2}$

$\Downarrow$

$\frac{\pi}{2}$

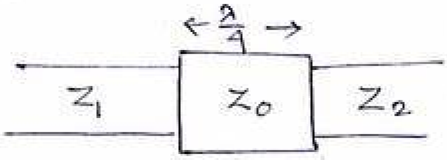
$jZ_0 = X_L \qquad -jZ_0 = X_C$ (capacitance)

Hence, if a line of length l is terminated in a short circuit or open circuit, the input impedance of transmission line is purely reactive.

Dt. 1/2/16

3) $\lambda/4$ line as a quarter wave impedance transfor:
When a section of transmission line is used either as reactance or as a resonant circuit, it is a 2 terminal network. However a section of line can be used as 4 terminal network, when it is inserted between the generator and the load. Such a line is known as

quaterwave transformer. Because it has the effect of transforming the load impedance in an inverse manner depending on the characteristic impedance of the line.



$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$$

$$= Z_0 \frac{\frac{Z_L}{\tan\beta l} + jZ_0}{\frac{Z_0}{\tan\beta l} + jZ_L}$$

$$= \frac{Z_0^2}{Z_L}$$

$$l = \frac{\lambda}{4}$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}$$

$$= \frac{\pi}{2}$$

$$\boxed{Z_0 = \sqrt{Z_{in} Z_L}}$$

Q7) Given $Z_L = 100\Omega$
LQ

SWR = 1.5

$Z_0 = ?$

$$S = SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$|\Gamma| = \frac{S-1}{S+1} = \frac{0.5}{2.5} = 0.2$$

$\Gamma = -0.2$

$\Gamma = +0.2$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$Z_0 = Z_L\left(\frac{1-\Gamma}{1+\Gamma}\right) = 100\left(\frac{0.8}{1.2}\right) = 66.67\Omega \quad [\because \Gamma = 0.2]$$

$$Z_0 = Z_L\left(\frac{1-\Gamma}{1+\Gamma}\right) = 100\left(\frac{1.2}{0.8}\right) = 150\Omega \quad [\because \Gamma = -0.2]$$

LQ 14=18

14) $z_0 = 75 + j0.01\,\Omega$

(b) $Z_L = 70 + j50$

$\Gamma = ?$ $T = ?$ SWR = ? Insertion loss = ?

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-5 + j\,49.99}{145 + j\,50.01}$$

$= 0.075 + 0.318j = 0.33\angle 76.73$

$$\frac{Pol(-5, 49.99)}{Pol(145, 50.01)} = \frac{50.24\angle 95.71^\circ}{153.38\angle 19.02^\circ} = 0.075 + 0.318j$$

$T = 1 + \Gamma$

$= 1 + 0.075 + j0.318$

$= 1.075 + j0.318$

$= 1.122\angle 16.47^\circ$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.33}{1 - 0.33} = 1.985$$

$IL = 1 - |\Gamma|^2$

$= 0.8911$

IL in dB $= 10\log(1 - |\Gamma|^2)$

$= -0.5\,dB$.

15) $l = 100m$
LQ
$L = 27.72\,\mu H$

$C = 18\,nF$

Lossless, so $\alpha = 0$

$V_P = ?$ $\beta = ?$ $f = 100kHz$ $z_0 = ?$

$$V_P = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

$= 1.41 \times 10^6\ m/s$

$\beta = \omega\sqrt{LC} = 2\pi f\sqrt{LC} = 2\pi \times 100 \times 10^3 \sqrt{27.72 \times 10^{-6} \times 18 \times 10^{-9}}$

$= 0.4438\ rad/m$.

$z_0 = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{L}{C}} = 39.243\,\Omega$

20) LQ $l = 25m$

$Z_L = 40 + j30\,\Omega$

$f = 10MHz$

$L = 310.4\,nH/m$

$C = 38.28\,pF/m$

$z_0 = ?$ $\beta = ?$

$z_0 = \sqrt{\frac{L}{C}} = 90.05\,\Omega$

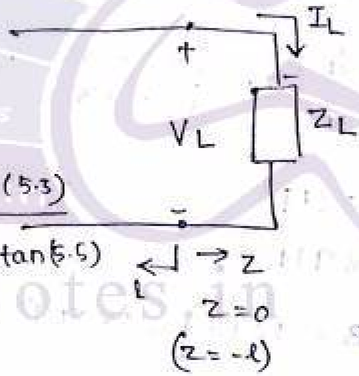
$\beta = 2\pi f\sqrt{LC}$

$= 2\pi \times 10 \times 10^{6}\sqrt{310.4\times10^{-9} \times 38.28\times10^{-12}}$

$= 0.22$ rad/m

$\beta l = 5.414$ rad

$Z_{in} = Z_0 \dfrac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$



$= 90.05 \dfrac{40+j30+j\,90.05\tan(5.3)}{90.05+j(40+j30)\tan(5.5)}$

$= 51.15\angle -37.78^\circ$

$= 0.449 - j0.348$

mid-point $\beta l = \beta \times \frac{25}{2} = 2.75$ rad

$Z_{in} = 35.26 - j0.63$

Q.27) LQ $P = 5kW$, $V = 500V$, $Z_L = 200 + j100\,\Omega$, Load disconnected

i) $\Gamma = ?$ (for Z_L) ii) $\Gamma = ?$ (no load)

i) $Z_0 = \dfrac{V^2}{P} = \dfrac{(500)^2}{5\times10^3} = 50\,\Omega$

$\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{200-50+j100}{200+50+j100} = \dfrac{150+j100}{250+j100}$

$\Gamma = 0.655 + 0.137j$

$= 0.67 \angle 11.885$

ii) $\Gamma = \dfrac{Z_L^{2\omega} - Z_0}{Z_L^{2\omega} + Z_0} = -1$

SWR $= \infty$.

Q.28) $Z_0 = 75\Omega$, $Z_L = 150 + j150$

$l = 0.375\lambda$

i) $\Gamma = ?$ ii) VSWR, Z_{in}, Z_{in} purely resistive, R

i) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{75 + j150}{225 + j150}$

$= 0.538 + 0.307j$

$= 0.62 \angle 29.76°$

ii) $VSWR = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = \dfrac{1 + 0.62}{1 - 0.62} = 4.263$

iii) $Z_{in} = Z_0 \dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$

$\beta l = \dfrac{2\pi}{\lambda} \times 0.375\lambda = 2.356$ rad.

$Z_{in} = 75 \times \left[\dfrac{150 + 150j + j75\tan(2.356)}{75 + j(150 + j150)\tan(2.356)}\right]$

$= 23.072 + 40.367j$

$= 46.495 \angle 60.25155$

iv) $V(l) = V_0^+ \left[1 + \Gamma e^{-2j\beta l}\right]$

$= V_0^+ \left[1 + |\Gamma| e^{j(\theta - 2\beta l)}\right]$

$\Gamma = |\Gamma| e^{j\theta}$

$\theta - 2\beta l = 0$

$l = \dfrac{\theta}{2\beta}$ rad

$= \dfrac{29.76 \times \frac{\pi}{180}}{2 \times 2\pi}$

$\Rightarrow l = 0.0413\lambda$

$Z_{in} = Z_0 \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$

$Z_{in_{max}} = Z_0 S = \dfrac{V_{max}}{I_{min}} = 319.725\Omega$

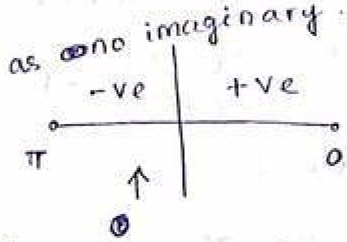
$Z_{in_{min}} = Z_0/S = \dfrac{V_{min}}{I_{max}} = \dfrac{V_0^+[1 - |\Gamma|]}{\frac{V_0^+}{Z_0}[1 + |\Gamma|]} = \dfrac{Z_0}{SWR}$

$I(l) = \dfrac{V_0^+}{Z_0}\left[1 - \Gamma e^{-2j\beta l}\right] = \dfrac{V_0^+}{Z_0}\left[1 - |\Gamma| e^{j(\theta - 2\beta)l}\right]$

Q18) For resistive termination on a lossless line
SR

$Z_L = R_L$ and $Z_0 = R_0$.

$\Gamma = \frac{R_L - R_0}{R_L + R_0}$. real value



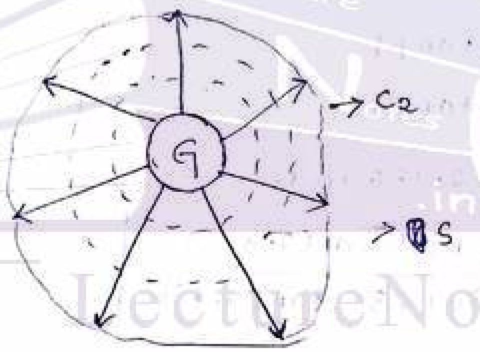
$R_L > R_0$

In this case, Γ is positive real and $\theta = 0^\circ$, so, $V(l)$ voltage is maximum and $I(l)$ current is minimum at the termination of the resistance.

$R_L < R_0$ then Γ is negative real and $\theta = \pi$ or $-\pi$ and voltage is minimum and current $I(l)$ maximum will occur at the terminating resistance.

Field Analysis of Transmission line :

(Derivation of Transmission Line parameters R, L, G & C in terms of electromagnetic field.)



Consider 1 metre section of uniform transmission line with fields $\vec{E}$ and $\vec{H}$ when S is the cross-sectional surface area of the line let voltage between the conductors be $V_0 e^{\pm j\beta z}$ and current be $I_0 e^{\pm j\beta z}$.

Step-1: To calculate inductance.

Magnetic energy of 1 metre section of the line is given by

$$w_m = \frac{\mu}{4} \int_S \vec{H} \cdot \vec{H}^* \, ds .$$

Circuit theory gives in terms of current :

$$w_m = \frac{1}{2} L I^2$$

$$= \frac{1}{2} L \left(\frac{I_0}{\sqrt{2}}\right)^2 = \frac{1}{4} L I_0^2$$

By comparing,

$$\boxed{L = \frac{\mu}{|I_0|^2} \int_S \vec{H} \cdot \vec{H}^* \, ds} \ \text{H/m}.$$

Step 2: To calculate capacitance.

The electrical energy stored per unit length for 1 metre section of the line can be given by,

$$W_e = \frac{\epsilon}{4} \int_S \vec{E} \cdot \vec{E}^* \, ds.$$

Circuit Theory says energy is given by,

$$W_e = \frac{1}{2} C V^2 = \frac{1}{2} C \left(\frac{V_0}{\sqrt{2}}\right)^2 = \frac{1}{4} C V_0^2$$

By comparing,

$$\boxed{C = \frac{\epsilon}{|V_0|^2} \int_S \vec{E} \cdot \vec{E}^* \, ds} \ \text{F/m}$$

$$P_c = \frac{R_s}{2} \int_{C_1 + C_2} \vec{H} \cdot \vec{H}^* \, dl$$

$C_1 + C_2 \rightarrow$ conductor boundaries

Circuit Theory,

$$P_c = R \frac{|I_0|^2}{2}.$$

$$R_s = \frac{1}{\sigma \delta_s}.$$

$$R = \frac{R_s}{|I_0|^2} \int_{C_1 + C_2} \vec{H} \cdot \vec{H}^* \, dl \ \ \Omega/\text{m}$$

$$P_d = \frac{\omega \epsilon''}{2} \int_S \vec{E} \cdot \vec{E}^* \, ds.$$

$$G = \frac{\omega \epsilon''}{|V_0|^2} \int_S \vec{E} \cdot \vec{E}^* \, ds \ \ \text{S/m}.$$

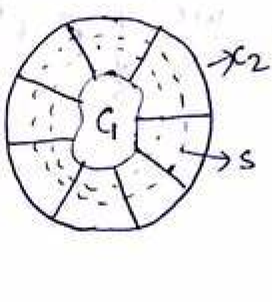
$$\epsilon = \epsilon' - j\epsilon'' = \epsilon'(1 - j\tan\delta)$$

where $P_d = \frac{G|V_0|^2}{2}$.

Dt. 5/2/16

Power loss per unit length due to finite conductivity of metallic conductor is given by,

$$P_c = \frac{R_s}{2} \int_{C_1 + C_2} \vec{H} \cdot \vec{H}^* \, dl$$



where R_s = surface resistivity

$$R_s = \frac{1}{\sigma \delta_s} = \sqrt{\frac{\omega \mu}{2\sigma}}$$

where δ_s = skin depth

$$= \frac{1}{\sqrt{\pi \mu \sigma f}}$$

$$= \sqrt{\frac{2}{\omega \mu \sigma}}$$

Note: Whenever electromagnetic wave penetrates a good conductor it decays experimentally. The distance below the conductor surface at which the current density as fall to $\frac{1}{e \text{ or } 37}$ of its initial value is known as skin depth.

To avoid, a very thin good conductor plating is done.

$$P_c = I^2 R$$

$$P_c = \left(\frac{I_0}{\sqrt{2}}\right)^2 R \text{ (for RMS value)}$$

$$P_c = \frac{1}{2} I_0^2 R$$

By comparing,

$$R = \frac{R_S}{I_0^2} \int_{C_1 + C_2} \vec{H} \cdot \vec{H}^* dL$$

For conductance power dissipated per unit length in long dielectric which is lossy can be given by:

$$P_d = \frac{\omega \epsilon''}{2} \int_S \vec{E} \cdot \vec{E}^* ds.$$

where, ϵ'' is imaginary part of complex dielectric

$$\therefore \epsilon = \epsilon' - j\epsilon'' = \epsilon'\left(1 - \frac{j\epsilon''}{\epsilon'}\right)$$

$$= \epsilon'(1 - j\tan\delta)$$

$$\tan\delta = \frac{\epsilon''}{\epsilon'} \text{ (loss tangent)}$$

$$P_d = GV^2 = \frac{1}{2} G V_0^2$$

$$G = \frac{\omega \epsilon''}{V_0^2} \int_S \vec{E} \cdot \vec{E}^* ds.$$

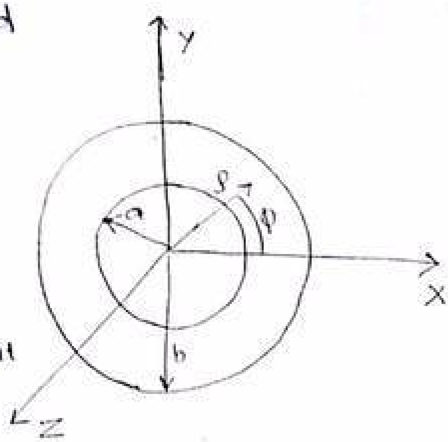
(30)

Q) The fields of a travelling TEM wave inside the co-axial line is shown in figure

The figure can be expressed

$$\bar{E} = \frac{V_0 \hat{\rho}}{\rho \ln b/a} e^{-\gamma z} \cdot \bar{H}$$

$$= \frac{I_0 \hat{\Phi}}{2\pi \rho} e^{-\gamma z}$$



$\gamma \rightarrow$ Propagation constant of the line.

The conductors are assumed to have a surface sensitivity R_s and the material feeling the space between the conductors. Assumed to have complex conductivity $\epsilon = \epsilon' - j\epsilon''$ and permeability $\mu = \mu_0 \mu_r$

Determine the transmision line parameter.

$$L = \frac{\mu}{|I_0|^2} \int_S \bar{H} \cdot \bar{H}^* ds \qquad ds = \rho d\phi \cdot d\rho$$

$$= \frac{\mu}{|I_0|^2} \iint \frac{I_0^2}{(2\pi\rho)^2} \rho \, d\phi \, d\rho$$

$$= \frac{\mu}{\cancel{I_0^2}} \int_0^{2\pi} d\phi \int_{\rho=a}^{\rho=b} \frac{\cancel{I_0^2}}{(2\pi\rho)^2} \rho \, d\rho$$

$$= \frac{\mu}{4\pi^2} \int_0^{2\pi} d\phi \int_{\rho=a}^{\rho=b} \frac{d\rho}{\rho}$$

$$= \frac{\mu}{4\pi^2} (2\pi - 0)(\ln b - \ln a)$$

$$= \frac{\mu}{2\pi} \ln \frac{b}{a} \quad H/m$$

Similarly $C = \frac{\epsilon'}{|V_0|^2} \int_S \bar{E} \cdot \bar{E}^* ds$

$$= \frac{\epsilon'}{\cancel{|V_0|^2}} \int_S \frac{\cancel{V_0^2}}{(\rho \ln b/a)^2} \rho \, d\phi \cdot d\rho$$

$$= \frac{\epsilon'}{(\ln b/a)^2} \iint \frac{1}{\rho} d\phi \cdot d\rho$$

$$= \frac{\epsilon'}{(\ln \frac{b}{a})^2} \int_0^{2\pi} d\phi \int_{\rho=a}^{\rho=b} \frac{d\rho}{\rho}$$

$$= \frac{2\pi\epsilon'}{\ln \frac{b}{a}} \text{ F/m}$$

$$R = \frac{R_s}{|I_0|^2} \int_{C_1+C_2} \bar{H}\cdot\bar{H}^* \, dl$$

$$= \frac{R_s}{|I_0|^2}\left[\int_0^{2\pi}\left(\frac{I_0}{2\pi\rho}\right)^2 a\, d\phi + \int_0^{2\pi}\left(\frac{I_0}{2\pi\rho}\right)^2 b\, d\phi\right]$$

$$= \frac{R_s}{2\pi}\left[\frac{1}{a}+\frac{1}{b}\right] \ \Omega/\text{m}$$

$$G = \frac{\omega\epsilon''}{(\ln b/a)^2} \int_{\phi=0}^{2\pi} \int_{\rho=a}^{b} \frac{1}{\rho^2}\, \rho\, d\rho\, d\phi$$

$$= \frac{2\pi\omega\epsilon''}{\ln \frac{b}{a}} \ \text{S/m}$$

Coaxial cable:

wave impedance $= \dfrac{E_\rho}{H_\phi} = \dfrac{\omega\mu}{\beta} = \sqrt{\dfrac{\mu}{\epsilon}} = \eta$

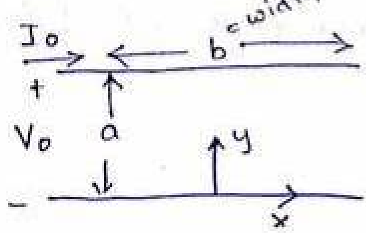
characteristic impedance $= \dfrac{V_0}{I_0} = \dfrac{E_\rho \ln b/a}{2\pi H_\phi} = \sqrt{\dfrac{\mu}{\epsilon}}\ \dfrac{\ln b/a}{2\pi}$

Power Flow $P = \frac{1}{2}\int_{S_0} \bar{E}\times\bar{H}^* \, ds$

$$= \frac{1}{2}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b} \frac{V_0 I_0^* \rho}{2\pi\rho^2 \ln b/a}\, d\rho\, d\phi$$

$$= \frac{1}{2} V_0 I_0^*$$

2) Parallel plate or Parallel wire T.L:



$E_y = \dfrac{-V_0}{a}$ V/m

$H_x = \dfrac{I_0}{b}$ A/m

$$L = \frac{\mu}{|I_0|^2}\int_S \bar{H}\cdot\bar{H}^* \, ds$$

$$= \frac{\mu}{|I_0|^2} \int_{x=0}^{a} \int_{y=0}^{b} \frac{I_0^2}{b^2}\, dx\, dy$$

$$= \frac{\mu a}{b} \ \text{H/m}$$

$$C = \frac{\epsilon}{|V_0|^2} \int_s \bar{E} \cdot \bar{E}^* \, ds$$

$$= \frac{\epsilon}{V_0^2} \int_0^a \int_0^b \frac{V_0^2}{a^2}\, dx\, dy$$

$$= \frac{\epsilon b}{a} \ \text{F/m}$$

$$R = \frac{R_s}{|I_0|^2} \int_{C_1 + C_2} (\bar{H})^2 \, dl$$

$$= \frac{2R_s}{|I_0|^2} \int (\bar{H})^2 \, dl$$

$$= \frac{2R_s}{I_0^2} \int_{x=0}^{b} \frac{I_0^2}{b^2}\, dx$$

$$= \frac{2R_s}{b} \ \Omega/\text{m}$$

$$G = \frac{\omega \epsilon''}{V_0^2} \int_s |\bar{E}|^2 \, ds$$

$$= \frac{\omega \epsilon''}{V_0^2} \int_0^a \int_0^b \frac{V_0^2}{a^2}\, dx\, dy$$

$$= \frac{\omega \epsilon'' b}{a} \ \mho/\text{m}$$

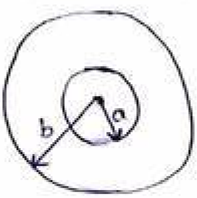
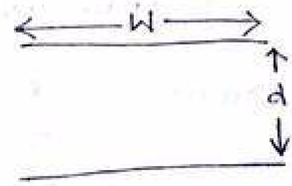
$$\eta = \frac{E_x}{H_x} = \sqrt{\frac{\mu}{\epsilon}}$$

characterstic impedance

$$Z_0 = \frac{V_0}{I_0} = \sqrt{\frac{L}{C}} = \eta \frac{a}{b}$$

Power flow , $P = \frac{1}{2} \int \bar{E} \times \bar{H}^* \, ds$

Transmission Line parameters for some common lines :

Co-axial	Two wire	Parallel Plate
		
$L = \frac{\mu}{2\pi} \ln \frac{b}{a}$	$L = \frac{\mu}{\pi} \cosh^{-1} \frac{D}{2a}$	$L = \frac{\mu d}{W}$.
$C = \frac{2\pi\epsilon'}{\ln b/a}$	$C = \frac{\pi\epsilon'}{\cosh^{-1} \frac{D}{2a}}$	$C = \frac{\epsilon' W}{d}$
$R = \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$	$R = \frac{\pi\epsilon'}{\cosh^{-1} \frac{D}{2a}}$	$R = \frac{2R_s}{W}$
$G = \frac{2\pi\omega\epsilon''}{\ln b/a}$	$G = \frac{\pi\omega\epsilon''}{\cosh^{-1} D/2a}$	$G = \frac{\omega\epsilon'' W}{d}$

Q4)
SQ $\mu = 2.3$, $\epsilon_r = 1.4$, $f = 5MHz$, $a = 0.3mm$, $b = 0.6mm$.

$z_i = ?$ $\beta = ?$ (lossless)

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}} \frac{\ln b/a}{2\pi} = \sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} \cdot \frac{\ln b/a}{2\pi}$$

$$= \sqrt{\frac{4\pi \times 10^{-7} \times 2.3}{8.854 \times 1.4 \times 10^{-12}}} \cdot \frac{\ln 0.6/0.3}{2\pi}$$

$= 53.97\Omega$.

$$\beta = \omega\sqrt{LC} = 2\pi f \sqrt{LC} = 2\pi \times 5 \times 10^6 \times \sqrt{4\pi \times 10^{-7} \times 2.3 \times 8.85 \times 10^{-12} \times \ldots}$$

$= 0.185$ rad/m .

Q2)
LQ $L = 10m$ (lossless) , $z_0 = 50\Omega$

$\epsilon_r = 3.5$, $\mu_r = 1$, $a = 1mm$, $b = ?$

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{\ln b/a}{2\pi}$$

$$\Rightarrow \ln b/a = 2\pi Z_0 \sqrt{\frac{\epsilon}{\mu}}$$

$$\ln \frac{b}{a} = 1.56$$

$b = ac^{1.56} \Rightarrow b = 4.7588 \times 1 \text{ mm} = 4.7588 \text{ mm}.$

Q3) LQ $\sigma = 4 \text{ S/m}$, $\epsilon_r = 80$, $\mu_r = 1$, $\alpha = ?$, $\gamma = ?$

$f = 1\text{kHz}, 100\text{kHz}, 1000\text{kHz}$ and δ_s at $100\text{kHz} = ?$

Condition, $\frac{\sigma}{\omega\epsilon} \gg 1 \rightarrow$ good conductor.

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi f\mu\sigma}{2}}$$

$$\alpha_{1kHz} = \sqrt{\frac{2\pi \times 1 \times 10^3 \times 4\pi \times 10^{-7} \times 1 \times 4}{2}} = 0.12566 \text{ Np/m}$$

$$\alpha_{100kHz} = 1.2566 \text{ Np/m}$$

$$\alpha_{1000kHz} = 3.9738 \text{ Np/m}.$$

$$\gamma = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}} = 3.974 \times 10^{-3}(1+j)\sqrt{f}$$

$$\gamma_{1kHz} = 0.1256 + j0.1256$$

$$\gamma_{100kHz} = 1.2567 + j1.2567$$

$$\gamma_{1000kHz} = 3.974 + j3.974$$

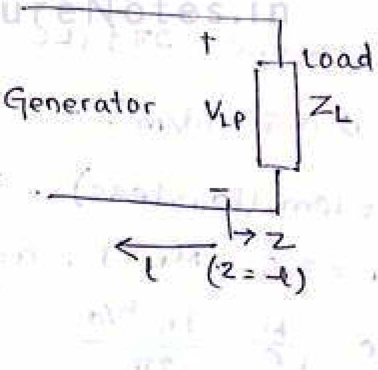
$$\delta_s = \sqrt{\frac{2}{\mu\omega\sigma}} = \frac{1}{\sqrt{\pi f\mu\sigma}} \quad \text{at } f = 100\text{kHz}$$

$$= \frac{0.795 \text{ m}}{0.00796\text{m}} = 7.96 \text{ mm}.$$

Q.11) LQ i) $Z_0 = 50\Omega$, $Z_L = 75\Omega$, $V_{LP} = 30\text{V}$.

$\Gamma = ?$ $V^+, V^- = ?$ $I^+, I^- = ?$

VSWR = ?



$$V(l) = V_0^+\left[1 + \Gamma e^{-2j\beta l}\right]$$

$$|V(l)| = V_{LP}$$

$$|V(l)| = V_0^+\left[1 + |\Gamma|\right]$$

$$\Rightarrow 30 = V_0^+\left[1 + \frac{1}{5}\right]$$

$$V_0^+ = 30 \times \frac{5}{6} = 25\text{V}$$

$$V_0^- = \Gamma \times V_0^+ \quad \text{as} \quad \frac{V_0^-}{V_0^+} = \Gamma$$

$$\Rightarrow V_0^- = \frac{1}{6} \times 25 = 5\text{V}$$

$I(z) = I^{+} \cdot e^{-\gamma z} + I^{-} e^{\gamma z}$

$I(z) = \frac{V_0^{+}}{Z_0}\left[1 - |\Gamma| e^{-j\beta l}\right]$

$I^{+} = \frac{V^{+}}{Z_0}$, $I^{-} = \frac{V^{-}}{Z_0}$

$I^{+} = \frac{25}{50} = \frac{1}{2} = 0.5$, $I^{-} = \frac{-5}{50} A = -0.1A$.

$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/5}{1-1/5} = \frac{6}{4} = \frac{3}{2} = 1.5$

Q19) $T_n = 100ns$
LQ.

$L = 0.20 \mu H/m$

$C = 60 pF/m$.

Length of cable.

$V_P = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.20 \times 10^{-6} \times 60 \times 10^{-12}}}$

$= 2.88 \times 10^{8}$ m/s

$\text{Velocity} = \frac{\text{length}}{\text{time}}$

$\text{Length} = \text{Velocity} \times T_n$

$= 2.88 \times 10^{8} \times 100 \times 10^{-9}$

$= 28.86 m$.

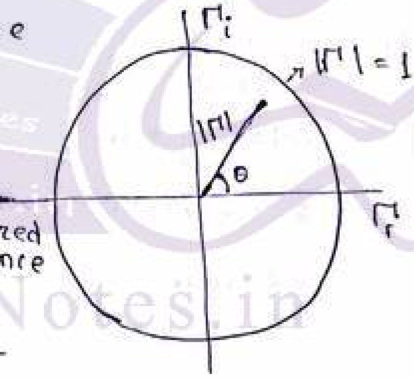
Smith Chart (Graphical solⁿ of T.L. Problem):

1939 - P. Smith Bell Teleplant Lab CAD.

- The polar co-ordinates of smith cart are magnitude and phase angle of reflection co-efficient.
- The cartessian co-ordinates are real and imaginary parts of reflection co-efficient.
- The entire chart lies within the unit circle i.e. reflection co-efficient = 1.
- The smith chart consist of a plot of normalized impedance or admittance with angle and magnitude of generalized complex reflection coefficient in a unit circle.
- The chart is applicable to analysis of lossless line as well as lossy line.

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\frac{Z_L}{Z_0} - 1}{\frac{Z_L}{Z_0} + 1}$$

Normal zero impedance



$$\frac{\Gamma}{1} = \frac{z_\ell - 1}{z_\ell + 1} \qquad \frac{Y_L}{Y_0} = \text{normalized admitance}$$

$$z_\ell = \frac{1+\Gamma}{1-\Gamma} \qquad \text{Let } \frac{Z_L}{Z_0} = z_L$$

$$z_\ell = r + jx = \frac{1 + \Gamma_r + j\Gamma_i}{1 - \Gamma_r - j\Gamma_i}$$

$$r + jx = \frac{1 + \Gamma_r^2 - \Gamma_i^2 + 2j\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$\frac{r}{1} = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} \qquad x = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

real part:

$$r(1-\Gamma_r)^2 + r\Gamma_i^2 = 1 - \Gamma_r^2 - \Gamma_i^2$$

$$\Gamma_r^2(r+1) + \Gamma_i^2(r+1) + r - 2r\Gamma_r = 1$$

$$\Gamma_r^2 - \frac{2r}{r+1}\Gamma_r + \left(\frac{r}{r+1}\right)^2 + \Gamma_i^2 = \frac{1}{r+1} - \frac{r}{r+1} + \left(\frac{r}{r+1}\right)^2$$

$$\Rightarrow \left(\Gamma_r - \frac{r}{r+1}\right)^2 + \Gamma_i^2 = \frac{1}{(r+1)^2}$$

$$(x - x_1)^2 + (y - y_1)^2 = a^2$$

centre $\left(\frac{r}{r+1}, 0\right)$ radius $= \frac{1}{r+1}$

$r = 0$, centre $(0,0)$ radius $= 1$

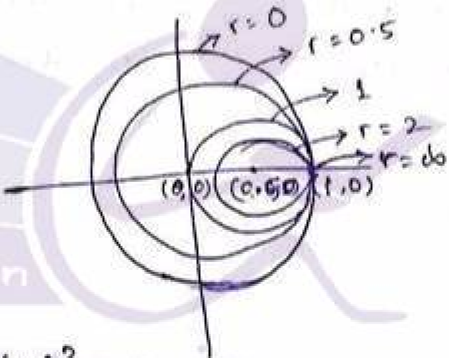
$r = \infty$, centre $(1,0)$ radius $= 0$

$r = 1$, centre $(\frac{1}{2}, 0)$ radius $= \frac{1}{2}$

$r = 2$, centre $(\frac{2}{3}, 0)$ radius $= \frac{1}{3}$

$\vdots$

$r = 0.5$, centre $(\frac{1}{3}, 0)$ radius $= \frac{1}{1.5} = 0.67$



Imaginary plot:

$$\frac{x}{1} = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$x(1-\Gamma_r)^2 + x\Gamma_i^2 = 2\Gamma_i$$

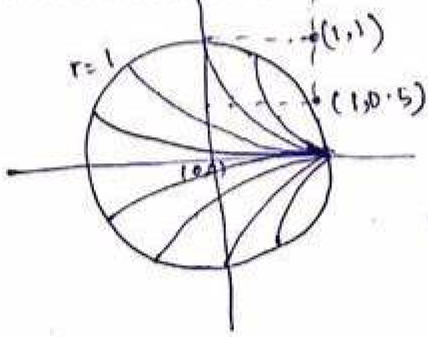
$$\Rightarrow (\Gamma_r - 1)^2 + \Gamma_i^2 - \frac{2\Gamma_i}{x} = 0$$

$$\Rightarrow (\Gamma_r - 1)^2 + \Gamma_i^2 - \frac{2\Gamma_i}{x} + \left(\frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

$$\Rightarrow (\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

centre $(1, \frac{1}{x})$ radius $= \frac{1}{x}$

$x = 0$	$(1, \infty)$	∞
$x = 1$	$(1, 1)$	1
$x = 2$	$(1, 0.5)$	0.5
$x = 0.5$	$(1, 2)$	2



Basic Features of Smith Chart:

1) 2 families of circles in the complex reflection coefficient frame $(|\Gamma|, \theta)$ which is called smith chart.

2) The characteristics of smith chart are described in terms of normalized impedance, i.e. $\frac{Z_L}{Z_0} = r + jx$, the normalized admitance $\frac{Y_L}{Y_0} = g + jb$, a normalize length i.e. $\frac{l}{\lambda}$.

3) Constant reactive circles with radius $\frac{1}{x}$, centre $(1, \frac{1}{x})$ ranges $-\infty \leq x \leq \infty$.

Constant resistance (r) circles with radius $\frac{1}{1+r}$, centre $(\frac{r}{1+r}, 0)$ on Γ_r axis $0 \leq r \leq \infty$.

4) One complete rotation covers a distance $\frac{\lambda}{2}$, along the line. The impedance and reflection coefficient repeat themselves at these intervals.

Dt 15/12/16

Q) Draw the following on the smith chart.

$Z_0 = 50\Omega$

a) $50 + j75\Omega$ b) $10 + j0$ c) $0 - j80$ d) $\Gamma_L = 0.3\angle 60°$

e) constant VSWR circle $\rho = 2.5$

f) Minimum resistance point on the constant VSWR circle $\rho = 1.5$.

...ing Matrix:

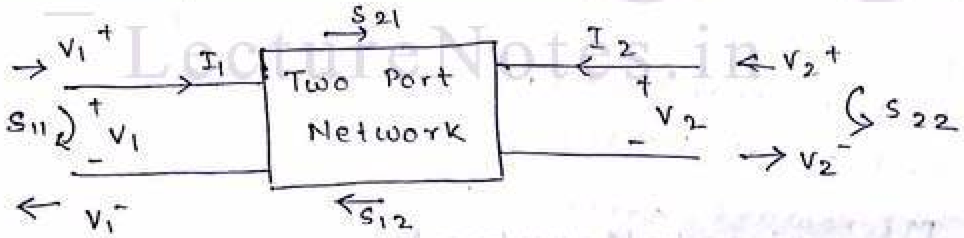
Scattering matrix is used in microwave analysis to overcome the problems which covers h, Y, Z parameters are used in high frequencies. 8C

1) Equipement is not readily avialable to measure total voltage and total current at the ports of network.
2) short and open circuit are difficult to achieve over a broadband of frequencies:
3) Active devices such as power transistors and tunnel diodes frequently would not have stability for a short and open circuit.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$



scattering matrix is the representation of direct measurement with the ideas of incident, reflected and transmitted wave.

$$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$$

$$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$$

$$\begin{bmatrix} V_1^- \\ V_2^- \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} V_1^+ \\ V_2^+ \end{bmatrix}$$

Two port

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \qquad S_{ij} = [S_{ij}] e^{j\theta_{ij}}$$

$$S_{ij} = \frac{V_i^-}{V_j^+} \qquad S_{ii} = \frac{V_i^-}{V_i^+}$$

= transfer coefficient

= Reflected coefficient

For 2port:

$$S_{11} = \frac{V_1^-}{V_1^+}\Big|_{V_2^+ = 0}$$

= reflection co-efficient at port1 with port 2 matched.

$$S_{12} = \frac{V_1^-}{V_2^+}\Big|_{V_1^+ = 0}$$

= reverse transmission coefficient with port 1 matched.

$$S_{21} = \frac{V_2^-}{V_1^+}\Big|_{V_2^+ = 0}$$

= Forward transmission coefficient with port 2 matched.

$$S_{22} = \frac{V_2^-}{V_2^+}\Big|_{V_1^+ = 0}$$

= Reflected coefficient at port 2 with port 1 matched.

- A scattering matrix is always a square matrix of order NXN. N = No. of ports.
- Elements of scattering matrix has the property of symmetry. This defines the reciprocity of symmetry. If port 1 & 2 are interchanged for 2-port network and performance of microwave device is still the same then we call that network as reciprocal network, passive components are reciprocical, whereas

active components such as amplifiers are reciprocal.)

$$S_{ij} = S_{ji}$$

- The product of conjugate transpose of scattering matrix with that matrix is equal to unitary matrix of order m.

$$[S^*]^T [S] = [I]$$

- This defines a lossless network:

$$[S^*]^T [S] = \begin{bmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$|S_{11}|^2 + |S_{21}|^2 = 1$$

$$|S_{12}|^2 + |S_{22}|^2 = 1$$

$$S_{11}^* S_{12} + S_{21}^* S_{22} = 0$$

$$|S_{11}| [S_{12}] = [S_{21}] [S_{22}]$$

$$-\arg S_{11} + \arg S_{12} = -\arg S_{21} + \arg S_{22} + \pi$$

$$S_{11} = S_{22} , \quad S_{12} = S_{21}$$

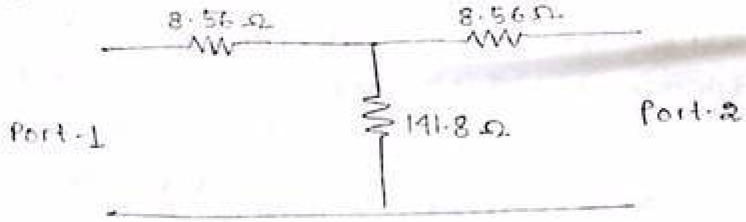
$$\boxed{S_{11} = \sqrt{1 - |S_{12}|^2}}$$

Sum of products between any row and column with complex conjugate of any other row or column is zero.

$$\sum_{i=1}^{n} S_{ij} S_{ik}^* = 0 \quad \text{where } j \neq k$$
$$j, k = 1, 2, 3 \ldots n$$

If any port move away from the junction by distance 'd', then the coefficient S_{ij} involving that particular port will be multiplied by the factor $e^{-j(2\pi d/\lambda)}$.

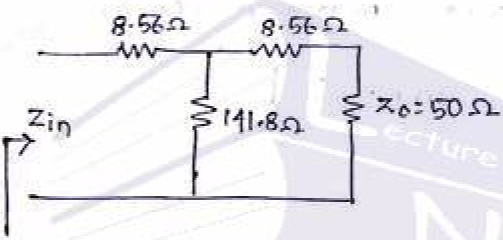
Q) Find the S-parameters of 3dB attenuator circuit shown in fig with 50Ω characteristic impedance.



$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$

$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{V_2^+=0}$ i.e. port-2 is connected to a matched load.

$$\Gamma^{(1)} = \frac{Z_{in}^{(1)} - Z_0}{Z_{in}^{(1)} + Z_0}$$



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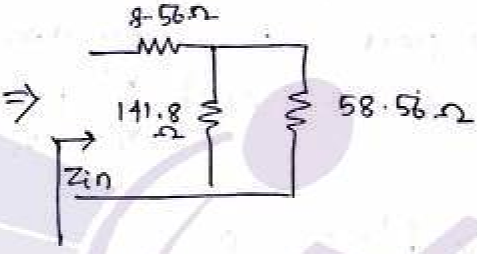


$Z_{in} = 8.56 + 141.8 \parallel 58.56 = 50\Omega$

$\Gamma^{(1)} = \frac{50-50}{50+50} = 0 \quad \therefore S_{11} = 0$

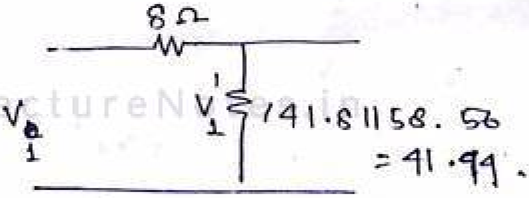
$S_{22} = 0$

due to symmetry.

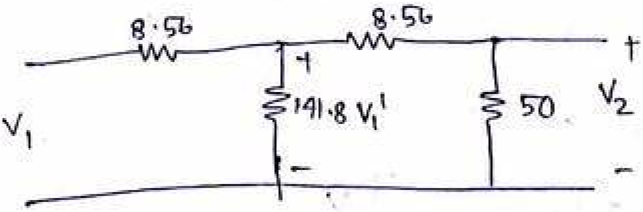
To calculate S_{21} & S_{12},

$V_2^- = S_{21}V_1^+ + S_{12}V_2^+$

$S_{21} = \left.\frac{V_2^-}{V_1^+}\right|_{V_2^+=0}$



$$V_1' = \frac{V_1 \times 41.44}{41.44 + 8.8}$$



$V_2 = \frac{V_1' \times 50}{50 + 8.56}$

$= 0.707\, V_1$

$S_{21} = 0.707$

$\Rightarrow S_{12} = 0.707 = \frac{1}{\sqrt{2}}$

due to symmetry.

$$S = \begin{bmatrix} 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & 0 \end{bmatrix}$$

2) A 2port network has the following matrix

$$[S] = \begin{bmatrix} 0.15\angle 0^\circ & 0.85\angle -45^\circ \\ 0.85\angle 45^\circ & 0.2\angle 0^\circ \end{bmatrix}$$

Determine whether the network is reciprocal loss-less if port-2 is terminated with matched load. Find out the return loss seen at port-1, if port-2 is terminated with short circuit. What will be the return loss seen from port-1.

$S_{12} \neq S_{21}$ not reciprocal.

$S_{ij} = S_{ji}$

$|S_{11}|^2 + |S_{21}|^2 = 1$

$|S_{12}|^2 + |S_{22}|^2 = 1$

$|0.15|^2 + |0.85|^2 \neq 1$ not lossless.

If port-2 is terminated with matched load,

$$S_{11} = 0.15\angle 0^\circ = \frac{V_1^-}{V_1^+}\Big|_{V_2^+ = 0} \text{ matched } = \Gamma$$

$\Rightarrow \Gamma = 0.15\angle 0^\circ$

$S_{11} = -20\log|\Gamma| = 16.5\text{dB}$

If port-2 is terminated with short circuit,

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1 = \frac{V^-}{V^+}$$

$\Gamma = 1$

$V_2^- = -V_2^+$

$V_2^- = S_{21} V_1^+ + S_{22} V_2^+$

$V_2^- - S_{22} V_2^+ = S_{21} V_1^+$

$\Rightarrow V_2^-[1 + S_{22}] = S_{21} V_1^+$

$V_1^- = S_{11} V_1^+ + S_{12} V_2^+$

$= S_{11} V_1^+ - S_{12} V_2^-$

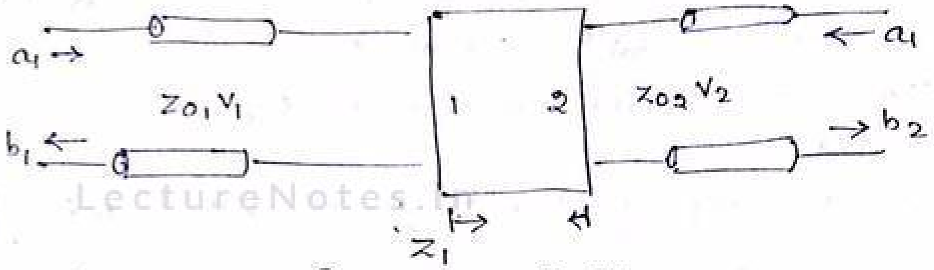
$$= S_{11} V_1^+ - S_{12}\left[\frac{S_{21}}{1 + S_{22}}\right] V_1^+$$

Port-2, short circuit:

$$\Gamma' = \frac{V_1^-}{V_1^+} = S_{11} - \frac{S_{12}S_{21}}{1+S_{22}}$$

$$= 0.15 - \frac{0.85\times0.85}{1+0.2} = -0.452$$

$$RL = -20\log|\Gamma'| = 6.9dB$$



$$V_i(z_i) = V_{i0}^+ e^{-\gamma_i z_i} + V_{i0}^- e^{\gamma_i z_i}$$

$$= V_i^+(z_i) + V_i^-(z_i)$$

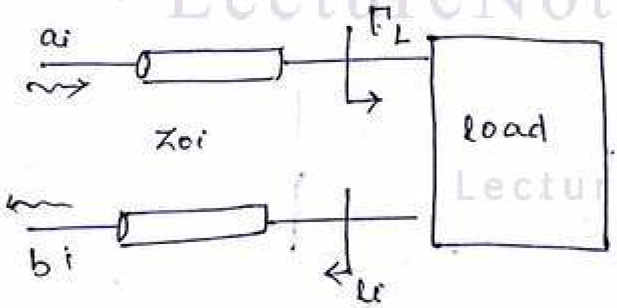
Similarly:

$$I(z_i) = \frac{V_i^+(z_i)}{Z_{0i}} - \frac{V_i^-(z_i)}{Z_{0i}} \quad (i=1,2)$$

Incoming wave function: $a_i(z_i) = \dfrac{V_i^+(z_i)}{\sqrt{Z_{0i}}}$

Out-going wave function $= b_i(z_i) = \dfrac{V_i^-(z_i)}{\sqrt{Z_{0i}}}$

For a 1-port Network:



$$\Gamma_L = \frac{\frac{V_0^-(0)}{\sqrt{Z_{0i}}}}{\frac{V_0^+(0)}{\sqrt{Z_{0i}}}} = \frac{b_i(0)}{a_i(0)} = S_{11}$$

$$b_i(0) = \Gamma_L a_i(0) = S_{11} a_i(0)$$

Two port network:

$$\begin{bmatrix} b_1(0) \\ b_2(0) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1(0) \\ a_2(0) \end{bmatrix}$$

Signal Flow Graph Analysis (Sfg):

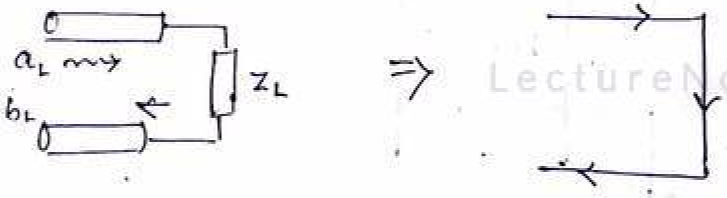
- This is convenient technique to & analyze circuit characterized by s-matrix.
- It allows one to see the flow of signals a circuit.
- Signals are represented by wave function i.e. {a_i & b_i}

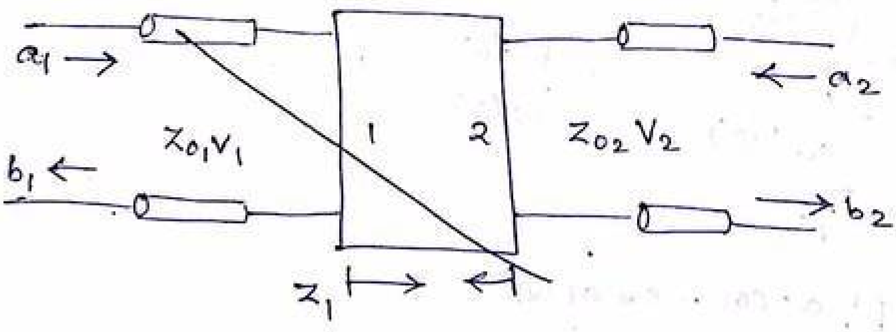
Construction Rules of Sfg:

- Each port 'i' of microwave network has 2 modes i.e. wave entering mode & wave reflecting mode i.e. each wave functions 'ai' & 'bi' is 2 node.
- S parameters are represented by branches between the nodes.
- Branches are unidirectional.
- A node value is equal to sum of branches entering it.

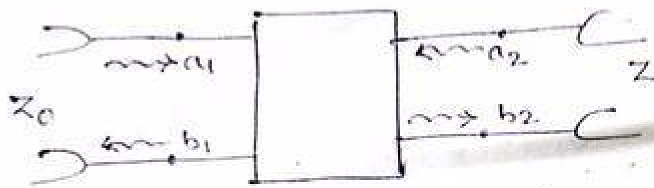
For example:

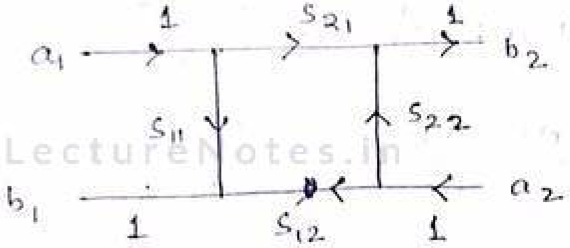
Single load:



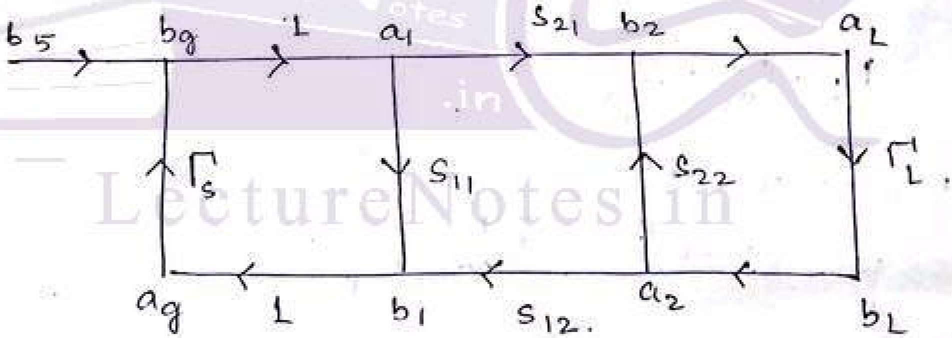


Example of 2-port device:





Complete SFG:



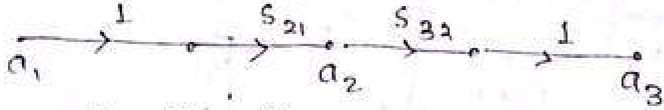
Internal 2 Dt. 5/1/16

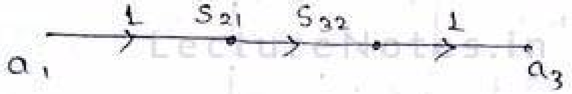
a4

Solving the SFG:

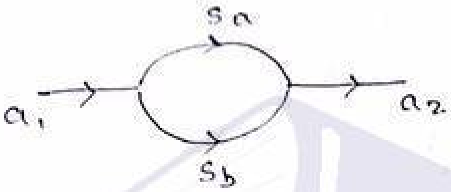
1) Mason's gain formula
2) Direct solution
3) Decomposition.

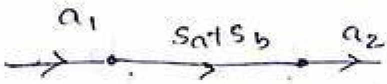
1) Series path:





2) Parallel path:





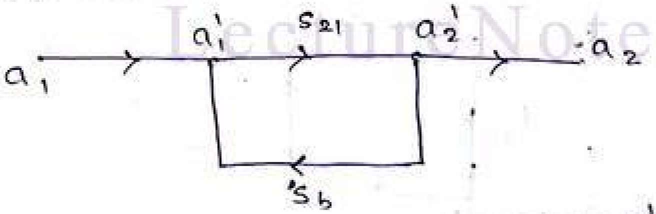
$a_2 = a_1 S_a + a_1 S_b = a_1 (S_a + S_b)$

$a_2 = 1 \times S_{21} a_1 = S_{21} a_1$

$a_3 = S_{32} a_2$

$\quad = S_{32} a_{21} a_2$

3) Self loop:



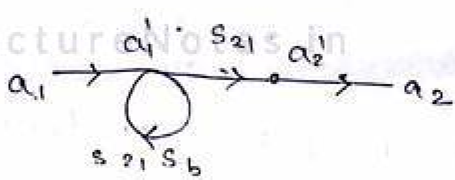
$a_1' = a_1 + a_2' S_b$

$a_2' = S_{21} a_1'$

$a_1' = a_1 + a_1' S_{21} S_b$

$a_1' = L a \qquad a_1'(1 - S_{21} S_b) = a_1$

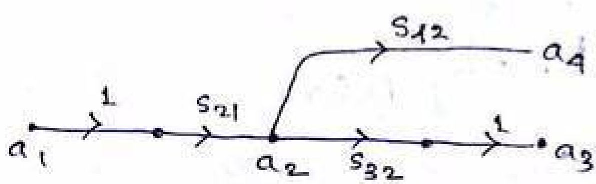
$L = \frac{1}{1 - S_{21} S_b}$



4) Splitting:

$a_4 = S_{42} a_2$

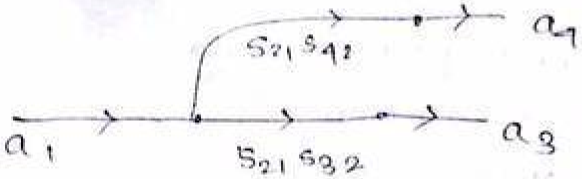
$a_3 = S_{32} a_2$

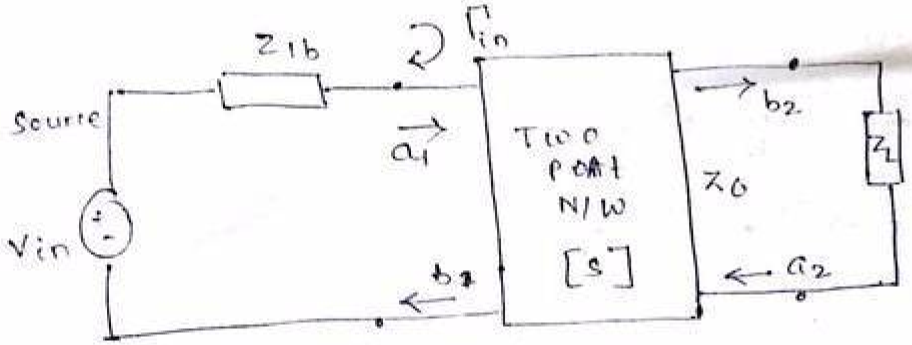


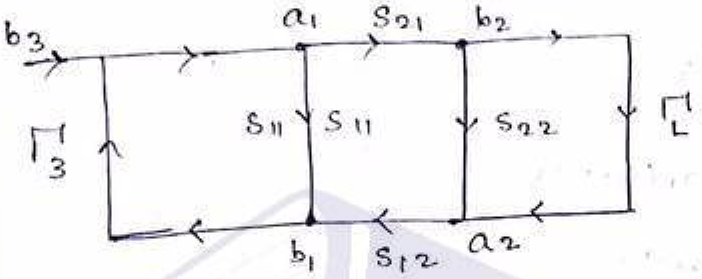
$a_2 = S_{21} a_1$

$\Rightarrow a_4 = a_{42} S_{21} a_1$

$a_3 = S_{21} S_{32} a_1$

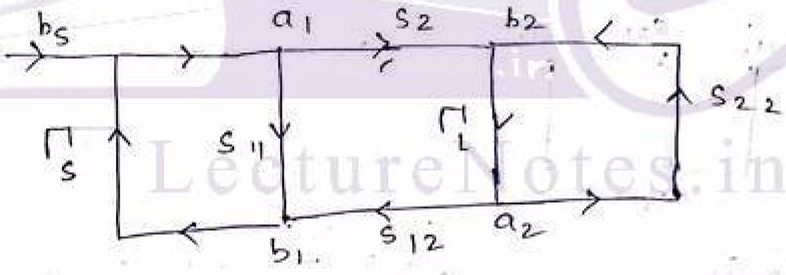




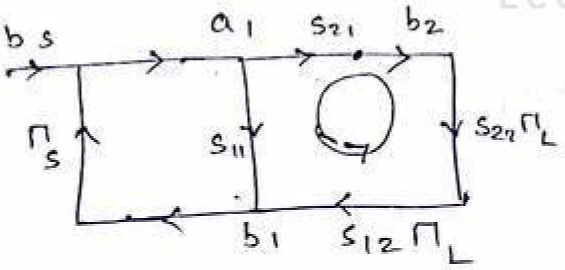


$$\Gamma_{in} = \frac{b_1}{a_1}$$

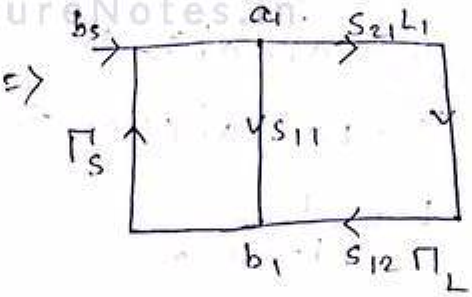
The ~~same~~ self loop at the end is rearranged.



Now apply selfloop formula to remove it:



$\Rightarrow$



$$L_1 = \frac{1}{1-\Gamma_L' S_{22}}$$

$$b_1 = a_1 S_{11} + a_1 (S_{21} L_1)(S_{12} \Gamma_L')$$

$$\frac{b_1}{a_1} = S_{11} + \frac{S_{21}}{1-\Gamma_L' S_{22}} (S_{12}\Gamma_L')$$

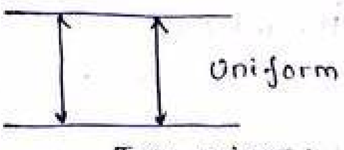
Wave Guide:

- Any system of material boundaries capable of guiding EM waves can be termed as wave guide.
- Wave guide provide an alternative two transmission lines for transmission of electrical energy at microwave frequencies 8 to 12GHz (X-band).
- Wave guides are relatively less lossy in comparision to the transmission line.

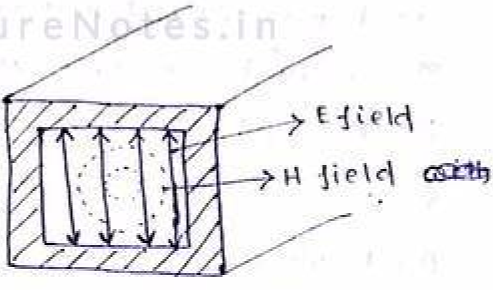
Advantages of wave guide over the conventional T.L.:

1. Wave guides are simple and rigid. Uniform crossection of a guide can be obtained with ~~ease~~ ease ~~than~~ done a uniform spacing bet. 2 conductors.
2. Since the fields are confined within the guides, there are no radiation losses.
3. The dielectric losses that limit the applicability of T.L. are totally avoided since there is no inner conductor.
4. Power dissipation due to ohmic loss in wave guides is also reduced as compared to the conventional lines due to greater current carrying over the wave guide ones.

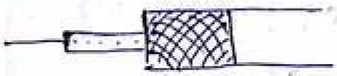
~~Rectangular & cylindrical wave guide:~~



Two wire



Rectangular waveguide



Coaxial

By placing two parallel conducting planes, it is seen that the electric field is normal to the plates and magnetic field is tangential, satisfying the boundary conditions.

- By placing two more conducting plates a rectangular tube is formed called a rectangular waveguide.
- In such a configuration, electric field varies with distance having max^m at the center. Magnetic lines per round and passed down the tube.

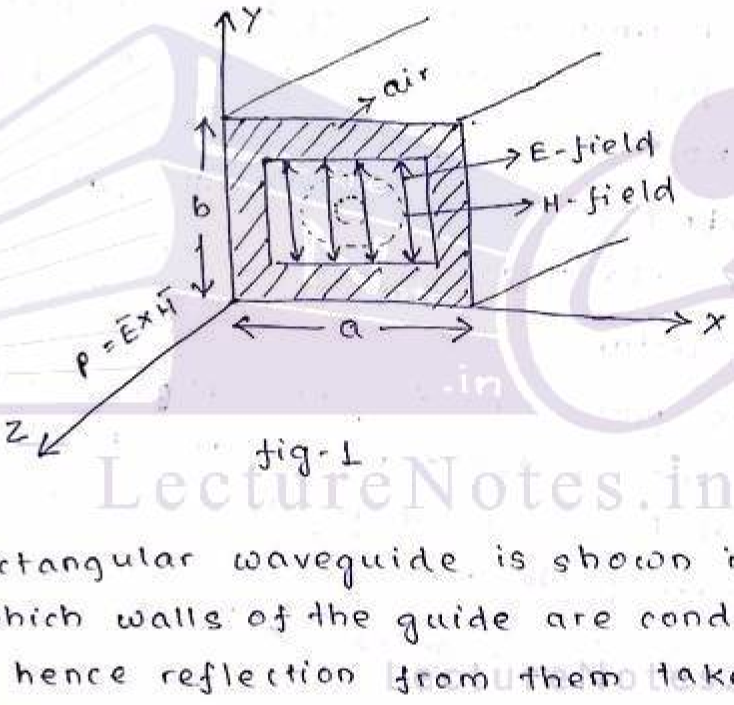


fig-1

A rectangular waveguide is shown in fig-1 in which walls of the guide are conductors and hence reflection from them take place. It may be noted that conduction of energy takes place not through wall, whose function is to confine it only.
But through the dielectric filling which is usually air.

Let us assume that waves are propagating along z-direction. The variation of field along this direction may be expressed as $e^{-\gamma z}$, where γ = propagation constant.

Further let us assume that the walls are perfectly conducting ($\sigma = \infty$), and dielectric inside the waveguide is lossless ($\sigma = 0$). (91)

- Since walls are assumed to be perfect conductor, two simple boundary conditionings are made.

i) Electric field must terminate normally on the conductor i.e. tangential component of electric field $E_t = 0$ (, E_n exists).

ii) Magnetic field must lie entirely tangentially along the wall surface. i.e. $H_n = 0$ (, H_t exits).

Let us assume source free region i.e. $J = 0$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial D}{\partial t} \qquad \nabla \times \bar{H} = \frac{\partial D}{\partial t} \qquad \frac{\partial}{\partial t} = j\omega$$

$$= \bar{J} + j\omega D \qquad = j\omega D \qquad \frac{\partial}{\partial z} - \gamma$$

LHS :

$$\nabla \times H = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \hat{y}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \hat{z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$= \hat{x}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) + \hat{y}\left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) + \hat{z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

RHS :

$$J\omega\epsilon\left[\bar{E}_x + \bar{E}_y + \bar{E}_z\right]$$

$$= J\omega\epsilon \bar{E}_x + J\omega\epsilon \bar{E}_y + J\omega\epsilon \bar{E}_z$$

Comparing LHS & RHS,

$$\frac{\partial H_z}{\partial y} + \gamma H_y = J\omega\epsilon E_x \quad \text{— (1)}$$

$$-\gamma H_x - \frac{\partial H_z}{\partial x} = J\omega\epsilon E_y \quad \text{— (2)}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J\omega\epsilon E_z \quad \text{— (3)}$$

$$\nabla \times \bar{E} = \frac{-\partial B}{\partial t}.$$

Similarly, we can get,

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \quad \text{—— (4)}$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = j\omega\mu H_y \quad \text{—— (5)}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad \text{—— (6)}$$

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Since the wave is propagated with delay.

∴ We have to consider time varying field as well.

$$\left[\text{i.e. } e^{\gamma z} A e^{j\omega t}\right]$$

$$\nabla \times \bar{E} = -\frac{\partial B}{\partial t}$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial D}{\partial t}$$

We can get the six time varying field equation.

$$\frac{\partial H_z}{\partial y} + \gamma H_y = j\omega\epsilon E_x \quad \text{—— (1)}$$

$$-\frac{\partial H_z}{\partial x} - \gamma H_x = j\omega\epsilon E_y \quad \text{—— (2)}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = j\omega\epsilon E_z \quad \text{—— (3)}$$

$$\frac{\partial E_z}{\partial y} + \gamma E_y = -j\omega\mu H_x \quad \text{—— (4)}$$

$$\frac{\partial E_z}{\partial x} + \gamma E_x = j\omega\mu H_y \quad \text{—— (5)}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -j\omega\mu H_z \quad \text{—— (6)}$$

Determination of field components in terms of E_z & H_z the component in the direction of propagation:

Comparing eq. (2) & (4),

$$\frac{\partial E_z}{\partial y} + \gamma\left[\frac{-1}{j\omega\epsilon}\frac{\partial H_z}{\partial x} - \frac{\gamma}{j\omega\epsilon} H_x\right] = -j\omega\mu H_x$$

$$\Rightarrow \frac{\partial E_z}{\partial y} - \frac{\gamma}{j\omega\epsilon}\frac{\partial H_z}{\partial x} = \frac{\gamma}{j\omega\epsilon} H_x - j\omega\mu H_x$$

$$\Rightarrow \frac{\partial E_z}{\partial y} - \frac{\gamma}{j\omega\epsilon}\frac{\partial H_z}{\partial x} = \left[\frac{\gamma^2 + \omega^2\mu\epsilon}{j\omega\epsilon}\right] H_x$$

$$\Rightarrow \boxed{H_x = \frac{j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial y} - \frac{\gamma}{k_c^2}\frac{\partial H_z}{\partial x}} \quad \text{--- (7)}$$

where $\boxed{k_c^2 = \gamma^2 + \omega^2\mu\epsilon}$ → cutoff wave number.

$$E_y = \frac{j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x} - \frac{\gamma}{k_c^2}\frac{\partial E_z}{\partial y} \quad \text{--- (8)}$$

$$H_y = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial x} - \frac{\gamma}{k_c^2}\frac{\partial H_z}{\partial y} \quad \text{--- (9)}$$

$$E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y} - \frac{\gamma}{k_c^2}\frac{\partial E_z}{\partial x} \quad \text{--- (10)}$$

Case:

(i) TEM mode: $E_z = H_z = 0$

For waveguide TEM mode doesn't exist since, the 4 eq.s (7), (8), (9), (10) vanishes.

(ii) TE mode: $E_z = 0$, $H_z \neq 0$

Since (7), (8), (9), (10) eq exists, when $E_z = 0$ & $H_z \neq 0$, so TE mode is possible.

(iii) TM mode: $E_z \neq 0$, $H_z = 0$

This mode is possible in rectangular waveguide.

(iv) Hybrid Mode: $E_z \neq 0$ $E_z \neq 0$

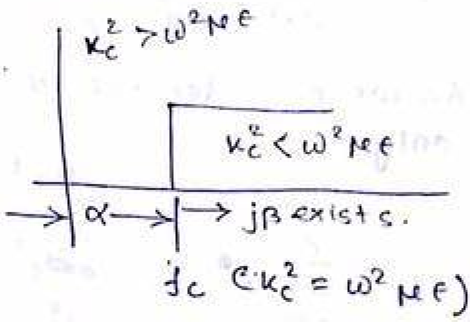
In this mode, the transmission is possible but often used in waveguide.

Cutoff wave number:

$$\boxed{k_c^2 = \gamma^2 + \omega^2\mu\epsilon}$$

$$\boxed{\gamma = \sqrt{k_c^2 - \omega^2\mu\epsilon}}$$

$$= \underbrace{\alpha}_{\text{attenuation constant}} + j\underbrace{\beta}_{\text{phase constant}}$$



case ①: $k_c^2 > \omega^2 \mu \epsilon \rightarrow$ real attenuation exist.

case ②: $k_c^2 < \omega^2 \mu \epsilon \rightarrow$ signal is propagated w/o attenuation $j\beta$ exists.

case ③: $k_c^2 = \omega^2 \mu \epsilon$

$$\Rightarrow \omega^2 = \frac{k_c^2}{\mu \epsilon}$$

$$\Rightarrow 2\pi f = \sqrt{\frac{k_c^2}{\mu \epsilon}}$$

$$\Rightarrow f_c = \frac{1}{2\pi}\sqrt{\frac{k_c^2}{\mu \epsilon}} \longrightarrow \text{cutoff frequency.}$$

The freq. after which the signal can be propagated without attenuation is called as cutoff frequency

Electromagnetic wave equation:

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \cancel{\sigma \bar{E}} + \frac{\partial \epsilon \bar{E}}{\partial t} \quad \text{— ①}$$

for perfect dielectric

$$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -\mu \frac{\partial \bar{H}}{\partial t} \quad \text{— ②}$$

$$\nabla \times (\nabla \times \bar{E}) = -\mu \frac{\partial}{\partial t} (\nabla \times \bar{H})$$

$$\nabla(\nabla \cdot E) - \nabla^2 E = -\mu \epsilon \frac{\partial^2 E}{\partial t^2}$$

Since $\nabla \cdot E = \frac{\rho}{\epsilon} = 0$ → free charge region.

$$\boxed{\nabla^2 E = \mu \epsilon \frac{\partial^2 E}{\partial t^2}}$$

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} = \mu \epsilon \frac{\partial^2 E}{\partial t^2} \quad \text{— *}$$

Similarly $\boxed{\nabla^2 H = \mu \epsilon \frac{\partial^2 H}{\partial t^2}}$

i.e. $\frac{\partial^2 H}{\partial x^2} + \frac{\partial^2 H}{\partial y^2} + \frac{\partial^2 H}{\partial z^2} = \mu \epsilon \frac{\partial^2 H}{\partial t^2}$ — **

As we consider the wave in propagating with z-axis only:

$e^{-\gamma z}$ $\quad$ $e^{j\omega t}$

$\frac{\partial}{\partial z} = -\gamma$ $\quad$ $\frac{\partial}{\partial t} = j\omega$

$\frac{\partial^2}{\partial z^2} = \gamma^2$ $\quad$ $\frac{\partial^2}{\partial t^2} = \omega^2$

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z = -\omega^2 \mu \epsilon E_z$$

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z = -\omega^2 \mu \epsilon H_z$$

These are the field equations.

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$$H_x = \frac{j\omega\epsilon}{K_c^2}\frac{\partial E_z}{\partial y} - \frac{\gamma}{K_c^2}\frac{\partial H_z}{\partial x}$$

$$E_y = \frac{j\omega\mu}{K_c^2}\frac{\partial H_z}{\partial x} - \frac{\gamma}{K_c^2}\frac{\partial E_z}{\partial y}$$

$$H_y = \frac{-j\omega\epsilon}{K_c^2}\frac{\partial E_z}{\partial x} - \frac{\gamma}{K_c^2}\frac{\partial H_z}{\partial y}$$

$$E_x = \frac{-j\omega\mu}{K_c^2}\frac{\partial H_z}{\partial y} - \frac{\gamma}{K_c^2}\frac{\partial E_z}{\partial x}$$

Wave Equation:

$$\left.\begin{aligned} \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \gamma^2 E_z &= -\omega^2 \mu \epsilon E_z \quad (1) \\ \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \gamma^2 H_z &= -\omega^2 \mu \epsilon H_z \end{aligned}\right\} (*)$$

Solution to Electromagnetic wave eq^n (*):

The wave equations in a waveguide are seem to be partial differential equations which can be solved by assuming product solution.

i) $E_z = XY$

X is function of x and Y of y.

substitute E_z in eq^n (1)

$$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} + \gamma^2 XY = -\omega^2 \mu \epsilon XY \quad (2)$$

Divide by XY:

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -(\gamma^2 + \omega^2 \mu \epsilon) = -(k_x^2 + k_y^2) = k_c^2$$

$$\because k_c^2 = -(k_x^2 + k_y^2)$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -k_x^2 \qquad \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -k_y^2$$

$X = c_1 \sin k_x x + c_2 \cos k_x x$

Similarly, $Y = c_3 \sin k_y y + c_4 \cos k_y y$.

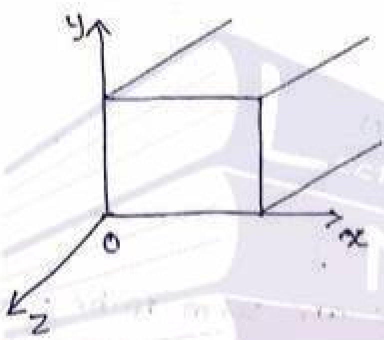
The final solution of E_z is given by, the product of individual solution of X and Y as

$$E_z = XY = (c_1 \sin k_x x + c_2 \cos k_x x)(c_3 \sin k_y y + c_4 \cos k_y y) \quad \text{③}$$

$$H_z = (c_5 \sin k_x x + c_6 \cos k_x x)(c_7 \sin k_y y + c_8 \cos k_y y) \quad \text{④}$$

Transverse Magnetic Mode (TM mode):

Applying boundary condition to eq. ③ to evaluate complex component.



$$E_z = \begin{cases} 0 \text{ at } x = 0, a \\ 0 \text{ at } y = 0, b \end{cases}$$

Case-1:

At $x = 0$, $E_z = 0$ this gives

$0 = E_z = [c_1 \sin k_x \cdot 0 + c_2 \cos k_x \cdot 0][c_3 \sin k_y y + c_4 \cos k_y y]$

$\Rightarrow E_z = c_2 (c_3 \sin k_y y + c_4 \cos k_y y)$

$\Rightarrow c_2 = 0$

Case-2:

At $y = 0$, $E_z = 0$.

$0 = (c_3 \sin k_y \cdot 0 + c_4 \cos k_y \cdot 0)(c_1 \sin k_x x + c_2 \cos k_x x)$

$\Rightarrow 0 = c_4 (c_1 \sin k_x x + c_2 \cos k_x x)$

$\Rightarrow c_4 = 0$

Putting c_2 and $c_4 = 0$ $\quad E_z = c_1 c_3 \sin k_x x \sin k_y y$ — ⑤

Case-3:

$E_z = 0$, at $x = a$

Substituting it in eqⁿ ⑤,

$0 = c_1 c_3 \sin k_x a \sin k_y y$.

$\sin k_x a = 0 = \sin m\pi$

$k_x a = m\pi \Rightarrow k_x = \frac{m\pi}{a}$ where $m = 0, 1, 2 \ldots$

case-4:

$E_z = 0$ at $y = b$ substituting in eq. 5,

$$0 = C_1 C_3 \sin k_x x \sin k_y b.$$

$\sin k_y b = 0 = \sin n\pi$

$k_y = \frac{n\pi}{b}$, where $n = 0, 1, 2, \ldots$

TM 6 eq.s:

$$E_z = C_1 C_3 \sin\left(\frac{m\pi}{a}x\right) \cdot \sin\left(\frac{n\pi}{b}y\right)$$

$$H_z = 0$$

$$H_x = \frac{j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial y} = \frac{j\omega\epsilon}{k_c^2} C_1 C_3 \frac{n\pi}{b} \sin\left(\frac{m\pi}{a}x\right) \cdot \cos\left(\frac{n\pi}{b}y\right)$$

$$H_y = \frac{-j\omega\epsilon}{k_c^2}\frac{\partial E_z}{\partial x} = \frac{-j\omega\epsilon}{k_c^2} C_1 C_3 \frac{m\pi}{a} \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi}{b}y\right)$$

$$E_x = \frac{-\gamma}{k_c^2}\frac{\partial E_z}{\partial x} = \frac{-\gamma}{k_c^2} C_1 C_3 \left(\frac{m\pi}{a}\right) x \sin\left(\frac{n\pi}{b}\right) y$$

$$E_y = \frac{-\gamma}{k_c^2}\frac{\partial E_z}{\partial y} = \frac{-\gamma}{k_c^2} C_1 C_3 \left(\frac{n\pi}{b}\right) \sin\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right)$$

These 6 eq.s are called field eq.s of TM waves.

Mode of operation (TM_{mn}):

1) $m = 0$ & $n = 0$

$E_z = 0$, $H_z = 0$, $E_x = 0$, $E_y = 0$, $H_x = 0$, $H_y = 0$

This is not possible (TM_{00}). No TM eq. exists for this mode. Hence this mode of operation is not possible.

2) $m = 0$ & $n = 1$

$E_z = 0$, $H_z = 0$, $E_y = 0$, $E_x = 0$, $H_x = 0$, $H_y = 0$

No eq. exist for this mode (M_{01}).

3) $m = 1$ & $n = 0$

T_{10} mode is also not possible.

4) $m = 1$ & $n = 1$

TM_{11} is possible, since all eq. exist; and this is the lowest order of the mode in which the ~~wave~~ wave is propagated. So, it is called the dominant mode.

Characteristics of TMmn modes: Dt. 23/2/16

1) Cutoff frequency:

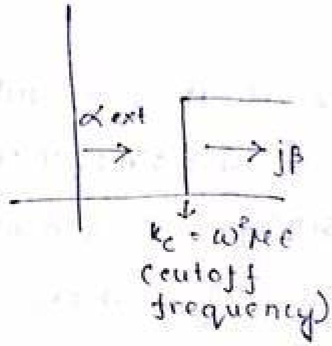
$$\gamma = \sqrt{k_c^2 - \omega^2 \mu\epsilon} = \sqrt{k_c^2 - k^2}$$

$$= \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - \omega^2\mu\epsilon}$$

$$\omega_c = \frac{\sqrt{k_c^2}}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

$$f_c = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

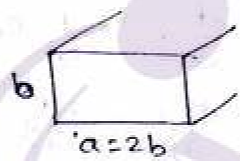
$$= \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$



Q) Putting $a = 2b$, taking air as medium, calculate the cutoff frequency in different modes of operation.

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{2b}\right)^2 + \left(\frac{n}{b}\right)^2}$$

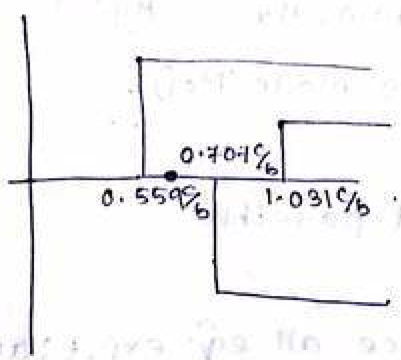


$$= \frac{c}{2b}\sqrt{\frac{m^2}{4} + n^2} \quad \left[\because c = \frac{1}{\sqrt{\mu\epsilon}}\right]$$

For $TM_{11} = \frac{c}{2b}\sqrt{\left(\frac{1}{4}+1\right)} = 0.559\frac{c}{b}$

$TM_{21} = \frac{c}{2b}\sqrt{1+1} = 0.707\frac{c}{b}$

$TM_{12} = \frac{c}{2b}\sqrt{\frac{1}{4}+4} = 1.031\frac{c}{b}$.



• in this frequency, if wave is being propagated, then the efficiency of the power loss is equal to zero.

2) Phase Constant:

$$\beta = \sqrt{\omega^2 \mu \epsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

cut off wavelength:

$$\lambda_c = \frac{c}{f_c} = \frac{c}{\frac{1}{2\pi\sqrt{\mu\epsilon}}\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]^{1/2}}$$

For air medium,

$$\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$$

wave impedance, $Z_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \beta/\omega\epsilon$

$$= \sqrt{\frac{\mu}{\epsilon}}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2}$$

where $\eta = \sqrt{\frac{\mu}{\epsilon}}$ = intrinsic impedance.

Phase velocity, $v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}}$

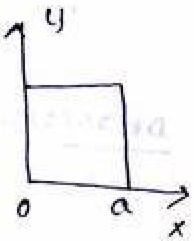
TM mode:

Transverse electric mode ($E_z = 0$, $H_z \neq 0$):

Boundary condition:

$E_x = 0$ at $y = 0, b$ $\Rightarrow \frac{\partial H_z}{\partial y} = 0$, at $y = 0, b$

$E_y = 0$ at $x = 0, a$ $\Rightarrow \frac{\partial H_z}{\partial x} = 0$, at $x = 0, a$



But $E_x = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial y}$

$E_y = \frac{-j\omega\mu}{k_c^2}\frac{\partial H_z}{\partial x}$

$$H_z = (C_5 \sin k_x x + C_6 \cos k_x x)(C_7 \sin k_y y + C_8 \cos k_y y)$$

C_5 & $C_7 = 0$ & $k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{a}$

TE mode field eq^n:

$$H_z = C_6 C_8 \cos\frac{m\pi}{a}x \cos\frac{n\pi}{b}y$$

$$E_x = \frac{j\omega\mu}{k_c^2}\left(\frac{n\pi}{b}\right) C_6 C_8 \cos\frac{m\pi}{a}x \sin\frac{n\pi}{b}y$$

$$E_y = \frac{-j\omega\mu}{k_c^2}\left(\frac{m\pi}{a}\right) C_6 C_8 \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

$$H_x = \frac{-\gamma}{k_c^2}\left(\frac{m\pi}{a}\right) C_6 C_8 \sin\left(\frac{m\pi}{a}\right)x \cos\left(\frac{n\pi}{b}\right)y$$

$$H_y = \frac{\gamma}{k_c^2}\left(\frac{n\pi}{b}\right) C_6 C_8 \cos\left(\frac{m\pi}{a}\right)x \sin\left(\frac{n\pi}{b}\right)y .$$

The characteristic impedance of TE mode is same as TM mode except the wave impedance i.e. $\gamma, f_c, \beta, \lambda_c, v_p$

Wave impedance:

$$Z_{TE} = \frac{E_x}{H_y} = \frac{-E_y}{H_x}$$

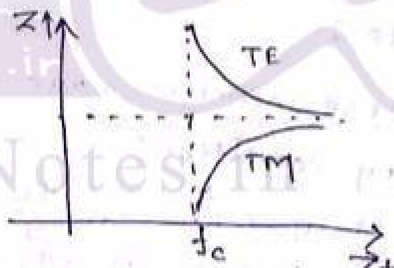
$$= \frac{\omega\mu}{\beta} = \sqrt{\frac{\mu}{\epsilon}} \times \sqrt{\frac{1}{1-(f_c/f)^2}}$$

$$= \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Comparing wave impedance of TE & TM modes:

$$Z_{TM} = \eta\sqrt{1-\left(\frac{f_c}{f}\right)^2} .$$

$$Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$



Dt. 26/2/16

Q) A rectangular waveguide with dimensions $a = 2.5$ cm, $b = 1$ cm, is to operate below 15.1 GHz. How many TE & TM modes gain the waveguide transmit if the guide is filled with a medium characterised by $\sigma = 0$, $\epsilon = 4\epsilon_0$, $\mu_r = 1$.

Calculate the cutoff frequencies of the mode.

TM_{11} $\quad TE_{10}$ ($a > b$) or TE_{01} ($b > a$) –

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$\mu = \mu_0\mu_r = \mu_0 \qquad \epsilon = 4\epsilon_0$

$\frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0 \cdot 4\epsilon_0}} = \frac{1}{2\sqrt{\mu_0\epsilon_0}} = \frac{c}{2}$

$f_c = \frac{c}{4}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{3\times10^8}{4}\sqrt{\left(\frac{A}{2.5\times10^{-2}}\right)^2+\left(\frac{B}{1\times10^{-2}}\right)^2}$

~~$TE_{10} = 3\times10^9$~~ (calc → A = 1 → B = 0 → Ans → ENG)

For TE_{10}, $f_c = 3\times10^9\ Hz = 3GHz$

TE_{20}, $f_c = 6GHz$

TE_{30}, $f_c = 9GHz$

TE_{40}, $f_c = 12GHz$

TE_{50}, $f_c = 15GHz$

TE_{01}, $f_c = 7.5GHz$

TE_{02}, $f_c = 15GHz$

$TE_{11} = TM_{11}$, $f_c = 8.077GHz$

$TE_{21} = TM_{21}$, $f_c = 9.6GHz$

$TE_{31} = TM_{31}$, $f_c = 11.71Hz$

$TE_{41} = TM_{41}$, $f_c = 14.15GHz$

$TE_{12} = TM_{12}$, $f_c = 15.296GHz$ (Not possible as the range is 15.1GHz.

Dominant Mode:
The minimum cutoff frequency 3GHz which is in TE_{10} mode, is called dominant mode.

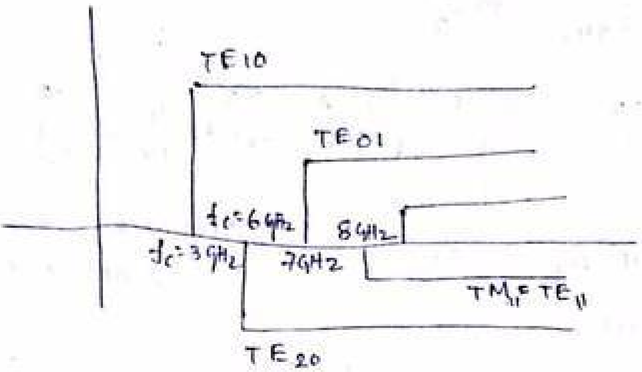
Degenerative Mode:
The cutoff frequencies where TE & TM modes are equal called degenerative mode.

If $a > b$ TE_{10} & TM_{11}

If $b > a$ TE_{01} & TM_{11}

Overmoded Mode:
A waveguide operating at a frequency where more than one mode propagates is said to be overmoded.

$f > f_c$, β only exist.



Evanescent mode/cut off mode :

In this mode, the frequency of operation is less than the cut off frequency. Here, the wave decays exponentially. So, only α exists.

Power Transmission in Rectangular waveguide :

Assume the guide ~~wave~~ is terminated in such a way that there is no reflection from the receiving end or the waveguide is infinitely long ~~component~~ compared with the wavelength.

A/c to Poynting theorm, the power transmitted by the guide is given by:

$$P_{tr} = \oint P \cdot ds = \oint \frac{1}{2}(E \times H^*)\, ds.$$

For lossless dielectric the time avg. power flow through a rectangular waveguide is given by-

$$P_{tr} = \frac{1}{2Z_g} \int_s |\bar{E}|^2 ds$$

$$= \frac{Z_g}{2} \int_s |\bar{H}|^2 ds$$

$$Z_g = \frac{E_x}{H_y} = \frac{-E_y}{H_x} \text{ - wave impedance.}$$

$$|\bar{E}|^2 = |E_x|^2 + |E_y|^2$$

$$|\bar{H}|^2 = |H_x|^2 + |H_y|^2$$

For TE_{mn} modes, the avg. power transmitted through a rectanuglar waveguide is given by

$$P_{tr} = \frac{\sqrt{1-(f_c/f)^2}}{2\eta} \int_0^b \int_0^a \left[|E_x|^2 + |E_y|^2 \right] dx\,dy$$

$$Z_g = \frac{\eta}{\sqrt{1-(f_c/f)^2}}$$

For TM_{mn} mode, the avg. power transmitted to rectangular waveguide is given by :-

$$P_{tr} = \frac{1}{2 Z_g} \int_0^b \int_0^a \left[|E_x|^2 + |E_y|^2 \right] dx\,dy$$

Q) A rectangular waveguide $a = 2.1$ cm, $b = 3.2$ cm is filled with
LQ dielectric of $\epsilon_r = 3.21$ and is operated at frequency
2015 9(a) $f = 50$ GHz. Calculate

i) cutoff frequency

ii) wave length

iii) phase constant

iv) phase velocity

v) group velocity

vi) wave impedance for TE_{22} mode.

i) $f_c = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$$= \frac{1}{2\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_r}} \times \sqrt{\left(\frac{2}{2.1\times10^{-2}}\right)^2 + \left(\frac{2}{3.2\times10^{-2}}\right)^2}$$

$$= 9.5327 \text{ GHz}$$

ii) $\lambda = \frac{c}{f} = \frac{3\times10^8}{50\times10^9} = 6\times10^{-3}$ cm

$$\lambda_c = \frac{c}{f_c} = \frac{c/f}{\sqrt{1-(f_c/f)^2}} = 3.145 \text{ cm}.$$

iii) $\beta = \sqrt{\omega^2\mu\epsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$

$$= \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 1.8426\times10^{22} \text{ rad/m}$$

iv) $\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}$

$$v_p = \frac{\frac{1}{\sqrt{\mu_0 \epsilon_0 \epsilon_r}}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = 1.7045\times10^8 \text{ m/s}$$

v) Group velocity:

$$v_g \times v_p = c^2$$

$$v_g > c > v_p \quad , v_g = \frac{c^2}{v_p} = 5.3\times10^8 \text{ m/s}$$

vi) TE_{22}:

$$\eta_{TE} = \frac{\eta}{\sqrt{1-(f_c/f)^2}} = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{\sqrt{\mu_0/\epsilon_0\epsilon_r}}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= 219.2\,\Omega.$$

2Q)

2015 7(b) If $a=2b$, TE_{10} mode is dominant mode.
& $a>b$

If $b=2a$, TE_{01} mode is dominant mode.
& $a<b$

for guide wavelength,

$$\lambda g = \frac{\lambda_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} \qquad f_c = \frac{c}{2a} = \frac{3\times10^8}{2\times4\times10^{-2}} = 3.75\,GHz$$

$$= \frac{c/f}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$= \frac{3\times10^8/3.75\times1.2\times10^9}{\sqrt{1-\left(\frac{1}{1.2}\right)^2}}$$

$$= 12.1\text{ cm}.$$

$$v_p = \frac{1/\sqrt{\mu_0\epsilon_0}}{\sqrt{1-(f_c/f)^2}} = \frac{c}{\sqrt{1-\left(\frac{1}{1.2}\right)^2}} = 5.43\times10^8 \text{ m/s}$$

$$v_g = \frac{c^2}{v_p} = 1.66\times10^8 \text{ m/s}.$$

Q) A plane EM wave is incident on a mat... of

2015 $\epsilon_r = 2.34$. Calculate the reflection coefficient

7(c)

$$\Gamma = \frac{\eta - \eta_0}{\eta + \eta_0} = \frac{\sqrt{\frac{\mu}{\epsilon}} - \sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{\frac{\mu}{\epsilon}} + \sqrt{\frac{\mu_0}{\epsilon_0}}} = \frac{\sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} - \sqrt{\frac{\mu_0}{\epsilon_0}}}{\sqrt{\frac{\mu_0 \mu_r}{\epsilon_0 \epsilon_r}} + \sqrt{\frac{\mu_0}{\epsilon_0}}}$$

$$= \frac{\frac{1}{\sqrt{\epsilon_r}} - 1}{\frac{1}{\sqrt{\epsilon_r}} + 1}$$ [$\because \mu_r$ is not given take it as 1]

$$= -0.209$$

$$VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 1.528$$

Q) In an air filled rectangular wave guide with

10 dimensions $\underset{b}{4} \times \underset{a}{8}$ ~~cm~~ operates in the $\underset{\text{as it is}}{TE_{10}}$ mode.

as it is TE_{10} mode.

Calculate i) f_c

ii) V_p at $f = 4GHz$.

i) $f_c = \frac{c}{2a} =$ ~~18.75GHz~~ $1.875GHz$

ii) $V_p = \frac{1/\sqrt{\mu_0 \epsilon_0}}{\sqrt{1 - (f_c/f)^2}} =$

13) Q) TE_{10} mode, $b = 1cm$, $\beta = 102.65\ rad/m$.

$f = 12GHz$ at TE_{10} mode.

i) $a = ?$ ii) wave impedance.

$$\beta = \omega\sqrt{\mu\epsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$= \frac{2\pi f}{c}\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$\Rightarrow f_c = 10.955GHz$

$TE_{10} \Rightarrow f_c = \frac{c}{2a} \Rightarrow a = \frac{c}{2f_c} = 1.4cm$

$$Z_{TE} = \frac{\eta}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}} = \frac{\sqrt{\mu/\epsilon}}{\sqrt{1 - \left(\frac{10.955}{12}\right)^2}} = \frac{377}{\sqrt{1 - \left(\frac{10.955}{12}\right)^2}} = 923.72\Omega$$

Dt. 8/3/16

Q) A rectangular waveguide is designed to propagate
6.7) dominant mode at a frequency of 5GHz. The cutoff
2013 frequency is 0.8 times the signal frequency. The ratio of
2(a) waveguide height to width is 2. The time avg. power flowing through the guide is 1kW. Determine the magnitude of electric and magnetic field intensities in the guide and indicate where they occur in the guide.

Dominant Mode is TE_{10}.

Given $f = 5GHz$, $f_c = 0.8f = 4GHz$

$\frac{a}{b} = 2$, $P_{av} = 10^3$ watt.

$$Z_{TE} = \frac{\eta \cdot 377}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

$$P_{av} = \frac{1}{2Z_g}\int_0^a\int_0^b \left[|E_x|^2 + |E_y|^2\right] dx\,dy$$

$$= \frac{\sqrt{1+\left(\frac{f_c}{f}\right)^2}}{2\eta}\int_0^a\int_0^b E_0^2 \sin^2\frac{\pi x}{a}\,dx\,dy$$

In TE_{10} mode $E_x = 0$

$$E_y = E_0 \sin\frac{\pi x}{a} e^{-j\beta z}$$

$$P_{av} = 10^3 = \frac{\sqrt{1-(0.8)^2}}{2\times 377} E_0^2 \int_0^a\int_0^b \sin^2\frac{\pi x}{a}\,dx\,dy$$

$$\Rightarrow 1000 = \frac{0.6}{4\times 377} E_0^2\, ab$$

$$= \frac{0.6}{4\times 377} E_0^2 \times 2b^2$$

TE_{10} mode, $f_c = \frac{c}{2a}$

$$\Rightarrow 4\times 10^9 = \frac{3\times 10^8}{2a}$$

$a = 0.0375\,m = 3.75\,cm$

$b = 0.01875 = 1.875\,cm$

$E_0 = 42.276\ kV/m$

$E_y = 42.276\times 10^3 \sin\frac{\pi x}{a} e^{-j\beta z}$

$\frac{E_y}{H_x} = Z_{TE}$

$H_x = \frac{E_y}{Z_{TE}} = \frac{E_y}{377/0.6}$

$H_x = 67.3\times10^{3} \sin\frac{\pi x}{a} e^{-j\beta z}$

14) $a = 3\,cm$, $b = 2\,cm$, TE mode at 6 GHz

loss tangent in air is 0.001.

$\sigma = 5.8\times10^{7}$ S/m.

Assuming dominant mode of TE mode i.e. TE_{10} mode,

$f_c = \frac{c}{2a} = \frac{3\times10^{8}}{2\times3\times10^{-2}} = 5\ GHz$

$\beta = \sqrt{\omega^2\mu\epsilon - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$

$= \sqrt{\omega^2\mu\epsilon - \left(\frac{\pi}{a}\right)^2}$

$= \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 69.46\ rad/s$

$\omega = \sqrt{\left(\frac{2\pi f}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2} = \sqrt{\left(\frac{2\pi\times6\times10^{9}}{3\times10^{8}}\right)^2 - \left(\frac{\pi}{3\times10^{-2}}\right)^2}$

For loss tangent:

$\tan\delta = 0.001$.

where δ = skindepth $= \tan^{-1} 0.001$

$= 0.001$

$\alpha_d = \frac{k^2 \tan\delta}{2\beta}$

$= \frac{(125.6)^2\times0.001}{2\times69.46}$

$= 0.1137\ np/m$

$k = \omega\sqrt{\mu\epsilon}$

$= \frac{2\pi f}{c} = \frac{2\pi\times6\times10^{9}}{3\times10^{8}}$

$= 125.6$

$1\,np = 8.686\,dB$

$\alpha_d = 0.9863\ dB/m$.

$$\alpha_c = \frac{R_s}{a^3 b \beta k \eta}(2b\pi^2 + a^3k^2) \quad \text{where } R_s = \sqrt{\frac{\omega \mu_0}{2a}}$$

$$= \sqrt{\frac{2\pi f \times \mu_0}{2a}}$$

135.6 3.77

$= 19.14 \times 10^3$ np/m. $\qquad = 202.09$ kΩ

8) In free space magnetic field is given as

2013 (20)

$H = 1.33\cos(8\times10^8 t - \beta z)\,a_x$ A/m.

Given expression for corresponding electric field, the propagation constant and the wave length. Compute also the powerflow per unit area. What is the direction of power flow?

$\frac{E_y}{H_x} = \eta = 377$ (for air)

$E = \eta H$

$= 1.33 \times 377 \cos(8\times10^8 t - \beta z)\,a_y$ V/m.

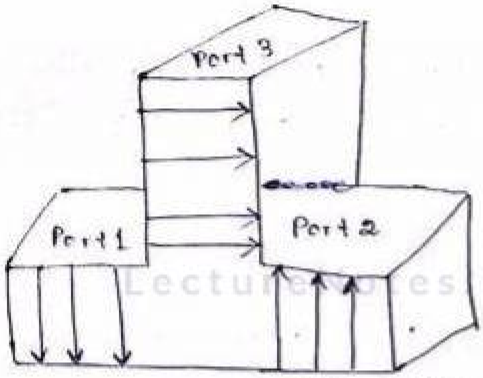
$\beta = \frac{\omega}{c} = \frac{8\times10^8}{3\times10^8} = 2.67$ rad/s.

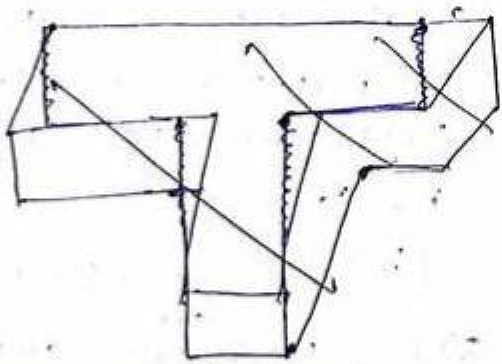
$\lambda = \frac{2\pi}{\beta} = 2.35$ m.

Power flow,

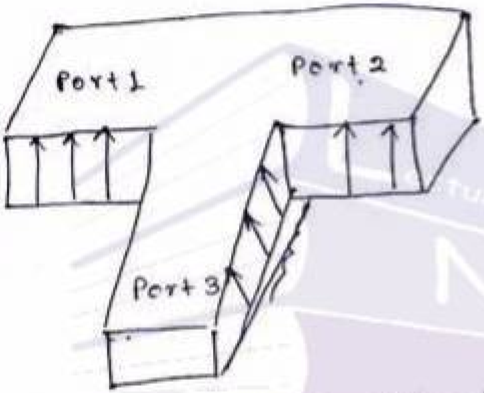
$P = \frac{1}{2}\oint E \times H$

$= \frac{1}{2}\int (1.33\times377)^2 \cos^2(8\times10^8 - \beta z)\,dx\,dy$.





E-plane Tee junction / section Tee (Voltage / series junction)



H-plane Tee Junction / shunt Tee (current junction → ∵ magnetic field H devides)

Scattering matrix of E-plane Tee:

- Intersection of 2 rectangular waveguide to form a 3-port network.
- seem like alphabet 'T', hence Tee junction.
- E-plane Tee junction is symetrical about port 3.
- signal fed in port-3 split into 2 equal parts at port 1 & 2 but 180° phase difference.
- Signal from same amplitude and opposite phase fed from 1 & 2 respectively, give rise to sum of both signals at port-3.

- Same amplitude and same phase if applied to port 1 & 2, then no signal appears at port 3. (17)

S-matrix:

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Conditions to evaluate S:

1) If port-3 is perfectly matched to the input field source, no reflection occurs at port 3.

i.e. $S_{33} = 0$

[$S_{11}, S_{22}, S_{33} \rightarrow$ reflection coefficient]

2) Port 1 & 2 are equal amplitude but opposite in phase, that indicate

$S_{13} = -S_{23}$

3) If passive device is symmetric, then

$S_{ij} = S_{ji}$

e.g. - $S_{21} = S_{12}$, $S_{31} = S_{13}$, $S_{23} = S_{32}$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix}$$

$S(S^*)^T = I$

$(S^*)^T S = I$

Applying unitary matrix:

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & -S_{13} \\ S_{13} & -S_{13} & 0 \end{bmatrix} \begin{bmatrix} S_{11}^* & S_{12}^* & S_{13}^* \\ S_{12}^* & S_{22}^* & -S_{13}^* \\ S_{13}^* & -S_{13}^* & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

R_1C_1: $|S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 = 1$ ← (1)

R_2C_2: $|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 = 1$ — (2)

R_3C_3: $|S_{13}|^2 + |S_{13}|^2 + 0 = 1 \Rightarrow S_{13} = \frac{1}{\sqrt{2}}$ ← (3)

R_1C_3: $\{S_{11}S_{13}^*\} - \{S_{12}S_{13}^*\} = 0$

$\Rightarrow S_{13}^{*}(S_{11} - S_{12}) = 0$

$\Rightarrow S_{13}^{*} \neq 0$ but $S_{11} = S_{12}$ — ③

① - ② :

$|S_{11}|^2 - |S_{22}|^2 = 0$

$\Rightarrow S_{11} = S_{22} = S_{12}$ — ⑤

$|S_{11}|^2 = \frac{1}{4}$

eq ①, $|S_{11}|^2 + |S_{11}|^2 + \frac{1}{2} = 1$

$\Rightarrow S_{11} = \frac{1}{2}$

$$S = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \end{bmatrix}$$

Scattering matrix of H-plane Tee (Current Junction). shunt

- Junction formed by cutting a slot in the narrow side of the waveguide and adding a rectangular waveguide to form a H-plane Tee.
- Port 1 & port 2 are collinear ports.
- Signal fed from port 3 is equally divided into port 1 & 2 with same phase.

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Condition:

1) Input ports P_1, P_2 Equally divided with same phase.

i.e. $S_{13} = S_{23}$ — ①

2) Port 3 is perfectly matched to input source.

i.e. $S_{33} = 0$ — ②

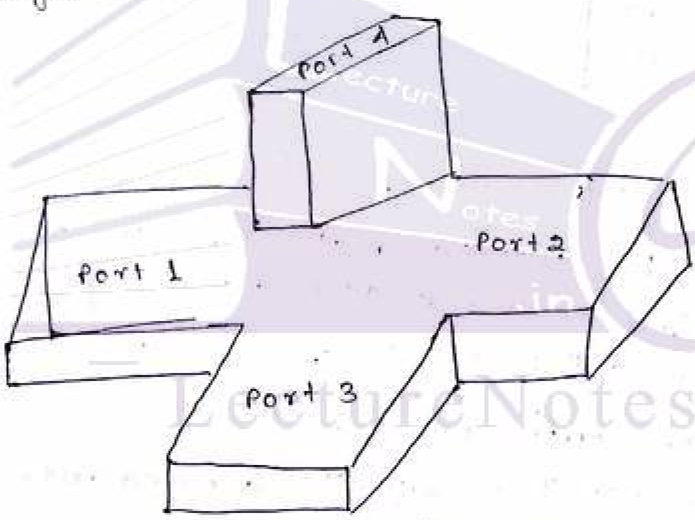
- Symmetrical device $S_{12} = S_{21}$, $S_{31} = S_{13}$, $S_{32} = S_{23}$.

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{13} \\ S_{13} & S_{13} & 0 \end{bmatrix}$$

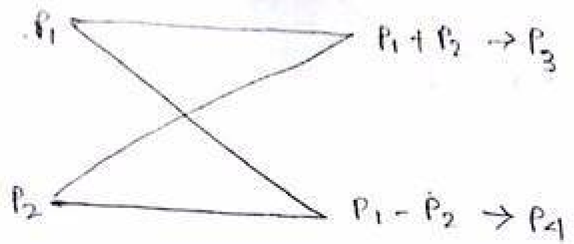
Applying unitary matrix,

$$S = \begin{bmatrix} 1/\sqrt{2} & -1/2 & 1/\sqrt{2} \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix}$$

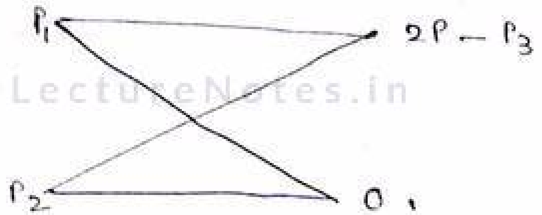
Magic TEE (E&H plane TEE):



- Combination of both E-plane, H-plane Tee is called magic Tee. Hence, obey the properties of both E & H plane, with all 4 matched ports.
- Signal from prot 3 (H-arm) → P_1, P_2 equally divided with same phase.
- Signal port 4 (E-arm) → P_1, P_2 equally divided with opposite plane.
- Signal is transmitted through P_1 & P_2 with different amplitude but same phase.



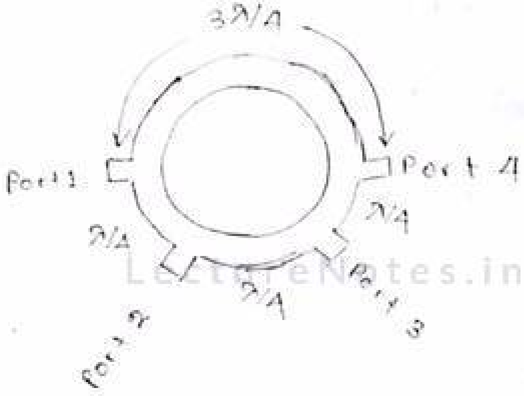
- Same signal with same phase



- Signal fed to port 1 doesn't appear at port 2 & vice-versa because E-arm causes a phase delay to the signal & H-arm causes a phase advance to the signal that results $S_{12} = S_{21} = 0$.

$$S = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \\ 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

Hybrid Ring (Rat Race):



Hybrid ring is used to combine or divide microwave power. The circular structure allows the total phase reversal (180° phase shift) of the microwave input signal. It consist of 3 λ/4 transmission line sections and 1 3λ/4 transmission line section. The phase delay is

Phase delay (in medium) $= \frac{2\pi}{\lambda} \times$ Path delay.

Let us consider input port is port1 or port 1 is excited. The equal output will appear at port 2 & port 4 and port 3 remain isolated.

Path length of port 2 $= (P_1 \leftrightarrow P_2) = \lambda/4$

port 4 $= (P_1 \leftrightarrow P_4) = 3\lambda/4$

If the characteristic impedance of all the ports is same, then power from port 1 will get divided between port 2 and port 4 but opposite in phase. Because phase difference,

$$\Delta\phi = \frac{2\pi}{\lambda}\left(\frac{3\lambda}{4} - \frac{\lambda}{4}\right) = \pi.$$

Similarly, at port 3 it receives 2 types of paths:

one via port 2 $= \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$

one via port 4 $= \frac{3\lambda}{4} + \frac{\lambda}{4} = \lambda$.

$$\Delta\phi = \frac{2\pi}{\lambda}\left(\lambda - \frac{\lambda}{2}\right) = \pi.$$

Wave reaching at port 3 opposite phase and equal magnitude, hence cancel each other. Therefore, no output is evaluated; therefore isolated.

If 2 signals 'a' & 'b' fed to port 1 & port 3, then
at port 2 = a+b
at port 4 = a-b

Input, Output & Isolated ports in Hybrid Ring:

Excited Port	Output Port	Isolated Port	Phase diff bet output port
1	2, 4	3	180°
2	1, 3	4	0°
3	2, 4	1	0°
4	3, 1	2	180°

Scattering Matrix:

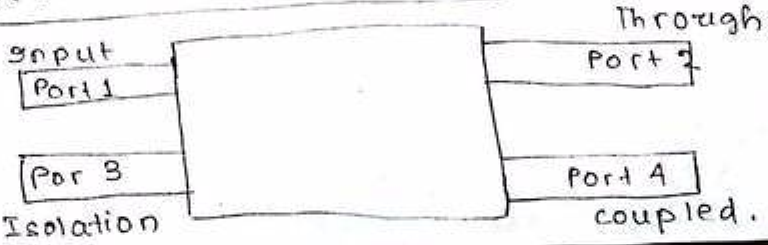
$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

1) All the ports of hybrid ring are perfectly matched.
i.e. $S_{11} = S_{22} = S_{33} = S_{44} = 0$ no reflection coefficient
2) The isolated ports,
$S_{13} = S_{24} = S_{31} = S_{42} = 0$.

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

Scattering matrix of rat race coupler.

Directional Coupler (M.I):



It is a flanged wave guide assembly used for microwave power measurement. It is a 4 port passive device in which reflection free transmission occurs between port 1 & port 2, if all the ports are terminated in their characteristic impedance.

- No coupling exists between the pair ports 1 & 3 & 2 & 4.

- The performance of the directional coupler is characterised by the following values.

Assume P_1 = i/p power

P_2 & P_4 = o/p power

P_3 = isolated (auxillary port)

coupling factor 'C' $= 10\log_{10}\frac{P_1}{P_4}$ dB

directivity 'D' $= 10\log_{10}\frac{P_4}{P_3}$

for ideal directional coupler $D \to \infty$

commercially 30-35 dB avialable.

Insertion loss $L = 10\log_{10}\frac{P_1}{P_2}$

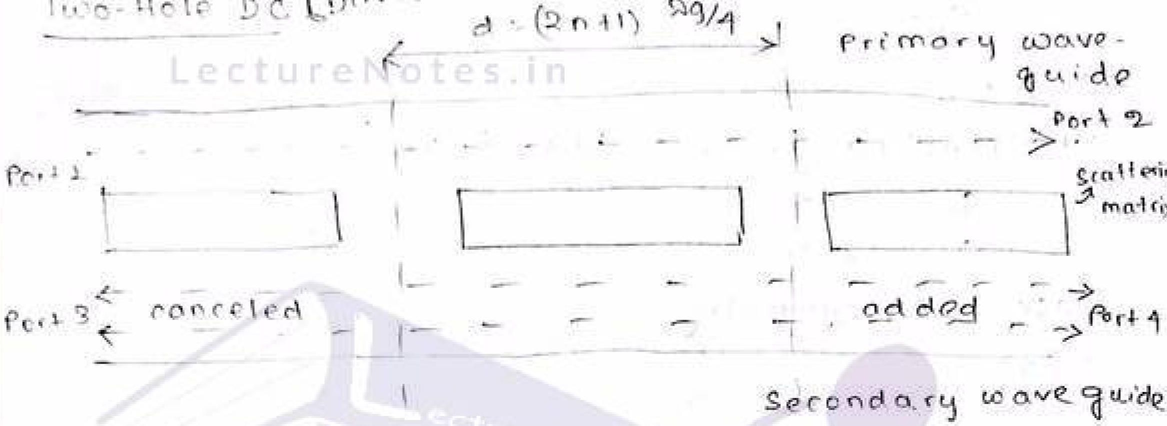
Isolation $I = 10\log_{10}\frac{P_1}{P_3}$ $I = \infty$ for ideal.

Prove $I = D + C$.

There are many directional couplers aviatable commercially. Some of them are

1. two-hole directional coupler
2. 4-hole " "
3. Bothe-hole " "
4. Schwinger coupler (reversal coupling of DC)

Two-Hole DC (Directional coupler):



Dt. 12/4/16

The spacing between two holes must be an odd multiple of quarter wavelength and can be written as

$d = 2(n+1)\lambda g/4$, where n = positive integer,
$\lambda g \rightarrow$ guided wavelength.

- Wave fed from port 1 → port 2

Port 4 — one through 1st hole, $d = (2n+1)\lambda g/4$
— one through 2nd hole, $d = (2n+1)\lambda g/4$.

Port 3 — 1st hole (0)
cancel each other due to opposite phase
— 2nd hole $[2d = 2(2n+1)\lambda g/4]$

$$\Delta\phi = \frac{2\pi}{\lambda} \times 2(2n+1)\frac{\lambda g}{4} = (2n+1)\pi.$$

Scattering matrix is written as

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix}$$

Phase difference is an odd multiple of π which was irrespective of hole size. These waves at port-3 always cancels out each other hence no power at port-3.

Case-1:
- All 4 ports are completely matched i.e.

$$S_{11} = S_{22} = S_{33} = S_{44} = 0.$$

Case-2:
- No coupling between (1 & 3) & (2 & 4)

$$S_{13} = S_{31} = 0$$
$$S_{24} = S_{42} = 0$$

Case-3: By symmetry
- $S_{13} = S_{31}$

$S_{12} = S_{21}$

$S_{14} = S_{41}$

$S_{23} = S_{32}$

$S_{34} = S_{43}$

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{12} & 0 & S_{23} & 0 \\ 0 & S_{23} & 0 & S_{34} \\ S_{14} & 0 & S_{34} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & 0 & S_{14}^* \\ S_{12}^* & 0 & S_{23}^* & 0 \\ 0 & S_{23}^* & 0 & S_{34}^* \\ S_{14}^* & 0 & S_{34}^* & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1C_1 : |S_{12}|^2 + |S_{14}|^2 = 1$ — ①

$R_2C_2 : |S_{12}|^2 + |S_{23}|^2 = 1$ — ②

$R_3C_3 : |S_{23}|^2 + |S_{34}|^2 = 1$ — ③

$R_1C_3 : S_{12}S_{23}^* + S_{14}S_{34}^* = 0$ — ④

From eq^n ① & ②, $S_{14} = S_{23}$

eq^n ② & ③, $S_{12} = S_{34}$

Let : $S_{12} = S_{34} = P = S_{34}^*$ (P is real & the no.)

(or) $P S_{23}^* + S_{23} P = 0$ $\quad [\because S \times (S)^* = I]$

$\Rightarrow S_{23}^* = -S_{23}$

Let $S_{23} = jq \quad S_{23}^* = -jq$

$$S = \begin{bmatrix} 0 & P & 0 & jq \\ P & 0 & jq & 0 \\ 0 & jq & 0 & P \\ jq & 0 & P & 0 \end{bmatrix}$$

Three Port Network:

- It is not possible to construct a 3 port network:

i) loseless ii) reciprocal iii) matched at all ports.

Let us consider a scattering matrix of 3 port network:

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Case - I :

If network is matched at every port then

$S_{11} = S_{22} = S_{33} = 0$ (refl. coefficient $= 0$)

Case - II :

If network is reciprocal i.e. due to symmetry then

$S_{ij} = S_{ji} \Rightarrow S_{12} = S_{21}$, $S_{31} = S_{13}$; $S_{32} = S_{23}$.

For a matched ratiprocal 3 port, the scattering matrix can be written as:

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

Case - III :

Lastly, if the network is Lossless, the product of scattering matrix with its conjugate is unitary matrix.

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{12}^* & 0 & S_{23}^* \\ S_{13}^* & S_{23}^* & 0 \end{bmatrix}$$

$$\left.\begin{aligned} R_1C_1 &: |S_{12}|^2 + |S_{13}|^2 = 1 \text{ --- } (1) \\ R_2C_2 &: |S_{12}|^2 + |S_{23}|^2 = 1 \text{ --- } (2) \\ R_2C_3 &: |S_{13}|^2 + |S_{23}|^2 = 1 \text{ --- } (3) \end{aligned}\right\} (*)$$

$$\left.\begin{aligned} S_{13}^* S_{23} &= 0 \\ S_{23}^* S_{12} &= 0 \\ S_{12}^* S_{13} &= 0 \end{aligned}\right\} (**)$$

Eq. (**) indicate that ~~all~~ atleast two of the 3 s-parameters must be equal to zero. But if this is true, the all eq. in (*) can't be satisfied, i.e.

e.g. - $S_{13} = 0$

(1) & (2) eq. (*) , $S_{12} = S_{23} = 1$.

Now in eq. ** 2nd eq. is not possible.

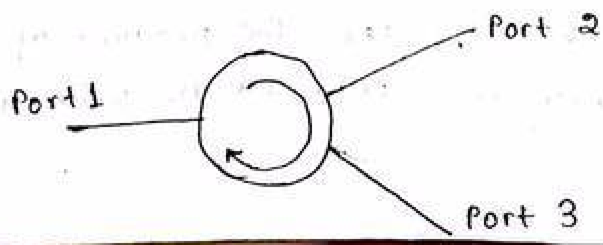
as $S_{23}^* S_{12} \neq 0$, (contradiction)

Therefore, we conclude 3-port network can't be lossless, reciprocal and matched at all ports. However, one can realise such a network if any 2 conditions are valid i.e.

Non-Reciprocal 3 port :

In this case a lossless 3-port i.e., matched at all the ports can be realised called as circulator.

$$S = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$



2) Matched only 2 ports of 3 ports:

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix}$$

3) Lossy Network:

In this case, all ports are simultaneously matched & reciprocal.

- DM Pozar
- 4 port n/w

Cavity Resonator:

- A cavity resonator is a space normally bounded by conducting surface (with short circuit. both end in which electromagnetic energy is stored as oscillating fields.
- The stored electric & magnetic energies inside the cavity resonator determine its equivalent inductance & capacitance.
- The energy dissipated by the finite conductivity of the finite one, determine its equivalent resistance.

Salient features of Cavity Resonator:

1) A cavity resonator is in the form of closed space and is a regional cavity/circuit.

2) It stores electromagnetic energy in the form of oscillatory fields.

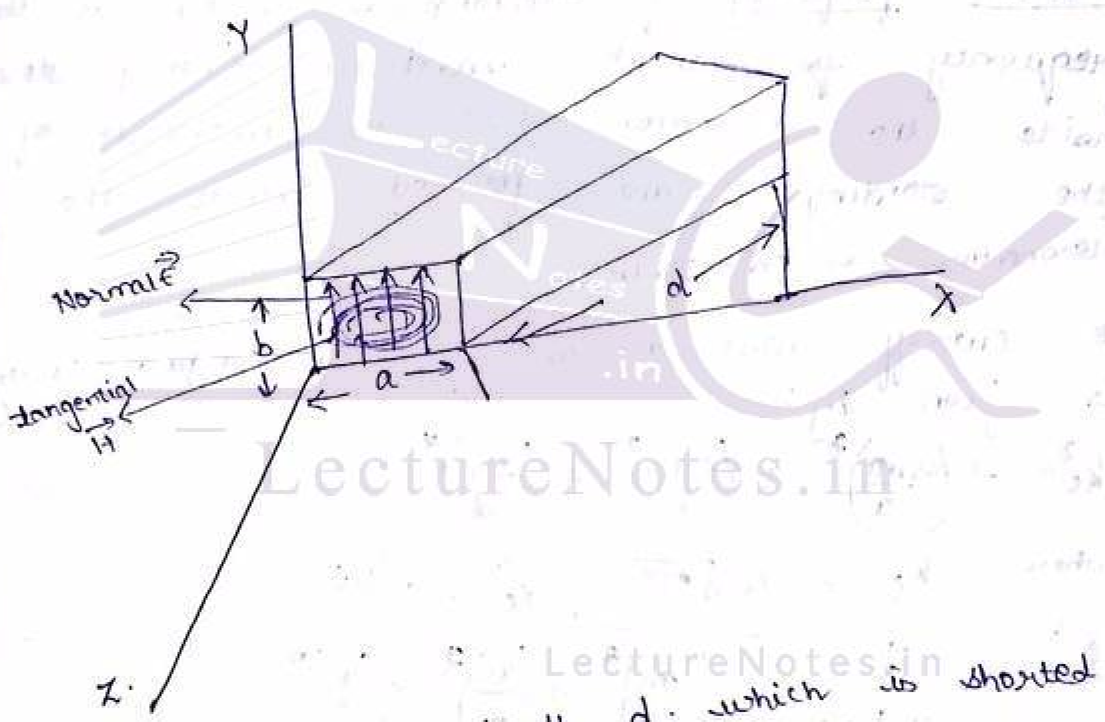
3) The resonant frequency depends on the shape & size of the cavity.

4) The cavities are commonly in the shape of rectangular, spherical or cylindrical.

5) The cavities are also designated as re-entrant cavity.

6) The probes, loops and slots are used to remove the electromagnetic energy from the cavities

<u>Rectangular Cavity Resonator</u> :-



It consist of a length d which is shorted at both ends (i.e. $z = 0$ to $z = d$). Generally, d is multiple of half guide wavelength.
i.e. $d = n\lambda_g/2$. The field inside the cavity must satisfy the following conditions

1. The tangential $\vec{E}$ field component to the walls must be zero.
2. The normal $\vec{H}$ field component to the walls must be zero.

The $\vec{z}$ component of the magnetic field in TE_{mnp} mode can be given by.

(60)

$$H_z = H_{oz} \cos\left(\frac{m\pi}{a}\right) \cos\left(\frac{n\pi}{b}\right) \sin\left(\frac{p\pi}{d}\right).$$

Similarly,

$\vec{x}$ Component of electric field in TE mnp mode can be given by :-

$$E_x = E_{oz} \sin\left(\frac{m\pi}{a}\right) \cdot \sin\left(\frac{n\pi}{b}\right) \cos\left(\frac{p\pi}{d}\right).$$

where, mnp = 0, 1, 2 . . . . no. of half waves in +x, +y and +z direction respectively.

<u>Resonant frequency</u> of Rectangular cavity is the frequency for which maximum energy stored inside the resonator i.e. the amplitude of the standing wave formed inside the resonator is maximum.

The cut-off wave number for rectangular cavity is given by :-

$$k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2.$$

where $k_c = \omega\sqrt{\mu\epsilon}$, $k_c^2 = \omega^2\mu\epsilon$.

$$f_r = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}.$$

$\underline{b < a < d}$ $\left.\begin{matrix} TE_{101} \\ TM_{110} \end{matrix}\right\}$ dominant mode.

<u>Quality factor</u> (Q-factor) :- It is a measure of frequency selectivity of resonant and non-resonant circuit.

$$Q = 2\pi f_r \frac{\text{max}^m \text{ Energy stored}}{\text{Energy dissipated per cycle}}$$

$$= \frac{\omega_r W_{e \& m}}{P_{\text{average Power}}}$$

At resonant frequency

$W_e = W_m$.

$$= \frac{\epsilon}{4} \int_V |E|^2 \cdot dv = \frac{\mu}{4} \int_V |H|^2 \, dv.$$

The average power loss;

$$P = \frac{R_s}{2} \int_S |H_t|^2 \, dS$$

where, $|H|^2 = |H_t|^2 + |H_n|^2$.

$$Q = \frac{\omega\mu \int_V |H|^2 \cdot dv}{2 R_s \int_S |H_t|^2 \, dS}$$

$$= \frac{\omega\mu}{2 R_s}$$

In half-power bandwidth or 3 dB bandwidth can be calculated as

$$Q_0 = \frac{f_r \text{ (resonant freq.)}}{\Delta f}$$

Δf = half power or 3 dB frequency.

<u>Unloaded Quality Factor (Q_0)</u> :-

When a cavity resonator is not connected with any external circuit or load; then the quality factor is said to be unloaded. This unloaded Q-factor associated with internal losses i.e. conduction losses, dielectric losses and radiation losses.

For rectangular cavity resonator having

dimensions a, b, d i.e.

$$Q_0 = \frac{\pi\eta}{4R_s} \times \frac{2b(a^2+d^2)^{3/2}}{ad(a^2+d^2) + 2b(a^3+d^3)}.$$

Q_0 will be maximum at $a = d$;

$$Q_{0_{max}} = \frac{\pi\eta}{R_s} \times \frac{1}{1+\frac{a}{2b}}.$$

The Q_{ext} (external quality factor)

The Q_{ext} is associated with only external loss (i.e. reflection losses due to impedance matching) occured due to presence of the external load.

$$Q_{ext} = \frac{Q_0}{k}$$

k = Coupling factor.

$k = 1$ for critical coupling.

$k < 1$ & $k = \frac{1}{SWR}$ for under-coupling.

$k > 1$ & $k = SWR$ for over-coupling.

$$Q_L = \frac{Q_0}{1+k}.$$

$$\therefore \frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}.$$

Q_L = loaded quality factor.

Q) An air filled rectangular cavity resonator has dimension $a = d = 5$ cm, $b = 3$ cm. Find out the dominating mode, resonant frequency, unloaded quality factor and half-power bandwidth

Assume that inner surfaces of cavity resonator are silver coated and conductivity $\sigma = 6.17 \times 10^7$ S/m

Dominant Mode :- TE_{101}.

(62)

$$f_r = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2 + \left(\frac{p}{d}\right)^2}$$

$$\therefore f_r = \frac{c}{2} \sqrt{\left(\frac{1}{5\times10^{-2}}\right)^2 + \left(\frac{0}{3\times10^{-2}}\right)^2 + \left(\frac{1}{5\times10^{-2}}\right)^2}$$

$$= 4.24 \text{ GHz}.$$

$$Q_0 = \frac{\pi \eta^{\to 377}}{4R_s} \times \frac{2b(a^2+d^2)^{3/2}}{ad(a^2+d^2) + 2b(a^3+d^3)}$$

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = 0.0164\,\Omega.$$

$$Q_{0max} = \frac{1.11\eta}{R_s} \times \frac{1}{1+\frac{a}{2b}}$$

$$= 13917.$$

$$\Delta f = \frac{f_r}{Q_{0max}} = \frac{4.24 \times}{13917} = 304.9 \text{ kHz}.$$

Smith Chart:

1Q) A 50Ω TL is terminated on a loop of 100 + j70 using smith chart, find

i) reflection coefficient

ii) transmission coefficient

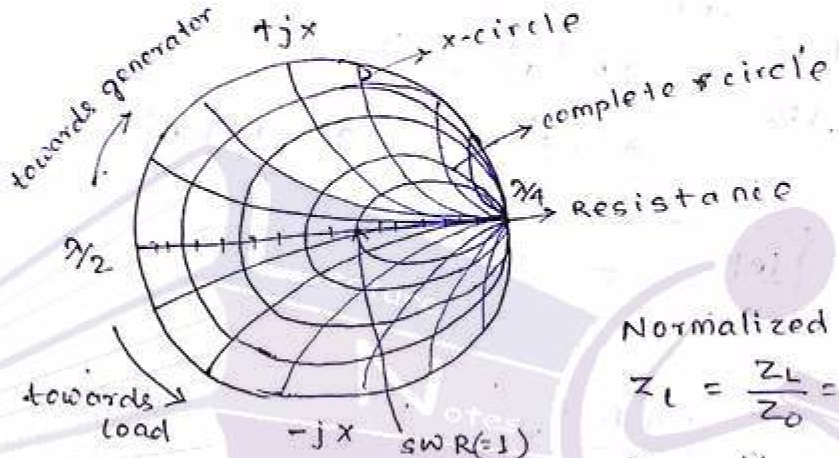
iii) VSWR

iv) i/p impedance at a distance 0.3λ from Load.

v) distance of the 1st minimum of the load.

Note:

What is smith chart?

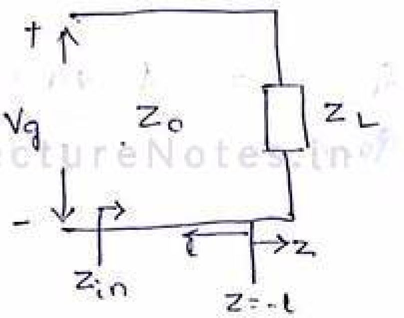


Normalized Value

$$z_l = \frac{Z_L}{Z_0} = \frac{1/Y_L}{1/Y_0}$$

$$Y_L = \frac{Y_0}{Y_L}$$

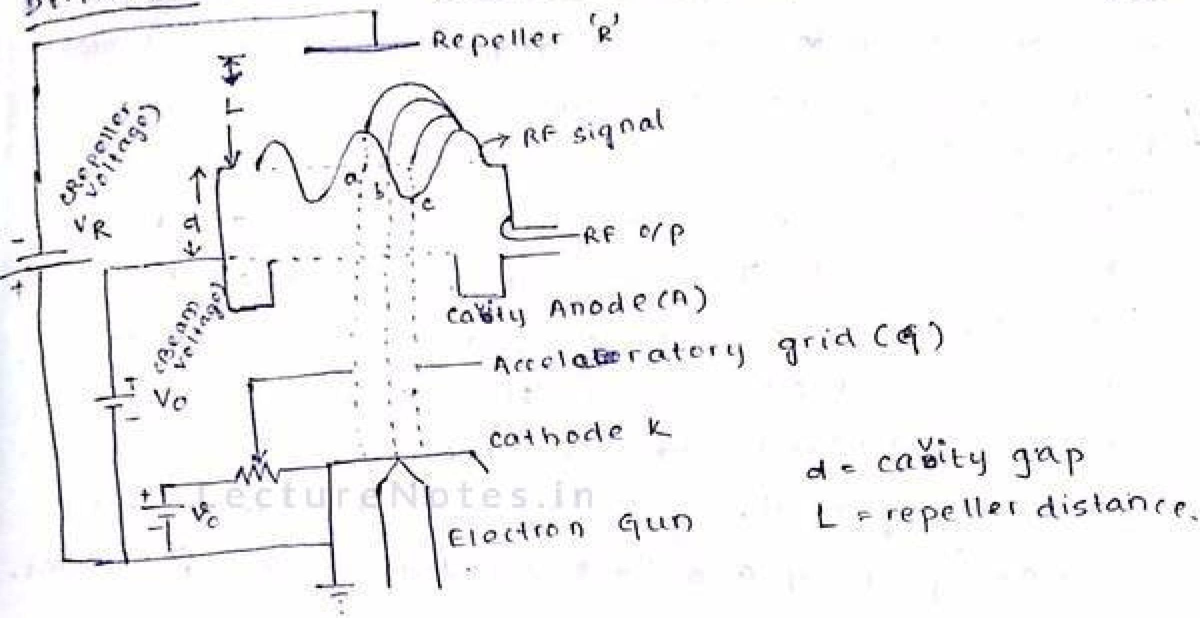
After every λ/2, characteristics repeates.



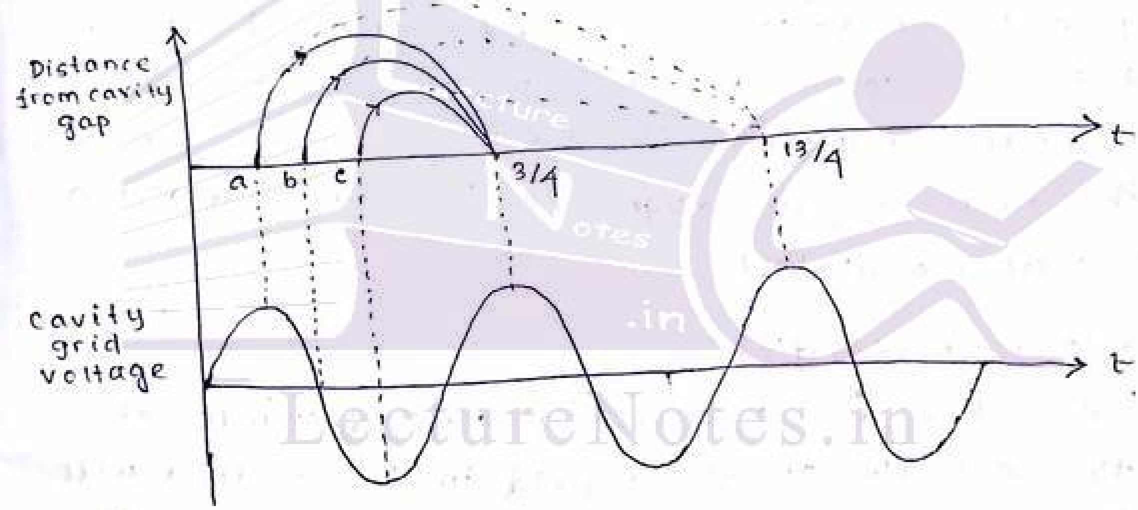
2) A line is terminated in a normalized admitance $Y_L = 0.2 - j0.5 = \frac{Y_0}{Y_L}$. Find the location of voltage max^n from the loaded also find the ~~location of~~ reflec^n coefficient, normalized impedance at a distance 0.12λ from the load. Also calculate normalized admitance 0.12λ from the load.

Dt. 15/3/16

Microwave Source:



(Reflex Klystron)



(Applegate diagram)

Construction:

A reflex klystron is a low power, low efficiency microwave oscillator shown in fig. 94 consists of an electron gun, a filament surrounded by cathod and a focusing electrode i.e. accelerating grid (G). The suitabley form electron beam is accelarated towards the cavity, which has a high positive voltage 'V_0' applied to it (Beam voltage) and thus act as anode.

After passing through the gap in the cavity, electrons travel towards repeller electron, which is at high -ve potential 'V_R'.

Repeller Space:

The electrons never reach this electrode ~~are~~ R because of the -ve field. The electrons are repelled by from the midway of the repeller space by the repeller are towards anode cavity. If the conditions are properly adjusted, then the returning electrons give energy to the gap that they took from it on forward journey and these oscillations are sustained.

Operation:

1) RF Noise:

Due to DC voltage in the cavity circuit, RF noise is generated in the cavity. This electromagnetic noise fed in the cavity becomes pronounced a cavity resonant frequency. The electrons passing through the cavity gap 'd' experience this RF field and are velocity modulated.

2) Velocity Modulation:

The electron 'a' shown in fig. which encounter the +ve halfcycle of the RF field in the cavity 'd' will be accelerated, those reference electrons like 'b' electron, which encounter zero RF field, will pass with unchanged original velocity and electron 'c' which encounter the -ve halfcycle will be retarded on entering the repeller space.

All those velocity modulated electrons will be repelled back to the cavity by the repeller due to its -ve potential. The repeller distance 'L' and the voltages can be adjusted to receive all the velocity modulated electrons at a same time on the +ve peak of the cavity RF voltage cycle. Thus the velocity modulated electrons are

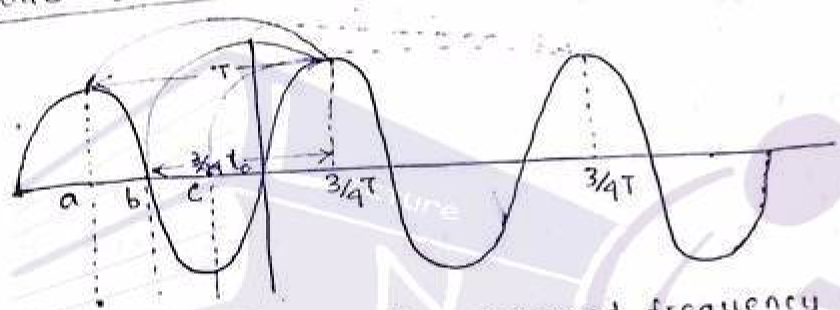
bunched together and loose their KE; when they (67)
encounter positive cycle of cavity RF field. This
loss of energy is thus transformed to conserve
the total power.

Dt. 18/3/16

Case-1: $P_{delivered}$ by bounded electron > P_{loss} in the cavity
Amplitude will increase to produce microwave oscillation

Case-2: $P_{delivered} = P_{loss}$ in the cavity.
A steady microwave oscillation is generated in resonant
frequency.

Mode of Oscillation:



Let T = time period at the resonant frequency
t_0 = time taken by the reference electron (b) to travel in the repeller space and returning to the cavity at the positive peak voltage on the formation of the bunch.

- Thus, by adjusting repeller voltage for a given dimensions of the reflex klystron, the bunching can be made to occur at $n = n + 3/4$ positive half cycle.
- Lowest order mode 3/4 occurs for maximum value of repeller voltage when transient time t_0 of the electron in the repeller space is minimum and the power output of lowest mode is maximum.

For calculation of RF power:

Assumption:

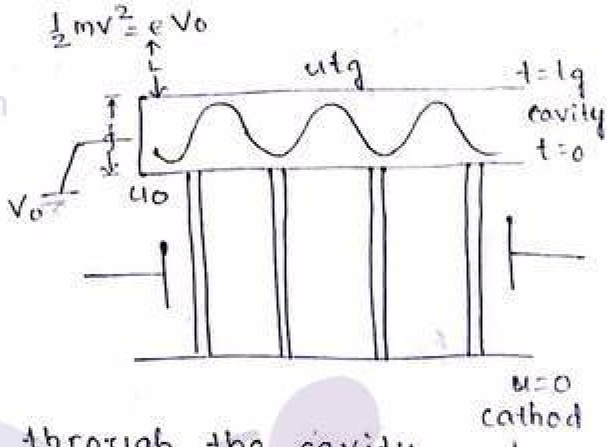
1) Cavity grids and repeller are plane parallel and very large in extent.
2) No RF field is excited in the repeller space.
3) Electrons are not intercepted by the cavity anode grid.

4) No debunching takes place in the repeller space.

5) The cavity RF gap voltage amplitude V_1 is small as compared to the DC beam voltage V_0 ($V_1 << V_0$).

Velocity Modulation:

The electron velocity 'u' attained due to the DC beam voltage V_0 while entering the cavity Gap at $t=0$ is uniform and is given by



$$u = u_0 = \sqrt{\frac{2eV_0}{m}}$$

$$= 5.93 \times 10^5 \sqrt{V_0} \ \text{m/sec}$$

The instantaneous cavity RF voltage can be written as :-

$$v(t) = V_1 \sin \omega t \qquad V_1 << V_0 .$$

The average transit time through the cavity. Gap 'd' and the transit angle can be written as

$$t_g = \frac{d}{u_0} \quad \text{and} \quad \theta_g = \omega t_g .$$

The average microwave voltage in the cavity gap can be written as;

$$V_{av} = \frac{1}{t_g} \int_0^{t_g} V_1 \sin \omega t \cdot dt$$

$$V_{av} = \frac{V_1}{\omega t_g} (-\cos \omega t_g + 1)$$

$$V_{av} = \frac{V_1}{\omega t_g} (1 - \cos \omega t_g)$$

$$V_{av} = \frac{V_1}{\omega t_g} \times 2 \sin^2 \frac{\omega t_g}{2}$$

$$= \frac{V_1}{\theta_g/2} \times \sin^2 \theta_g/2 .$$

$$\boxed{V_{avg} = V_1 \beta_1 \sin \theta_g/2}$$

where $\beta_1 = \dfrac{\sin \theta_g/2}{\theta_g/2}$ = de-coupling coefficient of the cavity gap.

$\beta_1 = 0$ for $d \Rightarrow 0$

Therefore, the coupling between the electron beam and cavity varies with the cavity gap 'd' in $\frac{\sin x}{x}$ form which is called as de-coupling coefficient.

The exist velocity from the cavity gap after velocity modulation is given by:

$$u_{tg} = \sqrt{\frac{2e(V_0 + V_{gv})}{m}}$$

$$= \sqrt{\frac{2e(V_0 + V_1\beta_1 \sin\theta_g/2)}{m}}$$

$$= \sqrt{\frac{2eV_0}{m}\left[1 + \frac{V_1\beta_1}{V_0}\sin\theta_g/2\right]}$$

where $\frac{\beta_1 V_1}{V_0}$ = depth of modulation

$$u_{tg} = \sqrt{\frac{2eV_0}{m}}\sqrt{1 + \frac{V_1\beta_1}{V_0}\sin\theta_g/2} = u_0\left(1 + \frac{V_1\beta_1}{2V_0}\sin\theta_g/2\right)$$

considering $\frac{V_1\beta_1}{V_0} << 1$

$$\Rightarrow u_{tg} = u_0\left[1 + \frac{\beta_1 V_1}{2V_0}\sin(\omega t_g - \theta_g/2)\right]$$

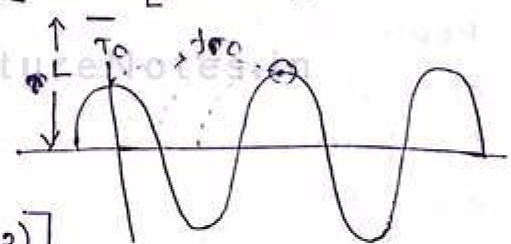
Transit Time: The round trip transit time in the repeller space is given by.

$$t_r = \frac{2 \times \text{velocity}}{\text{acceleration}}$$

$$F = ma = eE$$

$$\Rightarrow a = \frac{eE}{m} \text{ (Electric field) } = \frac{e}{m}\left(\frac{V_0 + V_r + V_1 \sin\omega t}{L}\right)$$

$$= \frac{e}{m}\left(\frac{V_0 + V_r}{L}\right)$$



$$t_r = \frac{2u(t_g)}{a}$$

$$= \frac{2u_0 mL\left[1 + \frac{\beta_1 V_1}{2V_0}\sin(\omega t_g - \theta_g/2)\right]}{e(V_0 + V_u)}$$

Since the reference electron doesn't under-go any velocity modulation its transit time in the repeller space can be written as;

$$t_0 = \frac{2u_0}{a} = \frac{2u_0 mL}{e(V_0 + V_r)} = NT = \frac{2\pi N}{\omega} \quad \text{—(1)}$$

$$\therefore t_r = t_0\left[1 + \frac{\beta_1 V_1}{2V_0}\sin(\omega t_g - \theta_g/2)\right]$$

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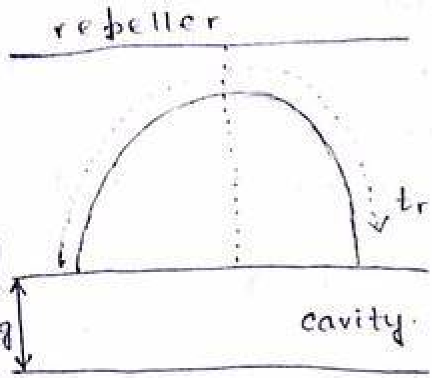
Density modulation & beam current :

The time of arrival of electron to the cavity gap can be expressed as



$$t_b = t_g + t_r$$

$$= t_g + t_0\left[1 + \frac{\beta_1 V_1}{2V_0}\sin(\omega t_g - \theta_g/2)\right]$$

$$= t_g + \frac{2\pi N}{\omega} + \frac{\pi N}{\omega}\frac{\beta_1 V_1}{V_0}\sin(\omega t_g - \theta_g/2)$$

$$\Rightarrow t_b = t_g + \frac{2\pi N}{\omega} + \frac{X}{\omega}\sin(\omega t_g - \theta_g/2) \quad \text{—— } *$$

where $X = \frac{\pi N \beta_1 V_1}{V_0}$ = Bunching parameter of Reflex Klystron —— (* *)

Diff. w.r.t. t_g, we have

$$\frac{dt_b}{dt_g} = 1 + X\cos(\omega t_g - \theta_g/2)$$

The bunched electrons on return constitute the bunched beam current i_b. Such that the conservation of charges gives

$$I_0 |dt_g| = i_b |dt_b|$$

I_0 → dc current.

$$i_b = \frac{I_0}{\left(\frac{dt_b}{dt_g}\right)} = I_0\left[1 + X\cos(\omega t_g - \theta_g/2)\right]^{-1}$$

From eqn *, since $V_1 << V_0$, $X << 1$, $t_b = t_g + \frac{2\pi N}{\omega}$

So that, $i_b = I_0\left[1 + X\cos(\omega t_b - 2\pi N - \theta_g/2)\right]^{-1}$.

By fourier expansion, the beam current of reflex klystron oscillator is

$$i_b = I_0 + 2I_0 \sum_{n=1}^{\infty} J_n(nX)\cos n(\omega t_b - 2\pi N - \theta_g/2)$$

$$= I_0 + 2I_0 J_1(X)\cos\omega(t_b - t_0 - \theta_g/2)$$
$$+ \sum_{n=2}^{\infty} 2I_0 J_n(nX)\cos n\omega(t_b - t_0 - t_g/2)$$

The fundamental component of RF induced current in the cavity can be written as

$$i_{RF} = \beta_1 2 I_0 J_1(X)\cos\omega(t_s - t_0 - t_g/2)$$

$= 2 I_0 \beta_1 J_1(X) \cos(\omega t_b - 2\pi N)$

($\because t_{g/2} << 2\pi N$, neglect the smallness)

Power Output :

The magnitude of fundamental RF current in the cavity is given by

$i_{RF} = 2 I_0 \beta_1 J_1(x)$.

The RMS RF power deliver to the cavity can be written as ~~power~~ P_{RF}.

$$P_{RF} = \frac{V_1}{\sqrt{2}} \cdot \times \frac{i_{RF}}{\sqrt{2}} = V_1 I_0 \beta_1 J_1(x) \quad \text{—— (1)}$$

From eq. (**)

$$P_{RF} = \frac{V_0 I_0 X}{\pi N} J_1(x)$$

From eq. (*)

$$\frac{2 N_0 M L}{e(V_0 + V_R)} = \frac{2\pi N}{\omega}$$

$$\Rightarrow \frac{2\sqrt{2eV_0/m}\, ML}{e(V_0 + V_R)} = \frac{2\pi N}{\omega}$$

$$P_{RF} = \frac{V_0 I_0 \times J_1(x)(V_0 + V_R)}{2\pi f L} \sqrt{\frac{e}{2mV_0}}$$

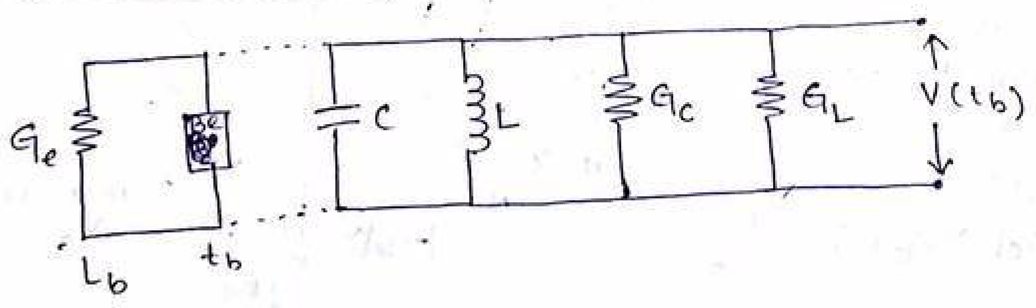
Efficiency:

Electric Efficiency $= \eta = \dfrac{P_{RF}}{P_{dc}}$

$P_{dc} = V_0 I_0$

$$\eta = \frac{x J_1(x)}{\pi N} = \frac{x J_1(x)(V_0 + V_R)}{2\pi f L} \sqrt{\frac{e}{2MV_0}}$$

Electronic Admittance (Y_e):



The electronic admittance (Y_e) of reflex klystron is defined by the ratio bunch beam current (i_b) and the cavity gap voltage at the time of bunching (t_b).

$$Y_e = \frac{i_{RF}(t_b)}{V(t_b)} = G_e + jB_e$$

$$= \frac{2 I_0 \beta_1 J_1(X) \cos(\omega t_b - \omega t_0)}{V_1 \sin \omega t_b}$$

where G_e - electronic conductance

B_e - electronic susceptance.

L & C - magnetic & electric storage of energy in the cavity

G_c - cavity loss conductance

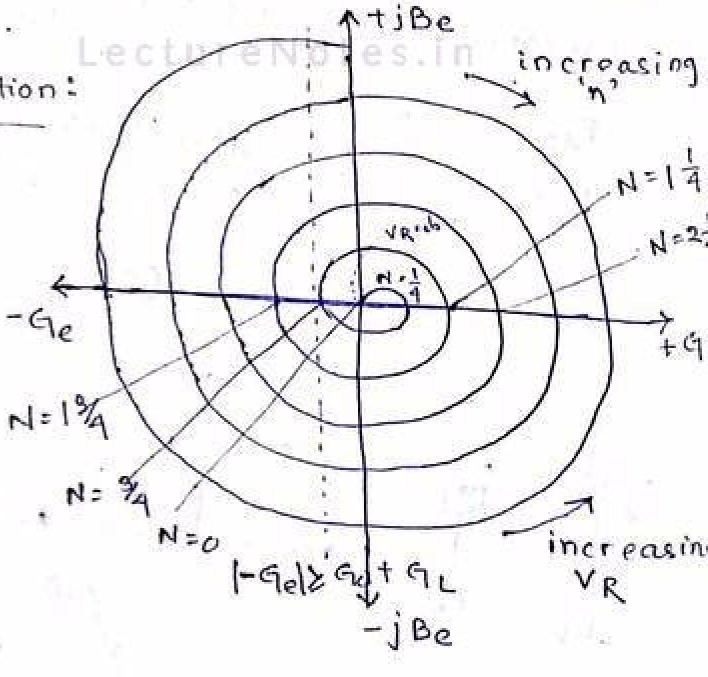
G_L - Load conductance.

- If the bunched electrons return to the cavity gap a little before reference transit time. $t_0 = (n + \frac{3}{4})T_0$, the AC beam current lags behind the field and an inductive reactance appears in the circuit for Be.
- If the bunched electrons return to the gap a little after t_0, the AC current leads the field and capacitive reactance is presented to the circuit for Be.

Condition for Oscillation:

The condition for oscillation is satisfied when G_e becomes negative and the total conductance in the circuit is negative.

$$|-G_e| \geq G_c + G_L = \frac{1}{R_{sh}}$$



$\therefore R_{sh}$: Effective Shunt Resistance.

This satisfies,

$$\omega t_0 = 2\pi(n + 3/4) = 2\pi N$$

$$n = 0, 1, 2, \ldots$$

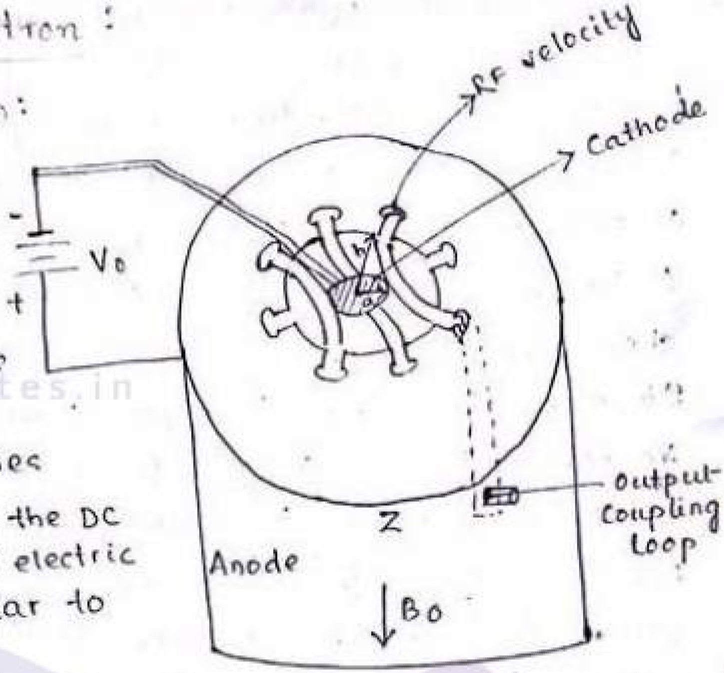
The variation of G_e and B_e in the complex Y_e plane is shown in figure above which forms a spiral from $V_R = \infty$, corresponding to the origin. The oscillation will occur for the values $2\pi N$ for the spiral lies in the area left of the plane $G_c + G_L$. If $Y_e = \pm jB_e$ oscillation takes place at a frequency lower and higher than the cavity resonant frequency.

Dt 25/3/16

Multicavity Magnetron :

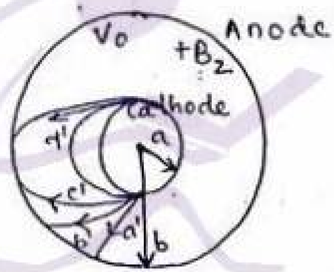
Principle of operation:

This is generated by Randall and boot. A magnetron oscillator is used to generate high peak microwave power. Magnetrons are cross-field tubes (M-type) in which the DC magnetic field and DC electric field are perpendicular to each other.



Construction:

It consists of a cylindrical cathode of finite length and radius 'a'. The cathode is surrounded by cylindrical anode of radius 'b'. The anode is a slow wave structure consisting of several re-enterant cavities equispaced around the circumstance and coupled together through the anode cathode space by means of slots.



Linear - O type
Crossfield - M type

DC voltage V_0 is supplied between anode and cathode which produces radial electric field. The DC magnetic flux denoted by V_0 is maintained in positive Z-direction by means of permanent magnet or an electromagnet. The electron emitted from cathode try to travel to the anode but the influence of $\vec{E}$ and $\vec{H}$ crossfield in the space between the anode and cathode experience a force on electron i.e.

$$F = -e\vec{E} - e(v \times \vec{B}).$$

Due to this, electron take curved trajectory which is moving with velocity 'v'.

Due to excitation of anode cavities by RF noise voltage in the biasing circuit, RF fields are freezed out of the slot to the space between anode and cathode. The accelerated electrons in the trajectory, when retarded by this RF field, transfer energy from the electron to the cavities to grow RF oscillations.

At zero magnetic field, due to influence of electric field only, electron take the straight path a' as shown in fig (2).

If magnetic field strength slightly increases keeping V_0 constant i.e. electric field constant then electron takes curve path b' to reach anode.

For a magnetic field strength B_c, the electrons just graze the anode surface and take the path c' to return to the cathode for a given voltage V_0. This B_c is called the cutoff magnetic flux density.

If the magnetic field is greater than B_c, the electrons return to the cathode by the path d' without reaching the anode.

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Operation of Magnetron Oscillator:

Under the magnetic field, the electron will rotate in circular path, and at any point this centrifugal force of electron will be balanced by the force ~~excited~~ exerted by the magnetic field.

$$\frac{mv^2}{r} = BeV$$

$$\Rightarrow V = \left(\frac{Be}{m}\right) r$$

We know $v = r\omega$

where $\omega \rightarrow$ cyclotron frequency.

$$\boxed{\omega = \frac{Be}{m}}$$

In cylindrical co-ordinate system the eq. of motion of an electron is given by

$$\frac{1}{r}\frac{d}{dt}\left(r^2 \frac{d\phi}{dt}\right) = \underbrace{\frac{e}{m}B}_{\omega} \frac{dr}{dt}$$

$$\frac{1}{r}\frac{d}{dt}\left(r^2 \frac{d\phi}{dt}\right) = \omega \frac{dr}{dt}$$

On integrating w.r.to t, we have

$$\int d\left(r^2 \frac{d\phi}{dt}\right) = \int \omega r dr$$

$$\Rightarrow r^2 \frac{d\phi}{dt} = \frac{\omega r^2}{2} + k$$

Boundary condition:

At surface of cathode, $r = a$

angular velocity $\frac{d\phi}{dt} = 0$.

$$\therefore 0 = \frac{\omega a^2}{2} + k$$

$$\Rightarrow \boxed{k = \frac{-\omega a^2}{2}}$$

$$\therefore r^2 \frac{d\phi}{dt} = \frac{\omega}{2}(r^2 - a^2) \quad \text{——— (1)}$$

9b v is the velocity of electron, under electric field,

$eV_{dc} = \frac{1}{2}mv^2$

But the velocity of electron under electric field has two components :

1. radial velocity
2. tangential velocity

$$v^2 = v_r^2 + v_\phi^2 = 2\frac{eV_{dc}}{m}$$

When an electron is just ~~crossing~~ grazing the anode, then

$r = b$ and $\frac{dr}{dt} = 0$.

$$\left(\frac{dr}{dt}\right)^2 + \left(r\frac{d\phi}{dt}\right)^2 = \frac{2eV_{dc}}{m}$$

$$\Rightarrow \left(b\frac{d\phi}{dt}\right)^2 = \frac{2eV_{dc}}{m}$$

$$\Rightarrow b\frac{d\phi}{dt} = \sqrt{\frac{2eV_{dc}}{m}} \quad \text{——} \; (2)$$

$$\Rightarrow b\frac{d\phi}{dt} = \frac{\omega}{2b}(b^2 - a^2) \quad \text{——} \; (3)$$

$$\Rightarrow \frac{\omega}{2b}(b^2 - a^2) = \frac{2eV_{dc}}{m}$$

$$\boxed{\frac{1}{2}\frac{Be}{m}b\left(1 - \frac{a^2}{b^2}\right) = \sqrt{\frac{2eV_{dc}}{m}}} \quad \text{——} \; (4)$$

Cutoff or critical magnetic field

$$B_c = \frac{1}{b}\sqrt{\frac{8V_{dc}m}{e}} \times \frac{1}{1 - \left(\frac{a}{b}\right)^2}$$

This means, if the applied magnetic field B is slightly greater than Bc, then for a given Vdc the electron will not reach the anode but will just graze the surface of the anode.

$V_{dc} = V_c$ = cutoff voltage ;

$$V_c = \frac{e}{8m}b^2\left(1 - \frac{a^2}{b^2}\right)^2 B_0^2$$

If the applied voltage V_{ac} is less than V_c, the electron will not reach the anode. (81)

Q.3) A 250KW pulsed cylindrical magnetron is operated with the following parameters

LA 7(c) 2015

anode voltage = 25kV

peak anode current = 25 A

magnetic induction = 0.35T

radius of cathode = 4cm

anode = 8cm.

Calculate:

i) efficiency of the magnetron.

iii) cutoff magnetic field

ii) cyclotrone frequency

iv) cutoff voltage.

i) $\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{V_0 I_0} \times 100\%$

$= \frac{250\times10^3}{25\times10^3\times25} \times 100 = 40\%$

ii) $\omega = \frac{Be}{m} = \frac{0.35\ wb/m \times 1.6\times10^{-19}}{9.1\times10^{-31}}$

$= 0.616\times10^{11}$ rad/s.

iii) $B_c = \frac{1}{8\times10^{-2}} \sqrt{\frac{8\times25\times10^3\times9.1\times10^{-31}}{1.6\times10^{-19}}} \times \frac{1}{1-\left(\frac{4}{8}\right)^2}$

$= 0.018\ wb/m^2$

$= 18\ m\frac{wb}{m^2}$

iv) $V_c = \frac{1.6\times10^{-19}}{8\times9.1\times10^{-31}} (8\times10^{-2})^2 \left(1-\frac{4^2}{8^2}\right)^2 \times(0.35)^2$

$= 9.692\times10^6$ m/s.

Q: A Linear magnetron has following parameters

V_0 = anode voltage = 20kV

cathode current = I_0 = 17A

magnetic field = B = 0.01 wb/m²

distance betⁿ cathode & anode = $d_{c \cdot a}$ = 5cm.

2013 6(a) Compute

i) the null cut-off voltage for a fixed B.

ii) the null cutoff magnetic field for a fixed V_0.

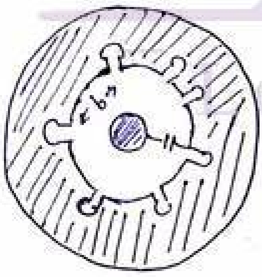
i) $B_c = \frac{1}{d}\sqrt{\frac{2m}{e}V_0}$

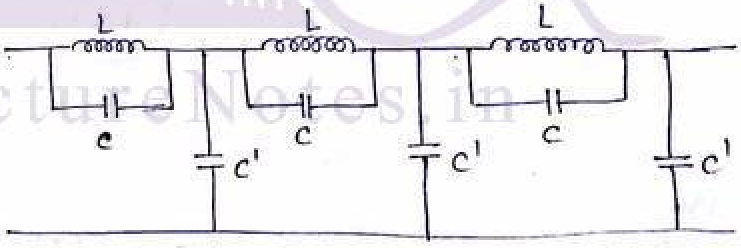
$= \frac{1}{5\times10^{-2}}\sqrt{\frac{2\times9.1\times10^{-31}}{1.6\times10^{-19}}\times20\times10^{3}}$

ii) $V_c = \frac{1}{2}\frac{e}{m}B^2d^2$

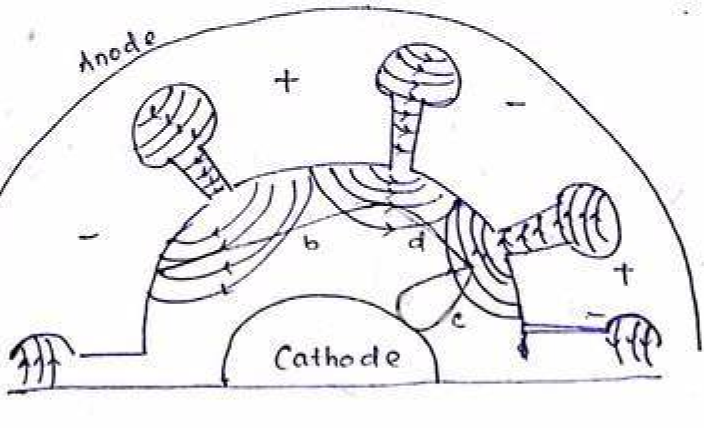
$= \frac{1}{2}\frac{1.6\times10^{-19}}{9.1\times10^{-31}}\times(0.01)^2\times(5\times10^{-2})^2$

Dt. 29/3/16

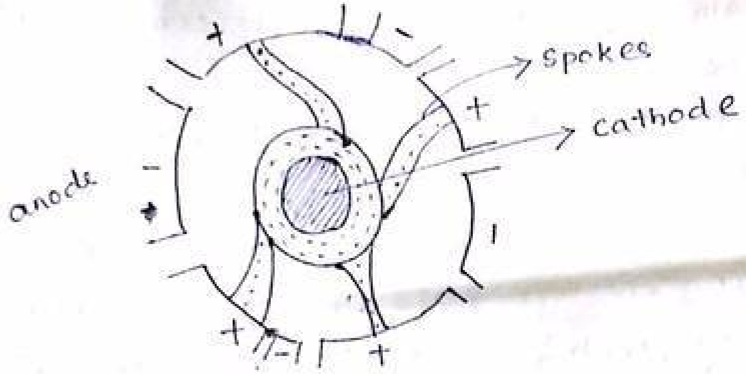




Band Pass Equivalent Circuit.



π - mode of operation



(Phase focusing)

The total phase shift around the internal periphery must be an integral multiple of two path for possible oscillations. The phase shift between two adjacent cavities is given by,

$$\phi_n = \frac{2\pi n}{N} , n = \pm 1, \pm 2, \cdots \pm N/2$$

For continuous interaction between e^- and RF field for transfer of energy, the anode DC voltage V_0 is adjusted to coincide the average rotational velocity of e^- with phase velocity of RF field in the interaction space.

$\phi_n = \pi$ or π-mode is commonly used for magnetron oscillator when $n = N/2$ to get max^m excitation in the cavity.

π - mode of operation:

- The electrons emitted from cathode try to travel towards anode with the influence of DC electric field $\vec{E}$ but take a curve trajectory with influence of DC magnetic field $\vec{B}$.

$$F = -[e\vec{E} + e(V \times \vec{B})] \text{ (Lorentz force)}$$

- The existance of RF oscillations in the resonant structure is due to noise voltages in the dc biasing circuit.
- The RF electric field at the resonant frequency of the π mode structure in slot of the cavities are

shown in the fig. with fringes out (comes out) in the cathode-anode space.

- In the absence of RF electric field, the e^- try to return back to the cathode follow the path d.
- Due to fringing RF electric field the e^-, 'a' experiences a retarding electric field. e^-, 'b' experiences accelerating electric field. The retarded e^-, 'a' experiences reduced magnetic force i.e. $-e(\vec{v} \times \vec{B})$ due to reduced velocity it moves towards anode.
- By adjusting the DC anode voltage and DC magnetic field; the circumferential velocity component of electron can be made such that e^- 'a' takes approximately one half cycle of the RF oscillation to travel from one slot position to the next. This makes e^- 'a' experience a retarding field and ultimately reaches the anode after continuously delivering energy to the RF oscillation (favoured e^-). On the other hand, the e^- 'b' is accelerated by the RF field to return quickly to the cathode causes heating loss in the cathode (about 5% of anode power) - unfavoured e^-.
- The oscillation amplitude grows due to retarded electrons and steady state is reached when the losses in the system are compensated by the RF oscillation continuously.

<u>Dt. 1/4/16</u>

The electron around 'a' such as 'c' & 'b' are acted upon both radial and tangential components of RF field in such a way that the electron 'c' moves faster than 'a' and that of 'd' moves slower than 'a' to form a bunch around 'a'.

- These electrons from 'c' to 'd' are confined to spokes or focussed and terminated to alternate anode. This is called <u>phase focusing</u>.

For π mode, these spokes have angular velocity equal to 2 anode poles/cycle. And the electron within the spokes deliver energy to oscillation before they are collected by anode. (85)

Frequency pulling & pushing:

The resonant frequency of magnetron can be altered by changing anode voltage.

The change in the anode voltage as the effect of alternating the orbital velocity of the electron clouds. This in turns alters the rate at which energy is given out to the resonators and therefore changes the oscillating frequencies. This method is called frequency pushing.

Frequency variation can take place by change in load impedance called as frequency pulling.

Performance & characteristics:

Frequency - 500MHz - 12GHz

Commercially - Peak power 3kW or higher.

Efficiency - 40 to 70%

Duty cycle = 0.1%

Application:

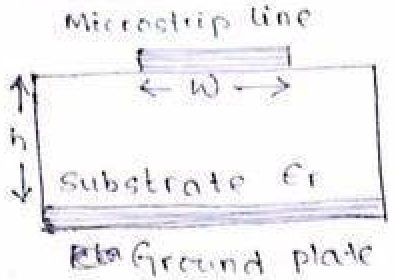
- Radar transmitter
- Industrial heating
- Microwave ovens.
- Pulsed Radar.

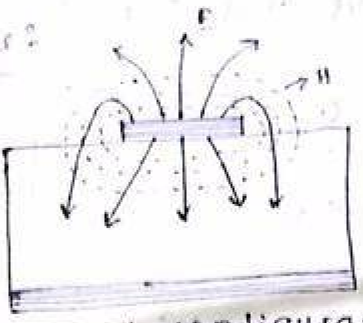
Scattering Matrix:

Scattering ma

Dt. 15/4/16

Microstrip Line / Strip Line:





Field configuration

The planar TL are important as these are widely used in microwave integrated ckt.
Some imp planar TLs which are printed or etched out on substrate are i) microstripline
ii) stripline
iii) slot line
iv) coplanar line

A micro stripline is evolved from 2 conductor transmission line which both the conducting surfaces are parallel to each other & separated by the substrate characteristic parameters of microstrip line are depend upon strip width (W), substrate height of thickness (h) & dielectric constant of substrate (ϵ_r).
The metal strip on the top of the substrate is not completely covered by the substrates, hence the wave propagating through microstrip doesn't encounter a homogenous medium. Therefore pure TEM mode propagation is not possible & hence <u>quasi TEM mode</u>.
Electromagnetic wave in microstrip line encounters two medium; one is air & other is substrate. To analyse the EM wave, we

have to consider homogeneous medium: (09)

$$\epsilon_{eff} = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r-1}{2} \times \frac{1}{\sqrt{1+12\frac{h}{w}}} \quad \text{i.e. } 1 < \epsilon_{eff} < \epsilon_r$$

phase velocity: $V_p = \frac{c}{\sqrt{\epsilon_{eff}}}$, $\lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{eff}}}$

$\lambda_0 = \frac{c}{f}$ = free space wave length.

Chr impedance.

$$Z_0 = \frac{\epsilon_0}{\sqrt{\epsilon_{eff}}} \ln\left\{ \frac{8h}{w} + \frac{w}{4h} \right\}, \text{ for } \frac{w}{h} \leq 1$$

$$= \frac{120\pi}{\sqrt{\epsilon_{eff}}} \left[\frac{w}{h} + 1.393 + 0.667 \ln\left(\frac{w}{d} + 1.444\right) \right], \text{ else.}$$

$$\underset{\text{dielectric}}{\alpha_d} = \frac{\beta_0 \epsilon_r (\epsilon_{eff} - 1)}{2\sqrt{\epsilon_{eff}}(\epsilon_r - 1)} \tan\delta$$

Substrate with high dielectric const. reduces the radiation losses -

$$\alpha_r = 60 \left(\frac{2\pi h}{\lambda}\right)^2 \left[1 - \frac{\epsilon_{eff} - 1}{2\sqrt{\epsilon_{eff}}} \log\left(\frac{\sqrt{\epsilon_{eff}} + 1}{\sqrt{\epsilon_{eff}} - 1} \right) \right], \text{ for } \epsilon_r \leq 5.$$

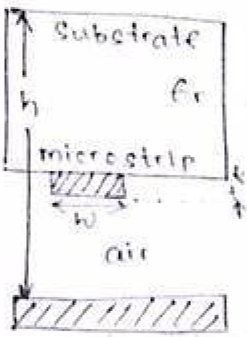
$$\underset{\text{conductor}}{\alpha_c} = \frac{R_s}{w Z_0}$$

$$\beta_0 = \frac{2\pi}{\lambda}$$

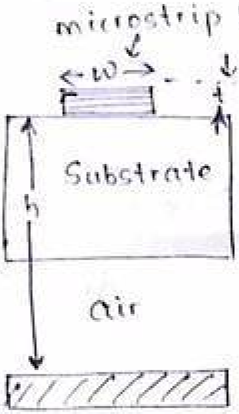
Q) A microstripline is printed on a RT Dura ed 6010LM substrate having relative dielectric const 10.9 & h = 2.5 moll (0.635 m), the chip width w = 0.5mm find:

i) ϵ_{eff} = 7.71 (A)
ii) Z_0 = 52.38 (Ω)
iii) λ = 0.022 m (m)

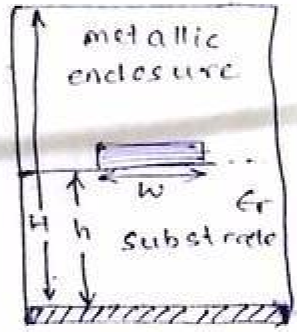
Configuration of microstrip line:



Ground plane
(Inverted MSL)



Ground plane
(Suspended MSL)



Ground plane
(Shelted MSL)

Inverted Microstrip Line: In this confg. the ground plane & microstrip line are in same plane but separated by air. ~~the air~~.

h = substrate thickness + air gap betn microstrip line & ground plane

Suspended MSL: In this confg. the ground plane & microstrip line are just opposite to each other. The air gap is kept betn. substrate and ground plane.

Shelted MSL: In this confg the metallic enclosure covers the whole microstrip line confg. This is most realistic configurn. because all the microstrip line based circuit required to be protected from environment and hence reduce the electromagnetic interference.

Advantages of microstrip line

1. Microstrip offers smallest size for microwave ckts. Since ckt size can be reduced by approximately the square root of dielectric const.
2. Easy to fabricate.

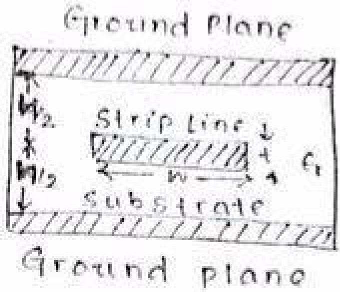
Disadvantages

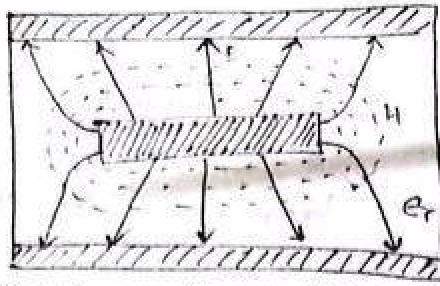
1. Unwanted radiations from microstrip line due to uncovered str.
2. High attenuation losses.
3. Poor isolations among the adjacent lines.

Q) Calculate the total attenuation for proper microstrip line given in the prev. prblm. The dissipation factor from RT Duraed of 6010CM substrate is 2.3×10^{-4} and conducting of Cu wire is 5.7×10^{7} mho/m.

Dt. 18/4/16

Strip Line :





Field configuration of strip line.

Strip line or triplate is a planar transmission line and can be seen as an extended version of microstrip line. It is basically a sandwich structure in which the ground planes are avialable *

EM wave propagate in a homogenous medium as compared to microstrip line. The strip line is etched out in one side of the grounded substrate and then covered by another grounded substrate.

Specific bonding film are used to attach two grounded substrate of same height. This process is complex and requires extreme care while fabricating the strip line.

The fields are confined within the substrate hence TEM mode can be achieve for the strip line configuration.

* in both side of the substrate, while the metal strip remains at the mid of the substrate.

Effective dielectric constant

$$\epsilon_{eff} = \epsilon_r$$

$$v_p = \frac{c}{\sqrt{\epsilon_{eff}}} \quad , \quad \lambda_g = \frac{\lambda_0}{\sqrt{\epsilon_{eff}}}$$

$$Z_0 = \frac{30\pi}{\sqrt{\epsilon_r}} \times \frac{1}{\frac{W_e}{h} + 0.441}$$

$$\frac{W_e}{h} = \frac{W_e}{h} \quad \text{if } \frac{W}{h} > 0.35$$

$$\frac{W_e}{h} = \frac{W_e}{h} - \left\{ 0.35 - \left(\frac{W}{h}\right)^2 \right\} \text{ if } \frac{W}{h} < 0.35$$

$$\alpha_c = \frac{0.16 R_s}{h Z_0} \times u \quad \text{for } \sqrt{\epsilon_r}\, Z_0 > 120$$

$$\alpha_c = \frac{Z_0 \epsilon_r \times 2.7 \times 10^{-3} \times R_s}{30\pi(h-t)} \times V \quad \text{for } \sqrt{\epsilon_r}\, Z_0 < 120.$$

where $u = 1 + \frac{h}{0.5W + 0.7t} \left[0.5 + \frac{0.414t}{W} + \frac{1}{2\pi} \ln \frac{4\pi W}{t} \right]$

$$V = 1 + \frac{2W}{h-t} \times \frac{1}{\pi} \left(\frac{h+t}{h-t}\right) \ln\left(\frac{2h-t}{t}\right)$$

Q) For a strip line, at 5 GHz. $\epsilon_r = 4$, $h = 3\text{mm}$, $W = 2.5\text{mm}$, $t = 0.01$ mm, $\tan\delta = 2 \times 10^{-3}$, $\sigma = 5.7 \times 10^7$ mho/m. Calculate $Z_0, \alpha_c, \alpha_d, \alpha$.

A) $\alpha = \alpha_c + \alpha_d$

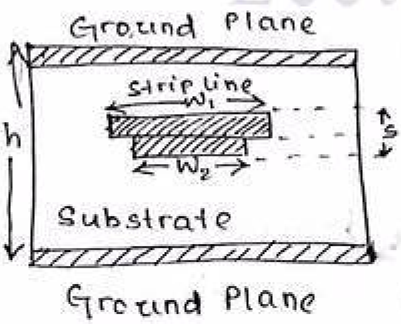
$Z_0 = 37\,\Omega$

$\alpha_c = 0.123$ N/m

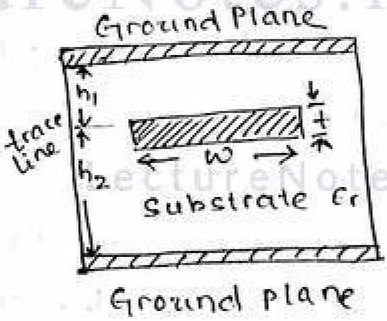
$\alpha_d = 0.209$ Np/m

$\alpha = 0.323$ Np/m.

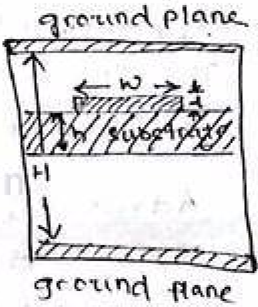
Different version of strip line:



Double conductor strip line



Offset strip line



Suspended strip line.

1) Double conductor SL: It is a combination of two equal or two unequal (trace) width. Division of zip line into two microstrip line. Division of zip line into two microstrip configuration.

2) **Offset Stripline:** This configuration can be achieved by joining the unequal height substrates. Also, in this configuration the centre conductor doesn't remain in ~~configura~~ the mid of total substrate height as in the case of basic strip line configuration.

3) **Suspended Stripline:** In this configuration the stripline is etched out on a thin substrate and the whole configuraⁿ is enclosed by a metallic enclosure. The stripline encounters air as a dielectric on both sides, because the substrate width is quite smaller.

Advantages:

(i) It increases the circuit dimension especially at milimeter wave frequencies because the dielectric used is air.

ii) No spurious radiation, as suspended stripline covered by metalic enclosure.

iii) It can operate over wide bandwidth.

iv) It offers low losses and high Q factor due to low dielectric and radiation pattern.

Disadvantages:

The only disadvantage is complex assembly and housing procedure.

Advantages of strip line:

1) Good EM shielding can be obtained since the strip line is covered by substrate and ground planes.

2) Low attenuation losses.

3) Wide bandwidth with lower cutoff frequency as it support TEM wave.

4) Better isolations betⁿ adjacent traces/lines due to non-interfacing natures.

Disadvantages:

1) complex & expensive to fabricate as it is a sandwich configuration.

2) The strip trace width (W) is smaller for a given impedance and substrate height as compared to micro stripline trace width.

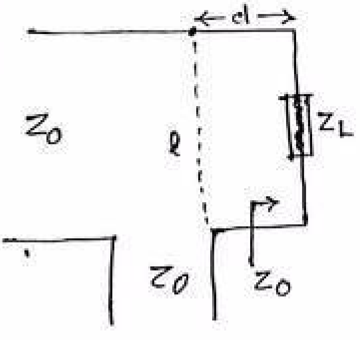
3) The tuning of stripline ckts are quite complex because tuning destroys the symmetry of strip line which in turn affects the mode of propagation of a EM wave in strip line.

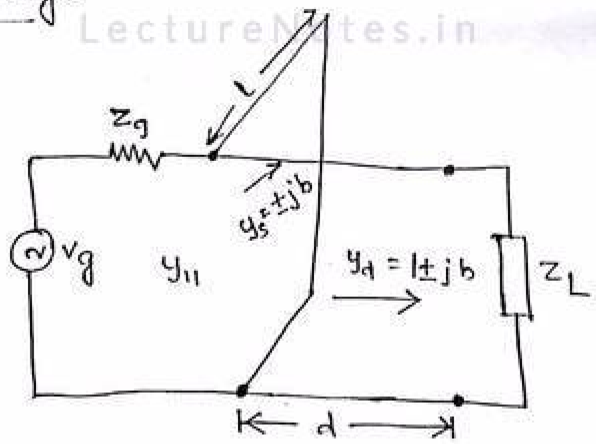
Dt. 19/4/16

Impedance matching
- Z_0 matching
- Complex conjugate matching
- Stub matching.

Stub:
- It is a piece of TL which may be either short circuited or open circuited.
- Stub matching having following advantages:
  i) length 'l' & characteristic impedance 'Z_0' remains unchanged.
  ii) mechanically it is possible to add adjustable susceptance in shunt with the line.
- Advantages of short ckt stub over open ckt stub:
  i) Short ckt stub radiates less power than open ckt stub. (Open ckted stub radiate from the open end.)
  ii) Effective length variation is possible by shorting a bar, therefore a sht circuited stub is invariably used, i.e. a good sht ckt is easier to obtain than a good open circuit.

Single Stub Matching:-





$y_{11} = y_s \pm y_d = 1$

For lossless line, Y_g is generalized as $Y_g = Y_0$

Maximum power transfer require: $Y_{11} = Y_0$.

where Y_{11} is the total admitance of the line and stub looking to the line at a point (1,1). The stub must be located at that point on the line where real part of the admittance looking towards load is Y_0. In normalized units, $y_{11} = \frac{Y_{11}}{Y_0}$ must be in the form $y_{11} = y_s + y_d = 1$.

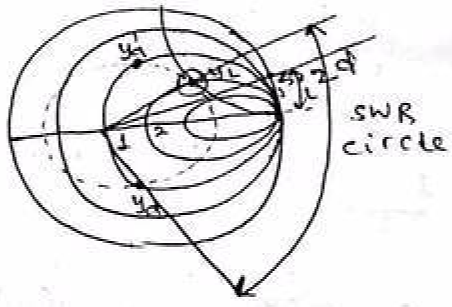
- The stub range is often adjusted so that the susceptance just cancels out the susceptance of the line at the junction.

Q) A lossless line of characteristic impedance $R_0 = 50\Omega$ is to be maintained to a load $Z_L = \left[\frac{50}{2+j(2+\sqrt{3})}\right]\Omega$ by means of lossless short-ckt stub. The characteristic impedance of stub $Z_{0S} = 100\Omega$.

A) $z_L = \frac{Z_L}{Z_0}$

$$y_L = \frac{1}{z_L} = \frac{Z_0}{Z_L} = \frac{50\Omega}{\frac{50}{2+j(2+\sqrt{3})}} = 2+j(2+\sqrt{3}) = 2+j3.732$$

1) Draw SWR circle through the pt. y_L so that a circle intersects the unit circle at the poit y_d & y_d'.



2) Since the characteristic impedance of stub is different from that of line, the condⁿ of impedance matching at the junction require ⇒ $Y_{11} = Y_d + Y_s$

$$y_{11} Y_0 = y_d Y_0 + y_s Y_0$$

$y_s = [1-(1-j2.6)]\frac{100}{50} = +j5.2$

Step-4) The distance bet^n load & stub position can be calculated from the distance scale.

$d = (Y_{s\lambda} - Y_{L\lambda}) = (0.302 - 0.215)$

$= 0.087\lambda$.

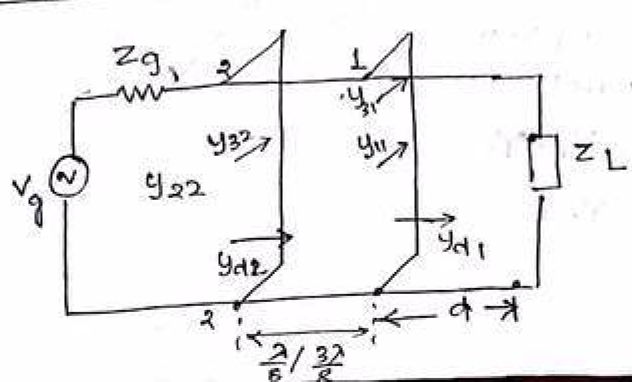
~~Step d~~

5) Length of the stub can be calculated from $r \to \infty$ to the $+j5.2$ position.

Demerits of Single Stub Matching:

1) The range of terminating impedance which can be transformed is limited. If the terminating impedance (i.e. input impedance of antenna say) changes then it is essential to adjust the position & length of the
It is easier for open wire line but is inconvinient in case of co-axial line.
2) It is useful for a fixed frequency only because as frequency varies, the position of the stub has to be varied. The change of susceptance however doesn't present any problem because shorting plug can be moved to req. position & hence single stub matching is narrow band system.

Double Stub Matching:

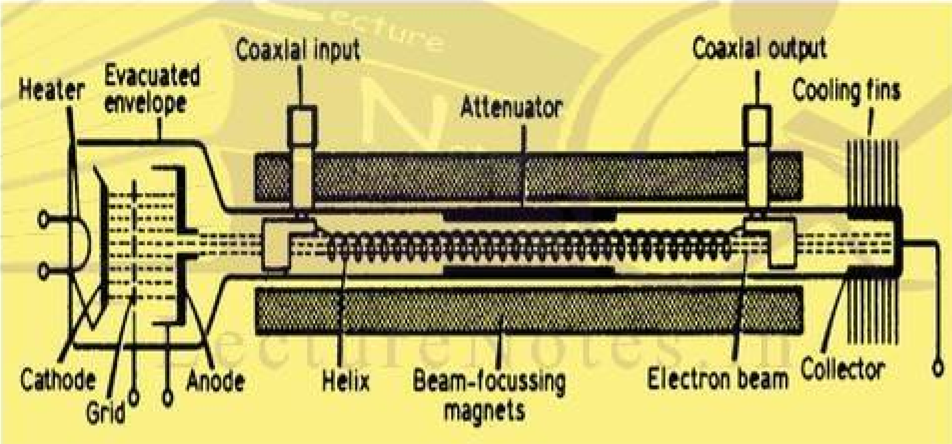


$y_{22} = y_{d2} \pm y_{s2} = 1$

$Y_{22} = Y_{d2} \pm Y_{s2} = Y_0$

Double stub devices consists of a short circuited stubs connected in parallel with the fixed length. The length of the fixed section is usually $\frac{1}{8}$, $\frac{3}{8}$ or $\frac{5}{8}$ of wavelength. The stub i.e. nearest to the load is used to adjust the susceptance & is located at a fixed wavelength from constant conductance unit circle i.e. $g=1$ on an appropriate constant SWR circle.

TRAVELING WAVE TUBE(TWT)



Write down the two differences between O type and M type tubes? Give proper examples.

O-type Microwave Tube	M-type Microwave tube
• Tubes in the O-type category are some times called linear or rectilinear beam tubes in recognition of the straight path taken by the electron beam. • In this class of devices, both velocity and density modulation take place, creating the bunching effect. The electron bundles thus created have a period in the microwave region. • Examples of O-type tubes include Klystron and travelling wave tubes (TWTA)	• A principle feature of such tubes is that electrons travel in a curved path. Those tubes were designated M-type. • These are crossed field devices where the static magnetic field is perpendicular to the electric field. • Example -Magnetrons

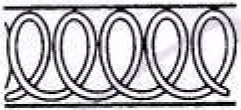
- The O- type tubes differ from M-type in that electrons travel in a straight line under the influence of parallel electric and magnetic fields.

- List the factors that limit the ordinary device to be ineffective at microwave frequencies.
- What are the limitations of conventional tubes at microwave frequencies

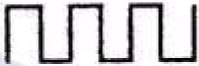
Ans-

Conventional tubes can't be used at microwave frequencies because of transit time effect. Lead inductance and inter electrode capacitance of the devices will finally limit the output which may even be zero.

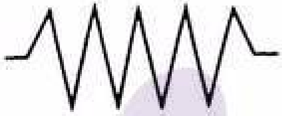
Types of slow wave structures



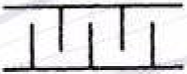
Helical line



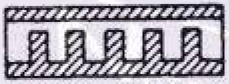
Folded back line



Zig-zag line



inter-digital line



corrugated wave guide

Slow wave structures are special circuits that are used in the microwave tubes to reduce wave velocity in a certain direction, so that the electron beam and signal wave can interact.

TRAVELING WAVE TUBE(TWT)

- The traveling wave tube is a form of thermionic valve or tube that is used for high power microwave amplifier designs.
- The travelling wave tube can be used for wideband RF amplifier designs where even now it performs well against devices using newer technologies.
- TWTs are used in applications including broadcasting, radar and in satellite transponders.
- The TWT is still widely used despite the fact that semiconductor technology is advancing all the time.
- Two types of TWT's are available
  - Low power TWT
  - High power TWT

- Low-power TWT for receivers
  - occurs as a highly sensitive, low-noise and wideband amplifier in radar equipments

Russian low-power twt UV-1B (cyrillic: УВ-1Б)

- High-power TWT for transmitters
  - These are in use as a pre-amplifier for high-power transmitters.

high-power twt VTR 572B

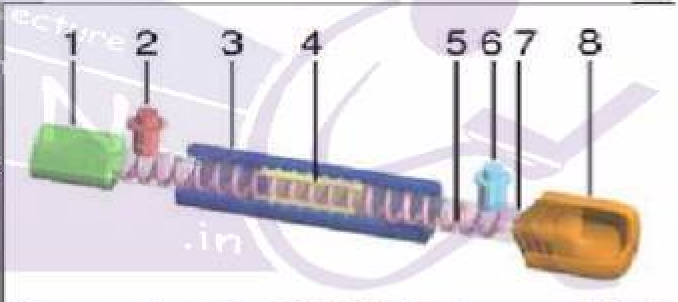
Silicon ...beyond teaching

Working Principle of TWT

- TWT is a microwave amplifier which makes use of distributed interaction between an electron beam and a travelling wave.
- Necessary condition for continuous Interaction between electron beam and RF field
  - ➢ Both should travel in the same direction and with the same velocity
  - ➢ But electron beam velocity = 0.1 X C and Travelling field (RF field) velocity = C (i.e. velocity of the light)
  - ➢ Interaction between RF field and moving electrons will takes place only when the velocity of RF field is retarded by some means (i.e. use of slow wave structure)

HELIX TWT CONSTRUCTION

- The Helix Travelling wave tube(TWT) , can be split into a number of separate major elements:
  - ➢ Vacuum tube
  - ➢ Electron gun
  - ➢ Magnet and focusing structure
  - ➢ RF input
  - ➢ Helix
  - ➢ RF output
  - ➢ Collector



Cutaway view of a TWT. (1) Electron gun; (2) RF input; (3) Magnets; (4) Attenuator; (5) Helix coil; (6) RF output; (7) Vacuum tube; (8) Collector.

- The detailed diagram of Helix TWT can be viewed as,

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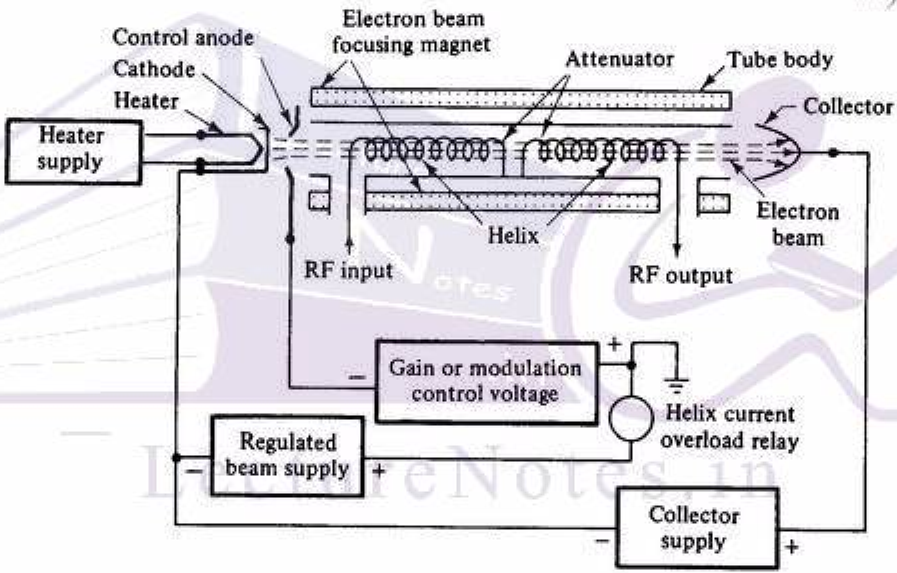
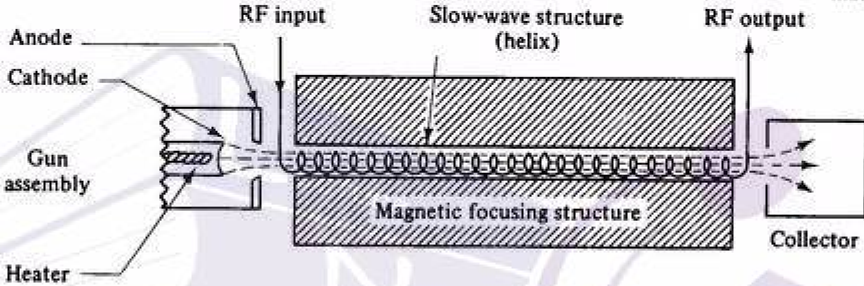
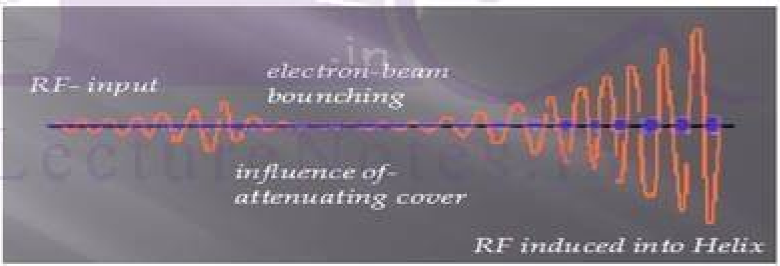
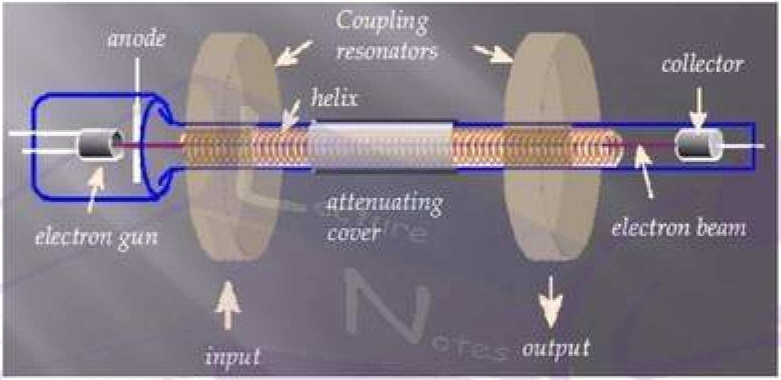


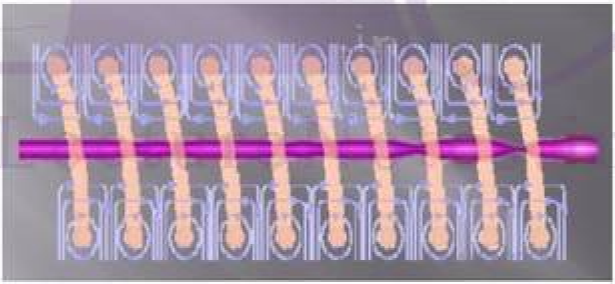
- The simplified circuit is,





Physical Construction Of TWT





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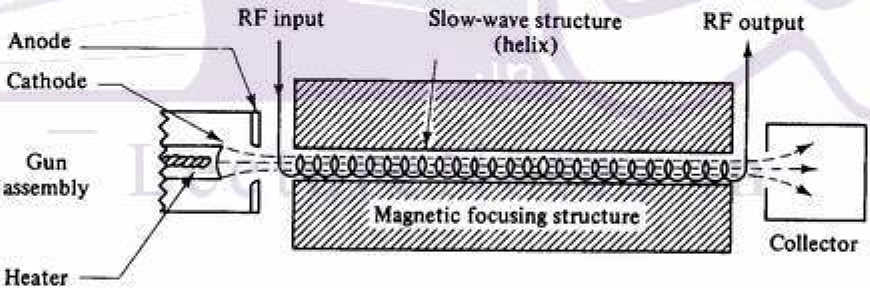
Working Operation:

- A Helix twt consists of an electron Gun and a Slow wave structure.
- First element-Electron gun comprising primarily of a heated cathode and grids. This produces and then accelerates a beam of electrons that travels along the length of the tube.
- The electron beam is focused by a constant magnetic field along the electron beam and the slow wave structure. This is termed as *O*-type traveling tube.

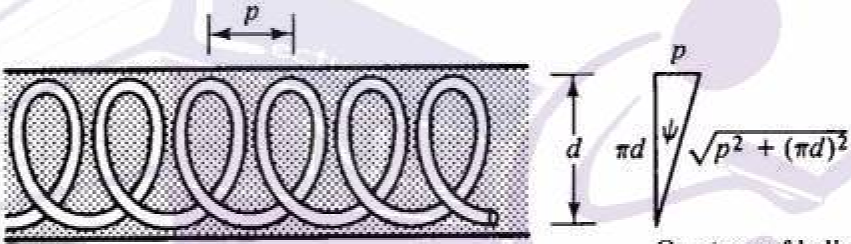
Electron Beam

- An electron gun is employed to produce very narrow electron beam and correctly focused through the center of long coaxial helix without touching it.
- The collector is made highly positive w.r.t. cathode, so that beam is being attracted to the collector with high velocity.
- The beam is kept from spreading with the help of d.c. axial



Signal (RF field)

- Signal is applied to the input end of the helix through wave guide/ coaxial line. This field propagate through helix with a speed that is less than speed of light.



One turn of helix

p = helix pitch
d = diameter of the helix
ψ = pitch angle

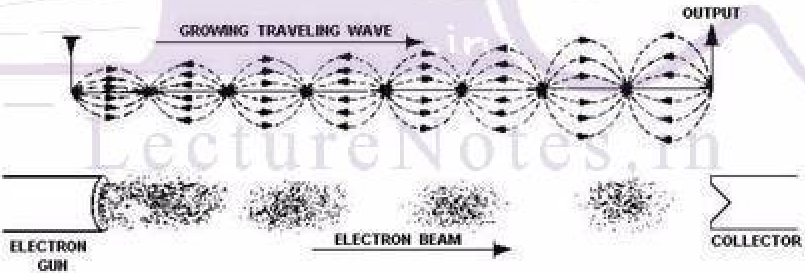
$$v_p \approx \frac{pc}{\pi d} = \frac{\omega}{\beta}$$

$$\text{Speed of electric field advances axially} = \text{velocity of light} \times \frac{\text{pitch of helix}}{\pi d(\text{helix circumference})}$$

- Briefly explain the amplification process of O-type Travelling Wave Tube (TWT) with proper diagram

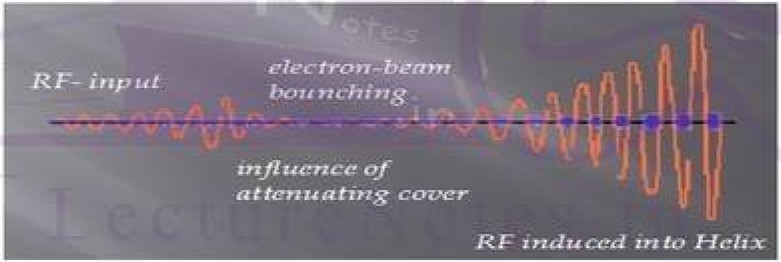
Interaction

- The dc beam voltage is adjusted so that the beam velocity is slightly greater than axial component of field on slow wave structure. The axial RF field and beam can now interact continuously.
- As the velocity of the beam is maintained slightly grater than phase velocity of travelling wave, more electrons face the retarding field than accelerating field and great amount of K.E is transferred from beam to RF field.
- This field amplitude increases forming more compact bunch and larger amplification of the signal voltage appears at the output end of the helix.



Reflection waves

- An attenuator placed midway along the helix, attenuates the reflected waves propagating from any mismatched load at the output end to prevent from reaching the input and causing oscillations.
- The attenuator will attenuate both forward and reflected waves on the helix without affecting the electron beam.



Characteristics of TWT:

- The Traveling Wave Tube (TWT) is a high-gain, low-noise , wide-bandwidth microwave amplifier.
- It is capable of gains greater than 40dB with bandwidths exceeding an octave. (A bandwidth of one octave is one in which the upper cutoff frequency is twice the lower cutoff frequency.)
- Traveling-wave tubes have been designed for frequencies a slow as 300Megahertz and as high as 50 Gigahertz.
- The TWT is primarily a voltage amplifier. The wide-bandwidth and low-noise characteristics make the TWT ideal for use as an RF amplifier in microwave equipment.
- TWT amplifiers and they are typically capable of developing powers of up to 2.5 kW. For narrowband RF amplifier applications it is possible to use coupled cavity TWTs and these can deliver power levels of up to 15 kW.
- Efficiency of 20 to 40 % is possible .

Remember

- The approximate solutions may be found by equating the dc electron beam velocity to the axial phase velocity of the travelling wave and the four propagation constants γ are given by

$$\gamma_1 = -\beta_e C\frac{\sqrt{3}}{2} + j\beta_e\left(1 + \frac{C}{2}\right)$$

$$\gamma_2 = \beta_e C\frac{\sqrt{3}}{2} + j\beta_e\left(1 + \frac{C}{2}\right)$$

$$\gamma_3 = j\beta_e(1 - C)$$

$$\gamma_4 = -j\beta_e\left(1 - \frac{C^3}{4}\right)$$

where C is the traveling-wave tube gain parameter and is defined as

$$C \equiv \left(\frac{I_0 Z_0}{4V_0}\right)^{1/3}$$

where $\beta_e = \omega/v_0$

is the phase constant of the velocity-modulated electron beam.

$$\frac{mv_0^2}{e} = 2V_0$$

The output power gain in decibels is defined as

$$A_p \equiv 10 \log \left|\frac{V(\ell)}{V(0)}\right|^2 = -9.54 + 47.3NC \qquad \text{dB}$$

Circuit length: N

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ASSIGNMENT PROBLEMS

13. A TWT operates at a beam current I_0=50 mA, beam voltage V_0=2.5kV, characteristic impedance of helix Z_0= 7.750., circuit length N = 45, and frequency 8GHz. Compute the gain parameter and all four propagation constants.

17. A Travelling Wave Tube (TWT) has the following characteristics: Beam voltage V_0 = 2KV, Beam current I_0 = 4mA, Frequency f = 8GHz, Circuit Length N = 50 in wavelength, Characteristics impedance Z_o=20Ω. Determine:

- the gain parameter
- power gain in dB

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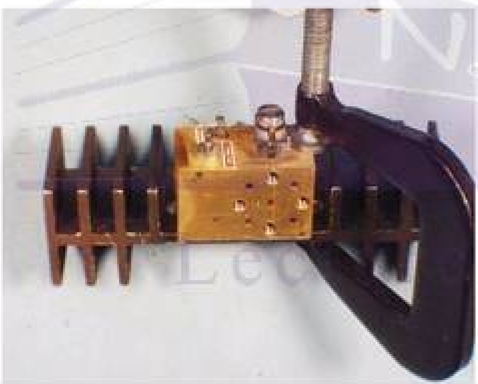
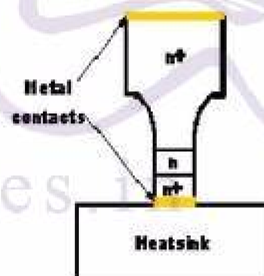
Differences Between TWT and Klystrons:

- The microwave circuit is non-resonant in TWT , while resonant circuits are used in klystrons.
- The interaction of electron beam and RF field in the TWT is continuous over the entire length of the circuit , but the interaction in the klystron occurs only at the gaps of a few resonant cavities.
- The wave in the TWT is a propagating wave , The wave in the klystron is not.
- In the couple cavity TWT there is coupling effect between the cavities, whereas each cavity in the klystron operates independently.

GUNN DIODE



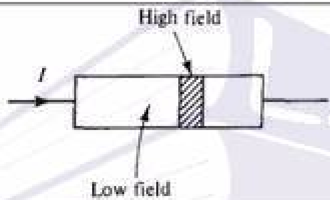
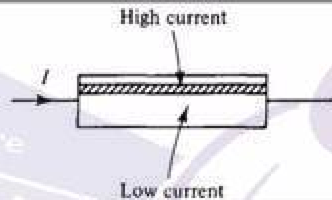
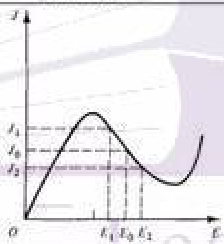
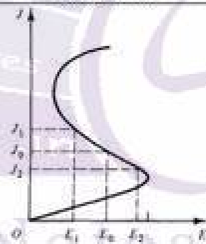


Contents

- Overview of The Gunn Diode
- Voltage/ Current controlled mode
- Positive/negative Resistance
- Microwave transistors/ Transferred Electron Devices (TEDs)
- Gunn diode basics
- Gunn diode construction
- Difference between Gunn diode and P-N junction
- Electrical equivalent circuit of a Gunn Diode
- Gunn Effect
- Two-Valley Model Theory
- High field domain
- Modes Of Operation
  Gunn-Oscillation
- Gunn Oscillation Modes

Overview of the Gunn Diode

- **What is it?**
  - The Gunn diode is used as local oscillator covering the microwave frequency range of 1 to 100GHz
  - Gallium Arsenide Gunn Diodes are made for frequencies up to 100GHz whereas Gallium Nitride can reach upto 3THz.
- **How it works?**
  - By means of the transferred electron mechanism, it has the negative resistance characteristic.
- **What's the applications?**
  - Local Oscillator and Avoid Collision Radar instead of Klystron etc..
- **What's the advantages?**
  - Low noise, High frequency operation and Medium RF Power.

Voltage-controlled mode	Current-controlled mode
The current density can be multi-valued.	The voltage value can be multi-valued.
High field domains are formed, separating two low field regions.	It splits the sample results in high current filaments running along the field directly.
	
	

Expressed mathematically, the negative resistance of the sample at a particular region is

$$\frac{dI}{dV} = \frac{dJ}{dE} = \text{negative resistance}$$

Positive Resistance	Negative Resistance
Current and voltage are in phase.	Current and voltage are in out of phase by 180^{o}.
Voltage drop across it is positive.	Voltage drop across it is negative.
A power of (I^2R) is dissipated in it (i.e. Positive resistance absorb power (passive devices)	A power of ($-I^2R$) is generated by the power supply. (i.e. Negative resistance generate power (active devices)

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Microwave transistors	Transferred Electron Devices (TEDs)
Transistors operate with either junctions or gates	These are bulk devices having no junctions or gates.
The majority of transistors are fabricated from the elemental semiconductors such as silicon or germanium.	TEDs are fabricated from the compound semiconductors such as gallium arsenide (GaAs), Indium phosphide (InP) and cadmium telluride (CdTe),
Transistors operate with "warm" electrons whose energy is not much greater than the thermal energy (0.026eV)	TEDs operate with "hot" electrons whose energy is very much greater than the thermal energy (0.026eV)

GUNN Diode Basics

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- The Gunn diode is a unique component - even though it is called a diode, it does not contain a PN diode junction. The Gunn diode or transferred electron device can be termed a diode because it does have two electrodes.
- It depends upon the bulk material properties rather than that of a PN junction. The Gunn diode operation depends on the fact that it has a voltage controlled negative resistance.
- The mechanism behind the transferred electron effect was first published by Ridley and Watkins in a paper in 1961. Further work was published by Hilsum in 1962, and then in 1963 John Battiscombe (J. B.) Gunn independently observed the first transferred electron oscillation using Gallium Arsenide, GaAs semiconductor.

Gunn diode construction

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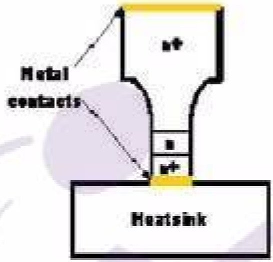
- Gunn diodes are fabricated from a single piece of n-type semiconductor because the transferred electron effect is only applicable to electrons and not holes found in a p-type material. it only utilises n-type semiconductor where electrons are the majority carriers.
- Within the device there are three main areas, which can be roughly termed the top, middle and bottom areas.
- The top n+ layer can be deposited epitaxially or doped using ion implantation. Both top and bottom areas of the device are heavily doped to give n+ material. This provides the required high conductivity areas that are needed for the connections to the device
- The active region is between a few microns and a few hundred micron thick. This active layer has a doping level between $10^{14}cm^{-3}$ and $10^{16}cm^{-3}$ - this is considerably less than that used for the top and bottom areas of the device. The thickness will vary according to the frequency required.
- The base also acts as a heat sink which is critical for the removal of heat. The connection to the other terminal of the diode is made via a gold connection deposited onto the top surface. Gold is required because of its relative stability and high conductivity.



A discrete Gunn diode with the active layer mounted onto a heatsink for efficient heat transfer

Gunn diode symbol

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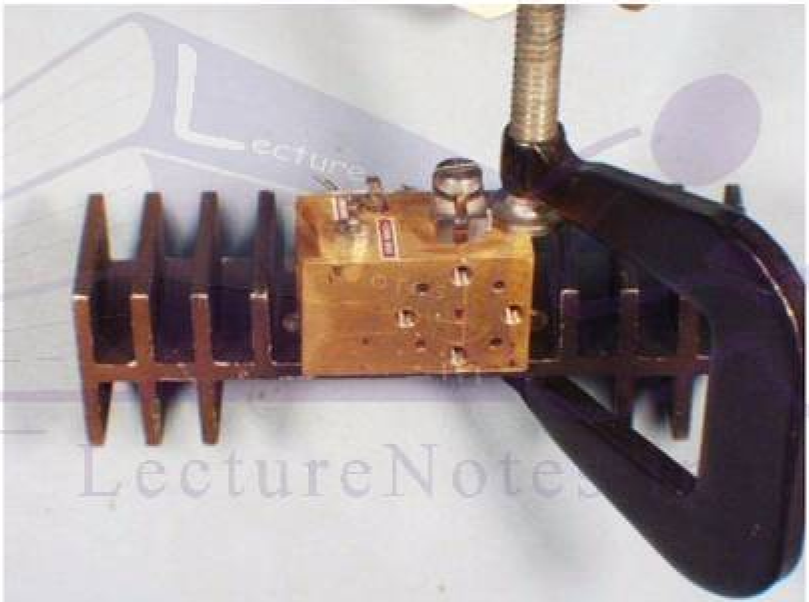

Difference between Gunn diode and P-N junction

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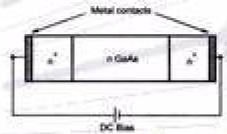
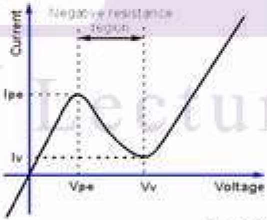
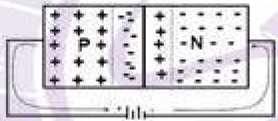
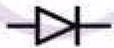
Gunn diode	P-N junction diode
• It only consists of N type semiconductor material • It has N+ n N+ material • No depletion region is formed	• It consists of P & N type semiconductor material • It has PN Junction • Depletion region between these materials.



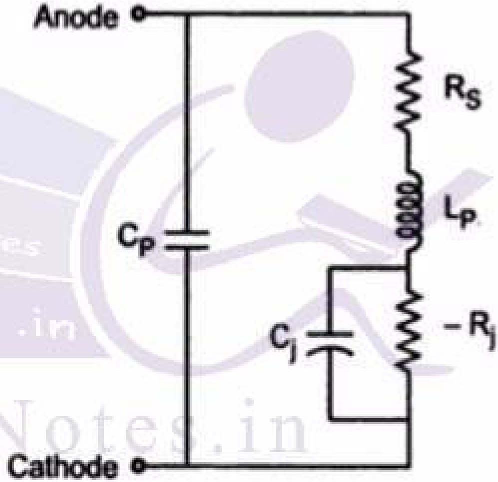




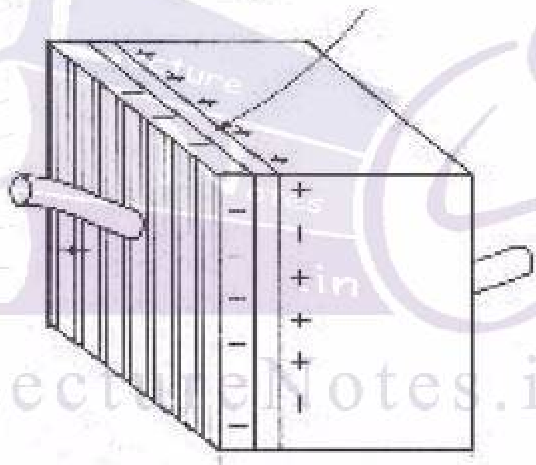


Electrical equivalent circuit of a Gunn Diode

- C_j and $-R_j$ are the diode capacitance and resistance respectively
- R_s includes the total resistance of lead, ohmic contacts, and bulk resistance of the diode
- C_p and L_p are the package capacitance and inductance respectively
- The negative resistance has a value that typically lies in the range –5 to –20 ohm



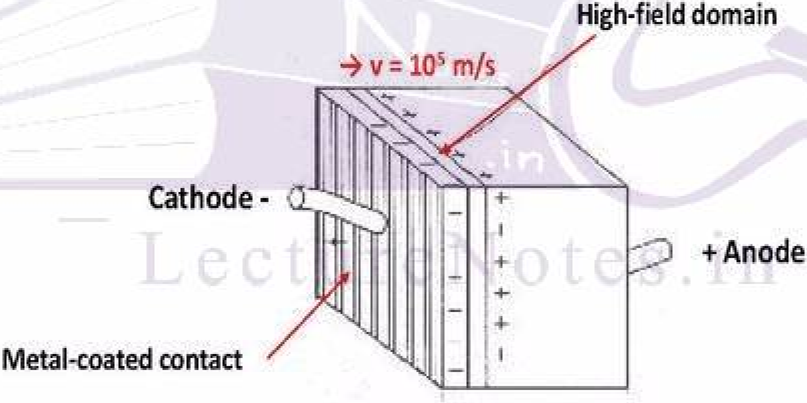
Gunn Effect



Gunn Effect

- **Gunn effect was discovered by J.B Gunn in IBM : 1963**

"Above some critical voltage, corresponding to an electric field of 2000~4000 V/cm, the current in every specimen (GaAs) became a fluctuating function of time"



Schematic diagram for n-type GaAs diode

Gunn Effect

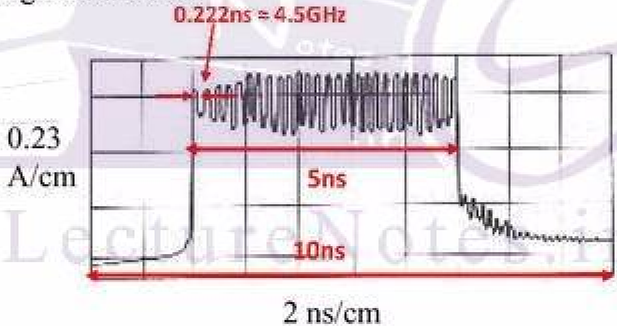
- In GaAs specimen this fluctuation took the form of periodic oscillation superimposed on pulse current.
- The frequency of oscillation depends upon the specimen length and not on external circuit parameters.
- The period of oscillation is proportional to $\frac{1}{\text{specimen length}}$ and closely equal to transit time of electrons between electrodes calculated from velocity ($=10^5$ m/s) . {Transit time is the time taken by the electrons to travel from cathode to anode.}
- Drift velocity (V_d) increases from 0 to maximum value corresponds to E varies from 0 to $E_{Threshold}$, when E > 3000V/cm (E_{th}) for n-GaAs, V_d decreases and diode exhibit negative resistance.

➢ A GaAs Gunn diode has a drift length of 10μm. Determine the frequency of oscillation.

Solution- f =V/L= 10^5 m/s / 10μm

Gunn Effect

- The current waveform was produced by applying a voltage pulse of 16V amplitude and 10ns duration to a specimen of n-type GaAs 2.5×10^{-3} in length.
- Oscillation frequency was 4.5Ghz
- The period of oscillation is equal to the transit time of electrons through the device.



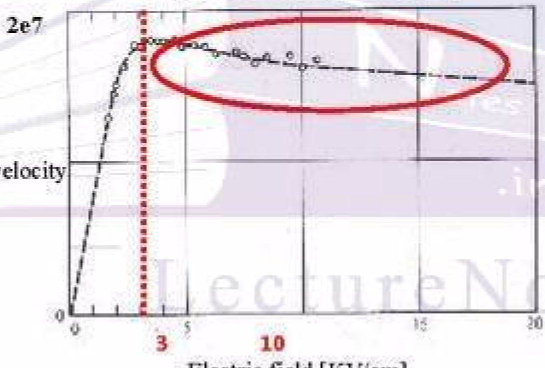
Current fluctuation of N-type GaAs reported by Gunn

Gunn Effect-Negative Differential Resistance

- **Drift velocity of electrons decrease when electric field excess certain value**
- **Threshold electric field about 3000V/cm for n-type GaAs.**

- For direct bandgap materials, like GaAs: v_d *vs.* E peaks before saturation & decreases again, after which it finally saturates.
- Because of this peak, there are regions in the v_d *vs.* E relationship that have:

Negative Differential Resistance



Drift velocity of electrons in GaAs bulk Vs electric field

$(dv_d/dE) < 0$

(for high enough E)

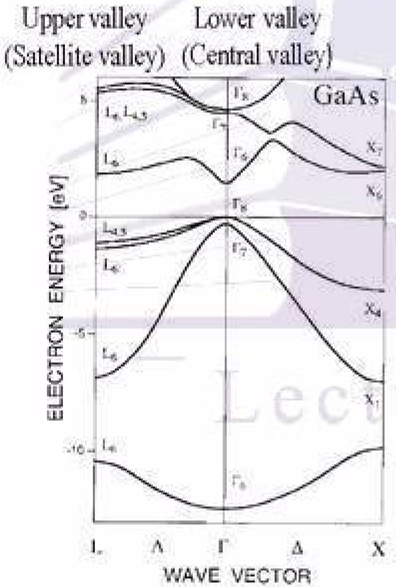
This effect is called

"Negative Differential Resistance"

or **"Negative Differential Mobility"**

or **"Negative Differential Conductivity"**

Two-Valley Model Theory



Some bulk semiconductor materials such as gallium arsenide(GaAs), Indium phosphide (InP) and cadmium telluride (CdTe), have two closely spaced energy bands in the conduction band.

Two-Valley Model Theory

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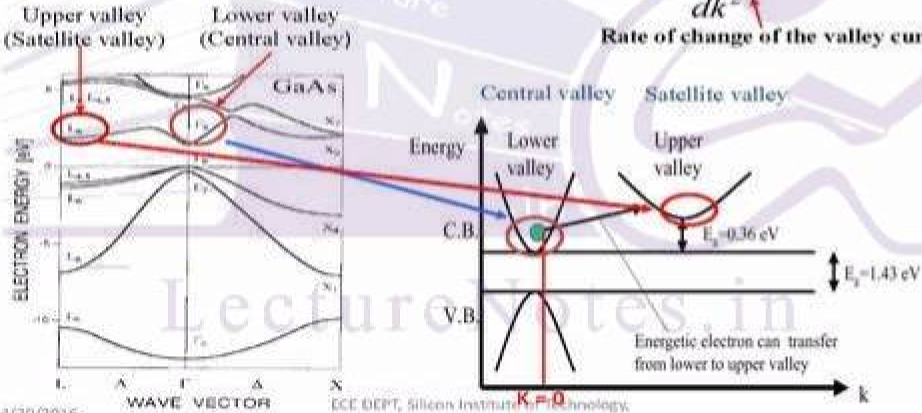
- According to the energy-band theory of n-type GaAs, there are two valleys in the conduction band
- Effective mass of electron is given by: $m^* = \dfrac{\hbar^2}{\dfrac{d^2E}{dk^2}}$

Rate of change of the valley curves slope



Two-Valley Model Theory

- **Effective mass of electron is given by:**

$$m^* = \frac{\hbar^2}{\dfrac{d^2E}{dk^2}}$$

Rate of change of the valley curves slope

- **Since the lower valley slope is shaper than the one in upper valley, thus electron effective mass in lower valley is lower than that in upper valley**
- **So that, the mobility of electron in upper valley is less due to the higher effective mass**

$$\mu_n = \frac{e\hat{\tau}}{m_n{}^*}$$

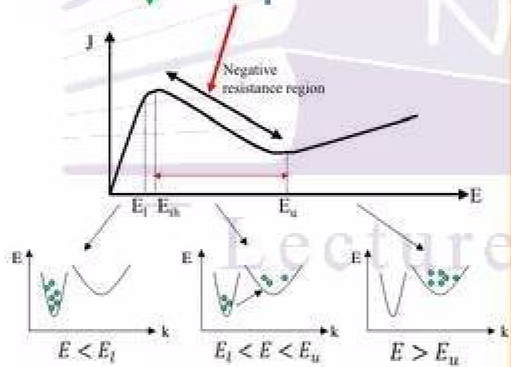
Valley	Effective mass M_e	Mobilityμ $Cm^2/V.s$
Lower	0.068	8000
Upper	1.2	180

* n-type GaAs

Two-Valley Model Theory

The current density v/s E-field according to equation

$$J = e(\mu_l n_l + \mu_u n_u)E \qquad \mu_l > \mu_u$$



Electron densities in the lower and upper valleys remain the same under an equilibrium condition.

- When the applied electric field is lower than the electric field of the lower valley ($E<E_l$), no electrons will transfer to the upper valley.
- When the applied electric field is higher than that of the lower valley and lower than that of the upper valley ($E_l < E < E_u$), electrons will begin to transfer to the upper valley.
- When the applied electric field is higher than that of the upper valley ($E_u < E$), all electrons will transfer to the upper valley.

Two-Valley Model Theory

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- Negative resistance : the current and voltage of a device are out of phase by 180degree → $P = -I^2 R$
- Conductivity of n-type GaAs is given by

$$\sigma = e(\mu_l n_l + \mu_u n_u)$$

$n_{l,u}$: *Electron density in lower/upper valley*

$\mu_{l,u}$: *Mobility in lower/upper valley*

- The differential resistance of the device is

$$\frac{d\sigma}{dE} = e(\mu_l \frac{dn_l}{dE} + \mu_u \frac{dn_u}{dE}) + e(n_l \frac{d\mu_l}{dE} + n_u \frac{d\mu_u}{dE})$$

➢ For an n-Type GaAs Gunn Diode: Electron density: $n = 10^{18}$ cm^{-3}, Electron density at lower valley: $n_l = 10^{10}$ cm^{-3}, Electron density at upper valley: $n_u = 10^8$ cm^{-3}, Temperature: $T = 300°$K. Determine the conductivity of the diode.

Ans:- 1.28mmho, Remember $\mu_l = 8000 \quad and \quad \mu_u = 180 \frac{cm^2}{V.s}$

Two-Valley Model Theory

- **According to Ohm's law:** $J = \sigma E$, $\dfrac{dJ}{dE} = \sigma + \dfrac{d\sigma}{dE}E$ *(2)*
- **rewrite equation 2:**

$$\frac{1}{\sigma}\frac{dJ}{dE} = 1 + \frac{\frac{d\sigma}{dE}}{\frac{\sigma}{E}} \quad (3)$$

- **Current density J must decrease with increasing field E**
- **Negative resistance occurs when**

$$-\frac{\frac{d\sigma}{dE}}{\frac{\sigma}{E}} > 1 \quad (4)$$

➢ A typical n-type GaAs Gunn diode has the following parameters: Threshold field E_{th} = 2800 V/cm, Applied field E = 3200 V/cm, Device length L = 10 μm, Doping concentration $n_o = 2 \times 10^{14}$ cm^{-3}, Operating frequency f = 10 GHz

a. Compute the electron drift velocity. $v_d = L \times f$

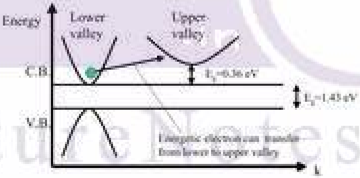
b. Calculate the current density. $J = q\, n_o v_d$

c. Estimate the negative electron mobility. $\mu_n = -\dfrac{v_d}{E}$

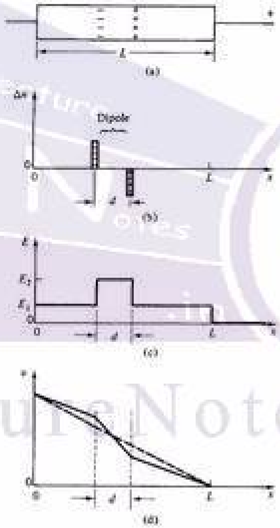
Two-Valley Model Theory- Ridley–Watkins–Hilsum theory

According to Ridley-Watkins-Hilsum (RWH) theory, the band structure of a semiconductor must satisfy the following three criteria in order to exhibit negative conductance

- The separation energy between the bottom of the lower valley and the bottom of the upper valley must be several times larger than the thermal energy at the room temperature which shall mean $\Delta E > kT$.
- The separation energy between the valleys must smaller than the band-gap energy between conduction and valence bands, which shall mean $\Delta E < E_G$. Otherwise, the semiconductor will breakdown to become high conductive device before electron can be transferred.
- Electron in the lower valley must have high mobility, small effective mass and a low density of state, whereas those in upper valley must have low mobility, large effective mass, and high density of state. In other word, electron velocity dE/dk must be larger in lower valley than in upper valley.



High-Field Domain

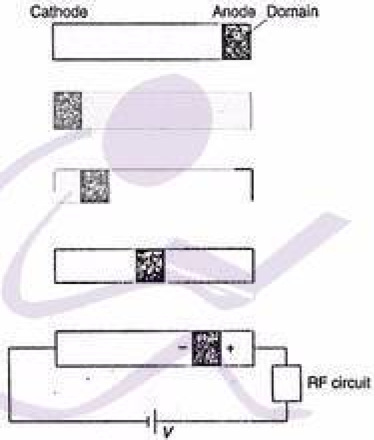


High-Field Domain

- When the applied voltage is above the threshold value, which was measured at about 3000 V/cm times the thickness of the GaAs diode, a high-field domain is formed near the cathode that reduces the electric field in the rest of the material and causes the current to drop to about two-thirds of its maximum value. This situation occurs because the applied voltage is given by

$$V = -\int_0^L E_x \, dx$$

- For a constant voltage V an increase in the electric field within the specimen must be accompanied by a decrease in the electric field in the rest of the diode. The high field domain then drifts with the carrier stream across the electrodes and disappears at the anode contact. When the electric field increases, the electron drift velocity decreases and the GaAs exhibits negative resistance.

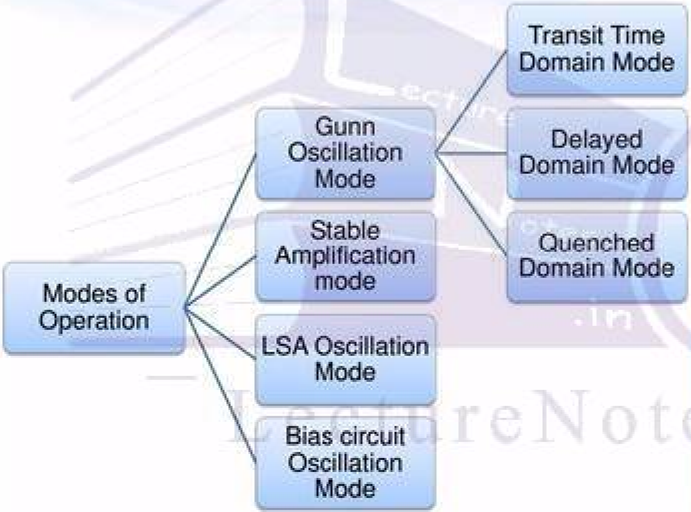


High field domain movement

High-Field Domain properties

1. A domain will start to form whenever the electric field in a region of the sample increases above the threshold electric field, the electron drift velocity decreases and the GaAs diode exhibits negative resistance.
2. If additional voltage is applied to a device containing a domain, the domain will increase in size and absorb more voltage than was added and the current will decrease.
3. A domain will not disappear before reaching the anode unless the voltage is dropped appreciably below threshold (for a diode with uniform doping and area).
4. The formation of a new domain can be prevented by decreasing the voltage slightly below threshold (in a non resonant circuit).
5. A domain will modulate the current through a device as the domain passes through regions of different doping and cross-sectional area, or the domain may disappear.
6. The domain's length is generally inversely proportional to the doping.
7. As a domain passes a point in the device, the domain can be detected by a capacitive contact, since the voltage changes suddenly as the domain passes. The presence of a domain anywhere in a device can be detected by a decreased current or by a change in differential impedance.

Modes Of Operation



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- **GaAs (Galliam Arsenide) has a property of negative resistance.**
- **The negative resistance in Gunn diode is due to**
  (a) electron transfer to a less mobile energy level (According to Einstein Equation $E=mc^2$)
  (b) high reverse bias
  (c) electron domain formation at the junction

Modes of operation

1. **Gunn Oscillation mode:**

- $fL = 10^7$ cm/s and $n_0L > 10^{12}$ cm/s^2
- Device is unstable because of the cyclic formation of either the accumulation layer or high field domain.
- Higher oscillation frequency can be obtained when the device is operated with high Q cavity resonator.

2. Stable Amplification mode:

$fL = 10^7$cm/s and $n_0L = 10^{11}$ cm/s^2 to 10^{12} cm/s^2

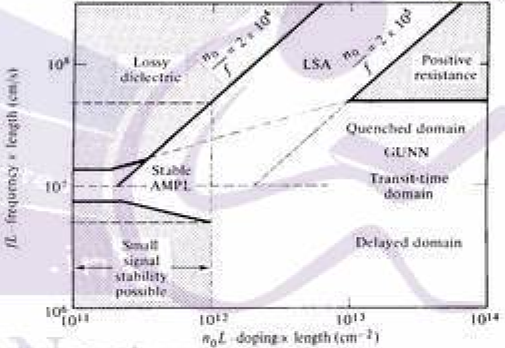
3. LSA Oscillation mode:

$fL > 10^7$cm/s, Quotient of doping divided by frequency is between 2×10^4 to 2×10^5

4. Bias-circuit oscillation mode:

This mode occurs only when there is either Gunn or LSA oscillation, and it is usually at the region where the product of frequency times length is too small in the figure to appear

Oscillation frequency in the bias circuit that are typically 1 KHz to 100 MHz.

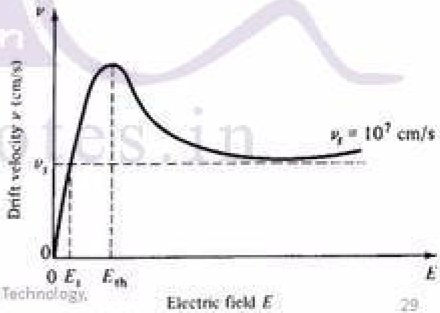


Gunn Oscillation mode

- If $E > E_{th}$ The high field domain drifts along the specimen until it reaches the anode or until the low-field value drops below the sustaining field E_s required to maintain v_S as shown in the figure

- Since the electron drift velocity v varies with E, there are 3 possible modes:

  1. Transit time domain mode
  2. Delayed domain mode
  3. Quenched domain mode



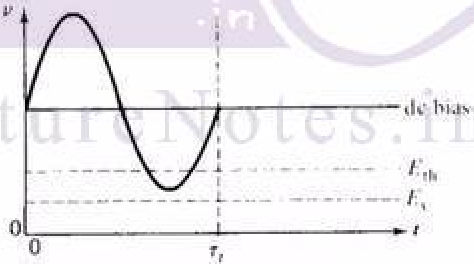
Gunn Oscillation mode- Transit time domain mode

- Operating frequency(f) x length (L)=10^7 cm/s
- Doping (n_0) x length (L)> 10^{12}/cm^2
- When the electron drift velocity v_d is equal to the sustaining the velocity v_s, high field domain is stable.
- The electron drift velocity is given by

$$v_d = v_s = fL \approx 10^7 \ cm/s \qquad f = \frac{v_s}{L}$$

- Oscillation period is equal to transit time $\tau = \tau_t = \dfrac{1}{f_{resonant}}$
- Low efficiency less than 10%

Transit-time mode
$\tau_0 = \tau_t$



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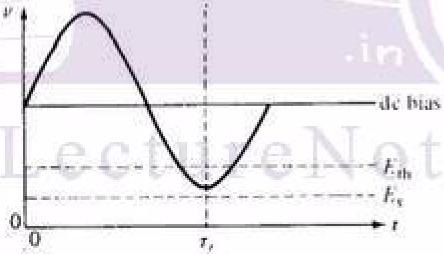
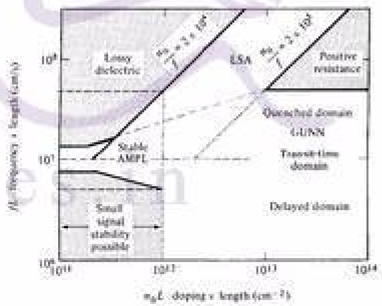
Gunn Oscillation mode- Delayed domain mode

- 10^6cm/s < fL< 10^7cm/s
- Also known as inhibited mode.
- When the transit time is chosen so that the domain is collected while $E < E_{th}$, a new domain cannot form until the field rises above threshold again.
- The oscillation period is greater than the transit time.
- There is an ohmic currents higher than domain currents.
- High efficiency up to 20%

Delayed mode
$\tau_0 > \tau_t$

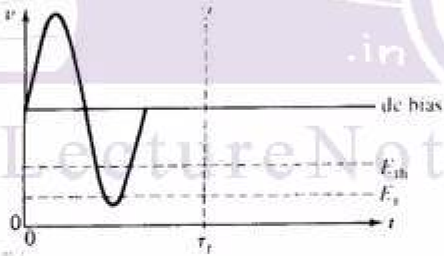


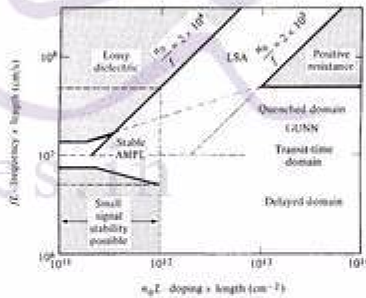


Gunn Oscillation mode- Quenched domain mode

- $10^7 < fL < 10^8$
- If $E < E_s$ domain collapses before it reaches the anode. When the bias field swings back above the threshold, a new domain is nucleated and the process repeats. Thus oscillation occurs at the frequency of resonant circuit rather than at the transit time frequency.
- The domain can be quenched before it is collected
- Efficiency can reach 13%.

Quenched mode
$\tau_{\theta} < \tau_t$





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LSA Mode- Limited Space charge Accumulation mode

- LSA Mode is the simplest mode of operation. $fL > 2 \times 10^7$ cm/s
- In LSA mode the Gunn diode is placed in a resonator which is tuned to a oscillator frequency of f_0

$$f_0 = \frac{1}{\tau_0} = \frac{1}{Oscillation\ Period}$$

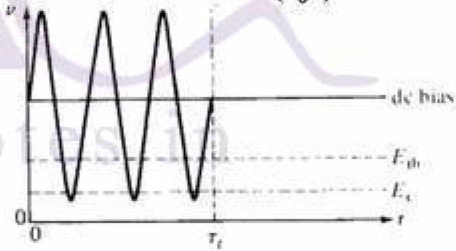
- The frequency is so high that domains have insufficient time to form while the field is above threshold. As a results, domains do not form.
- In this mode the device can be biased to several times the threshold voltage.

$2 \times 10^4 < (n_0/f) < 2 \times 10^5$

Advantage of LSA Mode:
- Efficiency is more than 20%

Limitation of LSA Mode:
- Very sensitive to load.
- Sensitive to operating temperature.
- Variation in doping concentration affect frequency stability.

LSA mode
$\tau_0 < \tau_t$
$\tau_0 = 3\tau_d$
dielectric relaxation time



LSA Mode- Limited Space charge Accumulation mode

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$$P = \eta I_o V_o$$

$$\boxed{P = \eta (ME_{th}L)(n_o \cdot ev_0 \cdot A)}$$

where
η = Conversion parameter of material
I_o = Operating current
V_o = Operating voltage
M = Multiplication factor of operating voltage above threshold.
E_{th} = Threshold electric field (kV/m)
L = Device length (μm)
n_o = Donor concentration (/ m^3)
e = Charge of electron (C)
v_0 = Average carrier velocity (m/s)
A = Device area (m^2)

The output power of LSA oscillator is ranging from 400 watts to 6 kW in pulsed mode.

- An LSA oscillator has the following parameters: Conversion efficiency: η= 0.06, Multiplication factor: M = 3.5 , Threshold field: E_{th} = 320 kV/m , Device length: L= 12μm, Donor concentration: $no = 10^{21}$ /m3 , Average carrier velocity: v_o= 1.5 x 10^5 m/s, Area: $A = 3 \times 10^{-8} m^2$. Determine the output power in milliwatts. **Ans-581mW**

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Criteria for classifying modes

$$\boxed{n_0 L > \frac{\varepsilon v_d}{e|\mu_n|}}$$

where ε = Semiconductor dielectric permittivity

v_d = Electron drift velocity

μ_n = Electron mobility

n_o = Doping concentration

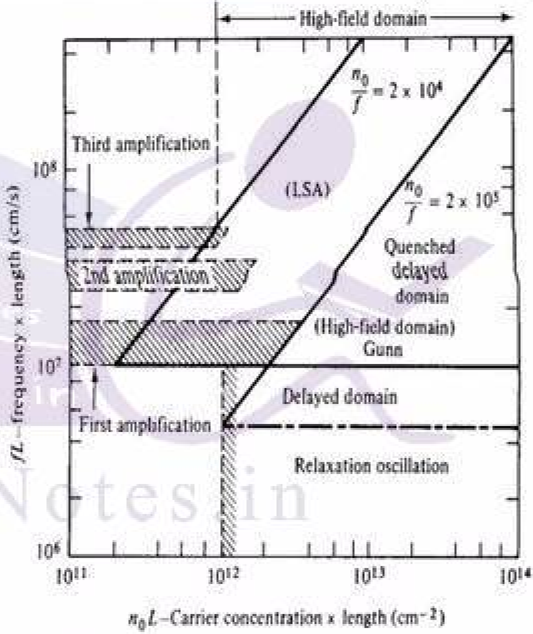
e = Charge on electron

- An n-type GaAs Gunn diode has the following parameters: Electron drift velocity: v_d = 2.5 x 105 *mis*, Negative electron mobility: $|\mu_n|$= 0.015 m²/V · S, Relative dielectric constant: ε_r = 13.1, Determine the criterion for classifying the modes of operation.

Ans—$n_0 L > 1.19 \times 10^{12}/cm^2$

Stable amplification mode

- When $n_0L < 10^{12}/cm^2$ the device exhibits amplification at the transit time frequency rather than spontaneous oscillation.
- This situation occurs because the negative conductance is utilized without domain formation.
- There are too few carriers for domain formation within the transit time.
- Therefore amplification of signals near the transit time frequency can be accomplished.



Summary of Gunn device modes

Sl. No.	Mode	Conditions of operations		Efficiency limit	Circuit conditions
		Transit time	Doping level (fL) and (n_0L)		
1	Transit time or resonant Gunn	$\tau_0 = \tau_t$	$fL = 10^7$ cm/s $n_oL = 10^{12}$ cm/s^2	10%	Non-resonant, Constant Voltage, Frequency determined by transit time, Low Q
2	Delayed domain or inhibited	$\tau_0 > \tau_t$	$10^6 < fL < 10^7$ $n_oL > 10^{12}$ cm/s^2	20%	Resonant, Constant impedance, High Q
3	Quenched domain	$\tau_0 < \tau_t$	$10^7 < fL < 10^8$ $n_oL > 10^{12}$ cm/s^2	13%	Resonant, Constant impedance, High Q
4	Limited Space charge Accumulation (LSA)	$\tau_0 << \tau_t$ $\tau_t = 3\tau_0$	$fL > 5x10^6 < 10^7$ $2x10^4 < (n_0/f) < 2x10^5$ $n_oL > 10^{12}$ cm/s^2	20%	Multiple resonant, high impedance, high dc bias
5	Stable amplification (multiple)	----	$10^7 < fL < 10^8$ $n_oL < 10^{12}$ cm/s^2	2-3%	Non resonant, Transit time (frequency independent of sample length)

ADVANTAGES

- It has much lower noise than IMPATT diodes
- Gunn amplifiers are capable of broad-band operation.
- Higher peak-to-valley ratio in its –ve resistance characteristics.
- High fundamental frequency operation.
- Increased efficiency.

APPLICATIONS

- Gunn diode oscillator as low & medium power oscillator in microwave receivers & instruments.
- As pump source in parametric amplifier.
- High-power Gunn oscillators (250-2000mW)are used as power output oscillators.
- Frequency modulator in low power transmitter.
- In police & CW-Doppler RADAR ,burglar alarms, aircraft rate-of-climb indicators.
- YIG (yttrium-iron garnet) -tuned Gunn VCOs for instrument applications.

ASSIGNMENT QUESTIONS

- What is LSA oscillation mode of Gunn diode?
- What are the Gunn domains? What do they do ?
- Explain the principle of operation of Gunn diode using two valley model theory. Give constructional details and the electrical equivalent circuit.
- Explain how a Gunn diode is used as an oscillator with the development of appropriate expressions (if any) and sketches.
- Explain three possible domain modes for Gunn oscillation mode with proper diagrammatical representation.
- Discuss the principles of the following terms

  (i)Gunn effect (ii) Two valley theory
- Describe, in brief, the limited space charge accumulation mode of operation for Gunn diodes.
- With the help of a circuit diagram explain how Gunn diode can be used as an oscillator and an amplifier

ASSIGNMENT QUESTIONS

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- Determine the conductivity of the n-type GaAs Gunn diode for the given parameters: Electron density at lower valley n1 = 10^{10} cm^{-3}, Electron density at upper valley nu = 10^{8} cm^{-3}, and Temperature T = 300°K.
- A GaAs Gunn diode has a drift length of 10μm. Determine the frequency of oscillation.

Assignment - 3 D: 22/3/16

1) For which condition a klystron amplifier act as
SQ oscillator? Write 2 application of klystron amplifier.

When the klystron amplifier is given a +ve feedback such that the overall phaseshift becomes zero/360° and $|A\beta| = 1$, then klystron amplifier acts as an oscillator.

Applications:

- UHF TV transmitter
- Long range RADAR
- Linear particle accelerator
- Tropo scatter links
- Earth station transmitter.

2) What is bunching parameter of reflex klystron
SQ oscillator?

$$X = \frac{\pi N \beta_i V_1}{V_0}$$

5&12) Under what conditions reflex klystron will work
SQ as an oscillator?

The necessary condition for oscillation is that magnitude of -ve real part of the electronic admittance should not be less than total conductance of the cavity circuit.

$$|-G_e| \geq (G_c + G_L)$$

$$G = G_e + G_b + G_L = \frac{1}{R_{sh}}$$

8) How does a reflex klystron differ from an amplified
SQ klystron?

Reflex klystron	Amplified Klystron
• One cavity	• Atleast 2 cavity
• Bunching & catching of electrons in the same cavity.	• Buncher & catcher cavities are different.

2) A reflex klystron operates under the following condition
i.e. $V_0 = 500V$, $R_{sh} = 20k\Omega$, $f_r = 8GHz$

2015 $L = 1mm \rightarrow$ spacing bet. repeller & cavity.

6(b) The tube is oscillating at a f_r the ~~~~ peak of $n=2$ mode. Assume that the transit time through gap and beam loading effect can be neglected.

i) the repeller voltage V_r.

ii) Direct current necessary to give microwave voltage of 200V.

iii) Electronic efficiency.

i) $t_0 = \frac{2u_0}{a} = \frac{2u_0 mL}{e(V_0+V_R)} = \frac{2\pi N}{\omega}$

where $u_0 = \sqrt{\frac{2eV_0}{m}}$

$$\frac{V_0}{(V_r+V_0)^2} = \frac{e}{m}\frac{(2\pi n - \pi/2)^2}{8\omega^2 L^2}$$

$V_r = 191.60V$ (calculate it)

ii) $V_2 = 2I_0 J_1(x') R_{sh}$ $\quad J_1(x') =$ Bessel's fun.

$i_0 = \frac{V_2}{2J_1(x') R_{sh}}$ $\quad = 0.582$ for $n=2$.

$x' = 1.841$ for $n=2$.

$= 8.59 mA$

iii) $\eta = \frac{2x' J_1(x')}{2\pi n - \pi/2} = 19.499\%$

11) A reflex klystron is operating at 4 GHz with following parameters:

2013 5(b)

$V_R = 2kV$, $V_0 = 500V$, $L = 2cm$, $n = 0$.

Calculate the change in frequency for a 4% change in repeller voltage.

$$\frac{V_0}{(V_0+V_r)^2} = \frac{e}{m}\frac{(2\pi n - \pi/2)^2}{8\omega^2 L^2}$$

$$\Rightarrow V_0 + V_r = \sqrt{\frac{mV_0}{e} \times \frac{8\omega^2 L^2}{(2\pi n - \pi/2)^2}} = \sqrt{\frac{8mV_0}{e}}\ \frac{2\pi f L}{2\pi n - \pi/2}$$

$$\frac{dV_r}{df} = \sqrt{\frac{8\ V_0}{}} \times \frac{2\pi L}{2\pi n - \pi/2}$$

$$df = \frac{dV_r}{P} \times \frac{1/100}{P}$$

10) An identical two cavity klystron amplifier operates at 4GHz, with $V_0 = 1kV$, $I_0 = 22mA$, gap (d) = 1mm, drift space = 3cm, It DC beam transconductance & catch up cavity total equivalent conductance are 0.25×10^{-4} ℧ & 0.3×10^{-4} ℧ respectively.

2013 5(a)

Calculate :

i) the ~~beam~~ decoupling coefficient, DC transist angle in drift space and the input cavity voltage magnitude for maximum output voltage.

ii) Voltage gain and efficiency, neglecting the beam loading.

i) $\beta_1 = \frac{\sin \theta g/2}{\theta g/2}$, $\theta g = \omega t g$ $\quad t g = \frac{d}{u_0}$

$= \frac{\omega d}{u_0}$ $\quad u_0 = \sqrt{\frac{2eV_0}{m}}$

$= \frac{\sin(38.2)^\circ}{0.6685}$ $\quad = \frac{2\pi f d}{u_0}$ $\quad = 0.593\times10^6\sqrt{V_0}$

$= 0.927$ $\quad = \frac{2\pi \times 4\times10^9 \times 1\times10^{-3}}{1.88\times10^7}$ $\quad = 1.88\times10^7 m/s$

$\theta = \frac{\omega L}{u_0} = 40.11 rad$ $\quad = 1.337\ rad = 76.6^\circ$

Cavity gap voltage magnitude:

$X = \frac{\pi N \beta_1 V_1}{V_0}$ $\qquad t_0 = \frac{2\pi N}{\omega}$

$\Rightarrow V_1 = \frac{2V_0 X}{\beta_1 \theta_0}$ $\qquad \Rightarrow \frac{\omega t_0}{2} = \pi N$

$\Rightarrow \frac{\theta_0}{2} = \pi N$

$= \frac{2\times10^3 \times 1.841}{0.427\times 40.11} =$

ii) Voltage Gain:

$$Av = \frac{\beta^2 \theta_0 J_1(x) I_0}{x\ V_0\ G_{sh}} = \frac{(0.927)^2 \times 40.11 \times 0.582 \times 22\times10^{-3}}{1.841\times0.55\times10^{-4}\times10^3}$$

$= 4.36 = 12.8dB$.

Catcher voltage $\quad V_2 = Av \times V_1$

$= 431.6 V$

$$\eta = \frac{P_{RF}}{P_{dc}} = \frac{\beta I_0 J_1(x) V_2}{I_0 V_0}$$

$$= \frac{\beta_1 J_1(x) V_2}{2V_0} = 23.28\%$$

Short Questions :

1) A lossless 100Ω transmission line is terminated in 50+j75. Find voltage reflection coefficient and VSWR.

Given $Z_0 = 100\Omega$, $Z_L = 50+j75$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -0.067 + 0.533j = 0.537\angle 97.16^\circ$$

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.537}{1-0.537} = 3.319$$

3) What is the input impedance of terminated transmission line for short circuited and open circuited line?

For short circuit line,

$$Z_L = 0, \quad \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$$

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \infty$$

$$V(z) = V_0^+ \left[e^{-j\beta z} - e^{j\beta z}\right] = -2j V_0^+ \sin\beta z$$

$$I(z) = \frac{V_0^+}{Z_0}\left[e^{-j\beta z} + e^{j\beta z}\right] = \frac{2V_0^+}{Z_0}\cos\beta z.$$

$$Z_{in} = \frac{V(-l)}{I(-l)} = \frac{2jV_0^+ \sin\beta l}{\frac{2V_0^+}{Z_0}\cos\beta l} \Rightarrow \boxed{Z_{in} = jZ_0 \tan\beta l}$$

For open circuit line.

$Z_L = \infty$, $\Gamma = 1$, $VSWR = \infty$

$$V(z) = V_0^+\left[e^{-j\beta z} + e^{j\beta z}\right] = 2V_0^+ \cos\beta z$$

$$I(z) = \frac{V_0^+}{Z_0}\left[e^{-j\beta z} - e^{j\beta z}\right] = -2j\frac{V_0^+}{Z_0}\sin\beta z$$

$$Z_{in} = \frac{V(-l)}{I(-l)} = \frac{2V_0^+ \cos\beta l}{2j\frac{V_0^+}{Z_0}\sin\beta l} \Rightarrow \boxed{Z_{in} = -jZ_0 \cot\beta l}$$

4) Calculate the characteristic impedance and propagation constant for a lossless co-axial line having inner radius 0.3 mm and outer radius 0.6mm with $\mu_r = 2.3$ and $\epsilon_r = 1.4$ at operating frequency 5MHz.

$\mu_r = 2.3$, $\epsilon_r = 1.4$, $f = 5MHz$; $a = 0.3mm$, $b = 0.6mm$

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}}\frac{\ln b/a}{2\pi} = \sqrt{\frac{\mu_0\mu_r}{\epsilon_0\epsilon_r}} \cdot \frac{\ln b/a}{2\pi} = 53.97\,\Omega$$

$\beta = \omega\sqrt{LC} = 2\pi f\sqrt{LC} = 0.185$ rad/m.

5) Write the expression for the attenuation constant (α), phase constant (β), characteristic impedance (Z_0) and phase velocity (V_P) for the lossless transmission line.

For lossless transmission line, $R = G = 0$.

$$\alpha = 0$$

$$\beta = \omega\sqrt{LC}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}}$$

$$V_P = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

6) The current in a transmission line is given as $i(t) = 1.2\cos(1.51 \times 10^{10} t - 80.3z)$. Determine the wavelength and phase velocity.

$\omega = 2\pi f = 1.5 \times 10^{10}$, $\beta = 80.3$

$\Rightarrow f = 2.4\,GHz$

$$\beta = \frac{2\pi}{\lambda}$$

$$\Rightarrow \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{80.3} = 0.0782\text{ m}.$$

$$V_\beta = \frac{\omega}{\beta} = \frac{1.5 \times 10^{10}}{80.3} = 1.88 \times 10^{8}\text{ m/s}.$$

8) What do you mean by voltage standing wave ratio (VSWR)? What is its significance?

When load mismatch ($Z_L \neq Z_0$), the presence of reflected wave leads to standing wave where the magnitude of the voltage on the line is constant.

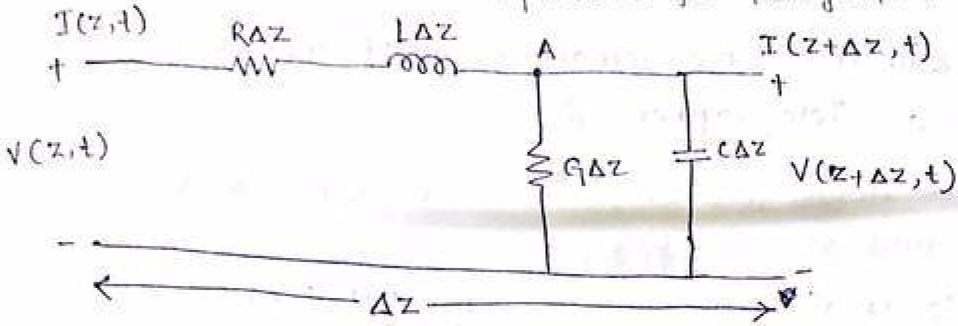
$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} \quad \text{for } 0 \leq |\Gamma| < 1.$$

$$1 < S < \infty$$

7) Why Z, Y & h parameters can't be measured at microwave frequencies?

At high frequency, standard circuit theory is invalid and Z, Y and h can only be measured at low frequency approximation so, it can't be measured at the microwave frequency.

11) Write mathematical expression for Telegrapher equation.



The series inductance L represents the total self inductance of the two conductors and shunt capacitance C is due to the close proximity of the two conductors. The series resistance R represents the resistance due to finite connectivity of the conductors and shunt conductance G is due to dielectric loss in the material between the conductors. R&G are loss elements of transmission line.

Applying KVL to the small circuit, we have

$$V(z,t) - R\Delta z\, I(z,t) - L\Delta z \cdot \frac{dI(z,t)}{dt} - V(z+\Delta z,t) = 0 \quad \text{—} ①$$

Applying KCL to node A.

$$I(z,t) - G\Delta z\, V(z+\Delta z,t) - C\,\frac{dV(z+\Delta z,t)}{dt} - I(z+\Delta z,t) = 0 \quad \text{—} ②$$

Dividing eq. ① & ② by Δz individually, and taking limit Δz→0 we have,

$$\lim_{\Delta z\to 0} \frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = \lim_{\Delta z\to 0} \frac{-R\Delta z\, I(z,t)}{\Delta z} - \lim_{\Delta z\to 0} \frac{L\Delta z\,\partial I(z,t)}{\Delta z}\cdot\frac{1}{\partial t}$$

$$\Rightarrow \frac{\partial V(z,t)}{\partial z} = -RI(z,t) - L\,\frac{\partial I(z,t)}{\partial t} \quad \text{—} ③$$

$$\lim_{\Delta z\to 0} \frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = \lim_{\Delta z\to 0} -G\Delta z\, V(z+\Delta z,t) - \lim_{\Delta z\to 0} C\Delta z\,\frac{\partial V(z+\Delta z,t)}{\partial t}$$

$$\Rightarrow \frac{\partial I(z,t)}{\partial t} = -G\,V(z,t) - C\,\frac{\partial V(z,t)}{\partial t} \quad \text{—} ④$$

Omitting the argument (z,t) from eq. ③ & ④ which is understood,

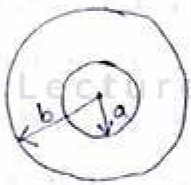
$$\frac{\partial V}{\partial z} = -RI - L\frac{\partial I}{\partial t} \quad \text{—} ⑤$$

$$\frac{\partial I}{\partial z} = -GV - C\frac{\partial V}{\partial t} \quad \text{--- (6)}$$

eq. (5) & (6) are time domain form of Transmission Line known as Telegrapher eq.

14) Write down two differences between a parallel wire line and co-axial line.

Co-axial Line



$L = \frac{\mu}{2\pi} \ln \frac{b}{a}$

$C = \frac{2\pi \epsilon'}{\ln b/a}$

$R = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$

$G = \frac{2\pi \omega \epsilon''}{\ln b/a}$

Parallel Line

$L = \frac{\mu d}{w}$

$C = \frac{\epsilon' w}{d}$

$R = \frac{2R_s}{w}$

$G = \frac{\omega \epsilon'' w}{d}$

18) Where do the minima of the voltage standing wave on a lossless line with a resistive termination occur, if $R_L > R_0$?

For resistive termination on a lossless line.

$Z_L = R_L$ and $Z_0 = R_0$

$\Gamma = \frac{R_L - R_0}{R_L + R_0}$, real value.

$R_L > R_0$, In this case, Γ is positive real and $\theta = 0°$, so, $V(l)$ voltage is maximum and $I(l)$ current is minimum at the termination of the resistance.

20) Show that a $\lambda/4$ long transmission line can be used on an impedance transformer. What is the bandwidth of this transformer?

When a section of transmission line is used either as reactance or as a resonant circuit, it is a 2 terminal network. However, a section of line can be used as 4 terminal network, when it is inserted between the generator and the load. Such a line is known as quaterwave transfermer. Because it has the effect of transforming the load impedance in an inverse manner depending on the characteristic impedance of the line.

(5)

Long Question:

1) A uniform transmission line has $R = 12m\,\Omega/m$, $G = 1.4\,\mu mho/m$, $L = 1.5\,\mu H/m$ and $C = 1.4\,nF/m$, at $70\,kHz$. Find (i) characteristic impedance, (ii) attenuation in decibels per kilometer, (iii) the velocity of propagation and (iv) propagation constant.

2015 2(a)

Given $R = 12\times10^{-3}\,\Omega/m$, $G = 1.4\times10^{-6}\,\mho/m$,

$L = 1.5\times10^{-6}\,H/m$, $C = 1.4\times10^{-9}\,F/m$, $f = 70\times10^{3}\,Hz$.

$\omega = 2\pi f = 4.39\times10^{5}$ rad/s.

i) characteristic impedance $(Z_0) = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$

$$= \sqrt{\frac{12\times10^{-3} + 4.39\times10^{5}\times1.5\times10^{-6}j}{1.4\times10^{-6} + 4.39\times10^{5}\times1.4\times10^{-9}j}}$$

$$= \sqrt{\frac{0.012 + 0.6585j}{1.4\times10^{-6} + 6.146\times10^{-4}j}} = \sqrt{\frac{0.6586\angle 88.95^\circ}{6.146\times10^{-4}\angle 89.87^\circ}}$$

$$= \sqrt{1071.45 - 17.2j} = \sqrt{1071.588\angle -0.92^\circ}\ \Omega = 32.73\angle -0.46\,\Omega$$

ii) $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$= \sqrt{(12\times10^{-3} + 4.39\times10^{5}\times1.5\times10^{-6}j)(1.4\times10^{-6} + 4.39\times10^{5}\times1.4\times10^{-9}j)}$$

$$= \sqrt{-4.046\times10^{-4} + 8.3\times10^{-6}j}$$

$$= \sqrt{4.046\times10^{-4}\angle 178.82}$$

$$= 0.02\angle 89.41^\circ = 2.05\times10^{-4} + 0.0199j$$

attenuation in decibels per kilometer (α)

$= \cancel{2.05\times10^{-4}\ \text{decibel/m}}\ \ 0.02\cos(89.41) = 2.06\times10^{-4}\ Np/Km$

iv) Propagation constant $(\beta) = 0.0199$.

iii) Velocity of propagation $(V_p) = \dfrac{\omega}{\beta} = \dfrac{4.39\times10^{5}}{0.0199}$

$= 22.06\times10^{6}$ m/s

2) The characteristic impedance of a 10m long lossless co-axial cable is 50Ω. The dielectric material between the inner and outer conductors of the cable has $\epsilon_r = 3.5$ and $\mu_r = 1$. If the radius of the inner conductor is 1mm. What should be the outer radius of this cable?

16
2015
2(b)
[3]

Given $l = 10m$ (lossless), $Z_0 : 50\Omega$, $\epsilon_r = 3.5$, $\mu_r = 1$,

$a = 1mm$, $b = ?$

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}} \cdot \frac{\ln b/a}{2\pi}$$

(6)

$$\Rightarrow \ln b/a = 2\pi Z_0 \sqrt{\frac{\epsilon}{\mu}} = 1.56$$

$$\Rightarrow b = a e^{1.56}$$

$$\Rightarrow b = 4.7588 \times 1\,mm$$

$$\Rightarrow b = 4.7588\,mm.$$

3) For sea water $\sigma = 4 S/m$ and $\epsilon_r = 80$, $\mu_r = 1$. Calculate, the attenuation constant (α) and propagation constant (γ) at frequencies 1kHz, 100kHz and 1000kHz and also find the skin depth (δ_s) at 100kHz.

2015
2(c)
[3]

Given $\sigma = 4 S/m$, $\epsilon_r = 80$, $\mu_r = 1$,

condition, $\frac{\sigma}{\omega\epsilon} >> 1 \rightarrow$ good conductor.

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi f \mu\sigma}{2}}$$

$$\alpha_{1kHz} = \sqrt{\frac{2\pi \times 1 \times 10^3 \times 4\pi \times 10^{-7} \times 1 \times 4}{2}} = 0.12566\ Np/m.$$

$$\alpha_{100kHz} = \sqrt{\frac{2\pi \times 100 \times 10^3 \times 4\pi \times 10^{-7} \times 1 \times 4}{2}} = 1.2566\ Np/m$$

$$\alpha_{1000kHz} = \sqrt{\frac{2\pi \times 1000 \times 10^3 \times 4\pi \times 10^{-7} \times 1 \times 4}{2}} = 3.9738\ Np/m$$

$$\gamma = (1+j)\sqrt{\frac{\omega\mu\sigma}{2}} = 3.974 \times 10^{-3} (1+j)\sqrt{f}$$

$$\gamma_{1kHz} = 0.1256 + j0.1256$$

$$\gamma_{100kHz} = 1.2567 + j1.2567$$

$$\gamma_{1000kHz} = 3.974 + j3.974.$$

$\delta_s = \sqrt{\dfrac{2}{\mu\omega\sigma}} = \sqrt{\dfrac{1}{\pi f \mu \sigma}}$

$= \cancel{0.0795}\ 7.96\,mm$

Q3 1) A lossless transmission line is terminated with 100Ω load. If SWR on the line is 1.5, find the two possible values of the characteristic impedance of the line.

2014 3(b)

Given $Z_L = 100\Omega$, $SWR = 1.5$

$S = SWR = \dfrac{1+|\Gamma|}{1-|\Gamma|}$

$\Rightarrow |\Gamma| = \dfrac{S-1}{S+1} = \dfrac{0.5}{2.5} = 0.2$

$\Gamma = \pm 0.2$

$\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$Z_0 = Z_L\left(\dfrac{1-\Gamma}{1+\Gamma}\right) = 100\left(\dfrac{0.8}{1.2}\right) = 66.67\Omega \quad (\because \Gamma = 0.2)$

$Z_0 = Z_L\left(\dfrac{1-\Gamma}{1+\Gamma}\right) = 100\left(\dfrac{1.2}{0.8}\right) = 150\Omega \quad (\because \Gamma = -0.2)$

11) A low loss co-axial cable of characteristic impedance of 50Ω is terminated in a resistive load of 75Ω. The peak voltage across the load is found to be 30 volts. Calculate

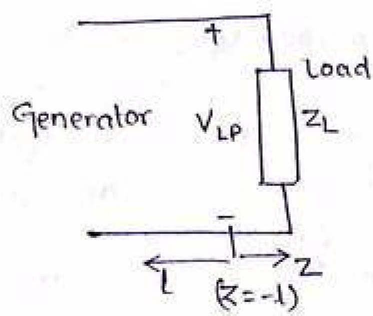
i) The reflection co-efficient of the load.

ii) The amplitude of the forward and reflected voltage waves

iii) The amplitude of the forward and reflected current waves

iv) The VSWR.

Given $Z_0 = 50\Omega$, $Z_L = 75\Omega$, $V_{LP} = 30V$.



$V(l) = V_0^+\left[1+\Gamma e^{-2j\beta l}\right]$

$|V(l)| = V_{LP}$

$|V(l)| = V_0^+\left[1+|\Gamma|\right]$

$\Rightarrow 30 = V_0^+\left[1+\frac{1}{5}\right] \Rightarrow V_0^+ = 25V$

$V_0^- = \Gamma \times V_0^+$.

$\Rightarrow V_0^- = 5V$

$I(z) = I^+ e^{-\gamma z} + I^- e^{\gamma z}$

$= \frac{V_0^+}{Z_0}\left[1 - |\Gamma| e^{-j\beta l}\right]$

$I^+ = \frac{V_0^+}{Z_0} = \frac{25}{50} = 0.5A$

$I^- = \frac{V_0^-}{Z_0} = \frac{-5}{50} = -0.1A$

$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+1/5}{1-1/5} = \frac{6}{4} = 1.5.$

18) A certain transmission line has a characteristic impedance of $75 + j0.01\Omega$ and is terminated in a load impedance of $70+j50\Omega$. Calculate
i) reflection coefficient
ii) SWR
iii) transmission coefficient
iv) insertion loss.

Given $Z_0 = 75 + j0.01\Omega$ $\quad Z_L = 70 + j50\Omega$

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-5 + j49.99}{145 + j50.01}$

$= 0.075 + 0.318j = 0.33\angle 76.73$

$T = 1 + \Gamma = 1.122\angle 16.47^\circ$

$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.33}{1-0.33} = 1.985$

$IL = 1 - |\Gamma|^2 = 0.8911$

$IL \text{ in dB} = 10 \log(1 - |\Gamma|^2)$

$= -0.5 dB.$

15) A 100m long lossless transmission line has a inductance and capacitance 27.72 μH & 18nF respectively. Determine,
i) Velocity of propagation
ii) phase constant for an operating frequency of 100kHz and
iii) the characteristic impedance of transmission line.

$l = 100m \quad L = 27.72\mu H \quad C = 18nF$

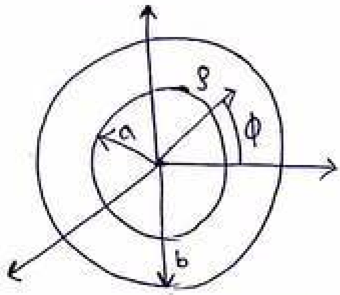
For lossless T.L., $\alpha = 0$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$
$$= 1.41 \times 10^6 \, m/s$$

$$\beta = \omega\sqrt{LC} = 2\pi f\sqrt{LC} = 0.4438 \text{ rad/m}$$

$$Z_0 = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{L}{C}} = 39.243\,\Omega.$$

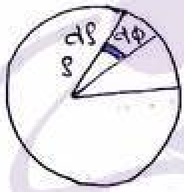
17) Find out the R, L, G, C parameters for co-axial transmission line.



$$L = \frac{\mu}{|I_0|^2}\int_S \bar{H}\cdot\bar{H}^* \, ds$$

$$= \frac{\mu}{|I_0|^2}\iint \frac{I_0^2}{(2\pi\rho)^2}\,\rho\, d\phi\, d\rho$$

$$= \frac{\mu}{I_0^2}\int_0^{2\pi} d\phi \int_{\rho=a}^{\rho=b} \frac{I_0^2}{(2\pi\rho)^2}\,\rho\, d\rho$$



$$= \frac{\mu}{4\pi^2}\int_0^{2\pi} d\phi \int_{\rho=a}^{b} \frac{d\rho}{\rho}$$

$$= \frac{\mu}{2\pi}\ln\frac{b}{a} \quad H/m$$

$$C = \frac{\epsilon'}{|V_0|^2}\int_S \bar{E}\cdot\bar{E}^* \, ds$$

$$= \frac{\epsilon'}{|V_0|^2}\int_S \frac{V_0^2}{(\rho \ln b/a)^2}\,\rho\, d\phi\, d\rho$$

$$= \frac{\epsilon'}{(\ln b/a)^2}\iint \frac{1}{\rho}\, d\phi\, d\rho$$

$$= \frac{\epsilon'}{(\ln b/a)^2}\int_0^{2\pi} d\phi \int_{\rho=a}^{b} \frac{d\rho}{\rho}$$

$$= \frac{2\pi\epsilon'}{\ln b/a} \quad F/m$$

$$R = \frac{R_s}{|I_0|^2}\int_{C_1+C_2} \bar{H}\cdot\bar{H}^* \, dl$$

$$= \frac{R_s}{|I_0|^2}\left[\int_0^{2\pi}\left(\frac{I_0}{2\pi\rho}\right)^2 a\, d\phi + \int_0^{2\pi}\left(\frac{I_0}{2\pi\rho}\right)^2 b\, d\phi\right]$$

$$= \frac{R_s}{2\pi}\left[\frac{1}{a}+\frac{1}{b}\right] \ \Omega/m$$

$$G = \frac{\omega \epsilon''}{(\ln b/a)^2}\int_{\phi=0}^{2\pi}\int_{\rho=a}^{b}\frac{1}{\rho^2}\rho\, d\rho\, d\phi$$

$$= \frac{2\pi\omega C''}{\ln b/a} \ S/m.$$

20) A 25m long lossless transmission line is terminated with a load having an equivalent impedance of $40+j30\Omega$ at 10MHz. The inductance and capacitance of the line are 310.4 nH/m and 38.28 pF/m respectively. Calculate, i) characteristic impedance, ii) phase constant, iii) input impedance at the sending end and iv) input impedance at midpoint of line.

$l = 25m$, $Z_L = 40+j30\Omega$, $f = 10MHz$, $L = 310.4nH/m$

$C = 38.28pF/m$

$$Z_0 = \sqrt{\frac{L}{C}} = 90.05\Omega$$

$$\beta = 2\pi f\sqrt{LC} = 0.22 rad/m$$

$$\beta l = 5.414 rad$$

$$Z_{in} = Z_0 \frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}$$

$$= 51.15\angle -37.78^\circ$$

$$= 0.449 - j0.348$$

mid-point $\beta l = \beta \times \frac{25}{2} = 2.75 rad$

$Z_{in} = 35.26 - j0.63$.

19) A lossless co-axial cable is used to delay a pulse by 100ns. The inductance and capacitance of the cable are 0.20µH/m and 60pF/m respectively. Determine the length of the cable.

$T_n = 100ns$ $L = 0.20\mu H/m$ $C = 60pF/m$

Length of cable:

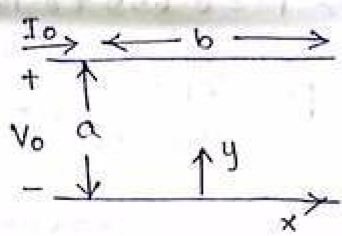
$$V_p = \frac{1}{\sqrt{LC}} = 2.88\times10^8 m/s$$

velocity $= \dfrac{\text{length}}{\text{time}}$

$\Rightarrow$ Length $= 2.88\times10^{8}\times100\times10^{-7}$

$= 28.86\,m.$

22) Derive the capacitance of a parallel wire transmission
13) line of width b and spacing a.



$E_y = \dfrac{-V_0}{a}$ V/m

$H_x = \dfrac{I_0}{b}$ A/m

$$L = \frac{\mu}{|I_0|^2}\int_s \bar{H}\cdot\bar{H}^*\,ds$$

$$= \frac{\mu}{|I_0|^2}\int_{x=0}^{b}\int_{y=0}^{a}\frac{I_0^2}{b^2}\,dx\,dy$$

$$= \frac{\mu a}{b}\ \text{H/m}$$

$$C = \frac{\epsilon}{|V_0|^2}\int_s \bar{E}\cdot\bar{E}^*\,ds$$

$$= \frac{\epsilon}{V_0^2}\int_0^{a}\int_0^{b}\frac{V_0^2}{a^2}\,dx\,dy$$

$$= \frac{\epsilon b}{a}\ \text{F/m}.$$

24) Define voltage reflection coefficient and current reflection coefficient.

Voltage reflection coefficient

$$\Gamma_V = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{V_0^-}{V_0^+}$$

Current reflection coefficient

$$\Gamma_I = \frac{Z_0 - Z_L}{Z_0 + Z_L}$$

27) A long transmission line carries 5kW at 500V to a matched load.

i) What is the reflection coefficient at the load end, when a load of impedance $200+j100\,\Omega$ is connected?

ii) What is the reflection coefficient at the load end when the load is disconnected?

$P = 5kW \quad V = 500V \quad Z_L = 200 + j100\,\Omega$

i) $Z_0 = \frac{V^2}{P} = \frac{(500)^2}{5\times10^3} = 50\,\Omega$

$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{150 + j100}{250 + j100}$

$\Rightarrow \Gamma = 0.655 + 0.137j = 0.67\angle 11.885$

ii) $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = -1$

28) A lossless 75 ohm transmission line is terminated by an impedance of $Z_L = 150 + j150\,\Omega$. The line is 0.375λ long. Find (i) reflection co-efficient Γ_L at the load, (ii) VSWR, (iii) Input impedance Z_{in}, (iv) the shortest length of line for which Z_{in} is purely resistive and (v) the value of this resistance.

Given $Z_0 = 75\,\Omega$, $Z_L = 150 + j150$, $l = 0.375\lambda$.

i) $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75 + j150}{225 + j150} = 0.62\angle 29.76°$

ii) $VSWR = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 4.263$

iii) $Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$

$\beta l = \frac{2\pi}{\lambda} \times 0.375\lambda = 2.356$ rad

$Z_{in} = 75 \times \left[\frac{150 + 150j + j75\tan(2.356)}{75 + j(150 + j150)\tan(2.356)}\right]$

$= 46.495\angle 60.25155$

$= 23.072 + 40.367j$

iv) $V(l) = V_0^+ \left[1 + \Gamma e^{-2j\beta l}\right]$

$= V_0^+ \left[1 + |\Gamma| e^{j(\theta - 2\beta l)}\right]$

$\Gamma = |\Gamma| e^{j\theta}$

$\theta - 2\beta l = 0$

$l = \frac{\theta}{2\beta}$ rad

$\Rightarrow l = 0.0413\lambda$

$$Z_{in} = Z_0 \frac{1+|\Gamma|}{1-|\Gamma|}$$

$$Z_{in\,max} = Z_0 S = 319.725\,\Omega$$

$$\cancel{Z_{in\,min} = \frac{Z_0}{S}}$$

$$\cancel{I(l) = \frac{V_0^+}{Z_0}\left[1 + \Gamma e^{-2j\beta l}\right]}$$

$$\cancel{= \frac{V_0^+}{Z_0}\left[1 - |\Gamma| e^{j(\theta - 2\beta l)}\right]}$$

2013)

6) b) A TWT operates at a beam current $I_0 = 50mA$, beam voltage $V_0 = 2.5kV$, characteristic impedance of helix $Z_0 = 7.75\Omega$, circuit length $N = 45$, and frequency 8GHz. Compute the gain parameter and all four propagation constants.

A) $I_0 = 50mA$, $V_0 = 2.5kV$, $Z_0 = 7.75\Omega$, $N = 45$, $f = 8GHz$.

The travelling-wave tube gain parameter is

$$C = \left(\frac{I_0 Z_0}{4 V_0}\right)^{1/3} = 0.0338 \simeq 0.034$$

The output power gain in dB is

$$A_P = 10\log\left|\frac{V(l)}{V(0)}\right|^2 = -9.54 + 47.3NC$$

$$= -9.54 + 47.3\times45\times0.034$$

$$= 62.5 dB.$$

The 4 propagation constants are:

$$\gamma_1 = -\beta_e C\frac{\sqrt{3}}{2} + j\beta_e\left(1+\frac{C}{2}\right)$$

$$\gamma_2 = \beta_e C\frac{\sqrt{3}}{2} + j\beta_e\left(1+\frac{C}{2}\right)$$

$$\gamma_3 = j\beta_e(1-C)$$

$$\gamma_4 = -j\beta_e\left(1-\frac{C^3}{4}\right)$$

where $\beta_e = \frac{\omega}{v_0}$ → phase constant of velocity modulated electron beam.

$$\frac{mv_0^2}{e} = 2V_0$$

$$\Rightarrow v_0 = \frac{2eV_0}{m} = \frac{2\times(1.6\times10^{-19})\times2.5\times10^3}{9\times10^{-31}}$$

$$= 8.89\times10^{14}$$

$$\beta_e = \frac{\omega}{v_0} = \frac{2\pi f}{v_0} = \frac{2\pi\times8\times10^9}{8.89\times10^{14}} = 5.65\times10^{-5}.$$

The propagation constants are:

$$\gamma_1 = -\beta_e C\frac{\sqrt{3}}{2} + j\beta_e\left(1+\frac{C}{2}\right) = -1.66\times10^{-6} + j5.75\times10^{-5}$$

$\gamma_2 = \beta_e c \frac{\sqrt{3}}{2} + j\beta_e \left(1 + \frac{c}{2}\right)$

$= 1.66 \times 10^{-6} + j5.74 \times 10^{-5}$

$\gamma_3 = j\beta_e (1 - c)$

$= j5.46 \times 10^{-5}$

$\gamma_4 = -j\beta \left(1 - \frac{c^3}{4}\right)$

$= -j5.65 \times 10^{-5}$

Q) A linear magnetron has the following parameters: anode voltage $V_0 = 20kV$, cathode current $I_0 = 17A$, magnetic field $= 0.01 Wb/m^2$, distance between cathode anode $= 5cm$. compute

i) the hull cut-off voltage for a fixed B_z.

ii) the hull cut-off magnetic field for fixed V_0.

A) i) The hull cut-off voltage is

$$V_c = \frac{1}{2}\frac{e}{m} B^2 d^2$$

$$= \frac{1}{2} \times \frac{1.6 \times 10^{-19}}{9.1 \times 10^{-31}} \times (0.01)^2 \times (5 \times 10^{-2})^2$$

$$= 21.978\ kV$$

ii) The hull cutoff magnetic field is

$$B_c = \frac{1}{d}\sqrt{\frac{2m}{e} V_0}$$

$$= \frac{1}{5 \times 10^{-2}} \left(\sqrt{\frac{2 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}}} \times 20 \times 10^3\right)$$

$$= 1.349\ wb/m^2$$

2) a) A rectangular waveguide is designed to propagate the dominant mode at a frequency of 5GHz. The cutoff frequency is 0.8 times the signal frequency. The ratio of the waveguide height to width is 2. The time average power flowing through the

guide is 1kW. Determine the magnitude of electric and magnetic intensities in the guide and indicate where these occur in the guide.

A) The dominant mode is TE_{10}.

Given $f = 5\,GHz$, $f_c = 0.8f = 4\,GHz$.

$\frac{a}{b} = 2$, $P_{av} = 10^3$ Watt

Wave impedance, $Z_{TE} = \dfrac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \dfrac{377}{\sqrt{1-(0.8)^2}}$

$= 628.33\,\Omega$

In TE_{10} mode, $E_x = 0$

$$E_y = E_0 \sin\frac{\pi x}{a} e^{-j\beta z}$$

$$P_{av} = \frac{1}{2Z_g}\int_0^a\int_0^b \left[|E_x|^2 + |E_y|^2\right] dx\,dy$$

$$= \frac{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{2\eta}\int_0^a\int_0^b E_0^2 \sin^2\frac{\pi x}{a}\,dx\,dy$$

$$\Rightarrow 1000 = \frac{\sqrt{1-(0.8)^2}}{2\times 377} \times E_0^2 \times ab$$

$$= 1.59\times10^{-3} \times E_0^2 \times 2b^2$$

For TE_{10} mode, $f_c = \frac{c}{2a}$

$$\Rightarrow 4\times10^9 = \frac{3\times10^8}{2\times a}$$

$$\Rightarrow a = 0.0375 = 3.75\,cm$$

$$b = 0.01875 = 1.875\,cm$$

$\therefore$ ~~$E_0 = \sqrt{\dfrac{1000}{1.59\times10^{-3}\times2\times(1.875)^2}} = 289.24$~~

$$\therefore E_0 = \sqrt{\frac{1000}{1.59\times10^{-3}\times2\times(1.875\times10^{-2})^2}} =$$

$$\Rightarrow 1000 = \frac{\sqrt{1-(0.8)^2}}{2\times 377} \times E_0^2 \times ab$$

$$\Rightarrow 1000 = 7.96\times10^{-4} \times E_0^2 \times 2b^2$$

$$\Rightarrow E_0 = \sqrt{\frac{1000}{7.96\times10^{-4}\times 2\times b^2}}$$

For TE_{10} mode, $f_c = \frac{c}{2a}$

$$\Rightarrow 4\times10^{9} = \frac{3\times10^{8}}{2a}$$

$$\Rightarrow a = 0.0375 = 3.75\text{cm}$$

$$b = 1.875\text{cm}$$

$$\therefore E_0 = \sqrt{\frac{1000}{7.96\times10^{-4}\times2\times(1.875\times10^{-2})^2}}$$

$$= 42.269\ \text{kV/m}$$

$$E_y = E_0 \sin\frac{\pi x}{a} e^{-j\beta z}$$

$$\Rightarrow E_y = 42.269\times10^{3} \sin\frac{\pi x}{a} e^{-j\beta z}$$

$$\frac{E_y}{H_x} = Z_{TE} \Rightarrow H_x = \frac{E_y}{Z_{TE}} = \frac{E_y}{628.33}$$

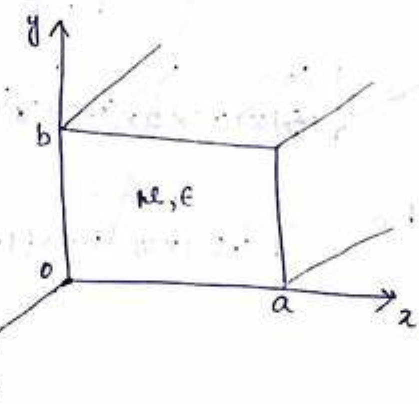
$$\therefore H_x = 67.3 \sin\frac{\pi x}{a} e^{-j\beta z}$$

b) Derive the electric and magnetic field intensities that you have used in the above problem. Write down two inferences of yours.

A) Assume a rectangular waveguide shown below: Filled with a material having permitivity ϵ & permeability μ.

& $a > b$.

The TE modes are characterised by fields with $E_z = 0$, while



H_z must satisfy the reduced wave eq.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) h_z(x,y) = 0 \text{ —— } ①$$

with $H_z(x,y,z) = h_z(x,y)e^{-j\beta z}$

& $k_c^2 = k^2 - \beta^2 \rightarrow$ cutoff wave number.

Substituting $h_z(x,y) = X(x)\,Y(y)$ in eq ①,

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + k_c^2 = 0$$

Putting $k_c^2 = k_x^2 + k_y^2$ where k_x & k_y are separation constants.

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + (k_x^2 + k_y^2) = 0$$

By usual separation of variables argument,

$$\frac{d^2X}{dx^2} + k_x^2 X = 0$$

$$\frac{d^2Y}{dy^2} + k_y^2 Y = 0$$

The general solⁿ for h_z can be written as

$$h_z(x,y) = (A\cos k_x x + B\sin k_x x)(C\cos k_y y + D\sin k_y y) \text{ — } ②$$

The boundary conditions are

$e_x(x,y) = 0$, at $y = 0, b$

$e_y(x,y) = 0$, at $x = 0, a$.

The general eq's of electric field components at TE mode are:

$$e_x = \frac{-j\omega\mu}{k_c^2}\frac{dH_z(x,y,z)}{dy}$$

$$= \frac{-j\omega\mu}{k_c^2}k_y\left[(A\cos k_x x + B\sin k_x x)(-C\sin k_y y + D\cos k_y y)\right] \quad ③$$

$$e_y = \frac{j\omega\mu}{k_c^2}\frac{dH_z(x,y,z)}{dx}$$

$$= \frac{-j\omega\mu}{k_c^2}k_x\left[(-A\sin k_x x + B\cos k_x x)(C\cos k_y y + D\sin k_y y)\right] \quad ④$$

Comparing boundary conditions with respective field eq.s, we see that

$$D=0 \text{ and } k_y = \frac{n\pi}{b} \quad \text{for } n=0,1,2\ldots$$

$$B=0 \text{ and } k_x = \frac{m\pi}{a} \quad \text{for } m=0,1,2\ldots$$

Now the final sol^n is

$$H_z(x,y,z) = A_{mn}\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z} \quad \text{— (5)}$$

where $A_{mn} \rightarrow$ arbitary constant.

The transverse field components of the TE_{mn} mode can be found by using all above eq^{n}s are:

$$E_x = \frac{j\omega\mu n\pi}{k_c^2 b}A_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}$$

$$E_y = \frac{-j\omega\mu m\pi}{k_c^2 a}A_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$$

$$H_x = \frac{j\beta m\pi}{k_c^2 a}A_{mn}\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}e^{-j\beta z}$$

$$H_y = \frac{j\beta n\pi}{k_c^2 b}A_{mn}\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}e^{-j\beta z}.$$

For dominant TE_{10} mode, $m=1$, $n=0$,

$$H_z = A_{10}\cos\frac{\pi x}{a}e^{-j\beta z}$$

$$E_y = \frac{-j\omega\mu a}{\pi}A_{10}\sin\frac{\pi x}{a}e^{-j\beta z}$$

$$H_x = \frac{j\beta a}{\pi}A_{10}\sin\frac{\pi x}{a}e^{-j\beta z},$$

$$E_x = E_z = H_y = 0.$$

where $k_c = \pi/a$

$$\beta = \sqrt{k^2 - (\pi/a)^2}.$$

2013) (1)

6) b) A TWT operates at a beam current $I_0 = 50mA$, beam voltage $V_0 = 2.5kV$, characteristic impedance of helix $Z_0 = 7.75\Omega$, circuit length $N = 45$, and frequency 8GHz. Compute the gain parameter and all four propagation constants.

A) $I_0 = 50mA$, $V_0 = 2.5kV$, $Z_0 = 7.75\Omega$, $N = 45$, $f = 8GHz$.

The travelling-wave tube gain parameter is

$$C = \left(\frac{I_0 Z_0}{4 V_0}\right)^{1/3} = 0.0338 \approx 0.034$$

The output power gain in dB is

$$A_P = 10 \log \left|\frac{V(l)}{V(0)}\right|^2 = -9.54 + 47.3NC$$

$$= -9.54 + 47.3 \times 45 \times 0.034$$

$$= 62.5 dB.$$

The 4 propagation constants are:

$$\gamma_1 = -\beta_e C \frac{\sqrt{3}}{2} + j\beta_e\left(1 + \frac{C}{2}\right)$$

$$\gamma_2 = \beta_e C \frac{\sqrt{3}}{2} + j\beta_e\left(1 + \frac{C}{2}\right)$$

$$\gamma_3 = j\beta_e (1 - C)$$

$$\gamma_4 = -j\beta_e\left(1 - \frac{C^3}{4}\right)$$

where $\beta_e = \frac{\omega}{v_0} \rightarrow$ phase constant of velocity modulated electron beam.

$$\frac{m v_0^2}{e} = 2V_0$$

$$\Rightarrow v_0 = \frac{2eV_0}{m} = \frac{2 \times (1.6 \times 10^{-19}) \times 2.5 \times 10^3}{9 \times 10^{-31}}$$

$$= 8.89 \times 10^{14}$$

$$\beta_e = \frac{\omega}{v_0} = \frac{2\pi f}{v_0} = \frac{2\pi \times 8 \times 10^9}{8.89 \times 10^{14}} = 5.65 \times 10^{-5}$$

The propagation constants are:

$$\gamma_1 = -\beta_e C \frac{\sqrt{3}}{2} + j\beta_e\left(1 + \frac{C}{2}\right) = -1.66 \times 10^{-6} + j5.75 \times 10^{-5}$$

$$\gamma_2 = \beta_e c \frac{\sqrt{3}}{2} + j\beta_e \left(1 + \frac{c}{2}\right)$$

$$= 1.66 \times 10^{-6} + j5.74 \times 10^{-5}$$

$$\gamma_3 = j\beta_e (1 - c)$$

$$= j5.46 \times 10^{-5}$$

$$\gamma_4 = -j\beta \left(1 - \frac{c^3}{4}\right)$$

$$= 0 - j5.65 \times 10^{-5}$$

Q) A linear magnetron has the following parameters: anode voltage $V_0 = 20\,kV$, cathode current $I_0 = 17\,A$, magnetic field $= 0.01\,Wb/m^2$, distance between cathode anode $= 5\,cm$. compute

i) the hull cut-off voltage for a fixed B_z.

ii) the hull cut-off magnetic field for fixed V_0.

A) i) The hull cut-off voltage is

$$V_c = \frac{1}{2} \frac{e}{m} B^2 d^2$$

$$= \frac{1}{2} \times \frac{1.6 \times 10^{-19}}{9.1 \times 10^{-31}} \times (0.01)^2 \times (5 \times 10^{-2})^2$$

$$= 21.978\,kV$$

ii) The hull cutoff magnetic field is

$$B_c = \frac{1}{d} \sqrt{\frac{2m}{e} V_0}$$

$$= \frac{1}{5 \times 10^{-2}} \left(\sqrt{\frac{2 \times 9.1 \times 10^{-31}}{1.6 \times 10^{-19}}} \times 20 \times 10^3 \right)$$

$$= 1.349\, wb/m^2$$

2) a) A rectangular waveguide is designed to propagate the dominant mode at a frequency of 5GHz. The cutoff frequency is 0.8 times the signal frequency. The ratio of the waveguide height to width is 2. The time average power flowing through the

guide is 1kW. Determine the magnitude of electric and magnetic intensities in the guide and indicate where these occur in the guide.

(3)

A) The dominant mode is TE_{10}.

Given $f = 5\,GHz$, $f_c = 0.8f = 4\,GHz$.

$\frac{a}{b} = 2$, $P_{av} = 10^3$ Watt

Wave impedance, $Z_{TE} = \frac{\eta}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{377}{\sqrt{1-(0.8)^2}}$

$= 628.33\,\Omega$

In TE_{10} mode, $E_x = 0$

$$E_y = E_0 \sin\frac{\pi x}{a} e^{-j\beta z}$$

$$P_{av} = \frac{1}{2Z_g}\int_0^a\int_0^b \left[|E_x|^2 + |E_y|^2\right] dx\,dy$$

$$= \frac{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}{2\eta}\int_0^a\int_0^b E_0^2 \sin^2\frac{\pi x}{a}\,dx\,dy$$

$\Rightarrow 1000 = \frac{\sqrt{1-(0.8)^2}}{2\times 377} \times E_0^2 \times ab$

$= 1.59\times10^{-3} \times E_0^2 \times 2b^2$

For TE_{10} mode, $f_c = \frac{c}{2a}$

$\Rightarrow 4\times10^9 = \frac{3\times10^8}{2\times a}$

$\Rightarrow a = 0.0375 = 3.75\,cm$

$b = 0.01875 = 1.875\,cm$

~~$\therefore E_0 = \sqrt{\frac{1000}{1.59\times10^{-3}\times2\times(1.875)^2}} = 229.24$~~

$\therefore E_0 = \sqrt{\frac{1000}{1.59\times10^{-3}\times2\times(1.875\times10^{-2})^2}} =$

4

$$\Rightarrow 1000 = \frac{\sqrt{1-(0.8)^2}}{2\times 377} \times E_0^2 \times ab$$

$$\Rightarrow 1000 = 7.96\times10^{-4} \times E_0^2 \times 2b^2$$

$$\Rightarrow E_0 = \sqrt{\frac{1000}{7.96\times10^{-4}\times 2\times b^2}}$$

For TE_{10} mode, $f_c = \frac{c}{2a}$

$$\Rightarrow 4\times10^9 = \frac{3\times10^8}{2a}$$

$$\Rightarrow a = 0.0375 = 3.75\,\text{cm}$$

$$b = 1.875\,\text{cm}$$

$$\therefore E_0 = \sqrt{\frac{1000}{7.96\times10^{-4}\times2\times(1.875\times10^{-2})^2}}$$

$$= 42.269\,\text{kV/m}$$

$$E_y = E_0 \sin\frac{\pi x}{a} e^{-j\beta z}$$

$$\Rightarrow E_y = 42.269\times10^3 \sin\frac{\pi x}{a} e^{-j\beta z}$$

$$\frac{E_y}{H_x} = Z_{TE} \Rightarrow H_x = \frac{E_y}{Z_{TE}} = \frac{E_y}{628.33}$$

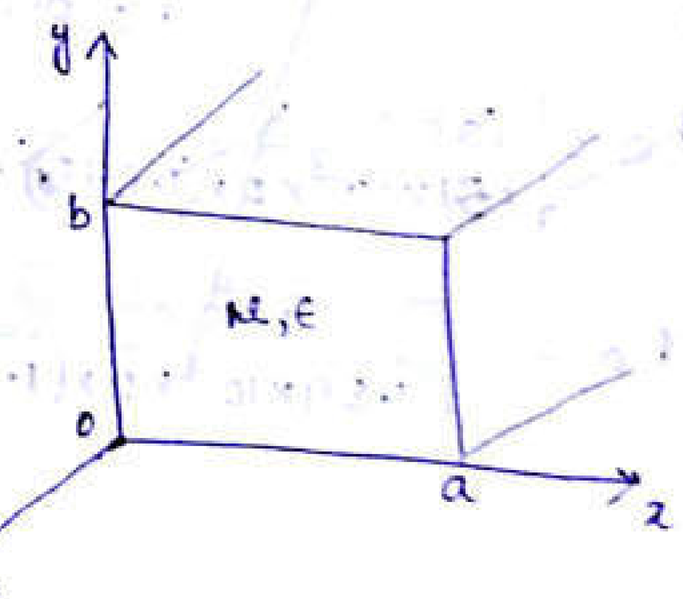
$$\therefore H_x = 67.3 \sin\frac{\pi x}{a} e^{-j\beta z}$$

b) Derive the electric and magnetic field intensities that you have used in the above problem. Write down two inferences of yours.

A) Assume a rectangular waveguide shown below:

Filled with a material having permitivity ϵ & permeability μ.

& $a > b$.



The TE modes are characterised by fields with $E_z = 0$, while

H_z must satisfy the reduced wave eq.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_c^2\right) h_z(x,y) = 0 \quad \text{--- (1)}$$

(5)

with $H_z(x,y,z) = h_z(x,y) e^{-j\beta z}$

& $k_c^2 = k^2 - \beta^2 \rightarrow$ cutoff wave number.

Substituting $h_z(x,y) = X(x)\,Y(y)$ in eq (1),

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + k_c^2 = 0$$

Putting $k_c^2 = k_x^2 + k_y^2$ where k_x & k_y are separation constants.

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} + (k_x^2 + k_y^2) = 0$$

By usual separation of variables argument,

$$\frac{d^2X}{dx^2} + k_x^2 X = 0$$

$$\frac{d^2Y}{dy^2} + k_y^2 Y = 0$$

The general sol^n for h_z can be written as

$$h_z(x,y) = (A\cos k_x x + B\sin k_x x)(C\cos k_y y + D\sin k_y y) \quad \text{--- (2)}$$

The boundary conditions are

$e_x(x,y) = 0$, at $y = 0, b$

$e_y(x,y) = 0$, at $x = 0, a$.

The general eq's of electric field components at TE mode are:

$$e_x = \frac{-j\omega\mu}{k_c^2}\frac{dH_z(x,y,z)}{dy}$$

$$= \frac{-j\omega\mu}{k_c^2} k_y \left[(A\cos k_x x + B\sin k_x x)(-C\sin k_y y + D\cos k_y y)\right] \quad \text{(3)}$$

$$e_y = \frac{j\omega\mu}{k_c^2}\frac{dH_z(x,y,z)}{dx}$$

$$= \frac{-j\omega\mu}{k_c^2} k_x \left[(-A\sin k_x x + B\cos k_x x)(C\cos k_y y + D\sin k_y y)\right] \quad \text{(4)}$$

Comparing boundary conditions with respective field eq^{n}s, we see that

(6)

$$D = 0 \text{ and } k_y = \frac{n\pi}{b} \text{ for } n = 0, 1, 2 \ldots$$

$$B = 0 \text{ and } k_x = \frac{m\pi}{a} \text{ for } m = 0, 1, 2 \ldots$$

Now the final sol^n is

$$H_z(x, y, z) = A_{mn} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z} \quad \text{— (5)}$$

where $A_{mn} \rightarrow$ arbitary constant.

The transverse field components of the TE_{mn} mode can be found by using all above eq^{n}s are:

$$E_x = \frac{j\omega\mu n\pi}{k_c^2 b} A_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}$$

$$E_y = \frac{-j\omega\mu m\pi}{k_c^2 a} A_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z}$$

$$H_x = \frac{j\beta m\pi}{k_c^2 a} A_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} e^{-j\beta z}$$

$$H_y = \frac{j\beta n\pi}{k_c^2 b} A_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} e^{-j\beta z}$$

For dominant TE_{10} mode, $m = 1$, $n = 0$,

$$H_z = A_{10} \cos\frac{\pi x}{a} e^{-j\beta z}$$

$$E_y = \frac{-j\omega\mu a}{\pi} A_{10} \sin\frac{\pi x}{a} e^{-j\beta z}$$

$$H_x = \frac{j\beta a}{\pi} A_{10} \sin\frac{\pi x}{a} e^{-j\beta z},$$

$$E_x = E_z = H_y = 0.$$

where $k_c = \pi/a$

$$\beta = \sqrt{k^2 - (\pi/a)^2}.$$

3) a) In free space, the magnetic field is given as $H = 1.33\cos(8\times10^8 t - \beta z)\, a_x$ A/m. Give the expressions for the corresponding electric field, the propagation constant and the wavelength. Compute also the power flow per unit area.

7

A) We know, $\frac{E_y}{H_x} = \eta = 377$ (for air)

$$\Rightarrow E_y = \eta H_x = 377 \times 1.33 \cos(8\times10^8 t - \beta z)$$

$$\Rightarrow E = 501.41 \cos(8\times10^8 t - \beta z)\, a_y \text{ V/m.}$$

$$\beta = \frac{\omega}{c} = \frac{8\times10^8}{3\times10^8} = 2.67 \text{ rad/s.}$$

$$\lambda = \frac{2\pi}{\beta} = 2.35 \text{ m.}$$

The power flow is

$$P = \frac{1}{2} \oint E \times H$$

$$= \frac{1}{2} \iint 377 \times \left[1.33 \cos(8\times10^8 t - \beta z)\right]^2 dx\, dy.$$

(8)

b) Derive an expression for the Q-factor of a rectangular cavity resonator.

A) A cavity resonator is a space normally bounded by conducting surface (short circuit both ends) in which electromagnetic energy is stored as oscillating fields.

<u>Quality Factor (Q)</u>: It is a measure of frequency selectivity of resonant and non-resonant circuit.

$$Q = \frac{\text{Max}^m \text{ energy stored}}{\text{Energy dissipated per cycle}}$$

$$= \frac{\omega_r W_{e\&m}}{P_{avg}}.$$

At resonant frequency,

$$W_e = W_m$$

$$= \frac{\epsilon}{4}\int_V |E|^2 \cdot dv = \frac{\mu}{4}\int_V |H|^2 dv$$

The avg. power loss,

$$P = \frac{R_s}{2}\int_S |H_t|^2 ds.$$

where, $|H|^2 = |H_t|^2 + |H_n|^2$

$$Q = \frac{\omega\mu \int_V |H|^2 \cdot dv}{2R_s \int_S |H_t|^2 ds} = \frac{\omega\mu}{2R_s}$$

In half-power bandwidth or 3dB bandwidth can be calculated as

$$Q_0 = \frac{f_r \text{ (resonant freq.)}}{\Delta f \text{ (half power or 3dB freq.)}}$$

<u>Unloaded Quality Factor (Q_0)</u>: When a cavity resonator is not connected with any external circuit or load; then the quality factor is

said to be unloaded. It is related to internal losses like conductor losses, dielectric losses and radiation losses.

For rectangular cavity resonator having dimension a,b,d :

$$Q_0 = \frac{\pi\eta}{4R_s} \times \frac{2b(a^2+d^2)}{ad(a^2+d^2)+2b(a^3+d^3)}$$

Q_0 will be maximum at a=d;

$$Q_{0max} = \frac{\pi\eta}{R_s} \times \frac{1}{1+\frac{a}{2b}}$$

External Quality Factor (Q_{ext}) : It is associated with only external losses (i.e. reflection losses due to impedance matching) occured due to presence of the external load.

$$Q_{ext} = \frac{Q_0}{K}$$

where k = coupling factor.

k=1 for critical coupling.

k<1 & $k = \frac{1}{SWR}$ for under coupling.

k>1 & k = SWR for over coupling.

$$Q_L = \frac{Q_0}{1+k}$$

$$\therefore \frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}}$$

where Q_L = loaded Quality factor.

5) a) An identical two-cavity klystron amplifier operates at 4GHz with $V_0 = 1kV$, $I_0 = 22mA$, gap 1mm, drift space 3cms. If dc beam transconductance and catcher cavity total equivalent conductance are 0.25×10^{-3} ℧s and 0.3×10^{-4} ℧s respectively, calculate

(i) the beam coupling coefficient, dc transit angle in drift space and the input cavity voltage magnitude for maximum output voltage and

(ii) voltage gain and efficiency, neglecting the beam loading.

A) i) The beam coupling coefficient is

$$\beta_1 = \frac{\sin \theta_g/2}{\theta_g/2}$$

where $\theta_g = \frac{\omega t_g}{}$

$$= \frac{\omega d}{u_0} = \frac{2\pi f d}{\sqrt{\frac{2eV_0}{m}}} = \frac{2\pi\times4\times10^{9}\times1\times10^{-3}}{1.88\times10^{7}\,(m/s)}$$

$$= 1.34\,\text{rad} = 76.77^\circ$$

$$\beta_1 = \frac{\sin\left(\frac{76.77}{2}\right)}{\left(\frac{1.34}{2}\right)} = 0.94$$

The dc transit angle in drift space is

$$\theta_0 = \frac{\omega L}{u_0} = \frac{2\pi f L}{1.88\times10^{7}}$$

$$= \frac{2\pi\times4\times10^{9}\times3\times10^{-2}}{1.88\times10^{7}} = 40.11\,\text{rad}$$

Cavity gap voltage magnitude:

$$X = \frac{\pi N \beta_1 V_1}{V_0} \qquad X = \frac{\pi N \beta_1 V_1}{V_0} \qquad V_1 = \frac{2V_0 X}{\beta_1 \theta_0}$$

$$t_0 = \frac{2\pi N}{\omega}$$

$$\Rightarrow \frac{\omega t_0}{2} = \pi N$$

$$\Rightarrow \frac{\theta_0}{2} = \pi N$$

$$\Rightarrow V_1 = \frac{2 V_0 X}{\beta_1 \times \theta_0}$$

$$= \frac{2 \times 1 \times 10^3 \times 1.841}{0.927 \times 40.11}$$

$= 99.02\ V$ [For identical two cavity klystron,

$X = 1.841$

& $J_1(X) = 0.582$]

ii) Voltage gain :

$$A_V = \frac{\beta^2 \theta_0 J_1(X) I_0}{X V_0 G_{sh}} = \frac{(0.927)^2 \times 40.11 \times 0.582 \times 22 \times 10^{-3}}{1.841 \times 1 \times 10^3 \times (0.25 + 0.3) \times 10^{-4}}$$

$= 4.36$

$(A_V)_{dB} = 20 \log(A_V) = 12.8\ dB.$

catcher voltage $= V_2 = A_V \times V_1$

$= 431.6\ V$

Efficiency, $\eta = \frac{P_{RF}}{P_{dc}} = \frac{\beta_1 I_0 J_1(X) V_2}{I_0 V_0}$

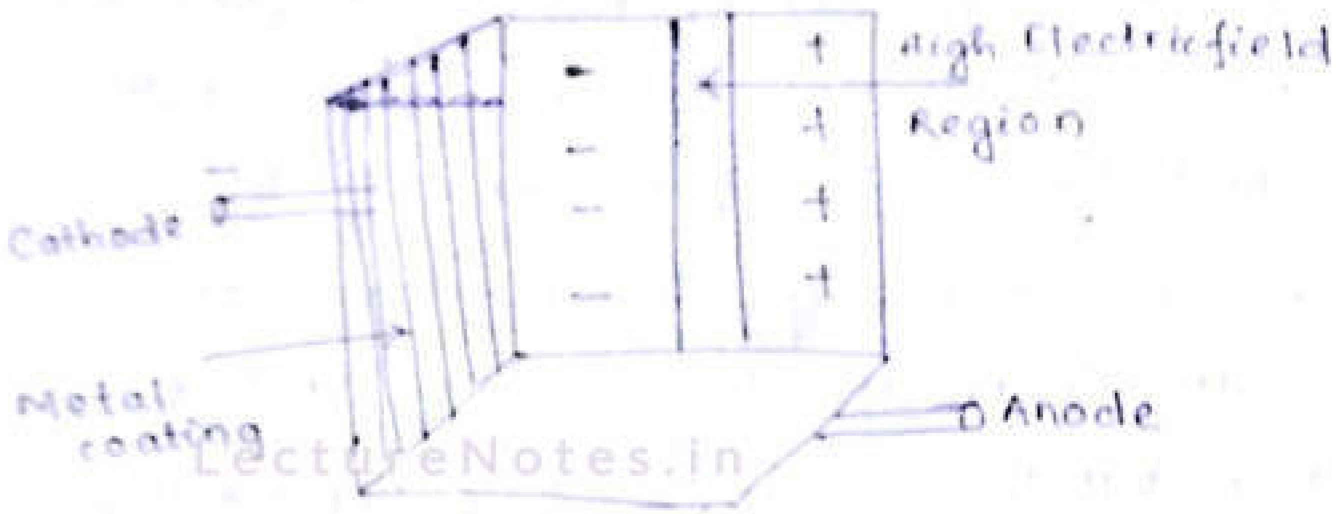
$$= \frac{\beta_1 J_1(X) V_2}{2 V_0} = 23.28\ \%.$$

b) A reflex klystron is operating at 4GHz with the following parameters $V_R = 2kV$, $V_0 = 500V$, $L = 2cm$, $n = 0$. Calculate the change in frequency for a 4% change in repeller voltage.

A) Power output :

$P_{RF} = V.$

7) a) Explain how a Gunn diode is used as an oscillator with the development of appropriate expressions and sketches. (12)



(n-type GaAs as Gunn)

- If a voltage of say about 50V is applied to a thin slice of GaAs, then -ve resistance will be encountered under certain conditions.
- Basically, these consist merely in applying a voltage gradient across the slice in excess of about 3,400 V/cm. Oscillations occur across the slice if connected to a suitably tuned circuit.
- Thus we see that the voltage gradient across the slice of GaAs is very high, giving rise to high electron velocity and so oscillations that occur are at microwave frequencies, with a cavity normally being used as a tuned circuit.
- It leads to continuous oscillation.

i) The frequency of oscillation depends on GaAs material

ii) Also this period of oscillation is inversely proportional to length of device

$$T \propto \frac{1}{l}$$

where $T \rightarrow$ Period of oscillation

$l \rightarrow$ length of GaAs.

So Gunn diode is used as oscillator with frequency

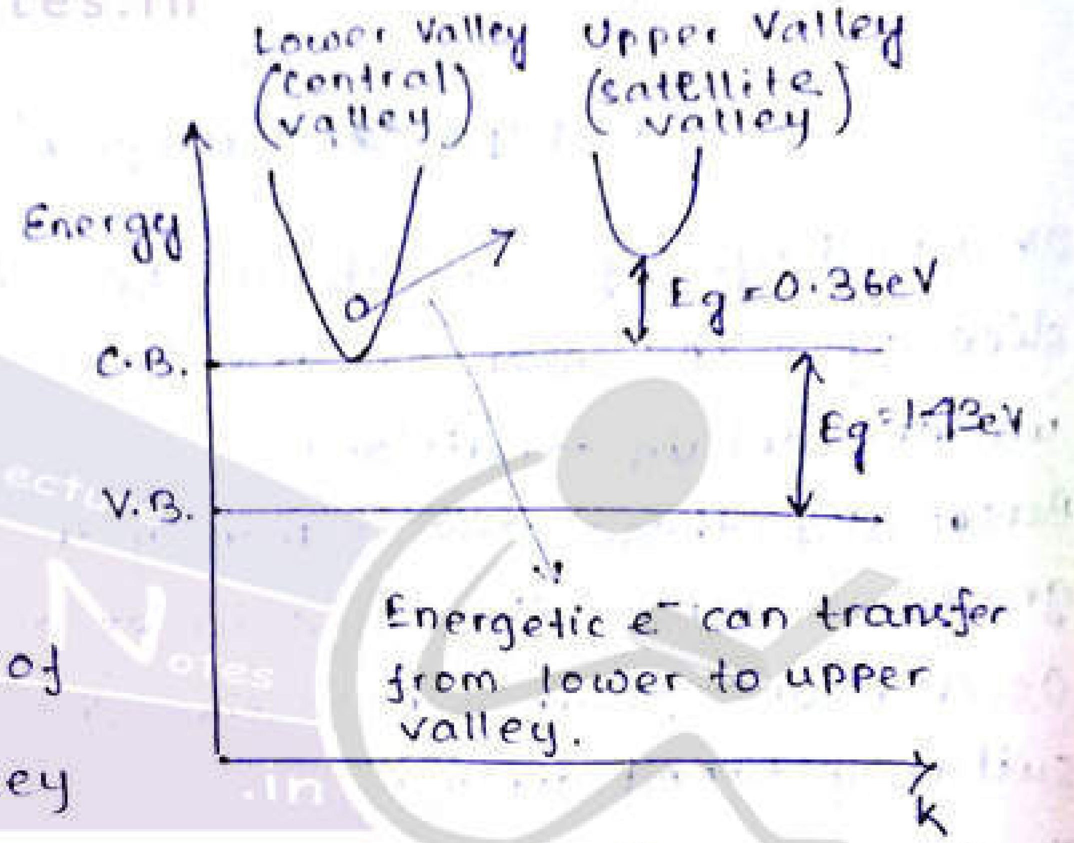
$$f = \frac{V_d}{L}$$

where standard value of drift velocity (V_d) $= 10^7 m/s$

L → effective length.

Q) Explain the principle of operation of Gunn diode using two valley model theory. Give constructional details and electrical equivalent circuit.

- According to the energy-band theory of n-type GaAs, there are two valleys in the conduction band.



- The effective mass of electron is given by,

$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

where $\frac{d^2E}{dk^2}$ → Rate of change of the valley curves slope.

- Since, lower valley slope is sharper than the one in upper valley, thus electron effective mass in lower valley is lower than that in upper valley. So, the mobility of electron in upper valley is less due to the higher effective mass.
- Electron densities in the lower and upper valleys remain the same under an equilibrium condition.
- When the applied electric field is lower than the electric field of the lower valley ($E < E_L$), no electrons will transfer to the upper valley.
- When $E_L < E < E_U$, electrons will begin to transfer to the upper valley.

- When $E_u < E$, all electrons will transfer to the upper valley.
- A/c to Ridley - Watkins - Hilsum TVM Theory, the band structure of a semiconductor must satisfy the following 3 criteria in order to exhibit -ve conductance.

i) The separation energy between the bottom of the lower valley and the bottom of the upper valley must be several times larger than the thermal energy at the room temperature which shall mean $\Delta E > kT$.

ii) The separation energy between the valleys must smaller than the band-gap energy between conduction and valence bands, which shall mean $\Delta E < E_g$. Otherwise, the semiconductor will breakdown to become high conductive device before the electron can be transferred.

iii) Electron in the lower valley must have high mobility, small effective mass and a low density of state, where as those in upper valley must have low mobility, high effective mass and high density of state. In other word, electro velocity dE/dk must be larger in lower valley than in upper valley.

Q) Determine the conductivity of the n-type GaAs Gunn diode for the given parameters: Electron density at lower valley $n_l = 10^{10}\ cm^{-3}$, upper valley $n_u = 10^8 cm^{-3}$, and temp. $T = 300^\circ K$.

A) Conductivity:

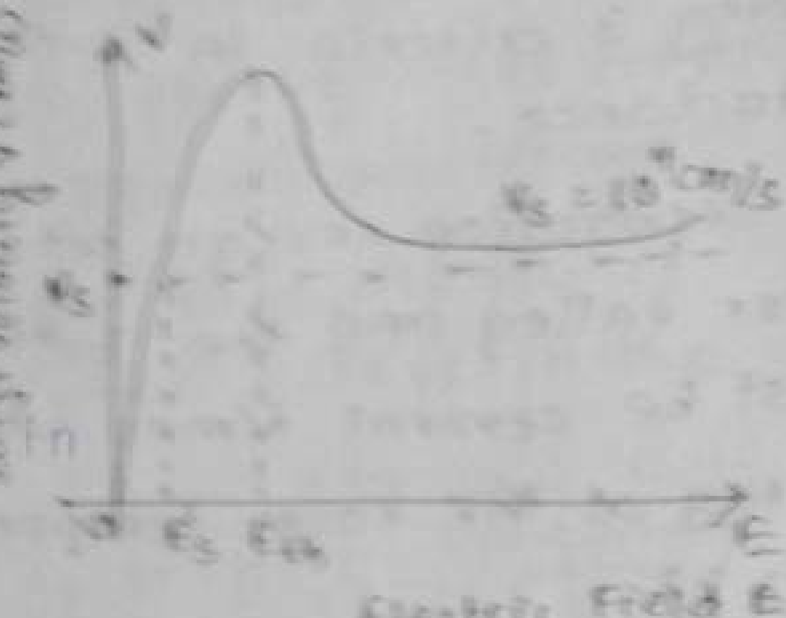
$$\sigma = e(\mu_l n_l + \mu_u n_u)$$

$= 1.6\times10^{-19}(8000\times10^{?} + 180\times10^{?})$ $\left[\because \mu_e = 8000\ cm^2/Vs,\ \mu_h = 180\ cm^2/Vs\right]$

$= 1.28\times10^{-5}$ ℧

Q) Short Note on Gunn Oscillation Mode.

A) If $E > E_{th}$, the high field domain drifts along the specimen until it reaches the anode or until the low-field value drops below the sustaining field E_s required to maintain v_s as shown above.



- Since the electron drift velocity v varies with E, there are 3 possible modes.
  1. Transit time domain mode.
  2. Delay domain mode.
  3. Quenched domain mode.

1. Transit time domain mode:

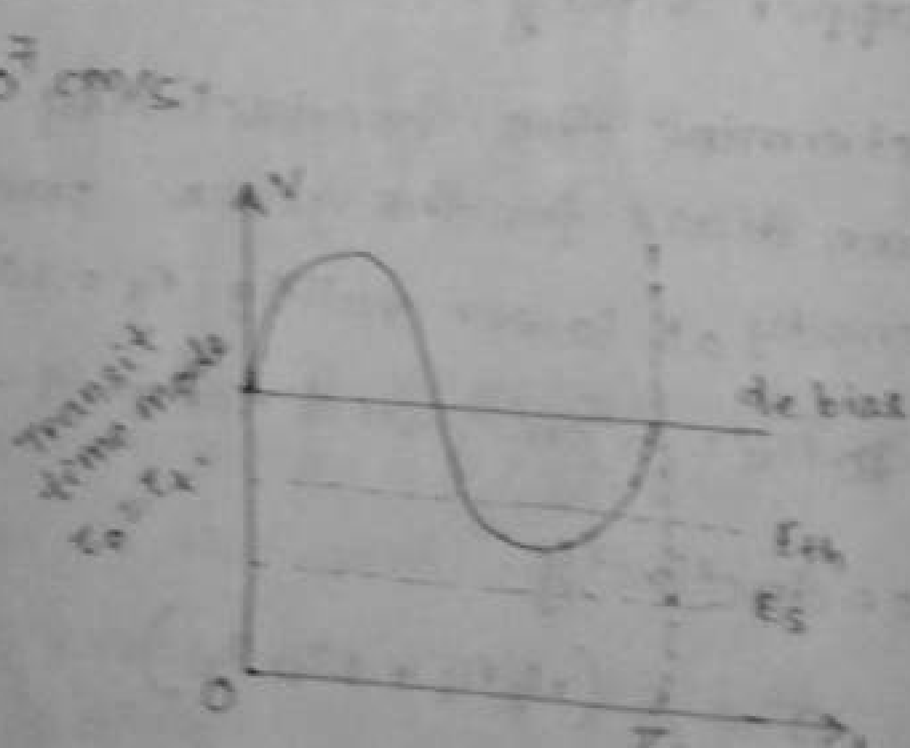
- Operating frequency (f) × length (L) = 10^7 cm/s.
- Doping (n_0) × length (L) $> 10^{12}$ cm^{-2}
- When drift velocity (v_d) = sustaining velocity (v_s) high field domain is stable.

  $\therefore v_d = v_s = fL \simeq 10^7$ cm/s.

- Oscillation period is equal to transit time

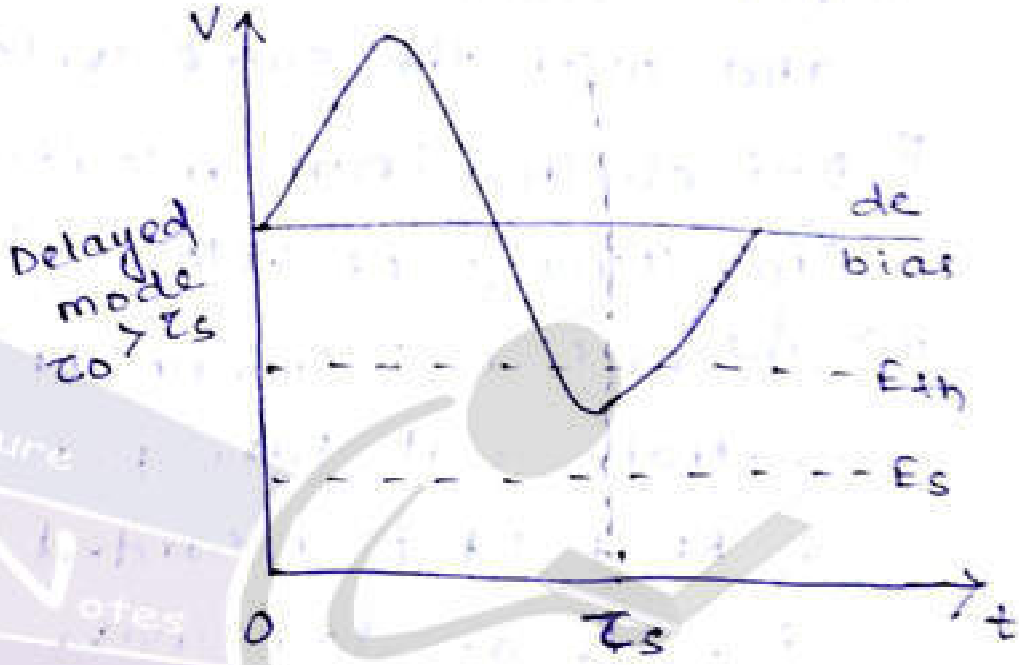
  $\tau = \tau_t = \dfrac{1}{f_{resonant}}$

- Efficiency < 10%.



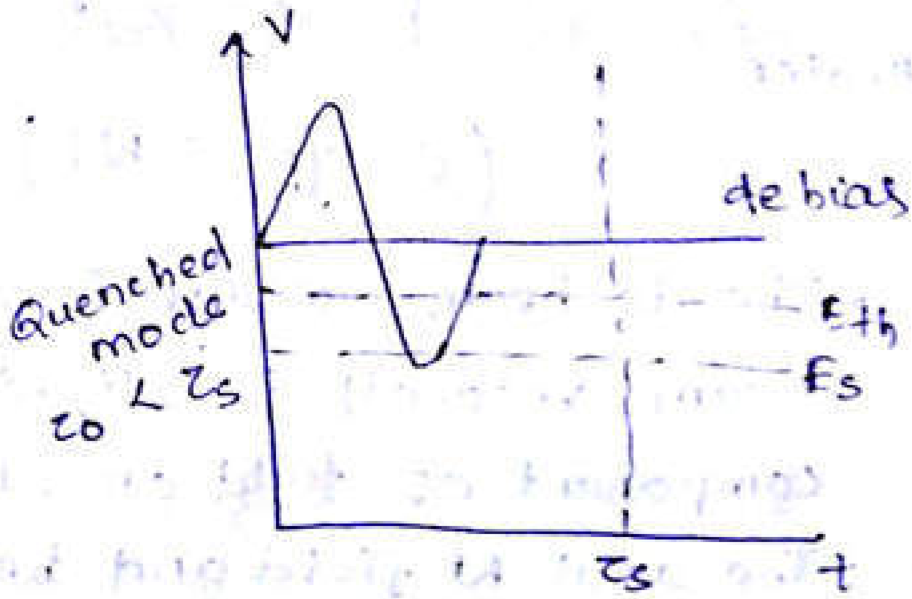
2) Delayed Domain Mode:

- $10^6 cm/s < fL < 10^7 cm/s$.
- It is also known as inhibited mode.
- When the transit time is choosen so that the domain is collected while $E < E_{th}$, a new domain can't form until the field rises above threshold again.
- The oscillation period is greater than transit time.
- There is an ohmic current higher than domain current.
- Efficiency is upto 20%.



3) Quenched Domain Mode:

- $10^7 cm/s < fL < 10^8 cm/s$
- If $E < E_s$, domain collapses before it reaches the anode. When the bias field swings back above the threshold, a new domain is nucleated and the process repeats. Thus oscillation occurs at the frequency of resonant circuit rather than at the transit frequency.
- The domain can be quenched before it is collected.
- Efficiency can reach 13%.



Q) Write a short note on working principle of TWT amplifier. (17)

A) TWT (Travelling Wave Tube) is a microwave amplifier which makes use of distributed interaction between an electron beam and a travelling tube.

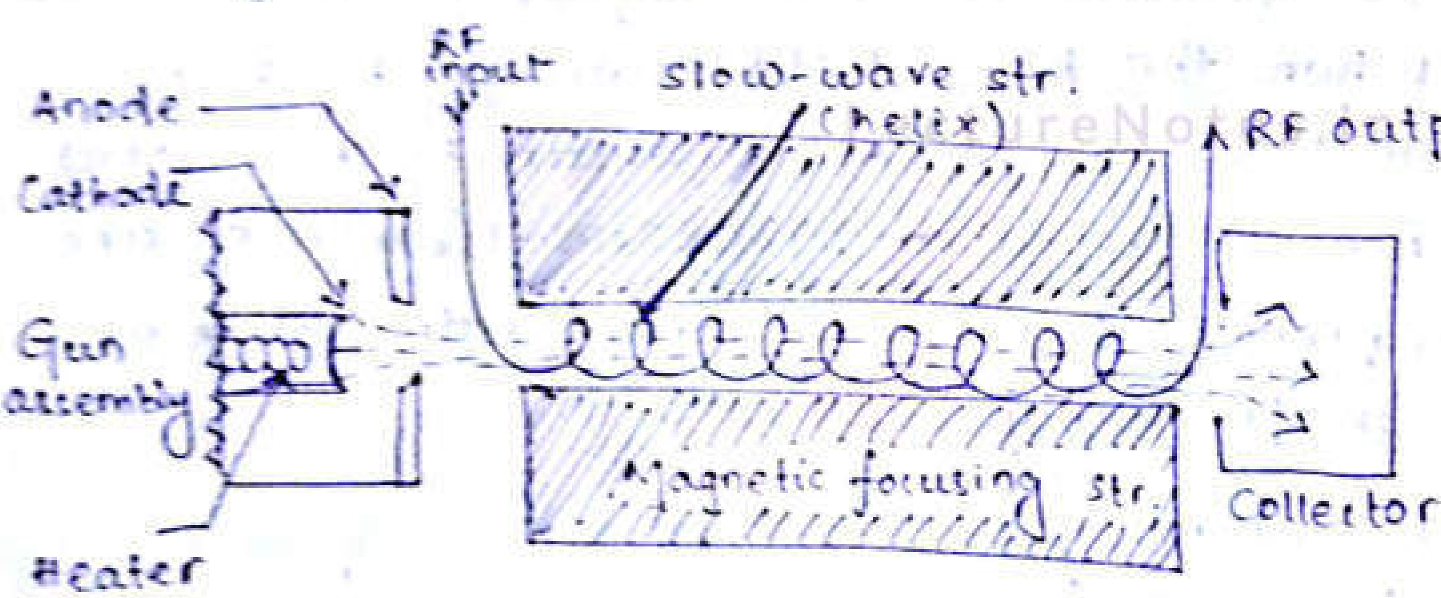
- Necessary condition for continuous interaction between electron beam and RF field:

i) Both should travel in the same direction and with the same velocity.

ii) But electron beam velocity = 0.1 × c and Travelling field velocity = c.

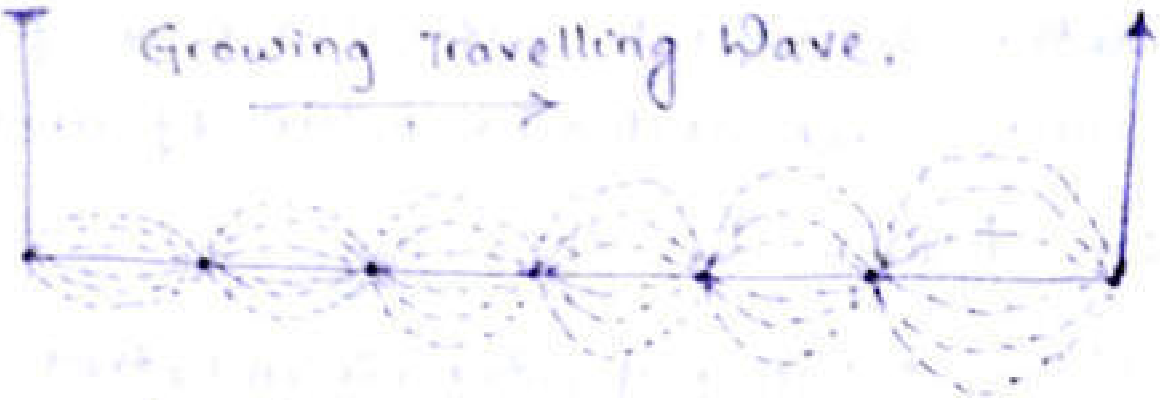
iii) Interaction between RF field and moving electrons will take place when velocity of RF field is retarded by some means (i.e. use of slow wave structure.).

Q) Briefly explain the amplification process of O-type Travelling Wave Tube with proper diagram.

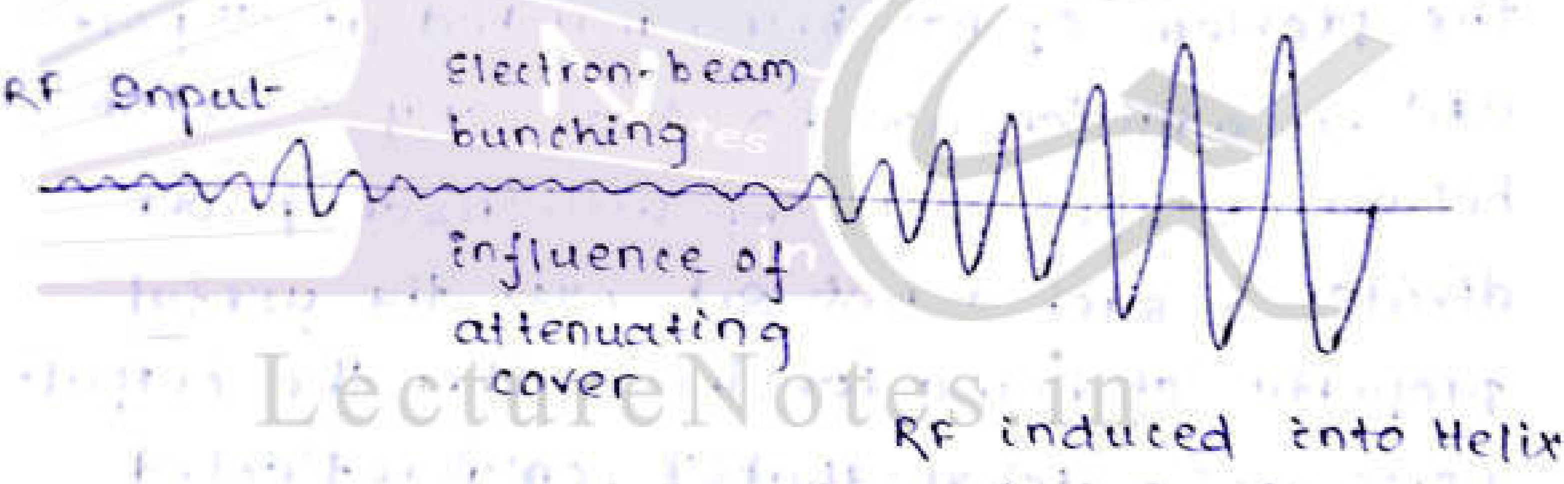


(O-Type TWT)

- The dc beam voltage is adjusted so that the beam velocity is slightly greater than axial component of field on slow wave structure. The axial RF field and beam can now interact continuously.



- As the velocity of the beam is maintained slightly greater than phase. velocity of travelling wave, more electrons face the retarding field than accelerating field and great amount of K.E. is transferred from beam to RF field.
- This field amplitude increases forming more compact bunch and larger amplification of the signal voltage appears at the output end of the helix.



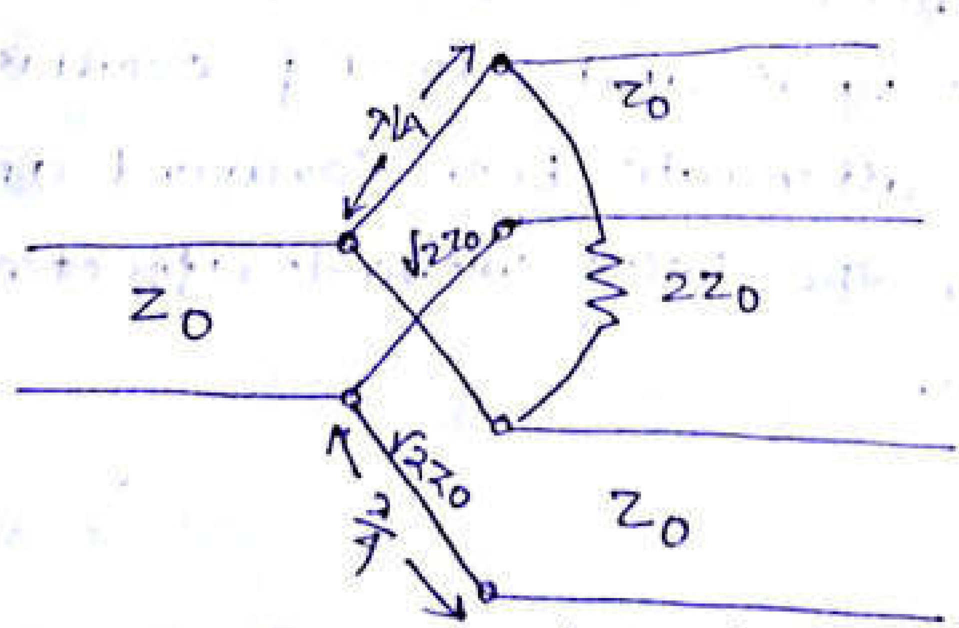
- An attenuator is placed midway along the helix, attenuates the reflected waves propagating from any mismatched load at the output end to prevent from reaching the input and causing oscillations.
- The attenuator will attenuate both forward and reflected waves on the helix without affecting the electron beam.

Q) What is Wilkinson power divider? Find the S-parameters of equal-split Wilkinson power divider using even and odd mode of analysis. (19)

A) Wilkinson Power Divider :

- It is a class of power divider circuit, that can achieve isolation between the output ports while maintaining a matched condition on all ports.
- It is used in radio frequency communication systems utilizing multiple channels since the high degree of isolation between the output ports prevents crosstalk between the individual channels.
- The lossless T-junction divider suffers from the problem of not being matched at all ports and, in addition, does not have any isolation between output ports. The Wilkinson power divider is such a network, with the useful property of being lossless when the output ports are matched; that is, only reflected power is dissipated.

Even-odd Mode Analysis :



(fig-1)

(Equal split Wilkinson Power Divider)

We have normalized all impedances to the characteristic impedance Z_0. The quarter wave line have a normalized characteristic impedance Z, and the shunt resistor has a normalized value of r; for equal split power divider, $Z = \sqrt{2}$ & $r = 2$.

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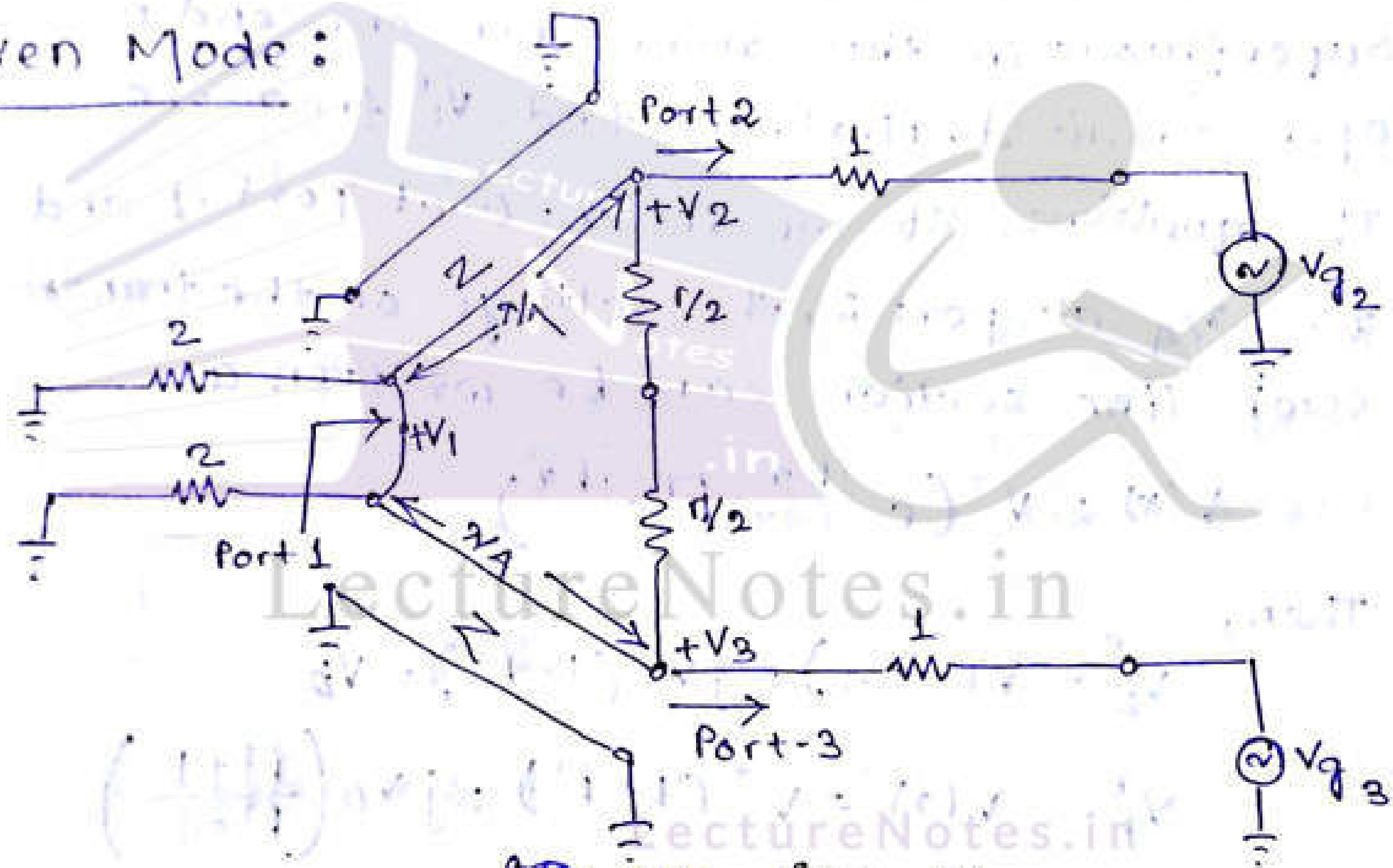
- For even mode, $V_{g_2} = V_{g_3} = 2V_0$

& odd mode, $V_{g_2} = -V_{g_3} = 2V_0$.

Then by superposition of these 2 modes, we effectively have an excitation of $V_{g_2} = 4V_0$, $V_{g_3} = 0$ from which we will find the s-parameters of the network.

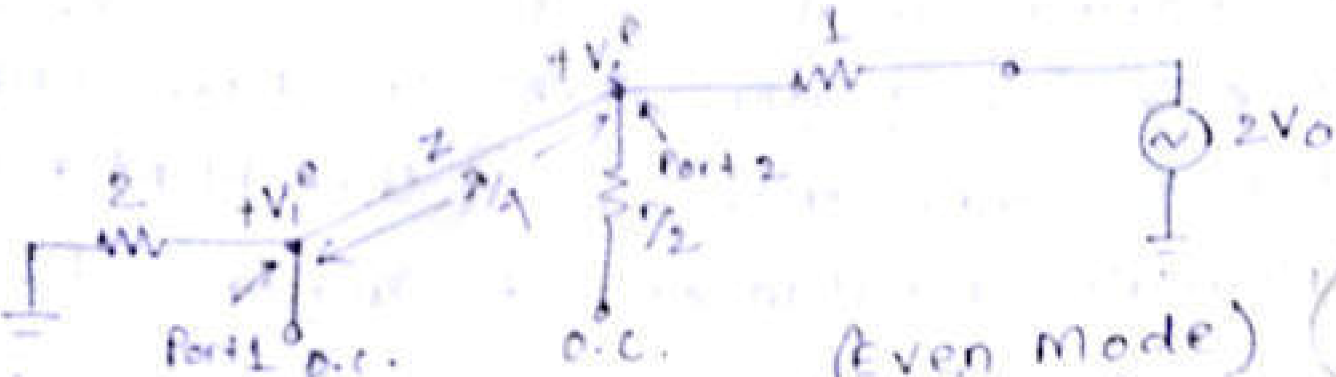
Even Mode:



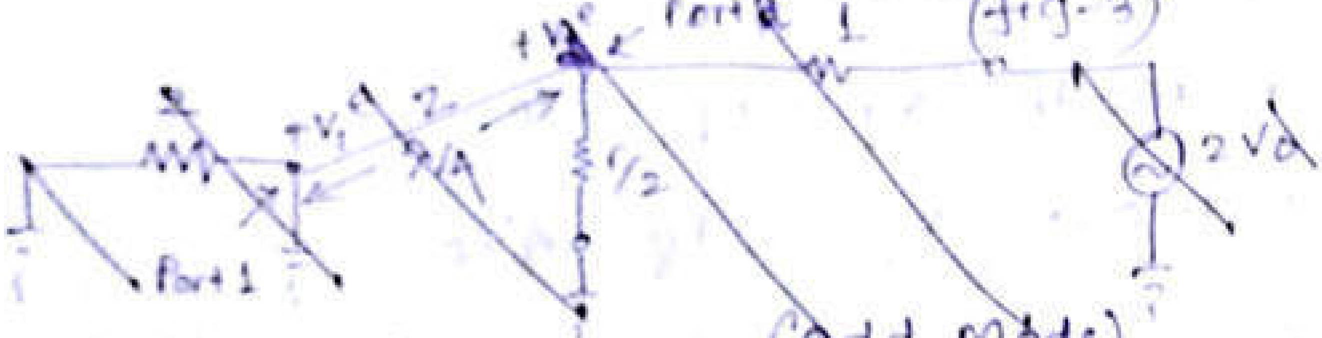
(fig-2)

For even mode excitation, $V_{g_2} = V_{g_3} = 2V_0$ & so $V_2^e = V_3^e$ and there is no current flow through the $r/2$ resistors or short ckt between the i/ps of the 2 transmission lines at port-1. Thus we can bisect the fig. with open ckts at these points to obtain the network. Then looking into port 2, we see an impedance,

$$Z_{in}^e = \frac{Z^2}{2}.$$



(Even Mode)
(fig-3)



(Odd Mode)

Since the transmission line looks like a quarter wave transformer. Thus, if $z = \sqrt{2}$, port 2 will be matched for even mode excitation; then $V_2^e = V_0$ since $Z_{in}^e = 1$. The $r/2$ resistor is superfluous in this case, since one end is open circuited. Next, we find V_1^e from the TL equations. If we let $x = 0$ at port-1 and $x = -\lambda/4$ at port 2, the voltage on the transmission line section can be written as

$$V(x) = V^+ (e^{-j\beta x} + \Gamma e^{j\beta x})$$

Then,

$$V_2^e = V(-\lambda/4) = jV^+(1 - \Gamma) = V_0$$

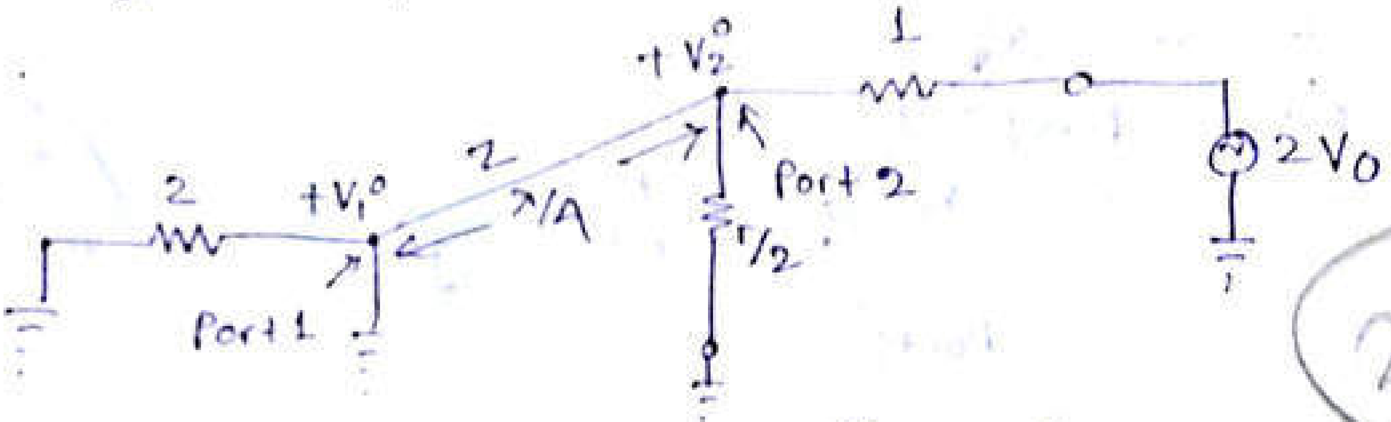
$$V_1^e = V(0) = V^+(1+\Gamma) = jV_0\left(\frac{\Gamma+1}{\Gamma-1}\right)$$

The reflection coefficient Γ is that seen at port 1, looking toward the resistor of normalized value 2, so

$$\Gamma = \frac{2-\sqrt{2}}{2+\sqrt{2}}$$

& $V_1^e = -jV_0\sqrt{2}$

Odd Mode: For odd mode excitation, $V_{g2} = -V_{g3} = 2V_0$ and so $V_2^o = -V_3^o$ and there is a voltage null along the middle of the circuit. ~~Looking into port 2, we see an impedance of r/2.~~



(Odd Mode) (fig-4)

Looking into port 2, we see an impedance of $r/2$, since the parallel connected TL is $\lambda/4$ long and shorted at port 1, and so looks like an open ckt at port 2. Thus, port 2 will be matched for odd mode excitation if we select $r=2$. Then $V_2^o = V_0$ & $V_1^o = 0$; for this mode of excitation all power is delivered to the $r/2$ resistors, with none going to port 1.

Now refering to the original fig-5, we have the $\|^r$ connection of 2 quarter-wave transformers terminated in loads of unity. The i/p impedance is

$$Z_{in} = \frac{1}{2}(\sqrt{2})^2 = 1.$$

The S-parameters for Wilkinson divider are:

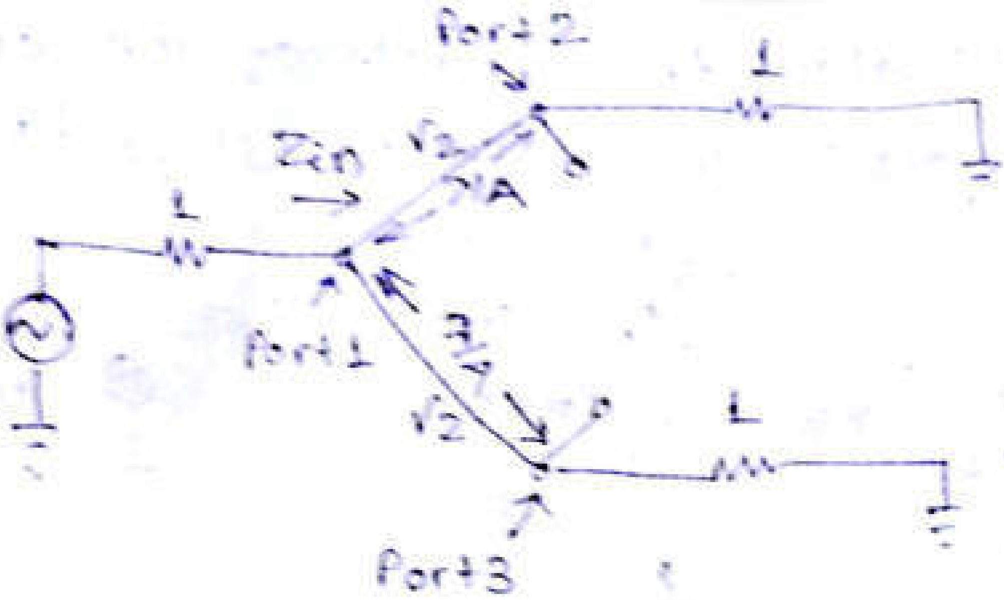
$S_{11} = 0$ ($Z_{in} = 1$ at port 1)

$S_{22} = S_{33} = 0$ (Ports 2 & 3 matched for even & odd modes)

$S_{12} = S_{21} = \dfrac{V_1^e + V_1^o}{V_2^e + V_2^o} = -j/\sqrt{2}$ (Symmetry due to reciprocity)

$S_{13} = S_{31} = -j/\sqrt{2}$ (Symmetry of ports 2 & 3)

$S_{23} = S_{32} = 0$ (due to short or open at bisection)



(fig-5)

Q) Discuss the hazards of EM radiation. Why are these labelled hazards?

- Microwaves are used in scientific and Industrial applications as well as military and civilian world. But high power microwave radiations have some adverse effect. These are called hazards.
- Different types of hazards are:
  - i) Hazards of Electromagnetic Radiation to Personnel (HERP)
  - ii) Hazards of Electromagnetic Radiation to Ordnance (HERO)
  - iii) Hazards of Electromagnetic Radiation to Fuel (HERF).

Hazards of Electromagnetic Radiation to Personnel:

- It is the potential of EM radiation to produce harmful biological effect in human.
- There is also possibility that the biological cellular process at the nucleus may affected by weak electric and magnetic fields from high power transmission lines.

- A close coupling between the body and the microwave field ~~to~~ produced the microwave frequencies for which the wavelength are the same order of magnitude as the dimensions of human body and large amount of heat can be generated to cause severe damage in the body.

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- When the body size is atleast 1/10 th of a wavelength then the significant energy is absorbed.

Hazards of Electromagnetic Radiation to Ordnance (HERO):

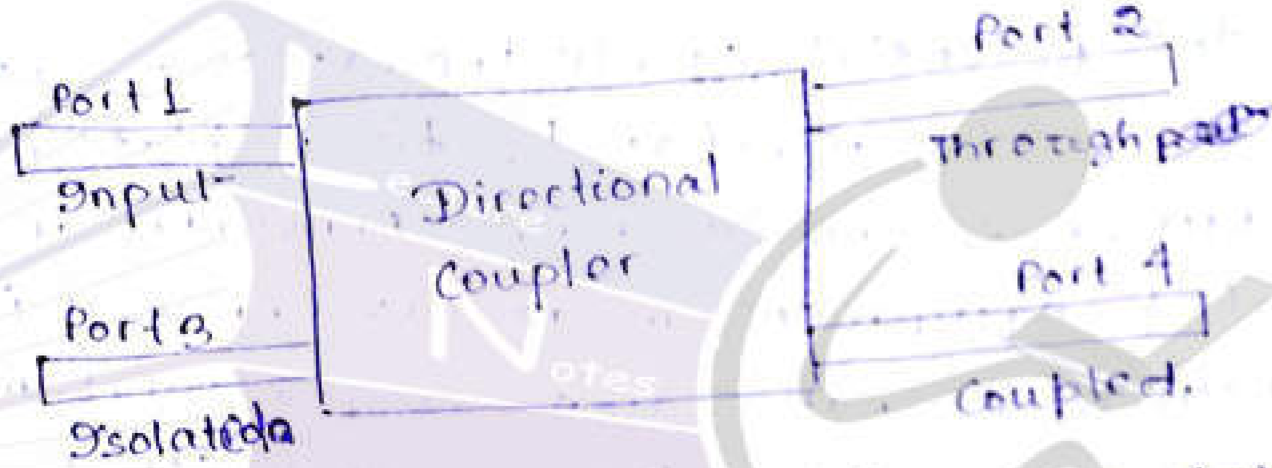
- It is the potential of electro explosive device to be adversely affected by EM radiation.
- In addition to the attending personnel and associated equipment, microwave energy is also dangerous to ordnance like weapon systems, safety and emergency devices and other equipment containing sensitive electro~~magnetic~~ explosive devices (EEDs).
- As ordnances donot have a circulatory system, they react to peak power where as human reacts to average power.
- However, EEDs can more easily be protected by enclosing them with metallic enclosures which reflect back the incident microwave energy from effects of RF energy than human.

Hazards of Electromagnetic Radiation to Fuel (HERF):

- It is the potential of EM radiation to cause spark ignition of ~~the~~ volatile combustibles such as vehicle fuels.

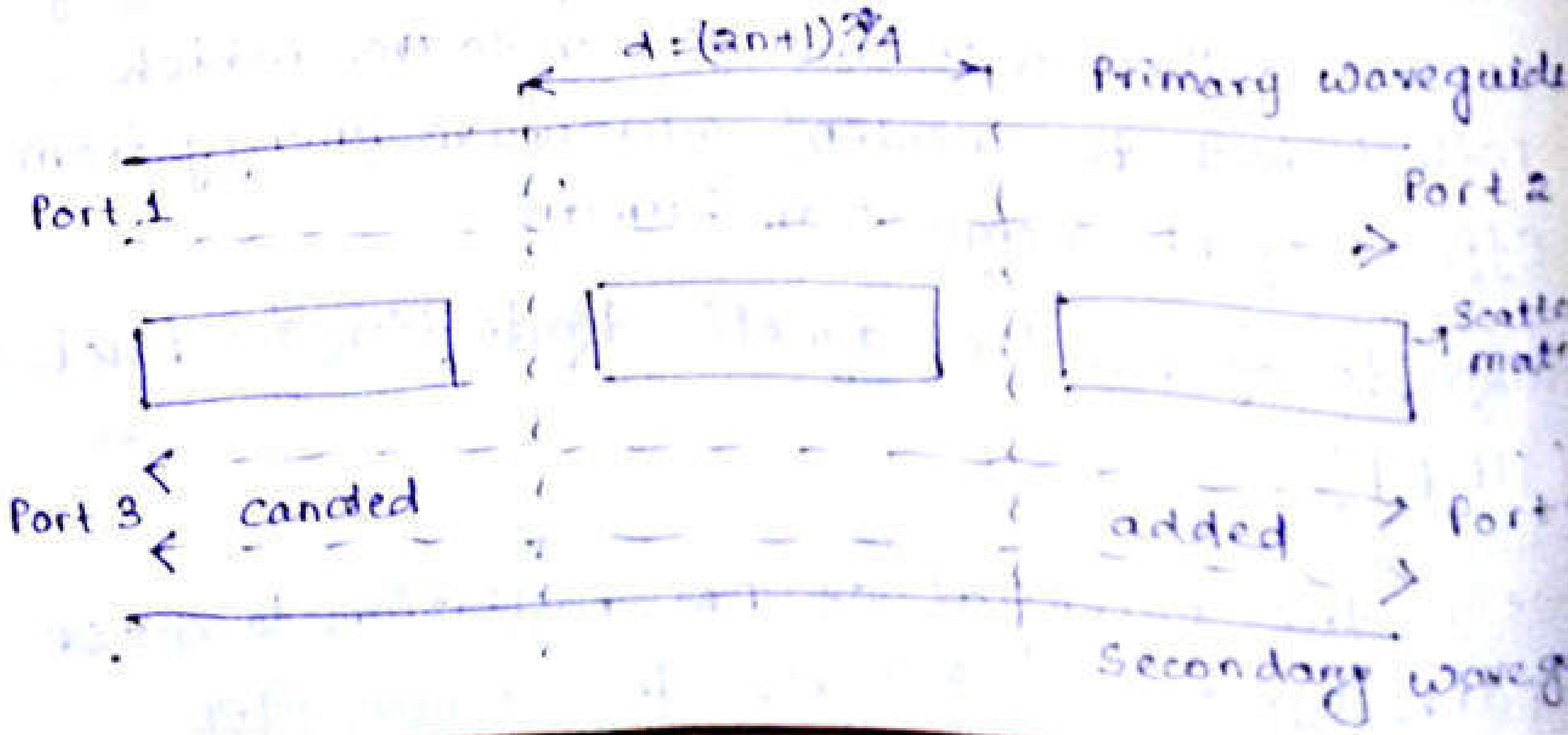
- The possibility of accidentally igniting fuel vapours by RF induced arcs ~~using~~ during fuel handling operations in proximity to high level RF fields causes HERF.
- The probability of ignition may be significant for more than 50 V-A arcs.

Q) Derive S-Parameter of a directional coupler. Write the expression for coupling factor, directivity and isolation.



The [S] matrix of a reciprocal 4-port network matched at all ports has the following form:

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{21} & 0 & S_{23} & S_{24} \\ S_{31} & S_{32} & 0 & S_{34} \\ S_{41} & S_{42} & S_{43} & 0 \end{bmatrix}$$



The spacing ~~betweeen~~ between 2 holes must be an odd multiple of quarter wavelength and can be written as

$d = (2n+1)\lambda g/4$ where n = positive integer
$\lambda g \rightarrow$ guided wavelength

~~Wave is fed from port 1 to port 2.~~

- The signals are added at port-4 due to same phase or forward direction and at port-3 are cancelled due to reverse direction or $180°$ phase change.
- Due to no coupling between port 1 & port 3, $S_{13} = S_{31} = 0$, also between port 2 & port 4, $S_{24} = S_{42} = 0$.

But here we will consider only S_{13} & S_{31} as port 1 is from input port so not for S_{24} & S_{42}.

Due to $S_{13} = S_{31} = 0$, cancellation occurs.

Due to less signal output at port 3, better directivity means maximum output will be avialable at port-4.

By symmetry, $S_{12} = S_{21}$
$S_{14} = S_{41}$
$S_{23} = S_{32}$
$S_{34} = S_{43}$.

Transmission Loss for DC = L
$= 10\log_{10}\frac{P_1}{P_2}$

$$S = \begin{bmatrix} 0 & S_{12} & 0 & S_{14} \\ S_{21} & 0 & S_{23} & 0 \\ 0 & S_{32} & 0 & S_{34} \\ S_{41} & 0 & S_{43} & 0 \end{bmatrix}$$

The characteristic eq. of directional coupler are:

Coupling = $C = 10\log\frac{P_1}{P_3} = -20\log\beta$ dB.

Directivity = $D = 10\log\frac{P_3}{P_4} = 20\log\frac{\beta}{|S_{14}|}$ dB

Isolation = $I = 10\log\frac{P_1}{P_4} = -20\log|S_{41}|$ dB

Q) Write Short note on Bethe hole directional coupler. (27)

- The directional property of all directional coupler is produced through the use of 2 separate wave components, which add in phase at the coupled port and cancelled at the isolated port.
- One of the simplest way to do this is to couple one waveguide to another through a single small hole in the common board wall between the two guides. Such as coupler is known as Bethe-hole coupler.
- By adjusting the relative amplitude of the equivalent sources, we can cancel the radiation in the direction of the isolated port, while enhancing the radiation in the direction of the coupler.

Q) Prove that a 3 port microwave device can't be lossless, reciprocal and matched at all points.

A) Let us consider a scattering matrix of 3port network:

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix}$$

Case-1: If the network is matched at every port then.

$S_{11} = S_{22} = S_{33} = 0$ [Reflection coefficient = 0]

Case-2: If the network is raciprocal i.e. due to symmetry, then

$$S_{ij} = S_{ji} \Rightarrow S_{12} = S_{21},\ S_{31} = S_{13},\ S_{32} = S_{23}.$$

For a matched reciprocal 3 port, the scattering matrix can be written as

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix}$$

Case-3: Lastly, if the network is lossless, the product of scattering matrix with its conjugate is unitary matrix. i.e.

$$S = \begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{12} & 0 & S_{23} \\ S_{13} & S_{23} & 0 \end{bmatrix} \begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{12}^* & 0 & S_{23}^* \\ S_{13}^* & S_{23}^* & 0 \end{bmatrix}$$

$$\left.\begin{aligned} R_1C_1 &: |S_{12}|^2 + |S_{13}|^2 = 1 \text{ —— } (1) \\ R_2C_2 &: |S_{12}|^2 + |S_{23}|^2 = 1 \text{ —— } (2) \\ R_2C_3 &: |S_{13}|^2 + |S_{23}|^2 = 1 \text{ —— } (3) \end{aligned}\right\} (*)$$

$$\left.\begin{aligned} S_{13}^* S_{23} &= 0 \\ S_{23}^* S_{12} &= 0 \\ S_{12}^* S_{13} &= 0 \end{aligned}\right\} (**)$$

Eq. (**) indicate that atleast two of the 3 S-parameters must be equal to zero. But if this is true, then all eq.s in (*) can't be satisfied.

e.g. $-\ S_{13} = 0$

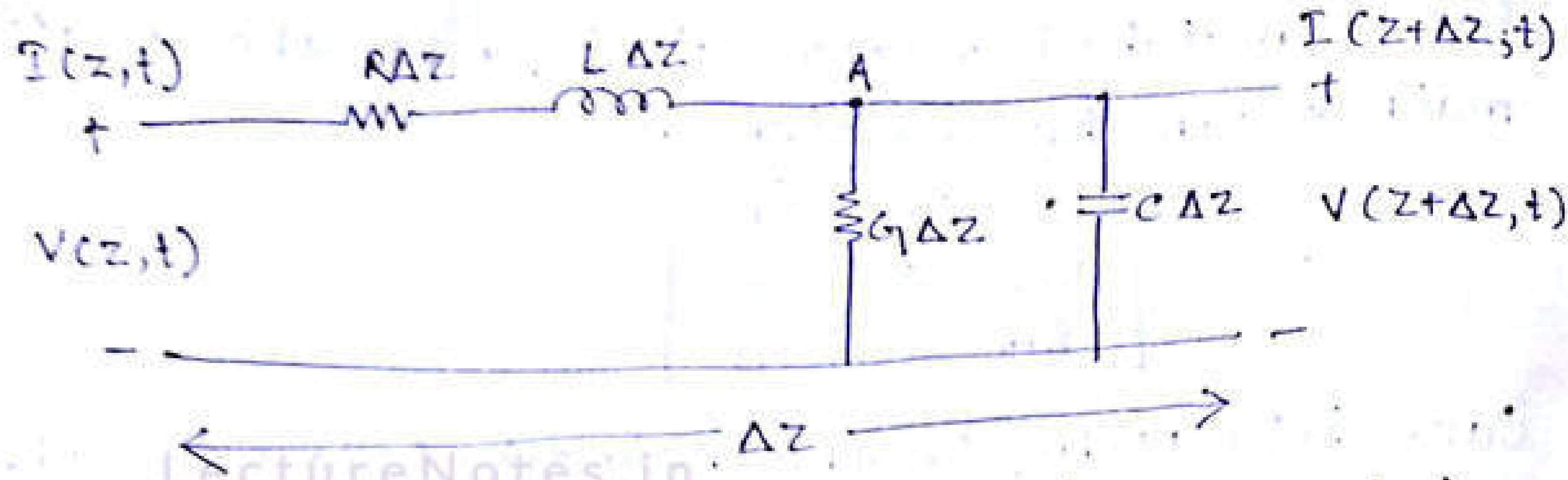
(1) & (2) eq. (*), $S_{12} = S_{23} = 1$

Now in eq. (**) 2nd eq. is not possible,

as $S_{23}^* S_{12} \neq 0$ (contradiction)

Therefore, we conclude that a 3-port network can't be lossless, raciprocal and matched at all ports. ~~However, one can realise such~~ a

Q) Derive Telegrapher equation from the lumped element equivalent circuit diagram of a transmission line.



The series inductance L represents the total self inductance of the two conductors and shunt capacitance C is due to the close proximity of the two conductors.

The series resistance R represents the resistance due to finite conductivity of the conductors and shunt conductance G is due to dielectric loss in the material between the conductors.

R & G are loss elements of Transmission line.

Applying KVL to the small circuit, we have

$$V(z,t) - R\Delta z I(z,t) - L\Delta z \cdot \frac{dI(z,t)}{dt} - V(z+\Delta z,t) = 0 \quad \text{— ①}$$

Applying KCL to node A,

$$I(z,t) - G\Delta z V(z+\Delta z,t) - C\frac{dV(z+\Delta z,t)}{dt} - I(z+\Delta z,t) = 0 \quad \text{— ②}$$

Dividing eqⁿ ① & ② by ΔZ individually and taking limit $\Delta z \to 0$, we have

$$\lim_{\Delta z\to 0} \frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = \lim_{\Delta z \to 0} \frac{-R\Delta z I(z,t)}{\Delta z}$$

$$- \lim_{\Delta z\to 0} \frac{L\Delta z \partial I(z,t)}{\Delta z}$$

$$\Rightarrow \frac{\partial v(z,t)}{\partial z} = -RI(z,t) - L\frac{\partial I(z,t)}{\partial t} \quad \text{——} (3)$$

$$\lim_{\Delta z \to 0} \frac{I(z+\Delta z, t) - I(z,t)}{\Delta z} = \lim_{\Delta z \to 0} -G\Delta z V(z+\Delta z, t) - \lim_{\Delta z \to 0} C\Delta z \frac{\partial V(z+\Delta z, t)}{\partial t}$$

$$\Rightarrow \frac{\partial I(z,t)}{\partial t} = -GV(z,t) - C\frac{\partial V(z,t)}{\partial t} \quad \text{——} (4)$$

Omitting the argument (z, t) from eq. (3) & (4) which is understood,

$$\frac{\partial V}{\partial z} = -RI - L\frac{\partial I}{\partial t} \quad \text{——} (5)$$

$$\frac{\partial I}{\partial z} = -GV - C\frac{\partial V}{\partial t} \quad \text{——} (6)$$

eq. (5) & (6) are time domain form of TL known as Telegrapher eq..

For sinusoidal steady state condition with cosine based phasors, we can derive eq. (5) & (6) as

$$\frac{dV}{dz} = -RI - j\omega L I = -(R + j\omega L)I = ZI \quad \text{——} (7)$$

$$\frac{dI}{dz} = -(G + j\omega C)V = YV \quad \text{——} (8)$$