CSCI-505-WRITTEN ASSINGMENT-II
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Q:1: The given code is iterating through all n values of the array. (For Loop). Inside, the while loop the code is iterating across indices which the value of the current index points to until, one reaches the value (element of array) ~~@~~ is same as the starting index. Since, every element points to a different index value, and all the elements which have value same as their index will never occur anywhere else, we can say that number of iterations inside while loop will be atmost n without getting stuck in any infinite loop.
So, the average time complexity will be $O(n^2)$. $\underline{n*n}$
For, the array to be completely sorted, every element pointing to its index, it will be order $O(n)$, since it will never go inside for loop.

For, the array (Sorted) shifted by 1 cyclically array being [2 3 4 ... n-1 n 1]. It will take n operations inside while loop each time and thus $O(n^2)$, which turns out to be worst case for the code.

Q:2: RECURSION TREE METHOD $\lceil x \rceil$: Round off to nearest integer.

a.) $T(n) = 2^n + T(\lceil n/2 \rceil)$, $T(1) = 1$

b.) $T(n) = n + T(\lceil n/2 \rceil)$, $T(1) = 1$

c.) $T(n) = n + 3T(\lceil n/2 \rceil)$, $T(1) = 0$

d.) $T(n) = n^2 + 2T(\lceil n/3 \rceil)$, $T(1) = 0$

e.) $T(n) = n + 2T(\lceil n/4 \rceil)$, $T(1) = 0$

5. a) $T(n) = 2^n + T(\lceil n/2 \rceil)$, $T(1) = 1$.

IS

n

$\lceil \frac{n}{2} \rceil$

$\left\lceil \frac{\lceil \frac{n}{2} \rceil}{2} \right\rceil$

$\vdots$

1

$\lceil \log_2 n \rceil$ operations

Because $\frac{n}{2^{\log_2 n}} = 1$

TREE

$$\begin{tikzcd} n \arrow[d] \\ \lceil \frac{n}{2} \rceil \arrow[d] \\ \left\lceil \frac{\lceil \frac{n}{2} \rceil}{2} \right\rceil \arrow[d, dashed] \\ 1 \end{tikzcd}$$

WORK (Total)

2^n

$2^{\lceil n/2 \rceil}$

$2^{\lceil \lceil n/2 \rceil /2 \rceil}$

$\vdots$

1

So, Total Work Done $= \underbrace{2^n + 2^{\lceil n/2 \rceil} + 2^{\lceil \lceil n/2 \rceil /2 \rceil} \cdots 1}_{\lceil \log_2 n \rceil \text{ : no. of terms}}$

$\approx 2^n + 2^{n/2} + 2^{n/4} \cdots 1 \leq 2^n + 2^{n-1} \cdots 1$

Total work $\leq 2^n \frac{(1 - 2^{-(n+1)})}{1 - 2^{-1}} \simeq \underline{2^{n+1}}$

Total work $\leq 2^{n+1}$ Total Work $\geq 2^n$

$\therefore$ Total work $= O(2^n)$

b) $T(n) = n + T(\lceil n/2 \rceil)$, $T(1) = 1$

IS		TREE	WORK (total)
n	$\lceil \log_2 n \rceil$ operations	n	n
$\lceil \frac{n}{2} \rceil$	Because	$\lceil \frac{n}{2} \rceil$	$\lceil \frac{n}{2} \rceil$
$\left\lceil \frac{\lceil \frac{n}{2} \rceil}{2} \right\rceil$	$\frac{n}{2^{\lceil \log_2 n \rceil}} = 1$	$\left\lceil \frac{\lceil \frac{n}{2} \rceil}{2} \right\rceil$	$\left\lceil \frac{\lceil \frac{n}{2} \rceil}{2} \right\rceil$
1			

So, total work Done $= \underbrace{n + \lceil \frac{n}{2} \rceil + \left\lceil \frac{\lceil \frac{n}{2} \rceil}{2} \right\rceil + \dots 1}_{\lceil \log_2 n \rceil \text{ number of terms}}$

$= \cancel{2^n}\ \cancel{1/2^{\lceil n/2 \rceil}}\ n + \frac{n}{2} + \frac{n}{4} \dots 1 = n(2^0 + 2^{-1} \dots 2^{-\log_2 n})$

$= n \frac{\left(1 - 2^{-(\log_2 n + 1)}\right)}{(1 - 2^{-1})}$ $\quad = \frac{n}{(1 - 2^{-1})} = 2n \quad 2n(1 - 2 \times 2^{-\log_2 n})$

~~Total Work Done = 2n ∴ Total Work Done = O(n)~~

$= 2n(1 - 2 \times n^{-1}) = \underline{2n - 4}$ $\cancel{\approx 1}$ = Total Work Done

∴ ~~O(n)~~ $\underline{\text{Work Done} = O(n)}$

c) $T(n) = n + 3T(\lceil n/2 \rceil)$, $T(1) = 0$.

IS | TREE | WORK (total)

n | | n

$\lceil \frac{n}{2} \rceil$ | | $3\lceil \frac{n}{2} \rceil$

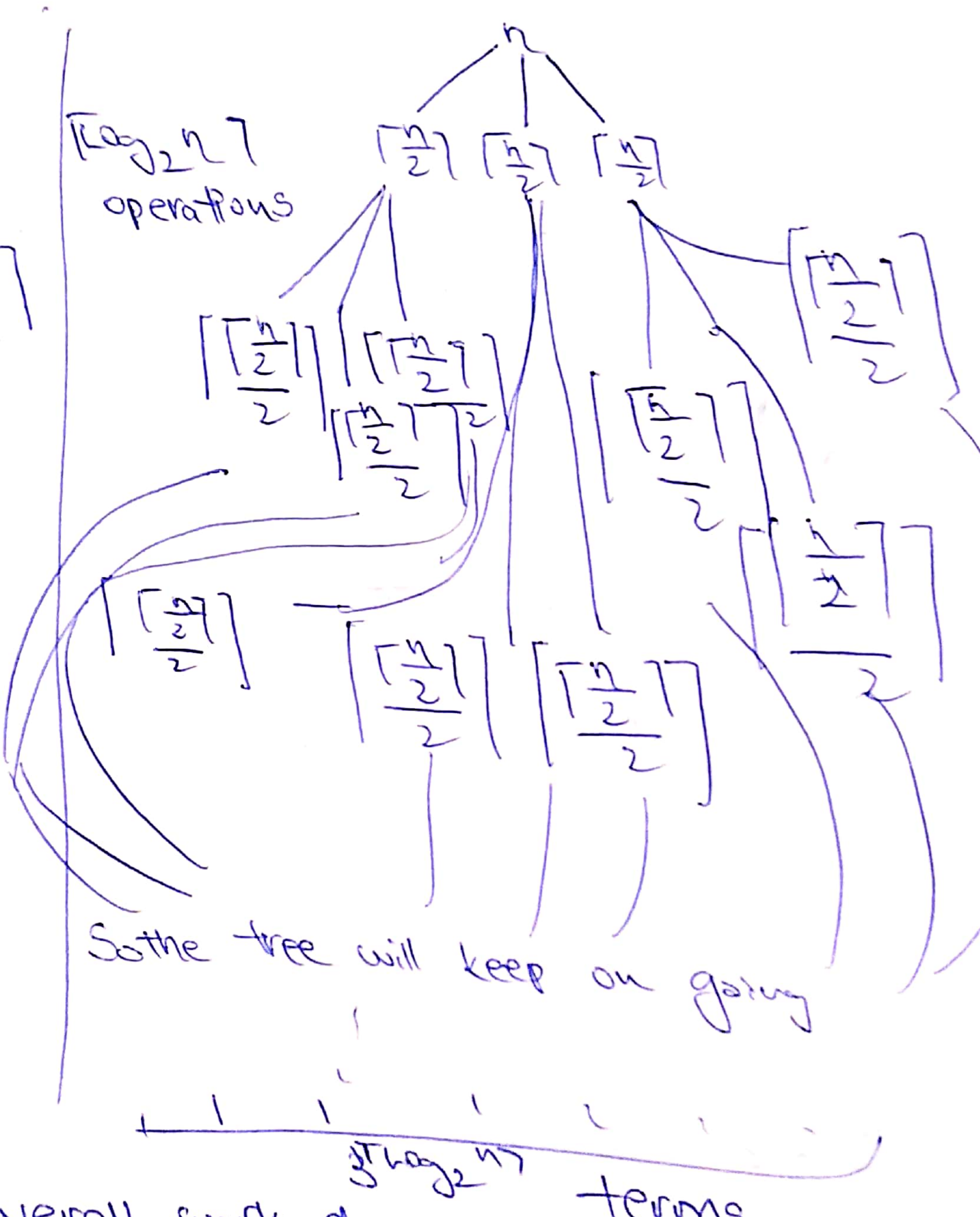
$\left\lceil \frac{\lceil \frac{n}{2} \rceil}{2} \right\rceil$ | | $3^2 \left\lceil \frac{\lceil \frac{n}{2} \rceil}{2} \right\rceil$

⋮

1 | | $\frac{3^{\lceil \log_2 n \rceil} n}{2^{\lceil \log_2 n \rceil}}$

$\downarrow$

$= 3^{\lceil \log_2 n \rceil} \times 0$


$\lceil \log_2 n \rceil$ operations
n
$\lceil \frac{n}{2} \rceil$ $\lceil \frac{n}{2} \rceil$ $\lceil \frac{n}{2} \rceil$
$\left\lceil \frac{\lceil \frac{n}{2} \rceil}{2} \right\rceil$
So the tree will keep on going
$3^{\lceil \log_2 n \rceil}$ terms

So, overall work done:

$= n + 3\lceil \frac{n}{2} \rceil + 3^2 \left\lceil \frac{\lceil \frac{n}{2} \rceil}{2} \right\rceil \ldots 3^{\lceil \log_2 n \rceil} \left(\frac{n}{2^{\lceil \log_2 n \rceil}} \right) + 0$ → roughly

~~$= n + 3\lceil \frac{n}{2} \rceil$~~

$\simeq \underbrace{n + \frac{3}{2}n + \left(\frac{3}{2}\right)^2 n \ldots \left(\frac{3}{2}\right)^{\log_2 n - 1} n}_{}$

$\lceil \log_2 n$ terms, since, $T(1) = 0$, so last term not included

Total Work $= n\left[1 + \left(\frac{3}{2}\right) + \left(\frac{3}{2}\right)^2 \cdots \left(\frac{3}{2}\right)^{\log_2 n - 1}\right]$

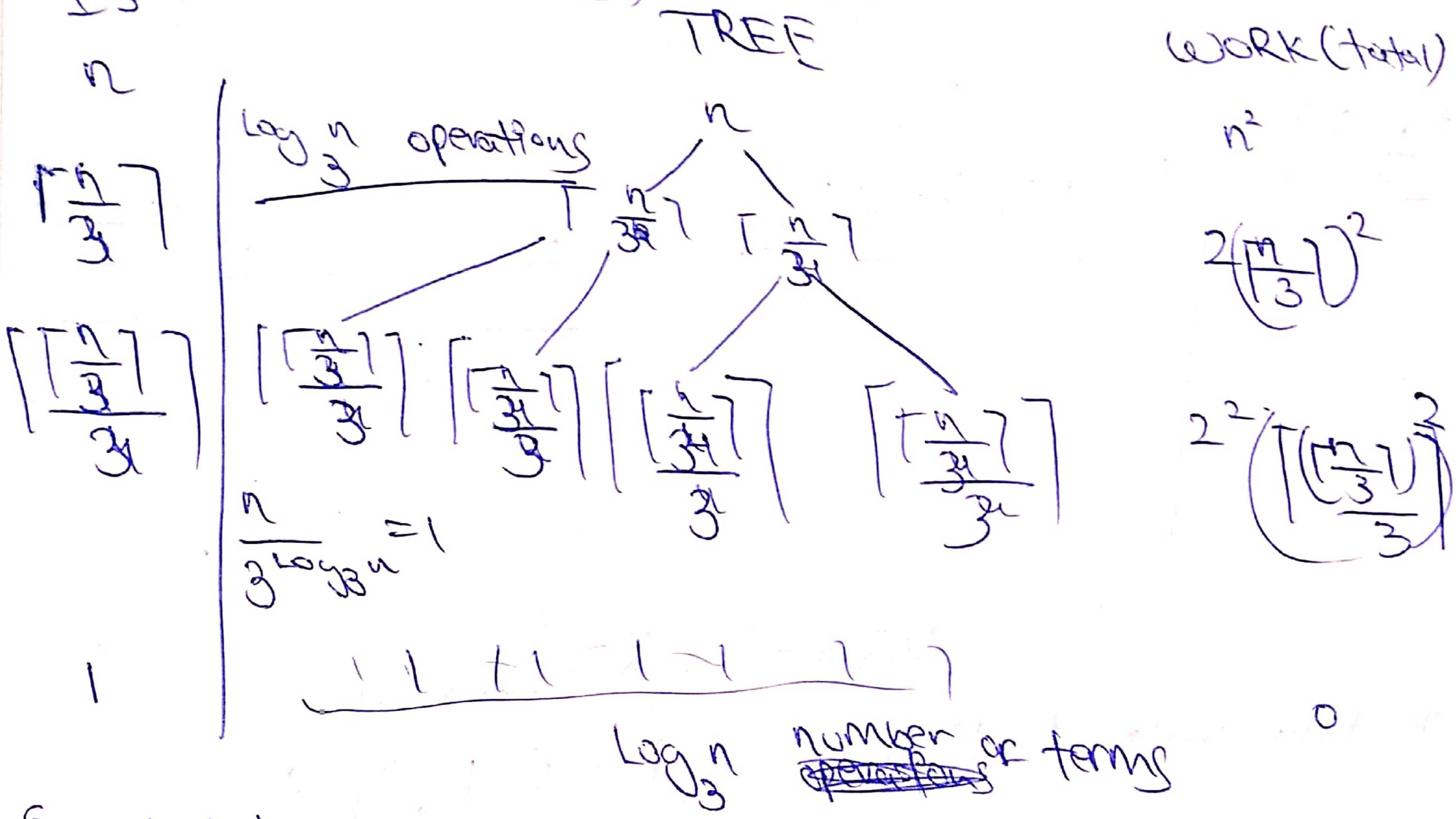
$= n\left[\frac{-1 + \left(3/2\right)^{\log_2 n}}{-1 + \left(3/2\right)}\right]$, $\left(\frac{3}{2}\right)^{\log_2 n} = \cancel{n \log}\ n^{\log_2 (3/2)}$

Ignoring den~~ominator constant~~ $= \frac{2}{\cancel{3}} n\left(-1 + n^{0.585}\right)$

$= \cancel{3} = \left(\frac{-2}{\cancel{3}} n + \frac{2}{\cancel{3}} n^{1.585}\right)$ $\therefore$ Total work $= O\left(n^{1.585}\right)$

$= O\left(n^{1 + \log_2 (3/2)}\right) = O\left(n^{\log_2 3}\right)$

d.) $T(n) = n^2 + 2T(\lceil n/3 \rceil)$, $T(1) = 0$


IS
n
$\lceil \frac{n}{3} \rceil$
$\left\lceil \frac{\lceil \frac{n}{3} \rceil}{3} \right\rceil$
1
TREE
$\log_3 n$ operations
n
$\lceil \frac{n}{3} \rceil$
$\frac{n}{3^{\log_3 n}} = 1$
$\log_3 n$ number ~~operations~~ of terms
WORK (total)
n^2
$2\left(\lceil \frac{n}{3} \rceil\right)^2$
$2^2\left(\left\lceil \frac{\lceil \frac{n}{3} \rceil}{3} \right\rceil\right)^2$
0

So, total Work Done $= n^2\left(1 + \left[\frac{2}{3^2}\right] - - \left(\frac{2}{3^2}\right)^{\log_3 n - 1}\right)$

$\lfloor \log_3 n \rfloor$ operations , since $T(1) = 0$

$= n^2 \cancel{[\ ]}$ using $\left(n^2 + 2\left\lceil \frac{n}{3} \right\rceil^2 + 2^2\left\lceil \frac{\lceil \frac{n}{3} \rceil}{3} \right\rceil^2 \cdots \frac{2}{3^{\log_3 n}}\right)$

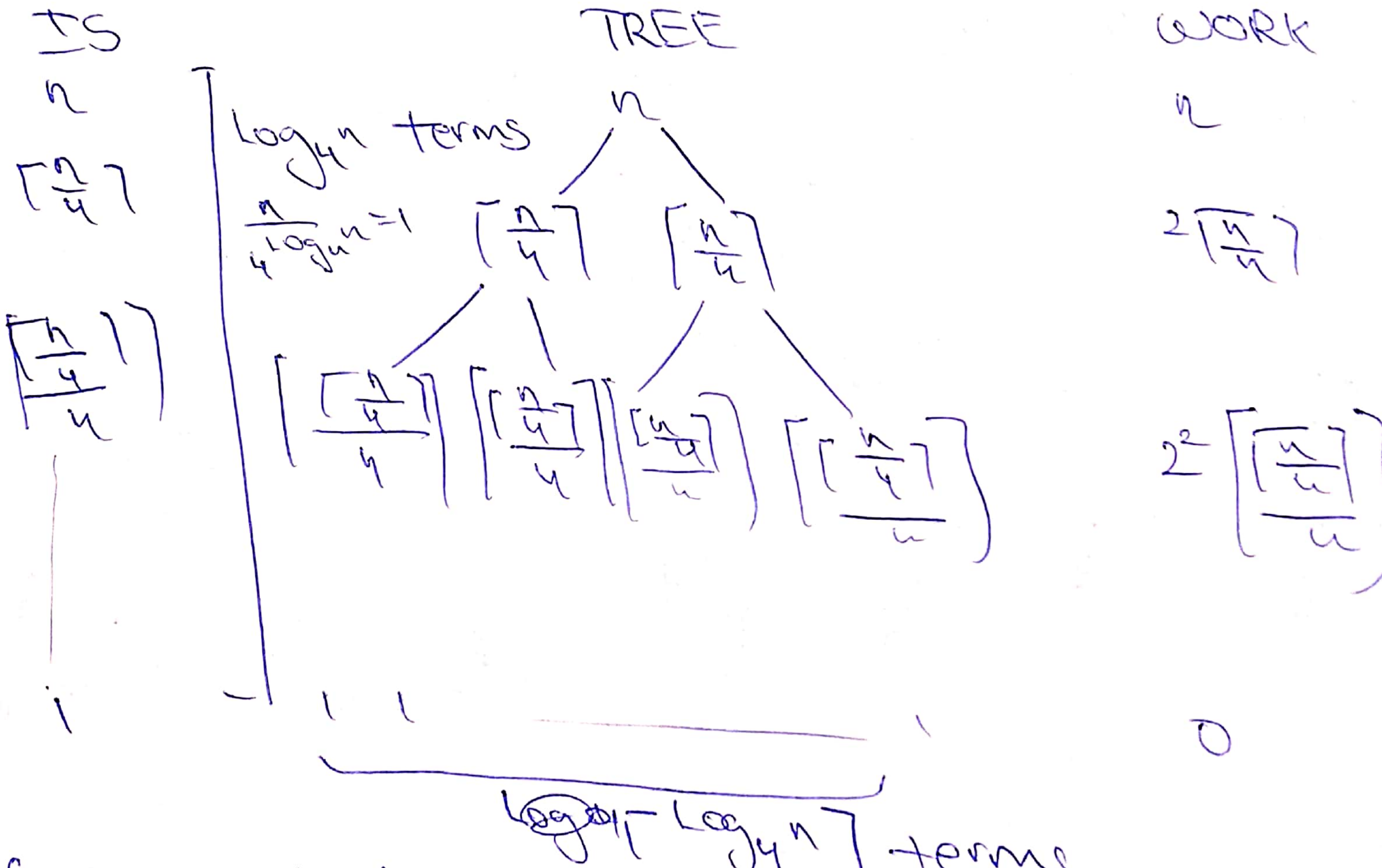
$= n^2 \frac{\left(1 - \left(\frac{2}{3^2}\right)^{\log_3 n}\right)}{1 - (2/3)}$ $\approx \cancel{9} = \cancel{3}\ 3n^2\left(1 - \left(\frac{2}{9}\right)^{\log_3 n}\right)$

$= 3n^2\left(1 - n^{\log_3(2/9)}\right) = 3n^2\left(1 - n^{-1.37}\right)$

$= 3n^2 - 3n^{0.63}$ = Total Work Done

$= \underline{O(n^2)}$

Qe.) $T(n) = n + 2T(\lceil n/4 \rceil)$, $T(1)=0$


IS
TREE
WORK
n
$\lceil \frac{n}{4} \rceil$
$\left\lceil \frac{\lceil \frac{n}{4} \rceil}{4} \right\rceil$
1
$\log_4 n$ terms
$\frac{n}{4^{\log_4 n}} = 1$
n
$2\lceil \frac{n}{4} \rceil$
$2^2 \left\lceil \frac{\lceil \frac{n}{4} \rceil}{4} \right\rceil$
0
$\lceil \log_4 n \rceil$ terms

So, total work done $= \underbrace{n + 2\lceil \frac{n}{4} \rceil + 2^2 + \left\lceil \frac{\lceil \frac{n}{4} \rceil}{4} \right\rceil + \ldots}_{\lfloor \log_4 n \rfloor \text{ terms}}$, since $T(1)=0$

$\cong n\left(1 + \frac{2}{4} + \frac{2^2}{4^2} \cdots \left(\frac{2}{4}\right)^{\log_4 n - 1}\right) = n\frac{\left(1 - 2^{-\log_4 n}\right)}{\left(1 - 2^{-1}\right)}$

$= 2n(1 - n^{-1/2}) = 2n - 2n^{-0.5}$ = Total Work Done

Total Work Done $= O(n)$